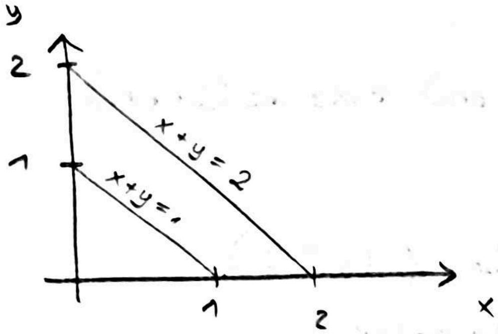
EX 4 p1

$Z = X + Y$, $p_{X,Y} = p_X \cdot p_Y$

Show that $p_Z(z) = \int_{x \in \mathcal{X}} p_X(x)\, p_Y(z-x)\, dx$

$= \int_{y \in \mathcal{Y}} p_Y(y)\, p_X(z-y)\, dy$



$z = x + y \Rightarrow y = z - x$

$\int_{x \in \mathcal{X}} p_{X,Y}(x,y)\, dx = p_Y(y)$

"chose z" ; chose x ; sum over all possible x $y = f(z,x) = z - x$

V1 $\mathbb{P}[X + Y = z] = \sum_x \mathbb{P}[X = x, Y = z - x]$

DISCRETE $p_Z(z) = \sum_x p_{X,Y}(x, z-x) = \sum_x p_X(x)\, p_Y(z-x)$

CONT. $p_Z(z) = \int_x p_{X,Y}(x, z-x)\, dx = \int_x p_X(x)\, p_Y(z-x)\, dx$

$F_Z(z) = \mathbb{P}[X + Y \leq z] = \int_{x=-\infty}^{\infty} \int_{y=-\infty}^{z-x} f_{X,Y}(x,y)\, dx\, dy$

V2

$f_Z(z) = \frac{dF_Z(z)}{dz} = \frac{d}{dz} \int_{x=-\infty}^{\infty} \int_{y=-\infty}^{z-x} f_X(x) f_Y(y)\, dx\, dy$

$= \int_{x \in \mathcal{X}} f_X(x) \underbrace{\frac{d}{dz} \int_{y=-\infty}^{z-x} f_Y(y)\, dy}\, dx$

LEIBNITZ RUL

$\frac{d}{dz} \int_{g_1(z)}^{g_2(z)} f_Y(y)\, dy = f_Y(g_2(z))\, g_2'(z) - f_Y(g_1(z))\, g_1'(z)$

$= (*)$

EX 4 p2

$g_2(z) = z - x \quad , \quad g_1(z) = -\infty$ — unsicher, ob das geht

$(*) = f_Y(z-x)\cdot 1 - f_Y(-\infty)\cdot 0$ $\qquad g_1(z) = a$

$\lim_{a\to\infty}$ wrsl...

$$\Rightarrow f_Z(z) = \int_{x\in\mathcal{X}} f_X(x) f_Y(z-x)\,dx \to \text{and vice versa for } y$$

(V3) $\quad g: \mathbb{R} \to \mathbb{R} \qquad z = x + y \qquad g(x,y) = x+y$

$y = z - x \qquad y \xrightarrow{\quad} \boxed{g(x,y)} \xrightarrow{\quad} z$

$g^{-1}(x,z) = z - x$

$f_{XY}(x,y) = f_X(x)\, f_Y(y) \qquad f_Y(y) = \int_{x\in\mathcal{X}} f_{XY}(x,y)\,dx$

$$f_Z(z) = \int_x f_{X,Y}(x, z-x) \left| \frac{d}{dz}\left(g^{-1}(x,z)\right)\right| dx$$

$$= \int_x f_{X,Y}(x, z-x)\,dx$$

$$= \int_x f_X(x)\, f_Y(z-x)\,dx$$

fix a value of x and then change of variable formula

würd das nehmen

Leibniz Rule:

$$\frac{d}{dx}\left(\int_{a(x)}^{b(x)} f(x,t)\,dt\right) = f(x, b(x))\frac{d}{dx} b(x) - f(x, a(x))\frac{d}{dx} a(x) + \int_{a(x)}^{b(x)} \frac{\partial}{\partial x} f(x,t)\,dt$$

$$f_{X+Y}(\xi) = f_X(\xi) * f_Y(\xi)$$

$$\Phi_{X+Y}(\omega) = \Phi_X(\omega)\,\Phi_Y(\omega)$$

$$\ln(\Phi_{X+Y}(\omega)) = \ln(\Phi_X(\omega)) + \ln(\Phi_Y(\omega))$$

$$\left.\frac{d^p \ln(\Phi_{X+Y}(\omega))}{d(j\omega)^p}\right|_{\omega=0} = C_{X+Y,p} \quad \text{... CUMULANT}$$

$$\left.\frac{d^p}{d(j\omega)^p} \ln(\Phi_{X+Y}(\omega))\right|_{\omega=0} = \left.\frac{d^p}{d(j\omega)^p}\left[\ln(\Phi_X(\omega)) + \ln(\Phi_Y(\omega))\right]\right|_{\omega=0}$$

$$C_{X+Y,p} = C_{X,p} + C_{Y,p}$$