Execise 8

Theorem: Prove (from the definition of a limit of a sequence) that if the sequence $\{a_n\}_{n=1}^{\infty}$ tends to limit L as $n \to \infty$, then for any fixed number $M > 0$ the sequence $\{Ma_n\}_{n=1}^{\infty}$ tends to the limit ML.

Proof: We proof this theorem with the definition of a limit of a sequence. A sequence converges to a limit, if the following is true:

$$\forall n(|x_n - L| < \frac{\epsilon}{|M|}), where$$

$$\epsilon \in R, \epsilon > 0, n \in N, N \in N, n \geq N$$

Following this we get:

$$|Mx_n - ML| < M\frac{\epsilon}{|M|}$$

$$|M||x_n - L| < \epsilon$$

Hence, the sequence converges to ML.