Execise 4

Theorem: Every odd number is one of the form $4n + 1$ or $4n + 3$.

Proof: We proof this withn the division theorem The division theorem states, that every natural number can be expressed as $n = ab + r, with\, a, r \in N and\, b \in Z$ and $0 <= r < a$. Since $a = 4$ there are four possible cases, that describes any natural number. These are:

$$4b,\ 4b + 1,\ 4b + 2,\ 4b + 3$$

Since any natural numbers of the form $4b$ or $4b + 2$ always even due to the factor 4. Hence, the only cases that a odd natural number can be expressed in is $4b + 1$ or $4b + 3$, which are the forms in the theorem.