XI (ALAN) ZHANG, V. KUMAR, and KORAY COSGUNER*

Although email marketing is highly profitable and widely used by marketers, it has received limited attention in the marketing literature. Extant research has focused on either customers' email responses or the "average" effect of emails on purchases. In this article, the authors use data from a U.S. home improvement retailer to study customers' email open and purchase behaviors by using a unified hidden Markov and copula framework. Contrary to conventional wisdom, the authors find that email-active customers are not necessarily active in purchases, and vice versa. Furthermore, the number of emails sent by the retailer has a nonlinear effect on both the retailer's short- and long-term profitability. Through a counterfactual study, the authors provide a decision support system to guide retailers in making optimal email contact decisions. This study shows that sending the right number of emails is vital for long-term profitability. For example, sending four (ten) emails instead of the optimal number of seven emails can cause the retailer to lose 32% (16%) of its lifetime profit per customer.

Keywords: email marketing, hidden Markov models, dynamic programming, copula models, profitability

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Dynamically Managing a Profitable Email Marketing Program

Email marketing is a widely used marketing tool by most business-to-business (B2B) and business-to-customer (B2C) companies. A survey study by Ascend2 (2016) shows that 52% of B2B and B2C companies use email technology as part of their marketing strategies. In addition to its widespread usage, email marketing can be quite profitable. According to a recent study published by the Direct Marketing Association (UK) Ltd (2015), the average revenue-based return on investment (ROI) of email marketing increased to £38 for each £1 spent compared with the average ROI of £24.93 reported in 2013. In the same study, approximately one in every five companies reported an ROI of more than £70, which is three times higher than the figures from 2013. Without a doubt, firms aim to generate higher ROI levels by launching effective email marketing programs. Although this goal is evident, firms may find it difficult to manage effective email programs for multiple reasons.

First, customers who are active in opening a firm's emails may not be active in purchasing from the firm, or vice versa. For example, some heavy buyers may not open the firm's emails frequently because they already know the firm and its offerings quite well. This implies that the firm may not need to target such customers aggressively through email marketing, because this type of buyer may pay little to no attention to emails received. However, some light buyers might be quite responsive in opening the firm's emails because they gather information through emails. Even though this type of customer opens emails more frequently, from the profit perspective, targeting such customers might be suboptimal for the firm because the chances of converting them into repeat buyers are likely low.

Second, sending the right number of emails is critical for the firm's profitability, especially because most customers tend to complain about the large number of emails firms send. A survey study by BlueHornet (2013), a marketing firm focusing

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on email solutions, finds that email frequency might overtake content irrelevance as an important factor driving customers away from email marketing. Although sending the optimal number of emails is paramount, finding that magic number is very challenging for a firm because not only do its customers have different intrinsic preferences for emails (i.e., email open[1] frequencies differ across customers), but customer preferences might change dynamically over time (i.e., an individual customer's email open frequencies might differ over time). In other words, the optimal number of emails to send may differ across customers and over time. Consistent with this argument, a study by Return Path (2015), an industry expert on email optimization, suggests that email frequency optimization should depend on customers' engagement level. However, the study does not provide an actionable tool for firms to adopt to launch effective email marketing programs.

Third, managers tend to treat each email marketing campaign as an independent solicitation process and fail to consider its long-term impact on customers' email open and purchase behaviors. In addition to immediate impact, an email might also affect the future profitability of the firm by shifting the customer–firm relationship level (Luo and Kumar 2013). For example, customers opening emails might become more interested with the firm and its offerings, which might prompt them to purchase more frequently. In this case, ignoring emails' long-term effects might cause the firm to make suboptimal email marketing decisions.

Because of all these challenges, firms need guidance in the form of a decision support system (DSS) to determine the optimal number of emails to send to different customers over time. In this study, our objective is to provide such a DSS to firms through (1) linking customers' email open behaviors with their purchase behaviors, (2) capturing heterogeneity in email open and purchase behaviors across customers and over time, and (3) incorporating the long-term effects of email marketing on firm profitability.

Methodologically, we propose a unified framework combining hidden Markov (HMM) (Netzer, Lattin, and Srinivasan 2008) and copula (Danaher and Smith 2011) models. In the hidden Markov component of our model, we capture the dynamics in customers' purchase and email behaviors by allowing the latent relationship states to govern both behaviors. As we discuss subsequently, jointly modeling the evolution of purchase and email states is important in the context of email marketing because customers' responsiveness in opening emails might also affect their relationship with the firm. In this regard, our study differs from the other studies in the HMM domain that have focused only on the customer's purchase behavior and ignored the possibility that other nonpurchase activities may also affect the customer–firm relationship (except Schweidel, Park, and Jamal 2014) level. In addition, customers' email open and purchase behaviors might be correlated because of time-invariant factors such as customer unobserved heterogeneity and time-variant factors such as the unobserved customer–firm relationship states. In the copula component of our model, we use a bivariate Frank copula to capture that correlation. We model the email open behavior using a binomial distribution (BD) and the purchase behavior using a zero-inflated negative binomial distribution (ZINBD). In this regard, the copula component of our model also differs from most copula applications in marketing, which have used copulas to correlate variables coming from continuous distribution families (except Stephen and Galak (2012), wherein double Poisson variables are first converted into continuous ones and treated as continuous in the estimation).

Substantively, we aim to answer the following questions:

1. How do the latent Markov states govern customers' purchase and email open behaviors?
2. Is there any correlation between customers' email open and purchase behaviors?
3. How do email contacts (i.e., number of emails sent by a firm) affect customers' purchase behavior in both the short and long runs? and
4. What is the optimal number of emails to send to different customers over time to maximize the long-term firm profit?

The empirical study identifies three customer–firm relationship states governing customers' and email open and purchase behaviors: low open/medium purchase, high open/low purchase, and high open/high purchase states. In other words, email-active customers may not be active in purchases; and email-inactive customers may still be relatively active in their purchase behaviors. In addition, we find that the firm's email contact policy has nonlinear short- and long-term effects on customers' purchase and email open behaviors as well as on the retailer's profitability. Furthermore, from our copula component, we identify a positive correlation between the two customer behaviors. We next derive a dynamic email marketing resource-allocation policy using the estimates from our proposed HMM and copula model of customer purchase and email open behaviors. This way, we provide a DSS to the firm regarding the optimal number of emails to send to maximize its lifetime profit. Drawing on our DSS, we find that the optimal number of emails to send significantly differs not only across customers but also within the same customer over time. In addition, we find that sending the right number of emails is very critical from the firm's profit perspective. For instance, sending four (ten) emails rather than the optimal number of seven emails per month causes the retailer to lose 32% (16%) of its lifetime profit.

To reiterate our contribution, to the best of our knowledge, this is the first empirical study to jointly model customers' email open and purchase behaviors over time. Methodologically, our article is the first study to combine the HMM and copula approaches in a unified framework. Therefore, this research provides important implications for firms to understand customers' behavioral attitudes toward email marketing as well as an easy-to-implement modeling framework to marketing researchers in the field. The proposed framework is flexible, and marketing researchers in the field can use it to study other nonpurchase customer behaviors (including customers' responses to other electronic and potentially interactive communications) that are possibly correlated with purchase. The framework can incorporate unobserved behavioral heterogeneity across customers and over time. Finally, through our DSS, we provide substantive managerial guidelines to firms to implement an effective email-marketing program. Similarly, our suggested DSS is also versatile, and firms can use it to manage marketing decisions such as direct mailings, telephone calls, and salesperson contacts.

In the following sections, we first review the literature on email marketing and customer relationship dynamics in email marketing. Second, we describe our data and present descriptive statistics. Third, we discuss our modeling framework. Fourth, we present the

[1]For this study, we define "email open" as the action of opening an email message.

empirical results. Fifth, we derive the optimal email marketing contact policy for the firm. Sixth, we discuss the robustness of our proposed model. Finally, we summarize our findings and conclude with caveats and directions for future research.

LITERATURE REVIEW

Email Marketing

There are several reasons for email marketing's popularity. First, emails enable marketers to send messages to their customers at very low cost. Chittenden and Rettie (2003) demonstrate that the total cost for acquisition and retention campaigns using email is $26,500 per 5,000 customers, as compared with that of direct mail campaigns, at $69,600 per 5,000 customers. Second, email messaging requires less preparation and execution time. Industry practice has shown that an email marketing campaign targeting 50,000 customers needs only six hours to prepare and run, whereas a similarly sized direct mail campaign needs 17 days before it can reach a targeted customer's mailbox. Third, emails typically generate faster responses and create an opportunity for interactive communication with customers. For example, customers can respond to a email the moment they receive it by clicking the hyperlinks that direct them to the sender firm's website through their computer or mobile device.

In this study, we focus on permission-based email marketing. Permission-based email marketing requires marketers to seek the customers' permission before sending them email messages (Godin 1999; Kumar, Zhang, and Luo 2014). This type of email marketing intends to maintain a repeat purchase relationship with customers rather than getting customers to buy only once. In line with this idea, previous research has shown that email marketing has a positive effect on customer loyalty. Tezinde, Smith, and Murphy (2002) discover that email advertisements are valuable to firms by inducing customers to visit the physical store. Merisavo and Raulas (2004) find that email marketing can enhance customer attitudinal loyalty toward the brand. Their study shows that customers tend to recommend the email messages to their friends if they find the messages interesting and useful.

Although the overall influence of email marketing is positive, we argue that researchers and practitioners should examine the customers' response to email-based marketing messages through two perspectives. First, customers may open and read an email simply to keep track of the firm's products and offerings. This behavior does not necessarily indicate that they are actively looking for information to assist their purchase decisions. Bonfrer and Dréze (2009) study a series of email marketing campaigns and propose a bivariate hazard model to predict when customers open or click an email. Kumar, Zhang, and Luo (2014) examine the total number of emails that are opened and clicked and investigate their impact on the time the customers subscribe to the email program. Second, and more importantly, customers may make purchases because of email marketing messages. Sahni, Zou, and Chintagunta (2016) analyze 70 randomized field experiments and find that email promotions not only increase customers' average purchase spending during the promotion window but also carry over to the week after the promotion expires. Kumar, Zhang, and Luo find that the average email open rate has a positive effect on average purchase spending, whereas the effect of the average email click rate is not significant.

Although studies have considered each of these two perspectives—email open and purchase—separately, it is surprising that no study has investigated both behaviors together. Bonfrer and Dréze (2009) did not consider the possible link between email open/click rate and purchase behavior because of the limitation of their data. Kumar, Zhang, and Luo (2014) captured the "average" effect of the customer's response to emails on purchase, but they did not consider the dynamics and heterogeneity in both email open and purchase behaviors. Sahni, Zou, and Chintagunta (2016) conducted a post hoc analysis of the experiments to show the aggregate-level effects of emails on customer expenditure, but they did not quantify how email open behavior affects customer purchase, nor did they consider the dynamic effects.

Customer Relationship in Email Marketing

Previous research has shown that the customers' relationship with the firm evolves over their lifetime. Netzer, Lattin, and Srinivasan (2008) propose an HMM to study the customer transitions among their latent relationship states with the firm. Following Netzer, Lattin, and Srinivasan, HMMs were widely used by marketing researchers across a wide range of marketing settings (Kumar et al. 2011; Li, Sun, and Montgomery 2011; Luo and Kumar 2013; Montoya, Netzer, and Jedidi 2010). This stream of research primarily examines the evolution of customer–firm relationships by focusing on only the purchase behavior, whereas we argue that other customer activities, such as customers' email open behavior, can also reveal useful information. Conceptually, we claim that studying customers' email open and purchase behaviors jointly is necessary for multiple reasons. First, there is a possible correlation between the two behaviors. For example, customers who are active in purchasing from a firm may also actively search for and examine information about the firm and its offerings by opening and reading emails. In contrast, customers who are inactive in purchasing from the firm may not be interested in reading emails related to the firm. Second, the email open behavior might carry relevant information for the level of customer–firm relationship. In other words, for customers, being active or passive in opening emails might also affect their relationship level with the firm and, more importantly (through this effect on the relationship level), affect their purchase behavior. For example, assume we have two customers at the same relationship level with the firm, and each makes the same purchases in the current period. In this case, the purchase-only model would predict the same level for the customer–firm relationship for the next period. However, if one of the customers is also active in opening emails, because of her email activity, her customer–firm relationship level might be different from that of the other customer. A purchase-only model cannot capture this difference in the relationship level of the two customers. In line with this reasoning, we characterize the customer–firm relationship by jointly utilizing customer-level email open and purchase information.

To the best of our knowledge, Schweidel, Park, and Jamal (2014) is the only study that models the customer–firm relationship by utilizing both customer-level purchase and nonpurchase behaviors. The authors examine the dependence of two customer behaviors (i.e., digital purchase and digital posting) and the evolution of the associated customer–firm relationship. They find that there is a correlation between the latent attribution processes of the two activities, meaning a customer's involvement in one activity is informative of the other. Methodologically, we differ from Schweidel, Park, and Jamal in

Table 1
COMPARISON OF EXISTING STUDIES

Studies	Type of Data	Type of Models: Dynamics Component	Type of Models: Marginal Distribution Component	Modeling Customer Relationship over Purchase	Modeling Customer Relationship over Multiple Decisions	Provides a DSS?
Netzer, Lattin, and Srinivasan (2008)	Donation	HMM	Logit	Yes	No	No
Montoya, Netzer, and Jedidi (2010)	Prescription	HMM	Binomial	Yes	No	Yes
Kumar et al. (2011)	B2B	HMM	Multivariate Tobit	Yes	No	Yes
Li, Sun, and Montgomery (2011)	Banking	HMM	Multivariate probit	Yes	No	Yes
Luo and Kumar (2013)	B2B	HMM	Multivariate Tobit	Yes	No	No
Schweidel, Park, and Jamal (2014)	E-commerce	Latent changepoint	Bivariate choice	Yes	Yes	No
This study	Retailing	HMM	Copula of ZINBD and BDs	Yes	Yes	Yes

that (1) we use an HMM rather than a latent changepoint framework and (2) we use a copula model to capture the correlation between purchase and email open behaviors rather than a multivariate choice framework. More importantly, our research differs from Schweidel, Park, and Jamal because we also provide a DSS that enables firms to allocate resources by utilizing information from two or more customer-based activities. Thus, ours is the first empirical study to model customer relationship over multiple customer decisions and to provide a DSS to help the firm maximize its lifetime profit. We summarize the characteristics our study and the aforementioned studies in Table 1.

To provide an effective DSS to guide a firm to make profitable email marketing decisions, one needs a good predictive model (that will be an input to the DSS) of customer purchase and email open behaviors. A good predictive model must accommodate the following three critical factors. First, across customers, the intensity of purchase and email open behaviors might differ. For example, some customers might regularly check emails (possibly to keep themselves updated with the firm's new product offerings or recent promotional offers) but may not be active in purchasing. Conversely, some customers might be very active in purchasing but may not open emails frequently (they might already know the firm and its offerings well). Because of these inherent differences in behavior across customers, capturing the heterogeneity in customers' email open and purchase behaviors is critical in predicting their behaviors.

Second, customers' email responsiveness and purchase activeness may evolve over time because customers have varying interests and needs at different times. For example, at the initial period of joining the email program, customers could be extremely active in opening emails to learn more about the firm, but their purchase activities may not match their enthusiasm in opening emails. After spending some time and effort getting to know the firm, customers may become familiar with and favorable to the firm. At this point, they might reduce the amount of attention they pay to the emails but, unsurprisingly, increase their purchase activities. Therefore, it is also critical for firms to understand the dynamics of both behaviors to predict customers' email open and purchase levels. Finally, there is a possible correlation between customers' email open and purchase behaviors, and firms need to understand the dependence between the behaviors to predict them accurately. Consequently, a good predictive model of customers' purchase and email behaviors must incorporate (1) customer heterogeneity, (2) customer-relationship dynamics, and (3) the correlation among customers' behaviors.

Our article contributes to the customer relationship literature by jointly studying customers' email open and purchase behaviors in the same modeling framework. Methodologically, we propose a unified HMM and copula framework that can capture not only the correlation between customer email open and purchase behaviors but also the evolution of the customer–firm relationship states that control these two behaviors. Our model is flexible, such that purchase activeness does not necessarily align with email responsiveness. In other words, customers who are active in purchases may not be active in opening emails, or vice versa. In addition, we account for unobserved customer heterogeneity in both customer activities through our random coefficient specification. After we estimate the proposed model, the estimated model becomes an input to our dynamic programming model, in which we develop a DSS to guide the firm in making profitable email marketing decisions. In the following sections, we first describe our data set and discuss our modeling framework.

DATA DESCRIPTION

Our database comprises information from a home improvement retailer in the United States. The retailer sells products and services from multiple categories (e.g., kitchen, plumbing, electrical, flooring, paint, outdoors). The data set consists of information about customer purchase transactions,[2] the number of emails the firm sent to these customers, and the customers' email open histories. The average interpurchase time is approximately six months but typically ranges from a couple of weeks to a few months. Because of the variety and complexity of the products, the large number of categories, and the lack of transactional data at the category level, we study the overall purchase behavior instead of the purchase behavior at the category level.

To form a sample of data for model estimation, we randomly selected a cohort of 200 customers who opted in to the retailer's email program in February 2007. These customers remained in the email program throughout the observation window and received emails from the firm continuously.[3] Thus,

[2]Although customers have the option to buy online and offline, 95% of the observed purchase transactions are offline.

[3]We observe that less than 1% of the customers in our database opted out of the firm's email marketing program. We include only the customers who stayed in the email program during the observation window.

Table 2
DESCRIPTIVE STATISTICS AND CORRELATION MATRIX[a]

	Descriptive Statistics				*Correlation Matrix*		
	Mean	*SD*	*Lower 5%*	*Upper 95%*	*Number of Purchases*	*Number of Emails Sent*	*Number of Emails Opened*
Number of purchases (per month, per customer)	.69	1.63	0	4	1		
Number of emails sent (per month, per customer)	6.90	4.91	0	15	.013	1	
Number of emails opened (per month, per customer)	1.64	3.16	0	10	.050	.425	1
Time since last purchase (in months)	6.33	7.16	1	22			
Time since last email open (in months)	5.45	7.23	1	22.05			
Indicator of open (per month, per customer)	.40	.49	0	1			
Indicator of purchase (per month, per customer)	.29	.45	0	1			

[a]The monthly descriptive statistics are calculated from our data and consist of 200 randomly chosen customers (from the retailer's customer database) who are observed for a period of 39 months.

we have data comprising customers' email open and purchase activities over a period of 39 months. We do not have information on the content of the emails; according to the focal firm's management team, the type of content in these emails varies significantly. The emails could be informative of new products or sales events; persuasive, featuring key benefits of the products; or a combination of the two types. In any given email, the firm generally features products from multiple categories (e.g., kitchen, lumber, and flooring).

In Table 2, we report the descriptive statistics and the correlation matrix of the customer- (purchase and email open) and firm- (email contacts) level behaviors. On average, the retailer sent 6.90 emails to its customers per month. The customers opened 1.64 emails and made .69 purchases, on average, per month. Note that we count only unique email opens because most of the emails are opened only once, if at all. The correlation matrix shows that the number of purchases has a weak correlation with the number of emails sent and the number of emails opened.

To demonstrate the complexity of customers' purchase and email open behavior, we randomly selected three customers from our database and plotted their purchase count and email open frequency over the observation period of 39 months (see Figure 1). Customer 1 was not active in either purchase or opening emails. Customer 1 only made purchases in three of the months and opened emails in two of the months. The time elapsed between the first and second purchase is 21 months. Customer 1 ceased to purchase or open emails after month 26. In comparison, Customer 2 had a more active purchase behavior over the course of 39 months. However, Customer 2 is not equally active in opening emails, because (s)he only opened emails in two of the months (months 10 and 11). In addition, we find that Customer 2 decreased his or her purchase behavior after month 19, demonstrating a time-variant purchase behavior. Customer 3 was moderately active in both purchase and opening emails. We observe that the average interpurchase time of Customer 3 is approximately four months.

Figure 1 shows that customers' purchase and email open behaviors are both heterogeneous and time variant and are not perfectly aligned with each other. The observed pattern indicates that the two behaviors may correlate with each other. In the subsequent section of our modeling framework, we discuss how we model the heterogeneity, the dynamics, and the correlation in purchase and email open behaviors.

In addition, to understand the process of customer purchase and email open frequency, we plot the distributions of both behaviors (see Figure 2). The distribution of purchase count shows that a discrete distribution such as a Poisson process may be able to capture the data-generating process. However, the mean (.68) and the variance (2.64) of the purchase count variable suggest overdispersion, which violates the underlying assumption of Poisson distribution. Furthermore, we observe an excess of zero purchases (71%), which can affect the estimation of the Poisson model. To account for both the overdispersion and excess of zeros, we use the ZINBD to model the purchase count variable.

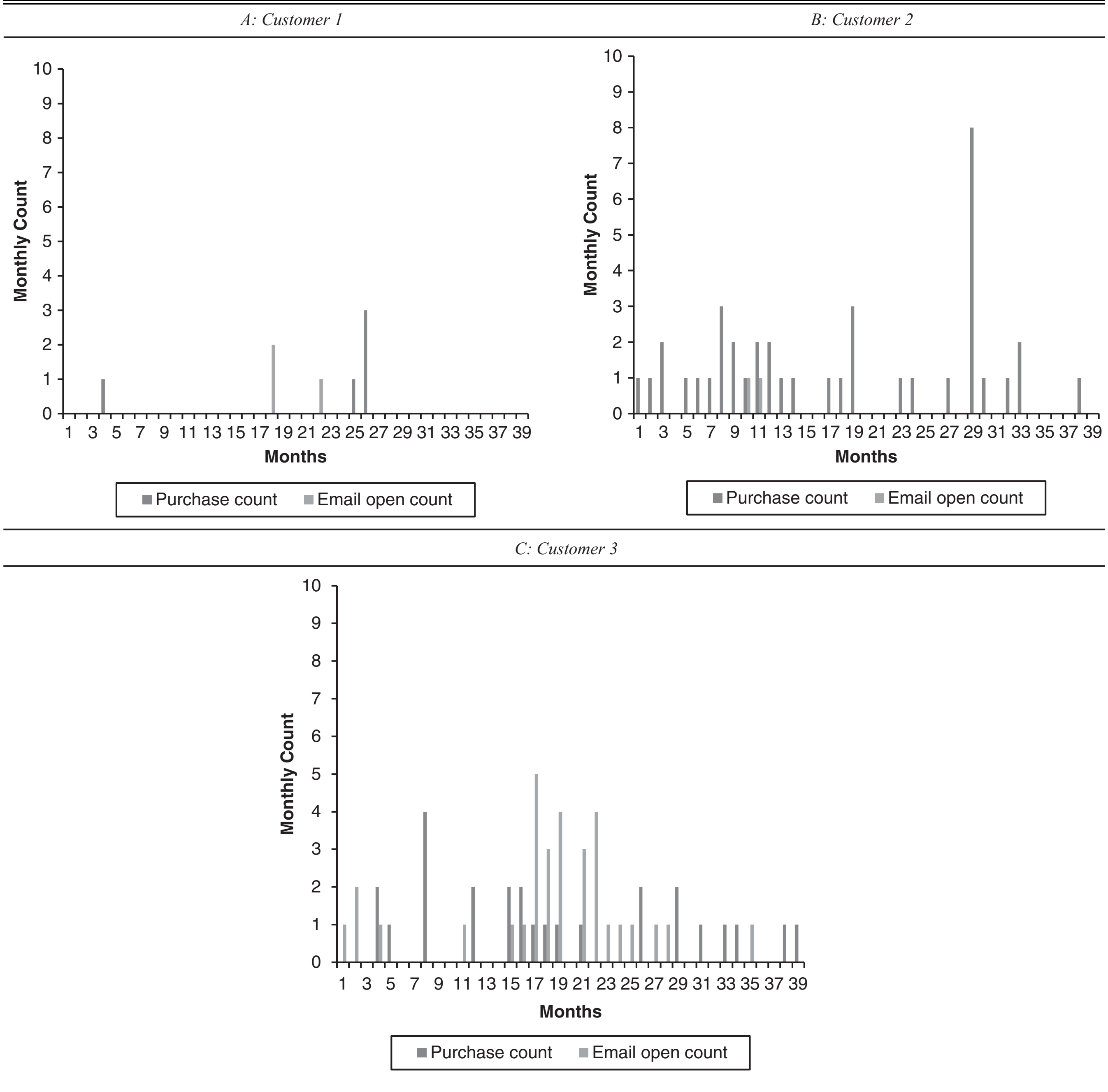
The distribution of email open count also suggests a discrete distribution. The (unique) email open count is the number of unique emails that are opened, conditional on the total number of emails received in each month. We observe that the maximum number of the emails sent (per customer and per month) in our data set is 20 emails. Therefore, the number of unique emails available to a customer to open cannot exceed 20. To capture this process, we use the typical BD, which captures the number of successes (email opens) in a sequence of events (receiving emails).

MODELING FRAMEWORK

In line with previous research, we use an HMM to identify customer–firm relationship states and the transitions among these states. An HMM describes a Markov process using discrete latent states. A HMM is a stochastic model used to capture the transition between these latent states and translate these states into observed behaviors. In the marketing literature, HMMs are widely used to study customer–firm relationships (e.g., Kumar et al. 2011; Luo and Kumar 2013; Montoya, Netzer, and Jedidi 2010; Netzer, Lattin, and Srinivasan 2008). In the context of email marketing, we use an HMM to study two observed customer behaviors jointly: purchases and email opens.[4] In our setting, the latent states

[4]In addition to our previous discussion (in the "Customer Relationship in Email Marketing" subsection) about the conceptual need of joint modeling, we provide two pieces of empirical evidence in our Web Appendix showing that the joint model is indeed an empirical requirement. First, we conduct a simulation study in which the joint model (bivariate email open and purchase) is compared with the purchase-only model (univariate purchase). We find that the estimated (from a simulated data set generated from the bivariate model) bivariate model recovers the assumed parameters more closely and efficiently than the estimated (from the same simulated data set) univariate model. Second, we show that based on the actual data fit, the estimated (from the observed empirical purchase data) univariate model's predictions were 16% worse than those of the estimated (from the observed empirical purchase and email open data) bivariate model. For further discussion, see the Web Appendix.

Figure 1
PURCHASE AND EMAIL OPEN COUNT OF THREE SELECTED CUSTOMERS



of the Markov process translate into different levels of customer–firm relationships that yield different purchase and email open activeness for customers. In addition, because there is a possible correlation between customers' purchase and email open behaviors, we capture the correlation between the two behaviors through a Frank copula function (for the graphical illustration of our proposed HMM and copula model of customer purchase and email open behaviors, see Figure 3).

In this section, we first discuss the primitives of our HMM. Second, we specify our conditional (on latent states of the Markov process) email open and purchase models. Third, we illustrate how we capture the correlation between state-dependent email open and purchase behaviors through our Frank copula component. Fourth, we discuss the model estimation. Finally, we discuss the model identification.

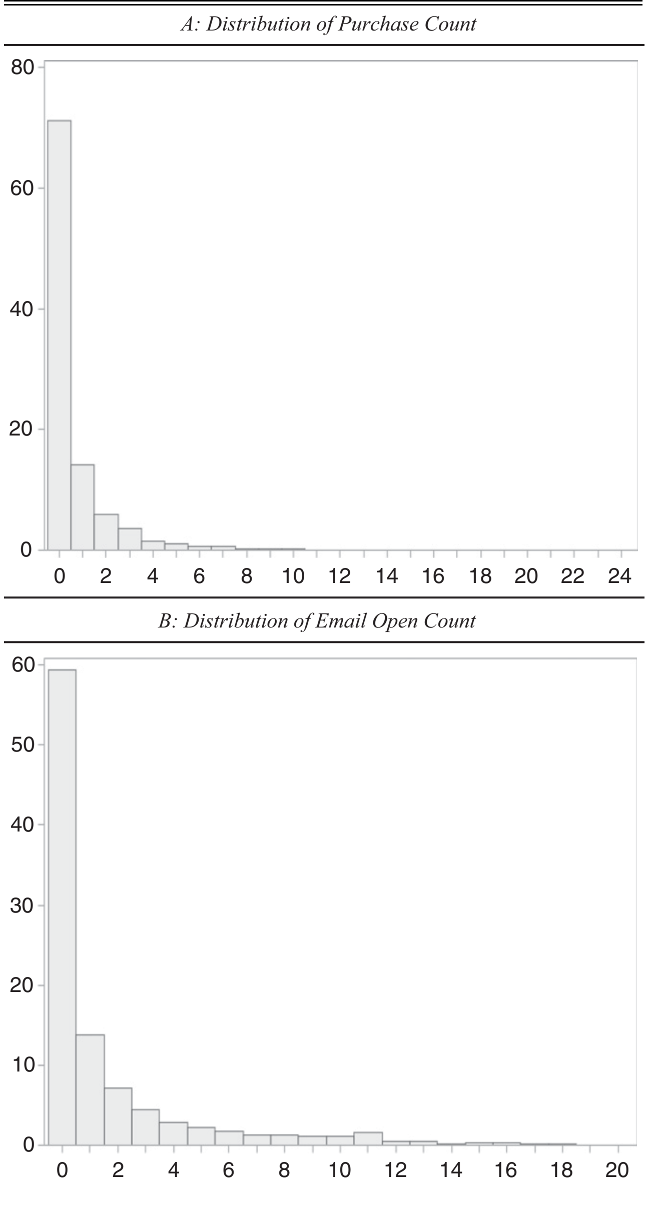
Overview of the Model

Let O_{it} be the number of emails customer i opens at time t.[5] Let Y_{it} be the number of purchases customer i makes at time t.

[5]The unit of time is a month in our study. Therefore, we use time t and month t interchangeably throughout the article.

Figure 2

DISTRIBUTIONS OF PURCHASE AND EMAIL OPEN COUNT



For customer i, we model the sequence of observations $[(Y_{i1} = y_{i1}, O_{i1} = o_{i1}), ..., (Y_{it} = y_{it}, O_{it} = o_{it})]$ using an HMM characterized by (1) the initial state distribution (ψ_i), (2) a sequence of transition probabilities (Q_{it}), and (3) a vector of probabilities that relate the latent states to the observed purchase and email open behaviors (H_{it}).

The Initial State Distribution

At any given month t, let s denote the level of customer i's relationship. Let ψ_{is} be the probability that customer i is initially in relationship state s, where $\psi_{is} \geq 0$ and $\sum_{s=1}^{NS} \psi_{is} = 1$, where NS is the number of the latent Markov states. In this study, we assume that all customers start at the lowest relationship state in the first month. Therefore, $\psi_i' = (\psi_{i1}, \psi_{i2}, ..., \psi_{i,NS}) = (1, 0, ..., 0)$.

The Markov Chain Transition Matrix

In our proposed HMM framework, we allow customers to transit to any relationship state s = 1, 2, ..., NS. Following Kumar et al. (2011), we use a multinomial logit specification to formulate this transition process. We define the transition matrix as follows:

$$(1)\qquad \mathbf{Q}_{i,t-1\to t} = \begin{array}{c|cccc} & & \text{State at t} & & \\ \text{State} & 1 & 2 & \cdots & NS \\ 1 & q_{it,1\to 1} & q_{it,1\to 2} & \cdots & q_{it,1\to NS} \\ 2 & q_{it,2\to 1} & q_{it,2\to 2} & \cdots & q_{it,2\to NS} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ NS & q_{it,NS\to 1} & q_{it,NS\to 2} & \cdots & q_{it,NS\to NS} \end{array},$$

where $q_{it,s\to s'}$ is the probability that customer i moves from state s at time t − 1 to state s′ at time t, where $0 \leq q_{it,s\to s'} \leq 1, \forall s, s'$, and $\sum_{s'} q_{it,s\to s'} = 1$. We specify the indirect transition utility of customer i for transitioning from the relationship state s at period t − 1 to state s′ at time t $(U_{it,s\to s'})$ as follows:

$$(2)\qquad U_{it,s\to s'} = V_{it,s\to s'} + e_{it,s\to s'},$$

where $e_{it,s\to s'}$ and $V_{it,s\to s'}$ denote the stochastic and deterministic components of the indirect utility, respectively. We assume that $e_{it,s\to s'}$ for i = 1, ..., N; t = 1, ..., T; s, s′ = 1, ..., NS are distributed i.i.d. Gumbel with location 0 and scale 1. We operationalize the deterministic component, $V_{it,s\to s'}$, as follows:

$$(3)\qquad V_{it,s\to s'} = \alpha_{s\to s'} + \gamma_{s\to s'} X_{i,t-1},$$

where $\alpha_{s\to s'}$ is the intrinsic utility of transitioning from relationship state s to s′ at time t. $X_{i,t-1}$ contains the following variables: $I[O_{i,t-1} > 0]$, $I[Y_{i,t-1} > 0]$, $EM_{i,t-1}$, $EM^2_{i,t-1}$, where I[A] is the indicator function that takes the value of 1 when event A occurs and the value of 0 otherwise; $O_{i,t-1}$ is the lagged email open count; $Y_{i,t-1}$ is the lagged purchase count; and $EM_{i,t-1}$ is the number of emails sent by the firm at time t − 1. $\gamma_{s\to s'} = [\gamma_{1,s\to s'}, \gamma_{2,s\to s'}, \gamma_{3,s\to s'}, \gamma_{4,s\to s'}]$ is the vector of corresponding response coefficients. We normalize the deterministic utility for customer i at time t to transition to the lowest state $(V_{it,s\to 1})$ to be zero for the identification purpose (i.e., $U_{it,s\to 1} = e_{it,s\to 1}$, where $e_{it,s\to 1}$ for i = 1, ..., N; t = 1, ..., T; s = 1, ..., NS are distributed i.i.d. Gumbel with location 0 and scale 1). Therefore, the transition probability for customer i, transitioning from state s to s′ at time t $(q_{it,s\to s'})$ becomes the following well-known multinomial logit share function:

$$(4)\qquad q_{it,s\to s'} = \frac{\exp(V_{it,s\to s'})}{1 + \sum_{k=2}^{NS} \exp(V_{it,s\to k})}.$$

Next, we discuss our conditional email open count model (CEOM) and purchase count model (CPM).

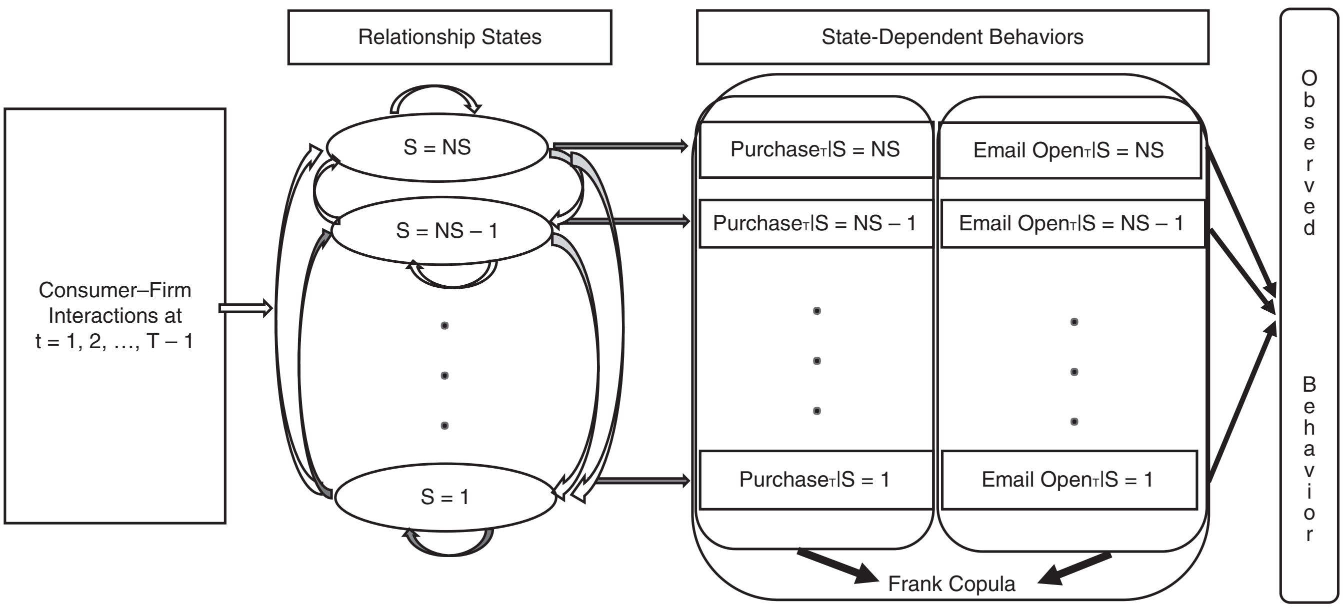
Conditional Email Open Count Model (CEOM)

We assume that the number of emails customer i (in state s) opens at time t follows a BD with parameters EM_{it} and $p_{it|s}$, given by

$$(5)\qquad P(O_{it} = o_{it}|s) = \binom{EM_{it}}{o_{it}} p_{it|s}^{o_{it}} \left(1 - p_{it|s}\right)^{EM_{it} - o_{it}},$$

where $p_{it|s}$ is the conditional probability that customer i (in state s) opens an email in month t. We model $p_{it|s}$ as a function of customers' past email open behavior as follows:

Figure 3
HMM AND COPULA MODEL OF CUSTOMER PURCHASE AND EMAIL OPEN BEHAVIORS



$$p_{it|s} = \frac{\exp\left[\alpha_{io|s} + \beta_{io}\ln(LO_{it})\right]}{1 + \exp\left[\alpha_{io|s} + \beta_{io}\ln(LO_{it})\right]}, \tag{6}$$

where $\alpha_{io|s}$ and β_{io} capture the intrinsic utility of opening an email and the effect of duration dependence (given state s), respectively. We capture the duration dependence through time since the last opened email variable (i.e., LO_{it}). We use the natural logarithm of LO_{it} to capture the diminishing effect. For identification purpose, we impose the following restrictions: (1) $\alpha_{io|s+1} = \alpha_{io|s} + \exp(\Delta\alpha_{io|s+1})$, where $\Delta\alpha_{io|s+1}$ is a parameter to estimate from the data, and (2) β_{io} is state invariant. These two restrictions guarantee that customers in a higher relationship state, all else being equal, have a higher probability of opening emails than those in a lower state. In addition, we allow both $\alpha_{io|s}$ and β_{io} to be customer specific to control for the unobserved customer heterogeneity. We assume that $\alpha_{io|s}$ and β_{io} are distributed with the following normal distributions.[6]

$$\alpha_{io|s} = \alpha_{o|s} + \nabla\alpha_{io}, \text{ and} \tag{7}$$

$$\beta_{io} = \beta_o + \nabla\beta_{io},$$

where $\nabla\alpha_{io} \sim N(0, \sigma^2_{\alpha_o})$ and $\nabla\beta_{io} \sim N(0, \sigma^2_{\beta_o})$.

Conditional Purchase Count Model (CPM)

Conditional on being in the relationship state s at time t, we assume that the number of purchases that customer i makes follows a ZINBD with parameters $\varphi_{it|s}$, $\lambda_{it|s}$, and r. For each observation y_{it}, ZINBD assumes that there are two data-generating processes (based on whether the outcome is equal to zero or greater than zero), which are defined as:

$$P(Y_{it} = y_{it}|s) = \begin{cases} \varphi_{it|s} + (1 - \varphi_{it|s})\left(1 + \frac{\lambda_{it|s}}{r}\right)^{-r} & \text{if } y_{it} = 0 \\ (1 - \varphi_{it|s})\frac{\Gamma(y_{it} + r)}{y_{it}!\Gamma(r)}\left(1 + \frac{\lambda_{it|s}}{r}\right)^{-r}\left(1 + \frac{r}{\lambda_{it|s}}\right)^{-y_{it}} & \text{if } y_{it} > 0 \end{cases}, \tag{8}$$

where r is a dispersion parameter that is assumed not to depend on covariates. The conditional mean and variance of the ZINBD are given as $E(y_{it}|s) = (1 - \varphi_{it|s})\lambda_{it|s}$ and $Var(y_{it}|s) = (1 - \varphi_{it|s})\lambda_{it|s}(1 + \varphi_{it|s}\lambda_{it|s} + \lambda_{it|s}|r)$, respectively. $\varphi_{it|s}$ and $\lambda_{it|s}$ capture the conditional zero-inflated probability and conditional expected purchase count for customer i (in state s) at time t, respectively.

To account for the excess of no purchases, we model the conditional zero-inflation component of the ZINBD ($\varphi_{it|s}$) as

$$\varphi_{it|s} = \frac{1}{1 + \exp\left[\delta_{0i|s} + \delta_{1i|s}\ln(LY_{it})\right]}, \tag{9}$$

where $\delta_{0i|s}$ and $\delta_{1i|s}$ capture the intrinsic utility of making a purchase and the effect of duration dependence, given state s, respectively. We capture the duration dependence through the time since the last purchase variable (i.e., LY_{it}). We use the natural logarithm of LY_{it} to capture the diminishing effect. To account for the unobserved heterogeneity, we allow $\delta_{ki|s}$ (for k = 0,1) to be customer specific. We assume that $\delta_{ki|s}$ (for k = 0,1) are normally distributed across customers as follows:

$$\delta_{ki|s} = \delta_{k|s} + \nabla\delta_{ki},\ k = 0, 1, \tag{10}$$

where $\nabla\delta_{ki} \sim N(0, \sigma^2_{\delta_k})$ for k = 0, 1.

We model the second component of the conditional purchase model (i.e., conditional expected purchase count [$\lambda_{it|s}$])

[6]We acknowledge that the random coefficient specification we implemented here might not fully capture all the cross-sectional heterogeneity that might exist. The use of customer-level fixed effects might better tease out the unobserved customer-level heterogeneity. However, we believe that the use of random coefficient specification is a reasonable compromise because of its high parsimony over the customer fixed effect specification.

as a function of the number of emails sent by the firm (EM_{it}), given by

$$\lambda_{it|s} = \exp\left(\alpha_{ip|s} + \beta_{1,ip|s}EM_{it} + \beta_{2,ip|s}EM_{it}^2\right). \quad (11)$$

Conditional on state s, $\alpha_{ip|s}$ is the intrinsic propensity to make purchases, and $\beta_{1,ip|s}$ and $\beta_{2,ip|s}$ are the corresponding response parameters. Note that, unlike the CEOM, we do not impose any identification restrictions on the parameters of the CPM. In other words, we do not make restrictions such as $\alpha_{ip|1} \le \alpha_{ip|2} \le ... \le \alpha_{ip|NS}$ and/or $\delta_{0i|1} \le \delta_{0i|2} \le ... \le \delta_{0i|NS}$. This restriction-free specification (in the CPM) allows our model to be flexible such that, for example, customers in a more active email-open state may be less likely to be active in purchases, or vice versa. In other words, instead of preimposing restrictions such that email-active customers must be active on purchases, we let the data show whether this is really the case.

To account for the unobserved heterogeneity, we allow $\alpha_{ip|s}$, $\beta_{1,ip|s}$, and $\beta_{2,ip|s}$ to be customer specific. We assume that $\alpha_{ip|s}$ and $\beta_{k,ip|s}$ (for k = 1,2) are normally distributed across customers as follows:

$$\alpha_{ip|s} = \alpha_{p|s} + \Delta\alpha_{ip}, \text{ and} \quad (12)$$

$$\beta_{k,ip|s} = \beta_{k,p|s} + \Delta\beta_{k,ip},\ k = 1,2,$$

where $\Delta\alpha_{ip} \sim N(0, \sigma^2_{\alpha_p})$ and $\Delta\beta_{k,ip} \sim N(0, \sigma^2_{\beta_{k,p}})$ for k = 1, 2.

The Correlation Between Purchase and Email Open Behavior

In each month, both the number of emails opened and purchases made by a customer might indicate the customer's interest in and level of interactions with the firm. Thus, we argue that there is a possible correlation between purchase and email open count distributions. Note that both the purchase count Y_{it} and the email open count O_{it} follow a discrete distribution. It is not easy to find a bivariate distribution that can capture the correlation between the ZINBD and the BD.

Danaher and Smith (2011) pioneer the use of the copula approach in marketing to link two marginal distributions that are not from the same family. Copula models are used in various studies to study multidimensional marketing problems (e.g., Glady, Lemmens, and Croux 2015; Kumar, Zhang, and Luo 2014; Park and Gupta 2012; Schweidel and Knox 2013; Stephen and Galak 2012). However, most studies have focused on using copula models to correlate variables from continuous distribution families. Stephen and Galak (2012) model multivariate count variables using a double Poisson model, but they first convert these discrete variables into continuous ones and use the Gaussian copula to correlate these converted variables. In the context of this study, we are dealing with two discrete variables that are characterized by two distinct distributions: ZINBD and BD. It is uncertain whether Stephen and Galak's approach will work with such distinct discrete distributions. In addition, for efficiency, it is more desirable to model the correlation between the two discrete variables directly rather than convert them to continuous ones first and treat them as continuous.

To tackle the challenge of correlating discrete distributions, we turn to the literature of mathematics and statistics. In line with Sklar's (1959) theorem—in a bivariate case, as an example—the cumulative distribution functions of any two variables can be connected using a copula function, and this copula function is unique if the two variables are continuous. With two continuous variables, the bivariate density can be derived from the partial derivatives of the chosen copula function. With two discrete variables, such as the bivariate count data in this study, although we cannot rely on partial derivatives, we can still obtain the bivariate probability mass function using finite differences of the chosen copula function. There are several statistical applications of the idea to model bivariate count data. Lee (1999) develops a bivariate negative BD to model rugby league scores using Frank copula. Song (2000) develops a multivariate dispersion model generated from Gaussian copula. Nikoloulopoulos and Karlis (2010) capture the association between the purchase counts of certain product categories. Following Nikoloulopoulos and Karlis (2010) and others, we use a copula to construct the bivariate probability mass function of O_{it} and Y_{it} as follows:

$$\begin{aligned} h(o_{it}, y_{it}) = {} & C[F_1(o_{it}), F_2(y_{it})] \\ & - C[F_1(o_{it} - 1), F_2(y_{it})] \\ & - C[F_1(o_{it}), F_2(y_{it} - 1)] \\ & + C[F_1(o_{it} - 1), F_2(y_{it} - 1)], \end{aligned} \quad (13)$$

where $C(\cdot)$ is the copula function and $F_1(o_{it})$ and $F_2(y_{it})$ are the distribution functions of O_{it} and Y_{it}, respectively. We use a Frank copula (e.g., Frank 1979; Genest 1987) in this context because of its flexibility to capture the full range of correlation. The Frank copula function is given by

$$C(u_1, u_2; \theta) = -\frac{1}{\theta}\log\left[1 + \frac{\left(e^{-\theta u_1} - 1\right)\left(e^{-\theta u_2} - 1\right)}{e^{-\theta} - 1}\right], \quad (14)$$

where u_1 and u_2 are the distribution functions (ZINBD and BD in our case) and θ is the Frank copula correlation parameter.

Endogeneity Correction

There is a possibility that the firm is targeting customers by sending a different number of emails on the basis of customers' past purchase activities and inherent characteristics (which we did not observe). In other words, the email contact variable (i.e., EM) might be endogenous. To solve this potential endogeneity problem in EM, we use a control function approach (Petrin and Train 2010), which is similar to the approach used in Villas-Boas and Winer (1999). This involves running a first-stage linear regression of EM on instruments,[7] which include customer-level purchase recency, purchase frequency (to capture customers' past purchase activeness), and customer-level fixed effects (to capture other customer-level characteristics that might be observed by the firm [to do targeted emailing] but not the researcher). From the first-step regression, we obtain an R^2 of 48.5%. In addition, we find the coefficients of purchase recency and frequency to be positive and statistically significant. In the second step, we incorporate the residuals from the first-stage regression as additional control variables into the HMM transition (Equation 3) and

[7]Because the variation in the EM variable is large, we treat it as a continuous variable rather than discrete in our application. In addition, we run a Poisson regression and find that residuals from that count regression and the linear regression are highly correlated (r = .97).

CPM (Equation 11) components of our proposed customer email open and purchase model.

Model Estimation

There are four sets of parameters to be estimated from our model: (1) parameters of the transition matrix (Equation 3), (2) parameters of the CEOM (Equations 6–7), (3) parameters of the CPM (Equations 9–12), and (4) the correlation parameter of the Frank copula function (Equation 14). Following Netzer, Lattin, and Srinivasan (2008), we write the vector of the bivariate probability mass function as a diagonal matrix $\mathbf{H_{it}}$. Given the proposed HMM structure, the likelihood function for a sequence of observations $[(Y_{i1} = y_{i1}, O_{i1} = o_{i1}), ..., ((Y_{it} = y_{it}, O_{it} = o_{it})]$ can be expressed as

$$(15) \quad L = \prod_{i=1}^{N} P[(Y_{i1} = y_{i1}, O_{i1} = o_{i1}), ..., (Y_{it} = y_{it}, O_{it} = o_{it})] = \prod_{i=1}^{N} \psi_i{}' \mathbf{H}_{i1} \prod_{t=2}^{T} \mathbf{Q_{it}H_{it}},$$

where 1 is an $S \times 1$ vector of ones.

Model Identification

Because all model components (transition matrix, CEOM, and CPM) always enter the likelihood calculation (see Equation 15) as a bundle, it is not possible to separately identify the coefficients of a specific variable if that variable is used repeatedly across different equations. Consequently, we choose and use a distinct set of variables in different components of our model. The chosen variables represent customers' past activities and the firm's past and current marketing activities; and these choices are in line with the customer relationship marketing literature (e.g., Kumar et al. 2011; Li, Sun, and Montgomery 2011; Netzer, Lattin, and Srinivasan 2008). Specifically, we model customer i's transition probability (from period t – 1 to t) among the states of the Markov process as a function of interactions between customer i and the firm at period t – 1. We use $O_{i,t-1}$, $Y_{i,t-1}$ to capture the recent interaction of the customer i with the firm, and $EM_{i,t-1}$ to capture the firm's recent interaction with customer i. We model the CEOM component as a function of the customer i's time since the last email open variable (i.e., LO_{it}). We model the CPM component as a function of customer i's time since last purchase variable (LY_{it}) and the firm's email marketing contact policy (EM_{it}). In addition, note that we use lagged and contemporaneous levels of customer- and firm-level behavioral variables in the transition and conditional behavior components (CEOM and CPM, respectively) of our model. Because we have sufficient within-variable variation (for these behavioral variables) over time, we are able to separately identify all three components of our model (i.e., the transition, CEOM, and CPM).

We identify the parameters of each of the components as follows. For the transition component itself, we observe significant within-customer behavior (email open and purchase) variation over time. This implies that customers' relationship levels with the firm vary significantly over time. This is the key for us to empirically identify parameters of the transition component (i.e., the parameters in Equation 3). Within relationship levels, we also observe customers behaving differently, and this helps us identify the unobserved customer heterogeneity parameters (i.e., the parameters of the heterogeneity distributions in Equations 7, 10, and 12). For the CEOM and CPM components, we observe a significant within-time variation across customers' email open and purchase behaviors. For example, at a given time, some customers are active in one behavior, some are active in both behaviors, and some are passive in both. In addition, within customers, the intensity of the two behaviors significantly vary over time. These variations help us identify the parameters of CEOM and CPM components (i.e., the parameters in Equations 6, 9, and 11). In the next section, we discuss our empirical results.

EMPIRICAL RESULTS

We estimate our HMM using the maximum likelihood estimation method. We maximize the simulated sample log-likelihood to estimate our model parameters. We decide on the number of HMM states using the Bayesian information criterion (BIC). In our application, we compare the performance of HMMs of up to four states (see Table 3). We find that the HMM with three states provides the best fit to the data as it gives the lowest BIC value. Thus, we choose the three-state HMM for further analysis.

In the following section, we first discuss the parameter estimates from our proposed HMM and copula model of customer purchase and email open behaviors. Second, we illustrate how the firm's email contact decision would affect the customer–firm relationship states—specifically, we discuss how it affects customers' transitions among different relationship states. Third, we discuss the distribution of customers across the three customer–firm relationship states. Specifically, we illustrate the proportion of customers who belong to different relationship states over time.

Parameter Estimates

Table 4 reports the parameter estimates for the three-state HMM. To label the states in terms of their purchase and email open behaviors, we calculate the predicted purchase and email open counts (at the average observed levels of customer- and firm-level behavioral variables such as LY, LO, EM, etc.). We find that customers in relationship states 1, 2, and 3 open .215, 2.745, and 2.764 emails (on average) per month, respectively. In terms of the purchase behavior, we find that customers in relationship states 1, 2, and 3 make .338, .148, and 1.219 purchases (on average) per month, respectively. From that calculation, we label the three latent relationship states, which govern the frequency of customers' email open and purchase in each month, as "low open/medium purchase," "high open/low purchase," and "high open/high purchase."[8] The results suggest that in state 3, the conventional wisdom holds—that is, the more customers make purchases, the more they open emails. However, such conventional wisdom does not hold in states 1 and 2. Specifically, email-active customers might be inactive in purchases (state 2), and email-inactive customers may still be reasonably active in purchases (state 1). This result underlines the importance of not restricting the parameters of the purchase model to follow the email open model (i.e., imposing the following restriction: higher the state, the higher the email open and purchase levels).

Table 4 shows that customers in all relationship states exhibit a negative purchase duration dependence. The longer

[8]For ease of presentation, we use purchase-only labels (i.e., low, medium, and high purchase) to refer states 2, 1, and 3, respectively, when we have a discussion related to purchases only. We use the full labels "low open/medium purchase," "high open/low purchase," and "high open/high purchase" to refer states 1, 2, and 3, respectively, when we have a discussion related to both purchase and email open behaviors.

Table 3
SELECTING THE NUMBER OF STATES

HMM States	*Log-Likelihood*	*BIC*
1	−17,956.41	36,064.73
2	−17,152.83	34,582.67
3	−15,119.06	30,658.11
4	−15,098.16	30,777.15

they go without making a purchase, the less frequently they make purchases. The effect is strongest for the low-purchase customers (−2.235), and it is the weakest for the high-purchase customers (−1.294). Similarly, customers also show a negative duration dependence for the email open behavior (−.637) (i.e., the longer they go without opening an email, the less frequently they open emails). The results suggest that making a purchase (opening emails) in the previous month increases (decreases) the customer's transition probability to the high open–high purchase state. The results also suggest that, across all relationship states, the number of emails sent by the firm initially increases, but then decreases (inverse U-shape), the short-term purchase count. In other words, the email contact has a nonlinear effect on the purchase count. In addition, the peak-point of the inverse U-shaped curve varies across different relationship states (i.e., customers respond in diverse ways to email contacts depending on their relationship state). This implies that there are different optimal numbers of emails to send to maximize the short-term purchase count across the three relationship states.

Results suggest that two of the five endogeneity correction parameters are significant. In other words, it is important to control for the endogeneity of EM to be able to accurately recover the short- and long-term effects of EM in customers' purchase behaviors and transitions among the relationship states. We also find significant heterogeneity in both the estimates of intercepts and the response coefficients of the CEOM and CPM components of our proposed model.

Transition Probability Matrix

The transition matrix from the HMM shows how customers evolve across different relationship states. We calculate the transition probabilities of a "typical" customer using Equations 1–4. We vary the number of emails received in the previous period and check the effect of email contacts on the state transitions (see Table 5).

When there is no email contact, the customers from medium-purchase and low-purchase states do not tend to move, whereas those from the high-purchase state tend to move down to a medium-purchase state or stay within the high-purchase state. Table 5 shows that email contact has diverse effects on the transition probabilities of the customers who are from different purchase states. For example, ten email contacts per month increases the likelihood that customers from the low-purchase state move up to the medium-purchase state (from 4.53% to 21.42%). However, one email contact per month only marginally increases such likelihood (to 5.67%). For medium-purchase customers, email contact increases their probabilities of staying in the same state. In summary, email contact has a positive long-term effect on either engaging customers with the firm for the first time or

Table 4
ESTIMATION RESULTS FOR THE THREE-STATE HMM

	Estimates	*SE*
Transition Matrix		
Intercept for transition (state 1 to 2)	−2.647***	.315
Intercept for transition (state 1 to 3)	−1.432***	.248
Intercept for transition (state 2 to 2)	2.829***	.352
Intercept for transition (state 2 to 3)	−3.815	4.554
Intercept for transition (state 3 to 2)	−2.398*	.953
Intercept for transition (state 3 to 3)	.061	.274
I[Lagged purchase > 0] on transition to state 2	.173	.218
I[Lagged purchase > 0] on transition to state 3	.363*	.150
I[Lagged open > 0] on transition to State 2	.422	.376
I[Lagged open > 0] on transition to State 3	−.486*	.216
Lag email sent on transition to state 2	−.245**	.088
Lag email sent on transition to state 3	−.110	.064
Lag email sent square on transition to state 2	.007	.005
Lag email sent square on transition to state 3	.001	.004
Lag email sent residual on transition to state 2	.144***	.038
Lag email sent residual on transition to state 3	.003	.033
Email Open Frequency (Binomial)		
Intercept state 1	−2.367***	.065
Intercept (additional state 2, exp)	1.105***	.016
Intercept (additional state 3, exp)	−4.450***	.969
Time since last open (log)	−.637***	.067
Variance for the intercept	1.817***	.064
Variance for time since last open	.388***	.062
Conditional Purchase Frequency (ZINBD)		
Purchase Count Equation		
Intercept (state 1)	−.716***	.105
Intercept (state 2)	−2.951***	.312
Intercept (state 3)	−.413	.317
Email sent (state 1)	.196***	.031
Email sent (state 2)	.470***	.011
Email sent (state 3)	.321***	.091
Email sent square (state 1)	−.013***	.002
Email sent square (state 2)	−.020***	.001
Email sent square (state 3)	−.027***	.007
Email sent residual (state 1)	−.039*	.016
Email sent residual (state 2)	.049	.037
Email sent residual (state 3)	−.002	.036
Variance for the intercept	1.021***	.049
Variance for email sent	.020*	.009
Variance for email sent square	.000	.001
Dispersion, exp	.655***	.071
Excess of Zeros Equation		
Intercept (state 1)	2.135***	.066
Intercept (state 2)	3.203***	.710
Intercept (state 3)	3.447***	.761
Time since last purchase (state 1) (log)	−1.528***	.073
Time since last purchase (state 2) (log)	−2.235***	.391
Time since last purchase (state 3) (log)	−1.294***	.338
Variance for the intercept	2.008***	.208
Variance for time since last purchase	.168	.091
Correlation (Email Open and Purchase)		
Frank copula correlation coefficient	.324***	.067
Likelihood	−15,119.06	
BIC	30,658.11	

*$p < .05$.
**$p < .01$.
***$p < .001$.
Notes: State 1 = "low open/medium purchase"; state 2 = "high open/low purchase"; state 3 = "high open/high purchase."

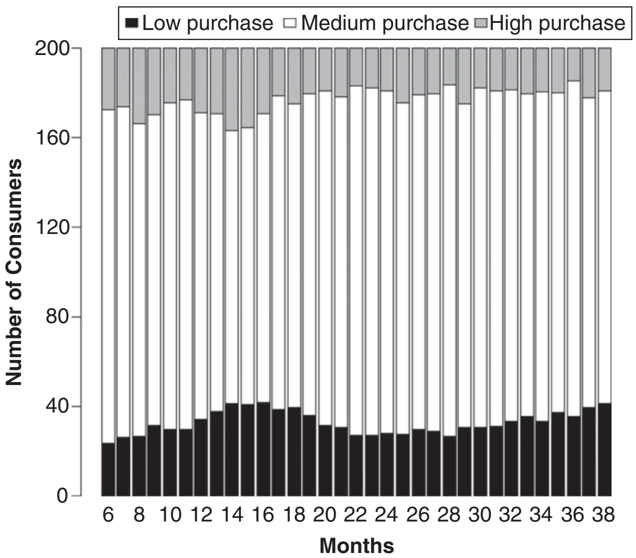
Table 5
TRANSITION PROBABILITY MATRIX OF THE HMM

	To Low Purchase	*To Medium Purchase*	*To High Purchase*
From Low Purchase			
Without email contact	95.38%	4.53%	.09%
With one email contact	94.22%	5.67%	.10%
With five email contacts	87.93%	11.92%	.14%
With ten email contacts	78.42%	21.42%	.16%
From Medium Purchase			
Without email contact	6.75%	76.53%	16.72%
With one email contact	5.49%	79.02%	15.48%
With five email contacts	2.66%	86.20%	11.14%
With ten email contacts	1.40%	91.27%	7.33%
From High Purchase			
Without email contact	5.43%	47.95%	46.62%
With one email contact	4.55%	50.99%	44.46%
With five email contacts	2.45%	61.93%	35.62%
With ten email contacts	1.43%	72.60%	25.97%

keeping them engaged. However, email contact has a negative long-term effect on the customers who already had a good relationship with the firm. For the customers from the high-purchase state, excessive email contacts move them down to lower-purchase states. While the likelihood of them staying in the high-purchase state is 46.62% in the no-email scenario, this likelihood decreases significantly to 25.97% in the ten-emails scenario. This nonlinear dependence between the email contacts and the transition probabilities among the purchase states motivates our policy simulation (in the next section), in which we determine the optimal number of emails to send to maximize the firm's lifetime profit per customer.

In Figure 4, we plot the average probabilities of customers across the three purchase states over time. We calculate the state membership distribution of each customer using the filtering approach (Montgomery et al. 2004; Netzer, Lattin, and Srinivasan 2008). Because we assume that each customer starts from the lowest state, we drop the first five periods as initialization periods and plot the states' evolutions over the rest of the 33 periods. We find that, on average, 74%, 12%, and 14% of customers started in the medium-, low-, and high-purchase states, respectively. Over the course of 33 months, the majority of the customers remained in the medium-purchase state, while the rest moved up or down to different states. In addition to learning the aggregate distribution of customers across the different relationship states (Figure 4), the firm might use the filtering approach to understand the state membership distribution at the customer level. Then, the firm could rely on this information to make targeted email contact decisions. We illustrate this idea in our policy simulation section next.

Figure 4
DISTRIBUTION OF CUSTOMERS' STATE MEMBERSHIP OVER TIME



Notes: On average, low-, medium-, and high-purchase customers make .148, .338, and 1.219 purchases per month, respectively.

OPTIMAL EMAIL MARKETING

The optimal marketing contact strategy is the focus of several prior studies (e.g., Khan, Lewis, and Singh 2009; Kumar et al. 2011; Li, Sun, and Montgomery 2011). In this study, because we find that sending more emails is not necessarily good for the firm to encourage its customers to make more purchases, determining the optimal number of emails might provide substantive profit gains for the firm. Therefore, our objective in this section is to provide a DSS that managers can rely on to maximize the firm's long-term profit by sending the optimal number of emails to their customers over time.

In this setting, at any given time t, the firm must decide how many emails to send. Given our estimated customer response function, the email contact decision has both short- and long-term implications on customer behavior. The short-term effect derives from the direct effect of emails on customer purchase and email open behaviors at time t (see Equations 5 and 11). The long-term effect comes from two sources, such that (1) customers' purchase and email open behaviors at time t affect the evolution of their customer relationship state from time t to t + 1 (see Equation 3) and (2) the email contacts at time t influence the relationship state transition from time t to t + 1 (see Equation 3). Because of these long-term effects, determining the optimal number of emails to send by the firm requires us to solve a dynamic programming problem.

From the retailer's perspective, the variable of interest is the number of times a customer purchases from the store each month. Under the assumptions of constant purchase amount per each purchase transaction and fixed gross margin for the retailer,[9] the purchase count is directly translated into the firm's profit (per customer and per transaction). For the firm's dynamic optimization problem, the payoff relevant state variables are (1) the probabilities that the customer exists in each of the customer–firm relationship states (p_{1t}, p_{2t}, $p_{3t} = 1 - p_{1t} - p_{2t}$), (2) the time since last purchase (LY), and (3) the time since last email open (LO). Therefore, the state vector at time t becomes $S_t = (p_{1t}, p_{2t}, LY_t, LO_t)$. Following Kumar et al. (2011), we assume the timing of the email contact decisions as follows. At the beginning of each month t, the firm predicts the probability that the customer exists in each of the three relationship states, p_{1t}, p_{2t}, and $p_{3t}(= 1 - p_{1t} - p_{2t})$. Next, based on the predicted p_{1t}, p_{2t}, and p_{3t}, as well as LY_t and LO_t, the firm decides how many emails to send to the customer. We use a multinomial logit share function to capture the state membership probabilities $(p_{1t}, p_{2t}, \text{and } p_{3t})$ and relate them to two parameters ω_{1t} and ω_{2t} as follows:

$$(16)\quad \begin{aligned} p_{1t} &= \frac{\exp(\omega_{1t})}{1 + \exp(\omega_{1t}) + \exp(\omega_{2t})} \\ p_{2t} &= \frac{\exp(\omega_{2t})}{1 + \exp(\omega_{1t}) + \exp(\omega_{2t})}. \\ p_{3t} &= 1 - p_{1t} - p_{2t} \end{aligned}$$

Let $S_t = (p_{1t}, p_{2t}, LY_t, LO_t)$ denote the state vector at time t. Time since last purchase (LY_t) and open (LO_t) states evolve on the basis of whether the customer makes purchases and opens emails at time t. If the customer makes purchases at time t, the corresponding state LY_{t+1} becomes 1, and if (s)he does not make any purchases at time t, LY_{t+1} becomes $LY_t + 1$. Similarly, if the customer opens emails at time t, the corresponding state LO_{t+1} becomes 1, and if (s)he does not open any emails, LO_{t+1} becomes $LO_t + 1$. Because we model customers' purchase and email open processes with ZINBD and BD, the time since last purchase and open states, LY_t and LO_t, evolve in a stochastic manner as follows:

$$(17)\quad LY_{t+1} = \begin{cases} 1, & \text{with } \Pr(Y_{it} > 0) \\ LY_t + 1, & \text{with } \Pr(Y_{it} = 0) \end{cases},$$

$$LO_{t+1} = \begin{cases} 1, & \text{with } \Pr(O_{it} > 0) \\ LO_t + 1, & \text{with } \Pr(O_{it} = 0) \end{cases}.$$

The evolution of the first two state variables $p_{1,t+1}$ and $p_{2,t+1}$, conditional on the firm's email contact decision EM_t, are given as

$$\begin{aligned} (18)\quad p_{s,t+1}\,|EM_t = {} & \Pr(Y_{it} > 0, O_{it} > 0)\sum_{k=1}^{S} p_{kt}\, q_{t+1,k\to s}(1,1,EM_t) \\ & + \Pr(Y_{it} = 0, O_{it} > 0)\sum_{k=1}^{S} p_{kt}\, q_{t+1,k\to s}(0,1,EM_t) \\ & + \Pr(Y_{it} > 0, O_{it} = 0)\sum_{k=1}^{S} p_{kt}\, q_{t+1,k\to s}(1,0,EM_t) \\ & + \Pr(Y_{it} = 0, O_{it} = 0)\sum_{k=1}^{S} p_{kt}\, q_{t+1,k\to s}(0,0,EM_t), \end{aligned}$$

where $q_{t+1,k\to s}[I(Y_t > 0), I(O_t > 0), EM_t]$ is the transition function from Equation 4, which is used to calculate the probability of transitioning customers from state k at time t to s at time t + 1, conditional on email contacts, purchase indicators, and email open indicators at time t.

At each time t, conditional on the state vector S_t, the firm's objective is to determine the optimal number of email contacts to maximize the discounted sum of expected future profits. Under some regularity conditions, this objective can be written in the following Bellman equation:

$$(19)\quad V(S_t) = \max_{EM_t}[\pi(EM_t, S_t) + \rho EV(S_{t+1}|S_t, EM_t)],$$

where ρ is the discount factor, $\pi(.)$ is the per period profit, and the expectation is over all the future states and actions of the firm.

To solve this dynamic optimization problem, we discretize the state space with ten levels for each state dimension p_{1t}, p_{2t}, LY_t, and LO_t yielding 10,000 state combinations. We use the value iteration algorithm (Rust 1987) to find the optimal mappings of the firm's email contacts to our chosen state combinations. Because of the discretization of the first two state dimensions, the value functions for the other points in the state space are computed through interpolation (Keane and Wolpin 1994). After we calculate the vector of the optimal mapping of email contacts (through the value iteration algorithm), we fit a multinomial logit model (MNL)[10] to predict optimal email contacts as a flexible function of the chosen state space. Next, we use this fitted MNL policy function to predict the optimal number of emails to send for any state combinations out of the chosen state space.

Drawing on the optimal mapping of email contacts to states from the value iteration algorithm, we find that the optimal email contact number ranges from 5 to 14. We observe a lot of heterogeneity in the ranges of optimal number of emails sent based on customers' relationship states. For instance, if the firm believes there is a greater than 30% probability that the customer is in the high-purchase state, the optimal number of emails to send ranges between five and seven (based on the value of the remaining state combinations). If the firm's believes that the customer is in the medium-purchase state with more than 30% probability, then the optimal number of emails to send ranges between six and ten. If the firm strongly believes (with more than 90% probability) that the customer is in the low-purchase state, the optimal number of emails to send ranges between 12 and 14. If the opposite is the case (i.e., the

[9]For confidentiality reasons, we did not observe customer-level purchase amount in our data set. However, from our communication with the managers of the firm, we are advised that the average purchase amount for each customer transaction is approximately \$80–\$110. We are also advised that the average profit margin is approximately 18%–22% across different product categories. Thus, in our profit and lifetime value calculations, we assume an average purchase amount of \$100 per customer, per transaction and a 20% profit margin for each customer purchase occasion. This assumption yields \$20 net profit per customer at each purchase occasion.

[10]We chose the MNL functional form for the policy function because the optimal mapping of the email contacts takes discrete and finite number of values (ranging between 5 and 14 emails to send, as we discuss next).

firm strongly believes that the customer is not in the low-purchase state), the optimal number of emails to send ranges between five and seven.

This heterogeneity in the optimal number of emails to send suggests that the firm should target different customers (by sending different number of emails) on the basis of the customers' observed behaviors (LY and LO) and the firm's belief about their relationship states. To illustrate how the firm might use our DSS to target its customers through email marketing, we randomly pick two customers from our database. We then calculate their HMM state membership probabilities over time through the filtering approach and use our optimal policy function to set the number of email contacts (based on their observed LY and LO levels in the data). We plot the optimal number of email contacts over time for these two customers in Figure 5. As Figure 5 shows, the optimal number of emails to send varies significantly not only across these two customers but also within the same customer over time. This exercise suggests that the firm can easily use our proposed DSS as an actionable management tool to dynamically and profitably manage its email contact decisions.

Next, we use our optimal policy function to simulate the firm's email contact decisions along with the customer purchase and email open responses over a long time horizon for three representative customers in the low-, medium-, and high-purchase states, respectively. This forward simulation exercise helps us understand how the customer lifetime value (CLV) of customers differs across three purchase states. In line with our assumptions (see footnote 9), we find that CLVs of customers in low-, medium-, and high-purchase states are \$1,333, \$1,411, and \$1,465, respectively. This exercise also reveals that, in the steady state, the optimal number of emails to send to each customer is seven per month. In addition, the discounted sum of lifetime purchase and email open counts are 73 and 118 per customer, respectively.

Finally, we conduct a what-if simulation study to measure how much profit the firm would lose if it deviated from the recovered optimal email policy function (see Figure 6). We use the steady-state distribution as the starting state combinations and test the alternative scenarios in which the firm deviates from sending the optimal number of seven emails. We test the scenarios in which the firm sends four, five, six, eight, nine, and ten emails instead of seven. Figure 6 shows that sending a suboptimal number of emails might cause the firm to lose significant amount of profit. For instance, sending four (ten) emails causes the firm to lose 32% (16%) of its lifetime profit per customer. This result suggests that sending the optimal number of emails is critical for the profitability of the firm's email marketing program.

ROBUSTNESS CHECKS

To check whether our model and findings are robust to alternative model specifications, we conduct four sets of robustness checks:

1. One set using alternative distributions to model the conditional purchase counts,
2. One set using different copula functions to model the correlation between customers' purchase and email open behaviors,
3. One set imposing HMM identification restriction on the CPM, and
4. One set including holiday dummies as additional control variables in the CPM.

As our first robustness check, we estimate three alternative model specifications in which the conditional purchase count is assumed to be distributed as Poisson (PD), zero-inflated Poisson (ZINPD), and negative binomial (NBD), respectively. We find that the estimates from these three alternative specifications look qualitatively similar to our CPM with ZINBD

Figure 5

WITHIN-CUSTOMER OPTIMAL EMAIL CONTACTS OVER TIME

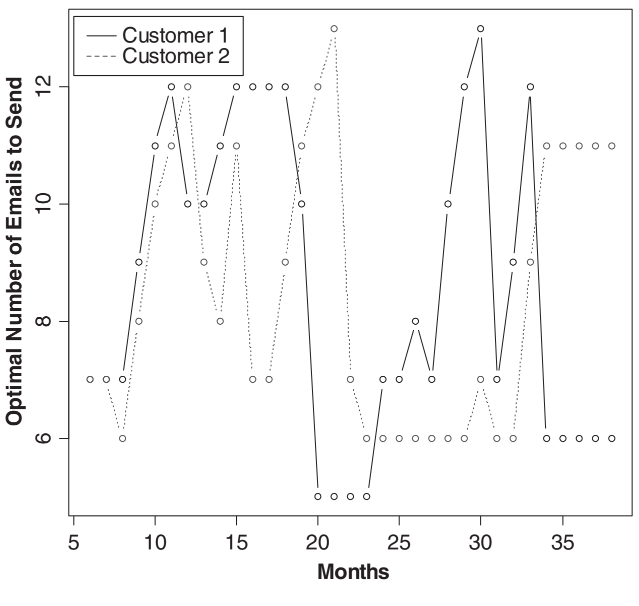
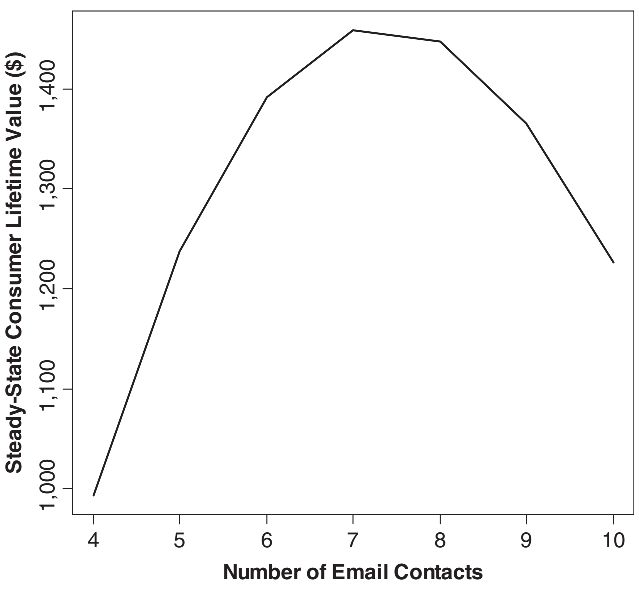


Figure 6

STEADY-STATE CLV VERSUS NUMBER OF EMAIL CONTACTS



specification. However, based on the BIC, we determine that our proposed ZINBD specification outperforms the alternative models (BIC_{ZINBD} = 30,658.11, BIC_{PD} = 33,440.23, BICZINPD = 32,201.11, BICNBD = 30,994.36). Therefore, we assume the conditional purchase count is distributed by ZINBD.

As our second robustness check, we estimate two alternative model specifications. Specifically, we estimate the alternative models with Clayton (1978) and Gumbel (1960) copula functions, which have been widely used in the statistics literature. Although all three specifications yield qualitatively very similar estimates, based on the BIC, we find that the Frank copula (BIC = 30,658.11) slightly outperforms the Clayton (BIC = 30,676.25) and Gumbel (BIC = 30,682.59) copula specifications. Therefore, we use the Frank copula as our proposed copula specification in the rest of our analysis.

For identification purposes, we could have imposed HMM restrictions on the CPM instead of the CEOM. However, this would have required us to impose 6 × (NS − 1) restrictions, including NS − 1 intercepts and NS − 1 response coefficients in the φ_{it} component and NS − 1 intercepts and 3 × (NS − 1) response coefficients in the λ_{it} component. In contrast, imposing HMM restrictions on the CEOM requires only 2 × (NS − 1) restrictions (NS − 1 intercepts and NS − 1 response coefficients in the p_{it} component); for this reason, we chose to impose restrictions on the CEOM. As our third robustness check, we estimate the alternative specification in which the identification restriction is imposed on the conditional purchase model. Using the BIC, our proposed model (BIC = 30,658.11) outperforms that alternative specification (BIC = 31,308.90). Therefore, we use our proposed model with HMM restrictions on CEOM as our main model in the rest of the analysis.

Finally, as our last robustness check, we incorporate the holiday dummies into our CPM as additional control variables. Essentially, customers might be more active in their purchase activities during the holidays. Because our data are at the monthly level, we define the months corresponding to Memorial Day, Fourth of July, Labor Day, Thanksgiving, and Christmas as holiday months and dummy-coded this holiday variable. From the comparison of our proposed model and the model with holiday dummies, we find that (1) all parameters look qualitatively very similar under both specifications; (2) coefficients for the holiday dummies are statistically insignificant; and (3) based on the BIC, our proposed model slightly outperforms the alternative model (30,658.11 vs. 30,661.85). As a result, we use our proposed model without holiday dummies as our main model in the rest of the analysis.

CONCLUSIONS, LIMITATIONS, AND FUTURE RESEARCH

Email marketing programs are used extensively in various industries to engage with customers. The general industry practice in measuring the effectiveness of an email marketing program is to examine customer responsiveness to emails such as email open rate. However, we demonstrate that considering only the email open rate could be misleading. Our empirical study shows that some of the very active email openers are the least active in their purchase behaviors. In addition, our findings show that some email-inactive customers are relatively active in their purchase behaviors. Thus, if firms focus on solely the email open rate to allocate their resources, they might overlook a pool of customers who are inactive in responding to emails but are relatively active in purchases.

To the best of our knowledge, substantively, this is the first empirical study to model the customer's email open and purchase behaviors jointly. In addition, methodologically, this is the first study to combines the HMM and copula models in a unified framework. In our HMM specification, we model the latent customer–firm relationship states that govern both customers' purchase activeness and email responsiveness. In our copula specification, we use a Frank copula to correlate the customer's email open and purchase behaviors.

Note that the purpose of this study is not to divert firms' attention from email open rates. Indeed, this study shows that, on average, there is a positive correlation between email open and purchase behaviors. However, we recommend that firms look at customers' purchase behaviors in addition to their email response rates. If the goal is to maximize long-term profitability, firms should be informed about the optimal level of email contacts they should make to their customers. Along these lines, we calculate the optimal email marketing contact policy by solving the firm's dynamic optimization problem. We propose an implementable framework to study an important substantive problem that can save firms millions of dollars.

In our specific application, although we use email open as the nonpurchase customer behavior, our framework is quite flexible in that it is applicable to other customer-level nonpurchase behaviors. In addition, our DSS is designed to guide the firm in determining how many emails to send; however, our proposed structure might assist with other firm-level marketing decisions without significant issues. Furthermore, as more information that might affect the purchase and nonpurchase behaviors becomes available, the firm could incorporate it by constructing additional control variables for the purchase and nonpurchase models. Among such variables, the ones that are relevant for the firm's decision in the supply side might be incorporated into the operationalization of the proposed DSS. This will allow the firm to target different customers at a given time and the same customers over time differently on the basis of additionally observed customer-level characteristics.

One of the limitations of this study is that we do not observe the content of the emails. Therefore, we do not focus on the emailing strategy paired with customers' contemporary needs, such as cross-selling and upselling. Previous research has investigated this important issue (e.g., Kumar, George, and Pancras 2008; Li, Sun, and Wilcox 2005). If the email content data are available, future research might not only consider the effect of emailing content on customers' response to emails and purchase behavior but also provide guidance to the firm about how to target email marketing on the basis of personalized customization of the email content.

Another limitation of this study is that we do not observe information from the firm's competitors. Customers might not respond to emails simply because they subscribe to many email programs from different firms. Each email delivered to the customer's inbox is a load of information. Customers who are not capable of processing the information will be overwhelmed and stop responding. If the firm is aware of its customers' inbox activity, incorporating this information into the study is imperative. However, because of the sensitivity of such information, it is unlikely that the retailer would be able to obtain it. Future studies should consider conducting field experiments to understand how competing emails affect customers' reactions to the focal firm's emails.

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