Execise 9

Theorem: Given an infinite collection $A_n, n = 1, 2, ...$ of intervals of the real line, their intersection is defined to be $\cap_{n=1}^{\infty} A_n = \{x|(\forall n)(x \in A_n)\}$. Give an example of a family of intervals $A_n, n = 1, 2, ...$ such that $A_{n+1} \subset A_n$ for all n and $\cap_{n=1}^{\infty} A_n = \emptyset$. Prove that your example has the stated property.

Proof: A family of intervalls with the mentioned properties are the following:

$$A_n = (0, \frac{1}{n}), n = 1, 2, ...$$

We proof the the first property by taking an $n \in N$.Then every element of those two intervall $(0, \frac{1}{n})$ and $(0, \frac{1}{n+1})$ are between 0 and $\frac{1}{n}$, respectively $\frac{1}{n+1}$. Following that $\frac{1}{n+1} < \frac{1}{n}$, A_{n+1} must be a subset of A_n. Now we have to proof the second property. As we learned from the first proof that the collection A_{n+1} is always a subset of A_n, we can conclude that all A_n are an intersection of A_∞ when $n \to \infty$. Since $\lim_{n\to\infty} \frac{1}{n} = 0$, the intervall $(0, 0)$ is an empty set, which proofs the second property.