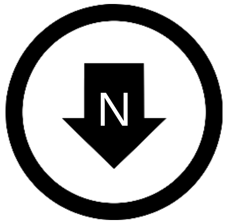
Shortcomings

1. Significant performance loss for sample sizes with **N << 5000**

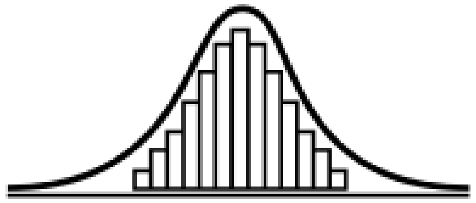
2. Unable to recover more than one **Gaussian signal**

3. For **SNR ≤ 20dB** high variance in performance
   ► Failures become increasingly likely

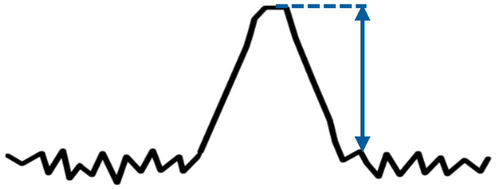
4. Inherently unrobust against outliers
   ► A **single outlier** will lead to failure of the method



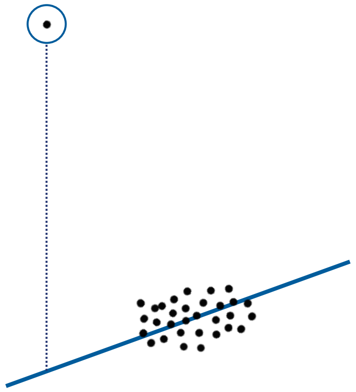
Samples



Gaussian



SNR ≤ 20dB



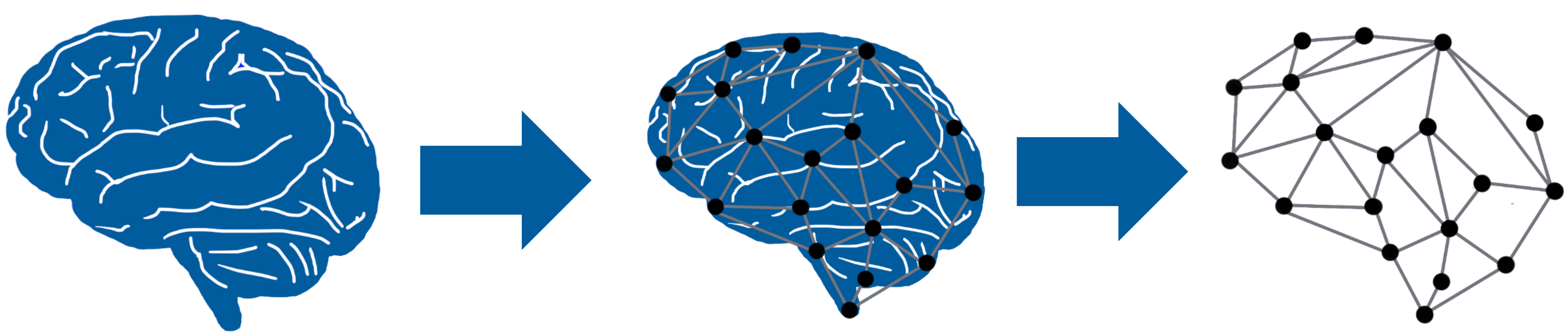
Outlier

Recent Developments

- In 2010: Blind Source Separation(BSS) using graph theory
- In 2020: First approaches to combine non-gaussianity based BSS with graph-based BSS (Jari Miettinen)

What is the idea?

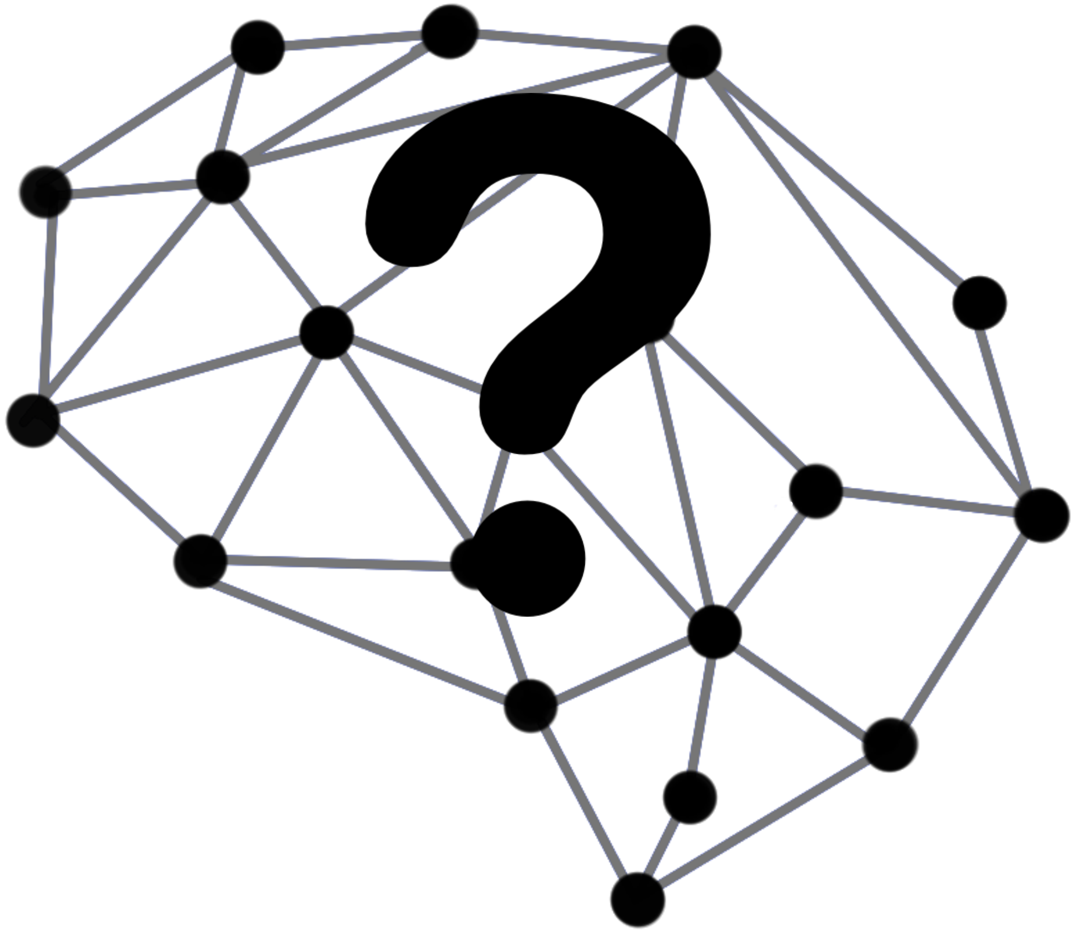
- Design a method that uses:
  a) Non-Gaussianity measure (classic ICA)
  b) Information about underlying graph structure



- Combine those in a way that they complement each other

Graph Blind Source Separation

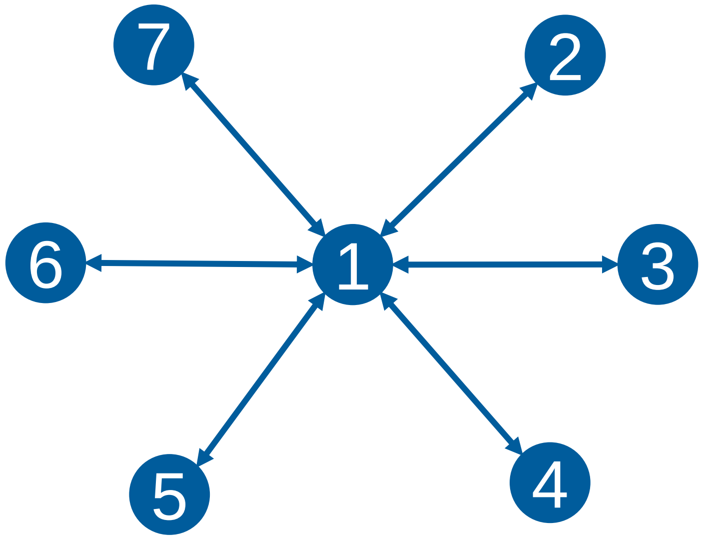
What are Graphs?



Graph Blind Source Separation
- What are Graphs?

- Graphs are structures build by vertices that are connected by edges
- Edges represent proximity between two vertices
- Graph structure can be stored in a matrix ⇒ The adjacency matrix **A**

$$\mathbf{A} = \begin{pmatrix} 0&1&1&1&1&1&1 \\ 1&0&0&0&0&0&0 \\ 1&0&0&0&0&0&0 \\ 1&0&0&0&0&0&0 \\ 1&0&0&0&0&0&0 \\ 1&0&0&0&0&0&0 \\ 1&0&0&0&0&0&0 \end{pmatrix}$$



Undirected unweighted graph

- Examples: Social networks, sensor networks, brain networks, etc.

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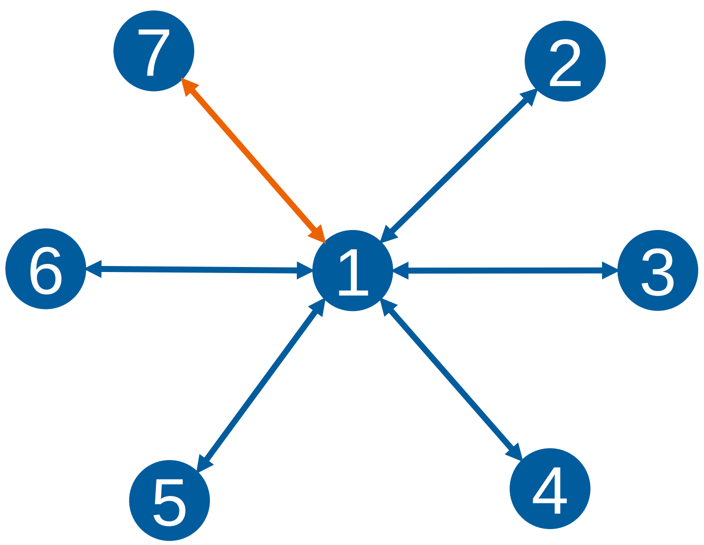
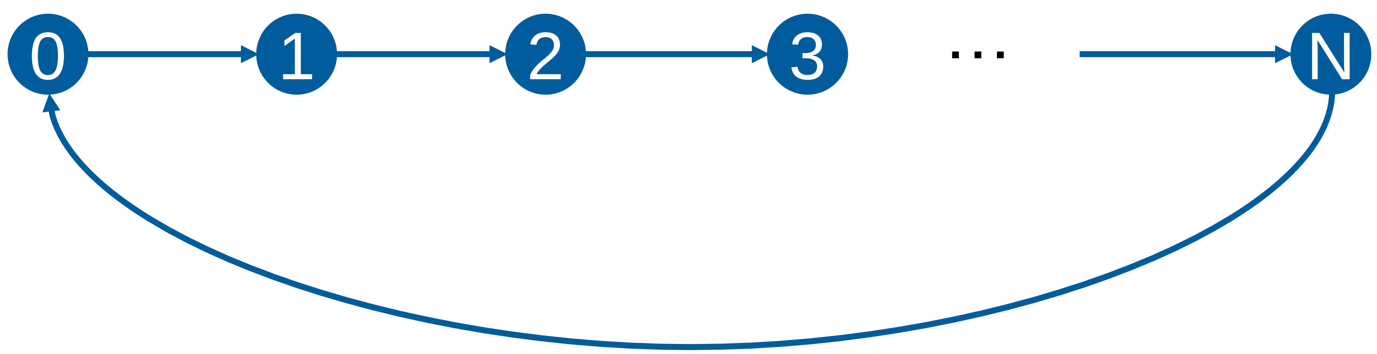


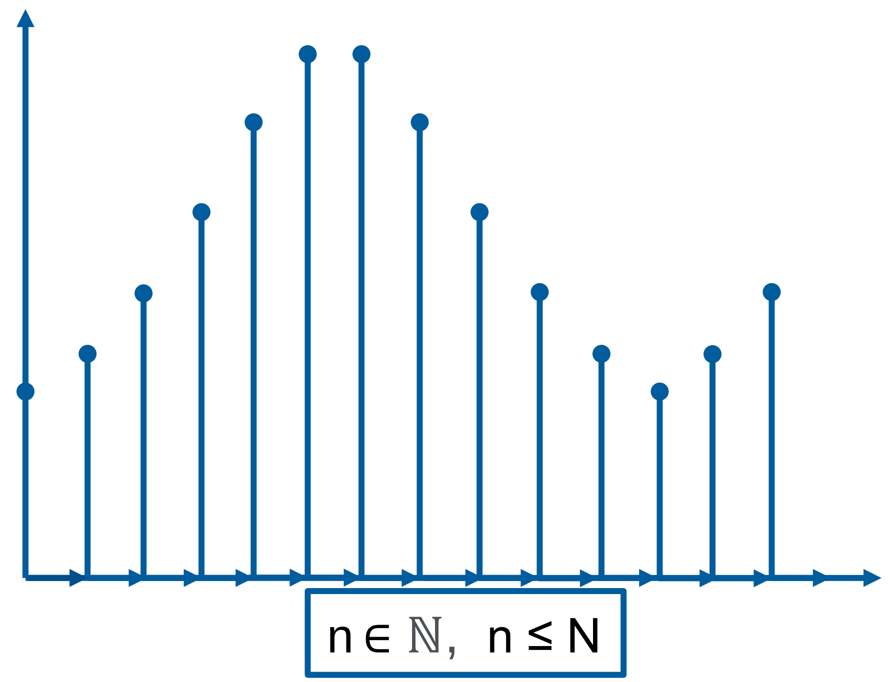
Undirected unweighted graph

- Examples: Social networks, sensor networks, brain networks, etc.

Graph Blind Source Separation
- What are Graph Signals?

- Analogy to discrete time signals
- Simplest case: Recursive Line Graph



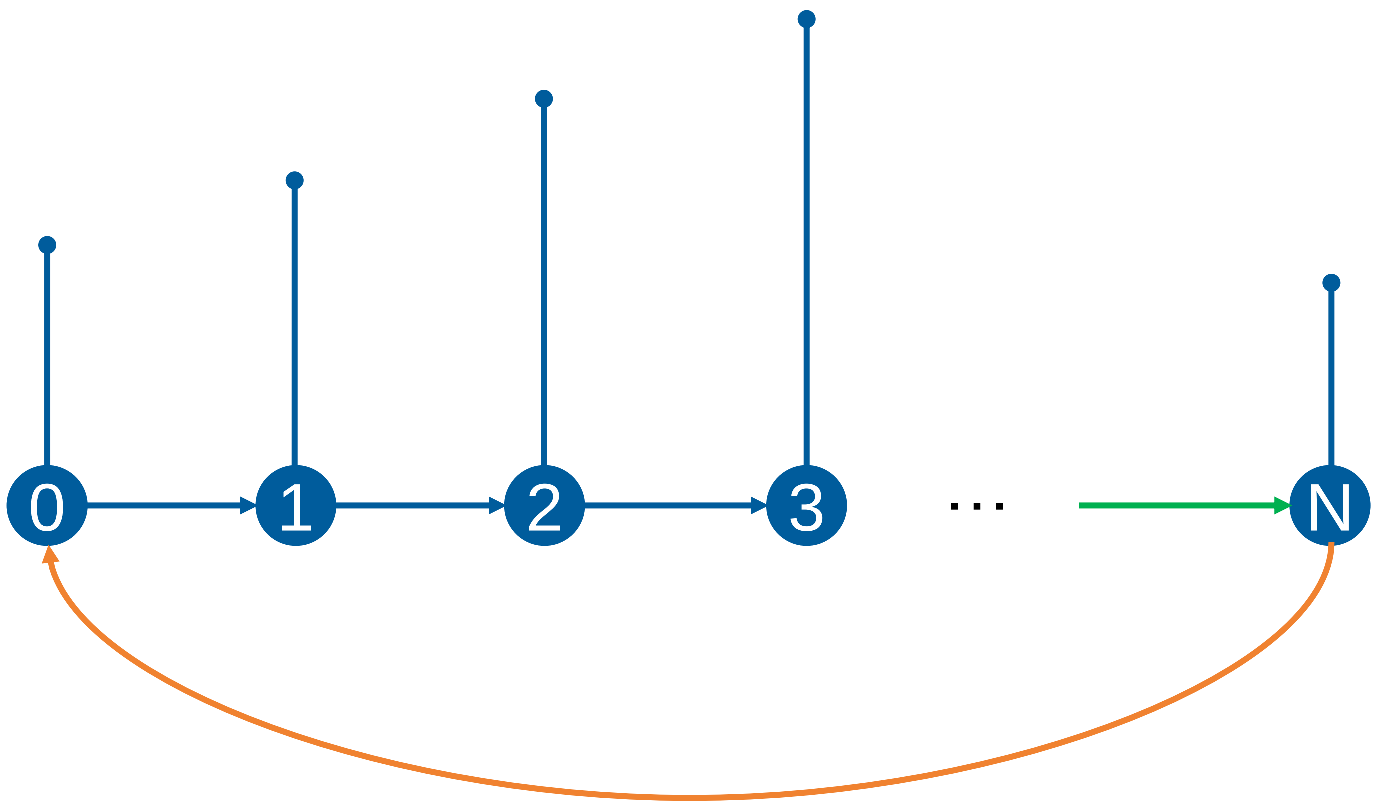


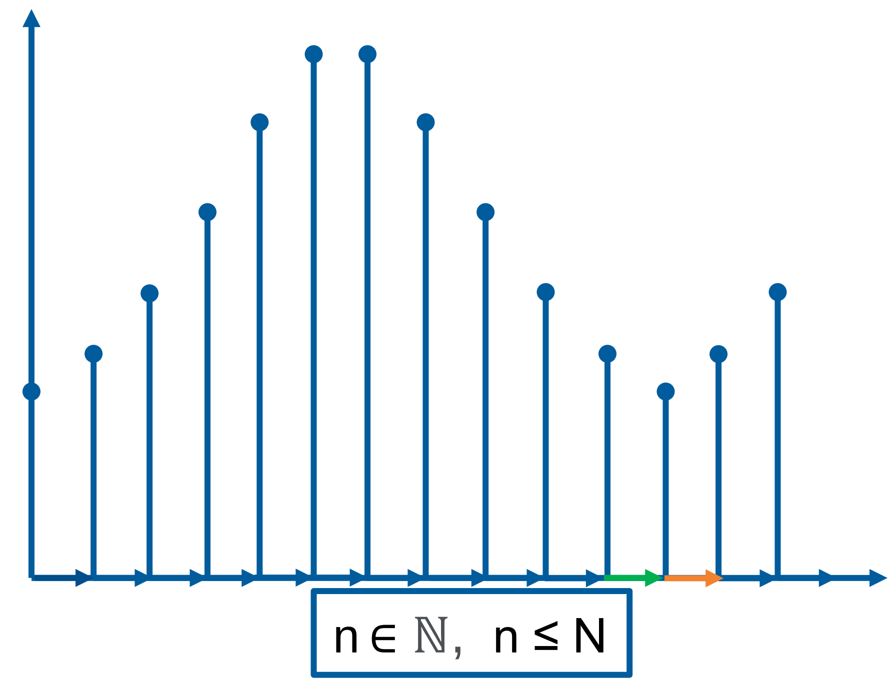
- Each node represents a sample point
- Directed graph ⇒ unsymmetric adjacency

$$\mathbf{A} = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & \dots & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & \dots & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & \dots & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & \dots & 0 \\ . & . & . & . & . & . & \dots & 0 \\ . & . & . & . & . & . & \dots & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & \dots & 0 \end{pmatrix}$$

Graph Blind Source Separation
- What are Graph Signals?

- Analogy to discrete time signals
- Simplest case: Recursive Line Graph



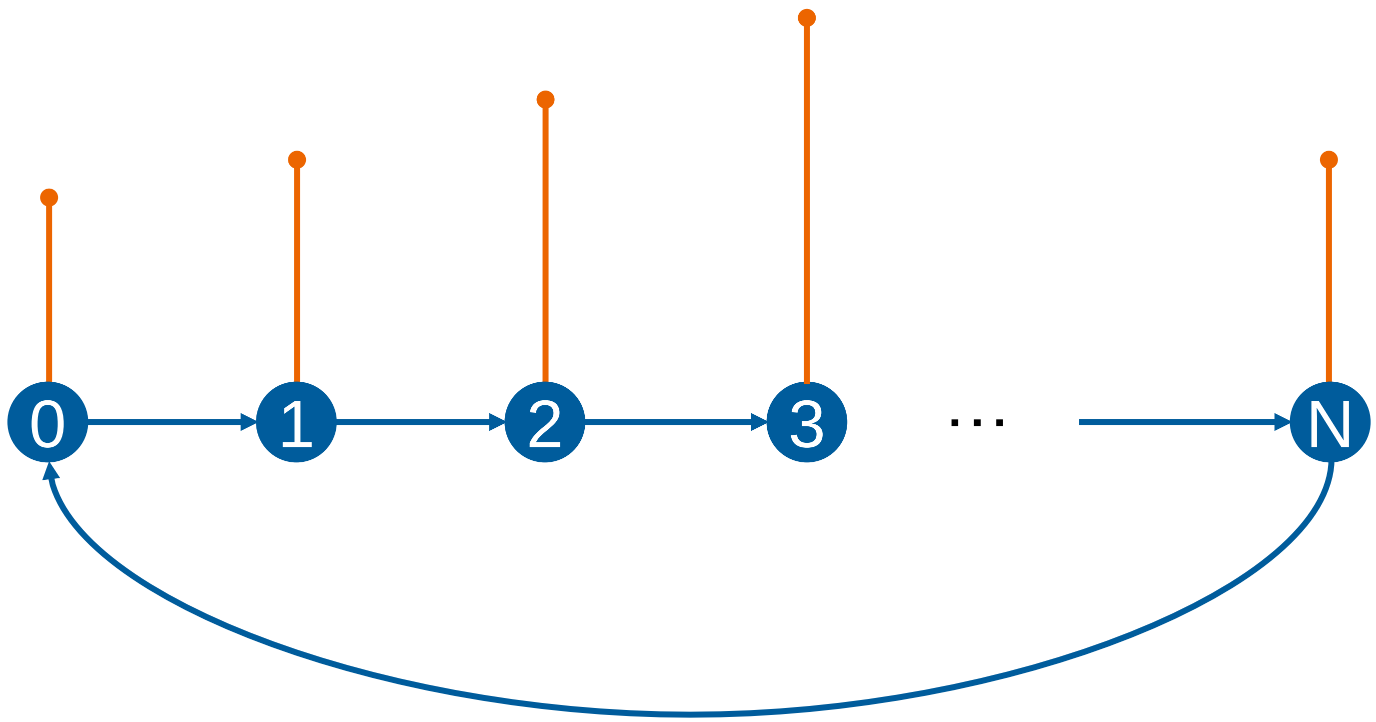


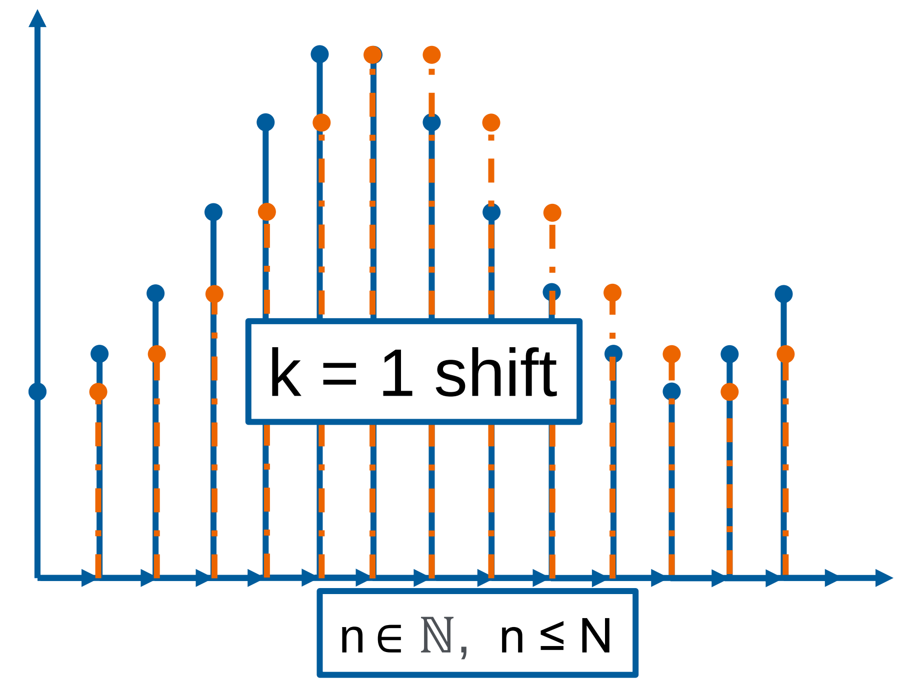
$$A = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & \dots & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & \dots & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & \dots & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & \dots & 0 \\ . & . & . & . & . & . & \dots & 0 \\ . & . & . & . & . & . & \dots & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & \dots & 0 \end{pmatrix}$$

- Multiple Signals can share the same Graph
- In this case: The class of all Periodic Signals with Period N

Graph Blind Source Separation
- What are Graph Signals?

- Analogy to discrete time signals
- Simplest case: Recursive Line Graph





- We call $\mathrm{X}\mathbf{A}^{\mathrm{k}} = \mathrm{X}^{(\mathrm{k})}$ a graph-shift by k
  - For Line Graph ⇔ Shift Operator as known from Digital Signal Processing
- *When a graph-shift is applied, the signal will shift along every **connected path** made up of the **k nearest edges**.*

Graph Blind Source Separation
- Graph-Autocorrelation

Using the introduced **graph-shift** **we define the graph-autocorrelation matrix analog to the sample autocorrelation:**

$$\mathbf{C}_{\mathrm{X}}^{Sample} \stackrel{\mathrm{def}}{=} \frac{1}{\mathrm{N}-1}\mathrm{X}\mathrm{X}^{\mathrm{T}} = \frac{1}{\mathrm{N}-1}\mathrm{X}\mathbf{I}\mathrm{X}^{\mathrm{T}}$$

$$\boldsymbol{A}^{k}\Big|_{\mathrm{k}=0} = \mathbf{I}$$

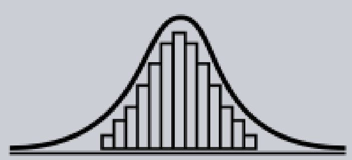
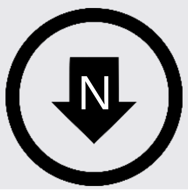
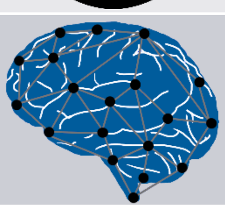
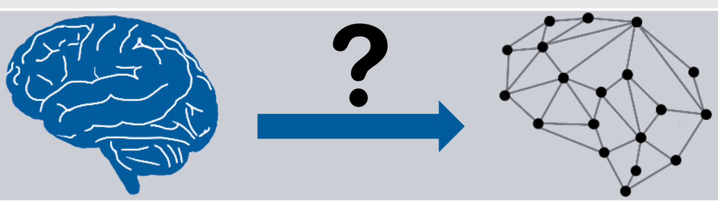
$$\mathbf{C}_{\mathrm{X}}^{\mathrm{Graph}}(\mathrm{k}) \stackrel{\mathrm{def}}{=} \frac{1}{\mathrm{N}-1}\mathrm{X}^{(\mathrm{k})}\mathrm{X}^{\mathrm{T}} = \frac{1}{\mathrm{N}-1}\mathrm{X}\mathbf{A}^{\mathrm{k}}\mathrm{X}^{\mathrm{T}}$$

- ► $\mathbf{C}_{\mathrm{X}}^{\mathrm{Sample}} \in \mathbb{R}^{\mathrm{N}\times\mathrm{N}}$: N dimensional square matrix of graph-autocorrelation
- ► $\mathbf{C}_{\mathrm{X}}^{\mathrm{Graph}}(\mathrm{k}) \in \mathbb{R}^{\mathrm{N}\times\mathrm{N}}$: N dimensional square matrix of graph-autocorrelation for k graph-shifts
- ► $\mathbf{A}^{\mathrm{k}} \in \mathbb{R}^{\mathrm{N}\times\mathrm{N}}$: k-th power of Adjacency matrix corresponding to k shifts along the graph
- ► $\mathrm{X} = (X_1, X_2, \dots X_D) \in \mathbb{R}^{N\times D}$: D signals stored as column vectors

- ► The **k-shift** along paths of an underlying Graph can provide **<u>additional information</u>** about independence between Signals!

Graph Blind Source Seperation
- Graph Decorrelation

- ► Find a matrix U that **diagonalizes** $\mathbf{U}\boldsymbol{C}_{\boldsymbol{X}}^{\boldsymbol{Graph}}\boldsymbol{U}^{\boldsymbol{T}}$ as much as possible
- ► The solution for U decorrelates the signals X in the graph-sense
  - ► U is a solution of the BSS problem for graph-signals

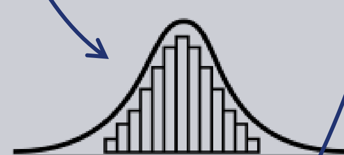
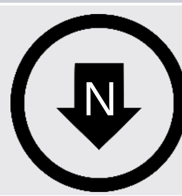
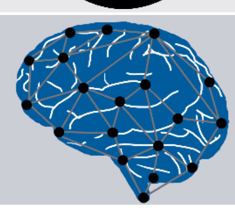
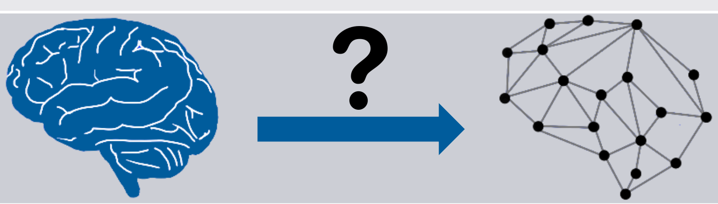
Advantages		**Disadvantages**
No assumptions about the distribution of the signals		Graph structure not always directly available
No Large sample size required		Highly dependent on accurately estimated graph
Graph structure in many real-world problems		

SPG

Graph Blind Source Seperation
- Graph Decorrelation

<u>PowerICA shortcomings:</u>

- Significant performance loss for sample sizes with **N << 5000**
- Unable to recover more than one **gaussian signal**

Advantages		Disadvantages
No assumptions about the distribution of the signals		Graph structure not always directly available
No Large sample size required		Highly dependent on accurately estimated graph
Graph structure in many real-world problems		

Graph based BSS and non-gaussianity based BSS seem to make up for each others weak points.

Graph Blind Source Seperation
- Composite Objective

How can we combine them?

- Approach from Jari Miettinen et al. (Aalto University)
- **Solution**: **composite objective function f :**