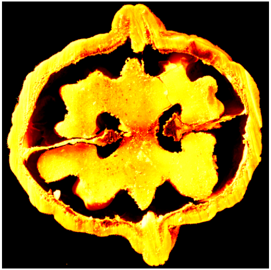
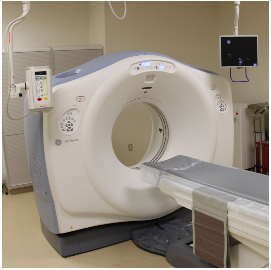
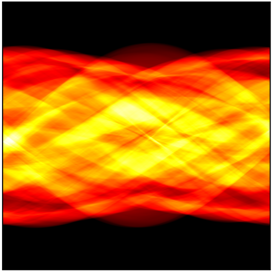
X-Ray Computed Tomography







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Mastermath Course
"Inverse Problems in Imaging"

March 26, 2019

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Centrum Wiskunde & Informatica (CWI)

- National research institute for mathematics and computer science, founded 1946.
- Focus: Fundamental research problems derived from societal needs.
- $\sim$200 people working in 15 research groups on 6 research themes: Software, Data, Networks, Computation, Quantum and Artificial Intelligence
- National and international industry and academic collaborations.
- 24 spin-off companies
- opportunities for MSc and PhD students

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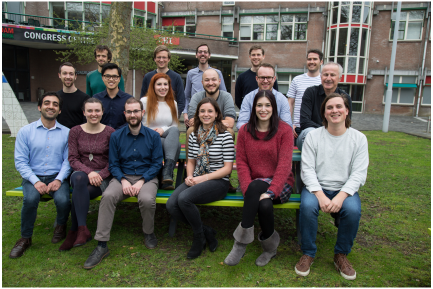
Computational Imaging @ CWI



- headed by Prof. Joost Batenburg (also Uni Leiden), 18 members
- mathematics, computer science & (medical) physics
- advanced computational techniques for 3D imaging
- (inter-)national collaborations from science, industry & medicine
- one of the two main developers of the ASTRA Toolbox
- FleX-ray Lab: custom-made, fully-automated X-ray CT scanner linked to large-scale computing hardware

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History of X-rays

Corresponding video by the ASTRA toolbox team: YouTube

(a) Wilhelm Röntgen (1845-1923)

(b) First X-ray image (1895)

source: Wikimedia Commons

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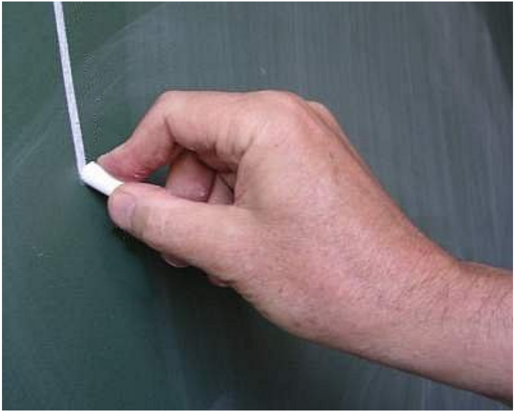
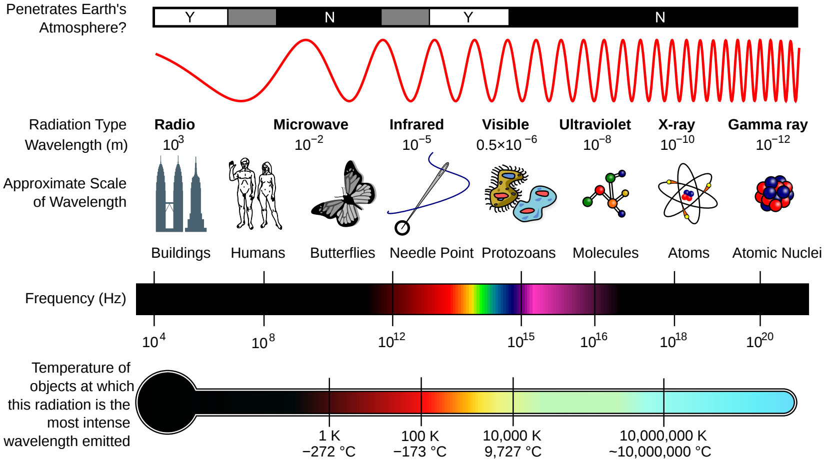
X-ray generation

Corresponding video by the ASTRA toolbox team: YouTube



What are X-rays?



source: Wikimedia Commons

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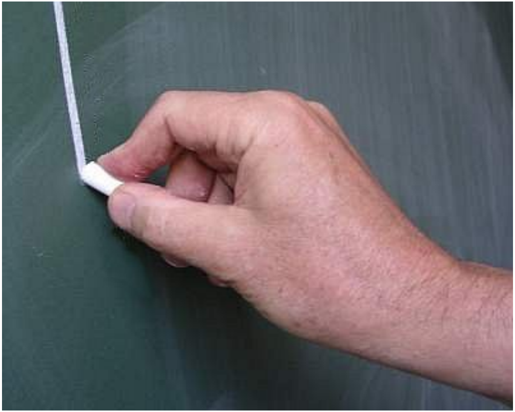
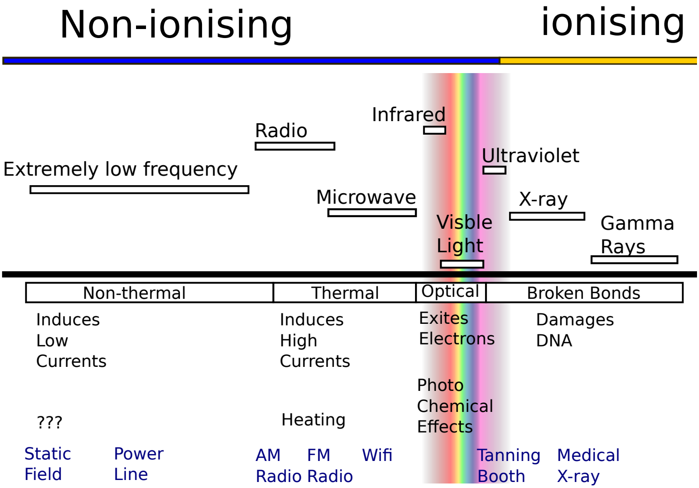
How do X-rays interact with materials?

Corresponding video by the ASTRA toolbox team: YouTube



How do X-rays interact with materials?

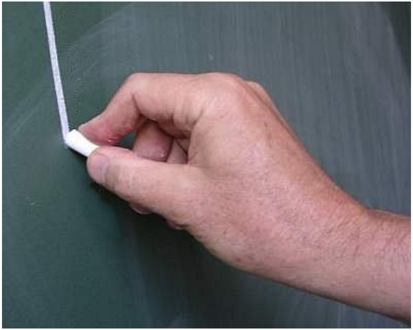


source: Wikimedia Commons

Mathematics of CT 1: Beer's Law

Summary:

- Lambert's law: The intensity of a ray that passes through a homogeneous medium of thickness s is given as $I_1 = I_0 \exp(-\mu s)$, where $\mu \geqslant 0$ is the effective absorption coefficient.
- Beer-Lambert's law: The intensity of a ray that passes through a heterogeneous medium is given by $I_1 = I_0 \exp\left(-\int_l \mu(x) dx\right)$.



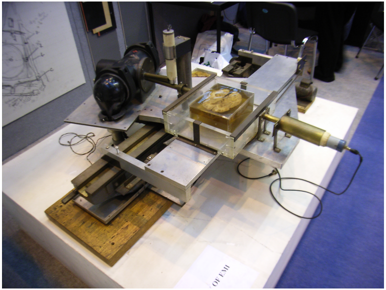
Radiography

An excellent video by Samuli Siltanen: YouTube

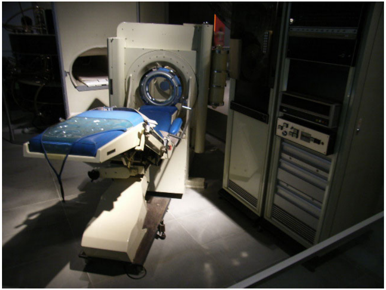
source: Wikimedia Commons

History of Computed Tomography (CT)

Godfrey Hounsfield and Allan McLeod Cormack developed X-ray tomography (Nobelprize 1979)



(c) CT prototype



(d) first comercial CT head scanner

source: Wikimedia Commons

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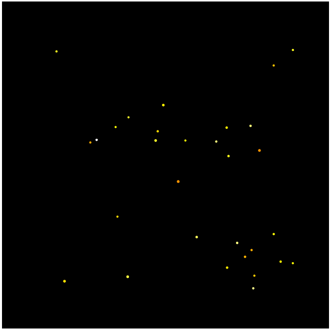
Modern CT Scanner

a video of a scanner during rotation YouTube

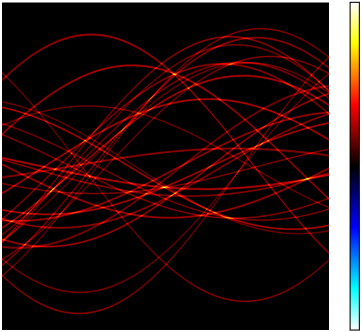
source: Wikimedia Commons

From projections to sinograms

Another excellent video by Samuli Siltanen: YouTube



(a) image



(b) sinogram

From projections to sinograms

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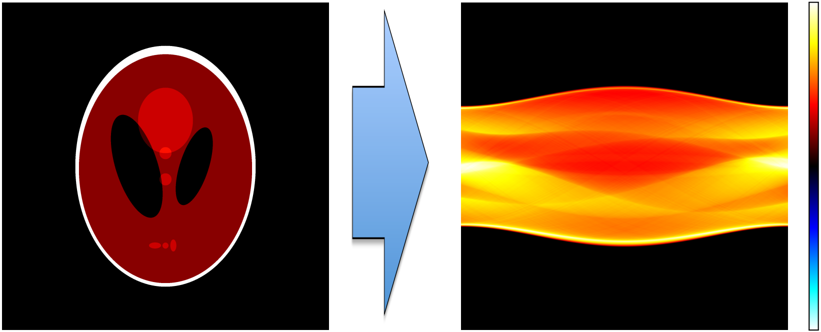
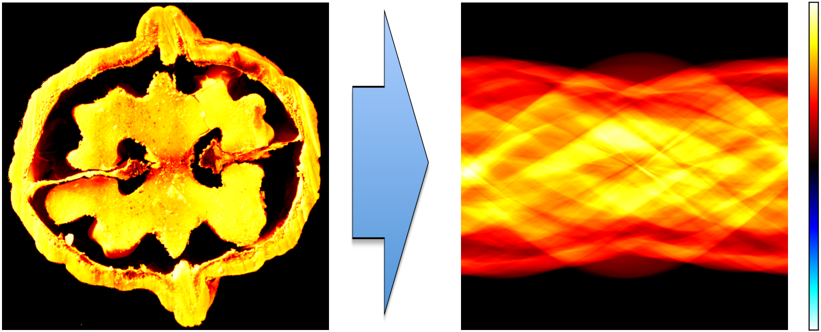


(a) image (b) sinogram

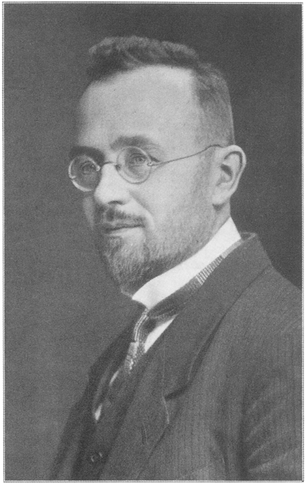
From projections to sinograms

Another excellent video by Samuli Siltanen:



(a) image (b) sinogram

Mathematics of CT 2: The Radon Transform

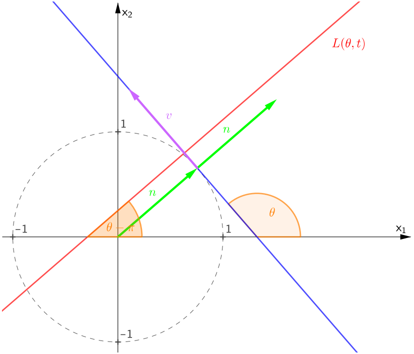


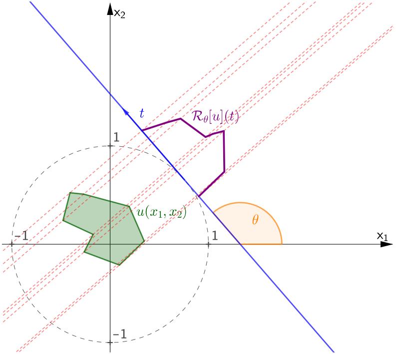
Johann Radon (1887-1956)

Videos by the ASTRA toolbox team:

- Radon transform YouTube
- Fourier slice theorem YouTube, proof YouTube
- Filtered backprojection YouTube

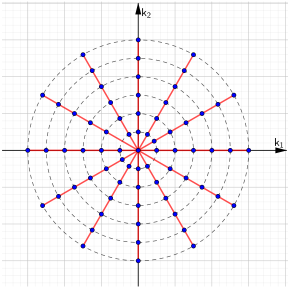
Mathematics of CT 2: Illustration Radon Transform



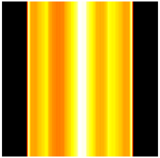


Problems of backprojection

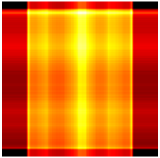
(a) true image



(b) Fourier sampling



(c) 1 angle



(d) 2 angles

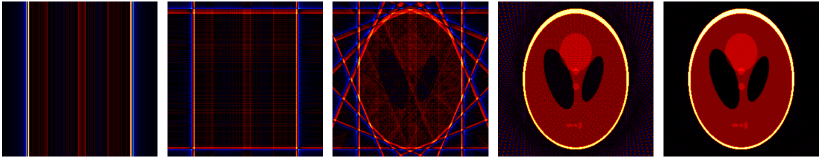
(e) 8 angles

(f) 64 angles

(g) 256 angles

Filtered backprojection

(a) 1 angle (b) 2 angles (c) 8 angles (d) 64 angles (e) 256 angles



(f) 1 angle (g) 2 angles (h) 8 angles (i) 64 angles (j) 256 angles

Just one more video by Samuli Siltanen: YouTube

Lab tour

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CT reconstruction methods

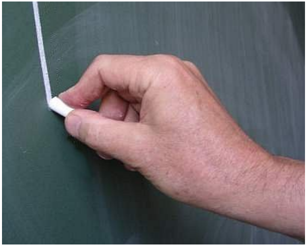
Analytical methods a la filtered backprojection:

- ✓ efficient to implement and execute
- ! lack of flexibility for unconventional scanning set-ups
- ! severe artifacts for limited / sparse projection data
- ! hard to introduce a-priori knowledge

Algebraic and variational methods (iterative methods):

- ! higher computational cost
- ✓ highly flexible, arbitrary geometries
- ✓ less artifacts for limited / sparse projection data
- ✓ introduction of a-priori knowledge possible

Mathematics of CT 3: Algebraic Reconstructions

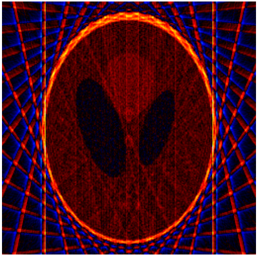


Videos by the ASTRA toolbox team:

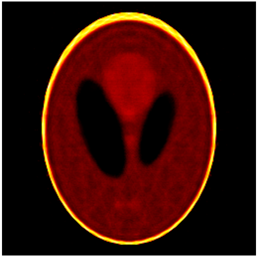
- Intro to algebraic methods YouTube
- Intro to iterative methods YouTube
- SIRT method YouTube
- other methods YouTube
- adding a-priori information YouTube

Iterative methods in action: 15 angles

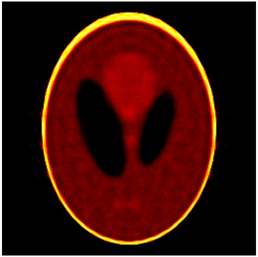
(a) true image



(b) FBP



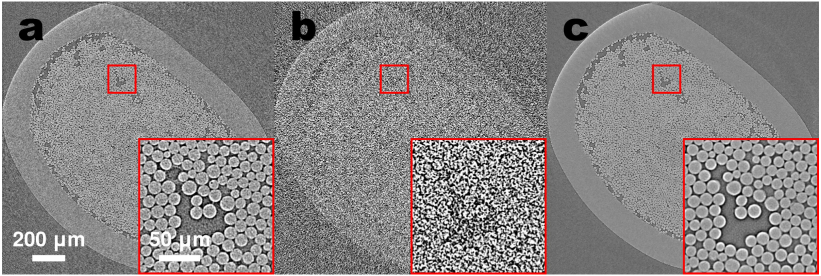
(c) ART



(d) SIRT

(e) TV regularization

Deep learning for low dose CT image reconstruction



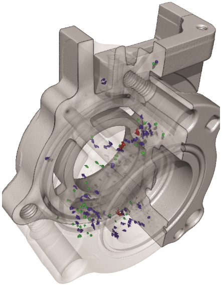
2560x2560 tomography images of fiber composite. Left: 1024 projections, middle/right: 128 projections

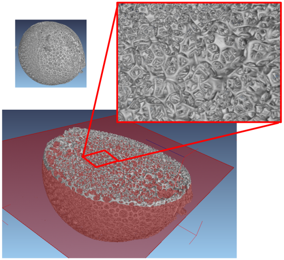
D. Pelt, J.A. Sethian, 2018. Mixed-scale dense network for image analysis, *PNAS 115 (2) 254-259.*

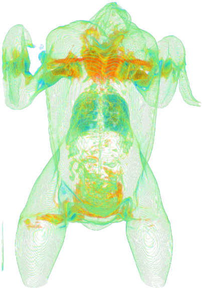
Example applications: Materials science, energy & biology

- Water within porous media YouTube
- Internal structure of Arundo donax YouTube
- Inside live flying insects – in 3D YouTube
- Movie of battery under load YouTube
- Metallic foam YouTube

Example applications: Industry & security







Airport baggage screening YouTube

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Some current developments

- Phase contrast X-ray imaging: Exploit phase shift in X-rays caused by material interaction to gain higher soft tissue contrast.
- Dynamic X-ray: Track fast dynamic processes in 3D (4D CT).
- Spectral CT: Use energy resolved detectors to improve analysis of complex materials and tissues.
- Scan adaptation: Make best possible use of given budget of radiation.
- Machine learning: Use deep learning to improve image reconstruction and analysis.

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Further reading

- **T. M. Buzug, 2008.** Computed Tomography - From Photon Statistics to Modern Cone-Beam CT, *Springer-Verlag Berlin Heidelberg.*
- **G. T. Herman, 2009.** Fundamentals of Computerized Tomography Image Reconstruction from Projections, *Springer-Verlag London.*
- **F. Natterer, 2001.** The Mathematics of Computerized Tomography, *Society for Industrial and Applied Mathematics.*