1. (a) Factor $2^{15} - 1 = 32{,}767$ into a product of two smaller positive integers.

$2^{15} - 1 = 32767 = 31 \cdot 1057$

$\cancel{2}$ $\cancel{3}$ $\cancel{5}$ $\cancel{7}$ $\cancel{11}$ $\cancel{13}$ $\cancel{17}$ $\cancel{19}$ $\cancel{23}$ $\cancel{29}$ 31 $\cancel{37}$

(b) Find an integer x such that $1 < x < 2^{32{,}767} - 1$ and $2^{32{,}767} - 1$ is divisible by x.

Conjectura: Se $m \in \mathbb{N}$, $m > 1$ e pode ser descrito por $m = a \cdot b$ com $a, b \in \mathbb{N}$, então $2^m - 1$ é divisível por $2^a - 1$ e $2^b - 1$

TESTE: $2^{32767} - 1 / 2^{31} - 1 = \sum_{n=0}^{1056} 2^{31n}$

$$\begin{array}{l|l} 2^{32767} - 1 & 2^{31} - 1 \\ \hline 2^{32736} - 1 & 2^{32736} + 2^{32705} + \dots + 2^{31} + 2^0 \\ 2^{32705} - 1 & \\ \vdots & \\ 2^{31} - 1 & \\ 0 & \end{array}$$

$2^{32767} - 1 / 2^{1057} - 1 = \sum_{n=0}^{30} 2^{1057n}$

$$\begin{array}{l|l} 2^{32767} - 1 & 2^{1057} - 1 \\ \hline 2^{31710} - 1 & 2^{31710} + 2^{30653} + \dots + 2^{1057} + 2^0 \\ 2^{30653} - 1 & \\ \vdots & \\ 2^{1057} - 1 & \\ 0 & \end{array}$$

PROVA:

Caso $1 < a < m$:

Somemos a expressão:

$2^m - 1 = 2^{a \cdot b} - 1$

Como $a < b$ e temos $1 < b$, então podemos somar zeros a expressão na forma $2^{na} - 2^{na}$ com n começando em 1 e terminando em $b-1$:

$$2^m - 1 = 2^{ba} + \overbrace{(2^a - 2^a) + (2^{2a} - 2^{2a}) + (2^{3a} - 2^{3a}) + \dots + (2^{(b-1)a} - 2^{(b-1)a})} - 1$$

Reorganizando os termos e colocando 2^{ba} ao final da sequência e -1 no começo temos:

$$2^m - 1 = (2^a + 2^{2a} + 2^{3a} + 2^{4a} + \dots + 2^{ba}) - (1 + 2^a + 2^{2a} + 2^{3a} + \dots + 2^{(b-1)a})$$

$$2^m - 1 = (2^a - 1)(2^{0a} + 2^{1a} + 2^{2a} + 2^{3a} + \dots + 2^{(b-1) \cdot a})$$

Como a soma $(1 + 2^a + 2^{2a} + 2^{3a} + \dots + 2^{(b-1)a})$ é um número inteiro e $2^a - 1$ também, podemos dizer que $(2^a - 1) \mid (2^m - 1)$

Caso $a = m$

Nesse caso temos que $m = a \cdot b = m \cdot b$, então $b = 1$

$2^m - 1 = 2^{a \cdot b} - 1$

$2^m - 1 = 2^m - 1$

$2^m - 1 = (2^m - 1) \cdot 1$

$2^m - 1 = (2^a - 1) \cdot 1 \longrightarrow (2^a - 1) \mid (2^m - 1)$

Caso $a = 1$

Nesse caso temos que $m = a \cdot b = 1 \cdot b$, então $b = m$ e $2^a - 1 = 1$

$2^m - 1 = (2^m - 1) \cdot 1$

$2^m - 1 = (2^m - 1) \cdot (2^a - 1) \longrightarrow (2^a - 1) \mid (2^m - 1)$

2. Make some conjectures about the values of n for which $3^n - 1$ is prime or the values of n for which $3^n - 2^n$ is prime. (You might start by making a table similar to Figure I.1.)

n	$3^n - 1$	$3^n - 2^n$
1	2	1
2	8	5
3	26	19
4	80	65
5	242	211
6	728	665
7	2186	2059
8	6560	6305
9	19682	19171
10	59048	58025
11	177146	175099

$3^n - 1$

- Aparentemente todo valor gerado a partir dessa expressão par.

$3^n - 2^n$

- Nada pode ser dito sobre a geração de números primos, porém é possível que caso n não seja primo, então $3^n - 2^n$ também não seja.

3. The proof of Theorem 3 gives a method for finding a prime number different from any in a given list of prime numbers.
   (a) Use this method to find a prime different from 2, 3, 5, and 7.
   (b) Use this method to find a prime different from 2, 5, and 11.

a) $2 \cdot 3 \cdot 5 \cdot 7 + 1 = 211$ b) $2 \cdot 3 \cdot 5 \cdot 7 \cdot 11 + 1 = 2311$

4. Find five consecutive integers that are not prime.

A partir do teorema 4 encontramos o primeiro elemento

$a_0 = (5+1)! + 2 = 722$ $S = \{722; 723; 724; 725; 726\}$

5. Use the table in Figure I.1 and the discussion on p. 5 to find two more perfect numbers.

n	Is n prime?	$2^n - 1$	Is $2^n - 1$ prime?
2	yes	3	yes
3	yes	7	yes
4	no: $4 = 2 \cdot 2$	15	no: $15 = 3 \cdot 5$
5	yes	31	yes
6	no: $6 = 2 \cdot 3$	63	no: $63 = 7 \cdot 9$
7	yes	127	yes
8	no: $8 = 2 \cdot 4$	255	no: $255 = 15 \cdot 17$
9	no: $9 = 3 \cdot 3$	511	no: $511 = 7 \cdot 73$
10	no: $10 = 2 \cdot 5$	1023	no: $1023 = 31 \cdot 33$

$2 \neq 1$
$3 \neq 1$
$4 \neq 2+1$
$5 \neq 1$
$6 = 3+2+1$
$7 \neq 1$
$8 \neq 4+2+1$
$9 \neq 3+1$
$10 \neq 5+2.1$
$11 \neq 1$
$12 \neq 6+4+2+1$
$13 \neq 1$
$14 \neq 7+2+1$
$15 \neq 5+3+1$
$16 \neq 8+4+2+1$
$17 \neq 1$
$18 \neq 9+6+3+1$
$19 \neq 1$
$20 \neq 10+5+2+1$
$21 \neq 7+3+1$
$22 \neq 11+2+1$
$23 \neq 1$
$24 \neq 12+8+4+2+1$
$25 \neq 5+1$
$26 \neq 13+2+1$
$27 \neq 9+3+1$
$28 = 14+7+4+2+1$

6. The sequence 3, 5, 7 is a list of three prime numbers such that each pair of adjacent numbers in the list differ by two. Are there any more such "triplet primes"?

Não existem triplets no formato $n, n+2, n+4$ além da sequência 3, 5, 7

7. A pair of distinct positive integers (m, n) is called *amicable* if the sum of all positive integers smaller than n that divide n is m, and the sum of all positive integers smaller than m that divide m is n. Show that (220, 284) is amicable.

$284 = 110+55+44+20+22+11+10+5+4+2+1$

$220 = 142+71+4+2+1$

1. Analyze the logical forms of the following statements:
   (a) We'll have either a reading assignment or homework problems, but we won't have both homework problems and a test.
   (b) You won't go skiing, or you will and there won't be any snow.
   (c) $\sqrt{7} \not\leq 2$.

a) P = have a reading assignment
Q = have a homework problems
R = have a test
$(P \vee Q) \wedge \neg(Q \wedge R)$

b) P = go skiing
Q = there will be snow
$\neg P \vee (P \wedge \neg Q)$

c) $\sqrt{7} \not\leq 2$ $\quad \neg(\sqrt{7} < 2) \vee (\sqrt{7} = 2))$

2. Analyze the logical forms of the following statements:
   (a) Either John and Bill are both telling the truth, or neither of them is.
   (b) I'll have either fish or chicken, but I won't have both fish and mashed potatoes.
   (c) 3 is a common divisor of 6, 9, and 15.

a) P = John is telling the truth
Q = Bill is telling the truth
$(P \wedge Q) \vee (\neg P \wedge \neg Q)$

b) P = have fish
Q = have chicken
R = have mashed potatoes
$(P \vee Q) \wedge \neg(P \wedge R)$

c) $(3|6) \wedge (3|9) \wedge (3|15)$

3. Analyze the logical forms of the following statements:
   (a) Alice and Bob are not both in the room.
   (b) Alice and Bob are both not in the room.
   (c) Either Alice or Bob is not in the room.
   (d) Neither Alice nor Bob is in the room.

P = Alice in the room
Q = Bob in the room

a) $\neg(A \wedge B)$
b) $\neg A \wedge \neg B$
c) $\neg A \vee \neg B$
d) $\neg A \wedge \neg B$

4. Analyze the logical forms of the following statements:
   (a) Either both Ralph and Ed are tall, or both of them are handsome.
   (b) Both Ralph and Ed are either tall or handsome.
   (c) Both Ralph and Ed are neither tall nor handsome.
   (d) Neither Ralph nor Ed is both tall and handsome.

P = Ralph is tall
Q = Ed is tall
R = Ralph is handsome
S = Ed is handsome

a) $(P \wedge Q) \vee (R \wedge S)$
b) $(P \vee R) \wedge (Q \vee S)$
c) $\neg(P \vee Q) \wedge \neg(R \vee S)$
d) $\neg(P \wedge R) \wedge \neg(Q \wedge S)$

5. Which of the following expressions are well-formed formulas?
   (a) $\neg(\neg P \vee \neg\neg R)$.
   (b) $\neg(P, Q, \wedge R)$.
   (c) $P \wedge \neg P$.
   (d) $(P \wedge Q)(P \vee R)$.

a) $\neg(\neg P \vee \neg\neg R) = \neg(\neg P \vee R) = P \wedge \neg R$ Well formed
b) $\neg(P, Q, \wedge R)$ Not well formed
c) $P \wedge \neg P$ = false Well formed
d) $(P \wedge Q)\_(P \vee R)$ Not well formed

6. Let P stand for the statement "I will buy the pants" and S for the statement "I will buy the shirt." What English sentences are represented by the following formulas?
   (a) $\neg(P \wedge \neg S)$.
   (b) $\neg P \wedge \neg S$.
   (c) $\neg P \vee \neg S$.

a) $P \wedge \neg S$ = I will buy the pants and won't buy the shirt
I won't buy the pants or I will buy the shirt.
b) I won't buy the pants and shirt.
c) I won't buy both pants and shirt – Não levarei calça e camisa juntos

7. Let S stand for the statement "Steve is happy" and G for "George is happy." What English sentences are represented by the following formulas?
   (a) $(S \vee G) \wedge (\neg S \vee \neg G)$.
   (b) $[S \vee (G \wedge \neg S)] \vee \neg G$.
   (c) $S \vee [G \wedge (\neg S \vee \neg G)]$.

a) Steve is happy or George is happy and Steve isn't happy or George isn't happy.
b) George is happy and Steve isn't happy or Steve is happy or George isn't happy
c) Steve is happy or George is happy and Steve is sad or George is sad

8. Let T stand for the statement "Taxes will go up" and D for "The deficit will go up." What English sentences are represented by the following formulas?
   (a) $T \vee D$.
   (b) $\neg(T \wedge D) \wedge \neg(\neg T \wedge \neg D)$.
   (c) $(T \wedge \neg D) \vee (D \wedge \neg T)$.

a) Taxes will go up or the deficit will go up
b) Taxes and deficit can't go up together, also taxes and deficit can't go down together.
c) Or taxes go up and deficit go down, or taxes go down and deficit go up.

9. Identify the premises and conclusions of the following deductive arguments and analyze their logical forms. Do you think the reasoning is valid? (Although you will have only your intuition to guide you in answering this last question, in the next section we will develop some techniques for determining the validity of arguments.)
(a) Jane and Pete won't both win the math prize. Pete will win either the math prize or the chemistry prize. Jane will win the math prize. Therefore, Pete will win the chemistry prize.
(b) The main course will be either beef or fish. The vegetable will be either peas or corn. We will not have both fish as a main course and corn as a vegetable. Therefore, we will not have both beef as a main course and peas as a vegetable.
(c) Either John or Bill is telling the truth. Either Sam or Bill is lying. Therefore, either John is telling the truth or Sam is lying.
(d) Either sales will go up and the boss will be happy, or expenses will go up and the boss won't be happy. Therefore, sales and expenses will not both go up.

a) $a(x)$ = x win the math prize.
$b(x)$ = x win the chemistry prize

$\neg(a(\text{Jane}) \wedge a(\text{Pete})) = \neg a(\text{Jane}) \vee \neg a(\text{Pete})$
$b(\text{Pete}) \vee a(\text{Pete})$
$a(\text{Jane})$
$b(\text{Pete})$

True

b) $B \vee F$
$P \vee C$
$\neg(F \wedge C) = \neg F \vee \neg C$
$\neg(B \wedge P) = \neg B \vee \neg P$

False

c) $a(x)$ = x is telling the truth
$a(\text{John}) \vee a(\text{Bill})$
$\neg a(\text{Bill}) \vee \neg a(\text{Sam})$
$a(\text{John}) \vee \neg a(\text{Sam})$

True

d) $a(x)$ = x will go up.
$b(x)$ = x will be happy

False

$(a(\text{sales}) \wedge b(\text{boss})) \vee (a(\text{expenses}) \wedge \neg b(\text{boss}))$
$\neg(a(\text{sales}) \wedge a(\text{expenses})) = \neg a(\text{sales}) \vee \neg a(\text{expenses})$

1. Make truth tables for the following formulas:
(a) $\neg P \vee Q$.
(b) $(S \vee G) \wedge (\neg S \vee \neg G)$.

a)

P	Q	¬P	¬P∨Q
V	V	F	V
V	F	F	F
F	V	V	V
F	F	V	V

b)

S	G	¬S	¬G	S∨G	¬S∨¬G	(S∨G)∧(¬S∨¬G)
V	V	F	F	V	F	F
V	F	F	V	V	V	V
F	V	V	F	V	V	V
F	F	V	V	F	V	F

2. Make truth tables for the following formulas:
(a) $\neg[P \wedge (Q \vee \neg P)]$.
(b) $(P \vee Q) \wedge (\neg P \vee R)$.

a)

P	Q	Q∨¬P	P∧(Q∨¬P)	¬[P∧(Q∨¬P)]
V	V	V	V	F
V	F	F	F	V
F	V	V	F	V
F	F	V	F	V

b)

P	Q	R	P∨Q	¬P∨R	(P∨Q)∧(¬P∨R)
V	V	V	V	V	V
V	V	F	V	F	F
V	F	V	V	V	V
V	F	F	V	F	F
F	V	V	V	V	V
F	V	F	V	V	V
F	F	V	F	V	F
F	F	F	F	V	F

3. In this exercise we will use the symbol + to mean *exclusive or*. In other words, $P + Q$ means "P or Q, but not both."
(a) Make a truth table for $P + Q$.
(b) Find a formula using only the connectives $\wedge$, $\vee$, and $\neg$ that is equivalent to $P + Q$. Justify your answer with a truth table.

a)

P	Q	P xor Q
V	V	F
V	F	V
F	V	V
F	F	F

b) $(P \wedge \neg Q) \vee (\neg P \wedge Q)$

P	Q	(P∧¬Q)	(¬P∧Q)	(P∧¬Q)∨(¬P∧Q)
V	V	F	F	F
V	F	V	F	V
F	V	F	V	V
F	F	F	F	F

4. Find a formula using only the connectives $\wedge$ and $\neg$ that is equivalent to $P \vee Q$. Justify your answer with a truth table.

$\neg(\neg P \wedge \neg Q)$

P	Q	¬P	¬Q	¬P∧¬Q	¬(¬P∧¬Q)
V	V	F	F	F	V
V	F	F	V	F	V
F	V	V	F	F	V
F	F	V	V	V	F

5. Some mathematicians use the symbol $\downarrow$ to mean *nor*. In other words, $P \downarrow Q$ means "neither P nor Q."
(a) Make a truth table for $P \downarrow Q$.
(b) Find a formula using only the connectives $\wedge$, $\vee$, and $\neg$ that is equivalent to $P \downarrow Q$.
(c) Find formulas using only the connective $\downarrow$ that are equivalent to $\neg P$, $P \vee Q$, and $P \wedge Q$.

a & b) $\neg P \wedge \neg Q = \neg(P \vee Q)$

P	Q	P↓Q
V	V	F
V	F	F
F	V	F
F	F	V

c) $\neg P = P \downarrow P$

$P \wedge Q = (P \downarrow Q) \downarrow (P \downarrow Q)$

$P \vee Q = \{[(P\downarrow P) \downarrow (Q \downarrow Q)] \downarrow [(P\downarrow P) \downarrow (Q\downarrow Q)]\} \downarrow \{[(P\downarrow P) \downarrow (Q\downarrow Q)] \downarrow [(P\downarrow P)\downarrow(Q\downarrow Q)]\}$
$[(\neg P \downarrow \neg Q) \downarrow (\neg P \downarrow \neg Q)] \downarrow [(\neg P \downarrow \neg Q) \downarrow (\neg P \downarrow \neg Q)]$
$(\neg P \wedge \neg Q) \downarrow (\neg P \wedge \neg Q)$
$\neg(\neg P \wedge \neg Q) = P \vee Q$

6. Some mathematicians write $P \mid Q$ to mean "P and Q are not both true." (This connective is called *nand*, and is used in the study of circuits in computer science.)
(a) Make a truth table for $P \mid Q$.
(b) Find a formula using only the connectives $\wedge$, $\vee$, and $\neg$ that is equivalent to $P \mid Q$.
(c) Find formulas using only the connective $\mid$ that are equivalent to $\neg P$, $P \vee Q$, and $P \wedge Q$.

a)

P	Q		$P \mid Q$
V	V		F
V	F		V
F	V		V
F	F		V

b) $\neg(P \wedge Q)$

c) $\neg P = P \mid P$

$P \wedge Q = \neg(P \mid Q) = (P \mid Q) \mid (P \mid Q)$

$P \vee Q = \neg(\neg P \wedge \neg Q) = (\neg P \wedge \neg Q) \mid (\neg P \wedge \neg Q) =$

$[(\neg P \mid \neg Q) \mid (\neg P \mid \neg Q)] \mid [(\neg P \mid \neg Q) \mid (\neg P \mid \neg Q)] =$

$\{[(P \mid P) \mid (Q \mid Q)] \mid [(P \mid P) \mid (Q \mid Q)]\} \mid \{[(P \mid P) \mid (Q \mid Q)] \mid [(P \mid P) \mid (Q \mid Q)]\}$

7. Use truth tables to determine whether or not the arguments in exercise 9 of Section 1.1 are valid.
(a) Jane and Pete won't both win the math prize. Pete will win either the math prize or the chemistry prize. Jane will win the math prize. Therefore, Pete will win the chemistry prize.
(b) The main course will be either beef or fish. The vegetable will be either peas or corn. We will not have both fish as a main course and corn as a vegetable. Therefore, we will not have both beef as a main course and peas as a vegetable.
(c) Either John or Bill is telling the truth. Either Sam or Bill is lying. Therefore, either John is telling the truth or Sam is lying.
(d) Either sales will go up and the boss will be happy, or expenses will go up and the boss won't be happy. Therefore, sales and expenses will not both go up.

a)
$\neg(J_m \wedge P_m)$
$P_m \vee P_c$
J_m
P_c

J_m	P_m	P_c		$\neg(J_m \wedge P_m)$		$P_m \vee P_c$
V	V	V		F		V
V	V	F		F		V
* V	F	V		V		V
V	F	F		V		F
F	V	V		V		V
F	V	F		V		V
F	F	V		V		V
F	F	F		V		F

P_c deriva das premissas

b) $M_B \vee M_F$
$V_p \vee V_c$
$\neg(M_F \wedge V_c)$
$\neg(M_B \wedge V_p)$

$\neg(M_B \wedge V_p)$ não deriva das premissas

M_B	M_F	V_P	V_C		$M_B \vee M_F$		$V_p \vee V_c$		$\neg(M_F \wedge V_c)$		$\neg(M_B \wedge V_p)$
V	V	V	V		V		V		F		F
V	V	V	F		V		V		V		F
V	V	F	V		V		V		F		V
V	V	F	F		V		F		V		V
V	F	V	V		V		V		V		F
V	F	V	F		V		V		V		F
V	F	F	V		V		V		V		V
V	F	F	F		V		F		V		V
F	V	V	V		V		V		F		V
F	V	V	F		V		V		V		V
F	V	F	V		V		V		F		V
F	V	F	F		V		F		V		V
F	F	V	V		F		V		V		V
F	F	V	F		F		V		V		V
F	F	F	V		F		V		V		V
F	F	F	F		F		F		V		V

c) $J \vee B$
$\neg B \vee \neg S$
$J \vee \neg S$

J	B	S		$J \vee B$		$\neg B \vee \neg S$		$J \vee \neg S$
V	V	V		V		F		V
V	V	F		V		V		V
V	F	V		V		V		V
V	F	F		V		V		V
F	V	V		V		F		F
F	V	F		V		V		V
F	F	V		F		V		F
F	F	F		F		V		V

$J \vee \neg S$ deriva das premissas

d) $(S_u \wedge B_H) \vee (E_u \wedge \neg B_H)$

$\neg(S_u \wedge E_u)$

Não deriva.

S_u	E_u	B_H		$S_u \wedge B_H$	$\vee$	$E_u \wedge \neg B_H$		$\neg(S_u \wedge E_u)$
V	V	V		V	V	F		F
V	V	F		F	V	V		F
V	F	V		V	V	F		V
V	F	F		F	F	F		V
F	V	V		F	F	F		V
F	V	F		F	V	V		V
F	F	V		F	F	F		V
F	F	F		F	F	F		V

8. Use truth tables to determine which of the following formulas are equivalent to each other:
(a) $(P \wedge Q) \vee (\neg P \wedge \neg Q)$.
(b) $\neg P \vee Q$.
(c) $(P \vee \neg Q) \wedge (Q \vee \neg P)$.
(d) $\neg(P \vee Q)$.
(e) $(Q \wedge P) \vee \neg P$.

a)

P	Q	$P \wedge Q$	$\neg P \wedge \neg Q$	$(P \vee Q) \vee (\neg P \wedge \neg Q)$
1	1	1	0	1
1	0	0	0	0
0	1	0	0	0
0	0	0	1	1

b)

P	Q	$\neg P \vee Q$
1	1	1
1	0	0
0	1	1
0	0	1

c)

P	Q	$\neg Q \vee P$	$\neg P \vee Q$	$(\neg Q \vee P) \wedge (\neg P \vee Q)$
1	1	1	1	1
1	0	1	0	0
0	1	0	1	0
0	0	1	1	1

d)

P	Q	$\neg(P \vee Q)$
1	1	0
1	0	0
0	1	0
0	0	1

e)

P	Q	$Q \wedge P$	$(Q \wedge P) \vee \neg P$
1	1	1	1
1	0	0	0
0	1	0	1
0	0	0	1

a e c são equivalentes
b e e são equivalentes

9. Use truth tables to determine which of these statements are tautologies, which are contradictions, and which are neither:
(a) $(P \lor Q) \land (\neg P \lor \neg Q)$.
(b) $(P \lor Q) \land (\neg P \land \neg Q)$.
(c) $(P \lor Q) \lor (\neg P \lor \neg Q)$.
(d) $[P \land (Q \lor \neg R)] \lor (\neg P \lor R)$.

a) —

P	Q	$P\lor Q$	$\neg P\lor\neg Q$	$(P\lor Q)\land(\neg P\lor\neg Q)$
1	1	1	0	0
1	0	1	1	1
0	1	1	1	1
0	0	0	1	0

b) Contradição

P	Q	$P\lor Q$	$\neg P\land\neg Q$	$(P\lor Q)\land(\neg P\land\neg Q)$
1	1	1	0	0
1	0	1	0	0
0	1	1	0	0
0	0	0	1	0

c) Tautologia

P	Q	$P\lor Q$	$\neg P\lor\neg Q$	$(P\lor Q)\lor(\neg P\lor\neg Q)$
1	1	1	0	1
1	0	1	1	1
0	1	1	1	1
0	0	0	1	1

d) Tautologia

P	Q	R	$Q\lor\neg R$	$P\land(Q\lor\neg R)$	$\neg P\lor R$	$[P\land(Q\lor\neg R)]\lor(\neg P\lor R)$
1	1	1	1	1	1	1
1	1	0	1	1	0	1
1	0	1	0	0	1	1
1	0	0	1	1	0	1
0	1	1	1	0	1	1
0	1	0	1	0	1	1
0	0	1	0	0	1	1
0	0	0	1	0	1	1

10. Use truth tables to check these laws:
(a) The second De Morgan's law. (The first was checked in the text.)
(b) The distributive laws.

a) $\neg(a\lor b)=\neg a\land\neg b$

a	b	$\neg(a\lor b)$	$\neg a\land\neg b$
1	1	0	0
1	0	0	0
0	1	0	0
0	0	1	1

b) $a\land(b\lor c)=(a\land b)\lor(a\land c)$

a	b	c	$a\land(b\lor c)$	$(a\land b)\lor(a\land c)$
1	1	1	1	1
1	1	0	1	1
1	0	1	1	1
1	0	0	0	0
0	1	1	0	0
0	1	0	0	0
0	0	1	0	0
0	0	0	0	0

c) $a\lor(b\land c)=(a\lor b)\land(a\lor c)$

a	b	c	$a\lor(b\land c)$	$(a\lor b)\land(a\lor c)$
1	1	1	1	1
1	1	0	1	1
1	0	1	1	1
1	0	0	1	1
0	1	1	1	1
0	1	0	0	0
0	0	1	0	0
0	0	0	0	0

11. Use the laws stated in the text to find simpler formulas equivalent to these formulas. (See Examples 1.2.5 and 1.2.7.)
(a) $\neg(\neg P \land \neg Q)$.
(b) $(P \land Q) \lor (P \land \neg Q)$.
(c) $\neg(P \land \neg Q) \lor (\neg P \land Q)$.

a) $\neg(\neg P\land\neg Q)=P\lor Q$

b) $(P\land Q)\lor(P\land\neg Q)=[(P\land Q)\lor P]\land[(P\land Q)\lor\neg Q]=(P\lor P)\land(Q\lor P)\land(P\lor\neg Q)\land(Q\lor\neg Q)=P\land(P\lor Q)\land(P\lor\neg Q)=P\land P\lor(Q\land\neg Q)=P$

c) $\neg(P\land\neg Q)\lor(\neg P\land Q)=\neg P\lor Q\lor(\neg P\land Q)=\neg P\lor[(Q\lor\neg P)\land(Q\lor Q)]=\neg P\lor(Q\lor\neg P)\land(\neg P\lor Q)=(\neg P\lor Q)\land(\neg P\lor Q)=\neg P\lor Q$

12. Use the laws stated in the text to find simpler formulas equivalent to these formulas. (See Examples 1.2.5 and 1.2.7.)
(a) $\neg(\neg P \lor Q) \lor (P \land \neg R)$.
(b) $\neg(\neg P \land Q) \lor (P \land \neg R)$.
(c) $(P \land R) \lor [\neg R \land (P \lor Q)]$.

a) $\neg(\neg P\lor Q)\lor(P\land\neg R)=\neg(\neg P\lor Q)\lor\neg(\neg P\lor R)=\neg[(\neg P\lor Q)\land(\neg P\lor R)]=\neg[P\lor(Q\land R)]$

b) $\neg(\neg P\land Q)\lor(P\land\neg R)=P\lor\neg Q\lor(P\land\neg R)=\neg Q\lor[(P\lor P)\land(P\lor\neg R)]=\neg Q\lor[P\land(P\lor\neg R)]=\neg Q\lor[(P\land P)\lor(P\land\neg R)]=\neg Q\lor P\lor(P\land\neg R)$

c) $(P\land R)\lor[\neg R\land(P\lor Q)]=(P\land R)\lor\neg[R\lor(\neg P\land\neg Q)]=\neg\{(\neg P\lor\neg R)\land[R\lor(\neg P\land\neg Q)]\}=\neg\{(\neg P\lor\neg R)\land(R\lor\neg P)\land(R\lor\neg Q)\}=\neg\{\neg P\lor(\neg R\land R)\land(R\lor\neg Q)\}=\neg\{\neg P\land(R\lor\neg Q)\}=$
$P\lor\neg(R\lor\neg Q)=P\lor(\neg R\land Q)$

13. Use the first De Morgan's law and the double negation law to derive the second De Morgan's law.
$\neg(P \land Q)$ is equivalent to $\neg P \lor \neg Q$.
$\neg(P \lor Q)$ is equivalent to $\neg P \land \neg Q$.

$\neg(P\lor Q)=\neg(\neg\neg P\lor\neg\neg Q)=\neg\neg(\neg P\land\neg Q)=\neg P\land\neg Q$

14. Note that the associative laws say only that parentheses are unnecessary when combining *three* statements with $\land$ or $\lor$. In fact, these laws can be used to justify leaving parentheses out when more than three statements are combined. Use associative laws to show that $[P \land (Q \land R)] \land S$ is equivalent to $(P \land Q) \land (R \land S)$.

$[P\land(Q\land R)]\land S=[P\land Q\land R]\land S=P\land Q\land R\land S=(P\land Q)\land(R\land S)$

ou $[(P\land Q)\land R]\land S=(P\land Q)\land[R\land S]=(P\land Q)\land(R\land S)$

15. How many lines will there be in the truth table for a statement containing n letters?

2^n linhas

16. Find a formula involving the connectives $\land$, $\lor$, and $\neg$ that has the following truth table:

P	Q	???
F	F	T
F	T	F
T	F	T
T	T	T

$\neg P\land\neg Q=$ verdadeiro
$P\land\neg Q=$ verdadeiro
$P\land Q=$ verdadeiro
$\neg P\land Q=$ falso

$\neg(\neg P\land Q)=P\lor\neg Q$
ou
$(\neg P\land\neg Q)\lor(P\land\neg Q)\lor(P\land Q)$

17. Find a formula involving the connectives $\land$, $\lor$, and $\neg$ that has the following truth table:

P	Q	???
F	F	F
F	T	T
T	F	T
T	T	F

$\neg P\land Q=$ verdadeiro
$P\land\neg Q=$ verdadeiro
$P\land Q=$ falso
$\neg P\land\neg Q=$ falso

$(\neg P\land Q)\lor(P\land\neg Q)$
ou
$\neg[(P\land Q)\lor(\neg P\land\neg Q)]=\neg(P\land Q)\land\neg(\neg P\land\neg Q)=(\neg P\lor\neg Q)\land(P\lor Q)$

18. Suppose the conclusion of an argument is a tautology. What can you conclude about the validity of the argument? What if the conclusion is a contradiction? What if one of the premises is either a tautology or a contradiction?

Princípio da explosão: De uma contradição é possível derivar qualquer coisa.

Tendo o princípio da explosão já em mente temos que mesmo que as premissas sejam contraditórias o argumento se manterá válido no caso da conclusão ser uma tautologia.
No caso de uma conclusão contraditória o argumento sempre será não válido, a não ser que as premissas sejam contraditórias.
No caso de uma das premissas ser uma tautologia a validade dependerá das demais premissas e conclusão.
No caso de uma contradição nas premissas o argumento é válido devido ao princípio da explosão.

1. Analyze the logical forms of the following statements:
(a) 3 is a common divisor of 6, 9, and 15. (Note: You did this in exercise 2 of Section 1.1, but you should be able to give a better answer now.)
(b) x is divisible by both 2 and 3 but not 4.
(c) x and y are natural numbers, and exactly one of them is prime.

$P(x,y)=x$ divide y
$Q(x)=x$ é primo

a) $P(3,6)\land P(3,9)\land P(3,15)$

b) $P(2,x)\land P(3,x)\land\neg P(4,x)$

b) $x\in\mathbb{N}\land y\in\mathbb{N}\land[(Q(x)\land\neg Q(y))\lor(\neg Q(x)\land Q(y))]$

2. Analyze the logical forms of the following statements:
(a) x and y are men, and either x is taller than y or y is taller than x.
(b) Either x or y has brown eyes, and either x or y has red hair.
(c) Either x or y has both brown eyes and red hair.

$H(x)=x$ é homem
$A(x,y)=x$ é mais alto que y
$M(x)=x$ tem olhos castanhos
$V(x)=x$ é ruivo

a) $H(x)\land H(y)\land[A(x,y)\lor A(y,x)]$

b) $(M(x)\lor M(y))\land(V(x)\lor V(y))$

c) $(M(x)\land V(x))\lor(M(y)\land V(y))$

3. Write definitions using elementhood tests for the following sets:
(a) {Mercury, Venus, Earth, Mars, Jupiter, Saturn, Uranus, Neptune}.
(b) {Brown, Columbia, Cornell, Dartmouth, Harvard, Princeton, University of Pennsylvania, Yale}.
(c) {Alabama, Alaska, Arizona, ..., Wisconsin, Wyoming}.
(d) {Alberta, British Columbia, Manitoba, New Brunswick, Newfoundland and Labrador, Northwest Territories, Nova Scotia, Nunavut, Ontario, Prince Edward Island, Quebec, Saskatchewan, Yukon}.

a) $P = \{p \mid p$ é um planeta do sistema solar$\}$
b) $U = \{u \mid u$ é universidade dos EUA$\}$
c) $E = \{e \mid e$ é um estado dos EUA$\}$
d) $P = \{p \mid p$ é uma província do Canadá$\}$

4. Write definitions using elementhood tests for the following sets:
(a) {1, 4, 9, 16, 25, 36, 49, ...}.
(b) {1, 2, 4, 8, 16, 32, 64, ...}.
(c) {10, 11, 12, 13, 14, 15, 16, 17, 18, 19}.

a) $A = \{x \in \mathbb{N}^+ \mid x = n^2$ com $n \in \mathbb{N}^+\}$
b) $B = \{x \in \mathbb{N}^+ \mid x = 2^K$ com $K \in \mathbb{N}\}$
c) $C = \{x \in \mathbb{Z} \mid 10 \leq x \leq 19\}$

5. Simplify the following statements. Which variables are free and which are bound? If the statement has no free variables, say whether it is true or false.
(a) $-3 \in \{x \in \mathbb{R} \mid 13 - 2x > 1\}$.
(b) $4 \in \{x \in \mathbb{R}^- \mid 13 - 2x > 1\}$.
(c) $5 \notin \{x \in \mathbb{R} \mid 13 - 2x > c\}$.

a) $A = \{x \in \mathbb{R} \mid x < 6\}$; logo é verdade que $-3 \in A$ — Variáveis livres: Nenhuma — Variáveis vinculadas: x
b) $B = \{x \in \mathbb{R}^- \mid x < 6\}$; logo não é verdadeiro que $4 \in B$ — Variáveis livres: Nenhuma — Variáveis vinculadas: x
c) $C = \{x \in \mathbb{R} \mid x < 6{,}5 - 0{,}5c\}$; a validade do argumento dependerá de c — Variáveis livres: c — Variáveis vinculadas: x

6. Simplify the following statements. Which variables are free and which are bound? If the statement has no free variables, say whether it is true or false.
(a) $w \in \{x \in \mathbb{R} \mid 13 - 2x > c\}$.
(b) $4 \in \{x \in \mathbb{R} \mid 13 - 2x \in \{y \mid y$ is a prime number$\}\}$. (It might make this statement easier to read if we let $P = \{y \mid y$ is a prime number$\}$; using this notation, we could rewrite the statement as $4 \in \{x \in \mathbb{R} \mid 13 - 2x \in P\}$.)
(c) $4 \in \{x \in \{y \mid y$ is a prime number$\} \mid 13 - 2x > 1\}$. (Using the same notation as in part (b), we could write this as $4 \in \{x \in P \mid 13 - 2x > 1\}$.)

a) $A = \{x \in \mathbb{R} \mid x < 6{,}5 - 0{,}5c\}$ — Variáveis livres: w e c
O valor será verdadeiro para — Variáveis vinculadas: x
$w < 6{,}5 - 0{,}5c$

b) $B = \{x \mid x \in \mathbb{R} \wedge 13 - 2x$ é primo$\}$; — Variáveis livres: nenhuma
logo é verdade que $4 \in B$, pois $13 - 2 \cdot (4) = 5$ é primo. — Variáveis vinculadas: x e y

c) $C = \{x \mid x$ é primo $\wedge\ x < 6\}$; — Variáveis livres: nenhuma
então é verdade que $4 \in C$ — Variáveis vinculadas: x e y

7. List the elements of the following sets:
(a) $\{x \in \mathbb{R} \mid 2x^2 + x - 1 = 0\}$.
(b) $\{x \in \mathbb{R}^+ \mid 2x^2 + x - 1 = 0\}$.
(c) $\{x \in \mathbb{Z} \mid 2x^2 + x - 1 = 0\}$.
(d) $\{x \in \mathbb{N} \mid 2x^2 + x - 1 = 0\}$.

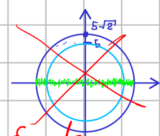
a) $\{x \in \mathbb{R} \mid 2x^2 + x - 1 = 0\} = \{1/2, -1\}$
b) $\{x \in \mathbb{R}^+ \mid 2x^2 + x - 1 = 0\} = \{1/2\}$
c) $\{x \in \mathbb{Z} \mid 2x^2 + x - 1 = 0\} = \{-1\}$
d) $\{x \in \mathbb{N} \mid 2x^2 + x - 1 = 0\} = \{\} = \emptyset$

8. What are the truth sets of the following statements? List a few elements of the truth set if you can.
(a) Elizabeth Taylor was once married to x.
(b) x is a logical connective studied in Section 1.1.
(c) x is the author of this book.

a) {Hilton Jr., Michael Wilding, Mike Todd, Eddie Fisher, Richard Burton, John Warner, Larry Fortensky}
b) $\{\neg; \vee; \wedge\}$
c) {Daniel J. Velleman}

9. What are the truth sets of the following statements? List a few elements of the truth set if you can.
(a) x is a real number and $x^2 - 4x + 3 = 0$.
(b) x is a real number and $x^2 - 2x + 3 = 0$.
(c) x is a real number and $5 \in \{y \in \mathbb{R} \mid x^2 + y^2 < 50\}$.

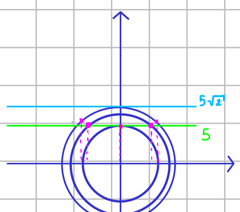
a) $\{1; 3\}$
b) $\emptyset$



c) ~~$\{y \in \mathbb{R} \mid x^2 + y^2 < 50\} = \{y \in \mathbb{R} \mid y$ seja um valor possível nas circunferências de raio menor que $5\sqrt{2}\}$~~

~~$\{x \in \mathbb{R} \mid 5 \in \{y \in \mathbb{R} \mid x^2 + y^2 < 50\}\} = \{x \in \mathbb{R} \mid y = 5$ pertença à circunferência de raio menor~~ que $5\sqrt{2}\} = (-5\sqrt{2}; 5\sqrt{2}) = \{x \in \mathbb{R} \mid |x| < 5\sqrt{2}\}$ Errado!

Do círculo de raio máximo $(R = 5\sqrt{2})$ até o círculo mínimo, onde y pode assumir o valor de 5 $(R = 5)$
o intervalo de x's possíveis que satisfazem a condição $x^2 + y^2 < 50$ vai se estreitando de:
$|x_{max}| = \sqrt{(5\sqrt{2})^2 - 5^2} = 5$
$|x_{max}| = \sqrt{5^2 - 5^2} = 0$
logo o conjunto é $\{x \in \mathbb{R} \mid |x| < 5\}$



Um determinado y_0 pode ser alcançado em um círculo de raio R conforme: $|x_{max}| = \sqrt{R^2 - y_0^2}$

Outra forma de pensar é que ao pegar um círculo de raio R sempre quando definirmos um $n \in \{y \in \mathbb{R} \mid x^2 + y^2 = R^2\}$ dois x's são definidos para o n escolhido, então ao pegar o contínuo dos círculos de R menor que $5\sqrt{2}$ o intervalo é definido também como um contínuo, nesse caso de $x_1 = 5$ e $x_2 = -5$ para $y = 5$ até $x_1 = x_2 = 0$ para $y = 5\sqrt{2}$, ou seja, $x < |5|$.

11. Let $A = \{1, 3, 12, 35\}$, $B = \{3, 7, 12, 20\}$, and $C = \{x \mid x$ is a prime number$\}$. List the elements of the following sets. Are any of the sets below disjoint from any of the others? Are any of the sets below subsets of any others?
(a) $A \cap B$.
(b) $(A \cup B) \setminus C$.
(c) $A \cup (B \setminus C)$.

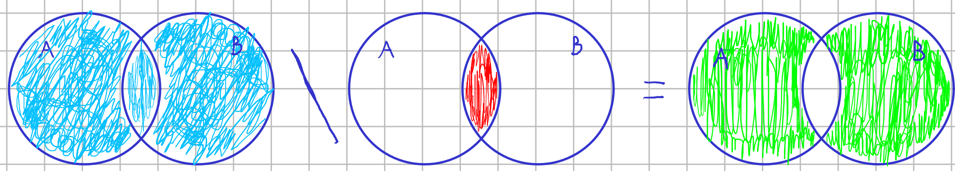
a) $A \cap B = \{3, 12\}$ — $A \cap B \subseteq A$; $A \cap B \subseteq B$
b) $A \cup B = \{1, 3, 7, 12, 20, 35\}$
$(A \cup B) \setminus C = \{1, 12, 20, 35\}$
c) $(B \setminus C) = \{12, 20\}$
$A \cup (B \setminus C) = \{1, 3, 12, 20, 35\}$ — $A \subseteq A \cup (B \setminus C)$; $(A \cup B) \setminus C \subseteq A \cup (B \setminus C)$; $A \cap B \subseteq A \cup (B \setminus C)$

12. Let $A = \{$United States, Germany, China, Australia$\}$, $B = \{$Germany, France, India, Brazil$\}$, and $C = \{x \mid x$ is a country in Europe$\}$. List the elements of the following sets. Are any of the sets below disjoint from any of the others? Are any of the sets below subsets of any others?
(a) $A \cup B$.
(b) $(A \cap B) \setminus C$.
(c) $(B \cap C) \setminus A$.

a) $A \cup B = \{$Estados Unidos, Alemanha, China, Austrália, França, Índia, Brasil$\}$
b) $A \cap B = \{$Alemanha$\}$
$(A \cap B) \setminus C = \emptyset$
c) $B \cap C = \{$Alemanha, França$\}$
$(B \cap C) \setminus A = \{$França$\}$

$(A \cap B) \setminus C \subseteq (B \cap C) \setminus A$; $(A \cap B) \setminus C \subseteq A \cup B$; $(B \cap C) \setminus A \subseteq A \cup B$
$(A \cup B)$ e $(A \cap B) \setminus C$ são disjuntos entre si, pois $(A \cup B) \cap (A \cap B) \setminus C = \emptyset$
$(B \cap C) \setminus A$ e $(A \cap B) \setminus C$ são disjuntos entre si, pois $(B \cap C) \setminus A \cap (A \cap B) \setminus C = \emptyset$

13. Verify that the Venn diagrams for $(A \cup B) \setminus (A \cap B)$ and $(A \setminus B) \cup (B \setminus A)$ both look like Figure 1.11, as stated in this section.



$(A \cup B) \setminus (A \cap B)$

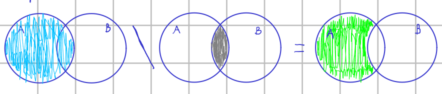
$(A \setminus B) \cup (B \setminus A)$

4. Use Venn diagrams to verify the following identities:

(a) $A \setminus (A \cap B) = A \setminus B$.

(b) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$.

a)



b)

5. Verify the identities in exercise 4 by writing out (using logical symbols) what it means for an object x to be an element of each set and then using logical equivalences.

a) $A\setminus(A\cap B)=\{x \mid x\in A\}\setminus\{x \mid x\in A \wedge x\in B\}=\{x \mid x\in A \wedge (x\notin A \vee x\notin B)\}=\{x \mid (x\in A \wedge x\notin A)\vee(x\in A \wedge x\notin B)\}=\{x \mid x\in A \wedge x\notin B\}=A\setminus B$

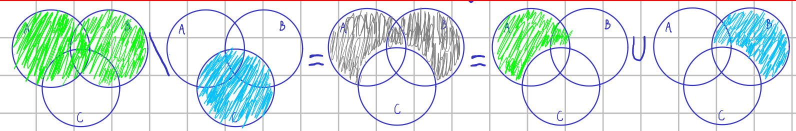
b) $A\cup(B\cap C)=\{x \mid x\in A\}\cup\{x \mid x\in B \wedge x\in C\}=\{x \mid x\in A \vee (x\in B \wedge x\in C)\}=\{x \mid (x\in A \vee x\in B)\wedge(x\in A \vee x\in C)\}=(A\cup B)\cap(A\cup C)$

6. Use Venn diagrams to verify the following identities:

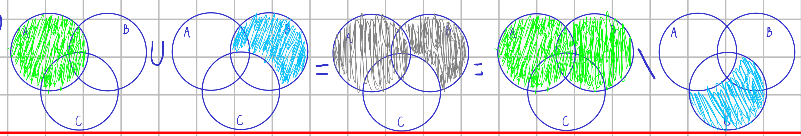
(a) $(A \cup B) \setminus C = (A \setminus C) \cup (B \setminus C)$.

(b) $A \cup (B \setminus C) = (A \cup B) \setminus (C \setminus A)$.

a)



b)



7. Verify the identities in exercise 6 by writing out (using logical symbols) what it means for an object x to be an element of each set and then using logical equivalences.

a) $(A\cup B)\setminus C=\{x \mid x\in A \vee x\in B\}\setminus\{x \mid x\in C\}=\{x \mid (x\in A \vee x\in B)\wedge(x\notin C)\}=\{x \mid (x\in A \wedge x\notin C)\vee(x\in B \wedge x\notin C)\}=(A\setminus C)\cup(B\setminus C)$

b) $A\cup(B\setminus C)=\{x \mid x\in A\}\cup\{x \mid x\in B \wedge x\notin C\}=\{x \mid (x\in A)\vee(x\in B \wedge x\notin C)\}=\{x \mid (x\in A \vee x\in B)\wedge(x\in A \vee \neg(x\in C))\}=\{x \mid (x\in A \vee x\in B)\wedge\neg(\neg(x\in A)\wedge x\in C)\}=$
$\{x \mid x\in A \vee x\in B\}\setminus\{x \mid x\in C \wedge x\notin A\}=(A\cup B)\setminus(C\setminus A)$

8. Use any method you wish to verify the following identities:

(a) $(A \setminus B) \cap C = (A \cap C) \setminus B$.

(b) $(A \cap B) \setminus B = \varnothing$.

(c) $A \setminus (A \setminus B) = A \cap B$.

a) $(A\setminus B)\cap C=\{x \mid x\in A \wedge x\notin B\}\cap\{x \mid x\in C\}=\{x \mid (x\in A \wedge x\notin B)\wedge(x\in C)\}=\{x \mid (x\in A \wedge x\in C)\wedge x\notin B\}=\{x \mid x\in A \wedge x\in C\}\setminus\{x \mid x\in B\}=(A\cap C)\setminus B$

b) $(A\cap B)\setminus B=\{x \mid x\in A \wedge x\in B\}\setminus\{x \mid x\in B\}=\{x \mid (x\in A \wedge x\in B)\wedge(x\notin B)\}=\{x \mid \text{falso}\}=\varnothing$

c) $A\setminus(A\setminus B)=\{x \mid x\in A\}\setminus\{x \mid x\in A \wedge x\notin B\}=\{x \mid x\in A \wedge \neg(x\in A \wedge x\notin B)\}=\{x \mid (x\in A \wedge x\notin A)\vee(x\in A \wedge x\in B)\}=\{x \mid x\in A \wedge x\in B\}=A\cap B$

9. For each of the following sets, write out (using logical symbols) what it means for an object x to be an element of the set. Then determine which of these sets must be equal to each other by determining which statements are equivalent.

(a) $(A \setminus B) \setminus C$.

(b) $A \setminus (B \setminus C)$.

(c) $(A \setminus B) \cup (A \cap C)$.

(d) $(A \setminus B) \cap (A \setminus C)$.

(e) $A \setminus (B \cup C)$.

a) $\{x \mid x\in A \wedge x\notin B\}\setminus\{x \mid x\in C\}=\{x \mid (x\in A \wedge x\notin B)\wedge(x\notin C)\}=\{x \mid x\in A \wedge x\notin B \wedge x\notin C\}$

b) $\{x \mid x\in A \wedge \neg(x\in B \wedge x\notin C)\}=\{x \mid x\in A \wedge (x\notin B \vee x\in C)\}$

c) $\{x \mid (x\in A \wedge x\notin B)\vee(x\in A \wedge x\in C)\}=\{x \mid x\in A \wedge (x\notin B \vee x\in C)\}$

d) $\{x \mid (x\in A \wedge x\notin B)\wedge(x\in A \wedge x\notin C)\}=\{x \mid x\in A \wedge x\notin B \wedge x\notin C\}$

e) $\{x \mid (x\in A)\wedge\neg(x\in B \vee x\in C)\}=\{x \mid x\in A \wedge x\notin B \wedge x\notin C\}$

a, d, e são equivalentes
b, c são equivalentes

10. It was shown in this section that for any sets A and B, $(A \cup B) \setminus B \subseteq A$.

(a) Give an example of two sets A and B for which $(A \cup B) \setminus B \neq A$.

(b) Show that for all sets A and B, $(A \cup B) \setminus B = A \setminus B$.

a) $(\{1,2,3\}\cup\{3,4\})\setminus\{3,4\}=\{1,2\}$
$(\{a,b\}\cup\{b,c\})\setminus\{b,c\}=\{a\}$

b) Dado dois conjuntos A e B quaisquer, temos que:
$A\cup B=\{x \mid x\in A \vee x\in B\}$ e $(A\cup B)\setminus B=\{x \mid x\in\{y \mid y\in A \vee y\in B\}\wedge x\notin B\}$, a segunda expressão pode ser reescrita como
$(A\cup B)\setminus B=\{x \mid (x\in A \vee x\in B)\wedge x\notin B\}$ que é o mesmo que $\{x \mid (x\in A \wedge x\notin B)\vee(x\in B \wedge x\notin B)\}=\{x \mid x\in A \wedge x\notin B\}$ que é a definição de $A\setminus B$, logo
$(A\cup B)\setminus B=A\setminus B$. ∎

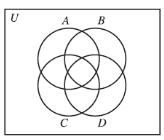
11. Suppose A and B are sets. Is it necessarily true that $(A \setminus B) \cup B = A$? If not, is one of these sets necessarily a subset of the other? Is $(A \setminus B) \cup B$ always equal to either $A \setminus B$ or $A \cup B$?

$(A\setminus B)\cup B=\{x \mid (x\in A \wedge x\notin B)\vee x\in B\}=\{x \mid (x\in A \vee x\in B)\wedge(x\notin B \vee x\in B)\}=\{x \mid x\in A \vee x\in B\}=A\cup B$

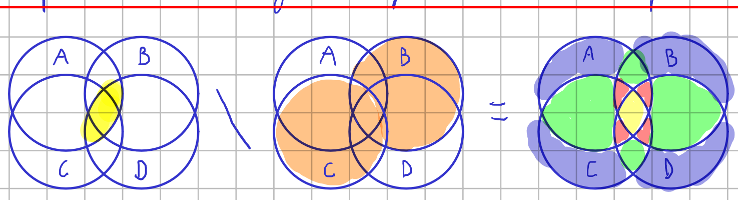
Para que $(A\setminus B)\cup B=A$ o conjunto B não pode conter elementos que não estejam em A, em outras palavras, $B\subseteq A$.

12. It is claimed in this section that you cannot make a Venn diagram for four sets using overlapping circles.

(a) What's wrong with the following diagram? (Hint: Where's the set $(A \cap D) \setminus (B \cup C)$?)



(b) Can you make a Venn diagram for four sets using shapes other than circles?



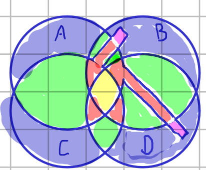
$A\cap D$ $\qquad$ $B\cup C$

$(x\in A \wedge x\in D)\wedge\neg(x\in B \vee x\in C)=x\in A \wedge x\notin B \wedge x\notin C \wedge x\in D$

Os conjuntos $\{x \mid x\in A \wedge x\notin B \wedge x\notin C \wedge x\in D\}$ e $\{x \mid x\notin A \wedge x\in B \wedge x\in C \wedge x\notin D\}$ não estão representados no diagrama.

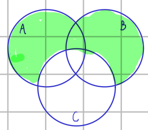
b) Sim é possível, assim como na figura abaixo:

OK!



A	B	C	D	
∈	∉	∉	∉	OK!
∉	∈	∉	∉	OK!
∉	∉	∈	∉	OK!
∉	∉	∉	∈	OK!
∈	∈	∉	∉	OK!
∈	∉	∈	∉	OK!
∈	∉	∉	∈	?
∉	∈	∈	∉	?
∉	∈	∉	∈	OK!
∉	∉	∈	∈	OK!
∈	∈	∈	∉	OK!
∈	∈	∉	∈	OK!
∈	∉	∈	∈	OK!
∉	∈	∈	∈	OK!
∈	∈	∈	∈	OK!

13. (a) Make Venn diagrams for the sets $(A \cup B) \setminus C$ and $A \cup (B \setminus C)$. What can you conclude about whether one of these sets is necessarily a subset of the other?
(b) Give an example of sets A, B, and C for which $(A \cup B) \setminus C \neq A \cup (B \setminus C)$.



$(A \cup B) \setminus C$

$(x \in A \vee x \in B) \wedge x \notin C$

$(x \in A \wedge x \notin C) \vee (x \in B \wedge x \notin C)$

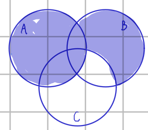
$[(x \in A \wedge x \notin C) \vee (x \in B)] \wedge [(x \in A \wedge x \notin C) \vee (x \notin C)]$

$[(x \in A \vee x \in B) \wedge (x \in B \vee x \notin C)] \wedge [(x \in A \vee x \notin C) \wedge (x \notin C \vee x \notin C)]$

$(x \in A \vee x \in B) \wedge (x \in A \vee x \notin C) \wedge (x \in B \vee x \notin C) \wedge (x \notin C \vee \text{falso})$

$[x \in A \vee (x \in B \wedge x \notin C)] \wedge [x \notin C \vee (x \in B \wedge \text{falso})]$

$[x \in A \vee (x \in B \wedge x \notin C)] \wedge x \notin C$



$A \cup (B \setminus C)$

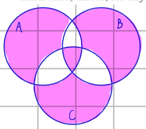
$x \in A \vee (x \in B \wedge x \notin C)$

$(x \in A \vee x \in B) \wedge (x \in A \vee x \notin C)$

a) Para que a igualdade valha $A \cap C = \emptyset$, ou seja, A e C são disjuntos.

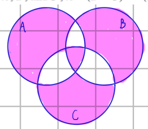
b) $(\{0,1,2\} \cup \{1,2,3\}) - \{2,3,4\} = \{0,1\} \neq \{0,1,2\} \cup (\{1,2,3\} - \{2,3,4\}) = \{0,2\}$

14. Use Venn diagrams to show that the associative law holds for symmetric difference; that is, for any sets A, B, and C, $A \,^2\, (B \,^2\, C) = (A \,^2\, B) \,^2\, C$.

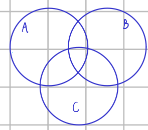


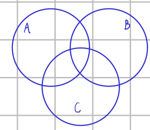
$A^2(B^2C)$

=



$(A^2B)^2C$





15. Use any method you wish to verify the following identities:
(a) $(A \,^2\, B) \cup C = (A \cup C) \,^2\, (B \setminus C)$.
(b) $(A \,^2\, B) \cap C = (A \cap C) \,^2\, (B \cap C)$.
(c) $(A \,^2\, B) \setminus C = (A \setminus C) \,^2\, (B \setminus C)$.

a) $(A^2B) \cup C = [(A \setminus B) \cup (B \setminus A)] \cup C = \{x \mid [(x \in A \wedge x \notin B) \vee (x \notin A \wedge x \in B)] \vee x \in C\}$

$[(A \cup C) \setminus (B \setminus C)] \cup [(B \setminus C) \setminus (A \cup C)] = \{x \mid [(x \in A \vee x \in C) \wedge \neg(x \in B \wedge x \notin C)] \vee [(x \in B \wedge x \notin C) \wedge \neg(x \in A \vee x \in C)]\} =$

$\{x \mid [x \in C \vee (x \in A \wedge x \notin B)] \vee [x \notin A \wedge x \in B \wedge x \notin C]\} = \{x \mid [(x \in A \wedge x \notin B)] \vee x \in C \vee [x \notin A \wedge x \in B \wedge x \notin C]\} =$

$\{x \mid [(x \in A \wedge x \notin B)] \vee [(x \in C \vee x \notin A) \wedge (x \in C \vee x \in B) \wedge (x \in C \vee x \notin C)]\} = \{x \mid [(x \in A \wedge x \notin B)] \vee [(x \in C \vee x \notin A) \wedge (x \in C \vee x \in B)]\} = \{x \mid [(x \in A \wedge x \notin B)] \vee [(x \notin A \wedge x \in B) \vee x \in C]$

$= \{x \mid [(x \in A \wedge x \notin B) \vee (x \notin A \wedge x \in B)] \vee x \in C\} = (A^2B) \cup C$

b)

$(A^2B) \cap C =$

$\{x \mid [(x \in A \wedge x \notin B) \vee (x \notin A \wedge x \in B)] \wedge x \in C\} = (A \setminus B \cup B \setminus A) \cap C =$

$[C \cap (A \setminus B)] \cup [C \cap (B \setminus A)] = \{x \mid [x \in C \wedge (x \in A \wedge x \notin B)] \vee [x \in C \wedge (x \in B \wedge x \notin A)]\}$

$[(x \in A \wedge x \in C) \wedge \neg(x \in B \wedge x \in C)] \vee [(x \in B \wedge x \in C) \wedge \neg(x \in A \wedge x \in C)] =$
$[x \in A \wedge x \in C \wedge (x \notin B \vee x \notin C)] \vee [x \in B \wedge x \in C \wedge (x \notin A \vee x \notin C)] =$
$[(x \in A \wedge x \notin B \wedge x \in C) \vee (x \notin B \wedge x \in C \wedge x \notin C)] \vee [(x \notin A \wedge x \in B \wedge x \in C) \vee (x \in B \wedge x \in C \wedge x \notin C)] =$
$[x \in C \wedge (x \in A \wedge x \notin B)] \vee [x \in C \wedge (x \in B \wedge x \notin A)]$

$(A \cap C)^2(B \cap C) =$

$\{x \mid [(x \in A \wedge x \in C) \wedge \neg(x \in B \wedge x \in C)] \vee [\neg(x \in A \wedge x \in C) \wedge (x \in B \wedge x \in C)]\} =$

$[(A \cap C) \setminus (B \cap C)] \cup [(B \cap C) \setminus (A \cap C)] = \{x \mid [x \in C \wedge (x \in A \wedge x \notin B)] \vee [x \in C \wedge (x \in B \wedge x \notin A)]\}$

Logo: $(A^2B) \cap C = (A \cap C)^2(B \cap C)$

c) $(A^2B) \setminus C = [(A \setminus B) \cup (B \setminus A)] \setminus C = \{x \mid [(x \in A \wedge x \notin B) \vee (x \notin A \wedge x \in B)] \wedge x \notin C\} = \{x \mid (x \in A \wedge x \notin B \wedge x \notin C) \vee (x \notin A \wedge x \in B \wedge x \notin C)\}$

$(A \setminus C)^2(B \setminus C) = [(A \setminus C) \setminus (B \setminus C)] \cup [(B \setminus C) \setminus (A \setminus C)] = \{x \mid [(x \in A \wedge x \notin C) \wedge \neg(x \in B \wedge x \notin C)] \vee [(x \in B \wedge x \notin C) \wedge \neg(x \in A \wedge x \notin C)]\} =$

$\{x \mid [(x \in A \wedge x \notin C) \wedge (x \notin B)] \vee [(x \in A \wedge x \notin C) \wedge (x \in C)] \vee [(x \in B \wedge x \notin C) \wedge (x \notin A)] \vee [(x \in B \wedge x \notin C) \wedge (x \in C)]\} = \{x \mid (x \in A \wedge x \notin B \wedge x \notin C) \vee (x \notin A \wedge x \in B \wedge x \notin C)\} = (A^2B) \setminus C$

16. Use any method you wish to verify the following identities:
(a) $(A \cup B) \,^2\, C = (A \,^2\, C) \,^2\, (B \setminus A)$.
(b) $(A \cap B) \,^2\, C = (A \,^2\, C) \,^2\, (A \setminus B)$.
(c) $(A \setminus B) \,^2\, C = (A \,^2\, C) \,^2\, (A \cap B)$.

a) $(A \cup B)^2C = [(A \cup B) \setminus C] \cup [C \setminus (A \cup B)] = \{x \mid [(x \in A \vee x \in B) \wedge x \notin C] \vee [x \in C \wedge \neg(x \in A \vee x \in B)] =$

$\{x \mid (x \in A \wedge x \notin C) \vee (x \in B \wedge x \notin C) \vee (x \notin A \wedge x \notin B \wedge x \in C)\}$

$(A^2C)^2(B \setminus A) = (A \setminus C \cup C \setminus A)^2(B \setminus A) = [(A \setminus C \cup C \setminus A) \setminus (B \setminus A)] \cup [(B \setminus A) \setminus (A \setminus C \cup C \setminus A)] = [[(x \in A \wedge x \notin C) \vee (x \in C \wedge x \notin A)] \wedge \neg(x \in B \wedge x \notin A)] \vee [(x \in B \wedge x \notin A) \wedge \neg[(x \in A \wedge x \notin C) \vee (x \in C \wedge x \notin A)]]$

$[(x \in A \wedge x \notin C) \vee (x \in C \wedge x \notin A)] \wedge \neg(x \in B \wedge x \notin A)$

$[(x \in A \vee x \in C) \wedge (x \in A \vee x \notin A) \wedge (x \notin C \vee x \in C) \wedge (x \notin C \vee x \notin A)] \wedge (x \notin B \vee x \in A)$

$[(x \in A \vee x \in C) \wedge (x \notin A \vee x \notin C)] \wedge (x \in A \vee x \notin B)$

$[(x \in A \wedge x \notin A) \vee (x \in A \wedge x \notin C) \vee (x \notin A \wedge x \in C) \vee (x \in C \wedge x \notin C)] \wedge (x \in A \vee x \notin B)$

$[(x \in A \wedge x \notin C) \vee (x \notin A \wedge x \in C)] \wedge (x \in A \vee x \notin B)$

$[(x \in A \vee x \notin B) \wedge (x \in A \wedge x \notin C)] \vee [(x \in A \vee x \notin B) \wedge (x \notin A \wedge x \in C)]$

$[(x \in A \wedge x \in A \wedge x \notin C) \vee (x \in A \wedge x \notin B \wedge x \notin C)] \vee [(x \in A \wedge x \notin A \wedge x \in C) \vee (x \notin A \wedge x \notin B \wedge x \in C)]$

$(x \in A \wedge x \notin C) \vee (x \in A \wedge x \notin B \wedge x \notin C) \vee (x \notin A \wedge x \notin B \wedge x \in C)$

$[x \in A \wedge x \notin C \wedge (x \in B \vee x \notin B)] \vee (x \notin A \wedge x \notin B \wedge x \in C) \vee (x \in A \wedge x \notin B \wedge x \notin C)$

$(x \in A \wedge x \in B \wedge x \notin C) \vee (x \in A \wedge x \notin B \wedge x \notin C) \vee (x \notin A \wedge x \notin B \wedge x \in C) \vee (x \in A \wedge x \notin B \wedge x \notin C)$

$(x \in A \wedge x \notin B \wedge x \notin C) \vee (x \in A \wedge x \in B \wedge x \notin C) \vee (x \in A \wedge x \notin B \wedge x \notin C) \vee (x \notin A \wedge x \notin B \wedge x \in C)$

$(x \in A \wedge x \in B \wedge x \notin C) \vee (x \in A \wedge x \notin B \wedge x \notin C) \vee (x \notin A \wedge x \notin B \wedge x \in C)$

$[x \in A \wedge x \notin C \wedge (x \in B \vee x \notin B)] \vee (x \notin A \wedge x \notin B \wedge x \in C)$

$(x \in A \wedge x \notin C) \vee (x \notin A \wedge x \notin B \wedge x \in C)$

$[(x \in B \wedge x \notin A) \wedge \neg[(x \in A \wedge x \notin C) \vee (x \in C \wedge x \notin A)]]$

$[(x \in B \wedge x \notin A) \wedge [(x \notin A \vee x \in C) \wedge (x \in A \vee x \notin C)]]$

$[(x \in B \wedge x \notin A) \wedge [(x \in A \wedge x \notin A) \vee (x \notin A \wedge x \notin C) \vee (x \in C \wedge x \in A) \vee (x \in C \wedge x \notin C)]]$

$[(x \in B \wedge x \notin A) \wedge [(x \notin A \wedge x \notin C) \vee (x \in C \wedge x \in A)]]$

$[[(x \in B \wedge x \notin A) \wedge (x \notin A \wedge x \notin C)] \vee [(x \in B \wedge x \notin A) \wedge (x \in C \wedge x \in A)]]$

$[(x \notin A \wedge x \in B \wedge x \notin C)]$

$$(x\in A\wedge x\notin C)\vee(x\notin A\wedge x\notin B\wedge x\in C)\vee(x\notin A\wedge x\in B\wedge x\notin C)=(x\in A\wedge x\notin C)\vee(x\notin A\wedge x\notin B\wedge x\in C)\vee(x\in B\wedge x\notin C)$$

$$(x\in A\wedge x\notin B\wedge x\notin C)\vee(x\in A\wedge x\in B\wedge x\notin C)\vee(x\notin A\wedge x\notin B\wedge x\in C)\vee(x\notin A\wedge x\in B\wedge x\notin C)=(x\in A\wedge x\notin C)\vee(x\notin A\wedge x\notin B\wedge x\in C)\vee(x\in B\wedge x\notin C)$$

$$(x\in A\wedge x\notin B\wedge x\notin C)\vee(x\in A\wedge x\in B\wedge x\notin C)\vee(x\in A\wedge x\in B\wedge x\notin C)\vee(x\notin A\wedge x\notin B\wedge x\in C)\vee(x\notin A\wedge x\in B\wedge x\notin C)=(x\in A\wedge x\notin C)\vee(x\notin A\wedge x\notin B\wedge x\in C)\vee(x\in B\wedge x\notin C)$$

$$(x\in A\wedge x\notin C)\vee(x\notin A\wedge x\notin B\wedge x\in C)\vee(x\in A\wedge x\in B\wedge x\notin C)\vee(x\notin A\wedge x\in B\wedge x\notin C)=(x\in A\wedge x\notin C)\vee(x\notin A\wedge x\notin B\wedge x\in C)\vee(x\in B\wedge x\notin C)$$

$$(x\in A\wedge x\notin C)\vee(x\notin A\wedge x\notin B\wedge x\in C)\vee(x\in A\wedge x\in B\wedge x\notin C)\vee(x\notin A\wedge x\in B\wedge x\notin C)=(x\in A\wedge x\notin C)\vee(x\notin A\wedge x\notin B\wedge x\in C)\vee(x\in B\wedge x\notin C)$$

$$(x\in A\wedge x\notin C)\vee(x\notin A\wedge x\notin B\wedge x\in C)\vee(x\in A\wedge x\in B\wedge x\notin C)\vee(x\notin A\wedge x\in B\wedge x\notin C)=(x\in A\wedge x\notin C)\vee(x\notin A\wedge x\notin B\wedge x\in C)\vee(x\in B\wedge x\notin C)$$

$$(x\in A\wedge x\notin C)\vee(x\notin A\wedge x\notin B\wedge x\in C)\vee[(x\in A\vee x\notin A)\wedge x\in B\wedge x\notin C]=(x\in A\wedge x\notin C)\vee(x\notin A\wedge x\notin B\wedge x\in C)\vee(x\in B\wedge x\notin C)$$

$$(x\in A\wedge x\notin C)\vee(x\notin A\wedge x\notin B\wedge x\in C)\vee(x\in B\wedge x\notin C)=(x\in A\wedge x\notin C)\vee(x\notin A\wedge x\notin B\wedge x\in C)\vee(x\in B\wedge x\notin C)$$

Logo $(A\cup B)^2C=(A^2C)^2(B\setminus A)$

b) $(A\cap B)^2C=[(A\cap B)\setminus C]\cup[C\setminus(A\cap B)]=\{x\mid[x\in A\wedge x\in B\wedge x\notin C]\vee[x\in C\wedge\neg(x\in A\wedge x\in B)]\}=\{x\mid(x\in A\wedge x\in B\wedge x\notin C)\vee(x\notin A\wedge x\in C)\vee(x\notin B\wedge x\in C)\}=$

$(A\cap B)^2C=\{x\mid(x\in A\wedge x\in B\wedge x\notin C)\vee(x\notin A\wedge x\in B\wedge x\in C)\vee(x\notin A\wedge x\notin B\wedge x\in C)\vee(x\in A\vee x\notin B\wedge x\in C)\vee(x\notin A\wedge x\notin B\wedge x\in C)\}=$

$(A\cap B)^2C=\{x\mid(x\in A\wedge x\in B\wedge x\notin C)\vee(x\notin A\wedge x\in B\wedge x\in C)\vee(x\notin A\wedge x\notin B\wedge x\in C)\vee(x\in A\vee x\notin B\wedge x\in C)\}=$

$(A^2C)^2(A\setminus B)=(A\setminus C\cup C\setminus A)^2(A\setminus B)=[(A\setminus C\cup C\setminus A)\setminus(A\setminus B)]\cup[(A\setminus B)\setminus(A\setminus C\cup C\setminus A)]=$

$\{x\mid[[(x\in A\wedge x\notin C)\vee(x\notin A\wedge x\in C)]\wedge\neg[x\in A\wedge x\notin B]]\vee[[x\in A\wedge x\notin B]\wedge\neg[(x\in A\wedge x\notin C)\vee(x\notin A\wedge x\in C)]]\}$

$[[(x\in A\wedge x\notin C)\vee(x\notin A\wedge x\in C)]\wedge\neg[x\in A\wedge x\notin B]]\vee[[x\in A\wedge x\notin B]\wedge\neg[(x\in A\wedge x\notin C)\vee(x\notin A\wedge x\in C)]]$

$[(x\in A\wedge x\notin C)\vee(x\notin A\wedge x\in C)]\wedge\neg[x\in A\wedge x\notin B]$

$[(x\in A\wedge x\notin C)\vee x\notin A]\wedge[(x\in A\wedge x\notin C)\vee x\in C]\wedge[x\notin A\vee x\in B]$

$(x\in A\vee x\notin A)\wedge(x\notin A\vee x\notin C)\wedge(x\in A\vee x\in C)\wedge(x\in C\vee x\notin C)\wedge(x\notin A\vee x\in B)$

$(x\notin A\vee x\notin C)\wedge(x\in A\vee x\in C)\wedge(x\notin A\vee x\in B)$

$[(x\notin A\wedge x\in A)\vee(x\notin A\wedge x\in C)\vee(x\in A\wedge x\notin C)\vee(x\notin C\wedge x\in C)]\wedge(x\notin A\vee x\in B)$

$[(x\notin A\wedge x\in C)\vee(x\in A\wedge x\notin C)]\wedge(x\notin A\vee x\in B)$

$[(x\notin A\wedge x\in C)\wedge(x\notin A\vee x\in B)]\vee[(x\in A\wedge x\notin C)\wedge(x\notin A\vee x\in B)]$

$(x\notin A\wedge x\notin A\wedge x\in C)\vee(x\notin A\wedge x\in B\wedge x\in C)\vee(x\notin A\wedge x\in A\wedge x\notin C)\vee(x\in A\wedge x\in B\wedge x\notin C)$

$(x\notin A\wedge x\in C)\vee(x\notin A\wedge x\in B\wedge x\in C)\vee(x\in A\wedge x\in B\wedge x\notin C)$

$(x\notin A\wedge x\in B\wedge x\in C)\vee(x\notin A\wedge x\notin B\wedge x\in C)\vee(x\notin A\wedge x\in B\wedge x\in C)\vee(x\in A\wedge x\in B\wedge x\notin C)$

$[x\in A\wedge x\notin B]\wedge\neg[(x\in A\wedge x\notin C)\vee(x\notin A\wedge x\in C)]$

$[x\in A\wedge x\notin B]\wedge[(x\notin A\vee x\in C)\wedge(x\in A\vee x\notin C)]$

$[[(x\notin A\vee x\in C)\wedge x\in A]\vee[(x\notin A\vee x\in C)\wedge x\notin C]]$

$[x\in A\wedge x\notin B]\wedge[(x\notin A\wedge x\in A)\vee(x\in C\wedge x\in A)\vee(x\notin A\wedge x\notin C)\vee(x\in C\wedge x\notin C)]$

$[x\in A\wedge x\notin B]\wedge[(x\in C\wedge x\in A)\vee(x\notin A\wedge x\notin C)]$

$(x\in A\wedge x\notin B\wedge x\in C)\vee(x\notin A\wedge x\in A\wedge x\notin B\wedge x\notin C)$

$(x\in A\wedge x\notin B\wedge x\in C)$

$(x\notin A\wedge x\in B\wedge x\in C)\vee(x\notin A\wedge x\notin B\wedge x\in C)\vee(x\notin A\wedge x\in B\wedge x\in C)\vee(x\in A\wedge x\in B\wedge x\notin C)\vee(x\in A\wedge x\notin B\wedge x\in C)=(x\in A\wedge x\in B\wedge x\notin C)\vee(x\notin A\wedge x\in B\wedge x\in C)\vee(x\notin A\wedge x\notin B\wedge x\in C)\vee(x\in A\vee x\notin B\wedge x\in C)$

$(x\in A\wedge x\in B\wedge x\notin C)\vee(x\notin A\wedge x\in B\wedge x\in C)\vee(x\notin A\wedge x\notin B\wedge x\in C)\vee(x\in A\wedge x\notin B\wedge x\in C)=(x\in A\wedge x\in B\wedge x\notin C)\vee(x\notin A\wedge x\in B\wedge x\in C)\vee(x\notin A\wedge x\notin B\wedge x\in C)\vee(x\in A\vee x\notin B\wedge x\in C)$

Logo $(A\cap B)^2C=(A^2C)^2(A\setminus B)$

c) $(A\setminus B)^2C=[(A\setminus B)\setminus C]\cup[C\setminus(A\setminus B)]=\{x\mid(x\in A\wedge x\notin B\wedge x\notin C)\vee[x\in C\wedge\neg(x\in A\wedge x\notin B)]\}=$

$\{x\mid(x\in A\wedge x\notin B\wedge x\notin C)\vee(x\notin A\wedge x\in B\wedge x\in C)\vee(x\notin A\wedge x\notin B\wedge x\in C)\vee(x\in A\wedge x\in B\wedge x\in C)\vee(x\notin A\wedge x\in B\wedge x\in C)\}=$

$\{x\mid(x\in A\wedge x\notin B\wedge x\notin C)\vee(x\notin A\wedge x\in B\wedge x\in C)\vee(x\notin A\wedge x\notin B\wedge x\in C)\vee(x\in A\wedge x\in B\wedge x\in C)\}$

$(A^2C)^2(A\cap B)=[(A\setminus C\cup C\setminus A)\setminus(A\cap B)]\cup[(A\cap B)\setminus(A\setminus C\cup C\setminus A)]=\{x\mid[[(x\in A\wedge x\notin C)\vee(x\notin A\wedge x\in C)]\wedge\neg[x\in A\wedge x\in B]]\vee[[x\in A\wedge x\in B]\wedge\neg[(x\in A\wedge x\notin C)\vee(x\notin A\wedge x\in C)]]\}$

$[[(x\in A\wedge x\notin C)\vee(x\notin A\wedge x\in C)]\wedge\neg[x\in A\wedge x\in B]]\vee[[x\in A\wedge x\in B]\wedge\neg[(x\in A\wedge x\notin C)\vee(x\notin A\wedge x\in C)]]$

$[(x\in A\wedge x\notin C)\vee(x\notin A\wedge x\in C)]\wedge\neg[x\in A\wedge x\in B]$

$[(x\in A\wedge x\notin C)\vee(x\notin A\wedge x\in C)]\wedge[x\notin A\vee x\notin B]$

$[[(x\in A\wedge x\notin C)\vee(x\notin A\wedge x\in C)]\wedge x\notin A]\vee[[(x\in A\wedge x\notin C)\vee(x\notin A\wedge x\in C)]\wedge x\notin B]$

$[(x\notin A\wedge x\in A\wedge x\notin C)\vee(x\notin A\wedge x\in C)]\vee[(x\in A\wedge x\notin B\wedge x\notin C)\vee(x\notin A\wedge x\notin B\wedge x\in C)]$

$(x\notin A\wedge x\in C)\vee(x\in A\wedge x\notin B\wedge x\notin C)\vee(x\notin A\wedge x\notin B\wedge x\in C)$

$(x\notin A\wedge x\notin B\wedge x\in C)\vee(x\notin A\wedge x\in B\wedge x\in C)\vee(x\in A\wedge x\notin B\wedge x\notin C)\vee(x\notin A\wedge x\notin B\wedge x\in C)$

$(x\notin A\wedge x\in B\wedge x\in C)\vee(x\in A\wedge x\notin B\wedge x\notin C)\vee(x\notin A\wedge x\notin B\wedge x\in C)$

$[x\in A\wedge x\in B]\wedge\neg[(x\in A\wedge x\notin C)\vee(x\notin A\wedge x\in C)]$

$[x\in A\wedge x\in B]\wedge[(x\notin A\vee x\in C)\wedge(x\in A\vee x\notin C)]$

$[x\in A\wedge x\in B]\wedge[[x\notin A\wedge(x\in A\vee x\notin C)]\vee[x\in C\wedge(x\in A\vee x\notin C)]]$

$(x\in A\wedge x\in B)\wedge[(x\notin A\wedge x\in A)\vee(x\notin A\wedge x\notin C)\vee(x\in C\wedge x\in A)\vee(x\in C\wedge x\notin C)]$

$(x\in A\wedge x\in B)\wedge[(x\notin A\wedge x\notin C)\vee(x\in C\wedge x\in A)]$

$[(x\in A\wedge x\in B)\wedge(x\notin A\wedge x\notin C)]\vee[(x\in A\wedge x\in B)\wedge(x\in C\wedge x\in A)]$

$(x\in A\wedge x\in B\wedge x\in C)$

$(x\notin A\wedge x\in B\wedge x\in C)\vee(x\in A\wedge x\notin B\wedge x\notin C)\vee(x\notin A\wedge x\notin B\wedge x\in C)\vee(x\in A\wedge x\in B\wedge x\in C)=(x\in A\wedge x\notin B\wedge x\notin C)\vee(x\notin A\wedge x\in B\wedge x\in C)\vee(x\notin A\wedge x\notin B\wedge x\in C)\vee(x\in A\wedge x\in B\wedge x\in C)$

$(x\in A\wedge x\notin B\wedge x\notin C)\vee(x\notin A\wedge x\in B\wedge x\in C)\vee(x\notin A\wedge x\notin B\wedge x\in C)\vee(x\in A\wedge x\in B\wedge x\in C)=(x\in A\wedge x\notin B\wedge x\notin C)\vee(x\notin A\wedge x\in B\wedge x\in C)\vee(x\notin A\wedge x\notin B\wedge x\in C)\vee(x\in A\wedge x\in B\wedge x\in C)$

Logo $(A\setminus B)^2C=(A^2C)^2(A\cap B)$

17. Fill in the blanks to make true identities:
(a) $(A\ {}^2\ B)\cap C=(C\setminus A)\ {}^2$ _______.
(b) $C\setminus(A\ {}^2\ B)=(A\cap C)\ {}^2$ _______.
(c) $(B\setminus A)\ {}^2\ C=(A\ {}^2\ C)\ {}^2$ _______.

16. Use any method you wish to verify the following identities:
(a) $(A\cup B)\ {}^2\ C=(A\ {}^2\ C)\ {}^2\ (B\setminus A)$.
(b) $(A\cap B)\ {}^2\ C=(A\ {}^2\ C)\ {}^2\ (A\setminus B)$.
(c) $(A\setminus B)\ {}^2\ C=(A\ {}^2\ C)\ {}^2\ (A\cap B)$.

15. Use any method you wish to verify the following identities:
(a) $(A\ {}^2\ B)\cup C=(A\cup C)\ {}^2\ (B\setminus C)$.
(b) $(A\ {}^2\ B)\cap C=(A\cap C)\ {}^2\ (B\cap C)$.
(c) $(A\ {}^2\ B)\setminus C=(A\setminus C)\ {}^2\ (B\setminus C)$.

(a) $(A\setminus B)\setminus C$. (d) $(A\setminus B)\cap(A\setminus C)$. (e) $A\setminus(B\cup C)$.
(b) $A\setminus(B\setminus C)$. (c) $(A\setminus B)\cup(A\cap C)$.

a)

$(A^2B)\cap C=[(A\setminus B)\cup(B\setminus A)]\cap C=\{x\mid[(x\in A\wedge x\notin B)\vee(x\notin A\wedge x\in B)]\wedge x\in C\}=\{x\mid(x\in A\wedge x\notin B\wedge x\in C)\vee(x\notin A\wedge x\in B\wedge x\in C)\}$

$(C\setminus A)^2P=[(C\setminus A)\setminus P]\cup[P\setminus(C\setminus A)]=\{x\mid[x\in C\wedge x\notin A\wedge x\notin P]\vee[x\in P\wedge\neg(x\in C\wedge x\notin A)]\}=\{x\mid(x\notin A\wedge x\in C\wedge x\notin P)\vee(x\notin C\wedge x\in P)\vee(x\in A\wedge x\in P)\}$

			$(x\in A\wedge x\notin B\wedge x\in C)\vee(x\notin A\wedge x\in B\wedge x\in C)$	$(x\notin A\wedge x\in C\wedge x\notin P)\vee(x\notin C\wedge x\in P)\vee(x\in A\wedge x\in P)$	
$x\in A$	$x\in B$	$x\in C$	0	$0\vee 0\vee x\in P=x\in P$	Não pertence
$x\in A$	$x\in B$	$x\notin C$	0	$0\vee x\in P\vee x\in P=x\in P$	Não pertence
$x\in A$	$x\notin B$	$x\in C$	1	$0\vee 0\vee x\in P=x\in P$	Pertence
$x\in A$	$x\notin B$	$x\notin C$	0	$0\vee x\in P\vee x\in P=x\in P$	Não pertence
$x\notin A$	$x\in B$	$x\in C$	1	$x\notin P\vee 0\vee 0=x\notin P$	Não pertence
$x\notin A$	$x\in B$	$x\notin C$	0	$0\vee x\in P\vee 0=x\in P$	Não pertence
$x\notin A$	$x\notin B$	$x\in C$	0	$x\notin P\vee 0\vee 0=x\notin P$	Pertence
$x\notin A$	$x\notin B$	$x\notin C$	0	$0\vee x\in P\vee 0=x\in P$	Não pertence

$(x\in A\wedge x\notin B\wedge x\in C)\wedge(x\notin A\wedge x\notin B\wedge x\in C)=x\in C\wedge x\notin B$

Conferindo: $(x\notin A\wedge x\in C\wedge x\notin P)\vee(x\notin C\wedge x\in P)\vee(x\in A\wedge x\in P)$

$[x\notin A\wedge x\in C\wedge\neg(x\in C\wedge x\notin B)]\vee[x\notin C\wedge(x\in C\wedge x\notin B)]\vee[x\in A\wedge(x\in C\wedge x\notin B)]$

$(x\notin A\wedge x\in C\wedge x\notin C)\vee(x\notin A\wedge x\in C\wedge x\in B)\vee(x\notin C\wedge x\in C\wedge x\notin B)\vee(x\in A\wedge x\in C\wedge x\notin B)$

$(x\notin A\wedge x\in C\wedge x\in B)\vee(x\in A\wedge x\in C\wedge x\notin B)$

$(x\in A\wedge x\notin B\wedge x\in C)\vee(x\notin A\wedge x\in B\wedge x\in C)$

$\boxed{P=C\setminus B}$

b) $C\setminus(A^2B)=C\setminus[(A\setminus B)\cup(B\setminus A)]=\{x\mid x\in C\wedge\neg[(x\in A\wedge x\notin B)\vee(x\notin A\wedge x\in B)]\}=\{x\mid x\in C\wedge[(x\notin A\vee x\in B)\wedge(x\in A\vee x\notin B)]\}=\{x\mid x\in C\wedge[(x\notin A\wedge x\notin B)\vee(x\in B\wedge x\in A)]\}=$
$\{x\mid(x\notin A\wedge x\notin B\wedge x\in C)\vee(x\in A\wedge x\in B\wedge x\in C)\}$

$(A\cap C)^2P=[(A\cap C)\setminus P]\cup[P\setminus(A\cap C)]=\{x\mid(x\in A\wedge x\in C\wedge x\notin P)\vee[x\in P\wedge\neg(x\in A\wedge x\in C)]\}=\{x\mid(x\in A\wedge x\in C\wedge x\notin P)\vee(x\notin A\wedge x\in P)\vee(x\notin C\wedge x\in P)\}$

			$(x\notin A\wedge x\notin B\wedge x\in C)\vee(x\in A\wedge x\in B\wedge x\in C)$	$(x\in A\wedge x\in C\wedge x\notin P)\vee(x\notin A\wedge x\in P)\vee(x\notin C\wedge x\in P)$	
$x\in A$	$x\in B$	$x\in C$	1	$x\notin P\vee 0\vee 0=x\notin P$	Não pertence
$x\in A$	$x\in B$	$x\notin C$	0	$0\vee 0\vee x\in P=x\in P$	Não pertence
$x\in A$	$x\notin B$	$x\in C$	0	$x\notin P\vee 0\vee 0=x\notin P$	Pertence
$x\in A$	$x\notin B$	$x\notin C$	0	$0\vee 0\vee x\in P=x\in P$	Não pertence
$x\notin A$	$x\in B$	$x\in C$	0	$0\vee x\in P\vee 0=x\in P$	Não pertence
$x\notin A$	$x\in B$	$x\notin C$	0	$0\vee x\in P\vee x\in P=x\in P$	Não pertence
$x\notin A$	$x\notin B$	$x\in C$	1	$0\vee x\in P\vee 0=x\in P$	Pertence
$x\notin A$	$x\notin B$	$x\notin C$	0	$0\vee x\in P\vee x\in P=x\in P$	Não pertence

$(x\notin A\wedge x\notin B\wedge x\in C)\vee(x\in A\wedge x\notin B\wedge x\in C)=x\notin B\wedge x\in C$

Conferindo: $(x\in A\wedge x\in C\wedge x\notin P)\vee(x\notin A\wedge x\in P)\vee(x\notin C\wedge x\in P)$

$(x\in A\wedge x\in C\wedge\neg(x\notin B\wedge x\in C))\vee(x\notin A\wedge x\notin B\wedge x\in C)\vee(x\notin C\wedge x\notin B\wedge x\in C)$

$(x\in A\wedge x\in C\wedge(x\in B\vee x\notin C))\vee(x\notin A\wedge x\notin B\wedge x\in C)$

$(x\in A\wedge x\in B\wedge x\in C)\vee(x\in A\wedge x\in C\wedge x\notin C)\vee(x\notin A\wedge x\notin B\wedge x\in C)$

$(x\in A\wedge x\in B\wedge x\in C)\vee(x\notin A\wedge x\notin B\wedge x\in C)$

$\boxed{P=C\setminus B}$

c) $(B\setminus A)^2C=[(B\setminus A)\setminus C]\vee[C\setminus(B\setminus A)]=\{x\mid[x\in B\wedge x\notin A\wedge x\notin C]\vee[x\in C\wedge\neg(x\in B\wedge x\notin A)]\}=\{(x\in B\wedge x\notin A\wedge x\notin C)\vee(x\notin B\wedge x\in C)\vee(x\in A\wedge x\in C)\}=$
$\{(x\in B\wedge x\notin A\wedge x\notin C)\vee(x\notin B\wedge x\in C)\vee(x\in A\wedge x\in C)\}=\{(x\notin A\wedge x\in B\wedge x\notin C)\vee(x\in A\wedge x\notin B\wedge x\in C)\vee(x\notin A\wedge x\notin B\wedge x\in C)\vee(x\in A\wedge x\in B\wedge x\in C)\vee\cancel{(x\in A\wedge x\notin B\wedge x\in C)}\}$

$(A^2C)^2P=[[(A\setminus C)\cup(C\setminus A)]\setminus P]\cup[P\setminus[(A\setminus C)\cup(C\setminus A)]]=\{x\mid[[(x\in A\wedge x\notin C)\vee(x\in C\wedge x\notin A)]\wedge x\notin P]\vee[x\in P\wedge\neg[(x\in A\wedge x\notin C)\vee(x\in C\wedge x\notin A)]]\}=$
$\{x\mid[(x\in A\wedge x\notin C\wedge x\notin P)\vee(x\notin A\wedge x\in C\wedge x\notin P)]\vee[x\in P\wedge[(x\notin A\vee x\in C)\wedge(x\in A\vee x\notin C)]]\}=$
$\{x\mid[(x\in A\wedge x\notin C\wedge x\notin P)\vee(x\notin A\wedge x\in C\wedge x\notin P)]\vee[(x\notin A\wedge x\notin C\wedge x\in P)\vee(x\in A\wedge x\in C\wedge x\in P)]\}=$
$\{x\mid(x\in A\wedge x\notin C\wedge x\notin P)\vee(x\notin A\wedge x\in C\wedge x\notin P)\vee(x\notin A\wedge x\notin C\wedge x\in P)\vee(x\in A\wedge x\in C\wedge x\in P)\}=$

17. Fill in the blanks to make true identities:
(a) $(A^2B)\cap C=(C\setminus A)^2$ _______.
(b) $C\setminus(A^2B)=(A\cap C)^2$ _______.
(c) $(B\setminus A)^2C=(A^2C)^2$ _______.

			$(x\notin A\wedge x\notin C\wedge x\in B)\vee(x\notin A\wedge x\in C\wedge x\notin B)\vee(x\in A\wedge x\in C\wedge x\notin B)\vee(x\in A\wedge x\in C\wedge x\in B)$	$(x\notin A\wedge x\notin C\wedge x\in P)\vee(x\notin A\wedge x\in C\wedge x\notin P)\vee(x\in A\wedge x\in C\wedge x\in P)\vee(x\in A\wedge x\notin C\wedge x\notin P)$
$x\in A$	$x\in B$	$x\in C$	1	$0\vee 0\vee P\vee 0=x\in P$
$x\in A$	$x\in B$	$x\notin C$	0	$0\vee 0\vee 0\vee\neg P=x\notin P$
$x\in A$	$x\notin B$	$x\in C$	1	$0\vee 0\vee P\vee 0=x\in P$
$x\in A$	$x\notin B$	$x\notin C$	0	$0\vee 0\vee 0\vee\neg P=x\notin P$
$x\notin A$	$x\in B$	$x\in C$	0	$0\vee\neg P\vee 0\vee 0=x\notin P$
$x\notin A$	$x\in B$	$x\notin C$	1	$P\vee 0\vee 0\vee 0=x\in P$
$x\notin A$	$x\notin B$	$x\in C$	1	$0\vee\neg P\vee 0\vee 0=x\notin P$
$x\notin A$	$x\notin B$	$x\notin C$	0	$P\vee 0\vee 0\vee 0=x\in P$

$(x\notin A\wedge x\in B\wedge x\notin C)\vee(x\notin A\wedge x\in B\wedge x\in C)\vee(x\in A\wedge x\notin B\wedge x\notin C)\vee(x\in A\wedge x\notin B\wedge x\in C)\vee(x\in A\wedge x\in B\wedge x\notin C)\vee(x\in A\wedge x\in B\wedge x\in C)$

$(x \notin A \wedge x \in B) \vee (x \in A \wedge x \notin B) \vee (x \in A \wedge x \in B) = (x \notin A \wedge x \in B) \vee x \in A = x \in A \vee x \in B$

Conferindo: $(x \in A \wedge x \notin C \wedge x \notin P) \vee (x \notin A \wedge x \in C \wedge x \notin P) \vee (x \notin A \wedge x \notin C \wedge x \in P) \vee (x \in A \wedge x \in C \wedge x \in P)$

$[x \in A \wedge x \notin C \wedge \neg(x \in A \vee x \in B)] \vee [x \notin A \wedge x \in C \wedge \neg(x \in A \vee x \in B)] \vee [x \notin A \wedge x \notin C \wedge (x \in A \vee x \in B)] \vee [x \in A \wedge x \in C \wedge (x \in A \vee x \in B)]$

$[x \in A \wedge x \notin C \wedge (x \notin A \wedge x \notin B)] \vee [x \notin A \wedge x \in C \wedge (x \notin A \wedge x \notin B)] \vee [x \notin A \wedge x \notin C \wedge x \in A] \vee [x \notin A \wedge x \notin C \wedge x \in B] \vee [x \in A \wedge x \in C \wedge x \in A] \vee [x \in A \wedge x \in C \wedge x \in B]$

$[x \notin A \wedge x \notin B \wedge x \in C] \vee [x \notin A \wedge x \in B \wedge x \notin C] \vee [x \in A \wedge x \notin B \wedge x \in C] \vee [x \in A \wedge x \in B \wedge x \in C]$

$\boxed{P = A \cup B}$