1/ We have

$$Y = f(X) = g(T) = T = \beta_0 + \sum_{m=1}^{M} \beta_m \sigma(\alpha_{0m} + \alpha_m X)$$

$$= \beta_0 + \sum_{m=1}^{M} \beta_m \sigma(\alpha_{0m} + \|\alpha_m\| (\frac{\alpha_m}{\|\alpha_m\|} X)$$

$$= \beta_0 + \sum_{m=1}^{M} \beta_m \sigma(\alpha_{0m} + \|\alpha_m\| (w_m X))$$

$$= \beta_0 + \sum_{m=1}^{M} \beta_m \left(\frac{1}{1 + e^{-(\alpha_{0m} + \|\alpha_m\|(w_m X))}} \right)$$

For each m from 1 to M, we have:

$$\lim_{(\alpha_{0m}, \|\alpha_m\|) \to (0,0)} \frac{1}{1 + e^{-(\alpha_{0m} + \|\alpha_m\|(w_m X))}}$$

$$= \frac{1}{1 + e^{-(0+0)}} = \frac{1}{1 + e^0} = \frac{1}{2}$$

So, we have:

$$\lim_{(\alpha_{0m}; \|\alpha_m\|) \to (0,0)} Y = \beta_0 + \frac{1}{2} \sum_{m=1}^{M} \beta_m$$

So input would approach linearity as α_m approach 0.