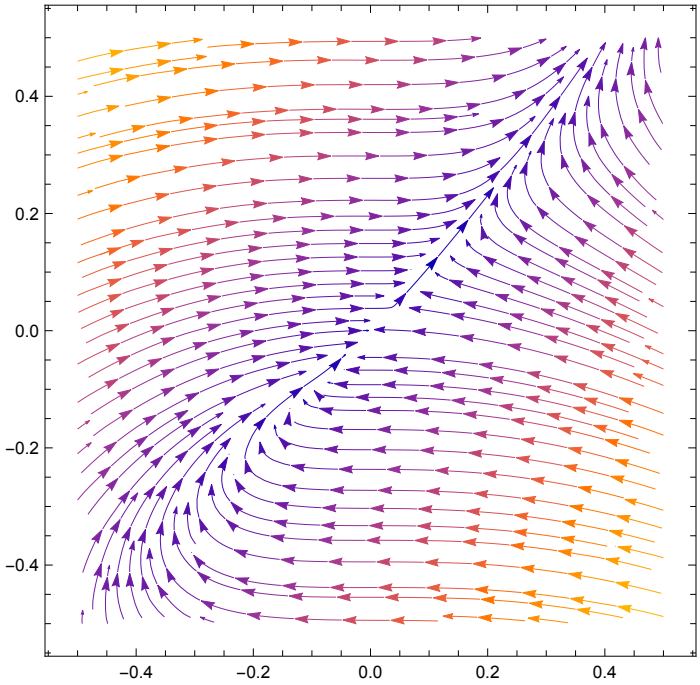
# HW3-1

## a) Give the index for the fixed point (0,0), x' = y-x; y' = x^2

```
ClearAll["Global'.*"];
eq1 = y - x;
eq2 = x^2;

StreamPlot[{eq1, eq2}, {x, -0.5, 0.5}, {y, -0.5, 0.5}, StreamPoints → Fine]
```

Out[ ]=




## b)

b) Give the index for the fixed point of the Cartesian system $\dot{x}$ and $\dot{y}$ corresponding to $\dot{r} = h(r)$, $\dot{\theta} = 0$, where $r$ and $\theta$ are polar coordinates. Let $h(r)$ be a smooth function with $h(r) \sim ar + O(r^2)$ for small values of $r$ and $a \neq 0$.

Polar to Cartesian:

$\dot{r} = h(r)$ $\quad h(r) = a \cdot r$ $\quad a \neq 0$ $\quad r$ small

$\dot{\theta} = 0$ $\quad \dot{r} = ar$

Polar to cartesian:

$x = r \cdot \cos(\theta)$ (I)

$y = r \cdot \sin(\theta)$ (II)

$\dot{u}v + u\dot{v}$

derive I & II with respect to t

$\dot{x} = a \cdot r \cdot \cos(\theta) - r \cdot \sin(\theta) \cdot \dot{\theta}$

$\dot{y} = a \cdot r \cdot \sin(\theta) + r \cdot \cos(\theta) \cdot \dot{\theta}$

with $\dot{\theta} = 0$,

$\dot{x} = a \cdot r \cdot \cos(\theta)$ (III)

$\dot{y} = a \cdot r \cdot \sin(\theta)$ (IV)

substitute (III) & (IV) with (I) & (II)

$\dot{x} = a \cdot x$

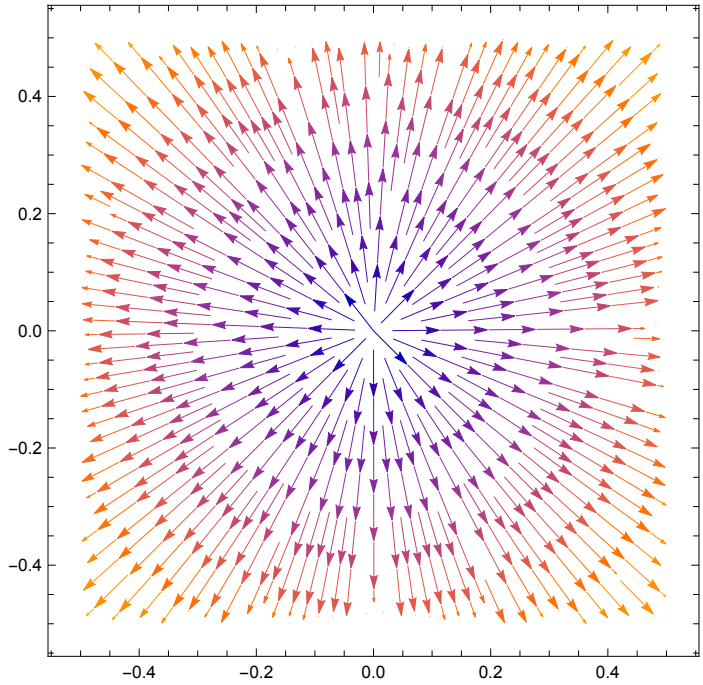
$\dot{y} = a \cdot y$

In[ ]:=

```
ClearAll["Global'.*"];
a = 1;
eq1 = a*x;
eq2 = a*y;

StreamPlot[{eq1, eq2}, {x, -0.5, 0.5}, {y, -0.5, 0.5}, StreamPoints → Fine]
```

Out[ ]=




Index is not depending on a

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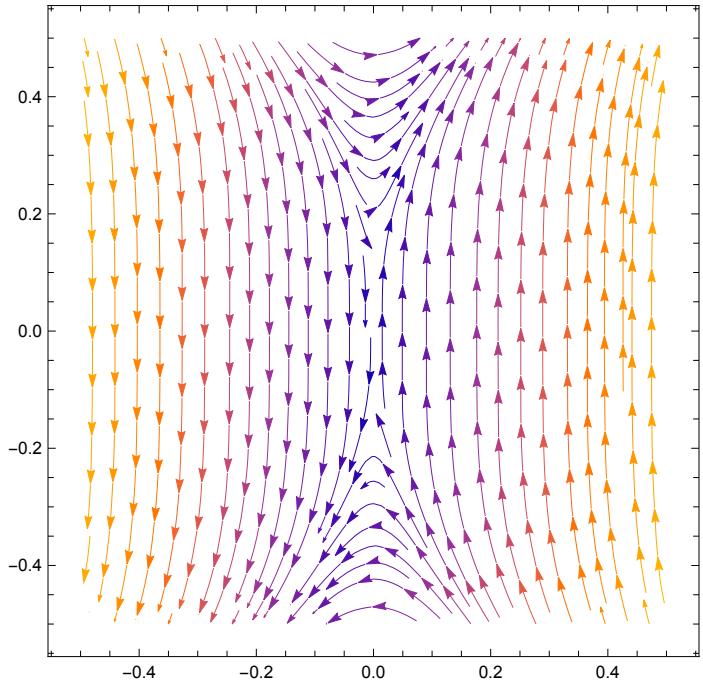
# c) Give the index for the fixed point of, x'=y^3 ; y' = x

In[ ]:=

```
ClearAll["Global'.*"];
eq1 = y^3;
eq2 = x;

StreamPlot[{eq1, eq2}, {x, -0.5, 0.5}, {y, -0.5, 0.5}, StreamPoints → Fine]
```

Out[ ]=



# d) Give the index for the fixed point of

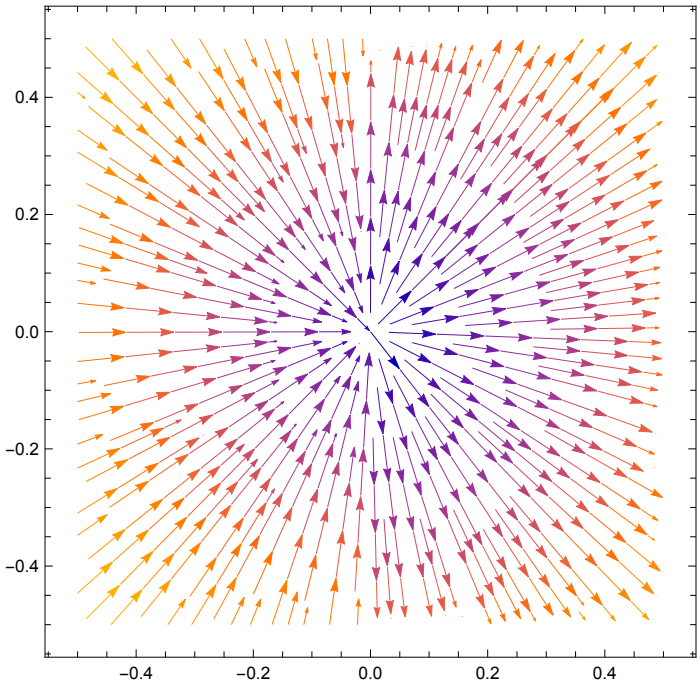
In[ ]:=
```
ClearAll["Global'.*"];
n = 1;
eq1 = (x^2 + y^2)^(Abs[n]/2)*Cos[n*ArcTan[y/x]];
eq2 = (x^2 + y^2)^(Abs[n]/2)*Sin[n*ArcTan[y/x]];

n = 3
eq3 = (x^2 + y^2)^(Abs[n]/2)*Cos[n*ArcTan[y/x]];
eq4 = (x^2 + y^2)^(Abs[n]/2)*Sin[n*ArcTan[y/x]];

StreamPlot[{eq1, eq2}, {x, -0.5, 0.5}, {y, -0.5, 0.5}, StreamPoints → Fine]
StreamPlot[{eq3, eq4}, {x, -0.5, 0.5}, {y, -0.5, 0.5}, StreamPoints → Fine]
```

Out[ ]= 3

Out[ ]=



Out[ ]=