Laboratory Work #3.

This laboratory work is focused on 3 different topics – systems with exits, iterative maps and synchronization of nonlinear oscillators. All three topics are related to apparently different processes – but do really have much in common. What is this linking substance – you will have a change to find out when you finish the tasks. Enjoy!

#1. System with escapes. Henoin – Heiles (HH) system. The governing equations of motion for the HH system read:

$$\begin{cases} \dfrac{d^2x}{dt^2} = -x - 2xy \\ \dfrac{d^2y}{dt^2} = -y - x^2 + y^2 \end{cases}$$

The total energy of the system is:

$$E = \frac{1}{2}\left(\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2\right) + \frac{1}{2}(x^2 + y^2) + x^2y - \frac{1}{3}y^3$$

Set $x(t_0 = 0) = 0$ and fix the energy level E. Then, you are free to select $y(t_0 = 0) = y_0$ and $\frac{dy}{dt}(t_0 = 0) = \dot{y}_0$. Integrate the system of ODE and plot the trajectories in $\left\{y, \frac{dy}{dt}\right\}$ phase plane. Find such energies when trajectories become unbounded. Try to plot the contours of the system energy and plot the trajectories on top of the pattern of contour lines. Determine such energies when all trajectories escape the central region. Associate different escape gates with different colors – and mark initial conditions by points with corresponding colors. Change the energy level and construct different plots of initial conditions. Discuss the results.

#2. The Logistic map $x_{k+1} = ax_k(1 - x_k); 0 \le a \le 4; k = 0,1,2, \ldots; 0 \le x_0 \le 4.$

Plot bifurcation diagram illustrating a cascade of period doubling bifurcations (assign parameter a to the x-axis and the values of the steady state process x_k to the y- axis).

Compute and plot the Lyapunov exponent for the Logistic map. The Lyapunov exponent for a discreet iterative map is defined as $\lambda = \lim\limits_{n\to\infty} \sum_{k=1}^{n} \ln \left|\frac{df}{dx}(x_k)\right|$, where f is the mapping function of the discreet iteravitive map.

Derive the analytic expression of the non-trivial period-1 orbit, investigate its stability. Set a to 1.5, 2.5 and 3.5. Use computational techniques to construct the set of non-asymptotic convergence to period-1 orbit.

#3. Synchronization of chaotic oscillators. Consider two coupled Rossler oscillators:

$$\frac{dx_1}{dt} = -y_1 - z_1 + \varepsilon(x_2 - x_1) \qquad \frac{dx_2}{dt} = -y_2 - z_2 + \varepsilon(x_1 - x_2)$$

$$\frac{dy_1}{dt} = x_1 + ay_1 \qquad \frac{dy_2}{dt} = x_2 + ay_2$$

$$\frac{dz_1}{dt} = b + z_1(x_1 - c) \qquad \frac{dz_2}{dt} = b + z_2(x_2 - c)$$

Set parameters $a = b = 0.2$ and $c = 7$. At $\varepsilon = 0$ two systems are uncoupled. Investigate a single uncoupled system by changing parameter c. Construct a graphical representation of the cascade of period doubling bifurcations leading to chaos (use Poincare sections).

Set such $\varepsilon > 0$ that the synchronization becomes phase-type synchronization and then complete synchronization.