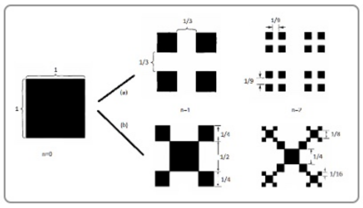
4.1 Box-counting dimension

Deadline: 20 Dec 23:59

(2 points)



The figure above shows two ways, (a) and (b), to evolve the unit square into two fractals sets (defined by iterating to generation $n = \infty$). In each generation in (a), every square is replaced by four squares, with side lengths 1/3 of the original square. In each generation in (b), every square is replaced by five smaller squares, four with side lengths 1/4 and one with 1/2 of the original square.

(a) Analytically find an expressioin for the box-counting dimension of the fractal obtained by evolving the unit square according to way (a) in the figure above.

(b) Analytically find an expressioin for the box-counting dimension of the fractal obtained by evolving the unit square according to way (b) in the figure above. Hint: The result is not $\frac{3}{2}$!

Navid:

$$N_{box} \sim A \cdot \varepsilon^{-D}$$

$-D$ → Dimension

ε → Size of small boxes

a) $m=4$

$r=\frac{1}{3}$

Similarity dimension $d=\frac{\ln(m)}{\ln(r)}$, hence $m=r^d$

m # number of copies

$r=$ scale factor.

$$d=\frac{\ln(4)}{\ln(3)}$$

"symmetric cantor set"

b) $m=5$ "asymmetric cantor set"

$r=\frac{1}{2}$ & $\frac{1}{4}$

1x 4x

N_{box}?

4 of length a

1 of length b

where $b=2\cdot a$

$N_{box} = 4\,N_a + 1\cdot N_b$ self similar

$$N_{box}(\varepsilon) = N_a\left(\frac{\varepsilon}{\varepsilon_a}\right) + N_b\left(\frac{\varepsilon}{\varepsilon_b}\right)$$

$\varepsilon_a=\frac{1}{4}$

$\varepsilon_b=\frac{1}{2}$

$$\cancel{A}\cdot\cancel{\varepsilon^{-D_0}} = 4\cdot\cancel{A}\left(\cancel{\frac{\varepsilon}{\varepsilon_a}}\right)^{-D_0} + 1\cdot\cancel{A}\cdot\left(\cancel{\frac{\varepsilon}{\varepsilon_b}}\right)^{-D_0}$$

$$1 = 4\left(\frac{1}{\varepsilon_a}\right)^{-D_0} + \left(\frac{1}{\varepsilon_b}\right)^{-D_0}$$

$$= 4\cdot\left(\frac{1}{4}\right)^{-D_0} + \left(\frac{1}{2}\right)^{-D_0}$$

$$1 = 4 \cdot \left(\frac{1}{2}\right)^{2D_0} + \left(\frac{1}{2}\right)^{D_0}$$

Real solution Approximate form Step-by-step solution

$$D = \frac{\log(1+\sqrt{17})}{\log(2)} - 1$$

$\log(x)$ is the natural logarithm