Example 1 - Birthday Problem

JPT

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The Birthday Problem

The birthday problem - the probability that at least two people in the room have an identical birth date.

Is it something like $\frac{1}{365} \times N = 0.063$?

Code for this: https://goo.gl/cf3w1Y

$$
\begin{aligned}
1 - \bar{p}(n) &= 1 \times \left(1 - \frac{1}{365}\right) \times \left(1 - \frac{2}{365}\right) \times \cdots \times \left(1 - \frac{n-1}{365}\right) \\
&= \frac{365 \times 364 \times \cdots \times (365 - n + 1)}{365^n} \\
&= \frac{365!}{365^n (365 - n)!} = \frac{n! \cdot \binom{365}{n}}{365^n} \\
p(n = 23) &= 0.507
\end{aligned} \tag{1}
$$

Simulate this stuff

1 - Simulate 10,000 rooms with $n = 23$ random birthdays, and store the results in matrix where each row represents a room.
2 - For each room (row) compute the number of unique birthdays.
3 - Compute the average number of times a room has 23 unique birthdays, across 10,000 simulations, and report the complement.

```r
birthday.prob = function(n.pers, n.sims) {
  # simulate birthdays
  birthdays = matrix(round(runif(n.pers * n.sims, 1, 365)),
                     nrow = n.sims, ncol = n.pers)
  # for each room (row) get unique birthdays
  unique.birthdays = apply(birthdays, 1, unique)
  # Indicator with 1 if all are unique birthdays
  all.different = (lapply(unique.birthdays, length) == n.pers)
  # Compute average time all have different birthdays
  result = 1 - mean(all.different)
return(result)
}
n.pers.param = n.pers
n.sims.param = 1e4
birthday.prob(n.pers.param,n.sims.param)
```

```
## [1] 0.5079
```

Results

- Many people originally think of a prob $\sim \frac{1}{365} \times N = 0.063$
- However the true probability is of $p(n = 23) = 0.507$
- And the simulated probability is of 0.5038