



Centrum Wiskunde & Informatica



**UCL**

# Image Reconstruction: A Playground for Applied Mathematicians

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Felix Lucka

**Partial Differential Equations and Applications Seminar**

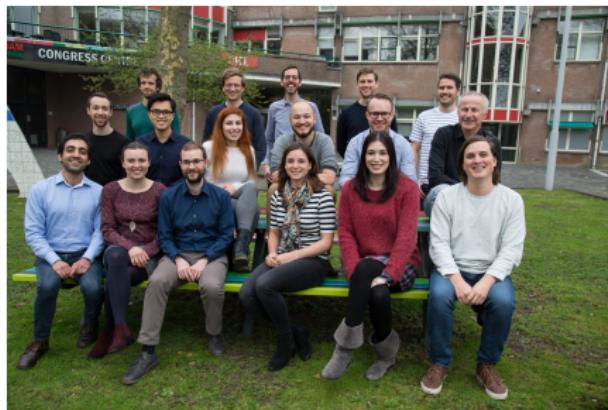
**TU Delft**

20 Feb 2020

## **Introduction and Overview**

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# Computational Imaging @ CWI



- headed by **Joost Batenburg**, 18 members
- mathematics, computer science & (medical) physics
- advanced computational techniques for 3D imaging
- (inter-)national collaborations from science, industry & medicine
- one of the two main developers of the **ASTRA Toolbox**
- **FleX-ray Lab:** custom-made, fully-automated **X-ray CT** scanner linked to large-scale computing hardware

# X-ray Computed Tomography (CT)



- X-rays (high-energy photons) get **attenuated** by matter
- 3D attenuation image **computed** from different 2D projections

# X-ray Computed Tomography (CT)



(a) Modern CT scanner

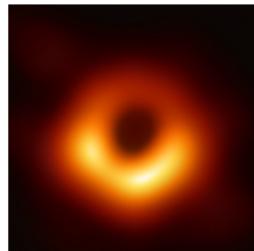


(b) CT scan of a patient's lung

Source: Wikimedia Commons

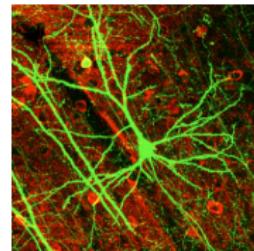
# Imaging Across Disciplines

**Observational astronomy**



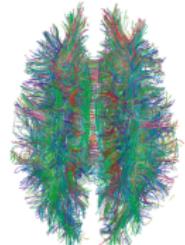
**Life and material science**

microscopy



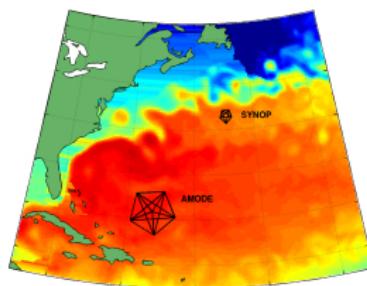
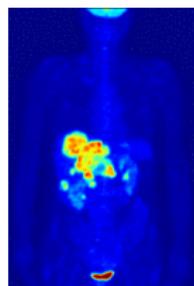
**Medical imaging**

CT, MRI, US, PET, SPECT...



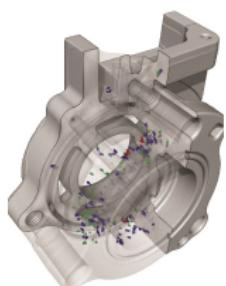
**Geophysical imaging**

(electrical) resistivity, seismic  
(ground-penetrating) radar...



**Remote sensing**

military/intelligence,  
earth/climate science

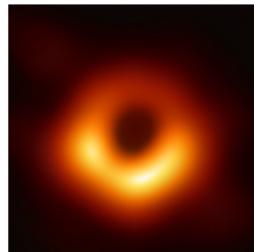


**Industrial process imaging**

Source: Wikimedia Commons

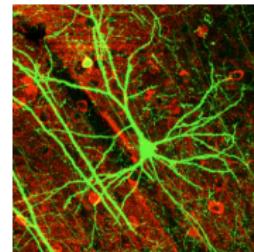
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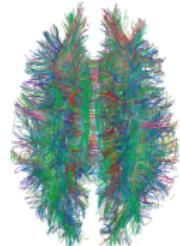
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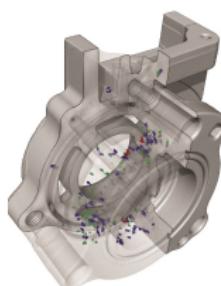
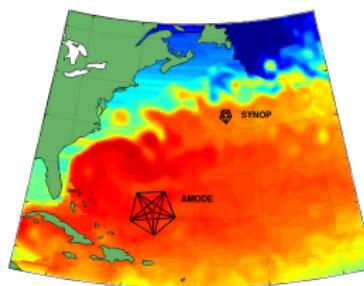
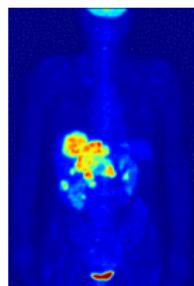
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**Industrial process imaging**

Source: Wikimedia Commons

**Mathematical Imaging:** *Reconstruct spatially distributed quantities of interest from indirect observations through algorithms derived from rigorous mathematics.*

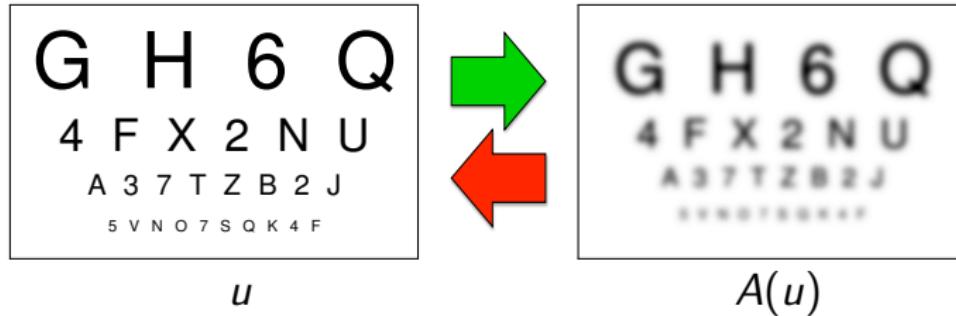
# Imaging: An Inverse Problem

**Inverse problem:** Recover **unknowns**  $u$  (image) from **data**  $f$  via

$$f = A(u) + \varepsilon$$

- **Forward operator**  $A$  solution of **PDE** modelling underlying physics.

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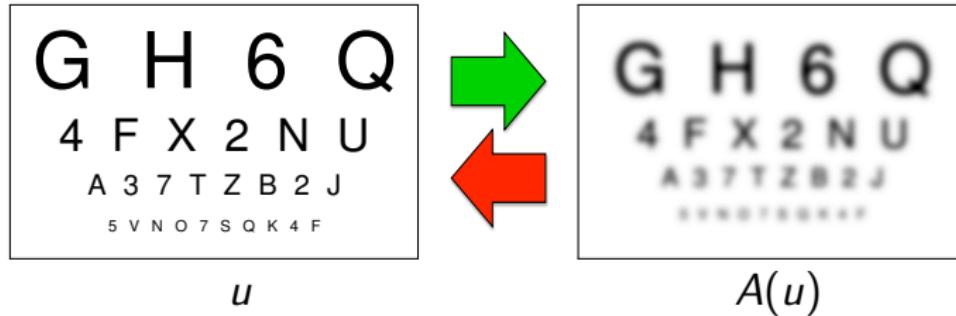


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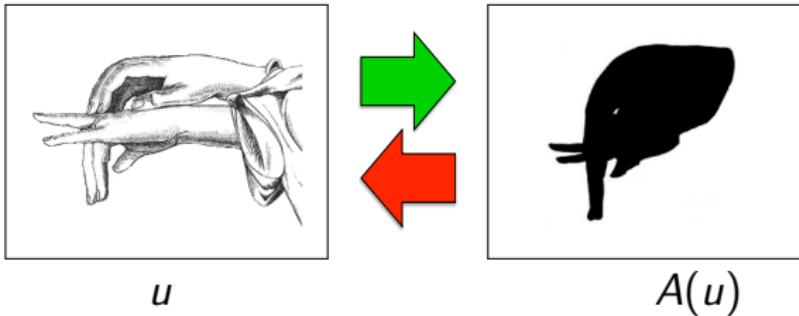


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# Overview Inverse Problems / Imaging Workflow

## mathematical modeling:

physics, PDEs, approximations

$$(s \cdot \nabla + \mu_a(x) + \mu_s(x)) \phi(x, s)$$

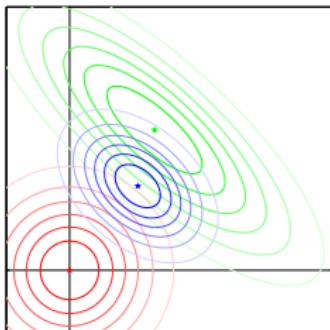
## reconstruction/inference approach:

regularization, statistical inference,  
machine learning

$$= q(x, s) + \mu_s(x) \int \Theta(s, s') \phi(x, s') ds'$$

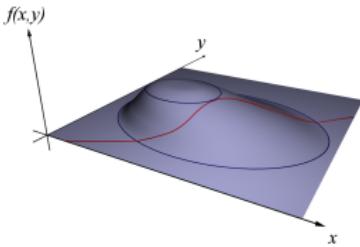
## theoretical analysis:

uniqueness, recovery conditions,  
stability



## reconstruction algorithm:

PDEs, numerical linear algebra,  
optimization, MCMC



## large-scale computing:

parallel computing, GPU computing

# Current Challenges in Computational Imaging

**core development for new modalities:**

hybrid imaging

**more from more:**

multi-spectral, multi-modal, high resolution

**same from less:**

low-dose, limited-view, compressed, dynamic

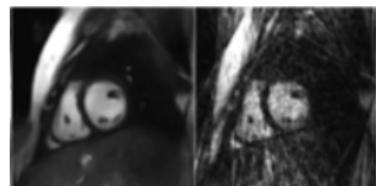
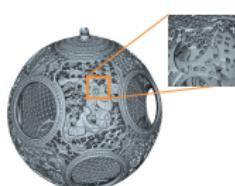
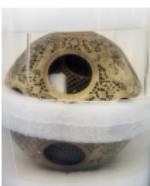
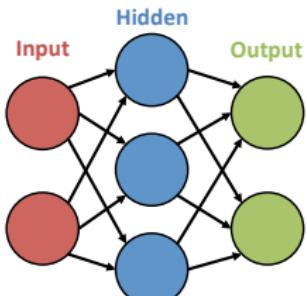
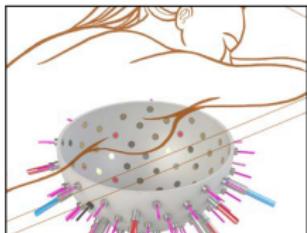
**break the routine:**

real-time, adaptive, explorative

**uncertainty quantification & quantitative imaging**

**machine learning:**

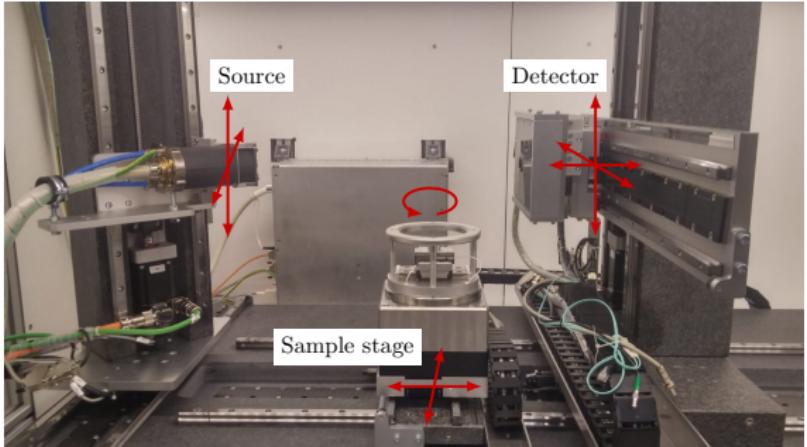
embedding, networks for 3D/4D, clinical training data



## Examples

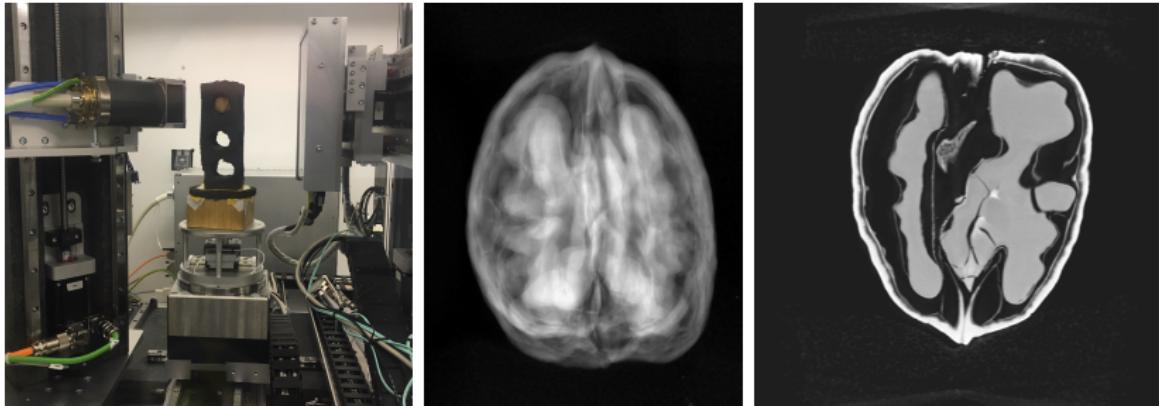
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# FleX-ray Lab



- custom-built, fully-automated, highly flexible
- **Aim: Proof-of-concept** experiments directly accessible to mathematicians and computer scientists.

# X-Ray Scan of Static Object



We share

- data sets on [zenodo.org](https://zenodo.org), community "CI-CWI"
- open data processing and reconstruction software:  
[astra-toolbox.com](https://astra-toolbox.com), [github.com/cicwi](https://github.com/cicwi)



**Der Sarkissian, L, van Eijnatten, Colacicco, Coban, Batenburg, 2019.**

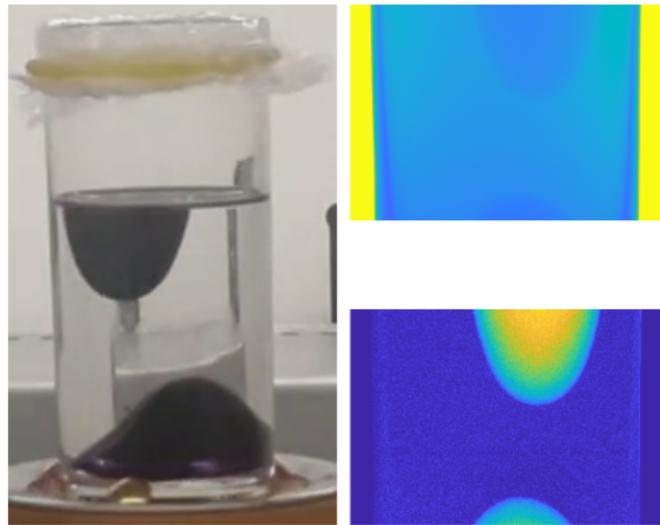
A Cone-Beam X-Ray CT Data Collection Designed for Machine Learning,  
[arXiv:1905.04787](https://arxiv.org/abs/1905.04787).

# X-Ray Scan of Dynamic Object



- canonical example of temperature-driven **two-phase flow instability**
- 120 projections per rotation → each projection averaged over  $3^\circ$
- 40ms exposure per projection → 4.8s per rotation

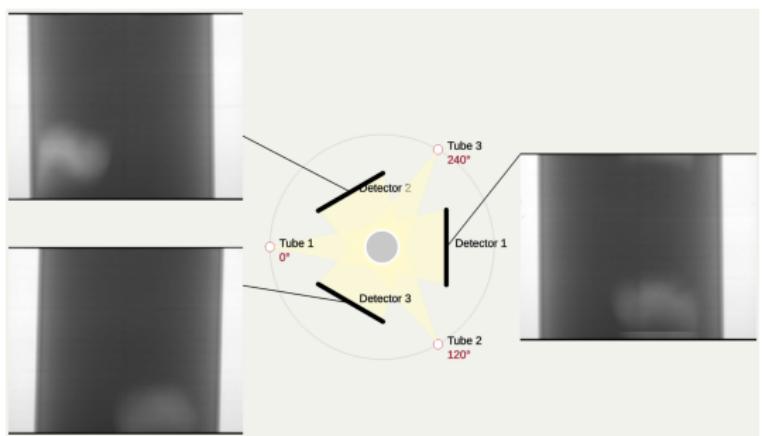
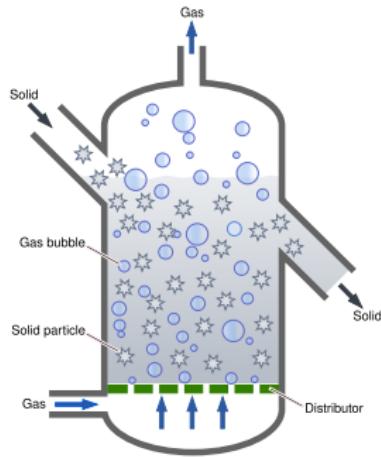
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# Example: Fluidized Bed Reactors

Collaboration with the Transport Phenomena group at TU Delft.



# Overview Dynamic Imaging

## Applications

- scientific, industrial and clinical
- vast range of dynamics (rigid motion, elastic deformation, fluid dynamics, crack formation, chemical kinetics, granular flows, ...)

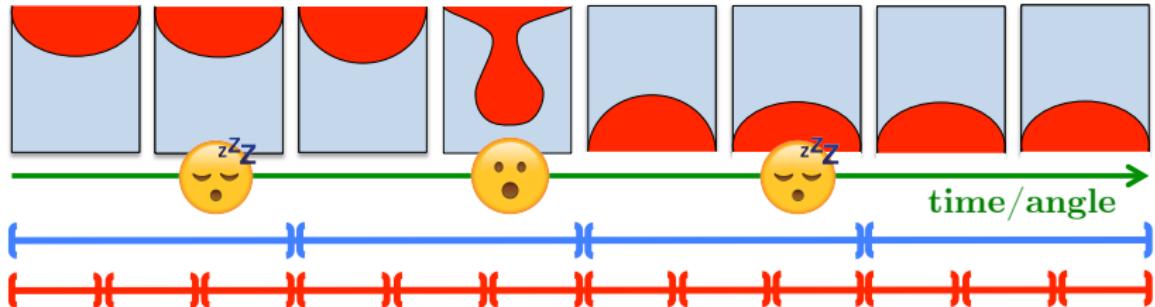
## Goals

- motion compensation
- gating
- full dynamic reconstruction (+ simultaneous motion estimation?)
- parameter identification in dynamical systems

## Challenges

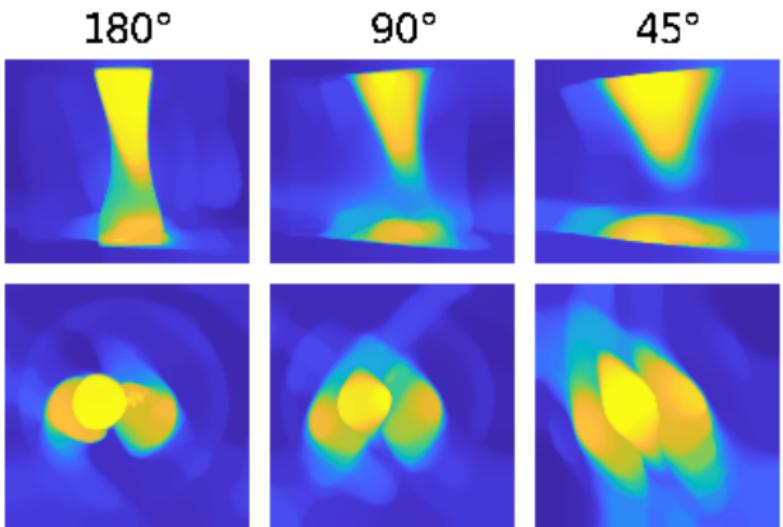
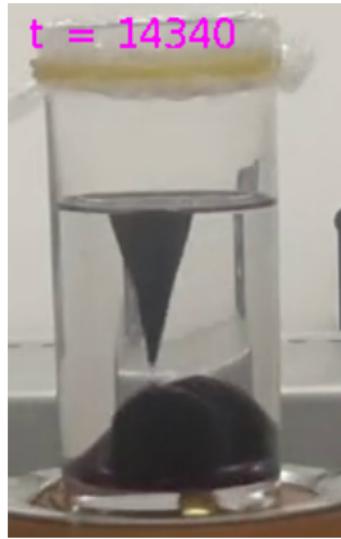
- dynamics too fast for high quality frame-by-frame reconstruction (motion artefacts, noise, low angular res,...)
- mathematical modeling of dynamics
- computational image reconstruction

# 4D Image Reconstruction Challenges



- binning:
  - large bins → motion artifacts
  - small bins → undersampling /limited view
- 4D is computationally heavier than 3D series
- No "golden bullet": different dynamics, different methods

# Lava Lamp: Frame-by-Frame Reconstruction



reconstruct image sequence  $u$  and motion fields  $v$  simultaneously

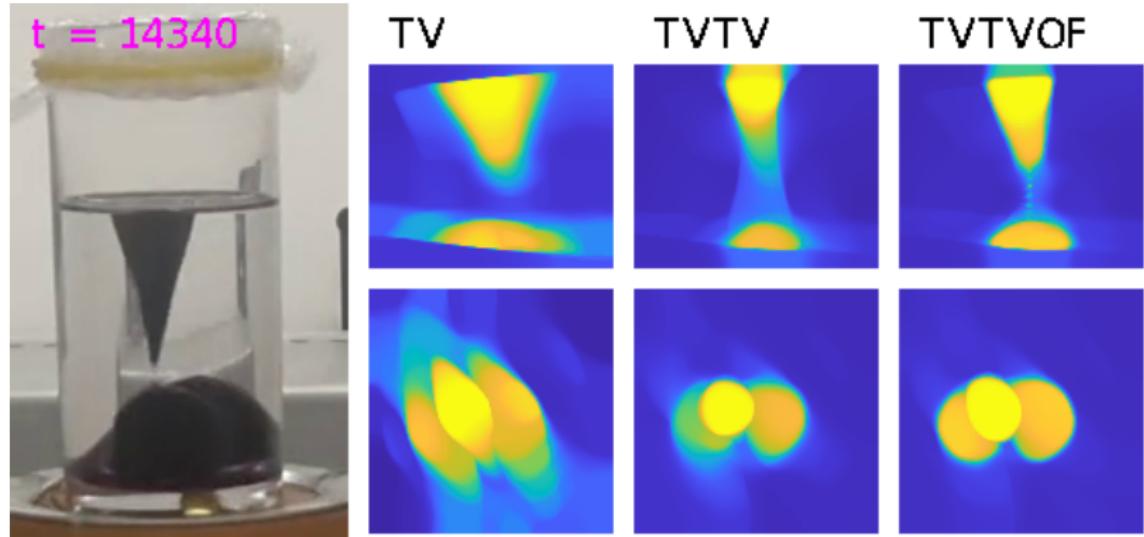
$$\min_{u,v} \sum_t \|A_t u_t - f_t\|_2^2 + \mathcal{J}(u_t) + \mathcal{M}(u,v) + \mathcal{H}(v)$$

- data discrepancy
- motion model (PDE)
- spatial assumptions on image
- spatial assumptions on motion

## numerical optimization

- alternate between image reconstruction and motion estimation
- image reconstruction **convex but non-smooth**  
primal-dual ("Chambolle-Pock"), augmented Lagrangian ("ADMM")
- motion estimation difficult, **non-convex, non-smooth**  
multi-resolution schemes (pyramids) with linearizations

# Lava Lamp: Spatio-Temporal Reconstruction

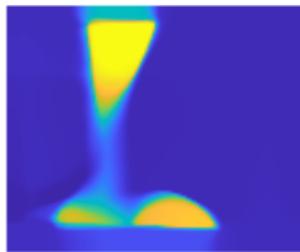


# Lava Lamp: Image and Motion Estimation

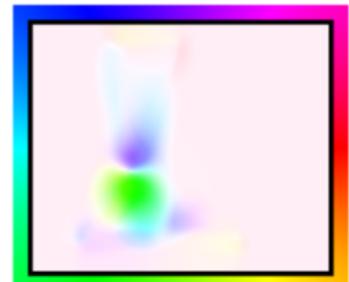
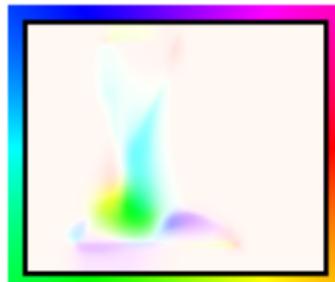
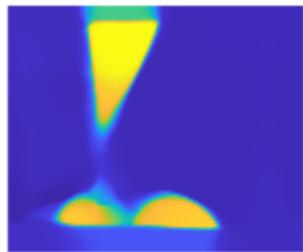


$t = 14340$

linear



non-linear



# Dynamic Compressed Sensing for Photoacoustic Tomography

X maxIP

Y maxIP

Z maxIP

full data, TV-fbf

16x, TV-fbf

16x, TTVL2

- **compressed sensing** data acquisition
- evaluation on experimental phantoms and in-vivo recordings



L, Huynh, Betcke, Zhang, Beard, Cox, Arridge, 2018. Enhancing Compressed Sensing 4D Photoacoustic Tomography by Simultaneous Motion Estimation, *SIAM Journal on Imaging Sciences* 11:4, 2224-2253.

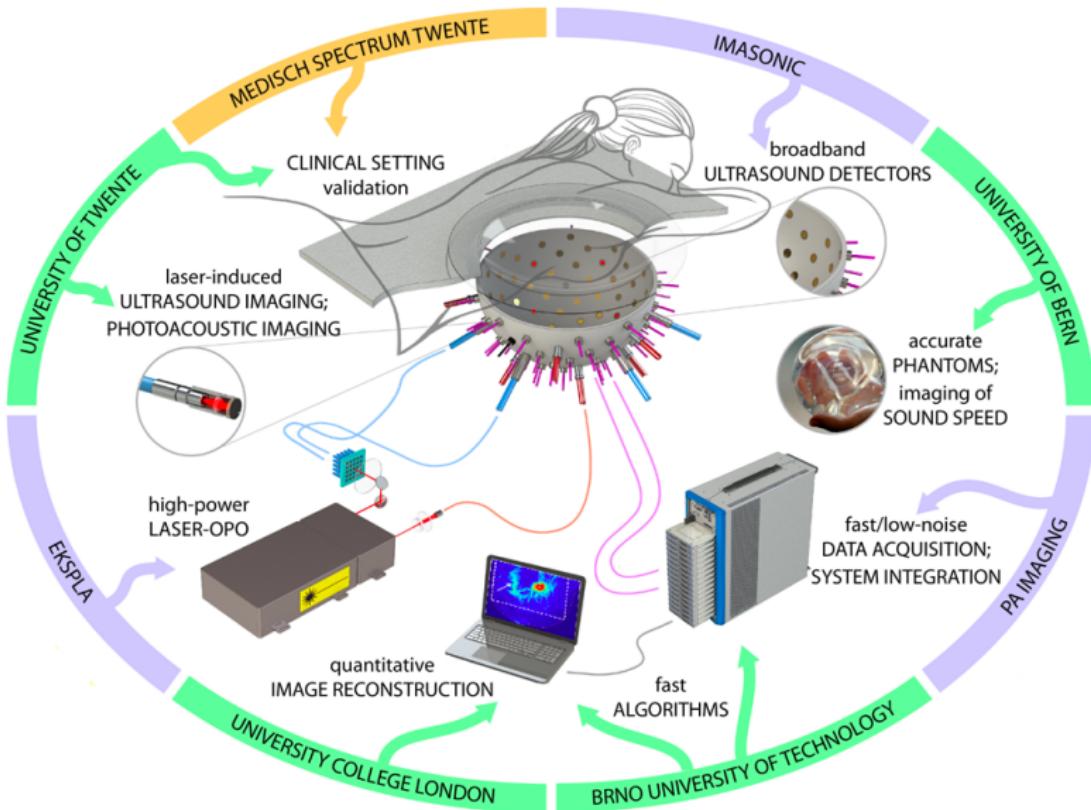
# Dynamic Compressed Sensing via Deep Learning



**Hauptmann, Arridge, L, Muthurangu, Steeden, 2018.** Real-time cardiovascular MR with spatio-temporal artifact suppression using deep learning - proof of concept in congenital heart disease, *Magnetic Resonance in Medicine*.

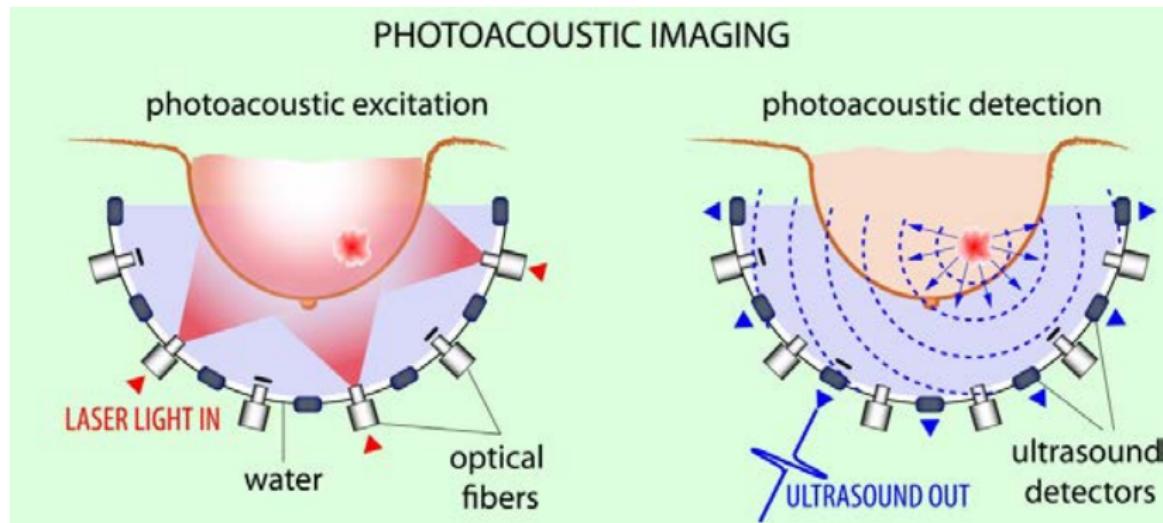
# H2020 Project: Novel Breast Imaging Scanner

## New diagnostic information from optical and acoustic properties

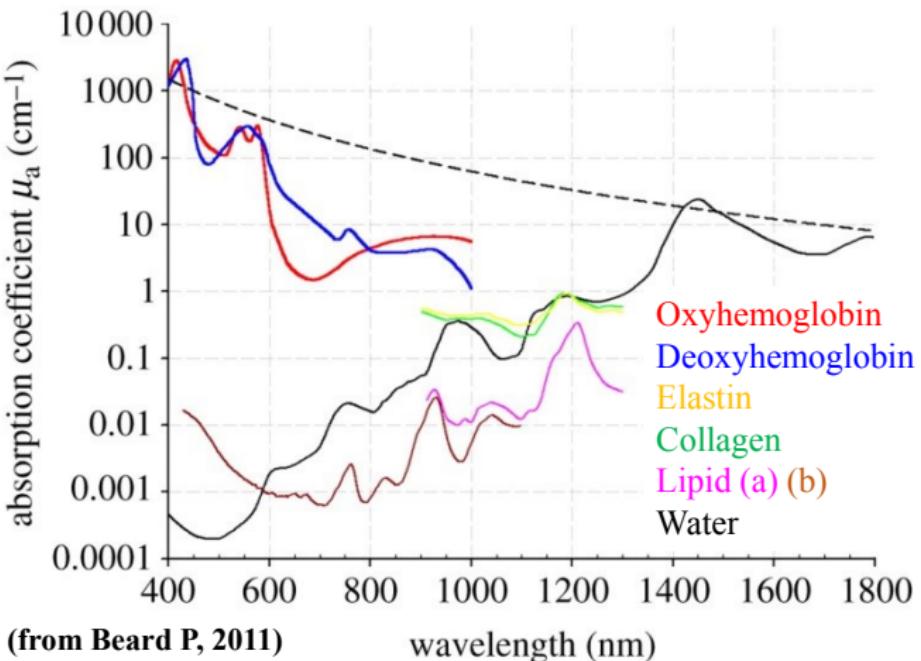


# Novel Modalities: Photoacoustic Tomography (PAT)

- hybrid imaging: "**light in, sound out**"
- quantitative images of optical properties
- **high res + high contrast**
- non-ionizing radiation



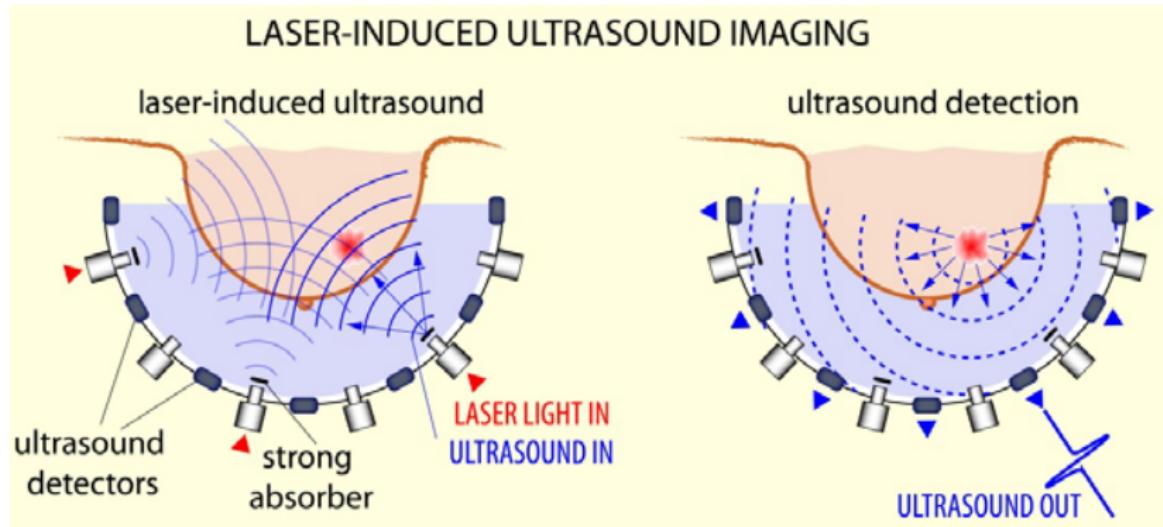
# Photoacoustic Imaging: Spectral Properties



- different wavelengths → **quantitative spectroscopic examinations.**
- gap between oxygenated and deoxygenated blood.

# Novel Modalities: Ultrasound Computed Tomography (USCT)

- "sound in, sound out"
- safe as conventional US
- quantitative images of acoustic properties
- novel diagnostic information



# Mathematical Modelling (simplified)

## Quantitative Photoacoustic Tomography (QPAT)

radiative transfer equation (RTE) + acoustic wave equation

$$(\nu \cdot \nabla + \mu_a(x) + \mu_s(x)) \phi(x, \nu) = q(x, \nu) + \mu_s(x) \int \Theta(\nu, \nu') \phi(x, \nu') d\nu',$$

$$p^{PA}(x, t = 0) = p_0 := \Gamma(x) \mu_a(x) \int \phi(x, \nu) d\nu, \quad \partial_t p^{PA}(x, t = 0) = 0$$

$$(c(x)^{-2} \partial_t^2 - \Delta) p^{PA}(x, t) = 0, \quad f^{PA} = Mp^{PA}$$

## Ultrasound Computed Tomography (USCT)

$$(c(x)^{-2} \partial_t^2 - \Delta) p^{US}(x, t) = s(x, t), \quad f^{US} = Mp^{US}$$

## Step-by-step inversion

1.  $f^{US} \rightarrow c$ : acoustic parameter identification from boundary data.
2.  $f^{PA} \rightarrow p_0$ : acoustic initial value problem with boundary data.
3.  $p_0 \rightarrow \mu_a$ : optical parameter identification from internal data.

# USCT Reconstruction Approaches

**Travel time tomography:** geometrical optics approximation.

- ✓ robust & computationally efficient
- ! valid for high frequencies ( $\rightarrow$  attenuation)
- ! low resolution, lots of data needed

**Reverse time migration:** forward wavefield correlated in time with backward wavefield (adjoint wave equation) via imaging condition

- ✓ 2 wave simulations, better quality
- ! approximation, needs initial guess, quantitative errors

**Full waveform inversion (FWI):** fit full model to all data.

- ✓ high resolution from little data,
- ✓ include transducer model, constraints, regularization
- ! lots of wave simulations
- ! complex numerical optimization, complex implementation
- low TRL but already used in 2D systems

# 3D Time Domain FWI for Breast USCT

## Starting point:



Pérez-Liva, Herraiz, Udías, Miller, Cox, Treeby 2017. Time domain reconstruction of sound speed and attenuation in ultrasound computed tomography using full wave inversion, JASA.

## Challenges and solutions for 3D:

- !  $2 \times N_{sources}$  wave simulations per gradient
- ! computationally & stochastically efficient gradient estimator
- ! memory requirements of gradient computation
- ! slow convergence and local minima
- ! computational resources

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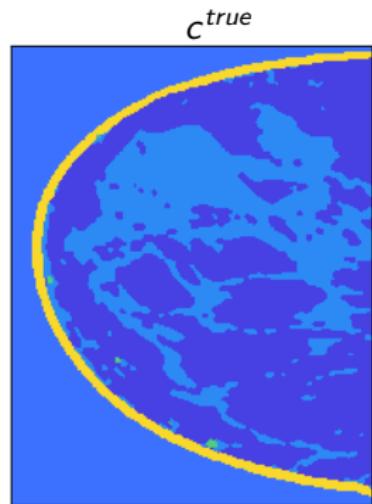
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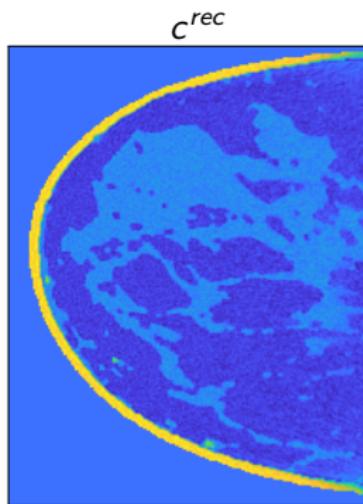
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  - **time-reversal based gradient computation**
- ! slow convergence and local minima
  - **coarse-to-fine multigrid schemes**
- ! computational resources
  - **runs on single GPU, can utilize multiple GPUs**

## Example

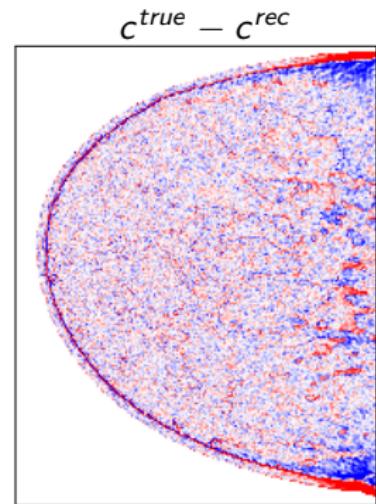
- 3D breast phantom at 0.5mm resolution, 1024 sources and receivers
- $442 \times 442 \times 232$  voxel, 3912 time steps
- multi-grid with 8 levels, coarsening factor  $\sqrt{2}$ .
- SL-BFGS (40 iter, 2d 4h on highest level), source encoding, 2 GPUs



color range 1450 to 1670 m/s



color range -50 to 50 m/s





## Summary

- imaging has broad range of applications
- mathematically: **inverse problem** of reconstructing distributed quantities from indirect observations
- **mathematical modeling**, (solving) **PDEs, numerical optimization**
- **challenges**: large-scale, optimization, uncertainty quantification, compressed sensing, dynamic/spectral imaging
- stable solution requires **a-priori information**
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**Thank you for your attention!**