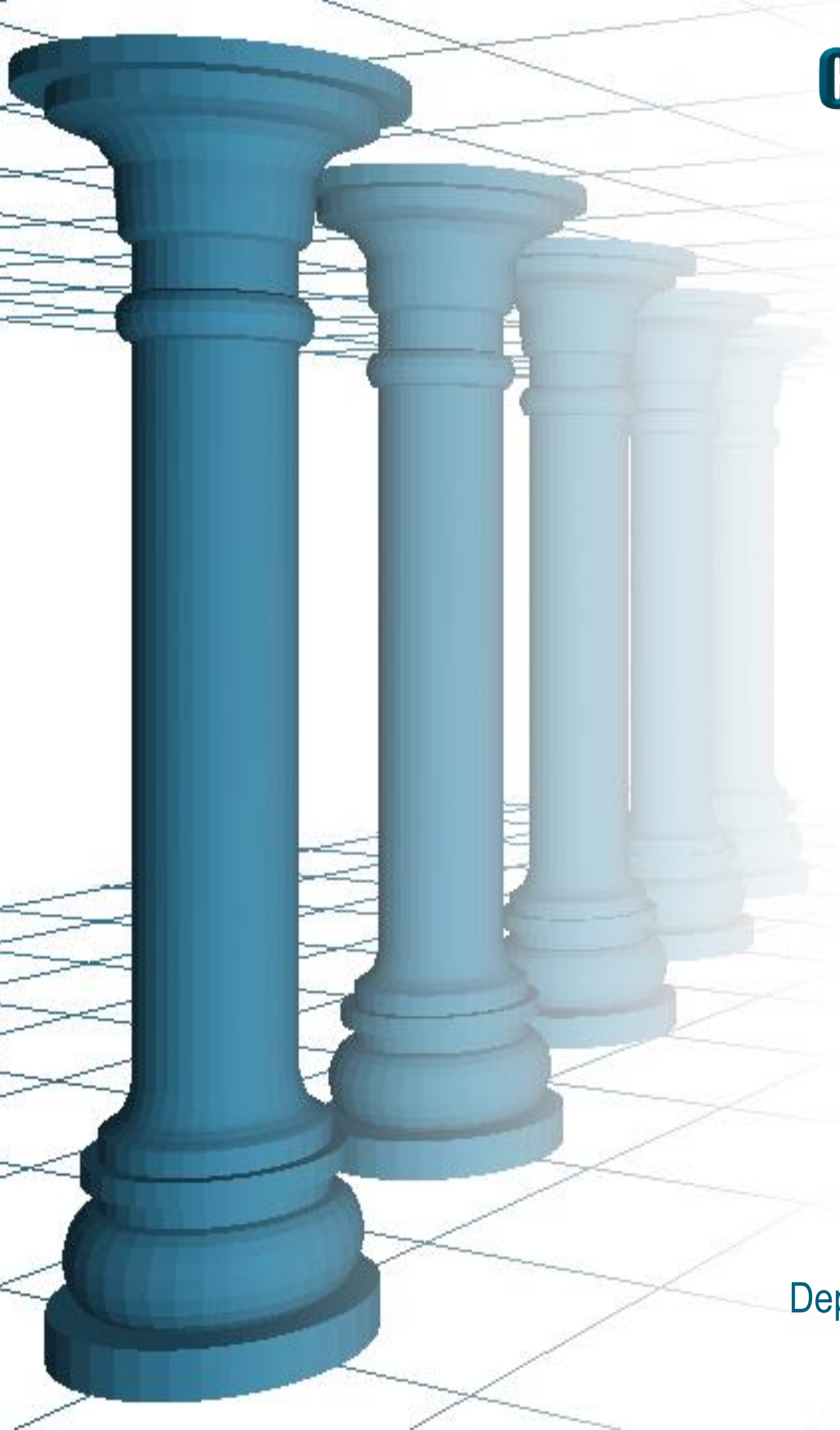

9

Ray Tracing

A transportation network for light

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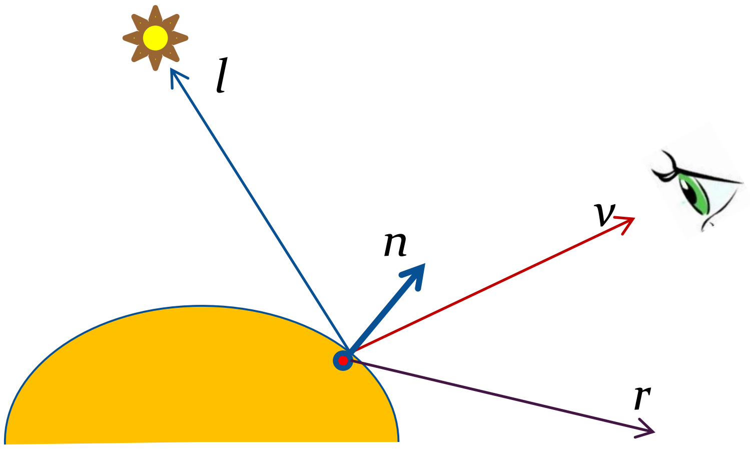
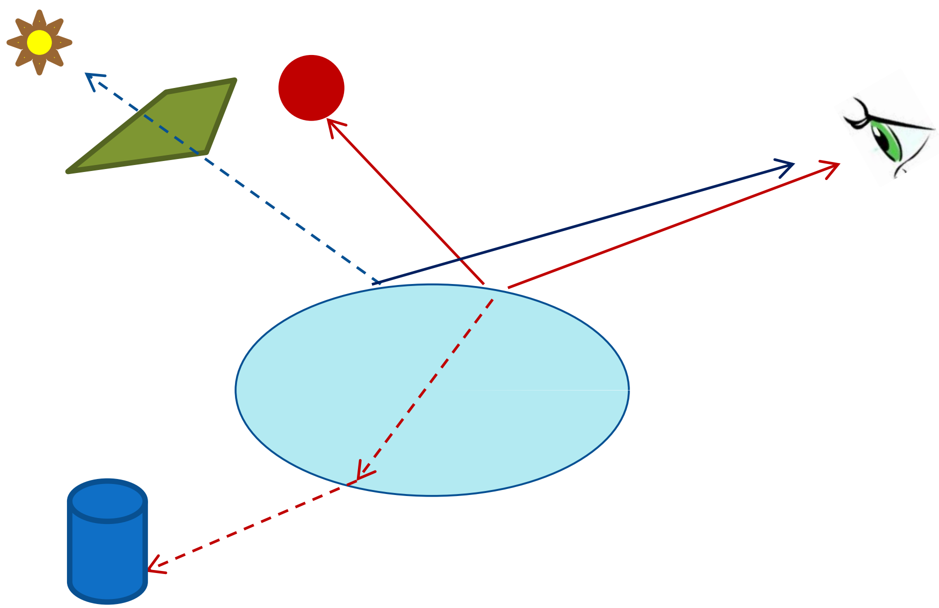
Local Illumination Model

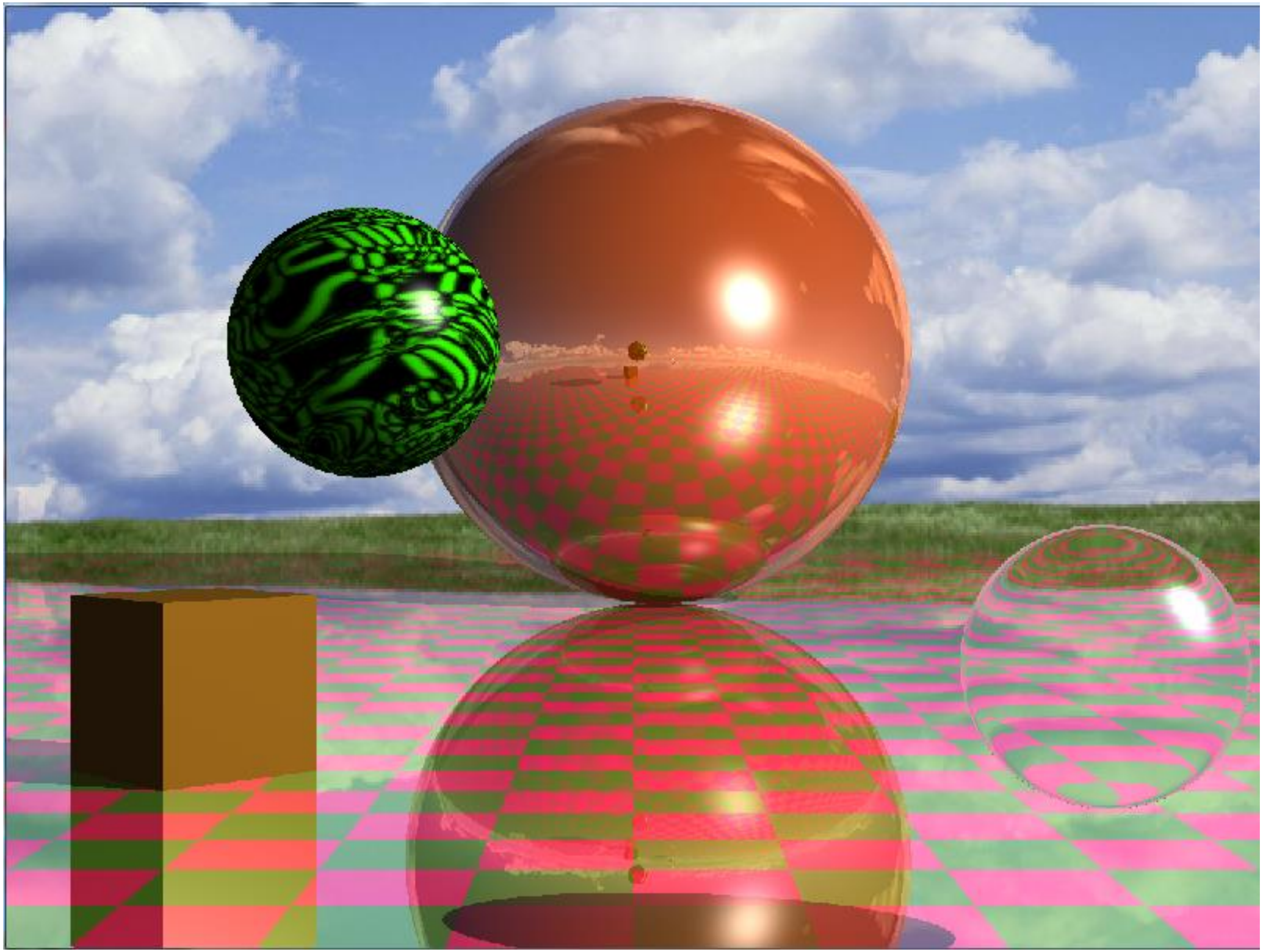
- A local illumination model considers only light travelling from a light source to a surface and then reflected off the surface to the eye.
  - Requires only the light source coordinates, local surface geometry and the material characteristics at a vertex.
  - Suitable for the hardware pipeline (OpenGL, Direct3D)
- Does not consider occlusions and transmittance of light.



Global Illumination

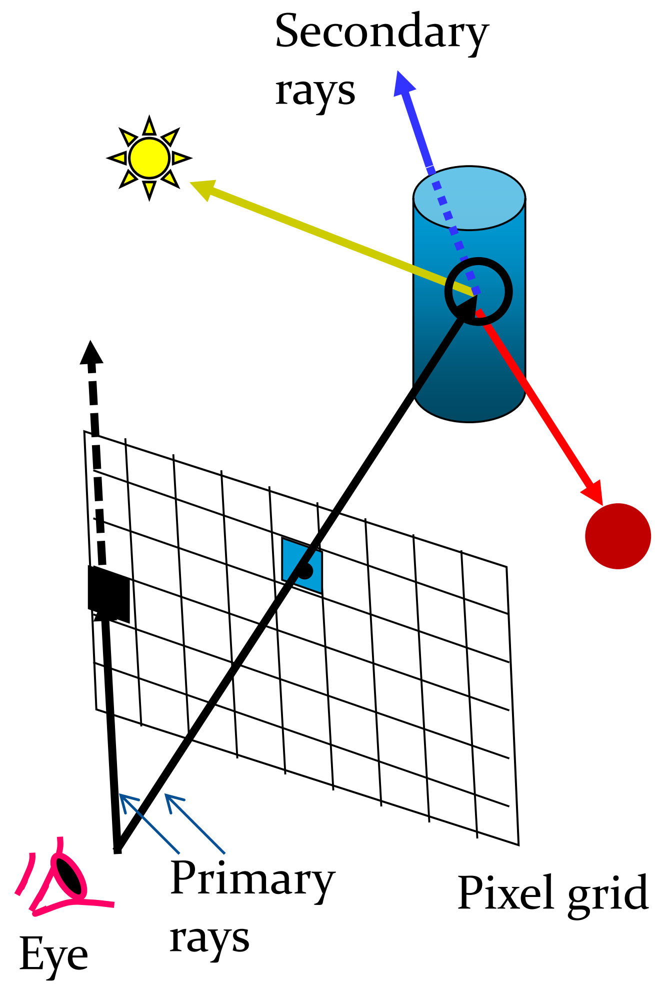
- The illumination at a given point is a combination of the light received from a source and the light reflected from other surfaces in the scene.
- Considers the effects of occlusions, surface reflections, light transmission through a medium (direct transmittance and refractions), and indirect illumination.





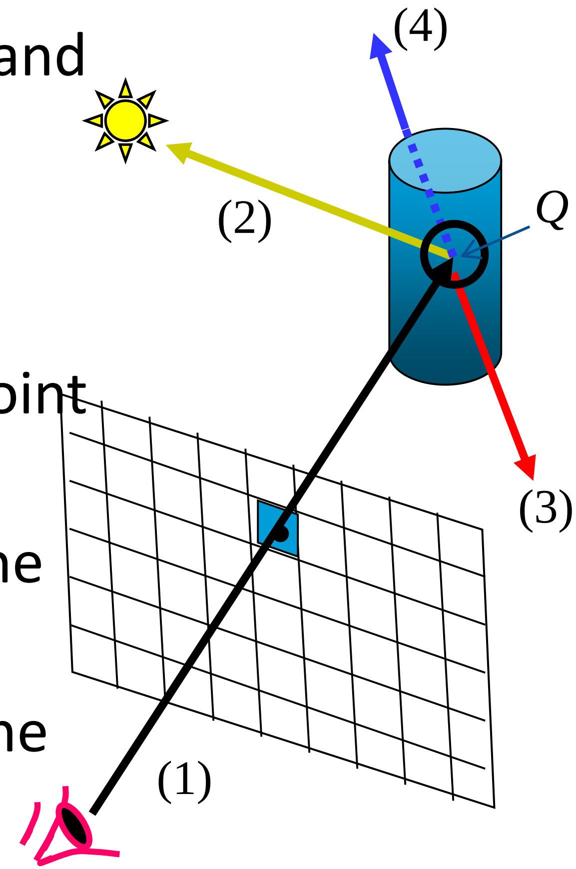
Ray Tracing (Backward Ray Tracing)

- Traces ray outwards from the eye to objects and light sources. This ray is called the primary ray.
- Use secondary rays to determine shadows and to model mirror reflection and refraction
- We can easily generate many global illumination effects
- But:
  - Doesn't handle diffuse inter-reflections
    - e.g. "colour bleed" from a bright red wall to an adjacent white wall
  - Computationally expensive



“Tracing” a ray

- Compare the ray will all scene objects and compute the closest point of intersection *Q*, and obtain the intersecting object’s index.
- Compute the colour value at the point of intersection *Q*.
- Generate a shadow ray to determine if the point *Q* is in shadow (2)
- If the surface is reflective, recursively trace the reflected ray at *Q* (3)
- If the surface is refractive, recursively trace the refracted ray at *Q* (4)
- Add the colour contributions from (2), (3) and (4), and return the colour value.

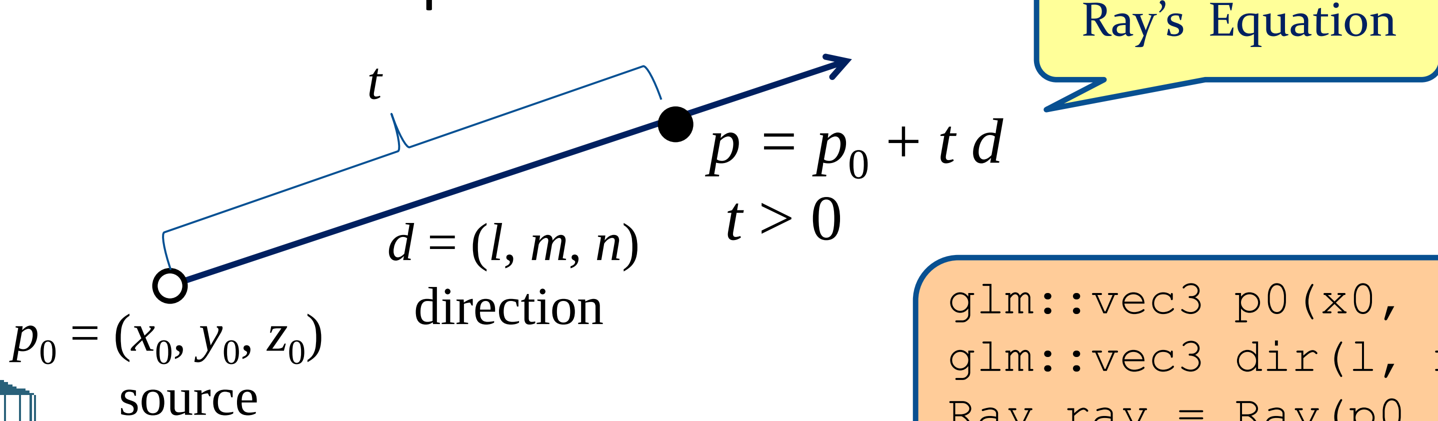


What is a "ray"?

A ray is specified using

- A point (the source of the ray): $p_0 = (x_0, y_0, z_0)$
- A vector (the direction of the ray): $d = (l, m, n)$
- The vector must be converted to a <u>unit</u> vector.

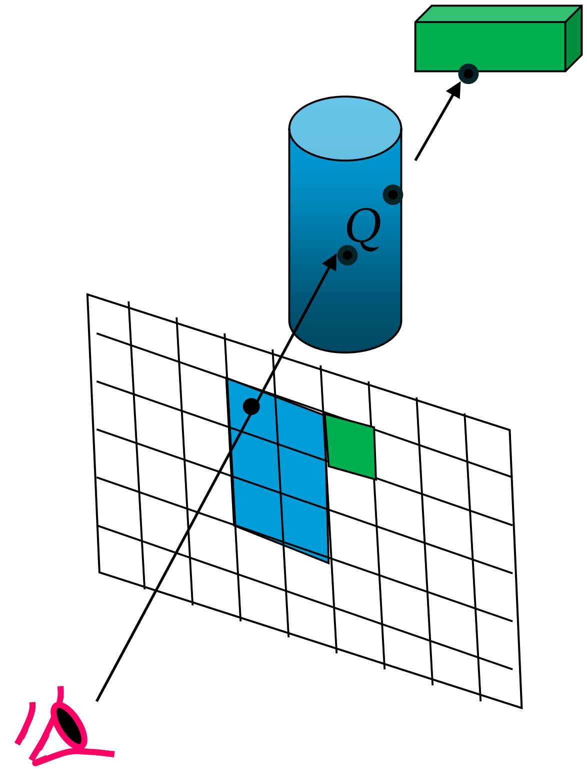
Any point on the ray can be represented using a single parameter t. The value of t denotes the distance from the source to that point.

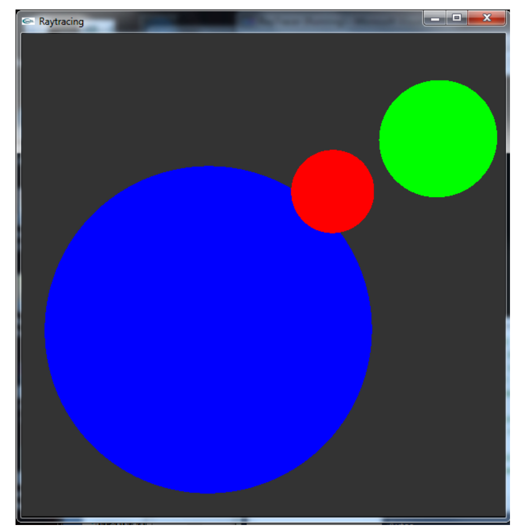



```
glm::vec3 p0(x0, y0, z0);
glm::vec3 dir(l, m, n);
Ray ray = Ray(p0,  dir);
ray.normalize();
```

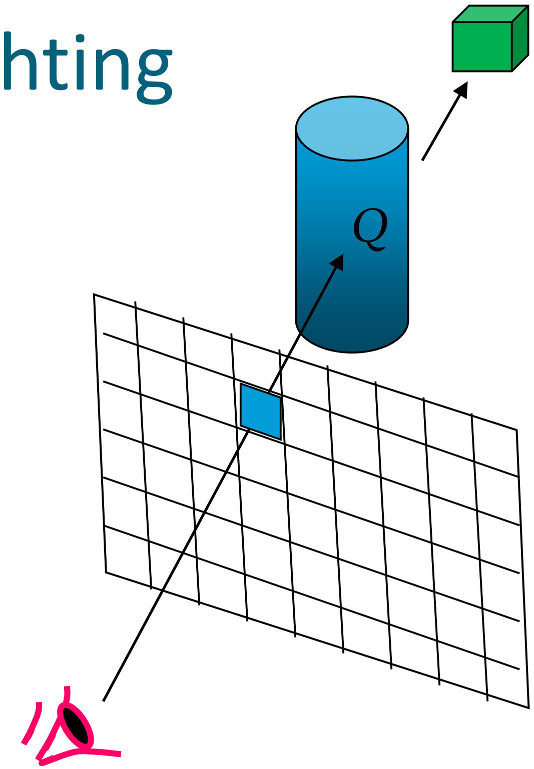
Ray Casting

- Ray tracing without secondary rays.
- Trace a ray from the view point (called the primary ray) through each "pixel" on the image plane
  - Test each surface to determine if it is intersected by the ray.
  - Compute the points of intersection on each primary ray.
  - Get the point of intersection Q that is closest to the eye.
  - Use the colour of the object on which Q lies as pixel colour.





Ray Casting + Phong Lighting



- At the point of intersection *Q*, compute the colour value using an illumination model.
- Simplified Phong Illumination:

  Assumptions:

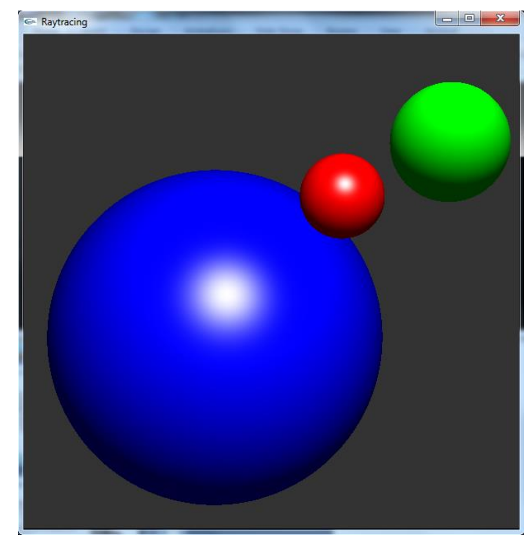
  Light's ambient color A = (0.2, 0.2, 0.2)

  Light's diffuse and specular color = (1,1,1)

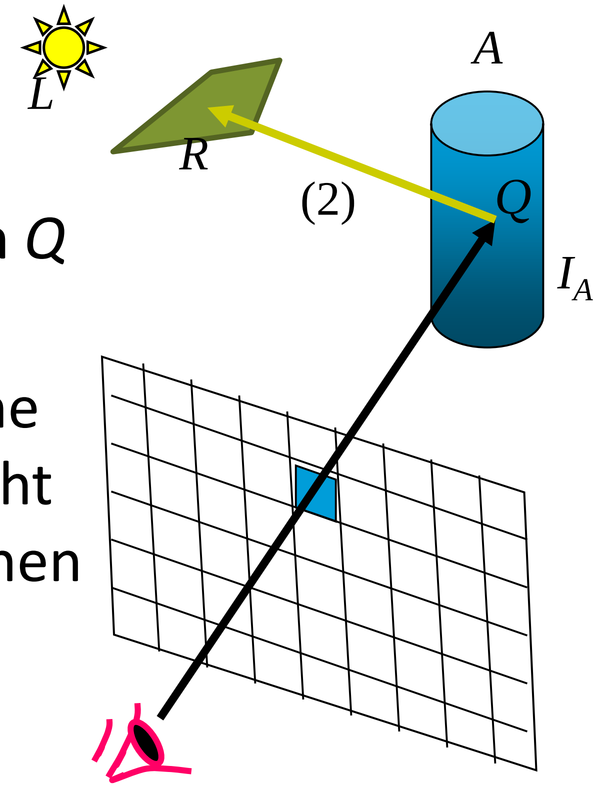
  M = Material Color (ambient and diffuse)

  Material's specular color = (1, 1, 1)

$$\text{Col} = \underbrace{AM}_{\text{Ambient term}} + \underbrace{(\boldsymbol{l} \bullet \boldsymbol{n})\, M}_{\text{Diffuse term}} + \underbrace{(\boldsymbol{r} \bullet \boldsymbol{v})^{f}}_{\text{Specular term}} (1,1,1)$$



Shadow Ray



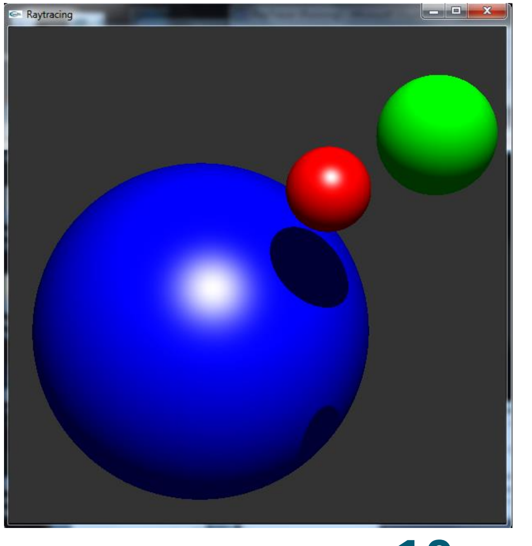
- Trace a ray from the point of intersection *Q* towards the light source *L* (2)
- If the shadow ray hits an object, and if the point of intersection *R* is between the light source and the object *A* (i.e., *RQ* < *LQ*), then the point *Q* is in shadow.

If *Q* is in shadow

I_A = AM

Else

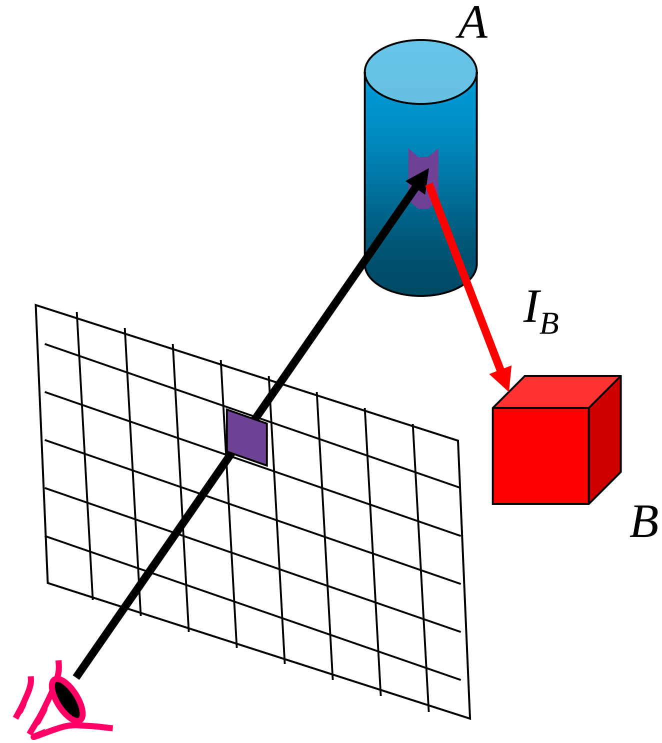
I_A = AM + M $(\boldsymbol{l} \bullet \boldsymbol{n})$ + (1, 1, 1) $(\boldsymbol{r} \bullet \boldsymbol{v})^f$

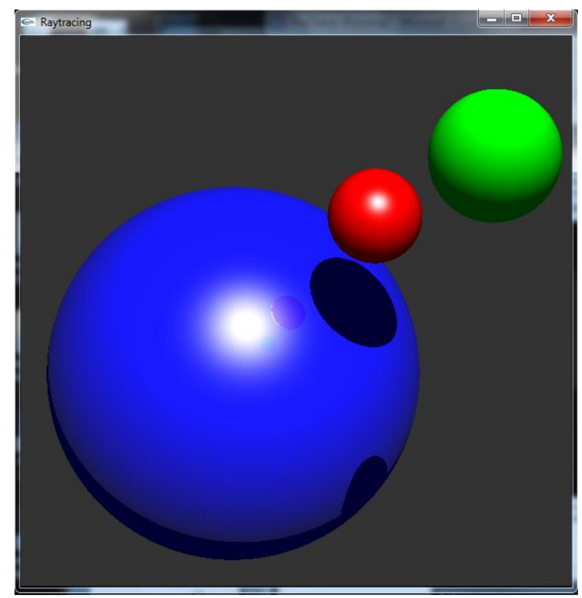


Reflections

- If the surface is reflective, then a secondary ray along the direction of reflection is traced.
- If this secondary ray meets a surface at a point with intensity I_B , then $\rho_r I_B$ is added to the pixel color
  - ρ_r is a scale factor (< 1), called the coefficent of reflection
  - ρ_r represents how much of colour I_B is reflected on the surface A.
- The colour of the pixel is now

$$I = I_A + \rho_r I_B$$

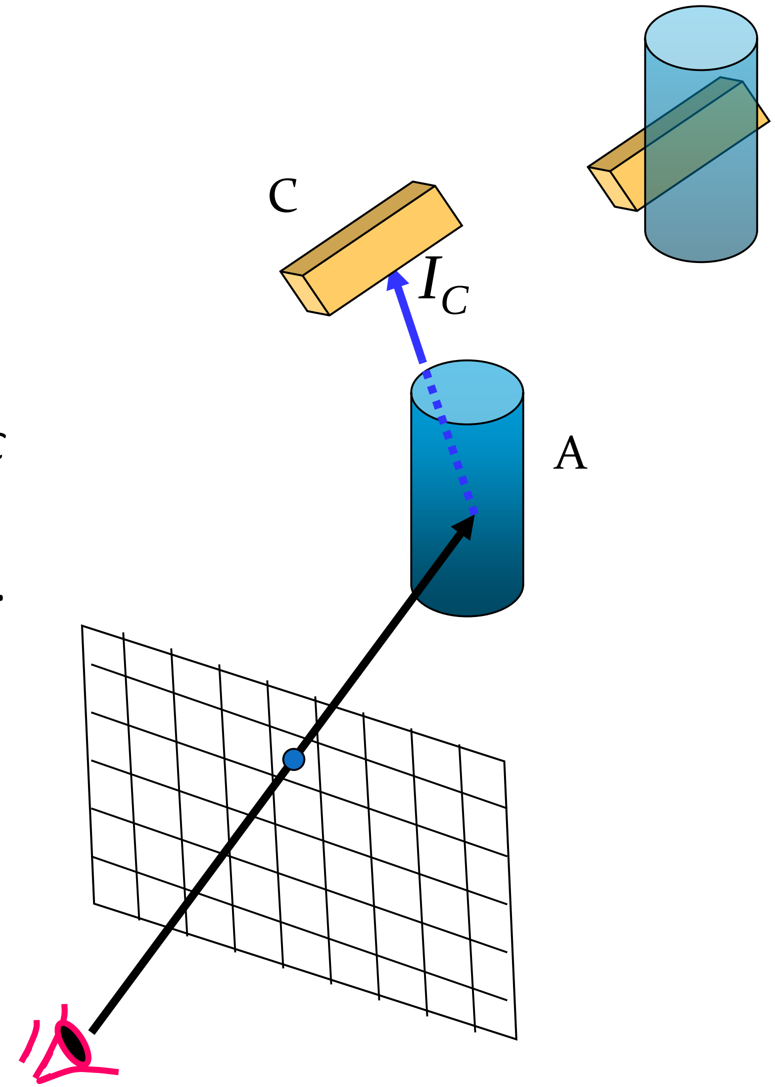




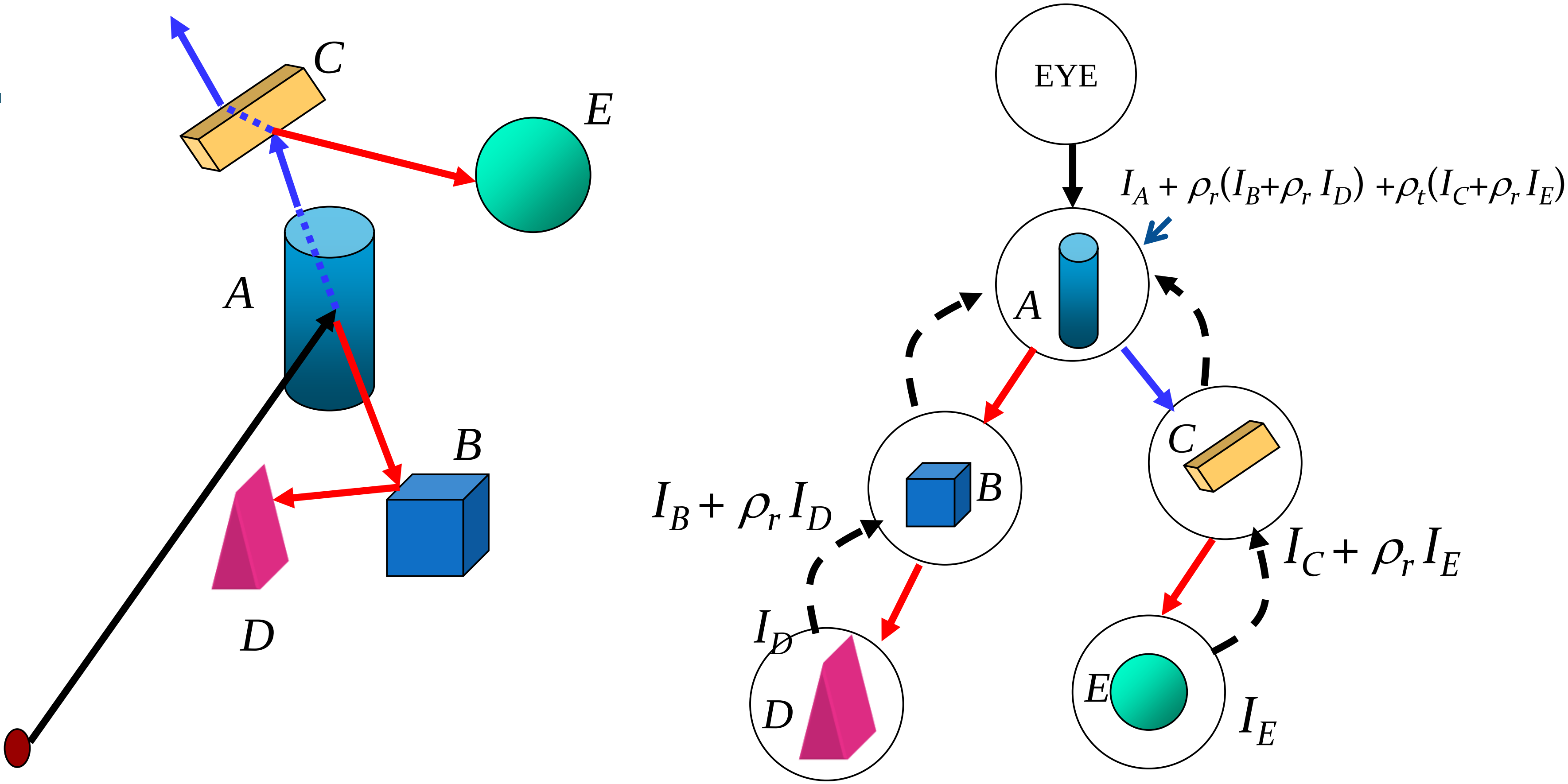
Refractions

- If the surface is transparent, a secondary ray along the direction of refraction is traced.
- If this secondary ray meets a surface at a point with intensity I_C, then $\rho_t I_C$ is added to the pixel color.
  - ρ_t is a scale factor (<1), called the coeff. of transmission
- The colour of the pixel is now

$$I = I_A + \rho_t I_c$$

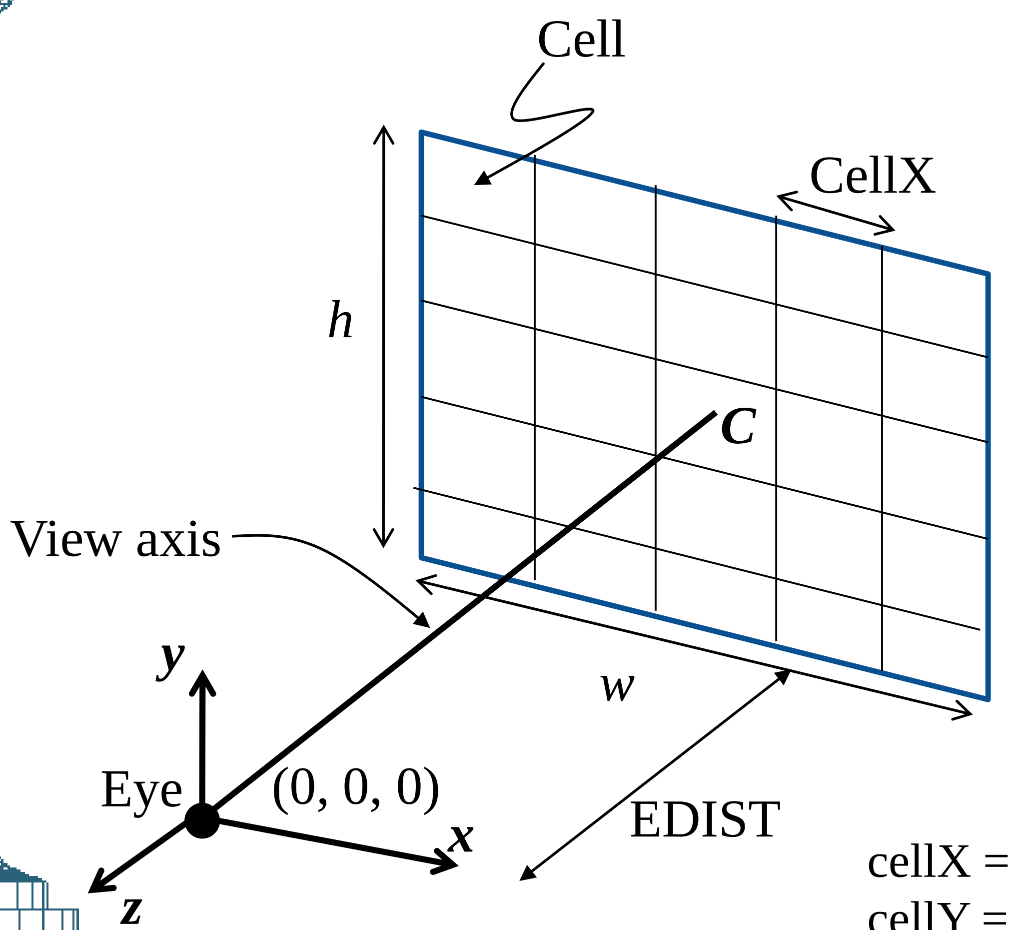


Binary Ray-tracing Tree



- Left branches represent reflections
- Right branches represent transmission paths

Ray Tracing Setup

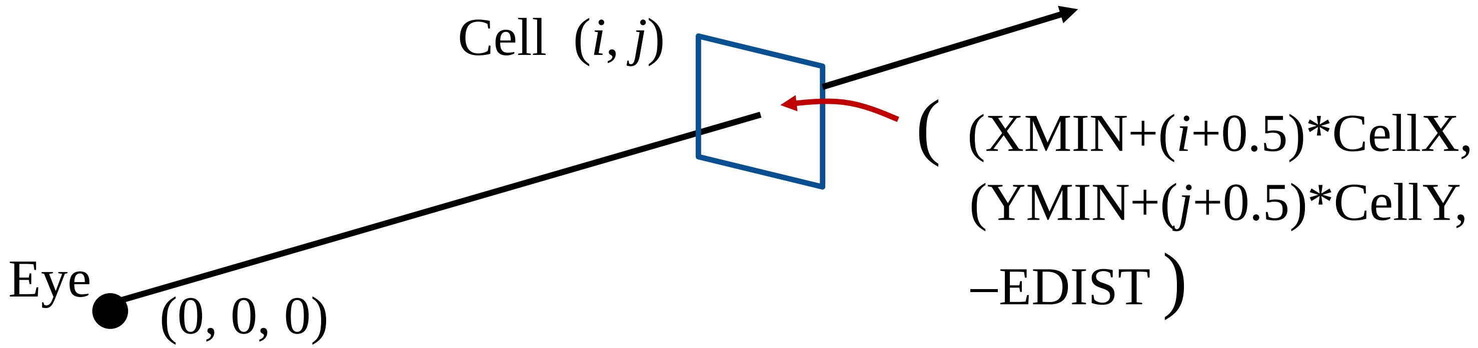


w = Width of screen in world units (eg. 5 units).
h = Height of screen in world units (eg. 5 units).
EDIST = Eye dist in world units (eg. 10 units)
XMIN = $-w/2$; XMAX = $w/2$;
YMIN = $-h/2$; YMAX = $h/2$;

NUMDIV = Number of subdivisions along x, y directions.

cellX = cell width = (XMAX-XMIN)/NUMDIV
cellY = cell height = (YMAX-YMIN)/NUMDIV
cell indices: i, j = 0, …, NUMDIV-1.

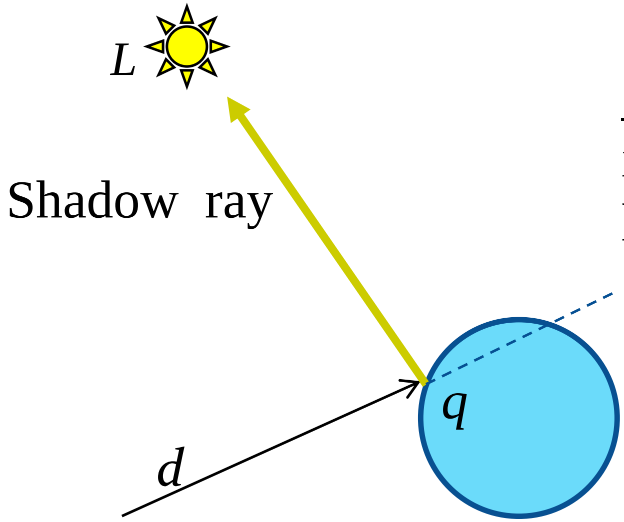
Primary Ray



- Ray position: (0, 0, 0)
- Ray direction *d*: ((XMIN+(i+0.5)*CellX, (YMIN+(j+0.5)*CellY, –EDIST)
- Normalize the above direction.

```
for(int i = 0; i < NUMDIV; i++) {
   xp = XMIN + i*cellX;
   for(int j = 0; j < NUMDIV; j++) {
         yp = YMIN + j*cellY;
         glm::vec3 dir(xp+0.5*cellX, yp+0.5*cellY, -EDIST);
         Ray ray = Ray(eye, dir);
         ray.normalize();
         ...
         ...
```

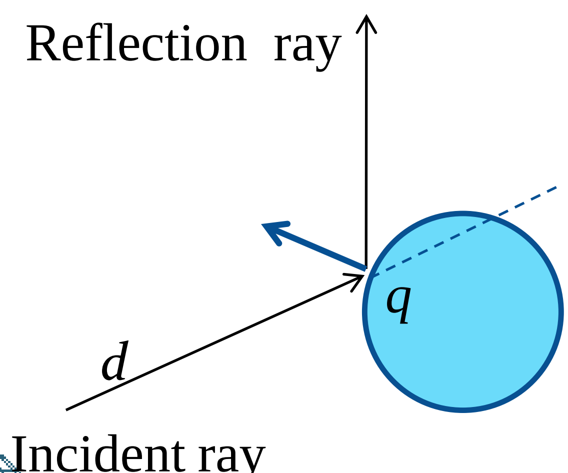
Secondary Rays



<u>Shadow ray</u>: A ray that originates at a point of intersection with an object, and directed towards a light source.

Position = q (point of intersection)

Direction = $L - q$ normalized.



<u>Reflection ray</u>: A ray from the intersection point towards the direction of reflection of the incident ray (Used only for reflective surfaces)

Position = q

Direction:

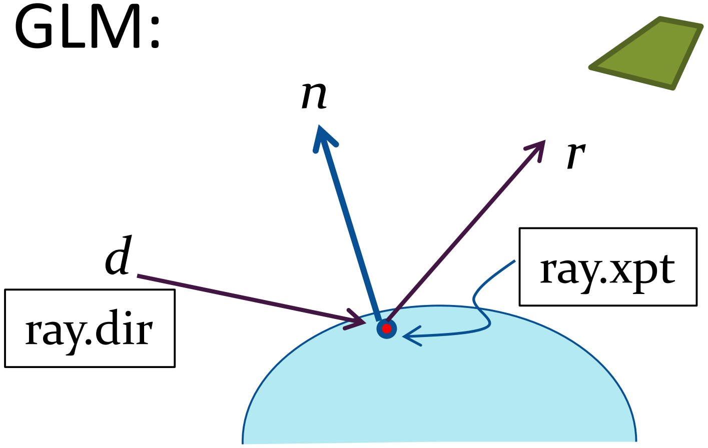
$r = -2(d \bullet n)n + d$ normalized.

(We will use a GLM function to compute r)

Reflections

Computation of reflected ray using GLM:

r = glm::**reflect**(***d***, ***n***);




```
if(ray.xindex == 0 && step < MAX_STEPS)
{
  glm::vec3 refl = glm::reflect(ray.dir, normalVector);
  Ray reflectedRay(ray.xpt, refl);
  glm::vec3 reflCol = trace(reflectedRay, step+1);
  colorSum = colorSum + (0.8f*reflCol);
}
```

Reflections

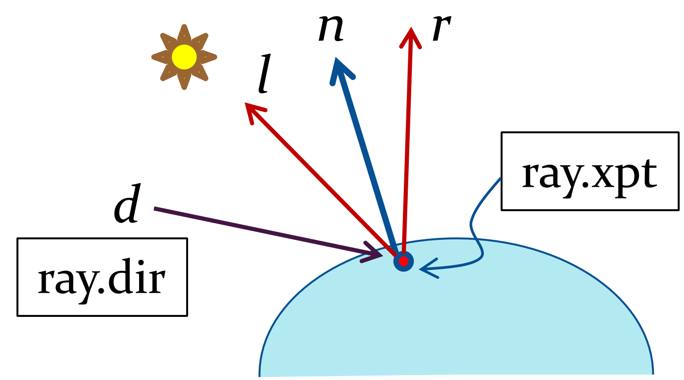
- Computation of specular reflections using GLM:

  r = glm::**reflect**(−***l***, ***n***);

  l: <u>Unit</u> light source vector

  Specular term = $(\mathbf{r}.\mathbf{v})^f$.

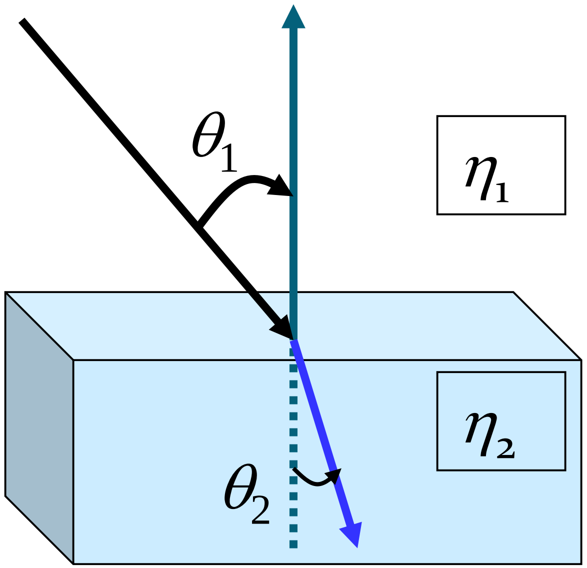
  v: View vector = −***d***



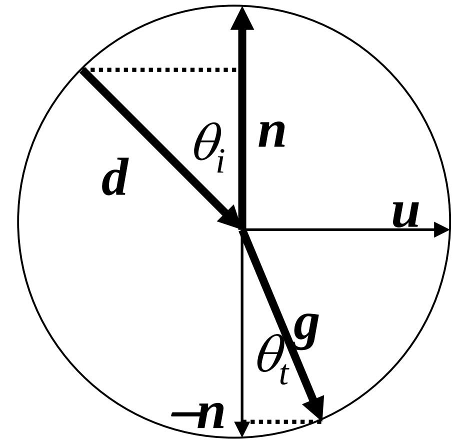

```
glm::vec3 viewVector = -ray.dir;        //View vector
glm::vec3 reflVector = glm::reflect(-lightVector, normalVector);
float rDotv = glm::dot(reflVector, viewVector);
float specularTerm;
if(rDotv < 0) specularTerm = 0.0;
else specularTerm = pow(rDotv, phong);
```

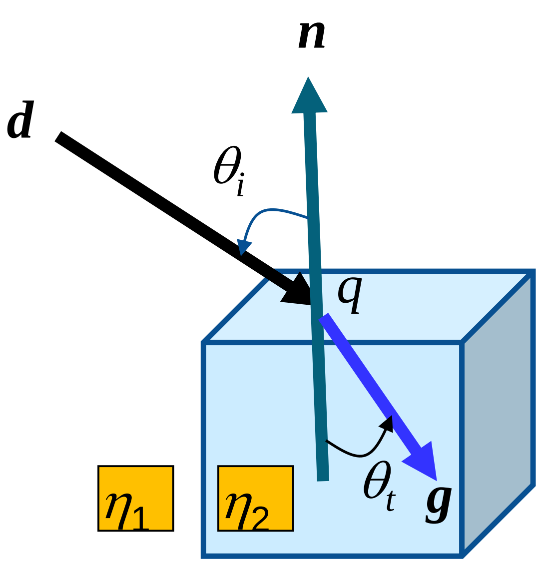
Index of Refraction

- Light travels at speed c/η in a medium with index of refraction η.
- Common values of index of refraction:
  - Air 1.
  - Water 1.33
  - Glass 1.5
  - Diamond 2.4
- Snell's Law of Refraction:
  - $\eta_1 \sin\theta_1 = \eta_2 \sin\theta_2$



Refracted Ray



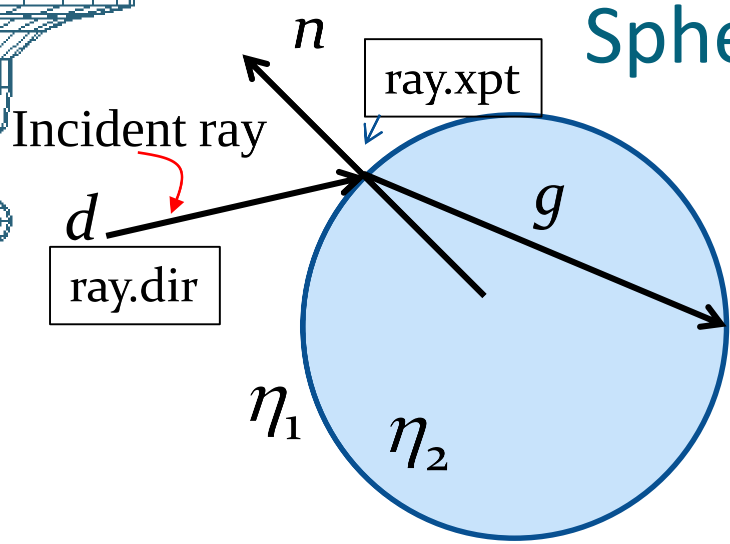


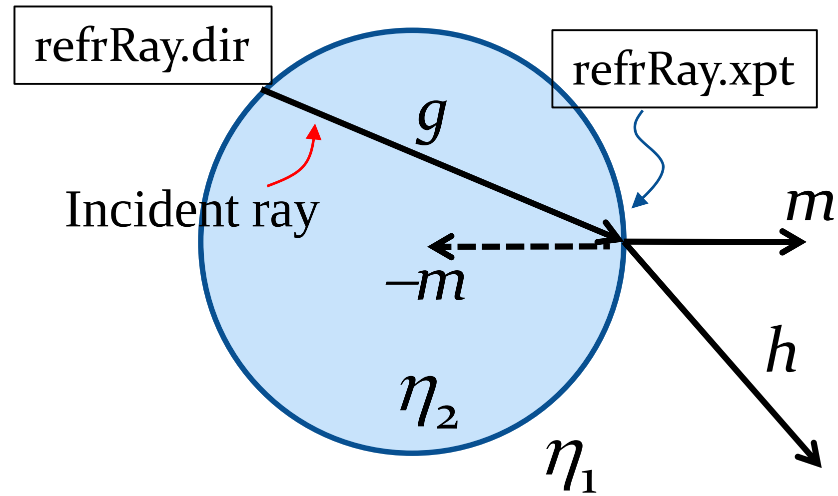
$$\boxed{\begin{array}{l} \boldsymbol{u}\sin\theta_i - \boldsymbol{n}\cos\theta_i = \boldsymbol{d} \\ \boldsymbol{u}\sin\theta_t - \boldsymbol{n}\cos\theta_t = \boldsymbol{g} \end{array}}$$

$$\boldsymbol{g} = \left(\frac{\eta_1}{\eta_2}\right)\boldsymbol{d} - \left(\frac{\eta_1}{\eta_2}(\boldsymbol{d.n}) + \cos\theta_t\right)\boldsymbol{n}$$

$$\cos\theta_t = \sqrt{\left(1 - \left(\frac{\eta_1}{\eta_2}\right)^2 \left(1 - (\boldsymbol{d.n})^2\right)\right)}$$

Sphere Refraction





n = sceneObjects[ray.xindex]->normal(ray.xpt);

g = glm::**refract**(***d***, ***n*,** eta);

Ray refrRay(ray.xpt, ***g***)

refrRay.closestPt(sceneObjects);

m = sceneObjects[refrRay.xindex]->normal(refrRay.xpt);

h = glm::**refract**(***g***, –***m*,** 1.0f/eta);

GLM Functions for Ray Tracing

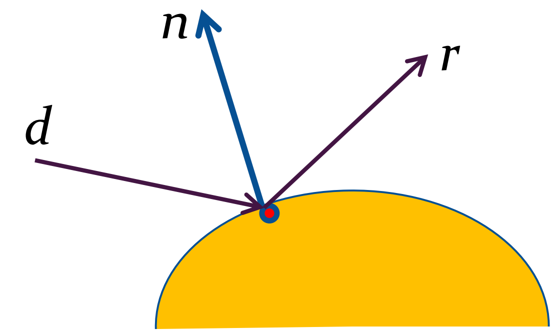
- Reflection:

  r = glm::**reflect**(***d***, ***n***);

  d: Unit incident vector

  n: Unit normal vector

  The reflection vector also is a unit vector.



- Refraction:

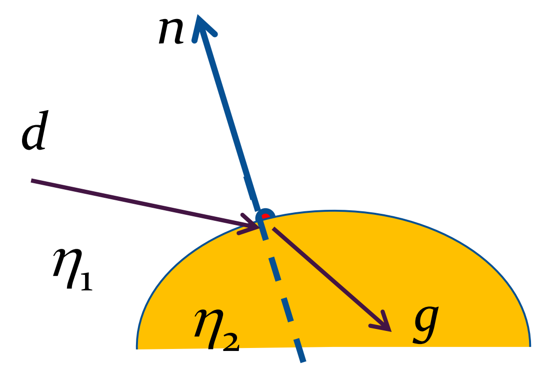
  g = glm::**refract**(***d***, ***n*****,** eta);

  d: Unit incident vector

  n: Unit normal vector

  The refraction vector also is a unit vector.

  eta = Ratio of refractive indices = η_1/η_2



Sphere Refraction

eta = 1/1.5

Sphere Refractions

eta = 1/1.01

eta = 1/1.003

Transparency and refractions

$\mu = 1.01$

Better

The spheres are also reflective

Better

With anti-aliasing

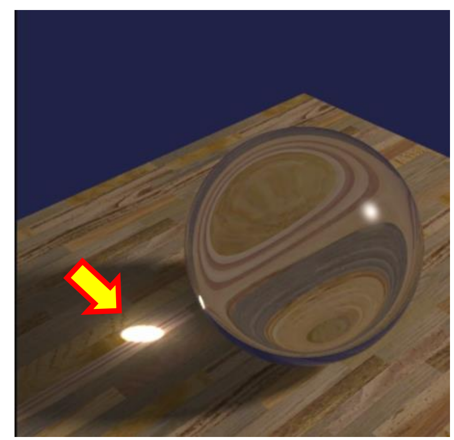
Shadows

Bad

Good

Better

Note: A concentration of light following refraction through a surface is known as a **caustic**. Caustics cannot be generated using simple shadow rays.



Ray-plane intersection

- Given a point a on a plane, and the normal vector n of the plane, the plane's equation can be written as

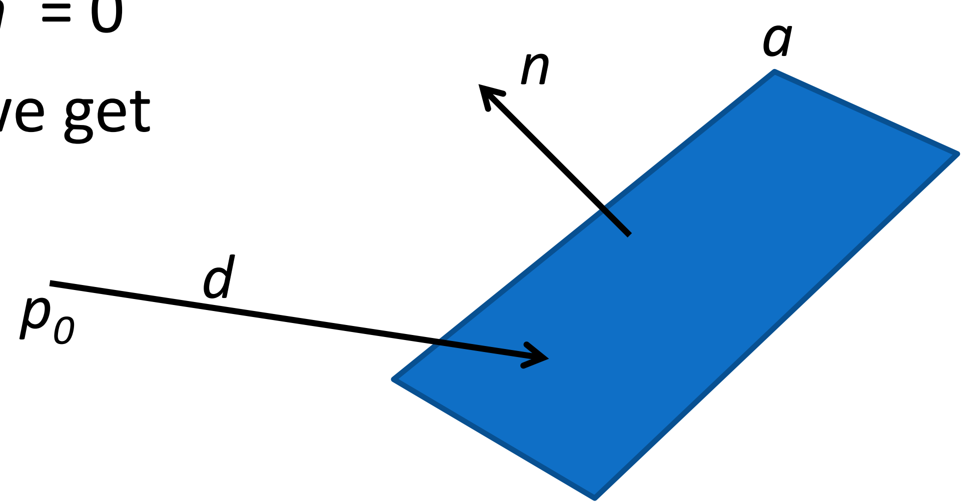
  $(p - a) \bullet n = 0$

- A ray is given by the equation

  $p = p_0 + t\, d.$

- At the point of intersection, both equations are true. Therefore, $(p_0 + t\, d - a) \bullet n = 0$
- From the above equation, we get

$$t = \frac{(a - p_0) \bullet n}{d \bullet n}$$



Ray-sphere intersection

- Equation of a sphere centred at C with radius r is

  $(p - C) \bullet (p - C) = r^2$

- Consider a ray given by the equation

  $p = p_0 + t\,d.$

- At the point of intersection, both equations are true.

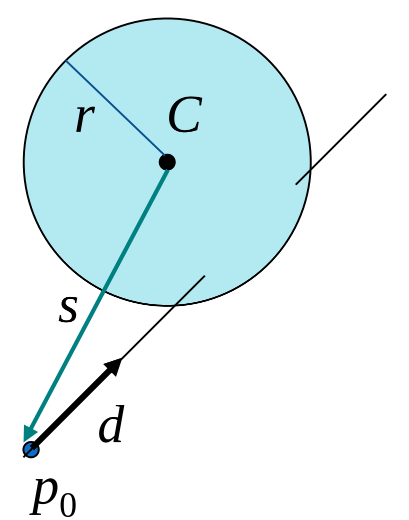
Therefore, $(p_0 + t\,d - C) \bullet (p_0 + t\,d - C) = r^2$

$(s + t\,d) \bullet (s + t\,d) = r^2$, where $s = p_0 - C$

$(d \bullet d)\, t^2 + 2\,(s \bullet d)\, t + (s \bullet s) - r^2 = 0.$

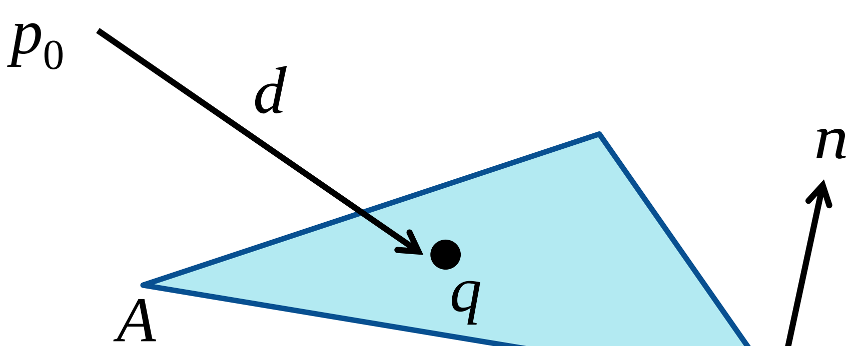
Since d is a unit vector, we get

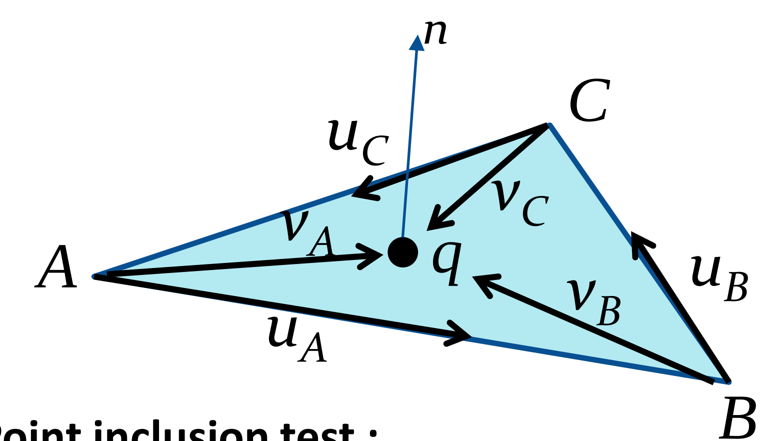
$$t = -(s \bullet d) \pm \sqrt{(s \bullet d)^2 - (s \bullet s) + r^2}$$



Ray intersection with polygonal objects

- Use the plane equation of each triangle to get the point of intersection q with the ray.
- Check if the point of intersection q lies within the triangle. This is done using a simple point inclusion test.





Intersection :

$$t = \frac{(A - p_0) \cdot n}{d \cdot n}$$

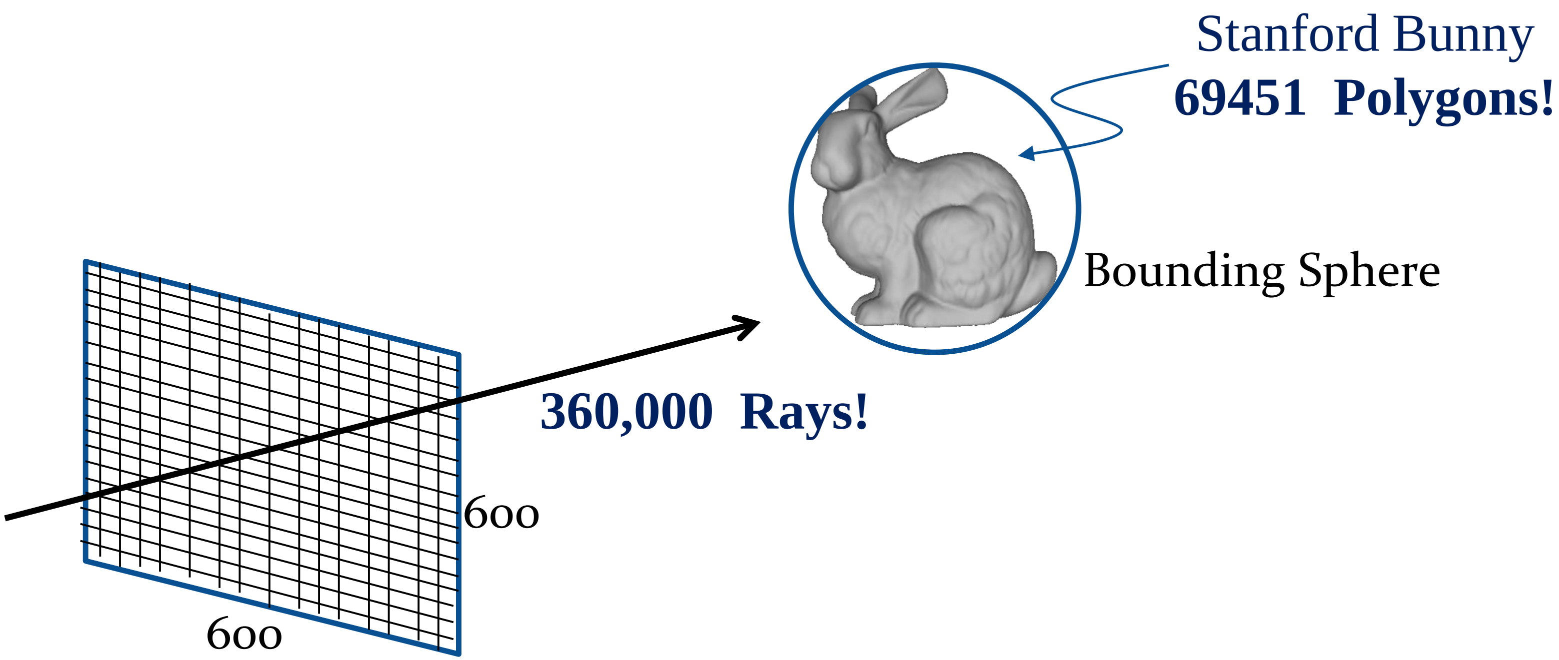
$$q = p_0 + t\, d$$

Point inclusion test :

If the cross products $(u_A \times v_A).n$, $(u_B \times v_B).n$, $(u_C \times v_C).n$ have the same sign, then the point q is inside the triangle.

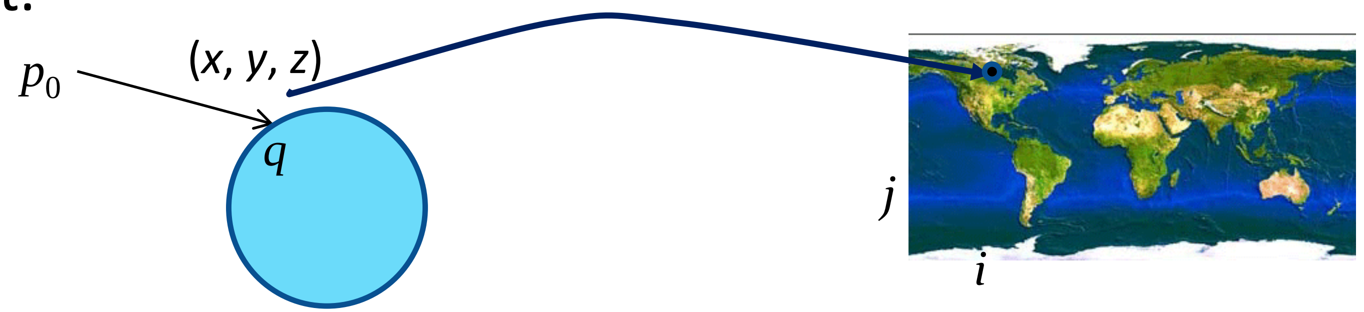
Ray intersection with polygonal objects

- Complex polygonal objects will require a large amount of ray-triangle intersection tests.
- Bounding volume hierarchies and spatial subdivision methods (*k*d-Trees, Octrees) are used to reduce the number of ray-primitive intersection tests.



Texture Mapping

- 2D texturing:
  - Similar to OpenGL texturing, but does not use texture coordinates or texture memory.
  - Map the coordinates of the point of intersection (*x*, *y*, *z*) to image coordinates, and assign the colour of the pixel to that point.



- Procedural texturing:
  - Define functions to map (*x*, *y*, *z*) coordinates to (*r*, *g*, *b*) colour values. $r = r(x, y, z)$; $g = g(x, y, z)$; $b = b(x, y, z)$;

Texture Mapping

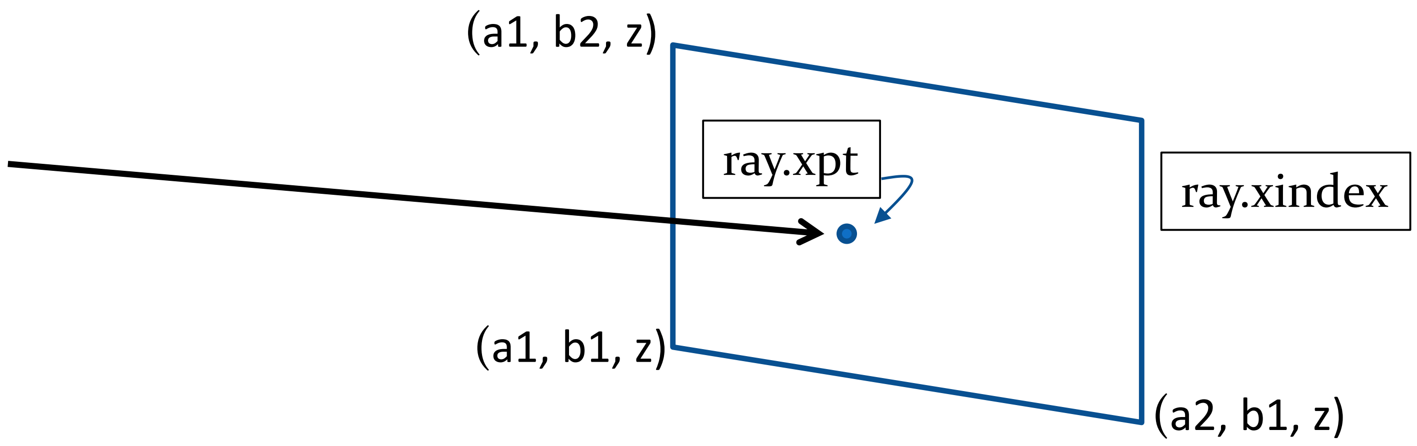
```
...
#include "Ray.h"
#include "TextureBMP.h"
#include <GL/glut.h>
using namespace std;
...
TextureBMP texture;
...
void initialize()
{
   ...
   texture = TextureBMP("myPicture.bmp");
}
...
glm::vec3 trace(Ray ray, int step)
{
   ...
   col = texture.getColorAt(s, t);
}
```

Lab08

TextureBMP.h
TextureBMP.cpp

Note:
$0 \le s \le 1$
$0 \le t \le 1$

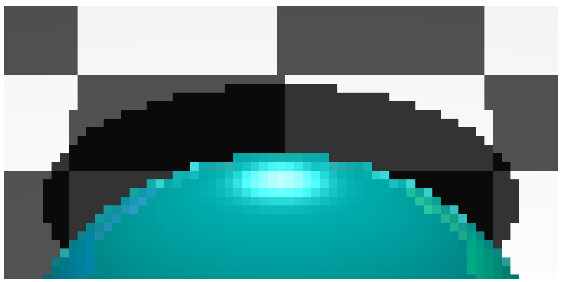
Texturing a Plane



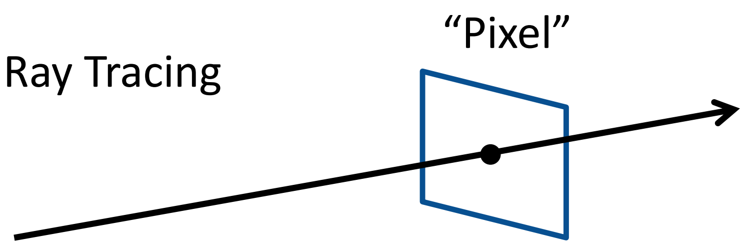

```
if(ray.xindex == 3)   //Assuming the plane to be textured has index 3
{
   texcoords = (ray.xpt.x - a1)/(a2-a1);
   texcoordt = (ray.xpt.y - b1)/(b2-b1);
   col = texture.getColorAt(texcoords, texcoordt);
}
```

Anti-Aliasing

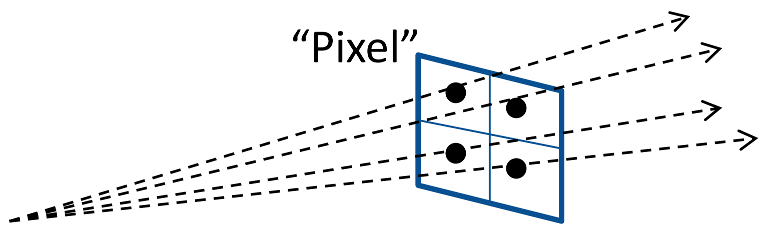
The ray tracing algorithm samples the light field using a finite set of rays generated through a discretized image space. This results in distortion artefacts such as jaggedness along edges of polygons and shadows.



“Normal” Ray Tracing

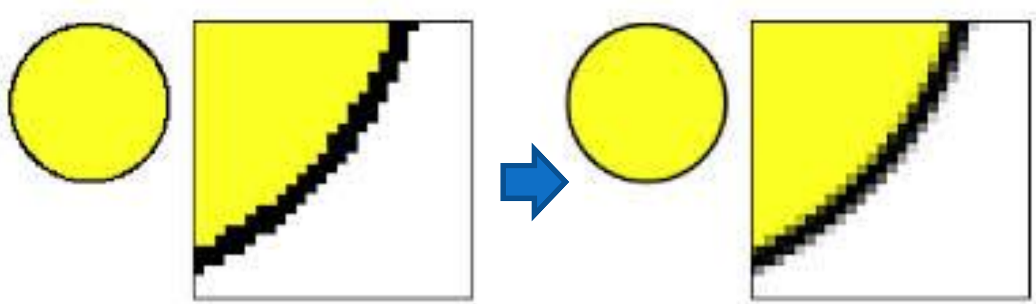


- Supersampling: Generate several rays through each square pixel (eg. divide the pixel into four equal segments) and compute the average of the colour values.

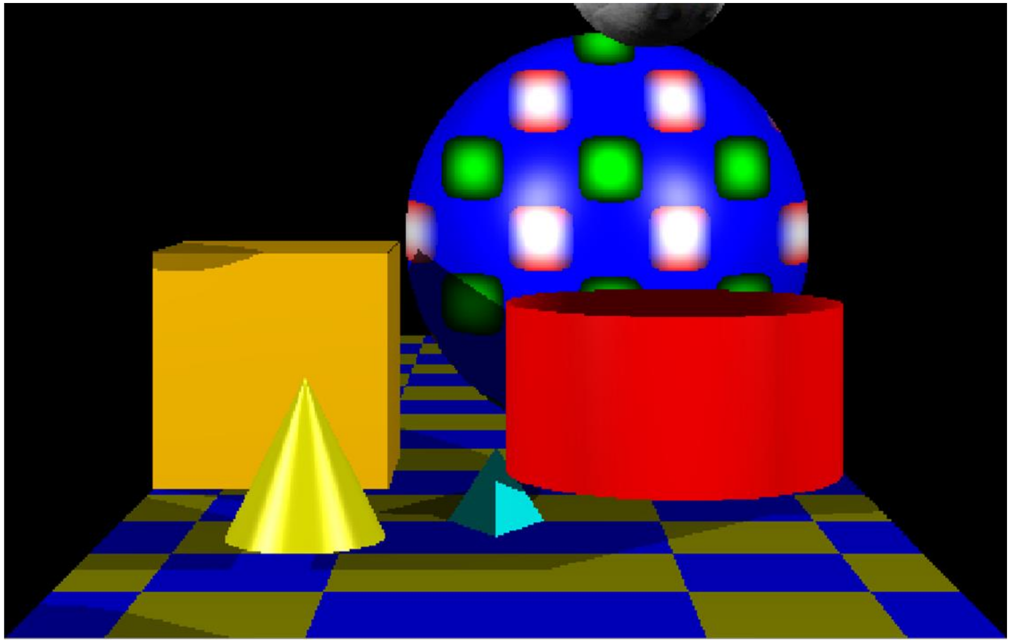


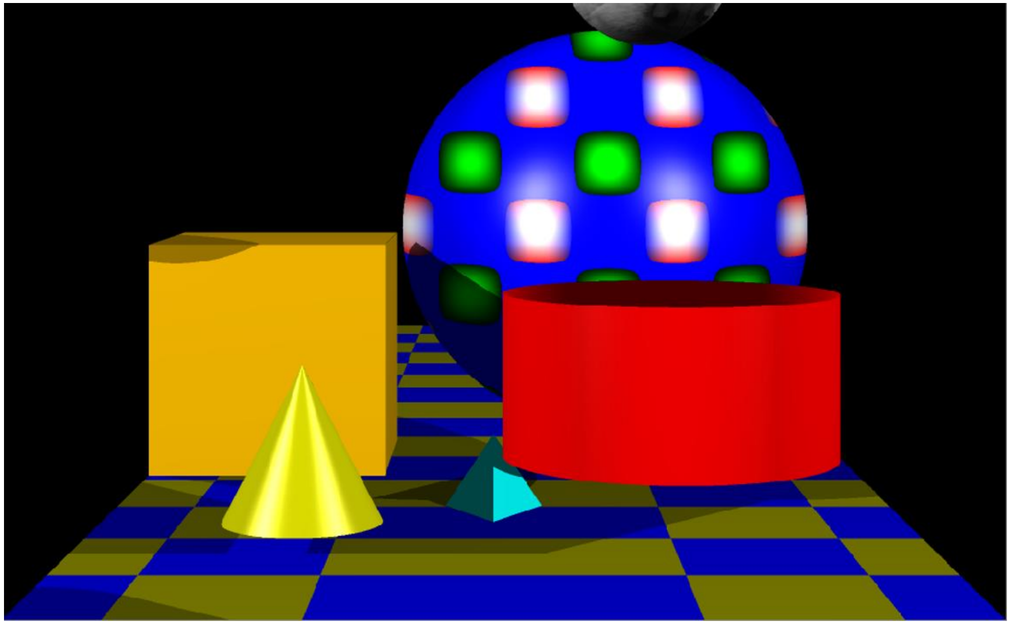
Anti-Aliasing

- Adaptive Sampling: As shown on the previous slide, each pixel is divided into four “sub-pixels”. Primary rays are generated through the centres of each sub-pixel. If the colour value along any ray varies significantly from the other three, that sub-pixel is split further into four sub-pixels, and more rays are generated through them.



Anti-Aliasing Example





Cylinder

- A cylinder at the origin with axis along the *y*-axis, radius *R* and height *h* is given by

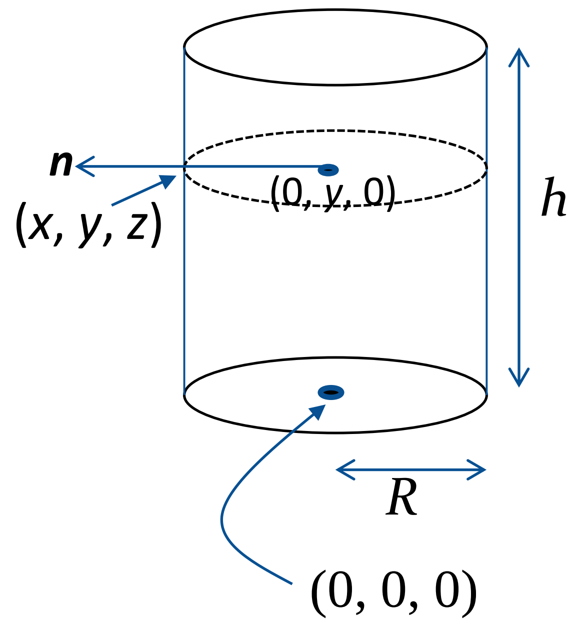
  $x^2 + z^2 = R^2$

  $0 \le y \le h$

- Normal vector at (x, y, z)

  (un-normalized) $n = (x, 0, z)$

  Normalized $n = (x/R,\ 0,\ z/R)$

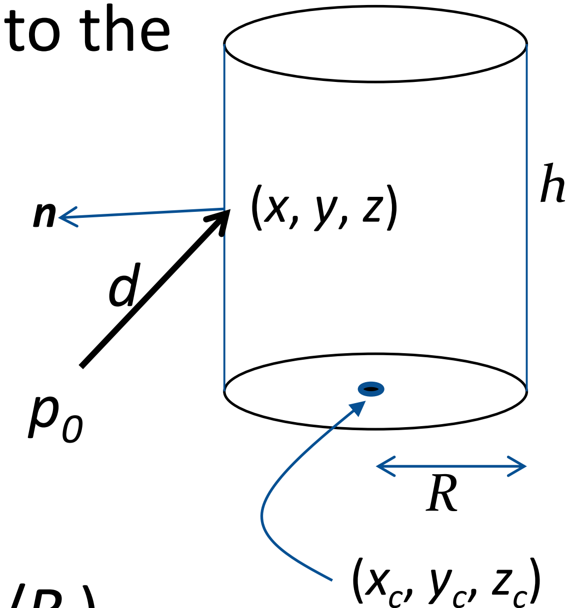


Ray – Cylinder Intersection

- A cylinder at (x_c, y_c, z_c), with axis parallel to the y-axis, radius R and height h is given by

  $(x - x_c)^2 + (z - z_c)^2 = R^2$

  $0 \le (y - y_c) \le h$



- Normal vector at (x, y, z)

  (un-normalized) $\boldsymbol{n} = (x - x_c,\ 0,\ z - z_c)$

  (normalized) $\boldsymbol{n} = ((x - x_c)/R,\ \ 0,\ \ (z - z_c)/R)$

Ray equation:

$x = x_0 + d_x\, t;\qquad y = y_0 + d_y\, t;\qquad z = z_0 + d_z\, t;$

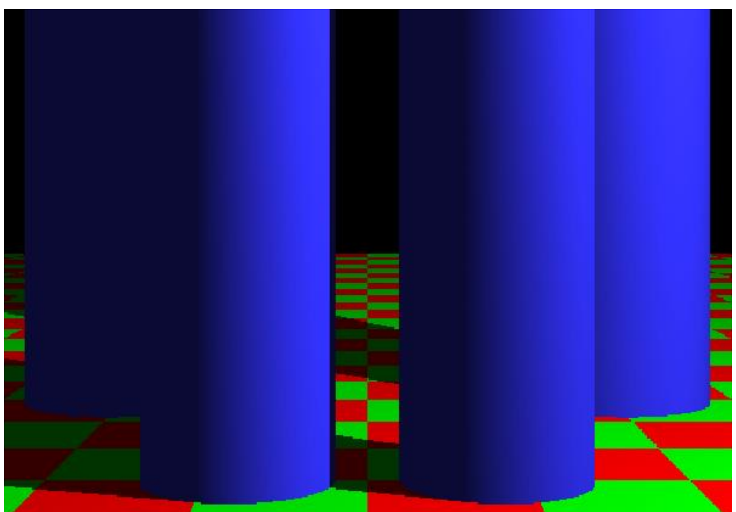
- Intersection equation:

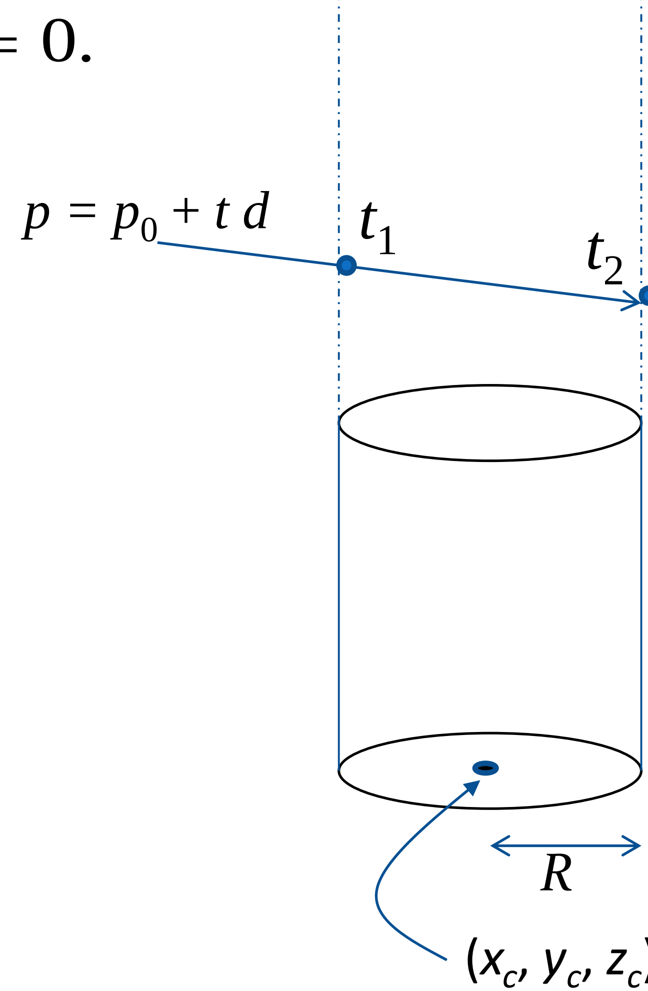
$$t^2\left(d_x^2 + d_z^2\right) + 2t\left\{d_x\left(x_0 - x_c\right) + d_z\left(z_0 - z_c\right)\right\} + \left\{\left(x_0 - x_c\right)^2 + \left(z_0 - z_c\right)^2 - R^2\right\} = 0.$$

Cylinder

$$t^2\left(d_x^2+d_z^2\right)+2t\left\{d_x\left(x_0-x_c\right)+d_z\left(z_0-z_c\right)\right\}$$
$$+\left\{\left(x_0-x_c\right)^2+\left(z_0-z_c\right)^2-R^2\right\}=0.$$

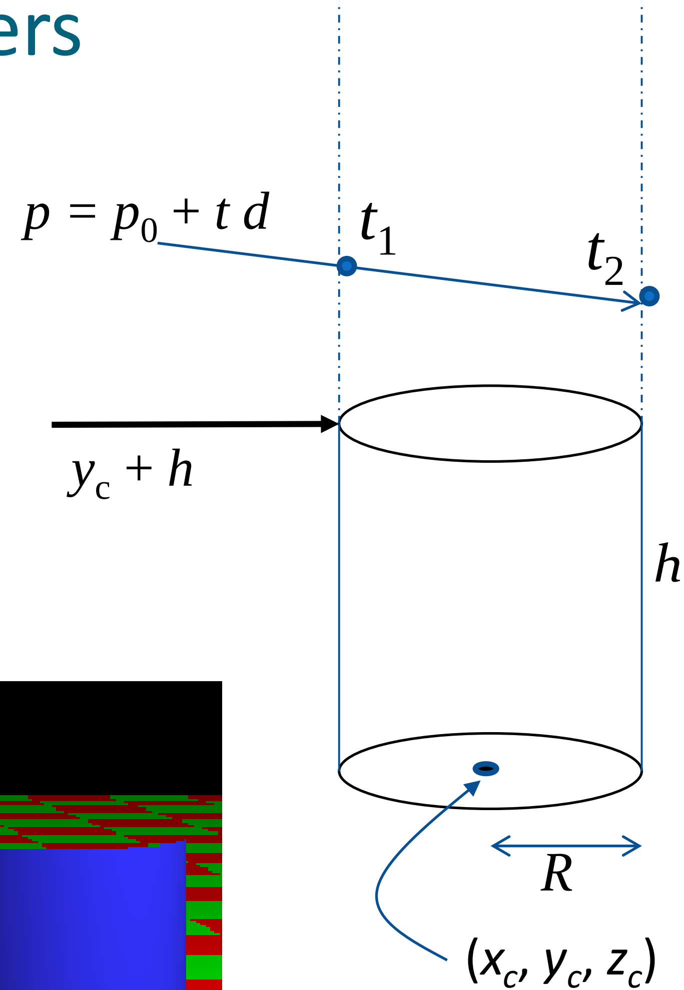
Using only the above equation to find the closest point of intersection gives cylinders of infinite height.

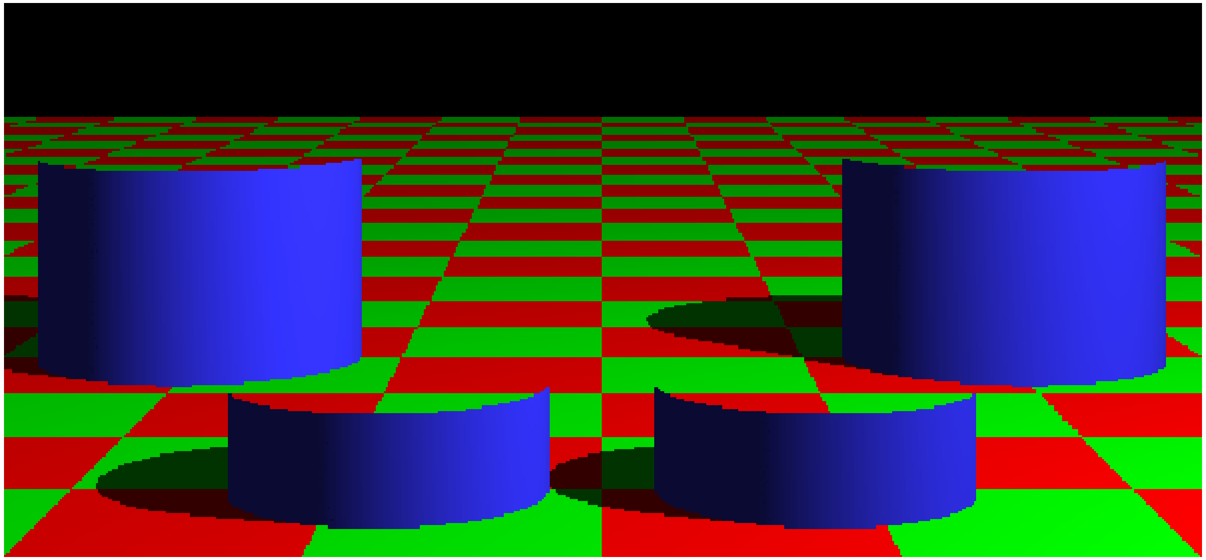




Broken Cylinders

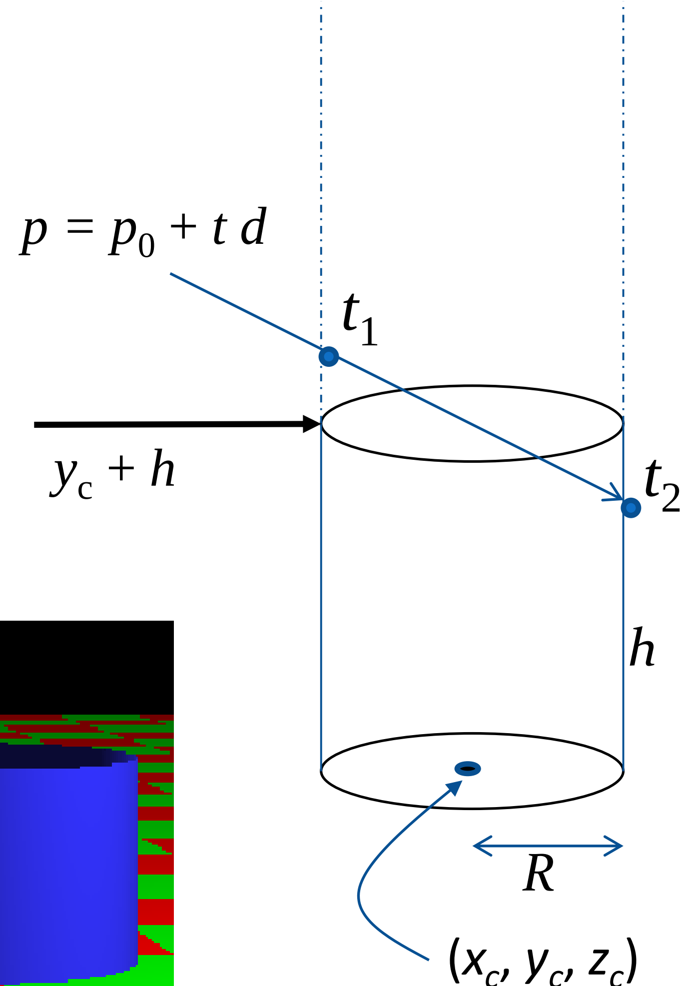
Should the closest point of intersection with the cylinder be discarded if it has a *y*-coordinate value greater than $y_c + h$?

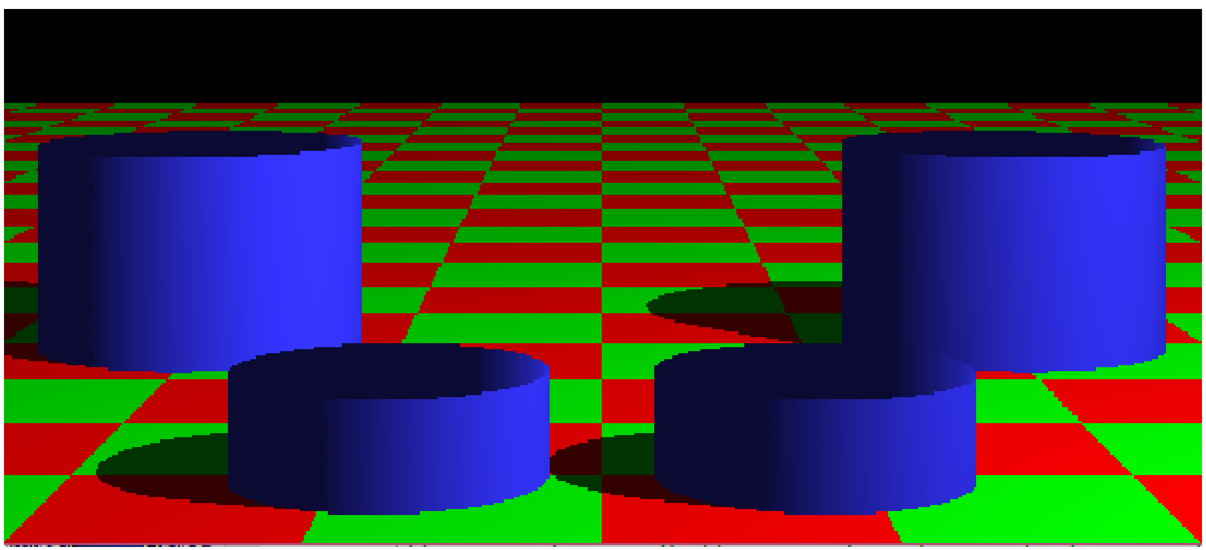




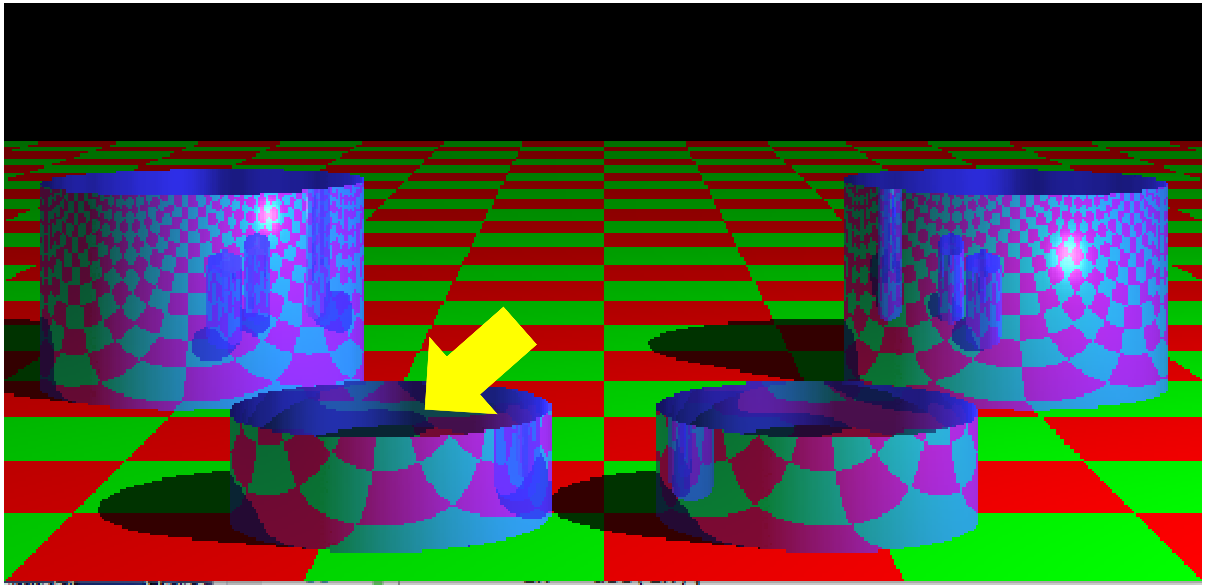
Cylinders

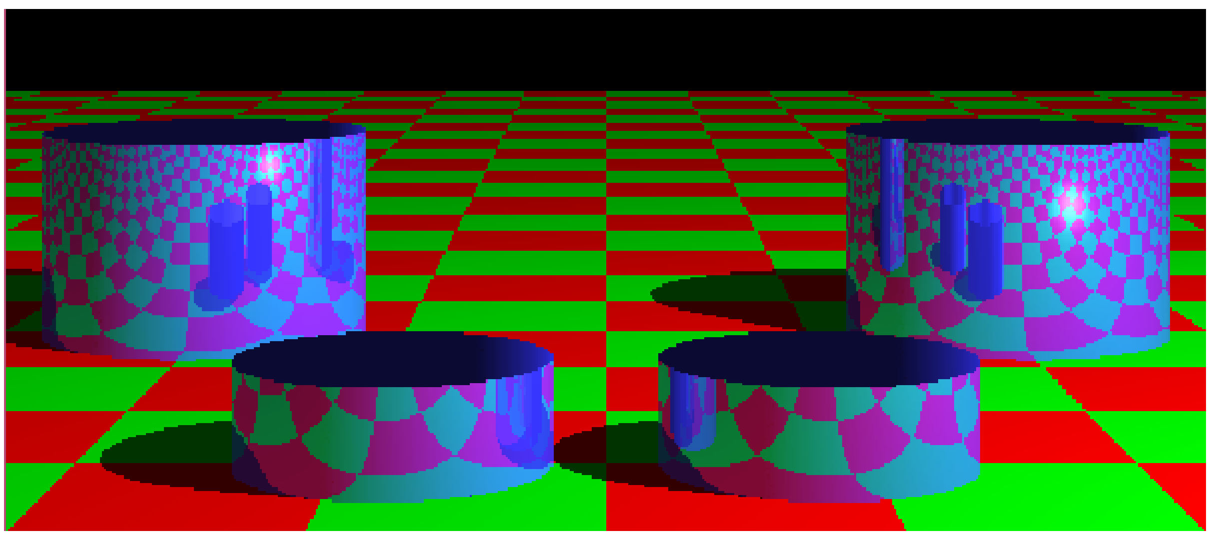
If the closest point of intersection is outside the cylinder, we should check if the second point of intersection is within the cylinder, and if so, return that value.





Cylinders with reflections



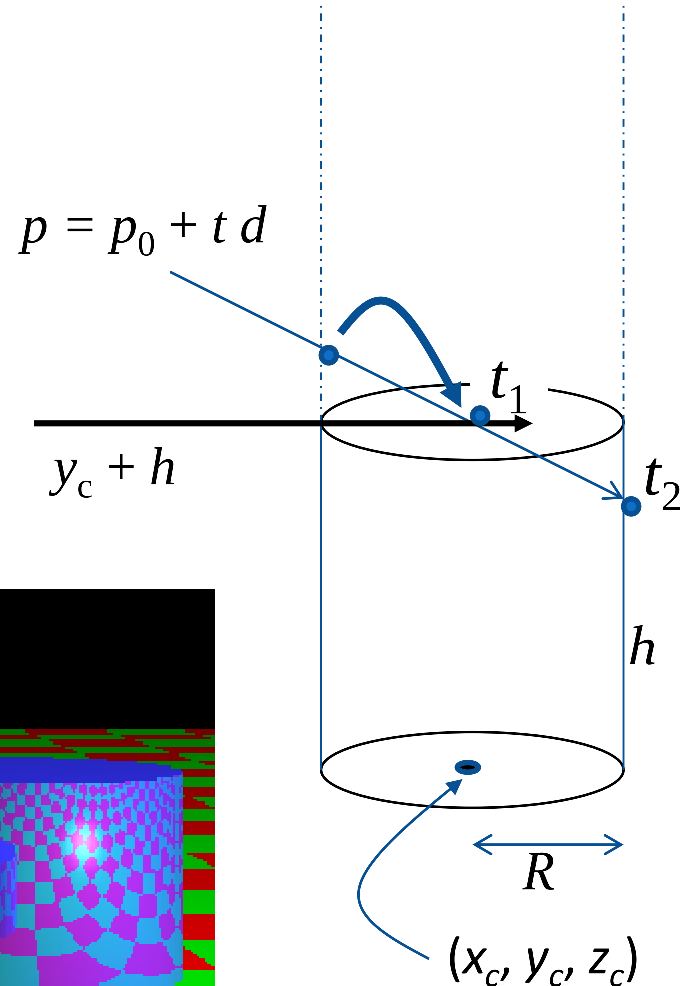


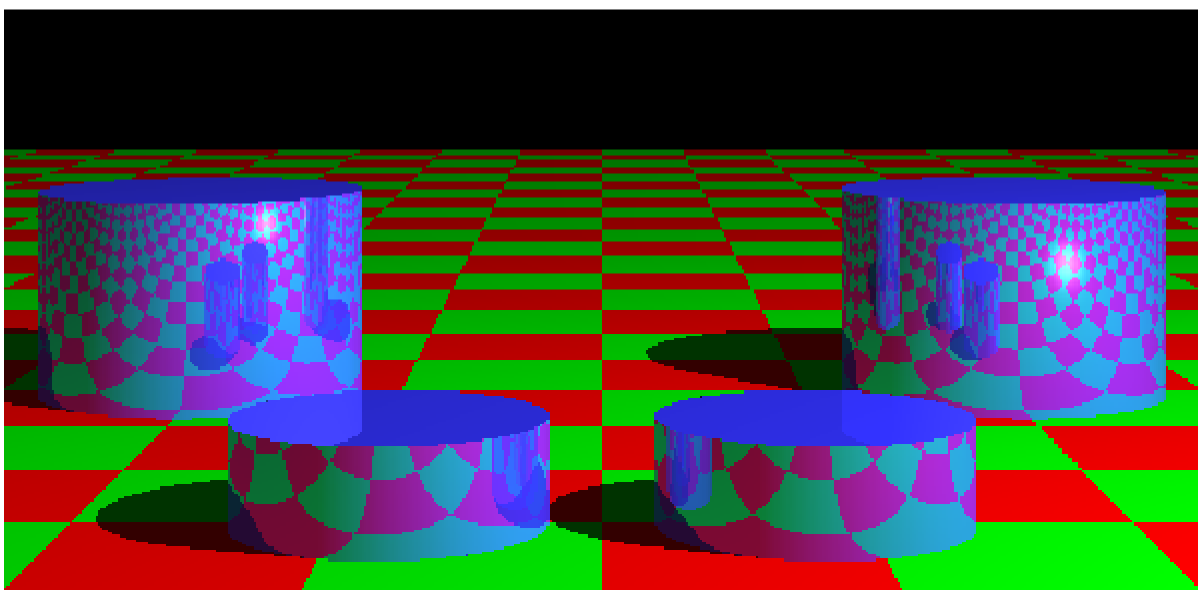
Better

Compute reflected colour only if normal.z > 0

Cylinders with cap

If the closest point of intersection is above the cylinder and the next point of intersection within the cylinder, we could compute the point on the ray which lies on the cap and return the corresponding *t* value.





Cone

- Consider a cone with centre of base at (x_c, y_c, z_c), axis parallel to the y-axis, radius R, and height h:
- Important equations:

  $\tan(\theta) = R/h$ (θ = half cone angle)

  For any point (x, y, z) on the cone,

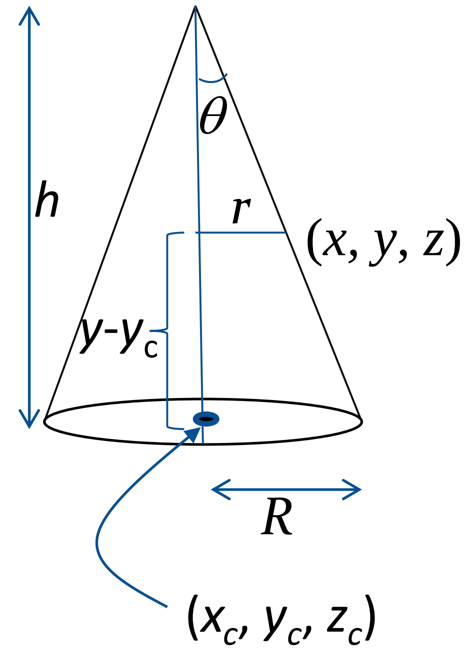
  $(x - x_c)^2 + (z - z_c)^2 = r^2$

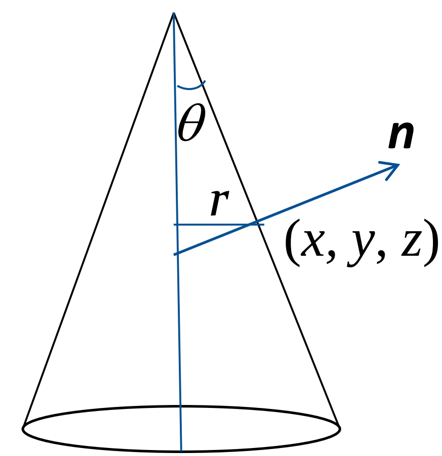
  where, $r = \left(\frac{R}{h}\right)(h - y + y_c)$

- Surface normal vector (normalized):

  $\boldsymbol{n} = (\sin\alpha\cos\theta, \sin\theta, \cos\alpha\cos\theta)$

  where $\alpha = \tan^{-1}\left(\frac{x - x_c}{z - z_c}\right)$

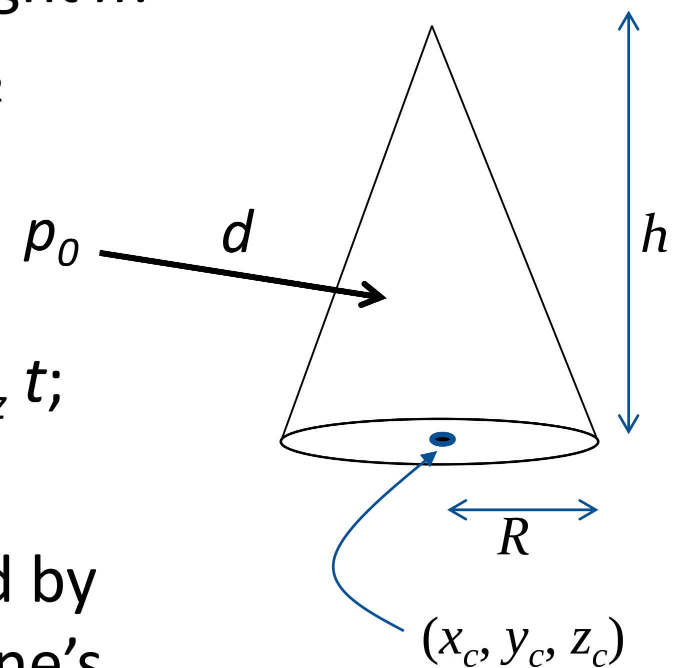




Ray-Cone Intersection

- Equation of a cone with base at (x_c, y_c, z_c), axis parallel to the *y*-axis, radius *R*, and height *h*:

$$(x - x_c)^2 + (z - z_c)^2 = \left(\frac{R}{h}\right)^2 (h - y + y_c)^2$$



- Ray equation:

$$x = x_0 + d_x\, t; \quad y = y_0 + d_y\, t; \quad z = z_0 + d_z\, t;$$

- The points of intersection are obtained by substituting the ray equation in the cone's equation and solving for *t*.

Common Mistakes

- Constructing a chequered pattern on the floor using multiple quads.

- Creating multiple quads for the floor leads to a large number of ray-plane intersection tests.
- A floor must be constructed using a single quad only.

Common Mistakes

- Computing the roots of the quadratic equation incorrectly.

```
float delta = b*b - (4*a*c);

if (fabs(delta) < 0.001) return -1.0;

float t1 = (-b - sqrt(delta)) / 2 * a;
float t2 = (-b + sqrt(delta)) / 2 * a;
```

✘

```
float delta = b*b - (4*a*c);

if (fabs(delta) < 0.001) return -1.0;

float t1 = (-b - sqrt(delta)) / (2 * a);
float t2 = (-b + sqrt(delta)) / (2 * a);
```

✓