Thesis Proposal: Quantum Algorithms for Partial Differential Equations

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Thesis Statement

This thesis investigates the computational complexity and practical implementation of high-precision quantum algorithms for solving linear partial differential equations (PDEs). Specifically, it aims to benchmark the exponential improvements in error dependence (polylog$(1/\epsilon)$) proposed by modern spectral methods against standard classical solvers.

Background & Motivation

Partial differential equations are ubiquitous in physics and engineering, but classical grid-based methods suffer from the "curse of dimensionality." While early quantum approaches (such as the HHL algorithm) offered potential speedups, they were limited by a polynomial dependence on the inverse error, denoted as $O(\text{poly}(1/\epsilon))$. This made high-precision solutions prohibitively expensive for near-term applications.

Recent literature, including the work of Childs et al. (2020) [1] and extensive analysis from the University of Waterloo [2], introduces a new class of algorithms utilizing spectral methods and Hamiltonian simulation. These methods theoretically allow for time evolution where the complexity scales logarithmically with the inverse error, $O(\text{polylog}(1/\epsilon))$. This project seeks to verify these bounds and explore the feasibility of these algorithms using current quantum simulation tools.

Methodology

The project will be conducted through a combination of theoretical analysis and numerical simulation:

- **Literature Review:** A comparative study of classical finite difference methods versus quantum spectral methods, specifically focusing on the complexity proofs provided in [1].
- **Implementation:** Implementation of toy-model PDEs (e.g., heat or wave equation) using quantum SDKs such as Qiskit or PennyLane to simulate Hamiltonian evolution.
- **Benchmarking:** Assessing the gate depth and resource requirements of the quantum approach relative to the desired precision ϵ.

Progression Plan & Milestones

Note: Weekly meetings between supervisor and student will be held from the start of Autumn 2026 until the thesis is finished. With some preliminary meetings in Spring 2026.

Spring 2026 — **Preparation Phase**
Completion of coursework (MAT 4430 and FYS 4415). Initial meetings regarding the topic and discussion of relevant research papers.

Autumn 2026 (Q3) — **Theoretical Exploration**
Theoretical exploration of PDEs and Quantum algorithms, specifically analyzing their strengths and weaknesses.

Autumn 2026 (Q4) — **Specialisation & Computation**
Specialisation of the research to a specific result or area within the theory. Execution of numerical computations and comparisons.

Spring 2027 (Q1) — **Deep Dive**
Finishing the deep dive in the specific area and resolving theoretical loose ends.

Spring 2027 (Q2) — **Writing & Compilation**
Compiling and rewriting the results derived from the previous three quarters into a consistent whole.

Relevant Curriculum

- **MAT 4430** – Quantum Information Theory
- **FYS 4415** – Quantum Computing and Quantum Information

References

[1] A. M. Childs, J. P. Liu, and A. Ostrander, "High-precision quantum algorithms for partial differential equations," *Quantum*, vol. 5, p. 574, 2021. [Online]. Available: arXiv:2002.07868.

[2] D. An, "Quantum algorithms for partial differential equations," Ph.D. dissertation, Univ. Waterloo, Waterloo, ON, Canada, 2022. [Online]. Available: UWSpace.