Execise 10

Theorem: Give an example of a family of intervals $A_n, n = 1, 2, ...$ such that $A_{n+1} \subset A_n$ for all n and $\cap_{n=1}^{\infty} A_n$ consists of a single real number. Prove that your example has the stated property.

Proof: This proof is similar to theorem 9 but with an closed intervall. A family of intervalls with the mentioned properties are the following:

$$A_n = [0, \frac{1}{n}], n = 1, 2, ...$$

We proof the the first property by taking an $n \in N$.Then every element of those two intervall $[0, \frac{1}{n}]$ and $[0, \frac{1}{n+1}]$ are $0 \leq x \leq \frac{1}{n}$, respectively $\frac{1}{n+1}$. Following that $\frac{1}{n+1} < \frac{1}{n}$, A_{n+1} must be a subset of A_n. Now we have to proof the second property. As we learned from the first proof that the collection A_{n+1} is always a subset of A_n, we can conclude that all A_n are an intersection of A_∞ when $n \to \infty$. Since $\lim_{n \to \infty} \frac{1}{n} = 0$, the intervall $[0, 0]$ is a set, which consists only of a singular real number 0.