Zad. 5. 316055

i	x_i	y_i	$f[x_{i-1},x_i]$	$f[x_{i-2},x_{i-1},x_i]$	$f[x_{i-3},\dots,x_i]$
0	-1	4	$\leftarrow b_0$		
1	0	4	0 $\leftarrow b_1$		
2	1	0	-4	2 $\leftarrow b_2$	
3	2	16	16	10	4 $\leftarrow b_3$

Wzór

$$\begin{cases} f[x_i] = f(x_i) = y_i \\ f[x_i, x_{i+1}, \dots, x_{j-1}, x_j] = \dfrac{f[x_{i+1}, \dots, x_j] - f[x_i, \dots x_{j-1}]}{x_j - x_i} \end{cases}$$

$$b_k = f[x_0, x_1, \dots, x_k]$$

Postać Newtona

$$L_n(x) = \sum_{k=0}^{n} f[x_0, \dots, x_k]\, p_k(x)$$

$$w(x) = 4 + 0\cdot(x-(-1)) - 2x(x-(-1)) + 4x(x-(-1))(x-1) =$$
$$= 4 + 2x(x+1) + 4x(x-1)(x+1)$$

Postać Lagrange'a

i	0	1	2	3
x_i	-1	0	1	2
y_i	4	4	0	16

$x - x_0 = (x+1)$

$x - x_1 = x$

$x - x_2 = (x-1)$

$x - x_3 = (x-2)$

$$L_3(x) = \sum_{k=0}^{3} y_k \lambda_k(x) \quad ; \quad \lambda_k = \prod_{\substack{j=0 \\ j\neq k}}^{3} \frac{(x-x_j)}{(x_k-x_j)}$$

$\lambda_0 = -\frac{1}{6}x(x-1)(x-2)$; $\lambda_1 = \frac{1}{2}(x+1)(x-1)(x-2)$; $\lambda_2 = -\frac{1}{2}x(x+1)(x-2)$; $\lambda_3 = \frac{1}{6}x(x+1)(x-1)$

$$L_3(x) = 4x^3 - 2x^2 + \tfrac{1}{3}x + 4 = -\tfrac{4}{6}x(x-1)(x-2) + \tfrac{4}{2}(x+1)(x-1)(x-2)$$
$$+ 0\cdot\lambda_2 + 16\cdot\tfrac{1}{6}\cdot x(x+1)(x-1)$$