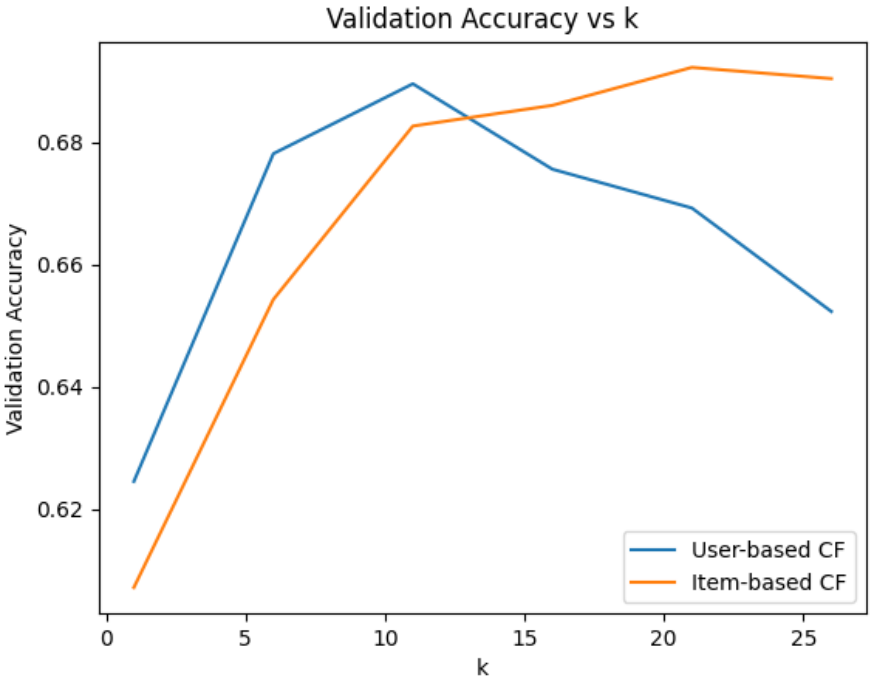
Question 1

(a) (b) (c) The accuracy on the validation data with $k \in \{1, 6, 11, 16, 21, 26\}$ on user-based and item-based collaborative filtering is as follows:



Test Accuracy on user-based CF with k* = 11: 0.6841659610499576

Test Accuracy on item-based CF with k* = 21: 0.6816257408975445

(d) The test on user-based CF is slightly better than item-based CF.

Additionally, the test accuracy on user-based CF cost less time than item-based CF.

Therefore, user-based CF is better than item-based CF in this case.

(e)
- The KNN algorithm is computational expensive for large datasets.
- The Curse of Dimensionality: In high dimensions, “most” points are approximately the same distance and the nearest neighbors are not very useful.

Question 2

(a) Given the probability that the question j is correctly answered by student i is:

$$p_{ij} = \frac{\exp(\theta_i - \beta_j)}{1 + \exp(\theta_i - \beta_j)}$$

The log-likelihood for all students is derived as follows:

$$\begin{aligned}\log p(\mathbf{C}|\boldsymbol{\theta}, \boldsymbol{\beta}) &= \sum_{i,j} (c_{ij} \log p_{ij} + (1 - c_{ij}) \log(1 - p_{ij})) \\ &= \sum_{i=1}^{n} \sum_{j=1}^{m} \left(c_{ij} \log \left(\frac{\exp(\theta_i - \beta_j)}{1 + \exp(\theta_i - \beta_j)} \right) + (1 - c_{ij}) \log \left(1 - \frac{\exp(\theta_i - \beta_j)}{1 + \exp(\theta_i - \beta_j)} \right) \right) \\ &= \sum_{i=1}^{n} \sum_{j=1}^{m} (c_{ij}(\theta_i - \beta_j) - \log(1 + \exp(\theta_i - \beta_j))),\end{aligned}$$

where c_{ij} is the binary response of student i to question j.

The log-likelihood with respect to θ_i is:

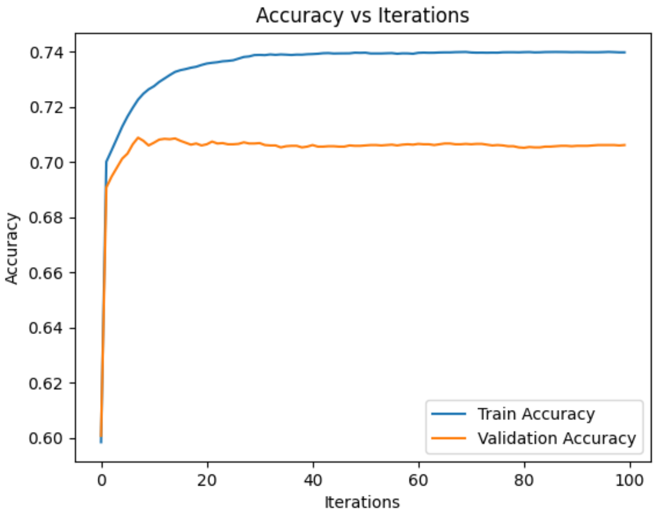
$$\begin{aligned}\frac{\partial \log p(\mathbf{C}|\boldsymbol{\theta}, \boldsymbol{\beta})}{\partial \theta_i} &= \sum_{j=1}^{m} \left(c_{ij} - \frac{\exp(\theta_i - \beta_j)}{1 + \exp(\theta_i - \beta_j)} \right) \\ &= \sum_{j=1}^{m} (c_{ij} - p_{ij}).\end{aligned}$$

The log-likelihood with respect to β_j is:

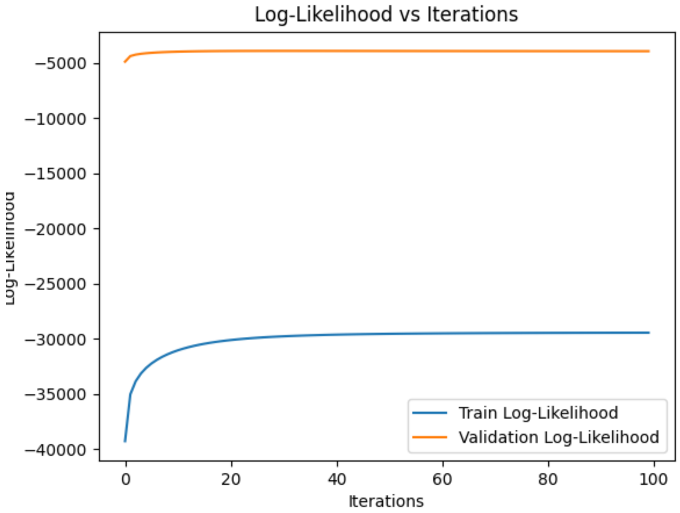
$$\begin{aligned}\frac{\partial \log p(\mathbf{C}|\boldsymbol{\theta}, \boldsymbol{\beta})}{\partial \beta_j} &= \sum_{i=1}^{n} \left(c_{ij} - \frac{\exp(\theta_i - \beta_j)}{1 + \exp(\theta_i - \beta_j)} \right) \\ &= \sum_{i=1}^{n} (c_{ij} - p_{ij}).\end{aligned}$$

(b) The hyperparameters I selected are: learning rate = 0.01 and iterations = 100.

The training and validation accuracies vs iterations are in the graph below:



The log-likelihoods vs iterations are in the graph below:

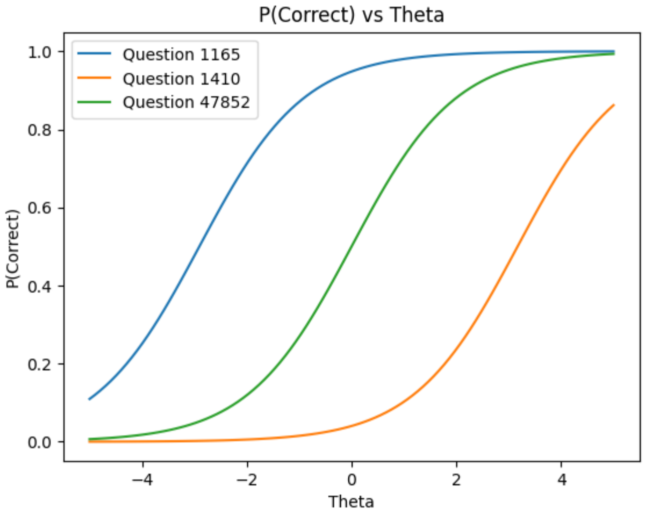


(c) The Final Validation Accuracy: 0.7063223257126728

The Final Test Accuracy: 0.707310189105278

(d) I select the lowest difficulty question j_1 (Question 1165), the highest difficulty question j_2 (Question 47852) and the average difficulty question j_3 (Question 1410).

The probability of the correct response is in the graph below:



(e) The shape of the curves are like the sigmoid function as expected.

Fix a question j. As θ_i increases, the probability of the correct response p_{ij} increases. This means if a student has a higher ability, the probability of the correct response increases.

Fix a student i. As β_j increases, the probability of the correct response p_{ij} decreases. This means if a question has a higher difficulty, the probability of the correct response decreases.

Q3.

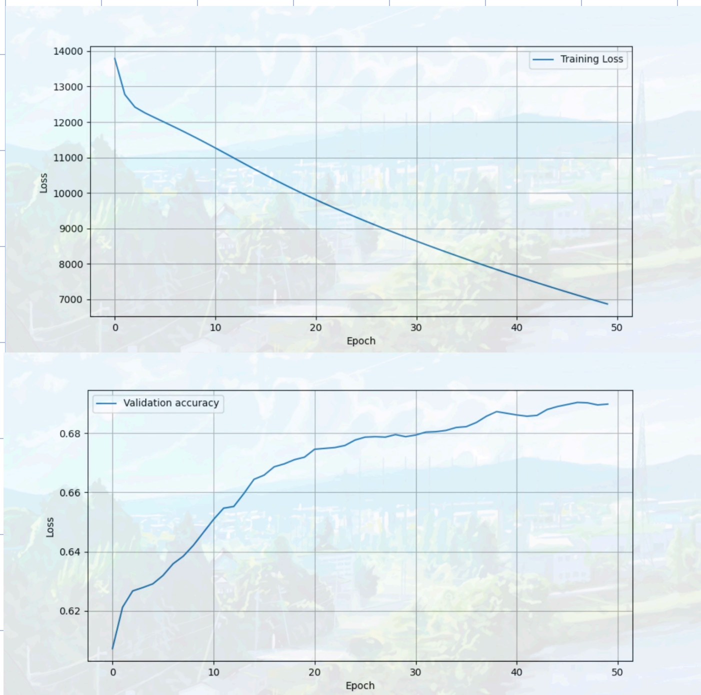
a). • ALS break down large matrix into lower-dimensional matrices, Neural network modeling non-linear relationship though layers.

• ALS is less flexible than Neural network since they are designed for matrix factorization where Neural network can modle non-linear relationship.

• ALS is more computationally efficient than Neural network for sparse dataset Neural network require significant computational resoures.

c) The optimization hyperparameter we have is:
k=50 Lr=0.01 num_epoch=50
and we got Validation Accuracy of 0.68981

d) plot with k=50 Lr=0.01 num_epoch=50:



Final Test accuracy is: 0.68558

e) The best regularization penalty is $\lambda=0.01$ with this λ,
final Validation Accuracy: 0.67824
final Test Accuracy: 0.68078

• The model didn't perform better with the regularization penalty, this may because that our modle already well-regularized and does not overfitting or negligible overfitting issue.

Q4. The final validation accuracy is : 0.66286

The final test accuracy is : 0.66949

Ensemble process:

We use three neural network models to implemented bagging ensemble We first randomly sample three sample with replacement from our training data. Then we train three different neural network independently for each training sample. These three neural network are complete independent and can run individually. After all models are trained, we use them to make prediction separately, finally we take the average of each of their predictions for our final prediction.

Better or Not :

No, the Bagging model is nearly the same performance as the single neural network model, so it doesn't improve proformance.

Reason:

Ensembling the same model that train on different data subset has lack model diversity, thus it does not always improve the model performance. Also, small traing subset could be another problem, when the traing set is small, there could be a issue that training subset are even smaller which make each model poor performance.