作业2 SA19225404 吴语港

写代码生成一个包含200条数据的数据集(数据集的具体要求见步骤1)), 再按照步骤2)~步骤4)的要求生成指定模型,训练模型、 并分别画出训练过程中模型在训练集和验证集上的学习曲线, 利用学习曲线判断模型是欠拟合状态、过拟合状态,还是表现正好。

```
In [35]: #准备工作
import d21zh as d21
#numpy
import numpy as np
#数据预处理
from sklearn import preprocessing
#需要先pip好mxnet, 再导入gluon, nd
from mxnet import autograd, gluon, nd
#导入数据,起小名
from mxnet.gluon import data as gdata, loss as gloss, nn
```

1)生成一个人工数据集。在训练数据集和测试数据集中,给定样本特征x,我们使用如下的三阶多项式函数来生成该样本的标签: $y = 1.2x - 3.4x2 + 5.6x3 + 5 + \epsilon$ 其中噪声项 ϵ 服从均值为0、标准差为0.1的正态分布。训练数据集和测试数据集的样本数都设为100。

```
In [49]: #训练集大小,测试集大小,多项式的系数(权重数组),常数项偏置
       n train, n test, true w, true b = 100, 100, [1.2, -3.4, 5.6], 5
       #【200x1矩阵】一维随机数正态分布,参数初始化,做输入x也就是features,一阶拟合
       features = nd.random.normal(shape=(n train + n test, 1))
       #【200×3矩阵】一次项,二次项,三次项,不带系数(权重),可以看成中间变量,三阶拟合
       poly features = nd.concat(features, nd.power(features, 2),nd.power(features, 3
       #【200x10矩阵】十阶拟合使用,容易nan,需要标准化或者调低学习率
       ten features = nd.concat(features, nd.power(features, 2),nd.power(features, 3
       ),
                            nd.power(features, 4),nd.power(features, 5),
                            nd.power(features, 6), nd.power(features, 7),
                            nd.power(features, 8),nd.power(features, 9),nd.power(
       features, 10))
       #标准化
       # a = nd.array(ten features, dtype=np.float64)
       # a = np.concatenate(a, axis=0)
       # preprocessing.scale(a)
       #【1×200矩阵】三阶多项式函数生成数据,带上系数计算,并加上常数偏置,进行求和
       #【1x200】加上方差为0.1的正态分布噪声
       labels += nd.random.normal(scale=0.1, shape=labels.shape)
```

```
# 【1x200】加上方差为0.1的正态分布噪声
labels += nd.random.normal(scale=0.1, shape=labels.shape)

In [37]: #查看样本全体,输入与输出
features[:], poly_features[:], ten_features[:], labels[:]

Out[37]: (
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-5.08783273e-02	-		
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[-2.26623344e+00	[2.75164723	e-01 7.57156238e-0	02 2.08342690e-02]
[-1.05594598e-01	[3.17283385	e-02 1.00668741e-0	03 3.19405190e-05]
[7.00263977e-01	[-2.26623344	e+00 5.13581419e+0	00 -1.16389532e+01]
[3.08013380e-01	[-1.05594598	e-01 1.11502195e-0	02 -1.17740291e-03]
[-5.78222096e-01	[7.00263977	e-01 4.90369648e-0	01 3.43388200e-01]
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[6.93279028e-01	[-5.78222096	e-01 3.34340781e-0	01 -1.93323240e-01]
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[-1.28509682e-02	[4.80031222	e-01 2.30429977e-0	01 1.10613585e-01]
[8.86878669e-01	[1.52610421	e+00 2.32899404e+0	00 3.55428767e+00]
[-3.11207384e-01	[-1.28509682	e-02 1.65147387e-0	04 -2.12230384e-06]
[1.18732512e+00	[8.86878669	e-01 7.86553800e-0	01 6.97577775e-01]
[3.75856906e-01	[-3.11207384	e-01 9.68500376e-0	02 -3.01404465e-02]
[3.12301755e-01	[1.18732512	e+00 1.40974092e+0	00 1.67382085e+00]
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[5.02259791e-01	[3.12301755	e-01 9.75323841e-0	02 3.04595362e-02]
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[1.15118575e+00	[5.02259791	e-01 2.52264887e-0	01 1.26702517e-01]
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[-5.50888956e-01 3.03478628e-01 -1.67183027e-01] [-5.30499160e-01 2.81429350e-01 -1.49298042e-01] [-4.11141425e-01 1.69037268e-01 -6.94982260e-02] [2.63828188e-01 6.96053132e-02 1.83638427e-02] [-6.45308495e-01 4.16423053e-01 -2.68721342e-01] [6.84398651e-01 4.68401521e-01 3.20573360e-01] [-4.89032641e-02 2.39152927e-03 -1.16953583e-04] [2.32864805e-02 5.42260183e-04 1.26273308e-05] [-1.60148084e-01 2.56474093e-02 -4.10738355e-03] [-7.80184686e-01 6.08688116e-01 -4.74889159e-01] [1.20582856e-01 1.45402253e-02 1.75330194e-03]	[3.45989734		
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   In [38]: #定义作图函数, y轴为对数尺度
            #x1值, y1值, x标注, y标注, x2值, y2值, 图例, 窗口大小
           def semilogy(x vals, y vals, x label, y label, x2 vals=None, y2 vals=None,
                       legend=None, figsize=(3.5, 2.5)):
               d21.set figsize(figsize)#设置窗口大小默认3.5*2.5
               d21.plt.xlabel(x label)#显示x轴意义
               d21.plt.ylabel(y label)#显示y轴意义
               d21.plt.semilogy(x vals, y vals)#进行第一条曲线的绘制
               if x2 vals and y2 vals: #如果有第二条曲线,就要画第二条线,并显示图例
                   d21.plt.semilogy(x2 vals, y2 vals, linestyle=':')#进行第二条曲线的绘制
                   d21.plt.legend(legend)#显示图例
   In [61]: #定义训练+显示一体化函数
           #批次数,误差,
           num epochs, loss = 100, gloss.L2Loss()
           #参数列表:训练输入,测试输入,训练输出,测试输出,学习率
           def fit and plot(train features, test features, train labels, test labels, lear
           nrate ):
               net = nn.Sequential()#一个有序的容器
               net.add(nn.Dense(1))
               net.initialize()
               batch size = min(10, train labels.shape[0])#批大小超过10个按10个记
               train iter = gdata.DataLoader(gdata.ArrayDataset(
                   train features, train labels), batch size, shuffle=True)
               #训练器采用随机梯度下降SGD训练,学习率为learnrate做输入参数
               trainer = gluon.Trainer(net.collect params(), 'sgd', {'learning rate': lear
           nrate })
               train ls, test ls = [], []#定义损失初值
               for in range (num epochs):#不同训练批次
                   for X, y in train iter:#监督学习不同的特征与标签
                      with autograd.record():
                          l = loss(net(X), y)
                      1.backward()
                      trainer.step(batch size)
                   #训练,每一批后的损失值,并外挂到train ls
                   train ls.append(loss(net(train features),
                                      train labels).mean().asscalar())
                   #测试,每一批后的损失值,并外挂到test 1s
                   test ls.append(loss(net(test features),
                                     test labels).mean().asscalar())
               #最后的训练与测试误差
               print('final epoch: train loss', train ls[-1], 'test loss', test ls[-1])
Loading [MathJax]/jax/output/HTML-CSS/jax.js 筷的结果,不同批次测试的结果,显示窗口
```

4.736012

4.519102

-11.413436

5.013267

5.7106915

-7.6337395

5.839211

1.9551197

5.0817366

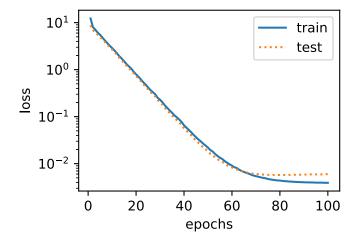
4.536426

- 2) 先使用与数据生成函数同阶的三阶多项式函数拟合。
- ①打印出最佳参数的取值,与真实参数值进行对比,并对结果进行分析点评;
- ②尝试画出训练过程中模型分别在训练集和验证集上的学习曲线;观察分析这2条学习曲线,判断该模型是欠拟合,还是过拟合,还是表现很好?请说明理由。
- ③尝试使用不同的学习率η(四种情况:过大,过小,正好),画出不同的学习率η下,模型在训练集上的学习曲线;对比观察这三条学习曲线的走势,分析学习率对训练过程的影响。

```
In [40]: #三阶拟合(模型复杂度刚刚好,学习率也刚刚好,最合适)学习率0.01 fit_and_plot(poly_features[:n_train, :], poly_features[n_train:, :], labels[:n_train], labels[n_train:], 0.01)
```

final epoch: train loss 0.003913792 test loss 0.0059956 weight: [[1.2012036 -3.4033813 5.6022115]]

bias: [5.0179133]



```
In [43]: #三阶拟合(模型复杂度刚刚好,学习率过大导致发散) 学习率0.1 fit_and_plot(poly_features[:n_train, :], poly_features[n_train:, :], labels[:n_train], labels[n_train:], 0.1)
```

final epoch: train loss 4.5542353e+34 test loss 1.6489693e+34 weight: [[-7.1200714e+16 6.7360635e+16 -4.5346428e+16]]

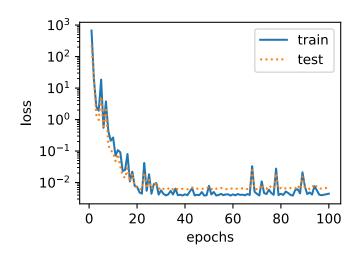
bias: [5.350943e+16]

In [44]: #三阶拟合(模型复杂度刚刚好,学习率稍大学习曲线有点震荡)学习率0.05 fit_and_plot(poly_features[:n_train, :], poly_features[n_train:, :], labels[:n_train], labels[n_train:], 0.05)

final epoch: train loss 0.0044334345 test loss 0.006539675

weight: [[1.2146236 -3.4069202 5.606834]]

bias: [5.0215087]

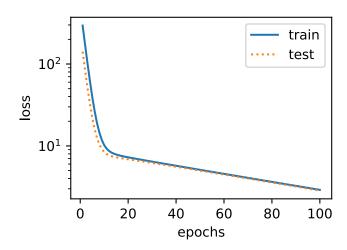


In [12]: #三阶拟合(模型复杂度刚刚好,不过过小的学习率导致学习太慢了)学习率0.001 fit_and_plot(poly_features[:n_train, :], poly_features[n_train:, :], labels[:n_train], labels[n_train:], 0.001)

final epoch: train loss 2.9103785 test loss 2.8517141

weight: [[0.9256708 -2.3574617 5.750919]]

bias: [1.9527975]



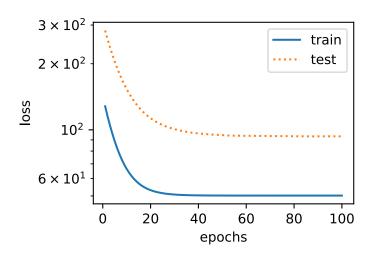
3) 再试试线性函数拟合。

尝试画出训练过程中模型分别在训练集和验证集上的学习曲线;观察分析这2条学习曲线,判断该模型是欠 拟合,还是过拟合,还是表现很好?请说明理由,并指出改进方案。

In [50]: #线性拟合(模型太简单,会导致欠拟合)训练和测试的loss都很差

final epoch: train loss 50.17723 test loss 93.4331

weight: [[14.462653]] bias: [2.2241619]



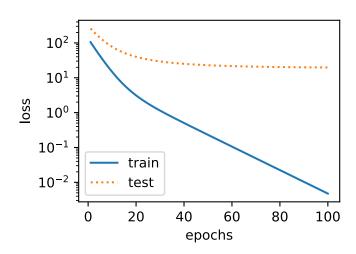
- 4) 使用三阶多项式函数模型来拟合,请注意,这次,只使用2个样本来训练模型。
- ①打印出最佳参数的取值,并与真实参数值进行对比,并对结果进行分析点评。
- ②尝试画出训练过程中模型分别在训练集和验证集上的学习曲线;观察分析这2条学习曲线,判断该模型是欠拟合,还是过拟合,还是表现很好?请说明理由,并指出改进方案。

In [51]: #三阶拟合,训练样本不足(欠拟合)需要更多的训练数据,后面的曲线应该是延伸的

final epoch: train loss 0.004758147 test loss 19.740759

weight: [[2.4169416 -0.21812136 4.527212]]

bias: [-0.44129366]



5) 使用10阶多项式函数模型来拟合。

- ②尝试画出训练过程中模型分别在训练集和验证集上的学习曲线;观察分析这2条学习曲线,判断该模型是欠拟合,还是过拟合,还是表现很好?请说明理由,并指出改进方案。
- ③尝试利用L2正则化技术,让该模型的表现好起来。画出训练过程中模型分别在训练集和验证集上的学习曲线,观察分析这2条学习曲线,判断该模型是欠拟合,还是过拟合,还是表现很好?并说明理由。
- ④打印出③中模型的最佳参数,并与①进行对比,尝试总结下L2正则化技术所带来的效果,并分析为何L2正则化技术可以解决过拟合问题。
- ⑤ 在③中使用不同的正则化强度,打印出不同正则化强度下的最佳参数,并对结果进行分析点评。

final epoch: train loss 41.39038 test loss 367.50558 weight: [[-0.06136507 0.00243929 -0.0610089 -0.00351013 0.05785299 -0.043 26928

0.0338949 -0.00164474 0.04598321 0.00667603]] bias: [6.871514e-05]

10² train test

epochs

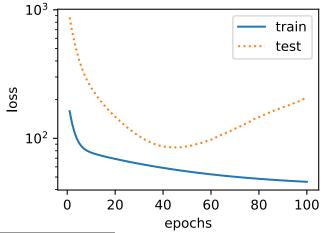
可以看出来最佳参数的取值,与真实参数值出入较大,而且有过拟和的现象,一般解决这种问题的方法如下

- 1.正则化方法,也就是权重衰减
- 2.加大训练数据
- 3.去掉神经网络的一部分进行训练,也就是丢弃法
- 4.提前终止

In [57]: #加入了L2正则化的拟合函数 #批次数, 误差.

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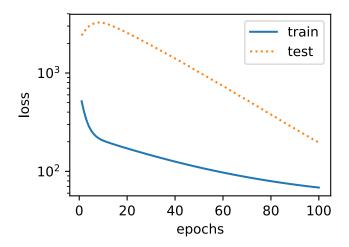
```
num epochs, loss = 100, gloss.L2Loss()
         #参数列表:训练输入,测试输入,训练输出,测试输出,学习率,惩罚项系数
        def fit and plot L2(train features, test features, train labels, test labels, l
        earnrate ,lambd):
            net = nn.Sequential()#一个有序的容器
            net.add(nn.Dense(1))
            net.initialize()
            batch size = min(10, train labels.shape[0])#批大小超过10个按10个记
            train iter = gdata.DataLoader(gdata.ArrayDataset(
                train features, train labels), batch size, shuffle=True)
            #训练器采用随机梯度下降SGD训练,学习率为learnrate做输入参数
            trainer = gluon.Trainer(net.collect params(), 'sgd', {'learning rate': lear
        nrate })
            train ls, test ls = [], []#定义损失初值
            for in range(num epochs):#不同训练批次
                for X, y in train iter:#监督学习不同的特征与标签
                   with autograd.record():
                       # 添加了L2范数惩罚项
                       1 = loss(net(X), y) + lambd * 12 penalty(net[0].weight.data().
        asnumpy())
                    1.backward()
                    trainer.step(batch size)
                #训练,每一批后的损失值,并外挂到train 1s
                train ls.append(loss(net(train features),
                                    train labels).mean().asscalar())
                #测试,每一批后的损失值,并外挂到test ls
                test ls.append(loss(net(test features),
                                   test labels).mean().asscalar())
            #最后的训练与测试误差
            print('final epoch: train loss', train ls[-1], 'test loss', test ls[-1])
            #不同批次训练的结果,不同批次测试的结果,显示窗口
            semilogy(range(1, num epochs + 1), train ls, 'epochs', 'loss',
                     range(1, num epochs + 1), test ls, ['train', 'test'])
            print('weight:', net[0].weight.data().asnumpy(),#输出训练后得出的系数(权重)
                  '\nbias:', net[0].bias.data().asnumpy())#输出训练后得出的常数项(偏置)
In [58]: # 十阶拟合(模型太复杂)加入了L2惩罚项, lambd=1, 还是有过拟和
        fit and plot L2(ten features[:n train, :],ten features[n train:, :], labels[:n
         train],
                    labels[n train:], 0.00000004, 1)
        final epoch: train loss 45.867153 test loss 205.78508
        weight: [[ 0.05440721  0.01373051 -0.00670504 -0.03651985  0.05950128  0.012
        35772
          -0.03963249 -0.04367105 0.04960401 0.0093249 ]]
        bias: [8.3577805e-05]
```



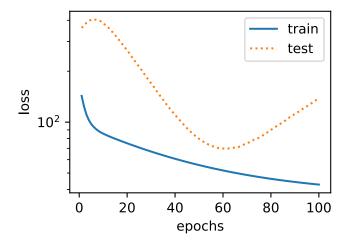
final epoch: train loss 68.418335 test loss 196.00613 weight: [[-0.00789979 0.0175912 -0.05790495 -0.01933222 0.03253733 0.05781509

-0.01514692 0.05282685 0.02166412 -0.01926502]]

bias: [8.2431376e-05]



```
In [63]: #十阶拟合(模型太复杂)加入了L2惩罚项,lambd=10,还是有过拟和,看来lambd不能太大fit_and_plot_L2(ten_features[:n_train,:],ten_features[n_train:,:],labels[:n_train],
labels[n_train:],0.00000004,10)
```



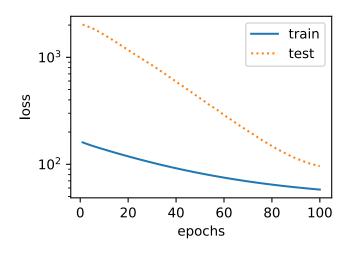
In [64]: #十阶拟合(模型太复杂)加入了L2惩罚项,lambd=0.01,调小了lambd更好一些了,目前是最好的情况fit_and_plot_L2(ten_features[:n_train, :],ten_features[n_train:, :], labels[:n_train],

labels[n_train:],0.00000004,0.01)

final epoch: train loss 58.013126 test loss 96.21356 weight: [[-0.00370966 0.02089359 -0.01593382 -0.06911948 0.07090588 0.066 69808

-0.03134783 0.02703604 0.03238817 -0.01092693]]

bias: [8.282152e-05]



L1和L2正则都是比较常见和常用的正则化项,都可以达到防止过拟合的效果。L1正则化的解具有稀疏性,可用于特征选择。L2正则化的解都比较小,抗扰动能力强。在求解过程中,L2通常倾向让权值尽可能小,最后构造一个所有参数都比较小的模型。因为一般认为参数值小的模型比较简单,能适应不同的数据集,也在一定程度上避免了过拟合现象。参数足够小,数据偏移得多一点也不会对结果造成什么影响,可以说"抗扰动能力强"。

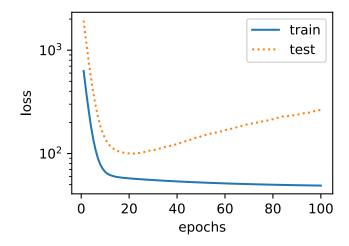
过拟合的时候,拟合函数的系数往往非常大,就是拟合函数需要顾忌每一个点,最终形成的拟合函数波动很大。在某些很小的区间里,函数值的变化很剧烈。这就意味着函数在某些小区间里的导数值(绝对值)非常大,由于自变量值可大可小,所以只有系数足够大,才能保证导数值很大。

而正则化是通过约束参数的范数使其不要太大,所以可以在一定程度上减少过拟合情况。

In [67]: #十阶拟合(模型太复杂)加入了L2惩罚项,lambd=0.0001,继续调小了,又出现了过拟和,看来lambd ± 0.01 的数量级比较合适

fit_and_plot_L2(ten_features[:n_train, :],ten_features[n_train:, :], labels[:n
train],

Loading [MathJax]/jax/output/HTML-CSS/jax.js | abels[n_train:],0.00000004,0.0001)



总的来说还是要模型好最重要,用**10**阶去拟合的话,正则化的效果都不如用三阶去拟合