Execise 3

Theorem: For any integer n, the number $n^2 + n + 1$ is odd.

Proof:

$$n^2 + n + 1 = q, q = 2p + 1$$

$$n^2 + n + 1 = 2p + 1$$

$$\frac{n^2 + n}{2} = p$$

$$\frac{n(n+1)}{2} = p$$

Since the multiplication with an even number always return an even number and the numerator must be even for every any integer n, p must be a natural number. Hence, the theorem is true.