

Reinforcement Learning

Fundamentals of Artificial Intelligence

Sequential Decision Making

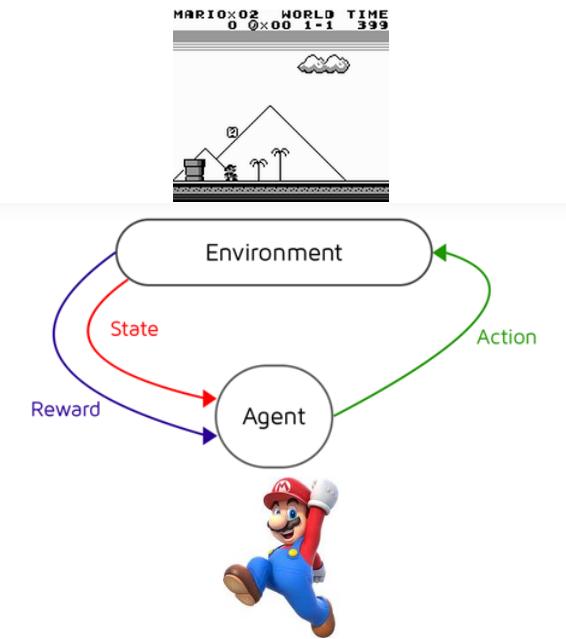
reinforcement learning (RL):

formalization of sequential decision making (action policy) of software agent interacting with environment

corresponds to search for best (or rather good) action policy to reach a given goal (e.g., win a game)

using learning from examples (data) to guide the search

RL usually more difficult (e.g., non-differentiable as a whole) than supervised learning (which can be seen as “generalized optimization”, often of proxy metric)



Main Elements of RL

goal: find action policy maximizing reward from environment

action policy: exploration-exploitation trade-off

- e.g., epsilon-greedy: random exploration at small fraction of the time
- off-policy instead of on-policy learning: policy for generating observations to learn from (exploration) independent from updated policy (current best)

feedback from environment: goal-directed, no supervision

- scalar reward signal
- cumulative and delayed rewards (credit assignment problem)

Optional Elements of RL

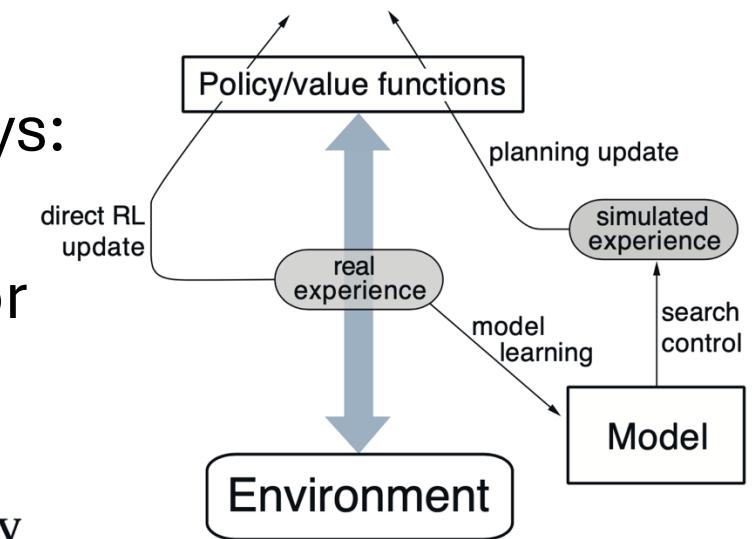
value functions for states or actions: improve efficiency of search in vast action policy space (alternative: direct policy search)

model of environment: (model-free) learning from trial-and-error or (model-based) planning

model of environment can be used in different ways:

- simulate experience from model (for learning)
- decision-time planning (e.g., heuristic search or model predictive control)

model $\xrightarrow{\text{planning}}$ policy



from Sutton

Markov Decision Process (MDP)

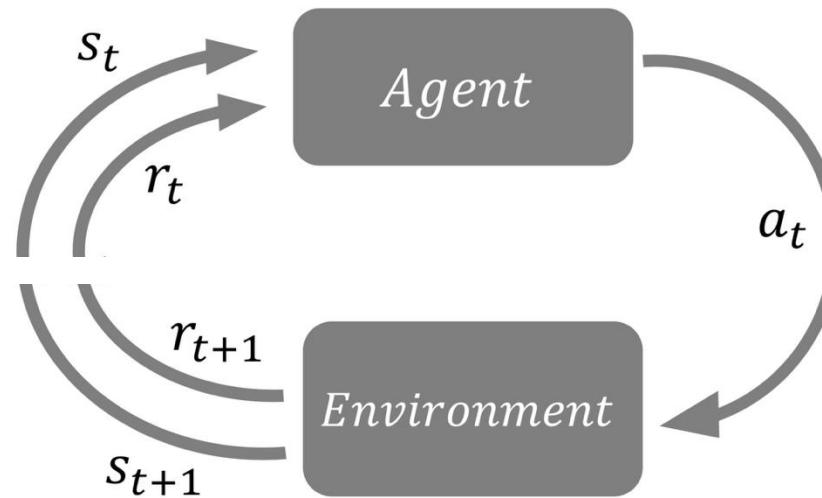
idea: current state includes all information about past

transition probabilities between states describe dynamics of given MDP

action policy: mapping from states to probabilities for selection of different actions

States, Actions, and Rewards

transition probabilities (model of environment): $p(s_{t+1}, r_{t+1} | s_t, a_t)$



reward hypothesis:

- reward as scalar signal
- goal: maximization of expected cumulative sum of received rewards

State and Action Values

state/action value: total amount of expected future reward starting from given state/action (usually with discounting of later steps)

→ indicating long-term desirability of states/actions

main motivation: improve efficiency of search in policy space
(for comparison: evolutionary methods search directly by evaluating entire policies)

State-Value Function

(needed for all states)

$$\begin{aligned} v_{\pi}(s_t) &= E_{\pi} \left[\sum_{k=0}^{\infty} \gamma^k r_{t+k+1} | s_t \right] = E_{\pi}[r_{t+1} + \gamma v_{\pi}(s_{t+1})|s_t] \\ &= \sum_{a_t} \pi(a_t | s_t) \sum_{s'_{t+1}, r_{t+1}} p(s'_{t+1}, r_{t+1} | s_t, a_t) [r_{t+1} + \gamma v_{\pi}(s'_{t+1})] \end{aligned}$$

policy: probability to take specific action being in a given state

transition probability (depending on environment) from state s_t to state s'_{t+1} for a given action

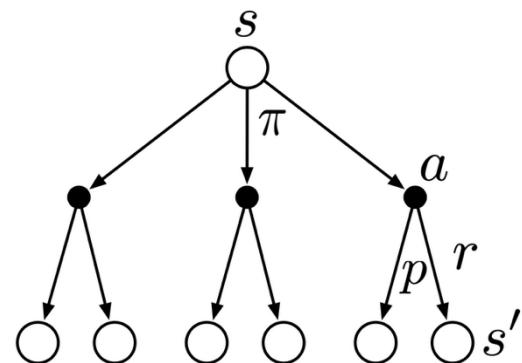
discount rate

Bellman (expectation) equation: recursion

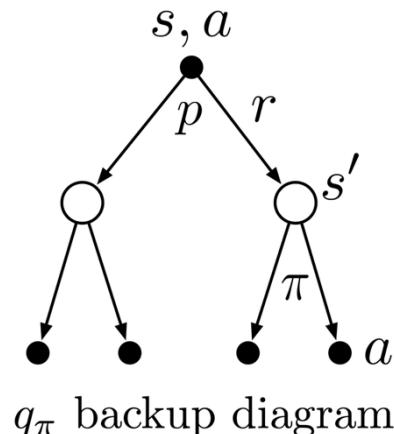
(sweep through entire state space)

Action-Value Function

$$\begin{aligned} q_{\pi}(s_t, a_t) &= E_{\pi} \left[\sum_{k=0}^{\infty} \gamma^k r_{t+k+1} | s_t, a_t \right] = E_{\pi}[r_{t+1} + \gamma q_{\pi}(s_{t+1}, a_{t+1}) | s_t, a_t] \\ &= \sum_{s'_{t+1}, r_{t+1}} p(s'_{t+1}, r_{t+1} | s_t, a_t) \left[r_{t+1} + \gamma \sum_{a'_{t+1}} \pi(a'_{t+1} | s'_{t+1}) q_{\pi}(s'_{t+1}, a'_{t+1}) \right] \end{aligned}$$



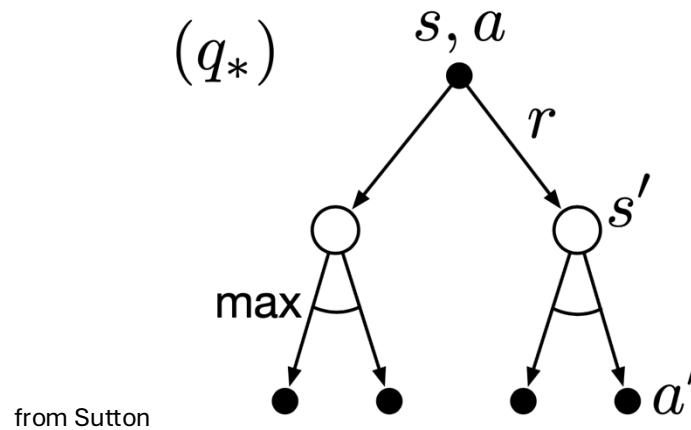
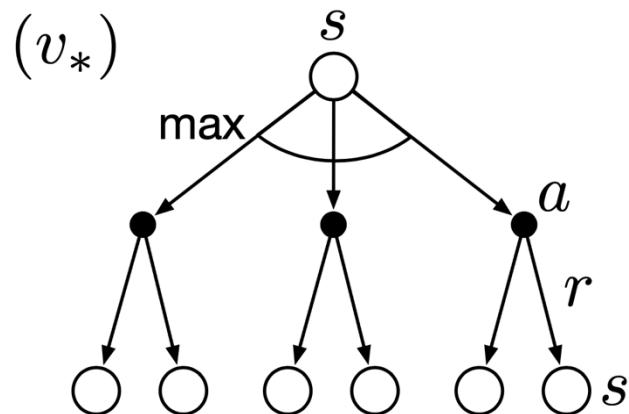
Backup diagram for v_{π}



q_{π} backup diagram

Bellman Optimality Equations

optimal solutions to Bellman equations (directly defining optimal policy):



rarely possible to find in practice (due to missing model of environment, invalid Markov property, limited computational resources)

→ approximate solutions

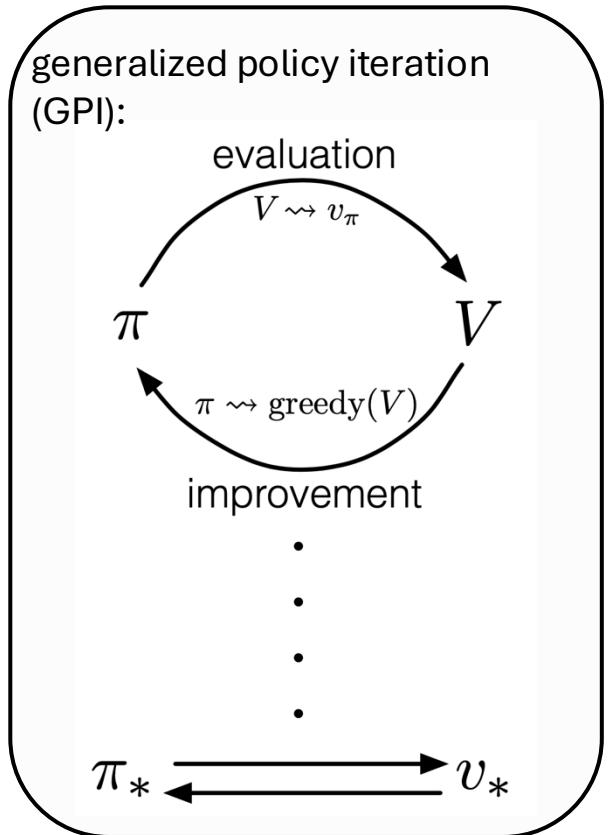
Dynamic Programming (DP)

iterative approaches to find approximations for optimal value functions

1. policy evaluation: calculate value function with current policy (Bellman equation as update rule)
2. policy improvement: adjusting policy to act greedy (pick actions with maximum values) with respect to value function of current policy

putting both components together:

- policy iteration: $\pi_0 \xrightarrow{E} v_{\pi_0} \xrightarrow{I} \pi_1 \xrightarrow{E} v_{\pi_1} \xrightarrow{I} \pi_2 \xrightarrow{E} \dots \xrightarrow{I} \pi_* \xrightarrow{E} v_*$
- value iteration: truncated policy evaluation using Bellman optimality equation as update rule (stopped after one update of each state)



from Sutton

GPI also followed by MC and TD methods ...

Limited Utility of DP

requires full model of environment

computationally expensive

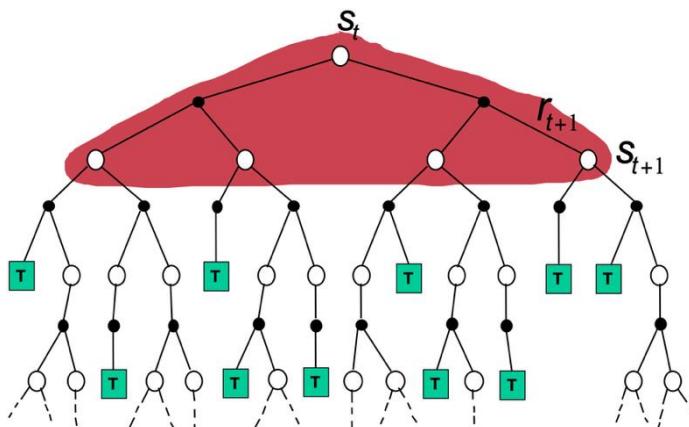
- expected update operation (based on values of all possible successor states and their probability)
- for each state (in potentially huge state space)
(asynchronous DP at least avoids systematic sweeps over entire state space)

→ need for more efficient methods achieving the same effect as DP, without (perfect) model of environment

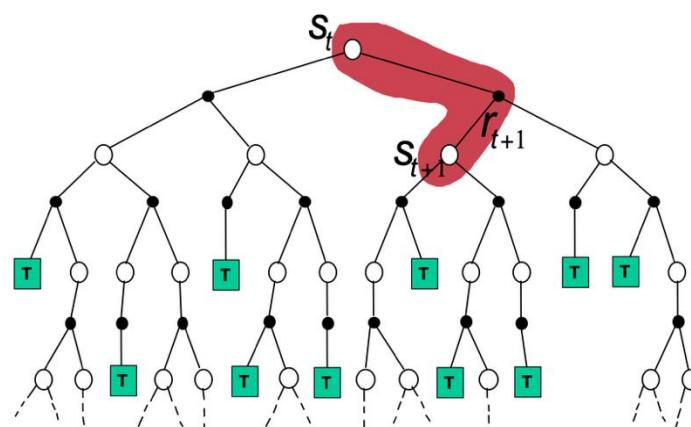
Bootstrapping and Sampling

bootstrapping: update estimates of state values based on estimates of values of successor states
sampling: experience of sample sequences (no need for complete knowledge of environment)

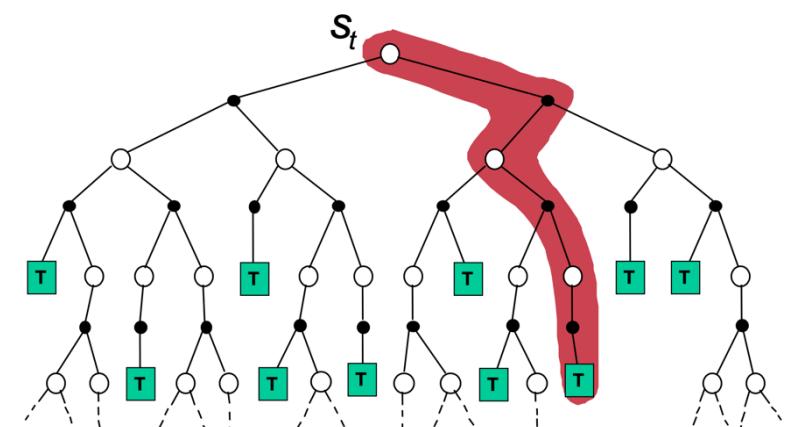
Dynamic Programming



Temporal Difference (TD) Learning



Monte Carlo (MC)



- bootstrapping
- no sampling → model-based
(transition probabilities needed)

- bootstrapping
- sampling → model-free

- no bootstrapping
- sampling → model-free

from Sutton

Sampling Update Rule

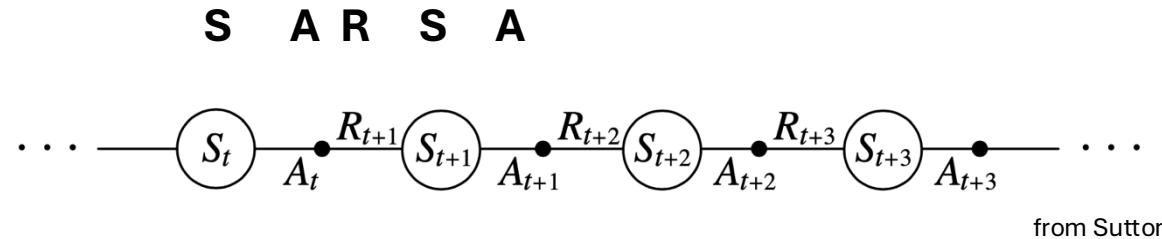
$$\text{NewEstimate} \leftarrow \text{OldEstimate} + \text{StepSize} [\text{Target} - \text{OldEstimate}]$$

MC: $v(s_t) \leftarrow v(s_t) + \eta \left[\sum_{k=0}^{\infty} \gamma^k r_{t+k+1} - v(s_t) \right]$

TD: $v(s_t) \leftarrow v(s_t) + \eta [r_{t+1} + \gamma v(s_{t+1}) - v(s_t)]$

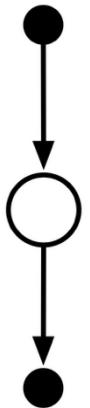
bootstrapping

On-Policy TD Control: SARSA



following pattern of GPI:

- estimate action-value function for current behavior policy
$$q_\pi(s_t, a_t) \leftarrow q_\pi(s_t, a_t) + \eta[r_{t+1} + \gamma q_\pi(s_{t+1}, a_{t+1}) - q_\pi(s_t, a_t)]$$
- change policy toward greediness with respect to q_π
(exploration for example via ε -greedy policy)

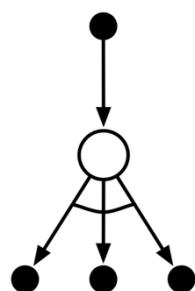


Off-Policy TD Control: Q-Learning

estimate action-value function directly approximating optimal one
(independent of behavior policy → potentially off-policy)

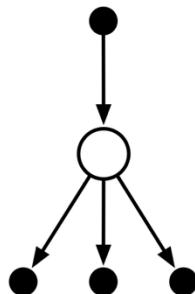
$$q(s_t, a_t) \leftarrow q(s_t, a_t) + \eta \left[r_{t+1} + \gamma \max_a q(s_{t+1}, a_{t+1}) - q(s_t, a_t) \right]$$

policy just determines which state-action pairs are visited and updated

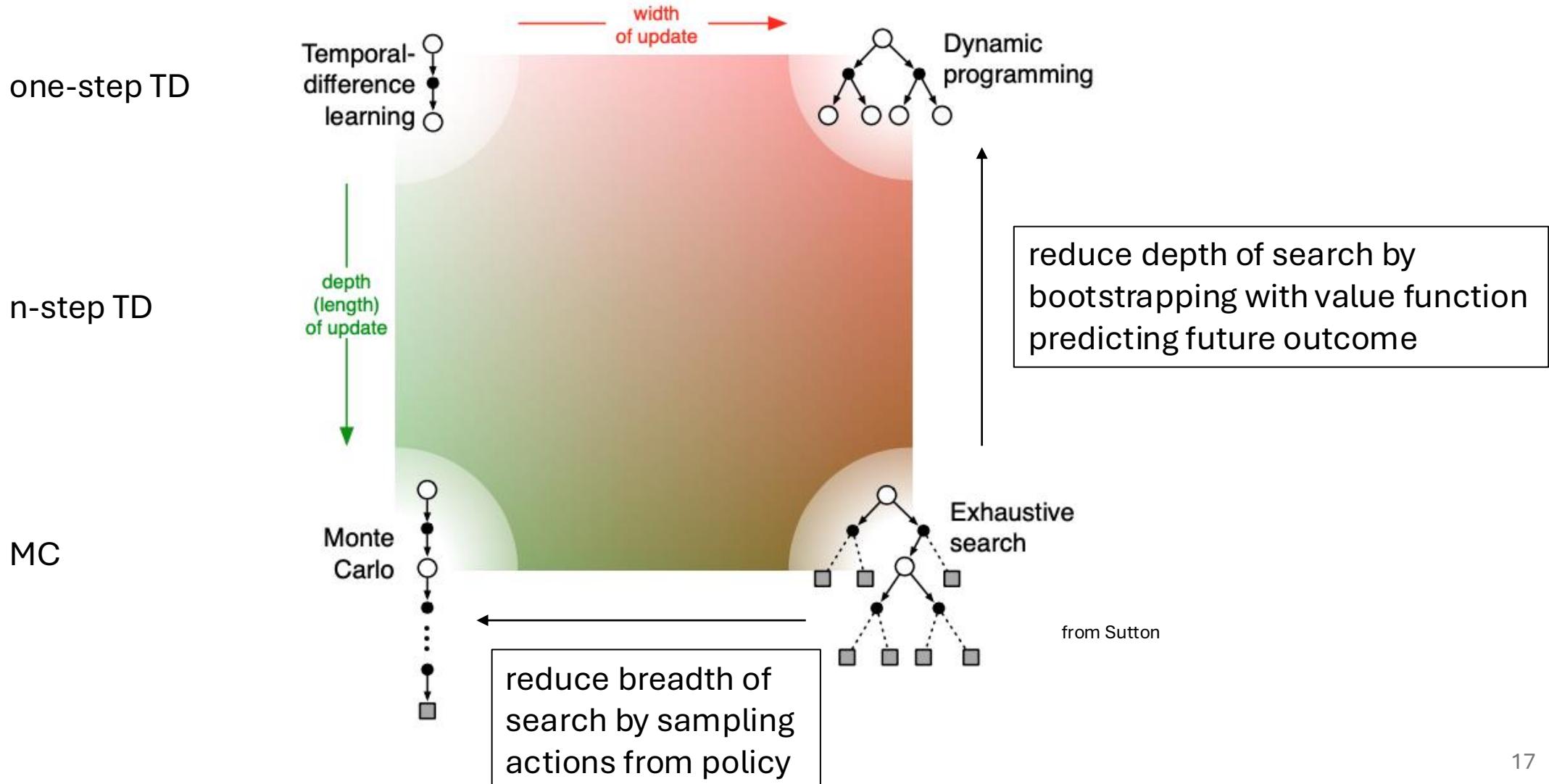


compare to expected Sarsa:

$$q(s_t, a_t) \leftarrow q(s_t, a_t) + \eta \left[r_{t+1} + \gamma \sum_a \pi(a_{t+1}|s_{t+1}) q(s_{t+1}, a_{t+1}) - q(s_t, a_t) \right]$$



Summary: Update Characteristics



Limitation of Tabular Methods

tabular methods (calculating values for each state/action) simply memorize observed data

problem with tabular solution methods in practice: large state/action spaces (kind of curse of dimensionality)

- need for generalization: supervised learning to the rescue
 - non-linear function approximation over state/action space
 - nowadays often deep learning methods → deep RL

Approximate Solution Methods

online setting: learning while interacting with environment
offline setting: learning from limited data without further interaction with environment

state/action values as parametrized function (instead of table)

- variables/features describing different states
- parameters (e.g., connection weights in neural network) to be learned

objective function for supervised learning (e.g., squared error loss):

$$J(\hat{\mathbf{w}}) = \sum_s (\nu_{\pi}(s) - \hat{\nu}(s; \hat{\mathbf{w}}))^2$$

parameters/weights to be optimized via (stochastic) gradient descent

→ RL problem expressed in supervised learning setup (potentially offline/batch data)

but $\nu_{\pi}(s)$ still calculated via RL methods (e.g., bootstrapping)

Deep Q-Network (DQN)

idea: deep neural network(s) approximating tabular action-value function (according to Q-learning): $q(s, a; \hat{w})$ as target of supervised learning model

key components to get it going (stable training):

- separate target network: weights only periodically updated with estimated Q-network weights → reducing correlations of Q-network with target (due to bootstrapping)
- experience replay: apply Q-learning updates on samples (or mini batches) of experience drawn at random from stored samples (agent's experiences) → removing correlations in observation sequence ("make it i.i.d.")

Famous Example of Deep RL: AlphaGo

Monte Carlo tree search (heuristic, lookahead search) for move (i.e., action) selection

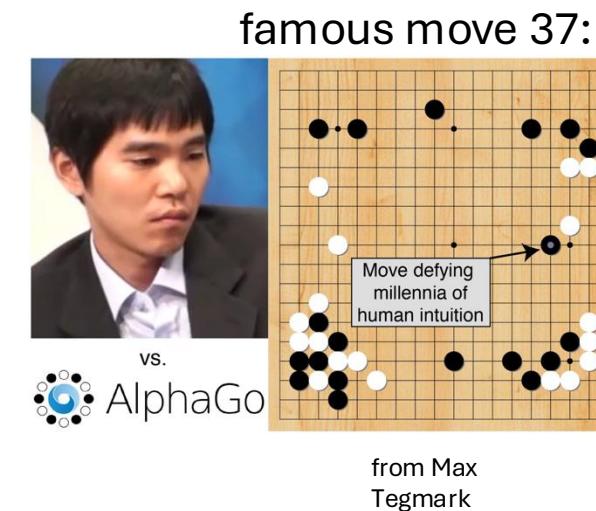
→ decision-time planning (advantage: focus on current state rather than full state space)

guided by deep convolutional neural networks for both value function and policy estimation

→ improving search efficiency

reduce depth of search tree by evaluating positions with **value function** (predicting outcome from given position → **bootstrapping**)

reduce breath of search tree by **sampling** actions using **policy network** (probability distribution over possible moves in given position)



Policy Gradient Methods

learning of parametrized policy (without value functions)

$\pi(a_t|s_t; \hat{\theta})$: probability to take different actions (target) given a state (variables/features) and parameters (e.g., neural network weights)

goal maximizing expected cumulative rewards

→ objective function corresponds to true state value: $J(\hat{\theta}) = v_\pi(s_t)$

policy gradient theorem:

$$\nabla_{\hat{\theta}} J(\hat{\theta}) \propto \sum_{a_t} q_\pi(s_t, a_t) \nabla_{\hat{\theta}} \pi(a_t | s_t; \hat{\theta})$$

REINFORCE

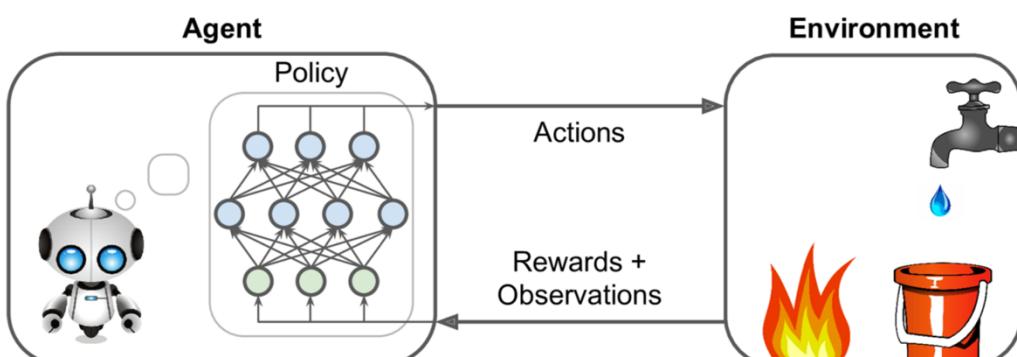
REINFORCE method (MC method following from policy gradient theorem):

$$\hat{\theta} \leftarrow \hat{\theta} + \eta \cdot \nabla_{\hat{\theta}} [\log \pi(a_t | s_t; \hat{\theta})] \cdot (r_{t+1} + \gamma r_{t+2} + \dots)$$

$$\nabla_{\hat{\theta}} J(\hat{\theta})$$

policy gradients → neural network gradients

“weighting” with observed
(discounted) return



policy gradient methods:
on-policy learning

REINFORCE with Baseline

policy gradient theorem unchanged by subtracting an action-independent baseline, e.g., an estimate of the state-value function:

$$\nabla_{\hat{\theta}} J(\hat{\theta}) \propto \sum_{a_t} [q_{\pi}(s_t, a_t) - \hat{v}(s_t; \hat{w})] \nabla_{\hat{\theta}} \pi(a_t | s_t; \hat{\theta})$$

$$\hat{\theta} \leftarrow \hat{\theta} + \eta \cdot \nabla_{\hat{\theta}} [\log \pi(a_t | s_t; \hat{\theta})] \cdot [(r_{t+1} + \gamma r_{t+2} + \dots) - \hat{v}(s_t; \hat{w})]$$

e.g., separate networks

hybrid between policy-based and value-based methods
→ reduction of variance

Actor-Critic Methods

using state-value function for bootstrapping → critic of policy:

$$\hat{\theta} \leftarrow \hat{\theta} + \eta \cdot \nabla_{\hat{\theta}} [\log \pi(a_t | s_t; \hat{\theta})] \cdot [(r_{t+1} + \gamma \hat{v}(s_{t+1}; \hat{w})) - \hat{v}(s_t; \hat{w})]$$


TD error

turning MC (observed return) into TD method

→ introduction of bias, but further reduction of variance

Synonym: Advantage Actor-Critic

for the critic of the action policy (actor):

interpret TD error $r_{t+1} + \gamma \hat{v}(s_{t+1}; \hat{w}) - \hat{v}(s_t; \hat{w})$

as advantage function $\hat{q}(s_t, a_t; \hat{w}) - \hat{v}(s_t; \hat{w})$

idea: calculates extra reward for specific action compared to average action in given state (expected state value)

Proximal Policy Optimization ([PPO](#)):

prominent advantage actor-critic method with some tricks

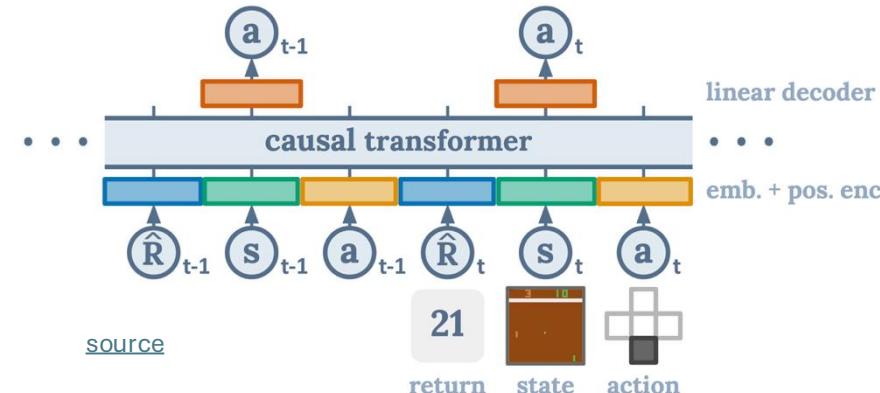
Sequence Modeling for Decisions/Actions

generative: transformer decoder architecture to autoregressively model trajectories

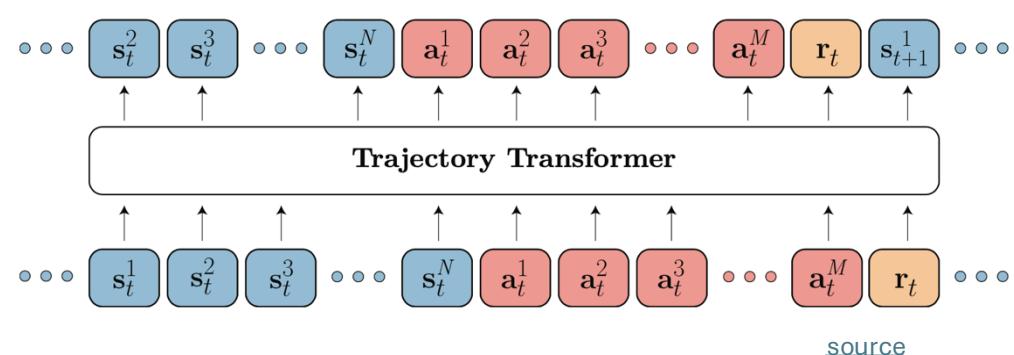
credit assignment directly via self-attention: implicitly forming state-return associations via similarity of query and key vectors (maximizing the dot product)

desired return tokens as prompt for action generation

Decision Transformer: conditioning on desired return, past states and actions to generate future actions



Trajectory Transformer: predicting also states and returns (adding model-based components, planning with beam search)

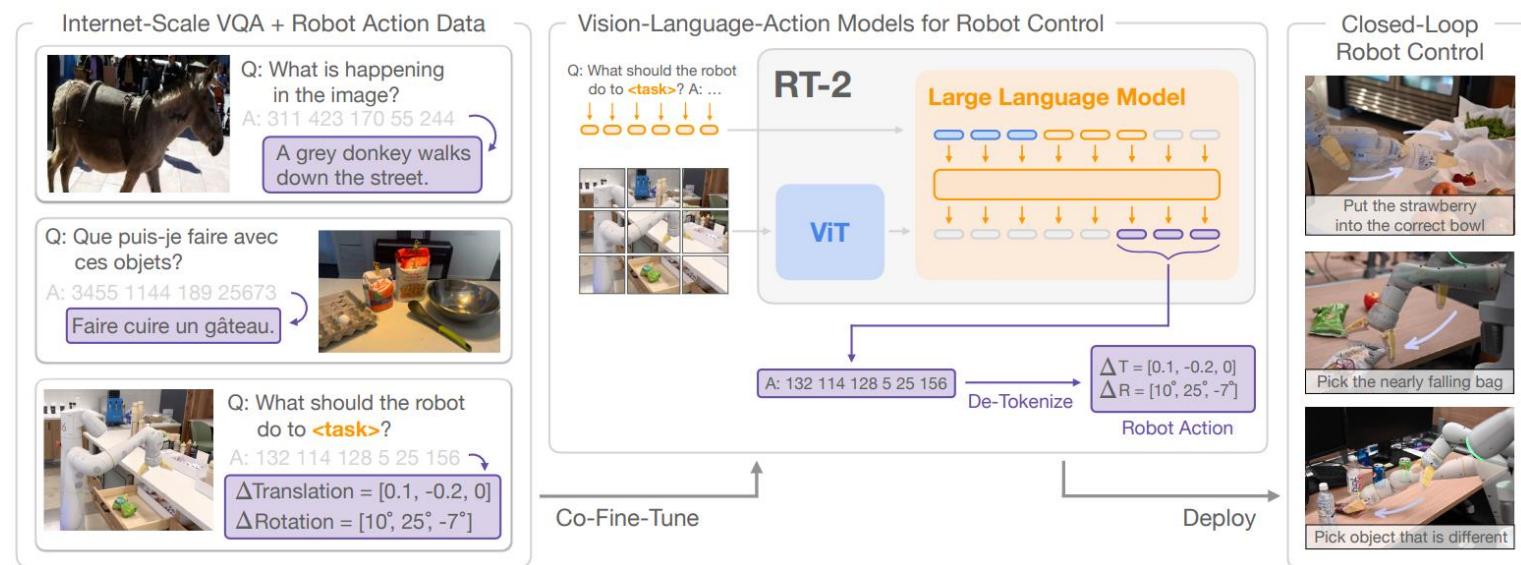


can also condition imitation learning diffusion objective for multimodal outputs

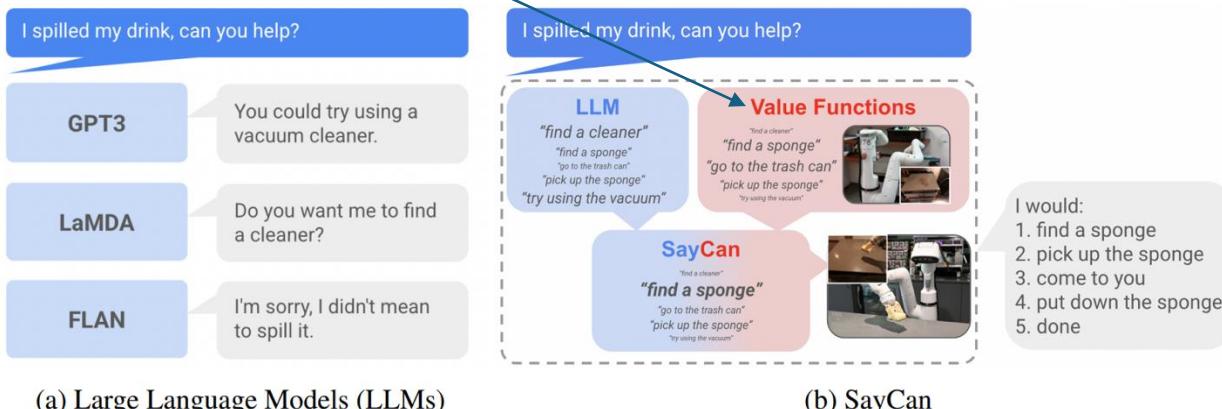
Robotic Control via LLMs (and Vision)

RT-2: vision-language-action model learning from web and robotics data

- representation of actions as tokens
- generalization by using pre-trained vision-language models



grounding with pre-trained skills (SayCan):



Code as Policies:

