Execise 5

Theorem: For any integer n, at least one of n, $n+2$ or $n+4$ is divisble by 3.

Proof: We proof this with the division theorem. The division theorem states, that every natural number can be expressed as $n = ab + r, with\, a, r \in N and\, b \in Z$ and $0 <= r < a$. Since $a = 3$ there are three possible cases, that describes any natural number.

$$3b,\ 3b+1,\ 3b+2$$

First case ($n = 3b$):

$$3/3b\ \vee\ 3/3b+2\ \vee\ 3/3b+4$$

$$True\ \vee\ False\ \vee\ False = True$$

Second case ($n = 3b+1$):

$$3/3b+1\ \vee\ 3/3b+1+2\ \vee\ 3/3b+1+4$$

$$False\ \vee\ True\ \vee\ False = True$$

Third case ($n = 3b+2$):

$$3/3b+1\ \vee\ 3/3b+2+2\ \vee\ 3/3b+2+4$$

$$False\ \vee\ False\ \vee\ True = True$$

Since for every of the third cases one of n, $n+2$, $n+4$ is true. Hence the theorem is true.