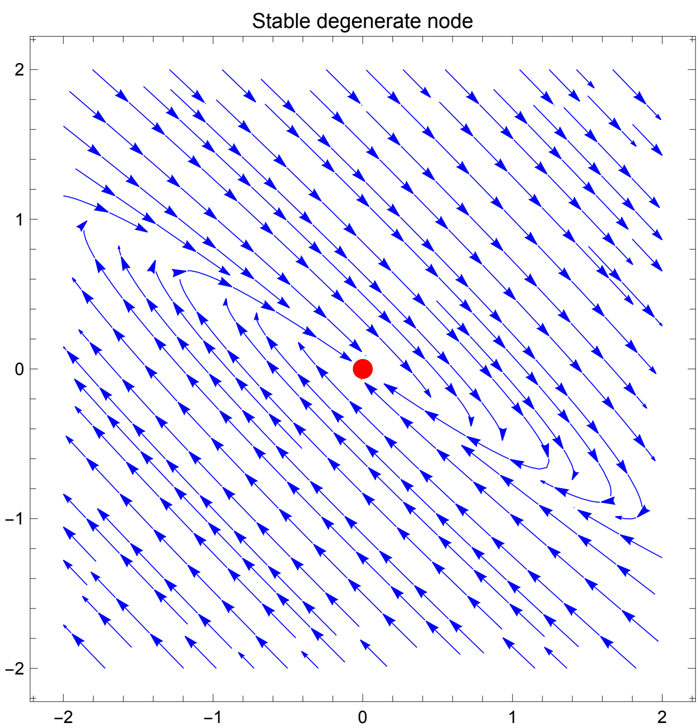
2.1 Degenerate System

a) Plot the trajectories for sigma = {-1,0,1}

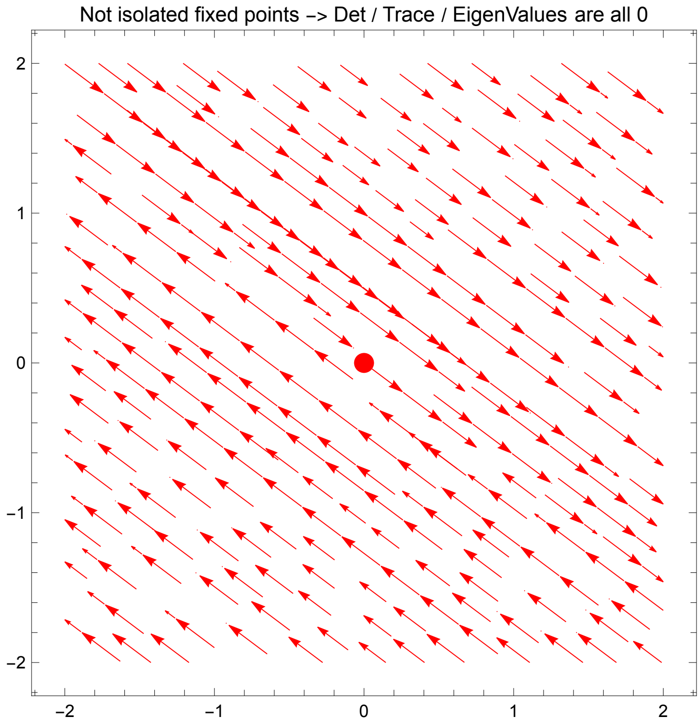
In[47]:=

```
(*sigma = -1*)
sigma = -1;
p1=StreamPlot[{(sigma + 3)*x +4*y , -(9/4)*x + (sigma-3)y},{x,-2,2},{y,-2,2},
StreamColorFunction→None , StreamStyle→Blue, PlotLabel→"Stable degenerate node", Epilo
sigma = 0;
p2=StreamPlot[{(sigma + 3)*x +4*y , -(9/4)*x + (sigma-3)y},{x,-2,2},{y,-2,2},
StreamColorFunction→None , StreamStyle→Red, PlotLabel→"Not isolated fixed points -> De
sigma = 1;
p3=StreamPlot[{(sigma + 3)*x +4*y , -(9/4)*x + (sigma-3)y},{x,-2,2},{y,-2,2},
StreamColorFunction→None , StreamStyle→Green, PlotLabel→"Unstable degenerate node", Ep
```

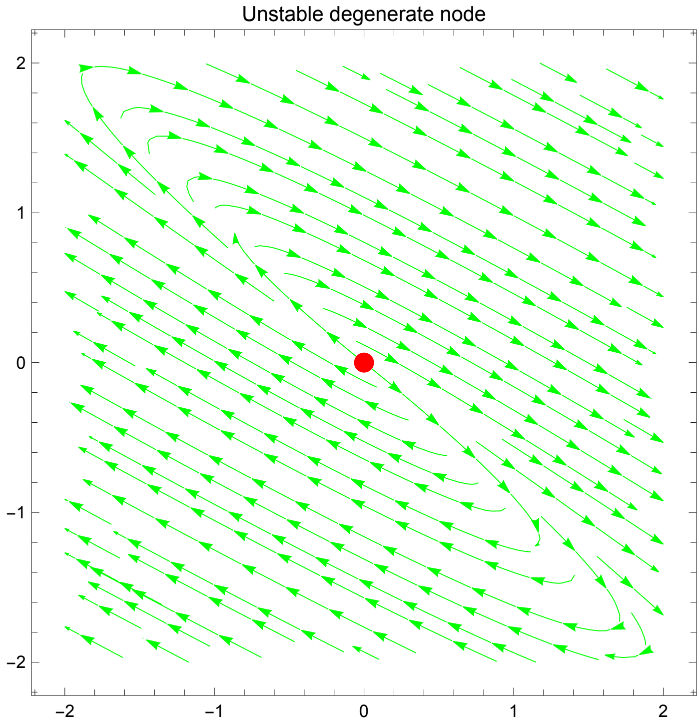
Out[48]=



Out[50]=



Out[52]=



In[19]:=

To determine the stability, check the determinant of A

In[20]:=
```
ClearAll["Global`*"]
Clear[sigma]
A={{(sigma+3),4},{-(9/4),sigma-3}};
det = Det[A]//Simplify
trac = sigma + 3 + sigma -3
Parabola = trac^2 - 4 * det
```

Out[23]= sigma^2

Out[24]= 2 sigma

Out[25]= 0

One sees that for all values of σ the equation of the parabola = 0. Therefore we always move on that parabola and have a stable σ<0, an unstable degenerate node σ>0 and a center for σ=0. The signs of the the eigenvalues λ determine if the flow is going inwards the stable point or outwards a)

$$\dot{x} = (\sigma+3)x + 4y$$
$$\dot{y} = -\frac{9}{4}x + (\sigma-3)y$$

$$A = \begin{pmatrix} \frac{\partial \dot{x}}{\partial x} & \frac{\partial \dot{x}}{\partial y} \\ \frac{\partial \dot{y}}{\partial x} & \frac{\partial \dot{y}}{\partial y} \end{pmatrix} = \begin{pmatrix} \sigma+3 & 4 \\ -\frac{9}{4} & \sigma-3 \end{pmatrix}$$

$\sigma = 0$

$$A(0) = \begin{pmatrix} 3 & 4 \\ -\frac{9}{4} & -3 \end{pmatrix}$$

$\tau = 0$

$\Delta = -9-(-9) = 0$

$\tau = 2\sigma$

$(\sigma+3)(\sigma-3) - (-\frac{9}{4})4$

$\sigma^2 - \cancel{3\sigma} + \cancel{3\sigma} - \cancel{9} + \cancel{9} = \Delta = \sigma^2$

$\sigma = -1$	$\sigma = 0$	$\sigma = 1$
$\Delta = 1 > 0$	$\Delta = 0$ $\tau = 0$	$\Delta = 1$
$\lambda_1 = -1$	$\lambda_1 = 0$	$\lambda_1 = 1$
$\lambda_2 = -1$	$\lambda_2 = 0$	$\lambda_2 = 1$
stable degenerate node	not isolated fixed point	unstable degenerate node

b) -> eigVal of A

In[26]:=
```
Clear[sigma];(*Clears any previous definition of sigma*)
A = {{(sigma + 3), 4}, {-(9/4), sigma - 3}}
Eigenvalues[A] // Simplify
```

Out[27]= $\left\{\{3+\text{sigma},4\},\left\{-\frac{9}{4},-3+\text{sigma}\right\}\right\}$

Out[28]= $\{\text{sigma},\text{sigma}\}$

c) -> eigVec of A

In[29]:=
```
Eigenvectors[A]
```

Out[29]= $\left\{\left\{-\frac{4}{3},1\right\},\{0,0\}\right\}$

In[30]:= $\left\{\left\{-\frac{4}{3},1\right\},\{0,0\}\right\}$

```
(*Rewrite by hand to norm → other PDF*)
eigV = {-4/3, 1};
n = Norm[eigV];
solC = (-1) * eigV / n
```

Out[30]= $\left\{\left\{-\frac{4}{3},1\right\},\{0,0\}\right\}$

Out[33]= $\left\{\frac{4}{5},-\frac{3}{5}\right\}$

d) -> A^-1

In[34]:=
```
A_inv = Inverse[A] // MatrixForm // Simplify
```

Out[34]//MatrixForm=

$$\begin{pmatrix} \frac{-3+\text{sigma}}{\text{sigma}^2} & -\frac{4}{\text{sigma}^2} \\ \frac{9}{4\,\text{sigma}^2} & \frac{3+\text{sigma}}{\text{sigma}^2} \end{pmatrix}$$

e) -> for which sigma is A singular -> det == 0

In[35]:=
```
Clear[sigma]
A = {{(sigma + 3), 4}, {-(9/4), sigma - 3}}
sigma_star = Solve[Det[A] == 0, sigma]
```

Out[36]= $\left\{\{3+\text{sigma},4\},\left\{-\frac{9}{4},-3+\text{sigma}\right\}\right\}$

Out[37]= $\{\{\text{sigma}\to 0\},\{\text{sigma}\to 0\}\}$

f) find the direction of the line o the fixed points

In[38]:=
```
B[sigma_] = {{sigma - c * d, d^2}, {-c^2, sigma + c * d}}
```

Out[38]= $\{\{-c\,d+\text{sigma}, d^2\}, \{-c^2, c\,d+\text{sigma}\}\}$

In[39]:=
```
{{-c d + sigma, d^2}, {-c^2, c d + sigma}}
sol1 = Eigenvectors[B[0]]
norm = Norm[sol1[[0]]]
```

Out[39]= $\{\{-c\,d+\text{sigma}, d^2\}, \{-c^2, c\,d+\text{sigma}\}\}$

Out[40]= $\left\{\left\{\frac{d}{c}, 1\right\}, \{0, 0\}\right\}$

Out[41]= Norm[List]

In[42]:=

$\left\{\left\{\frac{d}{c}, 1\right\}, \{0, 0\}\right\}$
```
eigVec = {d / c, 1}
```

Out[42]= $\left\{\left\{\frac{d}{c}, 1\right\}, \{0, 0\}\right\}$

Out[43]= $\left\{\frac{d}{c}, 1\right\}$

In[44]:=
```
n = Norm[eigVec]
```

Out[44]= $\sqrt{1+\text{Abs}\left[\frac{d}{c}\right]^2}$

In[45]:=

$\sqrt{1+\text{Abs}\left[\frac{d}{c}\right]^2}$
```
eigVec / n // Simplify
```

Out[45]= $\sqrt{1+\text{Abs}\left[\frac{d}{c}\right]^2}$

Out[46]= $\left\{\frac{d}{c\sqrt{1+\text{Abs}\left[\frac{d}{c}\right]^2}}, \frac{1}{\sqrt{1+\text{Abs}\left[\frac{d}{c}\right]^2}}\right\}$