Laboratory Work #2

General information about the second Laboratory Work

This is a little bit more demanding laboratory work if compared to the first one. We continue our journey through different bifurcations – and get a first experience with chaotic processes. We are still in the world of ODEs. The first problem is more “mathematical” – but the rest go deeper into computations. Almost all tasks have been discussed in lectures (with hints and examples in Moodle) – but it’s the first time when you are requested to tackle a “new” problem (task 2). Have fun!

The list of tasks

#1. Follow the Lindstedt’s method description in the lecture notes (in Moodle). Note, that the method is illustrated for Duffing’s equation and all computations are limited to x_1. Try to make one step forward – collect the terms at ε^2, use the expression of x_1, identify secular terms, and derive the expression of x_2. Symbolic algebra tools are highly recommended for the execution of the task. Try to visualize the derived solutions – do not forget that the time scale is transformed.

#2. Hopf bifurcation. Try to simulate a “real world” problem – the Brusselator model. Yes, you will not find the description of this model in our lecture notes – therefore this is a “real world” problem. Try to search the web – find the description of the model, construct the code for simulation, illustrate Hopf bifurcation. Remember that Newmark method will not work – you will not be able to transform a system of two first order ODEs into one second order ODE. Suggestion – use Euler, Adams or even RK integration method. What??? No problems – search the web. Yes, you will feel like immersed into a “real world” environment. BTW, you are not requested to simulate the Brusselator as a reaction diffusion system in two spatial dimensions. Self-organizing patterns is another topic which we will reach later in the course (assume now that the Brusselator model is described by a system of two first order ODEs with constant coefficients). Discuss what type of Hopf bifurcation you are able to observe using this model.

#3. Homoclinic bifurcation. Part 1. Remember the last task from Lab 1? It’s a high time to return to the same problem. Try to carefully illustrate the homoclinic bifurcation in this system. Explain in your own words – what happens when the homoclinic bifurcation occurs in this system. Part 2. Consider the following model: $\frac{d^2y}{dx^2} + A\frac{dy}{dx} + y = y^3 - y^2\frac{dy}{dx}$. Use numerical integration to identify the critical value of the parameter A where the homoclinic bifurcation occurs.

#4. The hysteresis effect (the forced Duffing oscillator).
The model reads: $\frac{d^2x}{dt^2} + 0.1\frac{dx}{dt} + x + \alpha x^3 = 0.5\cos(\omega t)$. Set $\alpha = 0.1$. Illustrate the jump effect by slowly varying ω starting from 0 to 2 – and then on the opposite – starting from 2 to 0. What does it mean “slowly varying”? It means that you are requested to run computational simulations and vary ω so slowly that it would not affect the transient processes. You could vary ω in every time step by a tiny fraction – so tiny that it does not show a vivid impact to the transient processes. What you are requested to plot? – ω on the horizontal axis – and the amplitude of the steady state oscillations on the vertical axis. Repeat the simulation at $\alpha = -0.1$.

#5. Period doubling bifurcations and road to chaos in a forced mathematical pendulum.
This paradigmatic model reads: $\frac{d^2x}{dt^2} + h\frac{dx}{dt} + \sin(x) = b\cos(\omega t)$. Set $\omega = 2/3$; $b = 2.048$ and illustrate the bifurcation diagram by slowly varying h from 1.05 to 1. What are you requested to plot? - h on the horizontal axis – and Poincare section points of $\frac{dx}{dt}$ on the vertical axis. Comment the results.