DEPARTMENT OF INFORMATICS

TECHNISCHE UNIVERSITÄT MÜNCHEN

Master's Thesis in Robotics, Cognition, Intelligence

Comparison of Controllers for Trunk Stabilization in a Bipedal Robot

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Vergleich von Reglern für die Rumpfstabilisierung in einem zweibeinigen Roboter

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I confirm that this master's thesis in robotics, cognition, intelligence is my own work and I have documented all sources and material used.

Munich, 15.03.2023 Felix Schausberger

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Abstract

Zusammenfassung

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Glossary

F Vector of the ground reaction force, i.e. the external force acting on the limb (N). ix, xi, 6, 7, 8

Fr Froude number. 7

P_{mech} Power of the limb acting on the center of mass of the body, i.e. the external mechanical power (J). 6, 7

V Velocity vector of the center of mass ($\frac{m}{s}$). ix, xi, 6, 7, 8

W_{mech} External mechanical work (W). 7

Λ Average angle associated with V (°). xi, 8

Φ Average collision angle (°). xi, 7, 8

Θ Average angle associated with F (°). xi, 8

κ Collision fraction, calculated as the quotient of Φ and the sum of Λ and Θ. ix, 7, 8

λ Instantaneous angle of V relative to horizontal (°). ix, 6, 7, 8

ϕ Instantaneous angle of deviation of perpendicularity of force and velocity vectors (collision angle) (°). ix, 6, 7, 8

θ Instantaneous angle of F relative to vertical (°). ix, 6, 7, 8

g Acceleration due to gravity ($\frac{m}{s^2}$). 7, 8

h Height of the center of mass. 7

m Mass of the body (kg). 8

n Number of sampled points in a stride period. 8

Acronyms

BSLIP Bipedal Spring-Loaded Inverted Pendulum 5, 6

CoM Center of Mass xi, 2, 4, 5, 6, 7, 8

CoT Cost of Transport 10

$\mathbf{CoT_{mech}}$ Mechanical Cost of Transport 8

DoF Degrees of Freedom 9

GRF Ground Reaction Force xi, 7

MCA Mechanical Cost Analysis ix, 6, 7

RIP Rigid Inverted Pendulum 5

SLIP Spring-Loaded Inverted Pendulum ix, 2, 5, 6, 7, 8

TUM Technical University of Munich iii

VLO Vertical Leg Orientation ix, 4

Chapter 1

Introduction

Robotics aims to build artificial cognitive systems that can act on their own to achieve some predefined goal. However, in order to be able to respond to and interact with one's environment, the necessity of grasping basic, ~~underlying~~ mechanisms of our world evolves. To master this, agents[1] are required to perceive their environment, anticipate the need to act, learn from experience, and adapt to changing circumstances [25]. Robots must navigate an increasingly complex, uncertain, unstructured and human-shaped environment. This can only be achieved by exhibiting some degree of cognition. Hereby, nature provides a remedy. The course of millions of years of evolution created an almost inexhaustible arsenal of potential solutions and highly optimized system processes, which frequently inspire engineers [23]. Studies of mechatronic systems inspired by biology can be categorized in respects of locomotion and mechanisms, actuation, sensing, and control [24]. Within this, one field which nature truly masters represents locomotion. Nature has evolved various biological forms and functions to maneuver energy efficiently, ~~agile~~ and safely through even the most hazardous environments. Subsequently, two types of terrestrial locomotion established in nature~~-~~ walking upright on at least two legs like humans and most mammals do, which facilitates fast locomotion~~,~~ and crawling low over the ground like reptiles, which usually tends to greater stability especially on rough terrain [24]. In general, bipedal runners achieve greater absolute stride lengths than quadrupeds of the same body mass [21]. This has been argued to be an advantage for persistence runners of our own species - for example, in endurance hunting of quadrupeds or in aggressive scavenging in competition with quadrupeds [4](Bramble and Lieberman, 2004) [18]. Energy comes at a premium not only for animals, wherein suitably fast and economical gaits are selected through organic evolution, but also for legged robots that must carry sufficient energy in their batteries. Although a robot's energy is spent at many levels from control systems to actuators, we suggest that the mechanical cost of transport is an integral energy expenditure for any legged system — and measuring this cost permits the most direct comparison between gaits of legged animals and robots. Although legged robots have matched or even improved upon total cost of transport of animals, this is typically achieved by choosing extremely slow speeds or by using regenerative mechanisms [18]. Since the beginning of mankind, legged locomotion has been of central importance to humans for hunting, agriculture, transportation, sport, and warfare [18]. Locomotion is fundamental for foraging, prey capture, predator evasion, securing territory, finding mates, and migration [18].

[1]Respectively any artificial entity displaying some degree of cognition [25].

Chapter 2

Fundamentals

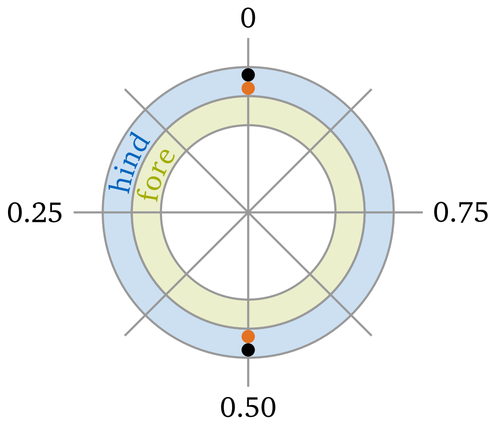
The analysis of terrestrial locomotion in the last half century has focused mainly on strategies for mechanical energy recovery during walking and running [16]. As early as the mid-1960s, Hildebrand [10] and others used cine films to distinguish gaits of terrestrial animals by such means as the duty cycle or limb contact phases. About a decade later, Cavagna, Heglund, and Taylor [5] revolutionized the understanding of animal locomotion by defining center of mass (CoM) mechanics using a point mass model and force plate measurements of the whole animal. Their comparison of walking and running reduced gait complexity by proposing that separate and mutually exclusive mechanisms act to exchange energy during the gait phase, such as anti-phase to in-phase kinetics and potential energy fluctuations. The publication induced today's widely accepted paradigm of conceptualizing walking as a rigid inverted pendulum and running as a compliant spring-loaded inverted pendulum (SLIP) [17]. While walking, the mechanical energy of the CoM remains nearly constant as kinetic and potential energies interchange oscillatory, causing the CoM to ascend over the support limb in mid-stance and fall forward into the next step. During running, however, this exchange is not possible because the CoM attains its lowest point in mid-stance, where the kinetic energy is also low. For this reason, running gaits assume the use of a spring-mass mechanism in which the interaction between the CoM and the ground allow for the storage and return of elastic strain energy in rather compliant legs [16].

2.1 Gaits

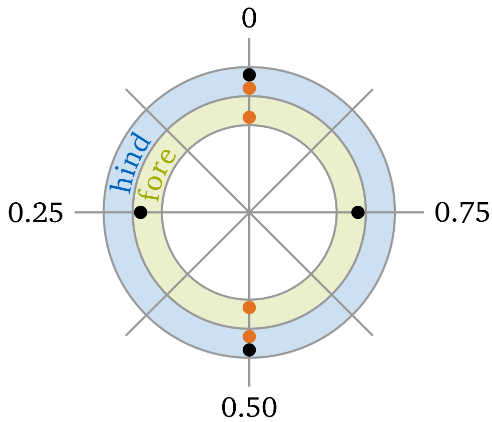
Over millions of years of evolution, living creatures have developed various modes of locomotion, so-called gaits, which can be distinguished by their movement, i.e. their temporal footfall patterns. For terrestrial animals, the footfall sequence represents the primary identifying feature of their gait, quantified by the phase relationship of the individual legs and expressed as a fraction or percentage of the time of foot contact to the stride period. For example, in bipedal walking, one foot lands at the beginning of the stride (i.e. at 0%) and the second foot lands at mid-stride (i.e. at 50%), representing one entire step [18].

2.1.1 Symmetrical vs. Asymmetrical Gaits

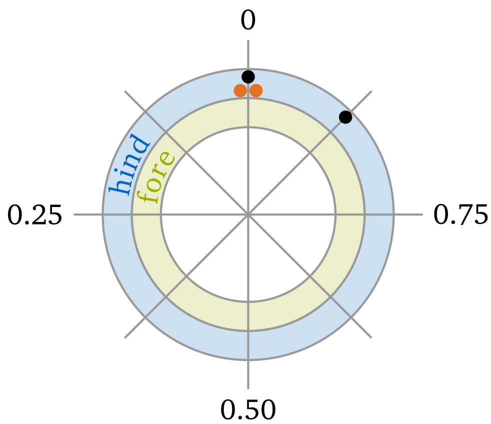
Legged gaits can be classified as symmetrical and asymmetrical, according to the phase relationship of the left-right pairs of legs, regardless of the number of pairs. If the left and right leg of a pair is one-half stride cycle apart out of phase, the gait is defined as symmetrical - if not, the gait is defined as asymmetrical. Examples of a symmetrical gait include the bipedal walking of humans, the quadrupedal trotting of dogs, the pacing of camels, and the six-legged trotting of cockroaches, where all left-right pairs of front, middle, and hind legs are one-half stride cycle out of phase with one another. The number of legs limits the number of leg sequencing options, such that bipedal striding gaits are restricted to symmetric (walking and running, see fig. 2.1a) and asymmetric gaits (hopping and skipping, see fig. 2.1c). Quadrupeds use five symmetrical gaits (lateral and provisional diagonal sequence walking, trotting (see fig. 2.1b), pacing and ambling) and six asymmetrical gaits (lope, transverse and rotary gallops, half-bound (see fig. 2.1d), bound, and pronk). However, note that these are broad definitions and that phase separations between foot contacts show substantial variation within gaits, as can be seen in Hildebrand's (1965, 1968) plots for the gaits of horses and dogs [18].



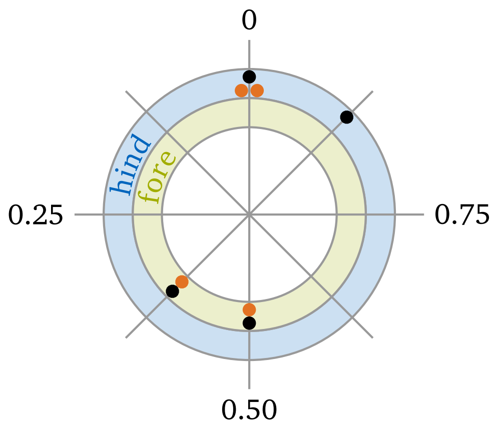
(a) walk (black), run (orange)



(b) walk (black), trot/pace (orange)



(c) skip (black), hop (orange)



(d) gallop (black), half-bound (orange)

Figure 2.1: Common gaits of bipeds (a, c) and quadrupeds (b, d). Stereotypical foot contact phases are represented as a fraction of stride period on polar plots. The outer ring represents the rear limb contacts (blue) and the inner ring represents the front limb contacts (green). Half a cycle out of phase of fore and hind legs indicates symmetrical gait (a, b), substantial deviations of this in either pair indicates asymmetrical gait (c, d). (Source: modified adopted from [18], p. 3)

2.1.2 Bipedal Gaits

Bipedal striding gaits, including that of our own genus, are symmetrical by definition 2.1a. These gaits can be found today mainly in birds and in earlier times in theropod dinosaurs, which also represent the greatest diversity of bipedal runners. Humans and most birds[1] walk at slow speeds and run at fast speeds. Some great apes and monkeys are volitional bipeds, but usually only for rather short distances. At top speed, some lizards [12] and cockroaches [8] can bring their bodies into an almost upright posture and thus achieve bipedal running movements, increasing speed by expanding stride length. In general, bipeds achieve greater absolute stride lengths than quadrupeds of the same body mass [21]. This has been argued to be an advantage for endurance runners as our own species, for example while engaging in persistence hunting quadrupeds or aggressive scavenging in competition with them [4].

2.1.3 Vertical Leg Orientation

The bipedal spring mass model is capable of periodic gait patterns such as walking and running. A gait pattern is fully described by the system parameters and initial conditions. The initial conditions are chosen so that the stance leg is in contact with the ground and vertically oriented, i.e. the CoM is exactly above the foot point ($x = x_{FP1}$), meaning that the horizontal position is zero with respect to the actual foot point, called vertical leg orientation (VLO). A single step is completed when the swing leg attains ground contact and the CoM is orthogonally above the second foot point ($x = x_{FP2}$), as shown in figure 2.2. These initial conditions can be used to reduce the number of independent initial conditions [22].

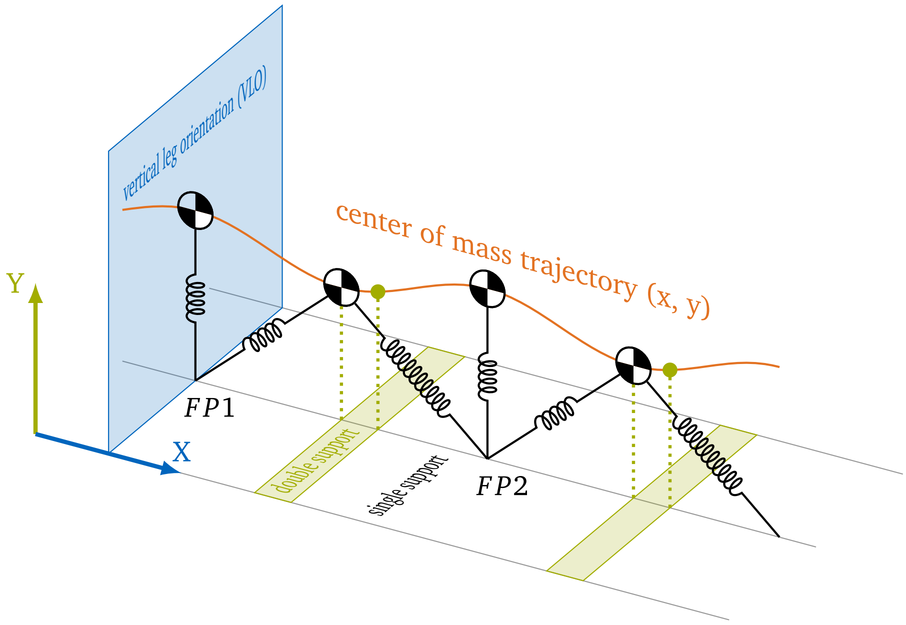


Figure 2.2: The bipedal spring mass model for walking. The simulation starts at the moment of vertical leg orientation (VLO) during single support phase and ends after one step is completed at VLO of the opposite one. (Source: modified adopted from [22], p. 1)

2.2 Gait Analysis

A wide variety of models ~~have~~ been developed to analyze terrestrial locomotion, which can be classified based on the phase relationship of the kinetic and potential energy oscillations.

[1]Except small songbirds, which typically have more of a hopping than walking gait [18].

The two most influential models for gait analysis for bipedal, quadrupedal, and multi-legged locomotion are the rigid inverted pendulum (RIP) model for walking, where the energy oscillations are out of phase, and the spring-loaded inverted pendulum (SLIP) model for running, where the energy oscillations are in phase [18]. Both methods model the subject as a point mass with oscillating legs, which allows the neglect of rotations around the CoM [13]. In bipedal gaits, such as humans do, the leading foot represents the braking force and serves as an anchor point for the body's next movement. In contrast, the trailing foot represents the propulsive force that adds energy to the system to vault over this same anchor point. The CoM reaches its lowest point in mid-stance when running, i.e. the minimum potential energy, and the provisional highest point when walking, i.e. the maximum potential energy. Since braking occurs in the first half of a leg's stance phase and propulsion in the second half, the CoM velocity, i.e. the kinetic energy, reaches its minimum at mid-stance for both walking and running [18].

2.2.1 Spring-Loaded Inverted Pendulum

The SLIP model treats the phase of the stride in which the CoM is vaulting over the stance foot as an inverted pendulum with springs added inline to the massless legs. Since the springs are able to store energy during collision with the ground (i.e. during heel strike) and return it to the CoM during toe off, this model is typically found in running gaits [3]. This spring-based, "bouncy" gait can also be seen in animals that frequently hop, such as kangaroos (Solis, 2020). SLIP-like gaits include bipedal running and hopping, as well as quadrupedal and multi-legged trotting, often described as "bouncing" gaits as the greatest leg compression occurs at about the same time as the greatest vertical force (McMahon and Cheng, 1990). When physical springs are present, energy savings can be achieved via elastic storage and proportional return of absorbing and generative work performed by muscles or actuators [3].

2.2.2 Rigid Inverted Pendulum

According to the conventional interpretation of "two basic mechanisms", which states that the potential energy tends to reach a maximum near mid-stance during walking, it is sufficient to describe walking dynamics with a RIP model. Nonetheless, experimental studies show that bipedal and quadrupedal walking dynamics (Lee and Farley, 1998; Griffin et al., 2004, Genin et al., 2009) do not reflect this well. This is unsurprising given that an actual rigid inverted pendulum (i.e., a mass on a massless rod of fixed length) would show a peak vertical force instead of a minimum vertical force in the mid-stance position, as described by Geyer, Seyfarth, and Blickhan (2006).

2.2.3 Bipedal Spring-Loaded Inverted Pendulum

The same authors propose an alternative compliant-legged model of walking called the bipedal spring-loaded inverted pendulum (BSLIP), which reproduces the characteristic m-shaped force with a minimum at mid-stance by providing a spring-loaded leg that introduces compliance. The BSLIP also includes summation of leading and trailing leg forces during double support of the step-to-step transition. Although this revealing model is widely used and highly cited, the BSLIP has yet to upend the RIP model of walking in most textbook explanations. This may be partly because the BSLIP is more challenging to simulate and perhaps also because its conservative leg springs limit its ability to match the full range of human walking speeds (Lipfert et al., 2012). Nonetheless, a recent bipedal robot demonstrates SLIP-like running and BSLIP- like walking, using the same spring-loaded legs (Hubicki et al., 2018). In principle, the BSLIP model is more correct than the unrealistic impulsive step-to-step transition of the RIP model because it captures the m-shaped force profile and allows for double support [18].

2.2.4 Mechanical Cost Analysis

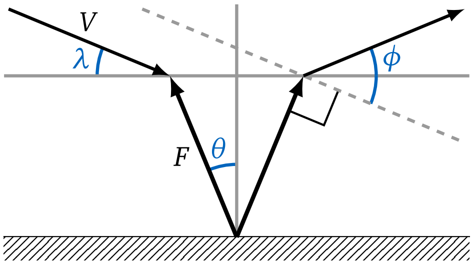
The MCA is a collision-based approach developed by Lee et al., which uses the same point mass model as SLIP-based methods, but describes the locomotion directly by analyzing the force and velocity vectors. The advantage of this is that, unlike to SLIP-based approaches, which are only approximations of a gait, no *a priori* model needs to be known, which holds the potential to distinguish different gaits as well as to discern their defined characteristics [16].

Fundamental Determinants of Center of Mass Dynamics

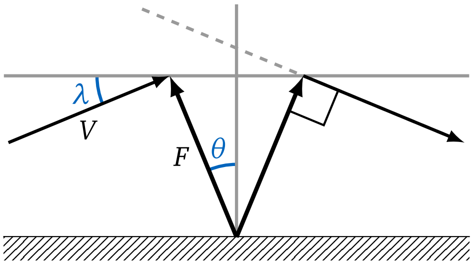
The central concept of MCA is D'Alembert's (1743) 'principle of orthogonal constraint', which shows that a mass can be redirected without mechanical work, as long as the constraint (i.e. the force vector) is perpendicular to the path (i.e. the velocity vector), such that their dot-product (i.e. the mechanical power) is zero [6]. However, this theoretical redirection with zero work cannot be implemented in real legged systems as terrestrial locomotion requires intermittent, discrete footfalls, which, among other things, constrains their ability to exert orthogonal forces by a leg's position with respect to the CoM, their kinematic range of motion and their force-torque capacity [18]. These "inelastic" collisions [15] by the limb with the ground preclude a consistent orthogonal relationship between the force and velocity vector, as their corresponding instantaneous angles θ (relative to vertical) and λ (relative to horizontal) [17] are of the same sign[2], as shown in Figure 2.3a. This results in a non-zero collision angle ϕ and abrupt, collision like changes in the CoM direction, which require mechanical work [16]. However, in the theoretical case, if the two vectors are perpendicular to each other, $\phi = 0$, meaning the angles θ and λ are equal and of opposite sign, which in turn means that no collision occurs, thus no work is done at the CoM, as for a wheel without rim but infinite spokes [2], as shown in 2.3b.
To enforce the principle of orthogonal constraint, that is, to keep the force and velocity vectors of the CoM as orthogonal as possible, both metrics must be measured at each instant of the stride. If this orthogonal relationship is violated, either generative or absorptive costs are incurred, depending on the sign of the collision angle ϕ, which is the summation of the force and velocity angles θ and λ. If ϕ is less than 90 degrees, the cost is generative, i.e., the two vectors point somewhat in the same direction and energy can be applied to move forward. Conversely, if the angle is greater than 90 degrees, the cost and energy are absorptive, as is the case with deceleration. Theoretically, provided both vectors are always exactly 90 degrees out of phase, it would hereby be possible to redirect the CoM with zero work [18]. The power of the limb acting on the CoM of the body, i.e. the external mechanical power, can thus be quantified as

$$P_{mech} = F \cdot V sin(\phi) \tag{2.1}$$

[2]This is consistent with compliant SLIP mechanics



(a) Compliant SLIP (i.e. $\phi \neq 0$ and $\kappa = 1$)



(b) Zero-collision (i.e. $\phi = 0$ and $\kappa = 0$)

Figure 2.3: Schematic representation of the mechanical cost analysis (MCA). Shown are the force vector F and velocity vector V with their corresponding angles, θ and λ of two isolated, hypothetical strides. The collision angle ϕ is the deviation of the orthogonal relation between F and V. The collision reduction is quantified by the collision fraction κ. Figure 2.3a shows the case for a compliant SLIP, figure 2.3b the idealized zero-collision case. (Source: modified adopted from [16], p. 4)

where F denotes the respective ground reaction force (GRF) vector, i.e. the external force acting on the limb, and V the velocity vector of the CoM. The mechanical work performed corresponds to the cumulative time integral over the duration of the impulse of the external mechanical power [16] [7]:

$$W_{mech} = \int P_{mech} dt. \tag{2.2}$$

Center of Mass Velocities

Another common dimensionless unit for comparing moving objects is the Froude number (Fr). The transition from walking to running happens at $Fr = 0.5$, as shown by [1]. The system has $Fr = 0.1409$, which is much lower than the transition value, meaning that the average walking speed can be improved by a better choice of parameters. The dimensionless velocity is the square-root of the Froude number.

$$\sqrt{Fr} = \sqrt{\frac{\bar{V}_y^2}{gh}} = \frac{\bar{V}_y}{\sqrt{gh}} \tag{2.3}$$

where $\bar{V}_y$ depicts the average forward velocity, g the acceleration due to gravity and h the height of the CoM [16].

Collision-Based Angles

The instantaneous collision angle ϕ is the deviation of perpendicularity of force and velocity vectors of the CoM [16], which is measured at each instance of the step and given by

$$\phi = arcsin\left(\frac{\sum |F \cdot V|}{\sum |F||V|}\right). \tag{2.4}$$

The collision angle over the contact periods of the entire stride Φ is given by the weighted average of ϕ, where the weights represent the magnitude of the force and velocity vectors at each instant:

$$\Phi = \frac{\sum |F||V|\phi}{\sum |F||V|}. \tag{2.5}$$

Substituting using the small angle approximation of equation 2.4 into equation 2.5, that is, when only small vertical undulations and fore-aft forces appear, shows that the collision angle Φ is a close approximation to the mechanical cost of transport (CoT_{mech}) [16] as can be seen in

$$CoT_{\text{mech}} = \frac{\sum |F \cdot V|}{n\bar{V}_y mg} = \frac{\sum |F \cdot V|}{\sum |F||V|} \tag{2.6}$$

where the CoT_{mech} is a dimensionless metric of the normalized mechanical power during the contact period of the gait when the limb redirects the CoM [17], i.e. the mechanical work at the center of mass (CoM) required to move a unit body weight a unit distance in the direction of travel [16] [5] and n is the number of samples in the stride period.

Collision Fraction

The collision reduction is directly correlated to the collision angle Φ and quantified by the collision fraction κ, which gets small if either the average? velocity angle Λ[3] or the force angle Θ is small, if there is a near perpendicularity of the velocity vector V and the force vector F throughout the stride or any combination thereof. The collision fraction is the actual collision relative to potential collision:

$$\kappa = \frac{\Phi}{\Lambda + \Theta}. \tag{2.7}$$

For compliant SLIP, $\phi = \lambda + \theta$, since the braking force yields a non-perpendicular angle with downward velocity whereas the propulsive force yields one with upward velocity, resulting in an instantaneous $\kappa = 1$, shown in figure 2.3a. Whenever F and V are oriented in the same direction, collisions are reduced and $\kappa < 1$ up to the idealized case in which F and V remain orthogonal throughout the entire stride and thus $\phi = |\lambda - \theta|$, leading to $\kappa = 0$, shown in 2.3b [16].

2.3 Trajectory Optimization

The term trajectory refers to the path an agent travels as a function of time. The term trajectory optimization is therefore the set of methods used to obtain the best trajectory, usually by selecting appropriate inputs to the system, i.e. controls, as functions of time. The comprehensive policy of optimization is to minimize the objective function subject to a number of constraints and restrictions.

Assumptions

Within this work it is assumed that the trajectory is single-phase and of continuous time, meaning the system dynamics are continuous throughout the entire trajectory. The dynamics,

[3] Which tends to be greater in SLIP-like gaits such as running, hopping or trotting due to increased vertical oscillations of the CoM during these "bouncing gaits" [17].

objective and constraints are smooth, consistent and potentially non-linear. Furthermore, the robot is considered to be left-right symmetric, allowing to search for a periodic walking gait with a single step instead of a stride[4]. A periodic gait requires that the joint trajectories, consisting of the joint angles, their rates and the associated torques, are the same for each successive step [14].

Constraints

The first and possibly most important constraint~~,~~ is the system dynamics. In addition, limits are defined for the boundary condition, restricting the initial and final states of the system, including upper and lower limits for the joint angles as well as voltage limits for the motors.

Nonlinear Programming

Most direct collocation methods transcribe a continuous-time trajectory optimization problem into a nonlinear program. A nonlinear program is a special name given to a constrained parameter optimization problem that has nonlinear terms in either its objective or its constraint function. A typical formulation for a nonlinear program is as follows:

$$\begin{aligned} \min_x \quad & f(x) \\ \text{subject to} \quad & h(x) = 0 \\ & g(x) \leq 0, \\ & x_l \leq x \leq x_u \end{aligned} \tag{2.8}$$

where x denotes the vector of design variables, $f(x)$ the objective function, $h(x)$ the equality constraint functions, $g(x)$ the inequality constraint functions and x_l, x_u the lower and upper bounds, respectively.

System Dynamics

During single stance, the system has five degrees of freedom (DoF): the absolute angles of both lower legs, i.e. of the knee joints (q_1 and q_5), both upper legs, i.e. of the hip joints (q_2 and q_4) as well as the torso (q_3), cumulated into the single vector q. Since it is a second order dynamical system, the derivative of the configuration, $\dot{q}$ must also be included. Thus, the state and dynamics can be described as:

$$x = \begin{bmatrix} q \\ \dot{q} \end{bmatrix}, \quad \dot{x} = f(x,u) = \begin{bmatrix} \dot{q} \\ \ddot{q} \end{bmatrix} \tag{2.9}$$

where q are the angles, $\dot{q}$ the angular rates and $\ddot{q}$ the accelerations, respectively.

[4] A stride consists of two consecutive steps.

Objective Function

To ensure that the walking gait is periodic, the sum of absolute deviations between the initial and final state is used as cost function:

$$f = \sum_{i=1}^{5} |q(T)_i - q(0)_i|. \tag{2.10}$$

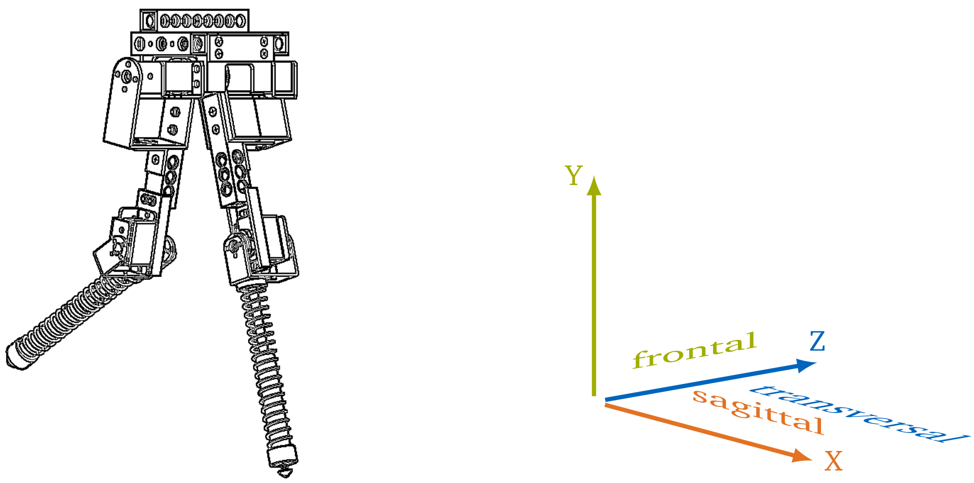
Choosing an appropriate cost function is desirable to obtain smooth, well-behaved solutions, ensuring good convergence of the nonlinear program. Among others, the CoT is widely used, which, however, is difficult to optimize over because the solutions tend to have discontinuities [14].

Chapter 3

Related work

3.1 The JenaFox Bipedal Walking Robot

The robotic test environment is the JenaFox robot, which is shown schematically in figure 3.1a. The bipedal robot consists of a torso connected to two segmented legs, each of which has an upper and a lower link. When taking a step, the stance leg supports the weight of the robot, while the swing leg is free to move above the ground. All limbs are connected via actuators, with sensors in each joint measuring angular position and velocity, except for the ankle joint, which is the only passive joint of the system. The trunk of the robot is attached to a boom via a freely rotating joint.



(a) Schematic representation of the JenaFox robot.

(b) Definition of the associated coordinate system.

Figure 3.1: Schematic representation of the JenaFox bipedal walking robot (a) with associated planes of motion (b). The X-axis points in the direction of motion, the Y-axis points upward. The planar motion is restricted to the sagittal plane (orange).

The tether mechanism constrains the motion of the robot on a sphere, without excessively ~~affect~~ its dynamics in the sagittal plane. The mechanism consists of an aluminum tube, a spherical pivot fixed to the floor and a tension cable. The tether is instrumented to provide measurements of the machine's three motions: vertical translation, forward translation, and rotation about the axis of the boom [19] [20].

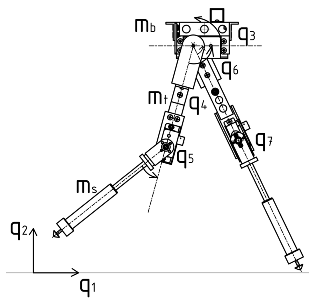


Figure 3.2: Degrees of freedom of the basic mechanical setup. The trunk (mass m_b) has three degrees of freedom (q_1, q_2, q_3), hip and knee of the right leg connect the thigh (m_t) to the body and shank (m_s) to the thigh respectively by one degree of freedom each (q_4, q_5), the same counts for the left side (q_6, q_7). (Source: modified adopted from [20], p.37)

Chapter 4

Solution approach

Chapter 5

Evaluation

Chapter 6

Future work

Chapter 7

Conclusion

Appendix A

Appendix 1

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