Execise 7

Theorem: Prove that for any natural number $2+2^2+2^3+...2^n = 2^{n+1}-2$

Proof: We proof this theorem by induction.

$$A(n) = \sum_{i=1}^{n}(2^i), B(n) = 2^{n+1} - 2$$

$$A(1) = B(1)$$

$$2^1 = 2^{1+1} - 2$$

$$2 = 2$$

The theorem is true for 1. Furthermore it must be true for $n+1$ to be true.

$$A(n+1) = B(n+1)$$

$$\sum_{i=1}^{n+1}(2^i) = 2^{n+2} - 2$$

Since we have a geometric series at the left hand side, we bring the sum into closed form first.

$$\frac{2(1-2^{n+2})}{1-2} = 2^{n+2} - 2$$

$$2^{n+2} - 2 = 2^{n+2} - 2, by\ Algebra$$

Hence the theorem is true.