HW2 2.3 Damped Pendulum

a)

```
In[53]:= A = {{0, 1}, {1, -s}}
Eigenvalues[A]
Eigenvectors[A]
```

Out[53]= {{0, 1}, {1, -s}}

Out[54]= $\left\{\frac{1}{2}\left(-s-\sqrt{4+s^2}\right), \frac{1}{2}\left(-s+\sqrt{4+s^2}\right)\right\}$

Out[55]= $\left\{\left\{\frac{1}{2}\left(s-\sqrt{4+s^2}\right), 1\right\}, \left\{\frac{1}{2}\left(s+\sqrt{4+s^2}\right), 1\right\}\right\}$

In[56]:= $\left\{\{1, 0\}, \left\{-\frac{1}{s}, 1\right\}\right\}$

Out[56]= $\left\{\{1, 0\}, \left\{-\frac{1}{s}, 1\right\}\right\}$

FP occurs when x' and y' = 0, this results in y = 0 and x therefore +- $\pi/2$

```
In[57]:= (* Determine type *)
Det[A]
Trace[A]
```

Out[57]= -1

Out[58]= {A, {{0, 1}, {1, -s}}}

```
In[59]:= {A, {{0, 1}, {1, -s}}}
```

Out[59]= {A, {{0, 1}, {1, -s}}}

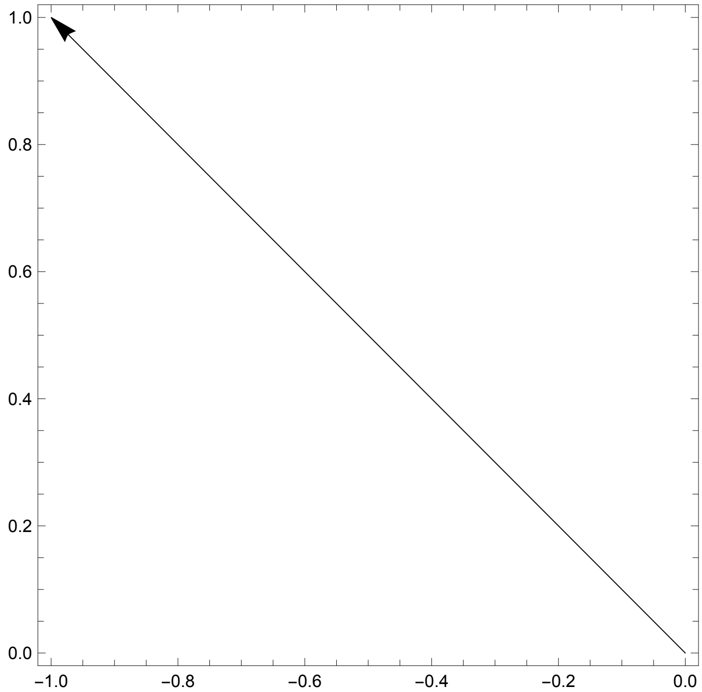
For σ=-2 at FP(0,0) check the Eigenvectors to find out if star or degenerate node

In[60]:=
```
A = {{0,1},{-1,-2}};
eigenVL = Eigenvalues[A]
eigenvectors = Eigenvectors[A]
Graphics[{
  Arrow[{{0, 0}, #}] & /@ eigenvectors
}, Frame → True, Axes → True, AspectRatio → 1, PlotRange → All]
```

Out[61]= {-1, -1}

Out[62]= {{-1, 1}, {0, 0}}

Out[63]=



Since only one Eigenvector for this case, σ=-2 at FP(0,0) is a degenerate node, since the Eigenvalues λ = -1 it is stable

$\dot{x} = y$

$\dot{y} = -\sin(x) - a y$

a) fix points:

In[64]:=

$\dot{x} = 0 \rightarrow y = 0$

$\dot{y} = 0$

$0 = -\sin(x)$;

FP for $x = \pi$ and $y = 0$, and $x = 0$ $y = 0$

$$A = \begin{pmatrix} 0 & 1 \\ -\cos(x) & -\sigma \end{pmatrix}$$

eval at $(\pm\pi, 0)$

$$A_1 = \begin{pmatrix} 0 & 1 \\ 1 & -\sigma \end{pmatrix}$$

$\det(A_1) = ad - bc = -1 = \Delta$

$\mathrm{trace}(A_1) = a + d = -\sigma = \tau$

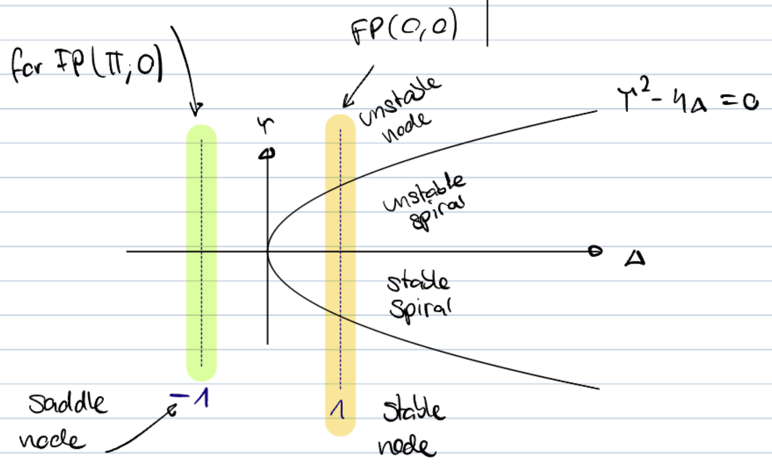
eval at $(0,0)$

$$A_2 = \begin{pmatrix} 0 & 1 \\ -1 & -\sigma \end{pmatrix}$$

$\det(A_2) = ad - bc = 1 = \Delta$

$\mathrm{trace}(A_2) = a + d = -\sigma = \tau$

;



FP	σ	FP-Type
$(\pi, 0)$	$\sigma \in \mathbb{R}$	saddle node
$(0,0)$	$0 < \sigma < 2$	stable spiral
$(0,0)$	$\sigma > 2$	stable node
$(0,0)$	$\sigma = 2$	stable degenerate node
$(0,0)$	$\sigma = 0$	center

$\tau^2 - 4 = 0$

$\tau^2 = 4$

$\tau = \pm 2$

In[66]:=

In[67]:=

In[68]:=

In[69]:=

In[70]:=

b)

In[71]:=
```
ClearAll["Global`*"]

(* Define the system of differential equations *)
sigma1 = 0; (* Saddle points *)
sigma21 = 1; (* Stable Spiral *)
sigma22 = 3; (* Stable Node *)
sigma23 = 2; (* Stable Degenerate Node *)
sigma24 = 0; (* Center *)
eq1 = x'[t] == y[t];
eq2 = y'[t] == -Sin[x[t]] - sigma1*y[t];
eq21 = y'[t] == -Sin[x[t]] - sigma21*y[t];
eq22 = y'[t] == -Sin[x[t]] - sigma22*y[t];
eq23 = y'[t] == -Sin[x[t]] - sigma23*y[t];
eq24 = y'[t] == -Sin[x[t]] - sigma24*y[t];

(* Fixed point and radius of the circular pattern *)
fixedPoint1 = {Pi, 0};
fixedPoint2 = {0, 0};
fixedPoint3 = {0, 0};

radius = 0.5;
radius3 = 0.2;

(* Define the system and the initial conditions for multiple trajectories in a circula
system1 = {eq1, eq2};
system21 = {eq1, eq21};
system22 = {eq1, eq22};
system23 = {eq1, eq23};
system24 = {eq1, eq24};

numTrajectories = 8;

initialConditions1 = Table[{x[0] == fixedPoint1[[1]] + radius Cos[2 Pi i/numTrajectories]
    y[0] == fixedPoint1[[2]] + radius Sin[2 Pi i/numTrajectories]}, {i, 0, numTrajectorie
initialConditions2 = Table[{x[0] == fixedPoint2[[1]] + radius Cos[2 Pi i/numTrajectories]
    y[0] == fixedPoint2[[2]] + radius Sin[2 Pi i/numTrajectories]}, {i, 0, numTrajectorie
initialConditions3 = Table[{x[0] == fixedPoint3[[1]] + radius3 *i,
    y[0] == fixedPoint2[[2]] + radius3 * i}, {i, 0, numTrajectories - 1}];

t0 = 0;
```

```
tMax = 40;

(* Solve the system of differential equations for multiple trajectories *)
sol1 = Table[NDSolve[{system1, initCond}, {x, y}, {t, t0, tMax}], {initCond, initialCo
sol21 = Table[NDSolve[{system21, initCond}, {x, y}, {t, t0, tMax}], {initCond, initial
sol22 = Table[NDSolve[{system22, initCond}, {x, y}, {t, t0, tMax}], {initCond, initial
sol23 = Table[NDSolve[{system23, initCond}, {x, y}, {t, t0, tMax}], {initCond, initial
sol24 = Table[NDSolve[{system24, initCond}, {x, y}, {t, t0, tMax}], {initCond, initial

(* Plot the phase portraint*)
ps10 = StreamPlot[{y, -Sin[x] - sigma1*y}, {x, -1, 3*Pi}, {y, -2, 2},
   PlotRange → All, ImageSize → Large, ColorFunction → (Black),
   StreamStyle → Directive[Black], StreamColorFunction→None];

(* Plot the phase portraint*)
ps21 = StreamPlot[{y, -Sin[x] - sigma21*y}, {x, -1, 1}, {y, -2, 2},
   PlotRange → All, ImageSize → Large, ColorFunction → (Black),
   StreamStyle → Directive[Black], StreamColorFunction→None];
ps22 = StreamPlot[{y, -Sin[x] - sigma22*y}, {x, -1, 1}, {y, -1, 1},
   PlotRange → All, ImageSize → Large, ColorFunction → (Black),
   StreamStyle → Directive[Black], StreamColorFunction→None];
ps23 = StreamPlot[{y, -Sin[x] - sigma23*y}, {x, -1, 1}, {y, -1, 1},
   PlotRange → All, ImageSize → Large, ColorFunction → (Black),
   StreamStyle → Directive[Black], StreamColorFunction→None];
ps24 = StreamPlot[{y, -Sin[x] - sigma24*y}, {x, -1, 1}, {y, -1, 1},
   PlotRange → All, ImageSize → Large, ColorFunction → (Black),
   StreamStyle → Directive[Black], StreamColorFunction→None];

(* Plot trajectories for the different systems *)
p10 = ParametricPlot[Evaluate[{x[t], y[t]} /. #] & /@ sol1, {t, t0, tMax}, PlotStyle →
p21 = ParametricPlot[Evaluate[{x[t], y[t]} /. #] & /@ sol21, {t, t0, tMax}, PlotStyle
p22 = ParametricPlot[Evaluate[{x[t], y[t]} /. #] & /@ sol22, {t, t0, tMax}, PlotStyle
p23 = ParametricPlot[Evaluate[{x[t], y[t]} /. #] & /@ sol23, {t, t0, tMax}, PlotStyle
p24 = ParametricPlot[Evaluate[{x[t], y[t]} /. #] & /@ sol24, {t, t0, tMax}, PlotStyle

(* Label the plot *)
Show[ps10, p10, FrameLabel → {"x", "y"}, PlotLabel→"Saddle Points"]
Show[ps21, p21, FrameLabel → {"x", "y"}, PlotLabel→"Stable Spiral"]
Show[ps22, p22, FrameLabel → {"x", "y"}, PlotLabel→"Stable Node"]
Show[ps23, p23, FrameLabel → {"x", "y"}, PlotLabel→"Stable Degenerate Node"]
Show[ps24, p24, FrameLabel → {"x", "y"}, PlotLabel→"Center (Neutrally stable)"]
```

••• ReplaceAll : {#1} is neither a list of replacement rules nor a valid dispatch table, and so cannot be used for replacing.

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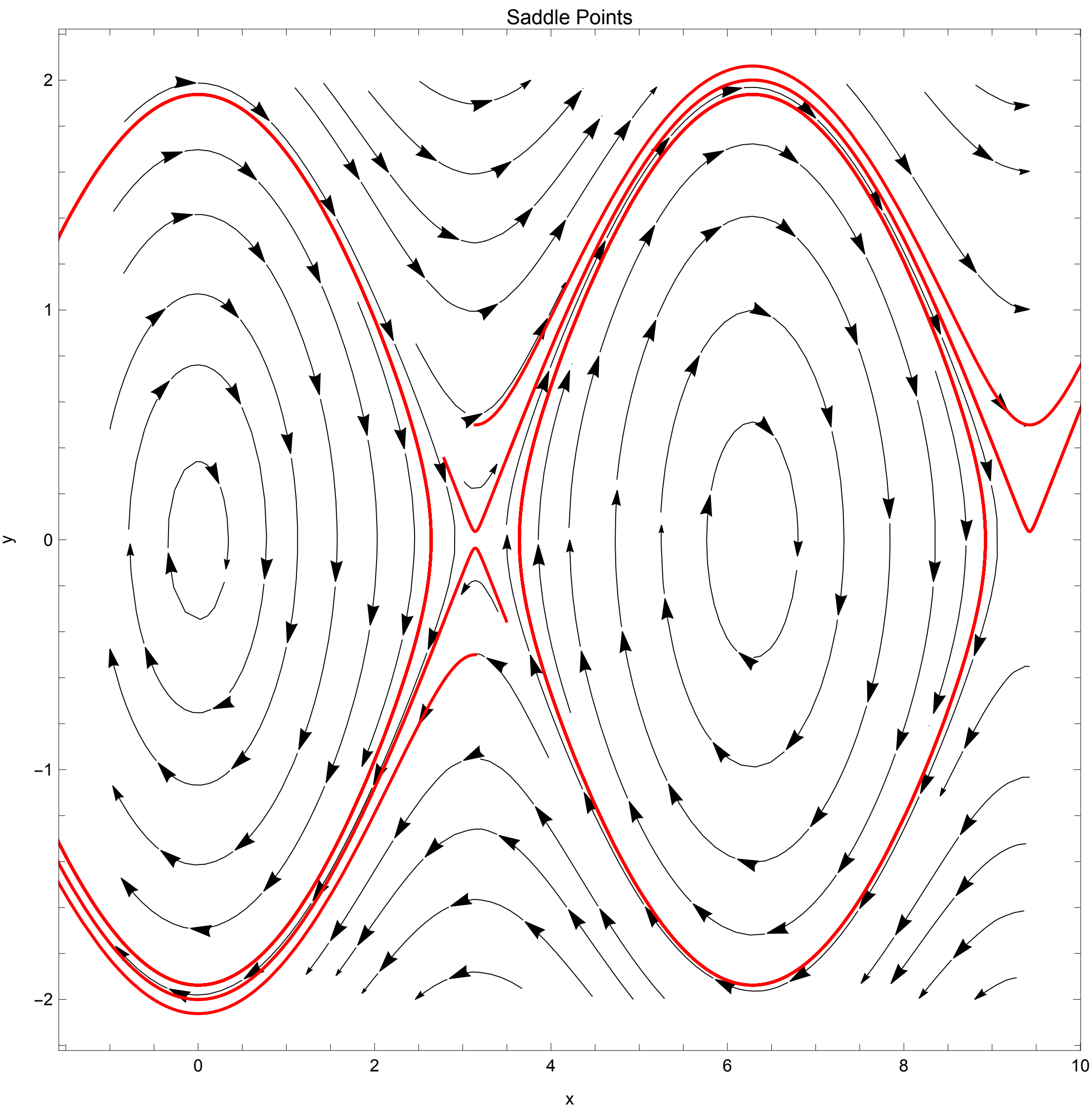
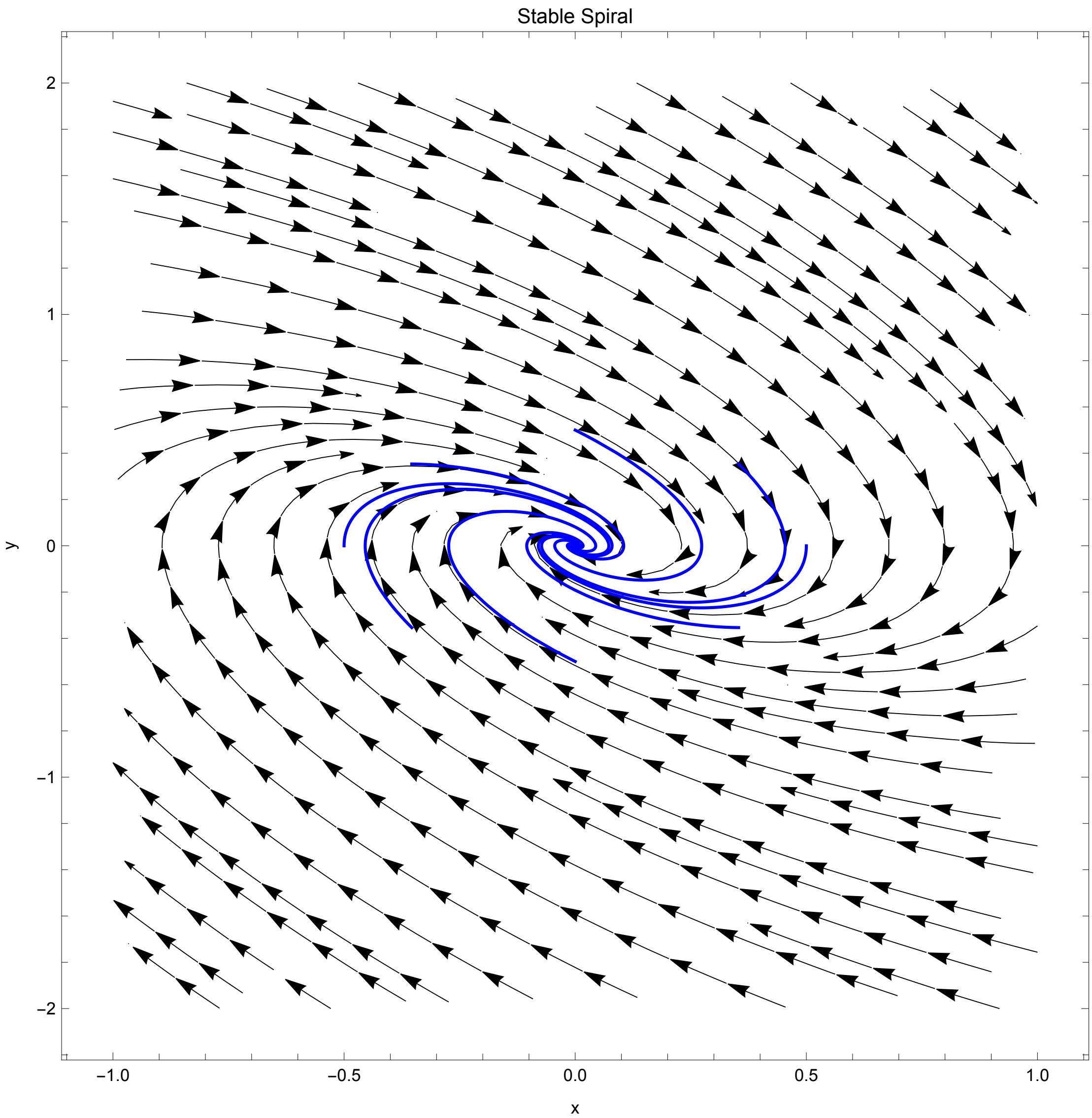
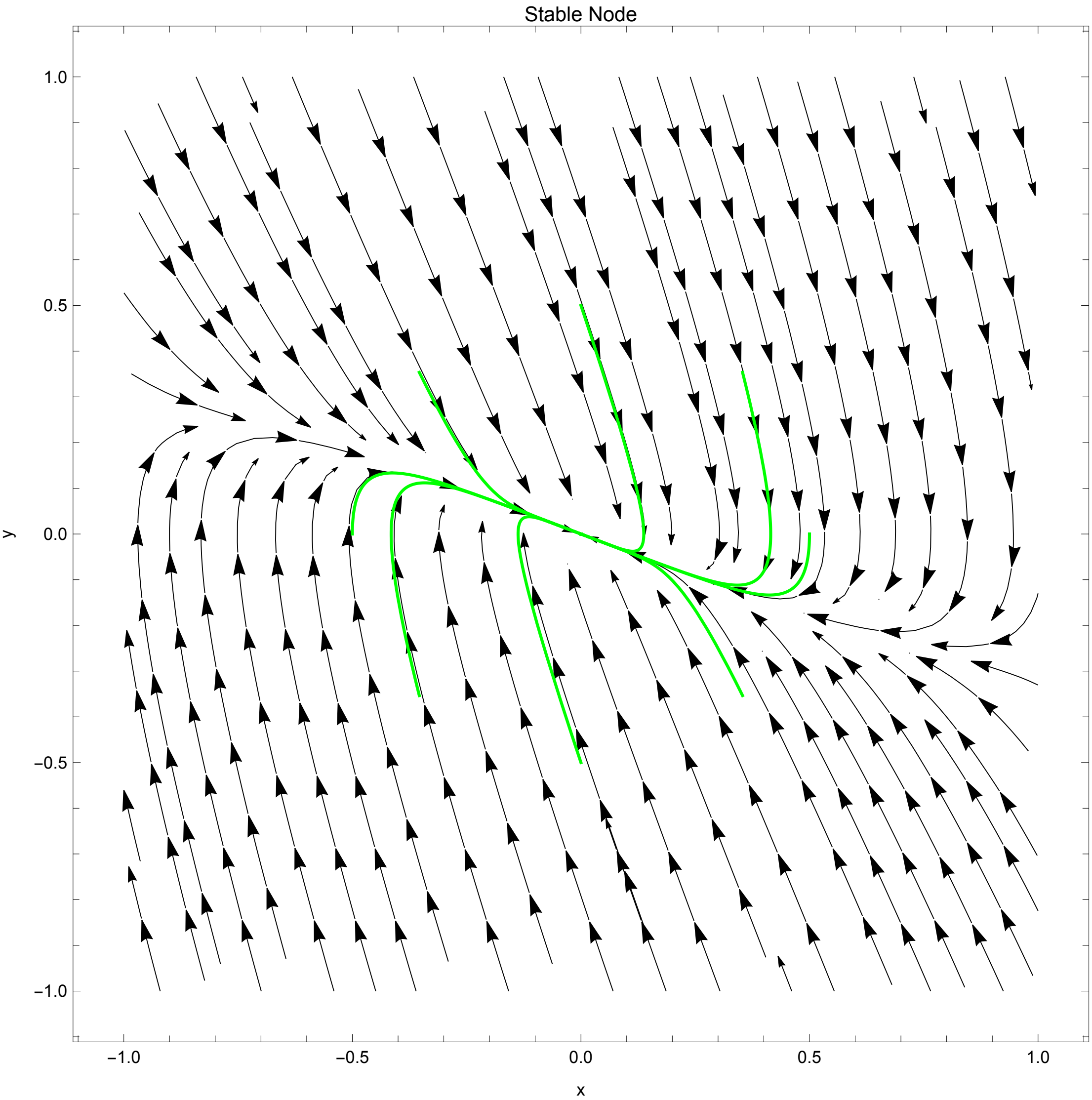
Out[114]=



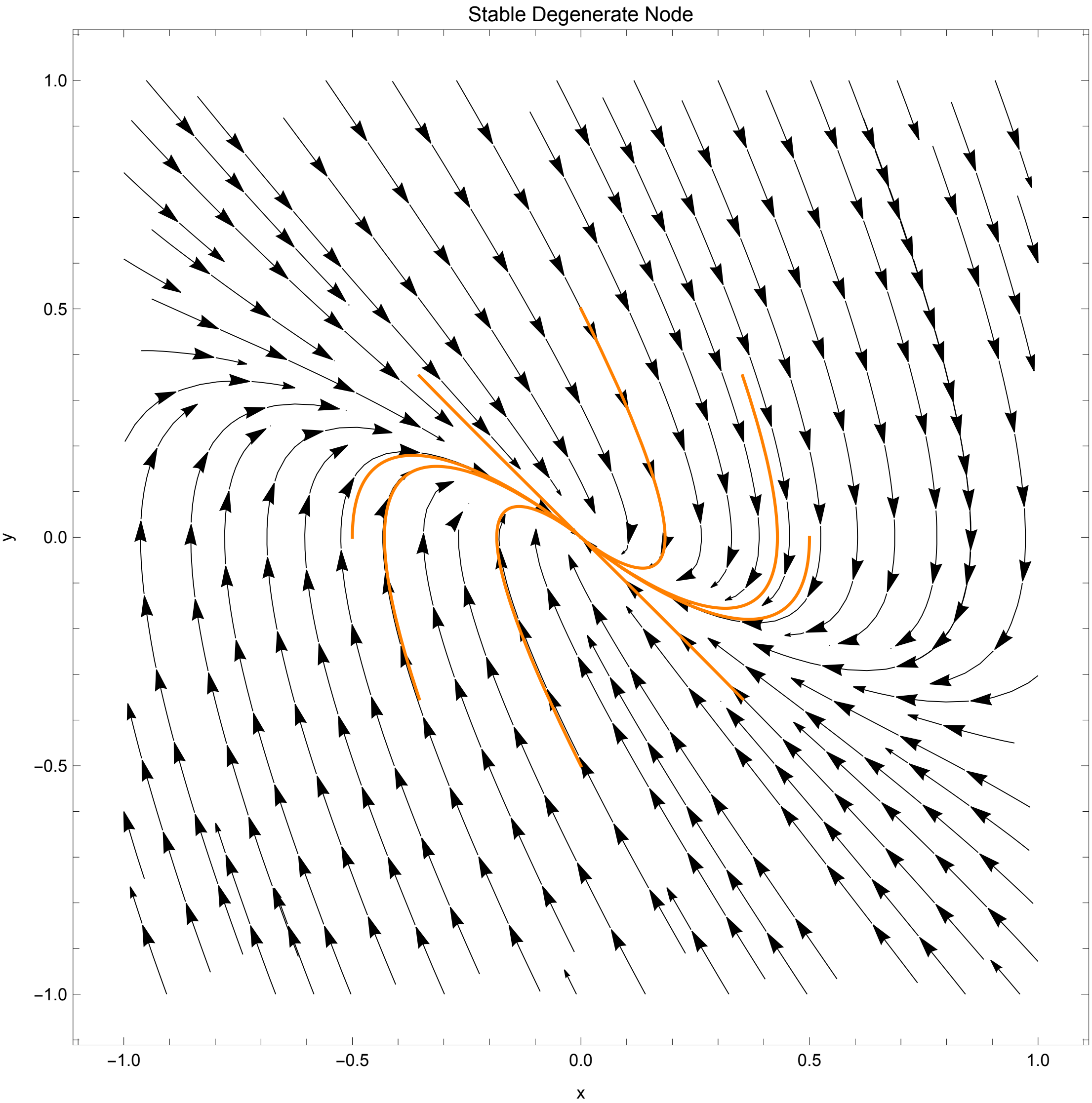
Out[115]=



Out[116]=



Out[117]=



Out[118]=