DEPARTMENT OF INFORMATICS

TECHNISCHE UNIVERSITÄT MÜNCHEN

Bachelor's Thesis in Informatics

Combatting the Precision Loss of Partial Contexts in Abstract Interpretation

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Bekämpfung des Präzisionsverlust durch partielle Kontexte in Abstrakter Interpretation

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I confirm that this bachelor's thesis in informatics is my own work and I have documented all sources and material used.

Munich, 15th of February 2023   Felix Sebastian Krayer

Acknowledgments

Abstract

Contents

1 Introduction

Structure: First we will introduce the basics of static analysis. This will go by introducing constraint systems and how these are used to gain information about the program statically. It will be accompanied by an example of a value-of-variables analysis acting on a toy language we will use for examples in this thesis. This will be extended to an interprocedural approach where partial context sensitivity will be introduced. Here the source of the precision loss will be pointed out. We then will propose an approach to combat this precision loss. The approach will first be introduced theoretically, after which we also present the challenges and results of implementing it in the GOBLINT analyzer. To give an evaluation to the proposed approach, a benchmark of the implementation will be performed and inspected. Our conclusions are presented in the last chapter.

2 Background

2.1 Static Analysis

Static analysis is defined by Rival [RY20] as "[...]an automatic technique that approximates in a conservative manner semantic properties of programs before their execution". This means that the program is analyzed just by the given source code without execution. The goal is to prove certain properties about the program in a "sound" manner i.e. any property that is proven to hold actually does hold. However, from failing to prove a property one cannot conclude that the given property does not hold.
In order to prove properties, e.g. finding that a program does not contain races or identifying dead code, we need to gain information about the program. This is done by performing various kinds of analyses. We will focus on flow sensitive analyses from now on i.e. analyses which find properties of the program dependent on the location within it. The semantic of these will be introduced in the following chapters.

2.1.1 Flow sensitive analysis

As noted above flow sensitive analyses find properties of the program dependent on the point within the program. Expressed differently this means a flow sensitive analysis will find an overapproximation of states the program may be in for any given point

```
1  int main() {
2    int x;
3    x = 0;
4    if (x == 0)
5      x = x + 1;
6  }
```
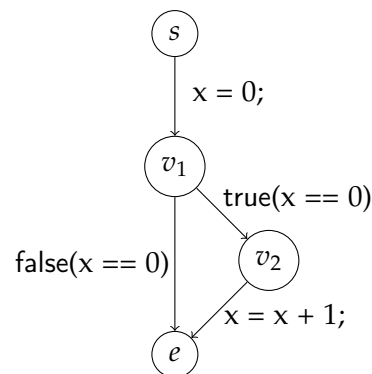
Figure 2.1: Example program (left) and corresponding CFG (right)

within the program or "program point". This state can describe many things dependent on the analysis performed.

First let us define what a program point is: Imagine a control flow graph (CFG), where nodes represent points between instructions within the program. Edges are labeled with instructions or checks (from now on collectively called "actions") and describe the transitions between these points (see example 2.1). Then any node on this CFG would be what we call a program point.

Concretely let N be the set of all program points. Furthermore, let $\mathbb{D}$ be a Domain containing abstract states describing concrete states of the program. This means that some $d \in \mathbb{D}$ can describe many states the program can be in.

Then an analysis is expected to find a mapping $\eta : N \to \mathbb{D}$ which maps program points to abstract states describing that location within the program i.e. for $[n] \in N$, $\eta [n]$ should be an abstract state describing all possible states (and possibly more) the program can be in at program point $[n]$.

As an example we will introduce a values-of-variables analysis for integers. This analysis finds a mapping from a set of variables X to abstractions of their possible values at any given program point. In the scope of this thesis we will focus on abstracting integer values by sets of integers. Thereby the goal of our values-of-variables analysis is to find a mapping $X \to 2^{\mathbb{N}}$ for each program point.

Combining this with the semantic of flow sensitive analysis from before, we get that the Domain $\mathbb{D}_{\mathsf{v}}$ for the values-of-variables analysis should be $\mathbb{D}_{\mathsf{v}} = X \to 2^{\mathbb{N}}$. Finally, the resulting $\eta_{\mathsf{v}} : N \to \mathbb{D}_{\mathsf{v}}$ for this analysis describes a mapping $\eta_{\mathsf{v}} [n]$ for some program point $[n] \in N$, where $\eta_{\mathsf{v}} [n]\, x$ is a set containing all values $x \in X$ may possibly hold at $[n]$. From this we can conclude that x cannot hold any value outside $\eta_{\mathsf{v}} [n]\, x$ at program point $[n]$.

2.1.2 Constraint systems

We now formulate a way in which we can describe an analysis in the form of constraints. For this we need a partial ordering $\sqsupseteq$ on the domain $\mathbb{D}$.

Then we create a system of constraints which can be solved for a solution. Consider the edges (u, A, v) of the CFG, where each edge denotes a transition from program point $[u]$ to program point $[v]$ via the action A. Now let each of these edges give raise to a constraint

$$\eta [v] \sqsupseteq [\![A]\!]^{\#} (\eta [u])$$

where $[\![A]\!]^{\#}$ denotes the abstract effect of the action A defining our analysis. In addition, we need a start state. This is given by $\mathsf{init}^{\#} : \mathbb{D}$ which is defined depending on the analysis. This gives raise to the start constraint $\eta [s] \sqsupseteq \mathsf{init}^{\#}$.

We will show these ideas with our example of the values-of-variables analysis: Let us define the partial ordering $\sqsupseteq_v$ that will be used in the constraints. We will do this by saying that a mapping $m_1 \in \mathbb{D}_v$ is ordered above another mapping m_2 iff for every variable the set it is mapped to in m_1 is a superset of the one the variable is mapped to in m_2. Formulated formally this is:

$$m_1, m_2 \in \mathbb{D}_v : m_1 \sqsupseteq_v m_2 \iff \forall x \in X : m_1 \ x \supseteq m_2 \ x$$

Next we define the start state $\text{init}^\# = m_\top$ for this domain as the mapping that maps every variable to the full set of integers $\mathbb{N}$ i.e. $\forall x \in X : m_\top \ x = \mathbb{N}$. This is because we assume variables to be randomly initialized in our toy language.

It remains to define the abstract effect of actions $[\![A]\!]_v^\#$ for our values-of-variables analysis. We will just show the effect of a simple variable assignment:

$$[\![(x = y;)]\!]_v^\# \ m = m \oplus \{x \mapsto (m \ y)\}$$

where $m \oplus \{x \mapsto s\}$ denotes that the mapping m is updated such that x will be mapped to the set s. The full definition of abstract effects of a values-of-variables analysis can be found at <// TODO>.

2.1.3 Interprocedural analysis

So far we only have defined how a program without procedure calls is analyzed. Now we want to introduce procedure calls of the form $f()$. Since a call has its own set of local variables to work with and a call stack can contain multiple of the same procedure (e.g. for recursion), we will analyze procedures in their own environment. However, we need to also consider global variables that may be affected by the analysis. The idea is to give procedures their own starting states and analyze them similarly as we have done before. The final state of the called procedure is then used to be combined back into the state of the caller before the call. Formalized for an edge $(u, f(), v)$ this looks as follows:

$$\eta \ [v] \sqsupseteq \text{combine}^\# \left((\eta \ [u]), (\eta \ [e_f]) \right)$$
$$\eta \ [s_f] \sqsupseteq \text{enter}^\# (\eta \ [u])$$

where $[s_f]$ and $[e_f]$ are the start and end node of the CFG for procedure $f()$. The functions $\text{combine}^\# : \mathbb{D} \times \mathbb{D} \to \mathbb{D}$ and $\text{enter}^\# : \mathbb{D} \to \mathbb{D}$ are defined by the analysis. These handle computing the start state for $f()$ and combining the caller state and the end state of the callee.

For our values

2.1.4 Context sensitivity

2.1.5 Partial context sensitivity

2.1.6 Precision loss

3 Main Contributions

3.1 Formal description

3.1.1 Taint analysis

In this section we will propose our approach to reduce the aforementioned precision loss 2. The basic idea is to track for each procedure which variables have been written or have possibly been altered in some other way. This information is then used in the values-of-variables analysis when combining the abstract state from the caller with the abstract return state given by the callee at the end of the procedure.
In the following we will call a variable that has been written or altered in the current procedure context "tainted". Therefore, we introduce a new taint analysis tracking which variables have been tainted within the context of the current procedure.
First we formalize our taint analysis in the syntax for flow sensitive analyses we have built in the previous chapter 2:
The Domain for this Analysis is the powerset of the set of variables X with the regular subset ordering:

$$\mathbb{D}_t = 2^X \text{ with } \sqsupseteq_t = \supseteq$$

From that follows that we seek to compute a mapping from program points to sets of variables i.e. $\eta_t : N \to \mathbb{D}_t$. To interpret this with the goal of our taint analysis in mind, we note that $\eta_t[n] = T$ will denote that T is the set of possibly tainted variables at program point $[n]$. Explained differently this means that for any variable $x \in T$ we cannot exclude that this variable was altered between the start of the procedure $[n]$ is in up until the program pint $[n]$.
It remains to define $\text{init}^\#$, $\text{enter}^\#$ and $\text{combine}^\#$ as well as the abstract effects of actions $[\![A]\!]^\#$. Recall that the notion of a "tainted" variable is defined in relation to the current procedure. This means we want to start fresh whenever we enter a procedure and start without any variable being initially tainted. Since the same holds for the initial state we have

$$\text{enter}^\# T = \text{init}^\# = \varnothing$$

Each edge $e = (u, A, v)$ introduces the constraint $\eta_t[v] \sqsupseteq [\![A]\!]^\#(\eta_t[u])$

$$[\![x = y]\!]^{\#} \ T = T \cup \{x\}$$
$$[\![^{*}x = y]\!]^{\#} \ T = T \cup \mathsf{MayPointTo}(x)$$

$$\mathsf{enter}^{\#} \ T = \varnothing$$
$$\mathsf{combine}^{\#} \ T_{\mathsf{cr}} \ T_{\mathsf{ce}} = T_{\mathsf{cr}} \cup T_{\mathsf{ce}}$$

3.1.2 Improving the values-of-variables analysis

$$
\begin{aligned}
\mathsf{combine}^{\#} \eta_{\mathsf{cr}} \ \eta_{\mathsf{ce}} =& \mathsf{let} \ \eta'_{\mathsf{cr}} = \eta_{\mathsf{cr}} \setminus T_{\mathsf{ce}} \ \mathsf{in} && (3.1) \\
& \mathsf{let} \ \eta'_{\mathsf{ce}} = \eta_{\mathsf{ce}} \cap T_{\mathsf{ce}} \ \mathsf{in} && (3.2) \\
& \eta'_{\mathsf{cr}} \cup \eta'_{\mathsf{ce}} && (3.3)
\end{aligned}
$$

3.2 Implementation

4 Evaluation

4.1 Testing

4.2 Benchmarking

5 Conclusion

Abbreviations

- CFG

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Bibliography

[RY20]   X. Rival and K. Yi. *Introduction to static analysis: an abstract interpretation perspective*. Mit Press, 2020.