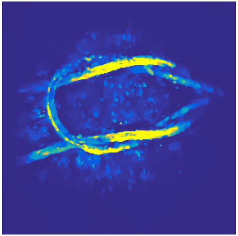
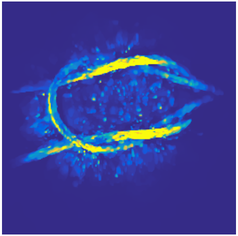
Enhancing Dynamic, Sub-Sampled 3D Photoacoustic Tomography by Simultaneous Motion Estimation





Felix Lucka
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joint with:
Simon Arridge, Paul Beard, Marta Betcke,
Ben Cox, Nam Huynh & Edward Zhang

Centre for Medical Image Computing

IMA Conference on Inverse Problems from Theory to Application
Cambridge, Sep 20, 2017.

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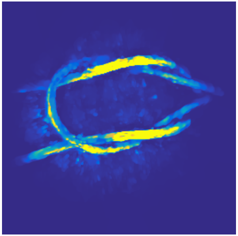

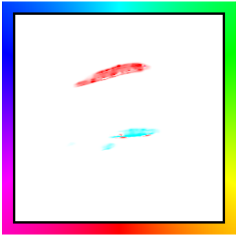

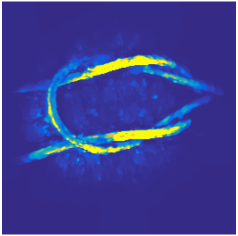

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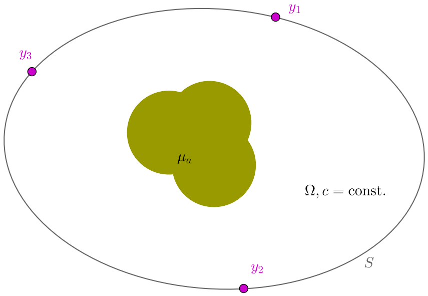
Optical Part

optical absorption coefficient: μ_a



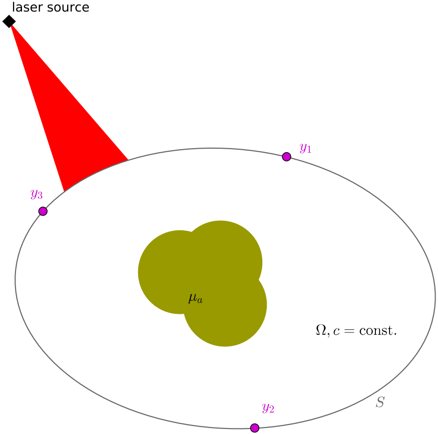
Acoustic Part

Basics of Photoacoustic Tomography (PAT)

Optical Part

optical absorption coefficient: μ_a

pulsed laser excitation: Φ



Acoustic Part

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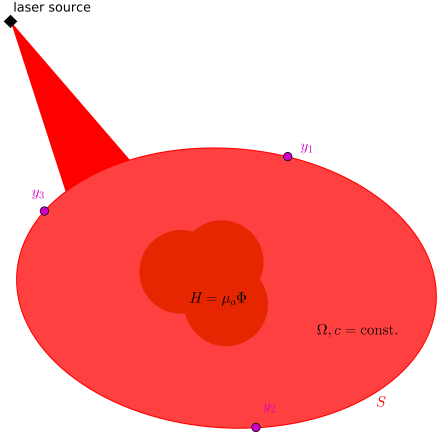
thermalization by chromophores: $H = \mu_a \Phi$



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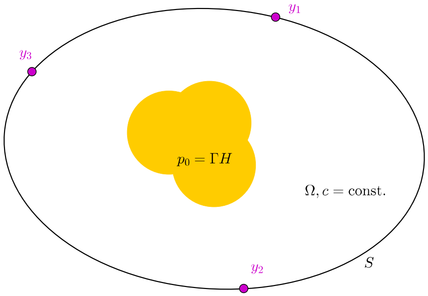
local pressure increase: $p_0 = \Gamma H$



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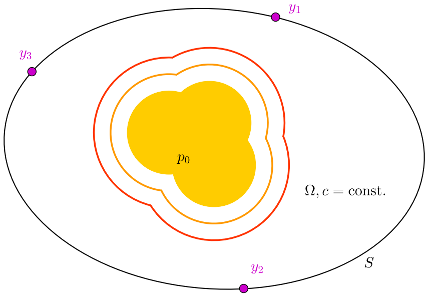
elastic wave propagation:

$$\Delta p - \frac{1}{c^2}\frac{\partial^2 p}{\partial^2 t} = 0$$

$$p|_{t=0} = p_0, \quad \frac{\partial p}{\partial t}|_{t=0} = 0$$



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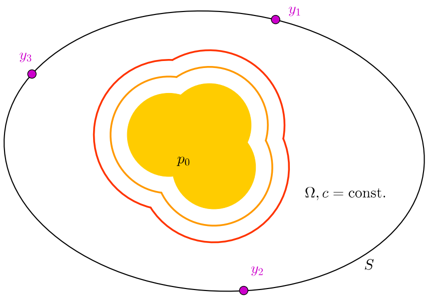

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measurement of pressure time courses:

$$f_i(t) = p(y_i, t)$$

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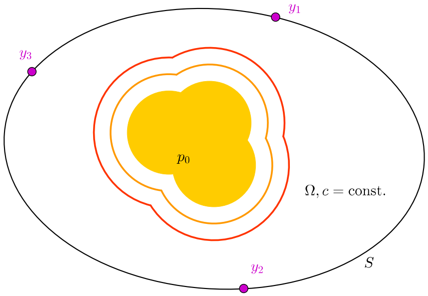

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Photoacoustic effect

- coupling of optical and acoustic modalities.
- "hybrid imaging"
- high optical contrast can be read by high-resolution ultrasound.

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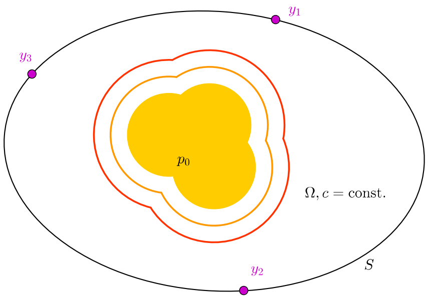

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Inverse problems:

! optical inversion (μ_a) from boundary data: severely ill-posed.

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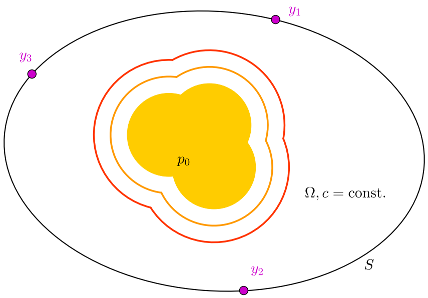

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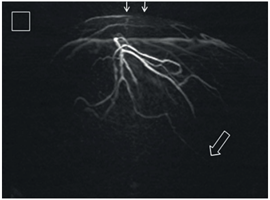
$$f_i(t) = p(y_i, t)$$

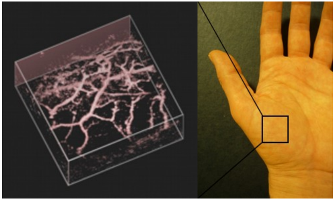
Inverse problems:

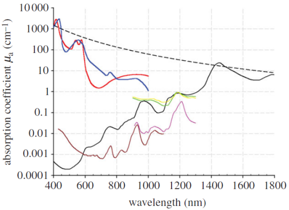
! optical inversion (μ_a) from boundary data: severely ill-posed.

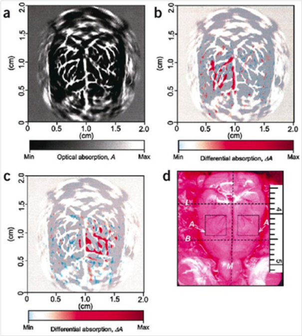
✓ acoustic inversion (p_0) from boundary data: moderately ill-posed.

✓ optical inversion (μ_a) from internal data: moderately ill-posed.





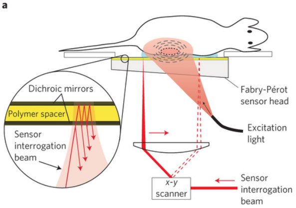


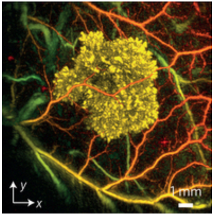


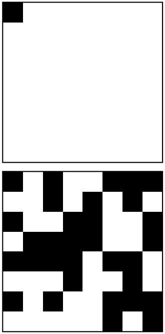
- Light-absorbing structures in soft tissue.
- High contrast between blood and water/lipid.
- Sensitive to blood oxygen saturation (SO_2).
- Different wavelengths allow quantitative spectroscopic examinations.
- Use of contrast agents for molecular imaging.
- Extremely promising future imaging technique!

sources: **Paul Beard, 2011**. *Biomedical photoacoustic imaging*, *Interface Focus.* Wikimedia Commons

Dynamic High Resolution Photoacoustic Tomography







Fabry Pérot (FB) interferometer:

✓ High spatial resolution

! Nyquist sampling leads to low temporal resolution

$\rightsquigarrow$ Beat Nyquist for sparse targets by incoherent sampling of each frame/wavelength i ("compressed sensing"):

$$f_i^c = C_i f_i = C_i(Ap_i + \varepsilon_i), \qquad i = 1, \dots, T$$

Image reconstruction:

$f_i^c \longrightarrow f_i$, $f_i \longrightarrow p_i$ by standard method.

$f_i^c \longrightarrow p_i$: standard or new method?

$F^c \longrightarrow P$: Full spatio-temporal method.

PAT Reconstruction & Numerical Wave Propagation

Variational regularization:

$$\hat{p} = \underset{p \geqslant 0}{\operatorname{argmin}} \left\{ \tfrac{1}{2} \| CAp - f^c \|_2^2 + \lambda \mathcal{J}(p) \right\}$$

! Iterative first-order methods require implementation of A and A^*.

✓ k-space pseudospectral time domain method for 3D wave propagation:

B. Treeby and B. Cox, 2010. *k-Wave: MATLAB toolbox for the simulation and reconstruction of photoacoustic wave fields*, Journal of Biomedical Optics.

✓ Derivation and discretization of adjoint PAT operator A^*:

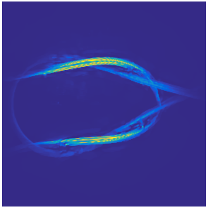
Arridge, Betcke, Cox, L, Treeby, 2016. *On the Adjoint Operator in Photoacoustic Tomography*, Inverse Problems 32(11).

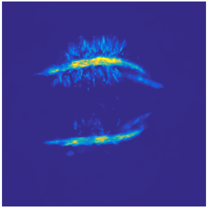
Accelerated 3D PAT via Compressed Sensing

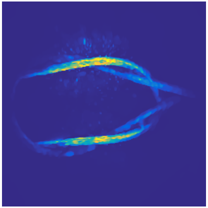
$$\hat{p}_i = \underset{p \geqslant 0}{\operatorname{argmin}} \left\{ \tfrac{1}{2} \| C_i A p - f_i^c \|_2^2 + \lambda \mathcal{J}(p) \right\}$$

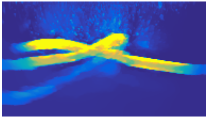
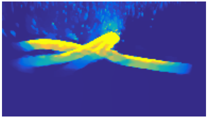
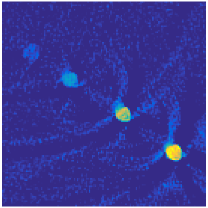
- ✓ combination of compressed sensing and sparsity-constrained image reconstruction
- ✓ generic total variation (TV) regularization enhanced by Bregman iterations
- ✓ extensive evaluation with realistic numerical phantom, experimental and *in-vivo* data
- ✓ significant acceleration with minor loss of quality.
- ! frame-by-frame reconstruction, only.

Arridge, Beard, Betcke, Cox, Huynh, L, Ogunlade, Zhang, 2016. *Accelerated High-Resolution Photoacoustic Tomography via Compressed Sensing*, *Physics in Medicine and Biology* 61(24).







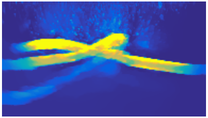




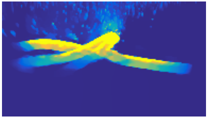
Continuous data acquisition

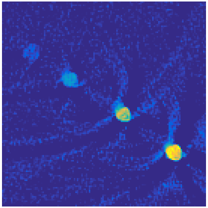
$\Longrightarrow$ tradeoff between spatial and temporal resolution.

Different dynamic models:

- Parametric models (shift, stretch, etc.): simple and nice if applicable.
- Structured Low-Rank (functional imaging with static anatomies/QPAT).
- Tracer uptake/wash-in models.
- Perfusion models.
- Needle guidance
- Intra-operative endoscopic imaging.
- Joint image reconstruction and motion estimation.







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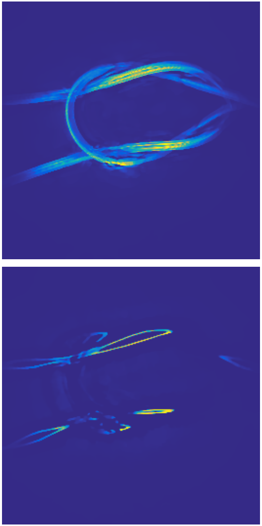
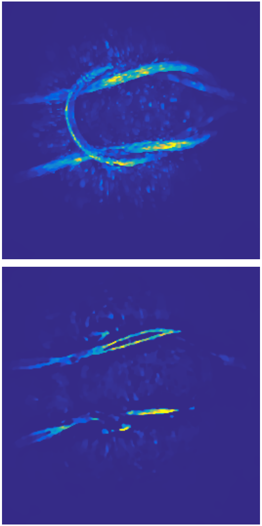
$$\hat{p}_i = \underset{p \geqslant 0}{\operatorname{argmin}} \left\{ \tfrac{1}{2} \| C_i A p - f_i^c \|_2^2 + \lambda TV(p) \right\}, \qquad \forall\, i = 1, \dots, T$$



full data



sub-sampled data (16x)

Non-parametric spatio-temporal regularization: Find $P \in \mathbb{R}^{N \times T}$ as

$$\hat{P} = \underset{P \geqslant 0}{\operatorname{argmin}} \left\{ \sum_{i}^{T} \frac{1}{2} \| C_i A \, p_i - f_i^c \|_2^2 + \lambda \mathcal{R}(P) \right\},$$

Lot's of possibilities, here: Implicit model formulated as joint image and motion estimation:

$$(\hat{P}, \hat{V}) = \underset{P \geqslant 0, V}{\operatorname{argmin}} \left\{ \sum_{i}^{T} \frac{1}{2} \| C_i A \, p_i - f_i^c \|_2^2 + \alpha \mathcal{J}(p_i) + \beta \mathcal{H}(v_i) + \gamma \mathcal{S}(P, V) \right\}$$

$\mathcal{S}(P, V)$ enforces motion PDE, e.g., optical flow equation:

$$\partial_t p(x,t) + (\nabla_x p(x,t)) \, v(x,t) = 0$$

Burger, Dirks, Schönlieb, 2016. *A Variational Model for Joint Motion Estimation and Image Reconstruction*, arXiv:1607.03255.

Example: TV-TV-Lp Regularization

$$\partial_t p(x,t) + (\nabla_x p(x,t))\, v(x,t) = 0$$

$\rightsquigarrow$ discretize and penalize deviation:

$$(\hat{P},\hat{V}) = \operatorname*{argmin}_{P \geqslant 0, V} \left\{ \sum_i^T \frac{1}{2}\| C_i A\, p_i - f_i^c \|_2^2 \right.$$
$$\left. + \alpha\, TV(p_i) + \beta\, TV(v_i) + \frac{\gamma}{p} \| (p_{i+1} - p_i) + (\nabla p_i) \cdot v_i \|_p^p \right\}$$

proximal-gradient-type scheme:

$$P^{k+1} = \mathbf{prox}_{\nu\mathcal{R}}\left(P^k - \nu A^T C^T \left(CAP^k - F^c\right)\right)$$

$$\mathbf{prox}_{\nu\mathcal{R}}(P) = \operatorname*{argmin}_{Q \geqslant 0} \left\{ \frac{1}{2}\| Q - P \|_2^2 + \nu \mathcal{R}(Q) \right\}$$
$$= \operatorname*{argmin}_{Q \geqslant 0} \left\{ \min_V \sum_i^T \frac{1}{2} \| q_i - p_i \|_2^2 \right.$$
$$\left. + \nu\alpha\, TV(q_i) + \nu\beta\, TV(v_i) + \frac{\nu\gamma}{p} \| (q_{i+1} - q_i) + (\nabla q_i) \cdot v_i \|_p^p \right\}$$

For $p \geqslant 1$, TV-TV-Lp denoising is a biconvex optimization problem:

$$\min_{Q \geqslant 0, V} S(Q, V) := \min_{Q \geqslant 0, V} \sum_{i}^{T} \frac{1}{2} \| q_i - p_i \|_2^2 + \nu\alpha \, TV(q_i) + \nu\beta \, TV(v_i) + \frac{\nu\gamma}{p} \| (q_{i+1} - q_i) + (\nabla q_i) \cdot v_i \|_p^p$$

Alternating optimization:

$$Q^{k+1} = \underset{Q}{\operatorname{argmin}} \, S(Q, V^k) \qquad \text{(TV-transport constr. denoising)}$$

$$V^{k+1} = \underset{V}{\operatorname{argmin}} \, S(Q^{k+1}, V) \qquad \text{(TV constr. optical flow estimation)}$$

! Both problems are convex but non-smooth.

! Need to ensure energy decrease.

Alternating optimization:

$$Q^{k+1} = \underset{Q}{\text{argmin}}\, S(Q, V^k) \qquad \text{(TV-transport constr. denoising)}$$

$$V^{k+1} = \underset{V}{\text{argmin}}\, S(Q^{k+1}, V) \qquad \text{(TV constr. optical flow estimation)}$$

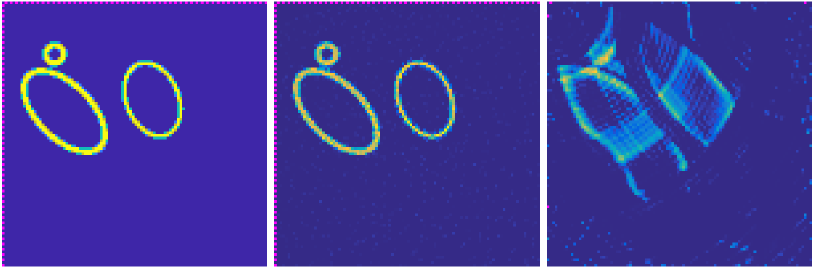
Primal-dual hybrid gradient for both: Too slow convergence in 3D.

Alternating directions method of multipliers (ADMM):

- ! More difficult to parameterize (to ensure monotone energy).
- ! Badly conditioned, large-scale least-squares problems.
- ! Crucial: Choice of iterative solver, preconditioning and stop criterion.
- ✓ Overrelaxed ADMM with step size adaptation and CG solver for Q.
- ✓ Overrelaxed ADMM with AMG-CG solver for V (frame-by-frame).
- ✓ Warm-start wherever possible.

Chambolle, Pock, 2016. *An introduction to continuous optimization for imaging*, Acta Numerica.

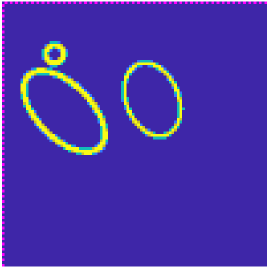
$$\hat{p}_i = \underset{p \geqslant 0}{\operatorname{argmin}} \left\{ \| C_i A p - f_i^c \|_2^2 \right\} \quad \forall \; i = 1, \ldots, T$$



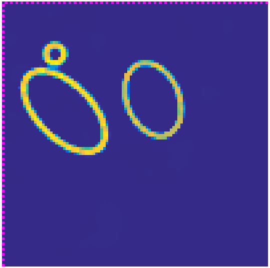
phantom   full data   sub-sampled (25x)

A 2D Example: Frame-by-Frame Total Variation

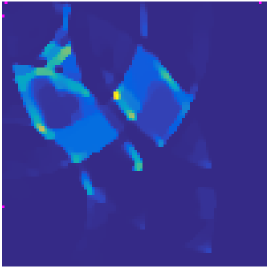
$$\hat{p}_i = \underset{p \geqslant 0}{\operatorname{argmin}} \left\{ \| C_i A p - f_i^c \|_2^2 + \lambda TV(p) \right\} \quad \forall\, i = 1, \ldots, T$$



phantom



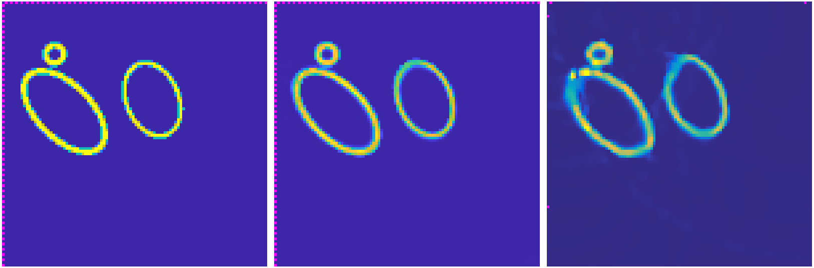
full data



sub-sampled (25x)

$$\left(\hat{P}, \hat{V}\right) = \underset{P \geqslant 0, V}{\operatorname{argmin}} \left\{ \frac{1}{2} \sum_i^T \| C_i A\, p_i - f_i^c \|_2^2 \right.$$
$$\left. + \alpha\, TV(p_i) + \beta\, TV(v_i) + \frac{\gamma}{2} \| (p_{i+1} - p_i) + \nabla p_i \cdot v_i \|_2^2 \right\}$$

$\alpha = \beta = \lambda_{TV}$, $\gamma = 1$



phantom | full data | sub-sampled (25x)

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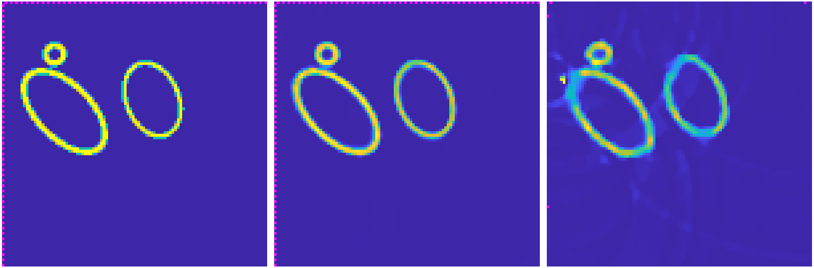
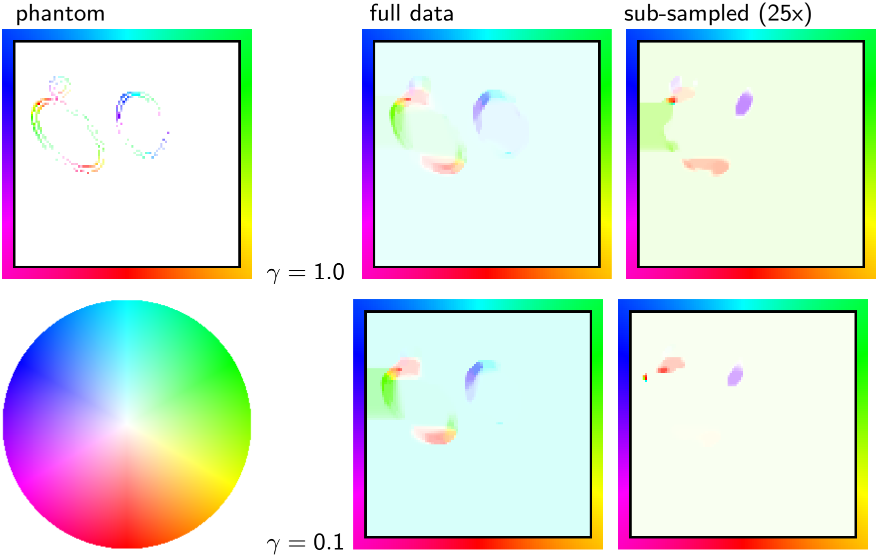
$\alpha = \beta = \lambda_{TV}$, $\gamma = 0.1$

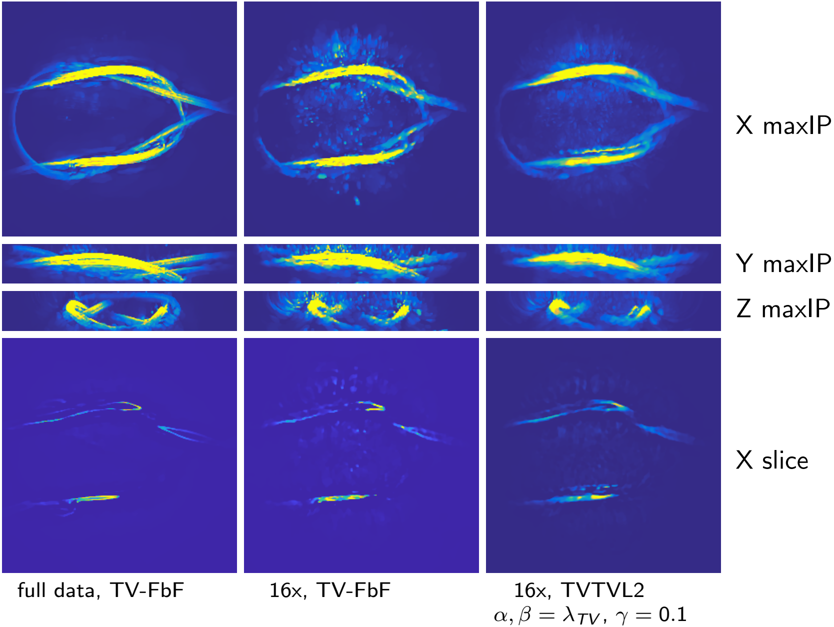


phantom | full data | sub-sampled (25x)

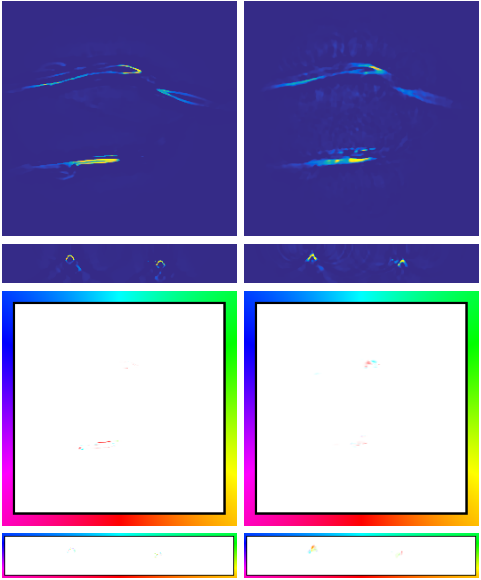
A 2D Example: Motion Estimation with TV-TV-L2



Artificially Sub-Sampled 3D Stop-Motion Data



Artificially Sub-Sampled 3D Stop-Motion Data



u - X slice

u - Z slice

$v - \bar{v}$ - X slice

$v - \bar{v}$ - Z slice

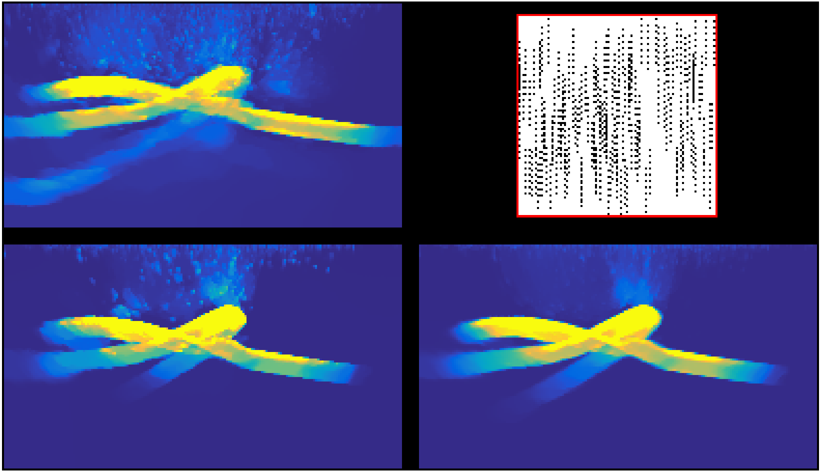
full data, TVTVL2

16x, TVTVL2

$\alpha, \beta = \lambda_{TV}$, $\gamma = 0.1$

$192 \times 190 \times 108$ voxels, 40 frames

sub-average over 8 frames



TV-FbF

TVTVL2, $\alpha = \beta = \lambda_{TV}$, $\gamma = 0.1$

Photoacoustic Tomography

- Imaging with laser-generated ultrasound ("hybrid imaging")
- High contrast for light-absorbing structures in soft tissue.

Challenges of fast, high resolution 4D PAT:

- Nyquist requires several thousand detection points $\rightsquigarrow$ slow.
- High computational load.

Acceleration through sub-sampling:

- Exploit low spatio-temporal complexity to beat Nyquist.
- Acceleration by sub-sampling the incident wave field to maximize non-redundancy of data.
- Adjoint PAT operator allows to use variational/iterative approaches.
- Sparse, spatio-temporal variational regularization: promising results, joint estimation of dynamic parameters?

- **Arridge, Beard, Betcke, Cox, Huynh, L, Zhang, 2017**. *Enhancing Compressed Sensing Photoacoustic Tomography by Simultaneous Motion Estimation*, *almost submitted.*
- **Huynh, L, Zhang, Betcke, Arridge, Beard, Cox, 2017**. *Sub-sampled Fabry-Perot photoacoustic scanner for fast 3D imaging*, *Proc. SPIE 2017.*
- **Arridge, Beard, Betcke, Cox, Huynh, L, Ogunlade, Zhang, 2016**. *Accelerated High-Resolution Photoacoustic Tomography via Compressed Sensing*, *Physics in Medicine and Biology* 61(24).
- **Arridge, Betcke, Cox, L, Treeby, 2016**. *On the Adjoint Operator in Photoacoustic Tomography*, *Inverse Problems* 32(11).

Centre for Medical Image Computing

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Thank you for your attention!

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k-Wave

EPSRC Engineering and Physical Sciences Research Council

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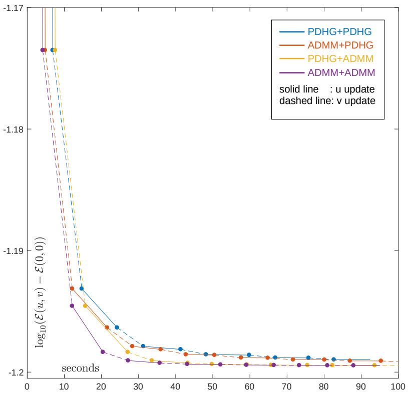
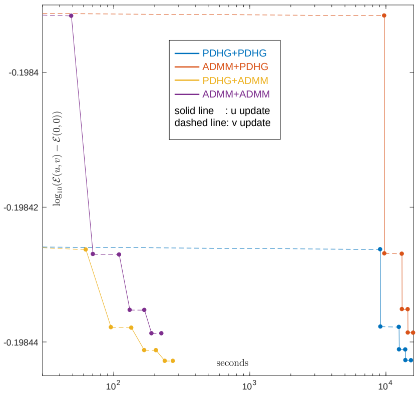
PDHG & ADMM in 2D & 3D





Preconditioning of the Least Squares Problem in ADMM