Name: _____ Multiple Choice #1-28 No calculator.

Scantron: Blue Side Key ID: A

1

$$\int_2^x \left(3t^2 - 1\right) dt =$$

(A) $x^3 - x - 6$ (B) $x^3 - x$ (C) $3x^2 - 12$ (D) $3x^2 - 1$ (E) $6x - 12$

2

What is the slope of the line tangent to the graph of $y = \ln(2x)$ at the point where $x = 4$?

(A) $\frac{1}{8}$ (B) $\frac{1}{4}$ (C) $\frac{1}{2}$ (D) $\frac{3}{4}$ (E) 4

3

If $f(x) = 4x^{-2} + \frac{1}{4}x^2 + 4$, then $f'(2) =$

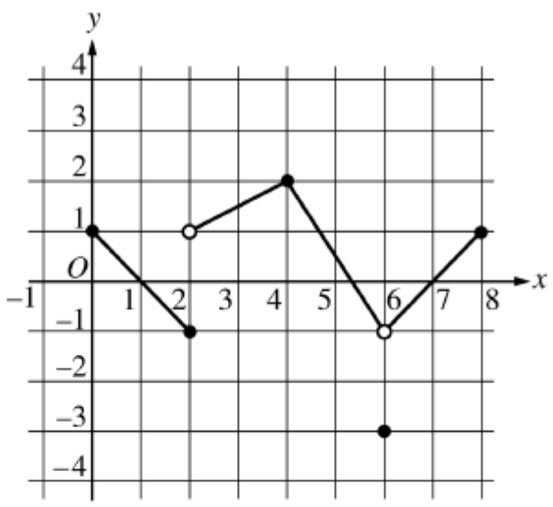
(A) -62 (B) -58 (C) -3 (D) 0 (E) 1

4

$$\int_1^2 \frac{dx}{2x+1} =$$

(A) $2\ln 2$ (B) $\frac{1}{2}\ln 2$ (C) $2(\ln 5 - \ln 3)$ (D) $\ln 5 - \ln 3$ (E) $\frac{1}{2}(\ln 5 - \ln 3)$

5



The figure above shows the graph of the function f. Which of the following statements are true?

I. $\lim_{x \to 2^-} f(x) = f(2)$

II. $\lim_{x \to 6^-} f(x) = \lim_{x \to 6^+} f(x)$

III. $\lim_{x \to 6} f(x) = f(6)$

(A) II only
(B) III only
(C) I and II only
(D) II and III only
(E) I, II, and III

6

$$\frac{d}{dx}\left(\sin^3\left(x^2\right)\right) =$$

(A) $\cos^3\left(x^2\right)$

(B) $3\sin^2\left(x^2\right)$

(C) $6x\sin^2\left(x^2\right)$

(D) $3\sin^2\left(x^2\right)\cos\left(x^2\right)$

(E) $6x\sin^2\left(x^2\right)\cos\left(x^2\right)$

7

$$\lim_{x\to\infty}\frac{x^3}{e^{3x}} \text{ is}$$

(A) 0 (B) $\frac{2}{9}$ (C) $\frac{2}{3}$ (D) 1 (E) infinite

8

Using the substitution $u = \sin(2x)$, $\int_{\pi/6}^{\pi/2}\sin^5(2x)\cos(2x)\,dx$ is equivalent to

(A) $-2\int_{1/2}^{1}u^5\,du$

(B) $\frac{1}{2}\int_{1/2}^{1}u^5\,du$

(C) $\frac{1}{2}\int_{0}^{\sqrt{3}/2}u^5\,du$

(D) $\frac{1}{2}\int_{\sqrt{3}/2}^{0}u^5\,du$

(E) $2\int_{\sqrt{3}/2}^{0}u^5\,du$

9

The function f has a first derivative given by $f'(x) = x(x-3)^2(x+1)$. At what values of x does f have a relative maximum?

(A) -1 only (B) 0 only (C) -1 and 0 only (D) -1 and 3 only (E) -1, 0, and 3

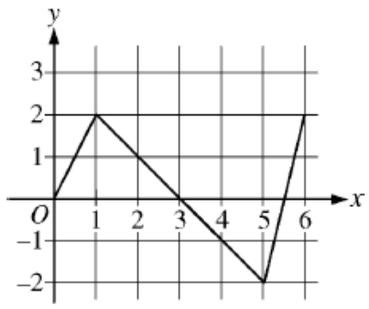
10

$$f(x) = \begin{cases} \dfrac{x^2-7x+10}{b(x-2)} & \text{for } x \neq 2 \\ b & \text{for } x = 2 \end{cases}$$

Let f be the function defined above. For what value of b is f continuous at $x = 2$?

(A) -3 (B) $\sqrt{2}$ (C) 3 (D) 5 (E) There is no such value of b.

11



Graph of f'

For $0 \le x \le 6$, the graph of f', the derivative of f, is piecewise linear as shown above. If $f(0) = 1$, what is the maximum value of f on the interval?

(A) 1 (B) 1.5 (C) 2 (D) 4 (E) 6

12

Let f be the function given by $f(x) = 9^x$. If four subintervals of equal length are used, what is the value of the right Riemann sum approximation for $\int_0^2 f(x)\,dx$?

(A) 20 (B) 40 (C) 60 (D) 80 (E) 120

13

$\dfrac{d}{dx}\left(\dfrac{x+1}{x^2+1}\right) =$ (B) $\dfrac{-x^2-2x+1}{x^2+1}$ (D) $\dfrac{3x^2+2x+1}{\left(x^2+1\right)^2}$

(A) $\dfrac{x^2+2x-1}{\left(x^2+1\right)^2}$ (C) $\dfrac{-x^2-2x+1}{\left(x^2+1\right)^2}$ (E) $\dfrac{1}{2x}$

14

The velocity of a particle moving along the x-axis is given by $v(t) = \sin(2t)$ at time t. If the particle is at $x = 4$ when $t = 0$, what is the position of the particle when $t = \dfrac{\pi}{2}$?

(A) 2 (B) 3 (C) 4 (D) 5 (E) 6

15

The function $y = g(x)$ is differentiable and increasing for all real numbers. On what intervals is the function $y = g\left(x^3 - 6x^2\right)$ increasing?

(A) $(-\infty, 0]$ and $[4, \infty)$ only

(B) $[0, 4]$ only

(C) $[2, \infty)$ only

(D) $[6, \infty)$ only

(E) $(-\infty, \infty)$

16

$\lim\limits_{x \to 3^-} \dfrac{|x-3|}{x-3}$ is

(A) -3 (B) -1 (C) 1 (D) 3 (E) nonexistent

17

If $f(x) = ae^{-ax}$ for $a > 0$, then $f'(x) =$

(A) e^{-ax}

(B) ae^{-ax}

(C) a^2e^{-ax}

(D) $-ae^{-ax}$

(E) $-a^2e^{-ax}$

18

A student attempted to solve the differential equation $\frac{dy}{dx} = xy$ with initial condition $y = 2$ when $x = 0$. In which step, if any, does an error first appear?

Step 1: $\int \frac{1}{y}\,dy = \int x\,dx$

Step 2: $\ln|y| = \frac{x^2}{2} + C$

Step 3: $|y| = e^{x^2/2} + C$

Step 4: Since $y = 2$ when $x = 0$, $2 = e^0 + C$.

Step 5: $y = e^{x^2/2} + 1$

(A) Step 2
(B) Step 3
(C) Step 4
(D) Step 5
(E) There is no error in the solution.

19

. For what values of x does the graph of $y = 3x^5 + 10x^4$ have a point of inflection?

(A) $x = -\frac{8}{3}$ only

(B) $x = -2$ only

(C) $x = 0$ only

(D) $x = 0$ and $x = -\frac{8}{3}$

(E) $x = 0$ and $x = -2$

20

. $\lim_{x \to 2} \frac{\ln(x+3) - \ln(5)}{x - 2}$ is

(A) 0 (B) $\frac{1}{5}$ (C) $\frac{1}{2}$ (D) 1 (E) nonexistent

21

Functions w, x, and y are differentiable with respect to time and are related by the equation $w = x^2y$. If x is decreasing at a constant rate of 1 unit per minute and y is increasing at a constant rate of 4 units per minute, at what rate is w changing with respect to time when $x = 6$ and $y = 20$?

(A) −384 (B) −240 (C) −96 (D) 276 (E) 384

22

. Let f be the function defined by $f(x) = 2x^3 - 3x^2 - 12x + 18$. On which of the following intervals is the graph of f both decreasing and concave up?

(A) $(-\infty, -1)$ (B) $\left(-1, \frac{1}{2}\right)$ (C) $(-1, 2)$ (D) $\left(\frac{1}{2}, 2\right)$ (E) $(2, \infty)$

23

$$f(x) = \begin{cases} 3x + 5 & \text{when } x < -1 \\ -x^2 + 3 & \text{when } x \geq -1 \end{cases}$$

If f is the function defined above, then $f'(-1)$ is

(A) −3 (B) −2 (C) 2 (D) 3 (E) nonexistent

24

Let f be the function defined by $f(x) = \frac{(3x+8)(5-4x)}{(2x+1)^2}$. Which of the following is a horizontal asymptote to the graph of f?

(A) $y = -6$

(B) $y = -3$

(C) $y = -\frac{1}{2}$

(D) $y = 0$

(E) $y = \frac{3}{2}$

25

If $y = x^2 - 2x$ and $u = 2x + 1$, then $\frac{dy}{du} =$

(A) $\frac{2(x^2 + x - 1)}{(2x + 1)^2}$ (B) $6x^2 - 3x - 2$ (C) $4x$ (D) $x - 1$ (E) $\frac{1}{x - 1}$

26

For $x > 0$, $\frac{d}{dx}\int_1^{\sqrt{x}} \frac{1}{1 + t^2} dt =$

(A) $\frac{1}{2\sqrt{x}(1 + x)}$ (B) $\frac{1}{2\sqrt{x}(1 + \sqrt{x})}$ (C) $\frac{1}{1 + x}$ (D) $\frac{\sqrt{x}}{1 + x}$ (E) $\frac{1}{1 + \sqrt{x}}$

27

A particle moves on the x-axis so that at any time t, $0 \le t \le 1$, its position is given by $x(t) = \sin(2\pi t) + 2\pi t$. For what value of t is the particle at rest?

(A) 0 (B) $\frac{1}{8}$ (C) $\frac{1}{4}$ (D) $\frac{1}{2}$ (E) 1

28

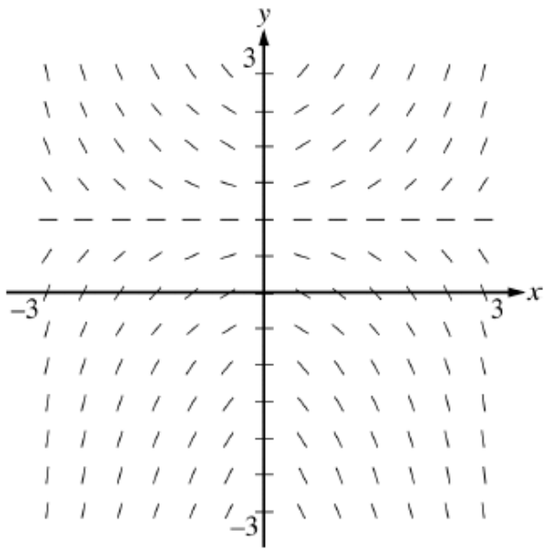
(A) $\frac{dy}{dx} = xy - x$

(B) $\frac{dy}{dx} = xy + x$

(C) $\frac{dy}{dx} = y - x^2$

(D) $\frac{dy}{dx} = (y - 1)x^2$

(E) $\frac{dy}{dx} = (y - 1)^3$



Shown above is a slope field for which of the following differential equations?

Name: _____ Multiple Choice #1-17 Calculator Required.

Scantron: Green Side Key ID: A

1

A particle moves along a straight line so that at time $t > 0$ the position of the particle is given by $s(t)$, the velocity is given by $v(t)$, and the acceleration is given by $a(t)$. Which of the following expressions gives the average velocity of the particle on the interval $[2, 8]$?

(A) $\frac{1}{6}\int_2^8 a(t)\,dt$

(B) $\frac{1}{6}\int_2^8 s(t)\,dt$

(C) $\frac{s(8) - s(2)}{6}$

(D) $\frac{v(8) - v(2)}{6}$

(E) $v(8) - v(2)$

2

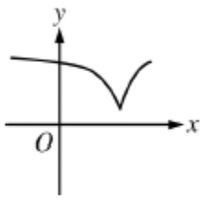
If $\sin\left(\frac{1}{x^2+1}\right)$ is an antiderivative for $f(x)$, then $\int_1^2 f(x)dx =$

(A) -0.281 (B) -0.102 (C) 0.102 (D) 0.260 (E) 0.282

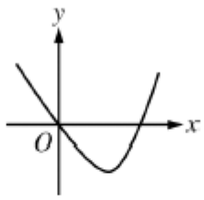
3

The function f is differentiable and increasing for all real numbers x, and the graph of f has exactly one point of inflection. Of the following, which could be the graph of f', the derivative of f?

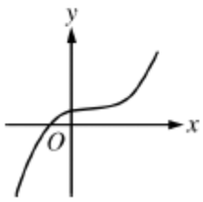
(A)



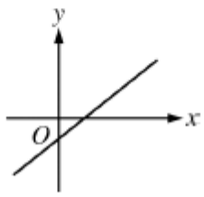
(B)



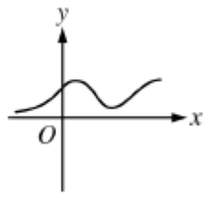
(C)



(D)



(E)



4

A vase has the shape obtained by revolving the curve $y = 2 + \sin x$ from $x = 0$ to $x = 5$ about the x-axis, where x and y are measured in inches. What is the volume, in cubic inches, of the vase?

(A) 10.716 (B) 25.501 (C) 33.666 (D) 71.113 (E) 80.115

5

x	$f(x)$
1	2.4
3	3.6
5	5.4

The table above gives selected values of a function f. The function is twice differentiable with $f''(x) > 0$. Which of the following could be the value of $f'(3)$?

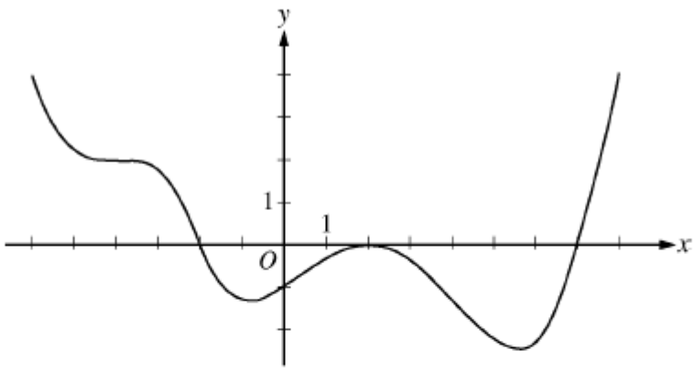
(A) 0.6 (B) 0.7 (C) 0.9 (D) 1.2 (E) 1.5

6

At time $t = 0$ years, a forest preserve has a population of 1500 deer. If the rate of growth of the population is modeled by $R(t) = 2000e^{0.23t}$ deer per year, what is the population at time $t = 3$?

(A) 3987 (B) 5487 (C) 8641 (D) 10,141 (E) 12,628

7



Graph of f'

. The figure above shows the graph of f', the derivative of function f, for $-6 < x < 8$. Of the following, which best describes the graph of f on the same interval?

(A) 1 relative minimum, 1 relative maximum, and 3 points of inflection

(B) 1 relative minimum, 1 relative maximum, and 4 points of inflection

(C) 2 relative minima, 1 relative maximum, and 2 points of inflection

(D) 2 relative minima, 1 relative maximum, and 4 points of inflection

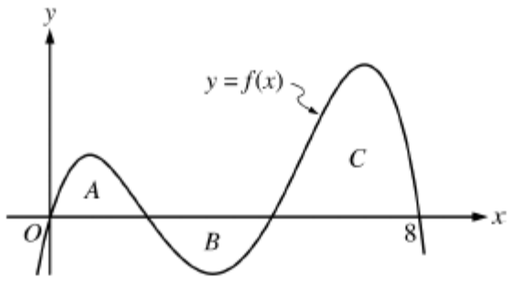
(E) 2 relative minima, 2 relative maxima, and 3 points of inflection

8

Let f and g be continuous functions such that $\int_0^6 f(x)\,dx = 9$, $\int_3^6 f(x)\,dx = 5$, and $\int_3^0 g(x)\,dx = -7$. What is the value of $\int_0^3 \left(\frac{1}{2}f(x) - 3g(x)\right)dx$?

(A) -23 (B) -19 (C) $-\frac{17}{2}$ (D) 19 (E) 23

9



. The regions A, B, and C in the figure above are bounded by the graph of the function f and the x-axis. The area of region A is 14, the area of region B is 16, and the area of region C is 50. What is the average value of f on the interval $[0, 8]$?

(A) 6 (B) 10 (C) $\frac{40}{3}$ (D) $\frac{80}{3}$ (E) 48

10

. A particle moves along the x-axis so that its velocity at time $t \geq 0$ is given by $v(t) = \frac{t^2 - 1}{t^2 + 1}$. What is the total distance traveled by the particle from $t = 0$ to $t = 2$?

(A) 0.214 (B) 0.320 (C) 0.600 (D) 0.927 (E) 1.600

11

. Line ℓ is tangent to the graph of $y = e^x$ at the point $\left(k, e^k\right)$. What is the positive value of k for which the y-intercept of ℓ is $\frac{1}{2}$?

(A) 0.405

(B) 0.768

(C) 1.500

(D) 1.560

(E) There is no such value of k.

12. A differentiable function f has the property that $f'(x) \le 3$ for $1 \le x \le 8$ and $f(5) = 6$. Which of the following could be true?

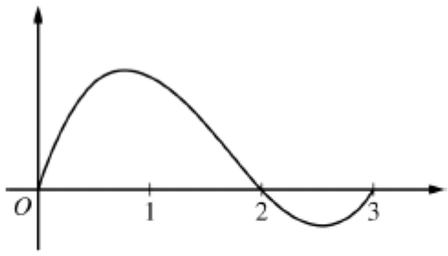
I. $f(2) = 0$

II. $f(6) = -2$

III. $f(7) = 13$

(A) I only
(B) II only
(C) I and II only
(D) I and III only
(E) II and III only

13.



Graph of f

The graph of the differentiable function f is shown in the figure above. Let h be the function defined by $h(x) = \int_0^x f(t)\,dt$. Which of the following correctly orders $h(2)$, $h'(2)$, and $h''(2)$?

(A) $h(2) < h'(2) < h''(2)$
(B) $h'(2) < h(2) < h''(2)$
(C) $h'(2) < h''(2) < h(2)$
(D) $h''(2) < h(2) < h'(2)$
(E) $h''(2) < h'(2) < h(2)$

14. What is the area of the region enclosed by the graphs of $y = e^x - 2$, $y = \sin x$, and $x = 0$?

(A) 0.239 (B) 0.506 (C) 0.745 (D) 2.340 (E) 3.472

15. A particle moves along a line so that its velocity is given by $v(t) = -t^3 + 2t^2 + 2^{-t}$ for $t \ge 0$. For what values of t is the speed of the particle increasing?

(A) $(0, 0.177)$ and $(1.256, \infty)$
(B) $(0, 1.256)$ only
(C) $(0, 2.057)$ only
(D) $(0.177, 1.256)$ only
(E) $(0.177, 1.256)$ and $(2.057, \infty)$

16. Let F be a function defined for all real numbers x such that $F'(x) > 0$ and $F''(x) > 0$. Which of the following could be a table of values for F?

(A)

x	$F(x)$
1	–3
2	–4
3	–6
4	–9

(B)

x	$F(x)$
1	–3
2	–1
3	3
4	19

(C)

x	$F(x)$
1	–3
2	0
3	3
4	6

(D)

x	$F(x)$
1	–3
2	5
3	11
4	13

(E)

x	$F(x)$
1	–3
2	–4
3	–3
4	0

17.

x	$f(x)$	$g(x)$	$f'(x)$
–4	0	–9	5
–2	4	–7	4
0	6	–4	2
2	7	–3	1
4	10	–2	3

The table above gives values of the differentiable functions f and g, and f', the derivative of f, at selected values of x. If $g(x) = f^{-1}(x)$, what is the value of $g'(4)$?

(A) $-\frac{1}{3}$ (B $-\frac{1}{4}$ (C) $-\frac{3}{100}$ (D $\frac{1}{4}$ (E) $\frac{1}{3}$