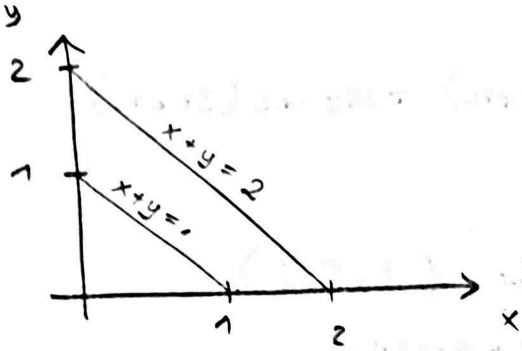
EX 4 p 1

$Z = X + Y$, $p_{X,Y} = p_X \cdot p_Y$

Show that $p_Z(z) = \int_{x\in\mathcal{X}} p_X(x)\, p_Y(z-x)\, dx$

$= \int_{y\in\mathcal{Y}} p_Y(y)\, p_X(z-y)\, dy$.



$z = x + y \Rightarrow y = z - x$

$\int_{x\in\mathcal{X}} p_{X,Y}(x,y)\, dx = p_Y(y)$

"chose z"   chose x ; sum over all possible x
$y = f(z,x) = z - x$

(V1) $\mathbb{P}[X+Y=z] = \sum_x \mathbb{P}[X=x, Y=z-x]$

DISCRETE $p_Z(z) = \sum_x p_{X,Y}(x, z-x) = \sum_x p_X(x)\, p_Y(z-x)$

CONT. $p_Z(z) = \int_x p_{X,Y}(x, z-x)\, dx = \int_x p_X(x)\, p_Y(z-x)\, dx$

$F_Z(z) = \mathbb{P}[X+Y \le z] = \int_{x=-\infty}^{\infty} \int_{y=-\infty}^{z-x} f_{X,Y}(x,y)\, dx\, dy$

(V2)

$f_Z(z) = \frac{dF_Z(z)}{dz} = \frac{d}{dz} \int_{x=-\infty}^{\infty} \int_{y=-\infty}^{z-x} f_X(x)\, f_Y(y)\, dx\, dy$

$= \int_{x\in\mathcal{X}} f_X(x)\, \frac{d}{dz} \overbrace{\int_{y=-\infty}^{z-x} f_Y(y)\, dy}^{F_Y(z-x)}\, dx$

LEIBNITZ
RUL

$\frac{d}{dz} \int_{g_1(z)}^{g_2(z)} f_Y(y)\, dy = f_Y(g_2(z))\, g_2'(z) - f_Y(g_1(z))\, g_1'(z)$

$= (*)$ $\frac{d}{dz} F_Y(z-x) = f_Y(z-x)$

läuft aufs Vgleiche

EX 4 p2

$g_2(z) = z - x \quad , \quad g_1(z) = -\infty$ ~ unsicher, ob das geht

$(*) = f_Y(z-x)\cdot 1 - f_Y(-\infty)\cdot 0$ $\quad g_1(z) = a$

$\lim_{a\to\infty}$ wrsl...

$\Rightarrow f_Z(z) = \int_{x\in\mathcal{X}} f_X(x) f_Y(z-x)\,dx \rightarrow$ and vice versa for y

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(V3) $\quad g: \mathbb{R} \to \mathbb{R} \qquad z = x+y \qquad g(x,y) = x+y$

$y = z - x$

$y \longrightarrow \boxed{g(x,y)} \longrightarrow z$

$g^{-1}(x,z) = z - x$

$f_{XY}(x,y) = f_X(x)\, f_Y(y) \qquad f_Y(y) = \int_{x\in\mathcal{X}} f_{XY}(x,y)\,dx$

$$f_Z(z) = \int_x f_{X,Y}(x, z-x) \left| \frac{d}{dz}\left(g^{-1}(x,z)\right)\right| dx$$

$$= \int_x f_{X,Y}(x, z-x)\,dx$$

$$= \int_x f_X(x)\, f_Y(z-x)\,dx$$

fix a value of x and then change of variable formula

würd das nehmen

Leibniz Rule:

$$\frac{d}{dx}\left(\int_{a(x)}^{b(x)} f(x,t)\,dt\right) = f(x, b(x))\frac{d}{dx}b(x) - f(x, a(x))\frac{d}{dx}a(x) + \int_{a(x)}^{b(x)} \frac{\partial}{\partial x} f(x,t)\,dt$$

EX 4 p3

$$f_{X+Y}(\xi) = f_X(\xi) * f_Y(\xi)$$

$$\phi_{X+Y}(\omega) = \phi_X(\omega)\,\phi_Y(\omega)$$

$$\ln(\phi_{X+Y}(\omega)) = \ln(\phi_X(\omega)) + \ln(\phi_Y(\omega))$$

$$\left.\frac{d^p \ln(\phi_{X+Y}(\omega))}{d(j\omega)^p}\right|_{\omega=0} = c_{X+Y,p} \quad \ldots \text{ CUMULANT}$$

$$\left.\frac{d^p}{d(j\omega)^p} \ln(\phi_{X+Y}(\omega))\right|_{\omega=0} = \left.\frac{d^p}{d(j\omega)^p}\Big[\ln(\phi_X(\omega)) + \ln(\phi_Y(\omega))\Big]\right|_{\omega=0}$$

$$c_{X+Y,p} = c_{X,p} + c_{Y,p}$$