2.2 2D linear System

a) EigValues of A

In[16]:=

```
A = {{sigma + 1, 3}, {-2, sigma - 1}} // Simplify
```

Out[16]= {{1 + sigma, 3}, {-2, -1 + sigma}}

In[17]:=

```
Eigenvalues[A] // Simplify
```

Out[17]= $\{-i\sqrt{5} + \text{sigma}, i\sqrt{5} + \text{sigma}\}$

b) Solve the dynamicals system

In[18]:=

```
ClearAll["Global'*"]
eq1=x'[t]==(sigma+1) x[t]+3 y[t];
eq2=y'[t]==-2 x[t]+(sigma-1) y[t];

initialConditions={x[0]==u,y[0]==v};

solution=DSolve[{eq1,eq2,initialConditions},{x,y},t];

(*Output the solution*)
solution // Simplify
```

Out[23]= $\{\{x \to \text{Function}[\{t\}, \frac{1}{5} e^{\text{sigma}\, t} (5 \text{Cos}[\sqrt{5}\, t] + 4\sqrt{5}\, \text{Sin}[\sqrt{5}\, t])],$
$y \to \text{Function}[\{t\}, -\frac{1}{5} e^{\text{sigma}\, t} (-5 \text{Cos}[\sqrt{5}\, t] + 3\sqrt{5}\, \text{Sin}[\sqrt{5}\, t])]\}\}$

c) Plot the found trajectories with simga = {-0.1, 0, 0.1}

In[24]:=

```
Clear[u, v, sigma]
```

In[25]:=

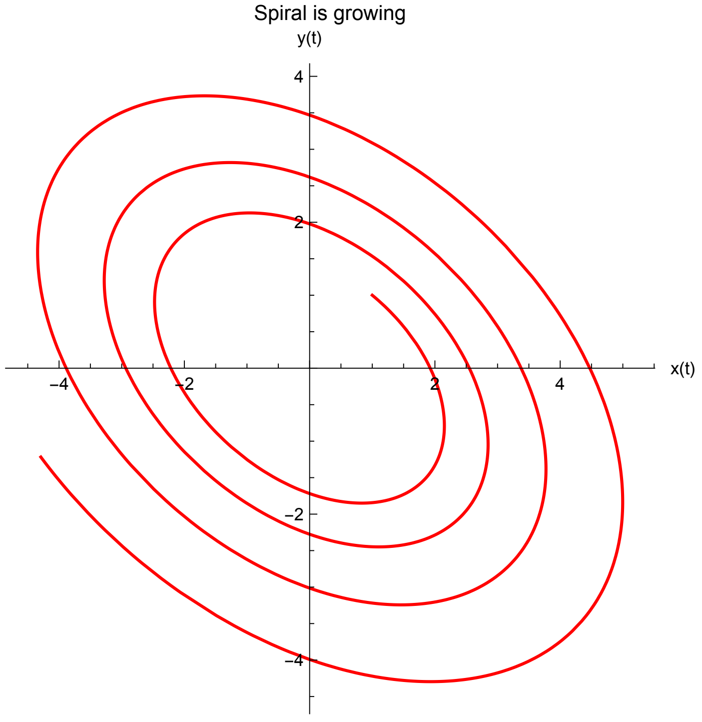
$$x[t\_] := \frac{1}{5} e^{\text{sigma } t} \left(5\, u \operatorname{Cos}\left[\sqrt{5}\, t\right] + \sqrt{5}\, u \operatorname{Sin}\left[\sqrt{5}\, t\right] + 3 \sqrt{5}\, v \operatorname{Sin}\left[\sqrt{5}\, t\right]\right);$$

$$y[t\_] := -\frac{1}{5} e^{\text{sigma } t} \left(-5\, v \operatorname{Cos}\left[\sqrt{5}\, t\right] + 2 \sqrt{5}\, u \operatorname{Sin}\left[\sqrt{5}\, t\right] + \sqrt{5}\, v \operatorname{Sin}\left[\sqrt{5}\, t\right]\right);$$

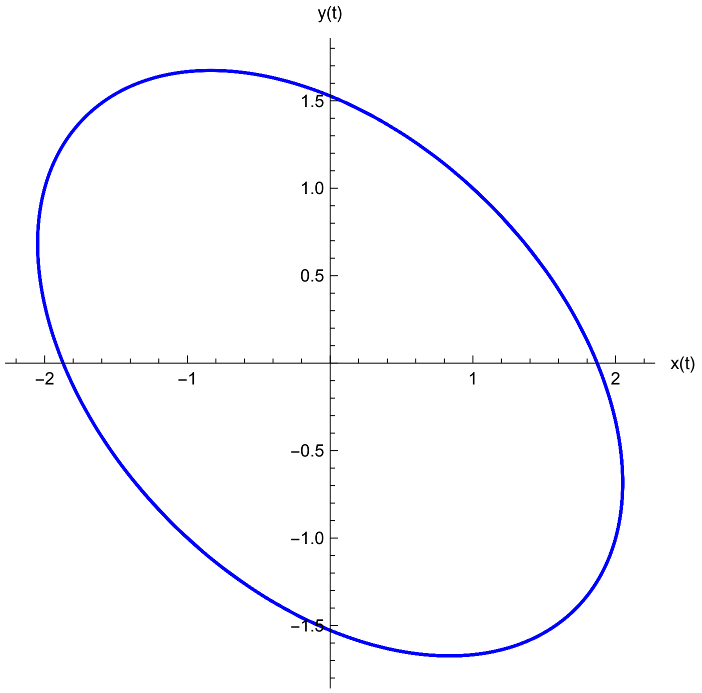
```
(* Plot the trajectory *)
(*ParametricPlot[{x[t],y[t]}/. {sigma→0.1},
  {t,0,10,PlotRange→All,AspectRatio→1,AxesLabel→{"x(t)","y(t)"}]*)
(*Define the time range for the plot*)tRange = {t, tMin, tMax};

(*Plot the trajectory*)
p1 = ParametricPlot[{x[t], y[t]} /. {sigma → 0.1, u → 1, v → 1}, {t, 0, 10},
  PlotRange → All, AspectRatio → 1, AxesLabel → {"x(t)", "y(t)"},
  PlotStyle → Red, PlotLabel → "Spiral is growing"]
p2 = ParametricPlot[{x[t], y[t]} /. {sigma → 0, u → 1, v → 1},
  {t, 0, 10}, PlotRange → All, AspectRatio → 1,
  AxesLabel → {"x(t)", "y(t)"}, PlotStyle → Blue]
p3 = ParametricPlot[{x[t], y[t]} /. {sigma → -0.1, u → 1, v → 1}, {t, 0, 10},
  PlotRange → All, AspectRatio → 1, AxesLabel → {"x(t)", "y(t)"},
  PlotStyle → Green, PlotLabel → "Spiral is shrinking"]
Show[p1, p2, p3];
```

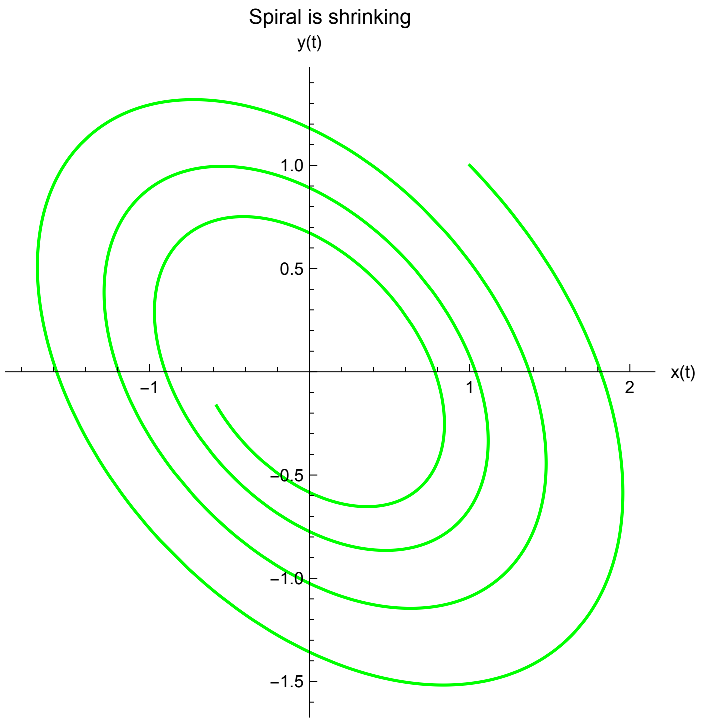
Out[28]=



Out[29]=



Out[30]=



In[ ]:=

In[ ]:=

In[ ]:=

d) for sigma = 0 compute the period of the ellipse

In[32]:=
```
(* For which period T is the *)
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In[33]:=
```
xs[t_] = x[t] /. sigma → 0
ys[t_] = y[t] /. sigma → 0
```

Out[33]= $\frac{1}{5}\left(5\,u\,\mathrm{Cos}\left[\sqrt{5}\,t\right]+\sqrt{5}\,u\,\mathrm{Sin}\left[\sqrt{5}\,t\right]+3\,\sqrt{5}\,v\,\mathrm{Sin}\left[\sqrt{5}\,t\right]\right)$

Out[34]= $\frac{1}{5}\left(5\,v\,\mathrm{Cos}\left[\sqrt{5}\,t\right]-2\,\sqrt{5}\,u\,\mathrm{Sin}\left[\sqrt{5}\,t\right]-\sqrt{5}\,v\,\mathrm{Sin}\left[\sqrt{5}\,t\right]\right)$

In[35]:=
```
Solve[xs[T] == xs[0], T]
Solve[ys[T] == ys[0], T]
```

Out[35]= $\left\{\left\{T\to \boxed{\frac{2\,\pi\,c_1}{\sqrt{5}}\ \text{if}\ c_1\in\mathbb{Z}}\right\},\left\{T\to \boxed{\frac{\mathrm{ArcTan}\left[\frac{4\,u^2-6\,u\,v-9\,v^2}{3\,(2\,u^2+2\,u\,v+3\,v^2)},\frac{2\,(\sqrt{5}\,u^2+3\,\sqrt{5}\,u\,v)}{3\,(2\,u^2+2\,u\,v+3\,v^2)}\right]+2\,\pi\,c_1}{\sqrt{5}}\ \text{if}\ c_1\in\mathbb{Z}}\right\}\right\}$

Out[36]= $\left\{\left\{T\to \boxed{\frac{2\,\pi\,c_1}{\sqrt{5}}\ \text{if}\ c_1\in\mathbb{Z}}\right\},\left\{T\to \boxed{\frac{\mathrm{ArcTan}\left[-\frac{2\,(u^2+u\,v-v^2)}{2\,u^2+2\,u\,v+3\,v^2},\frac{-2\,\sqrt{5}\,u\,v-\sqrt{5}\,v^2}{2\,u^2+2\,u\,v+3\,v^2}\right]+2\,\pi\,c_1}{\sqrt{5}}\ \text{if}\ c_1\in\mathbb{Z}}\right\}\right\}$

e) Compute the length ratio between the major and minor axes of the ellipse

assume (a*cos(t/T), b*sin(t/T)), for T = 2*pi/sqrt(5) is used. Now rotate the functions xs and ys so that the

In[37]:=
```
Clear[sigma]
u = 1;
v = 1;

r[t_] := Sqrt[(x[t])^2 + (y[t])^2];
sigma = 0;
(*Calculate the derivative of r[t] with respect to t*)
sol2 = D[r[t], t];

(*Solve for sol2==0*)
solutions = Solve[sol2 == 0, t]

(*Substitute the solutions back into
 r[t] to get the values of r at those points*)
rValues = r[t] /. solutions // Simplify;

(*Display the values of r at the corresponding points*)
minor = Min[rValues]
major = Max[rValues];
ratio = major / minor
```

Solve : Inverse functions are being used by Solve, so some solutions may not be found; use Reduce for complete solution information.

Solve : Unable to decide whether numeric quantities $\{(70+210\,i)-14\sqrt{-20+10\,i}\sqrt{11-2\,i},\ (10+30\,i)-2\sqrt{-20+10\,i}\sqrt{11-2\,i},\ (-5-15\,i)+\sqrt{-20+10\,i}\sqrt{11-2\,i},\ (70-210\,i)-14\sqrt{-20-10\,i}\sqrt{11+2\,i},\ (10-30\,i)-2\sqrt{-20-10\,i}\sqrt{11+2\,i},\ (-5+15\,i)+\sqrt{-20-10\,i}\sqrt{11+2\,i}\}$ are equal to zero. Assuming they are.

Out[43]=
$$\left\{\left\{t\to-\frac{\text{ArcCos}\left[-\sqrt{\frac{1}{14}\left(7-3\sqrt{5}\right)}\right]}{\sqrt{5}}\right\},\left\{t\to\frac{\text{ArcCos}\left[\sqrt{\frac{1}{14}\left(7-3\sqrt{5}\right)}\right]}{\sqrt{5}}\right\},\left\{t\to\frac{\text{ArcCos}\left[-\sqrt{\frac{1}{14}\left(7+3\sqrt{5}\right)}\right]}{\sqrt{5}}\right\},\left\{t\to-\frac{\text{ArcCos}\left[\sqrt{\frac{1}{14}\left(7+3\sqrt{5}\right)}\right]}{\sqrt{5}}\right\}\right\}$$

Out[45]=
$$\sqrt{\frac{7}{10}\left(5-\sqrt{5}\right)}$$

Out[47]=
$$\sqrt{\frac{5+\sqrt{5}}{5-\sqrt{5}}}$$

f) Compute the direction of the major

In[48]:=
```
u = 1;
v = 1;

ti = -(ArcCos[-Sqrt[1/14 (7-3 Sqrt[5])]]/Sqrt[5]);
xVal = x[ti];
yVal = y[ti];

vector = {xVal,yVal};
vector = Normalize[vector];
If[First[vector] < 0, vector = -vector];
vector//Simplify
```

Out[56]=
$$\left\{\frac{5\sqrt{7-3\sqrt{5}}+4\sqrt{5\left(7+3\sqrt{5}\right)}}{7\sqrt{5\left(5+\sqrt{5}\right)}},\frac{5\sqrt{7-3\sqrt{5}}-3\sqrt{5\left(7+3\sqrt{5}\right)}}{7\sqrt{5\left(5+\sqrt{5}\right)}}\right\}$$

In[57]:=

In[ ]:=

In[ ]:=