In[44]:=
```
(* Find all the zero positions *)
xstar = Solve[r * x + 4 * x^3 - 9 * x^5 == 0, x] // Simplify
```

Out[44]=
$$\left\{\{x \to 0\}, \left\{x \to -\frac{1}{3}\sqrt{2-\sqrt{4+9r}}\right\}, \left\{x \to \frac{1}{3}\sqrt{2-\sqrt{4+9r}}\right\}, \left\{x \to -\frac{1}{3}\sqrt{2+\sqrt{4+9r}}\right\}, \left\{x \to \frac{1}{3}\sqrt{2+\sqrt{4+9r}}\right\}\right\}$$

In[45]:=
```
xstar1[r_] = 0 * r;
xstar2[r_] = -1/3 Sqrt[2 - Sqrt[4 + 9 r]];
xstar3[r_] = 1/3 Sqrt[2 - Sqrt[4 + 9 r]];
xstar4[r_] = -1/3 Sqrt[2 + Sqrt[4 + 9 r]];
xstar5[r_] = 1/3 Sqrt[2 + Sqrt[4 + 9 r]];
xstar1[r_] = 0;
```

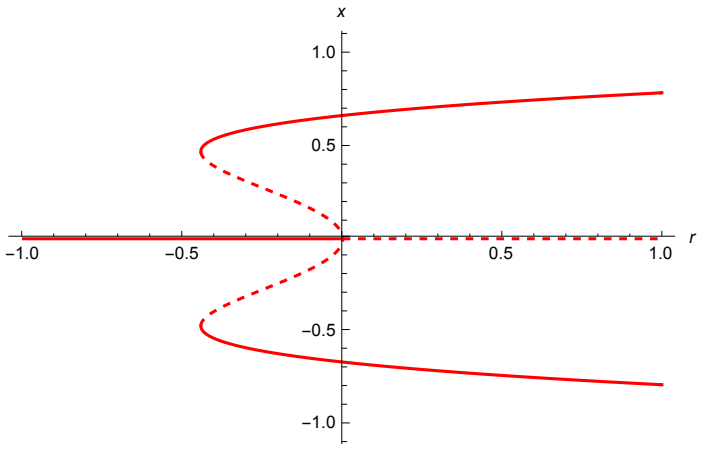
In[51]:=
```
(* Determine which xstars are stable and which are instable *)
```

In[52]:=
```
(* Determine a range for r*)
rmin = -1;
rmax = 1;
```

In[54]:=
```
(* First solution is for r < 0*)
plot1 = Plot[xstar1[r], {r, rmin, 0}, PlotStyle → {Red}];
plot11 = Plot[xstar1[r], {r, 0, rmax}, PlotStyle → {Red, Dashed}];
plot2 = Plot[xstar2[r], {r, rmin, rmax}, PlotStyle → {Red, Dashed}];
plot3 = Plot[xstar3[r], {r, rmin, rmax}, PlotStyle → {Red, Dashed}];
plot4 = Plot[xstar4[r], {r, rmin, rmax}, PlotStyle → {Red}];
plot5 = Plot[xstar5[r], {r, rmin, rmax}, PlotStyle → {Red}];
```

In[60]:=
```
(* Combine all plots together *)
Show[plot1, plot11, plot2, plot3, plot4, plot5,
 AxesLabel → {r, x}, PlotRange → {{rmin, rmax}, {rmin, rmax}}]
```

Out[60]=



In[64]:=
```
(* Find the saddle-node bifurcations analytically *)
df[x_, r_] = r + 12 x^2 - 45 x^4;
```

In[62]:=
```
b1[r_] = df[x, r] /. xstar[[3]]
```

Out[62]= $r + \frac{4}{3}\left(2 - \sqrt{4 + 9\,r}\right) - \frac{5}{9}\left(2 - \sqrt{4 + 9\,r}\right)^2$

In[63]:=
```
Solve[b1[r] == 0, r]
```

Out[63]= $\left\{\left\{r \to -\frac{4}{9}\right\}, \{r \to 0\}\right\}$