

(TEREC) CODING CLUB

Session 1 – 11/03/2024

OVERVIEW

OVERVIEW OF THIS SESSION

- Introduction
- Basics of Bayesian
- Brms
- INLA
- ABC
- Varia





BASICS OF BAYESIAN

Basics of Bayesian

Who has ever worked with Bayesian statistics?

Basics of Bayesian

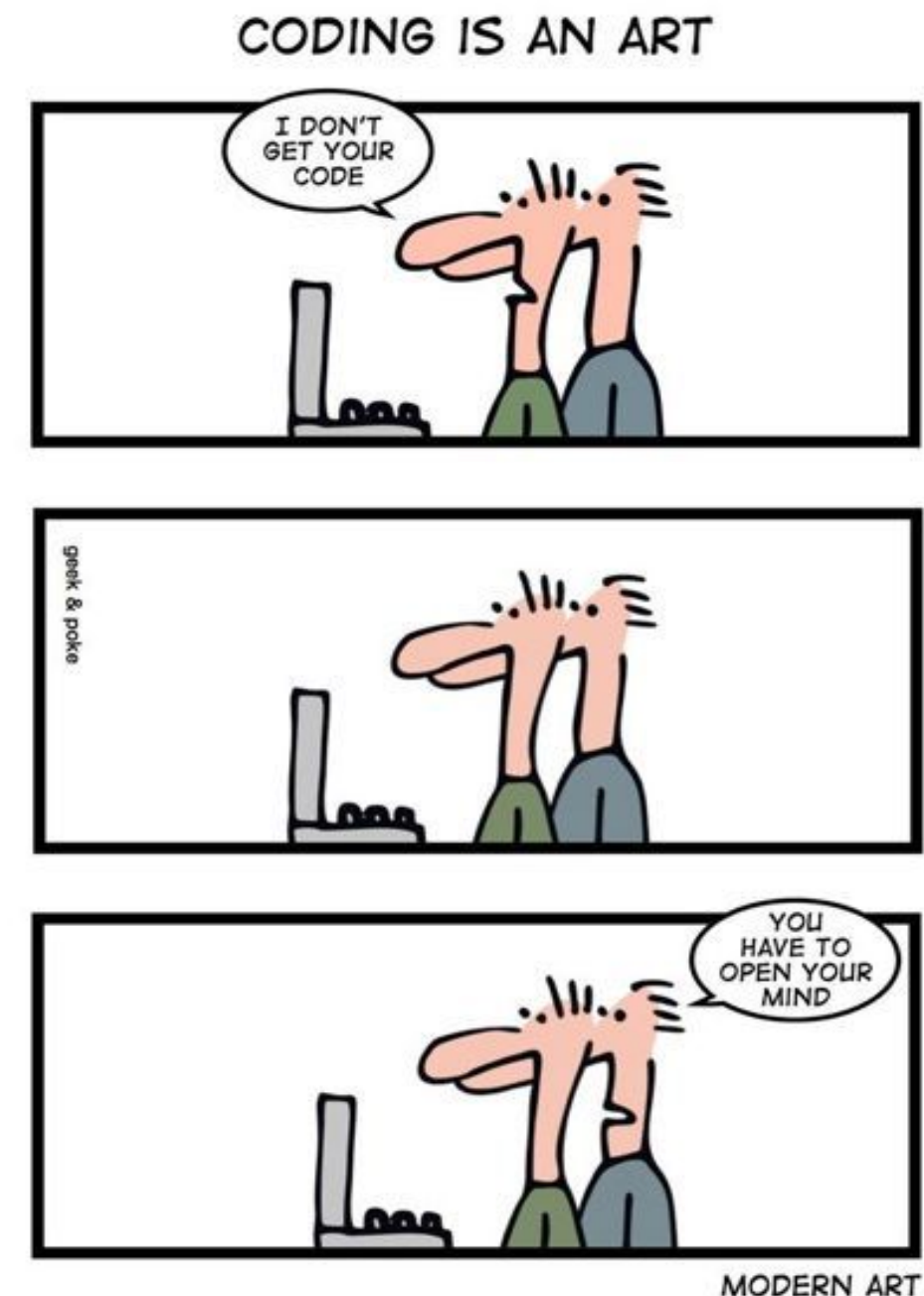
Who has ever worked with Bayesian statistics?

Who can explain what Bayesian is?

Basics of Bayesian

Who has ever worked with Bayesian statistics?

Who can explain what Bayesian is?



Basics of Bayesian

When tossing a coin, we say “chances of *head* are 0,5”.
How to you interprete this?

-*Raise hands on 3-*



Basics of Bayesian

When tossing a coin, we say “chances of *head* are 0.5”.
How to you interprete this?

-*Raise hands on 3-*

Left hand:

When I toss multiple times in a row, it will result in *head* in about 50% of the tosses.

Right hand:

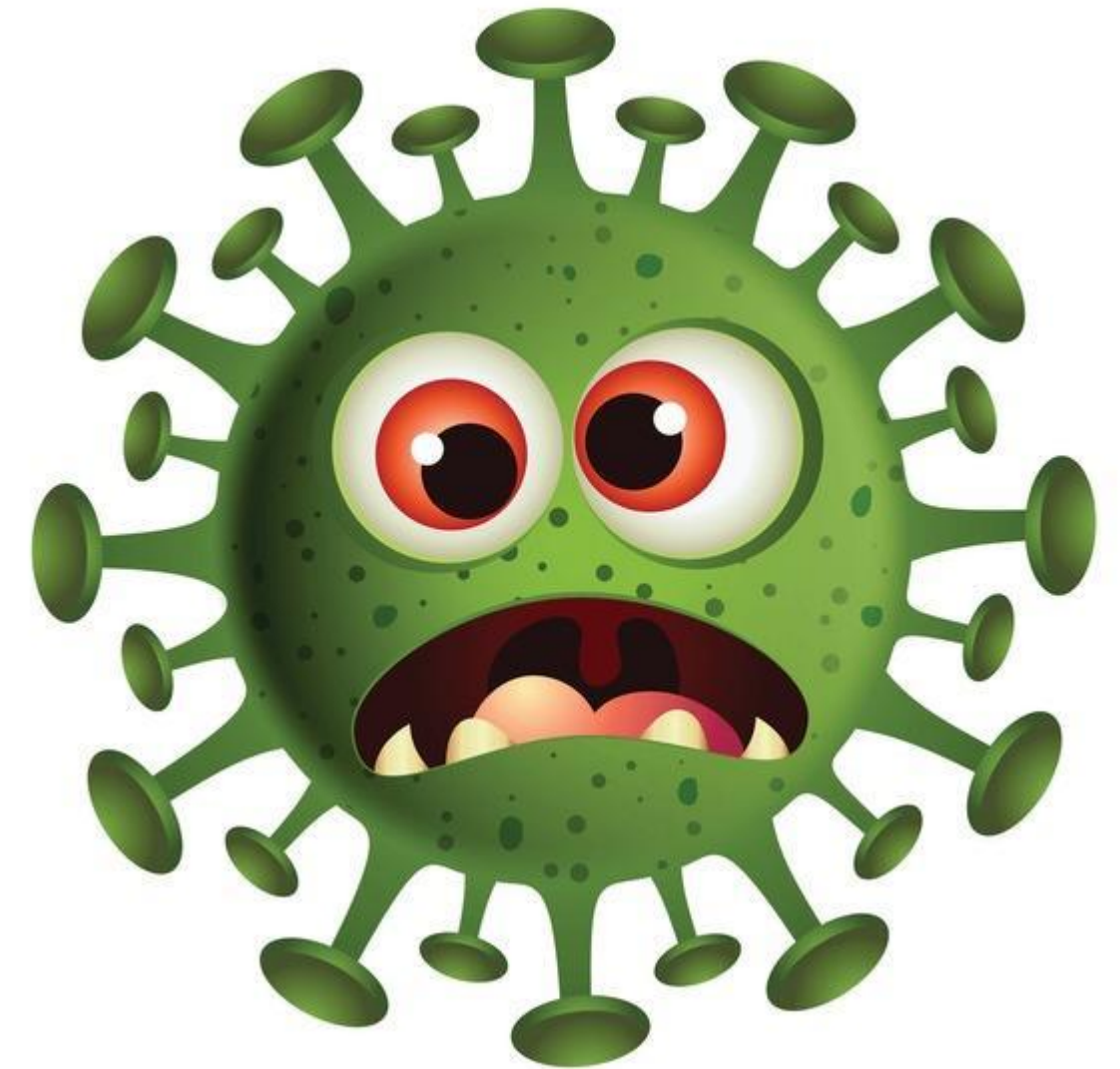
Chances of getting *head* or *coin* are equally probable.



Basics of Bayesian

Imagine going to the doctors. You were tested positive for a very rare disease. You can ask only one question, which would it be?

-Raise hands on 3-



Basics of Bayesian

Imagine going to the doctors. You were tested positive for a very rare disease. You can ask only one question, which would it be?

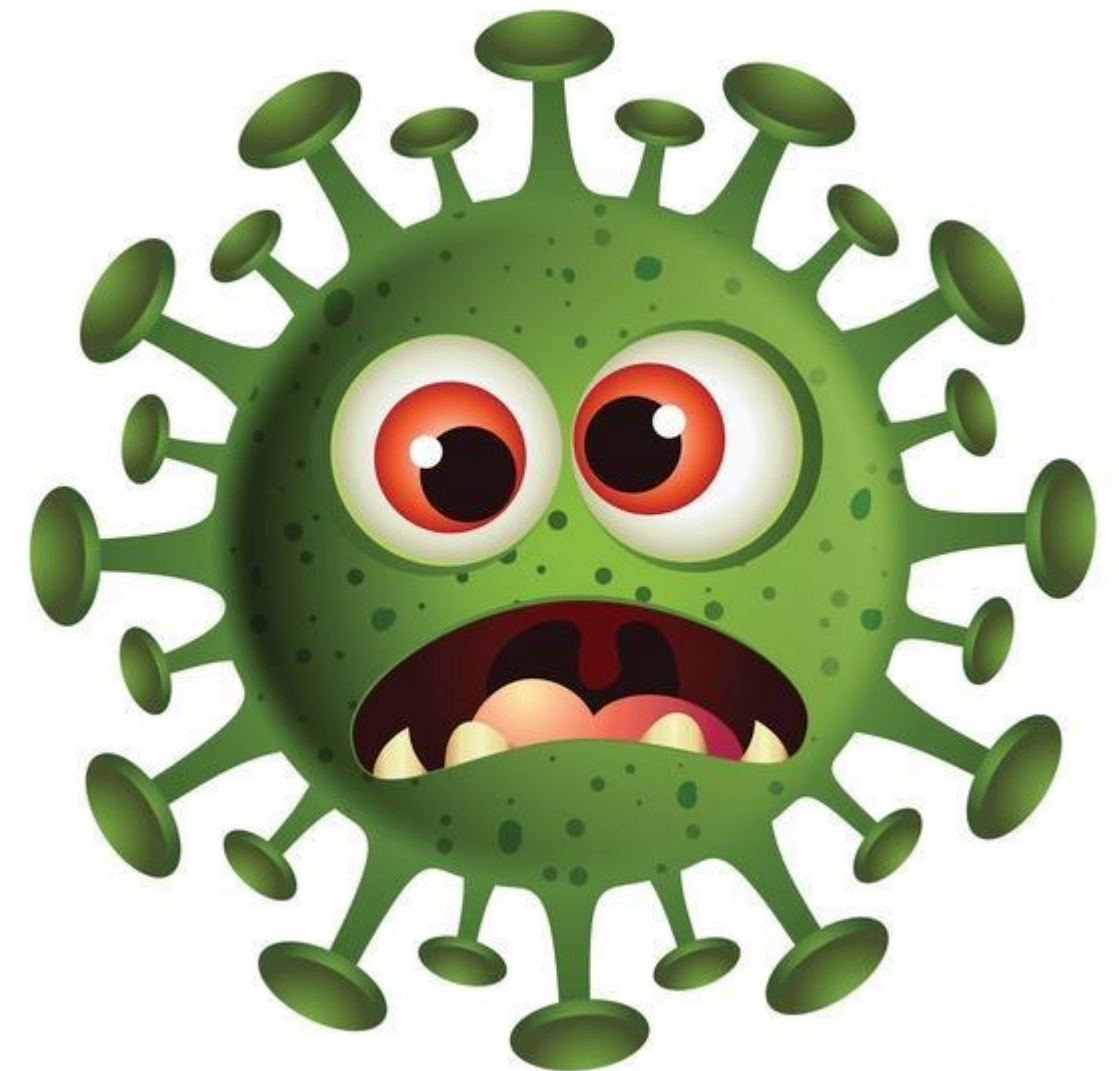
-Raise hands on 3-

Left hand:

If I don't have the disease, what would the chances be on a (false) positive result?

Right hand:

What are the chances I really have this disease?



Basics of Bayesian

What is it?

“Bayesian statistics apply probabilities to statistical problems to update prior beliefs in light of the evidence of new data. The probability expresses a degree of belief in a specific event.”



Basics of Bayesian

What is it?

“Bayesian statistics apply probabilities to statistical problems to update prior beliefs in light of the evidence of new data. The probability expresses a degree of belief in a specific event.”



- Used in physics, cancer research, ecology, psychology...
- Bayesian statistics vs. frequentist methods

Basics of Bayesian

What is it?

Bayesian statistics vs. frequentist methods

A chance or probability ...

□ *Bayesian: ...measures the relative probability of an event*

Basics of Bayesian

What is it?

Bayesian statistics vs. frequentist methods



A chance or probability ...

☐ *Bayesian: ...measures the relative probability of an event*

☐ *Frequentist: ... relative frequency on the long term of repeated events.*

Basics of Bayesian

What is investigated?

□ *Bayesian: What is the chance that a hypothesis is correct, given the data we observe?*

Basics of Bayesian

What is investigated?

- ☐ *Bayesian: What is the chance that a hypothesis is correct, given the data we observe?*
- ☐ *Frequentist: If the hypothesis is incorrect, what is the chance that we observe this or more extreme data?*

Basics of Bayesian

What is investigated?

- *Bayesian: There is prior information and conclusions are continuously adapted as soon as new data is available.*

Basics of Bayesian

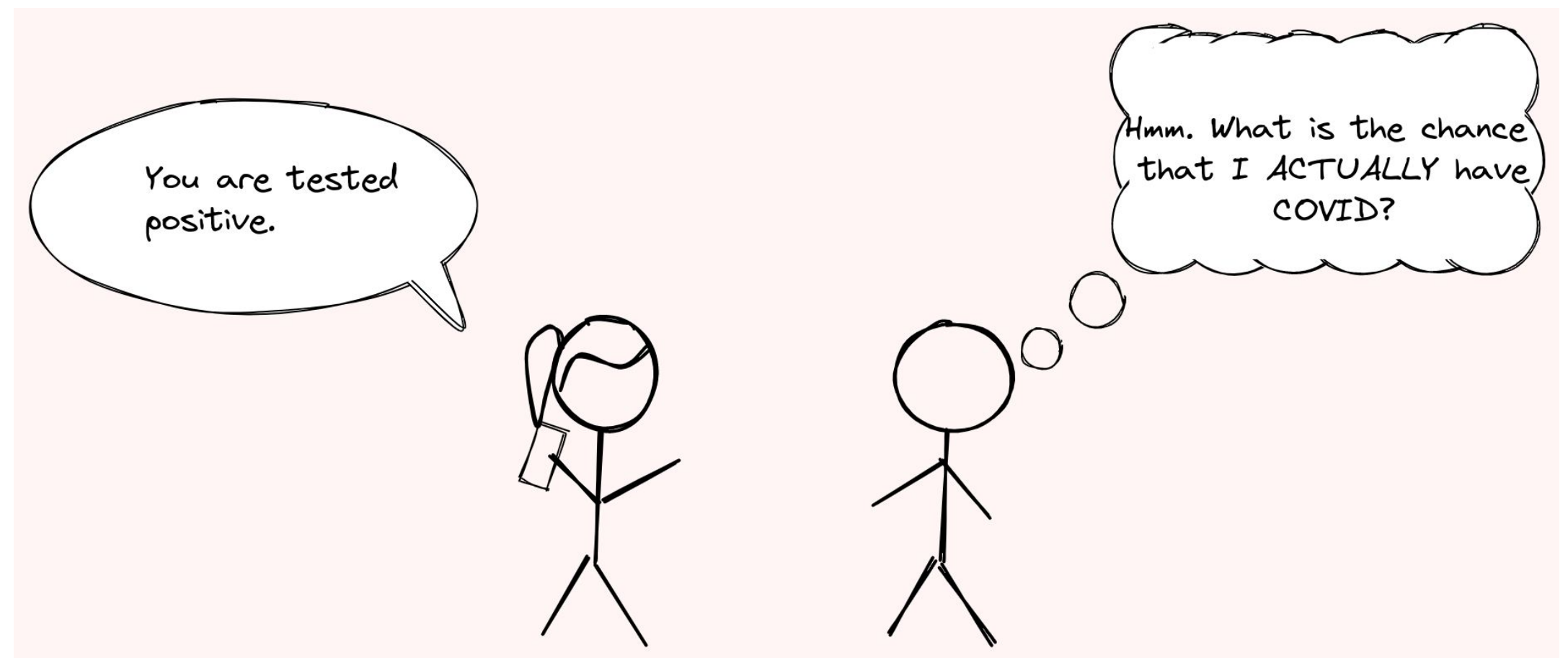
What is investigated?

- *Bayesian: There is prior information and conclusions are continuously adapted as soon as new data is available.*
- *Frequentist: Data is set and observed.*

Basics of Bayesian

Bayesian statistics vs. frequentist methods

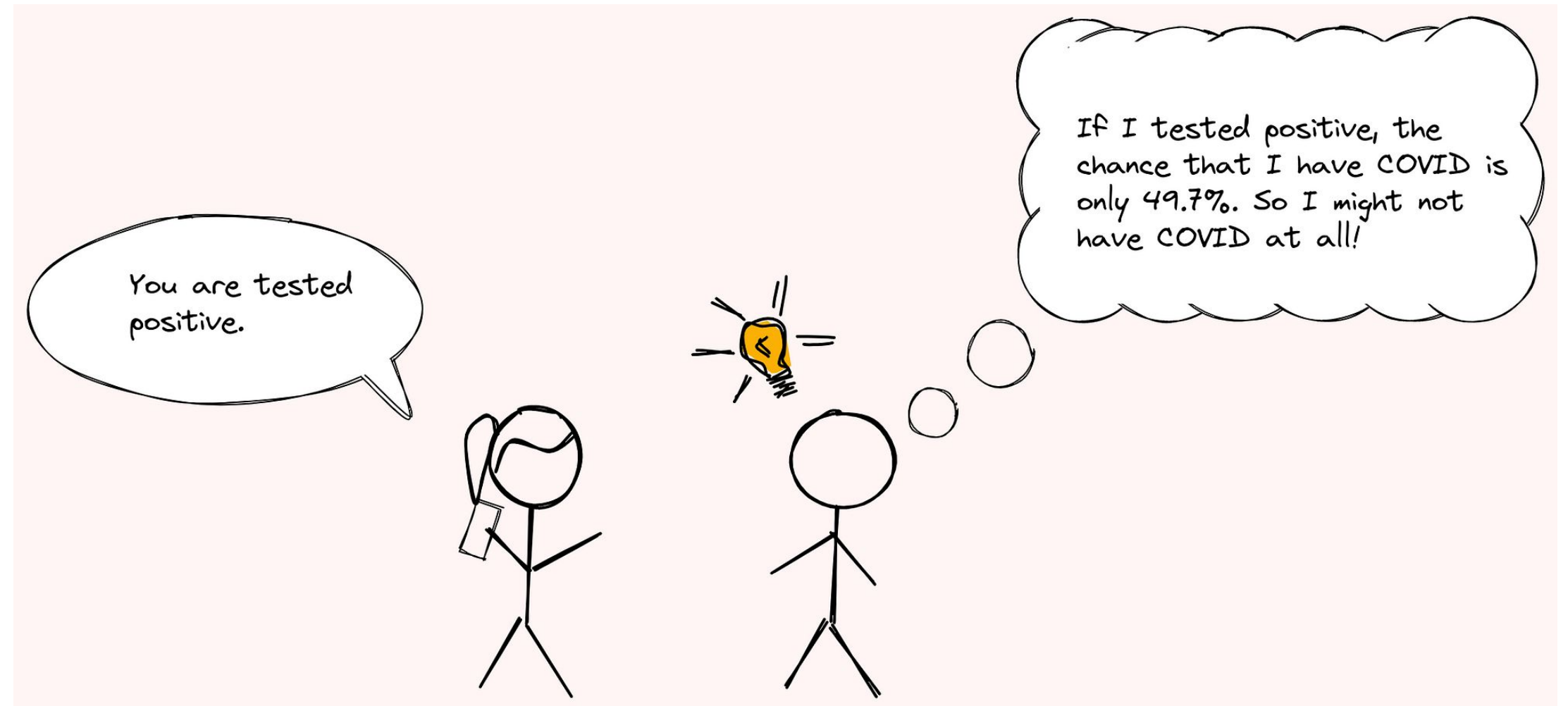
- Knowing these definitions
- Are our brains already Bayesian?
- Bayes' theorem



Basics of Bayesian

Bayesian statistics vs. frequentist methods

- Knowing these definitions
- Are our brains already Bayesian?
- Bayes' theorem



Basics of Bayesian

Terminology

Frequentist

P-value

Confidence interval

Power

Significant

Basics of Bayesian

Terminology

Frequentist

P-value

Confidence interval

Power

Significant

Terminology

Bayesian

Credible interval

Prior

Posterior

(Likelihood)

Basics of Bayesian

Terminology

Likelihood

Often associated with Bayesian

Helps to understand Bayesian

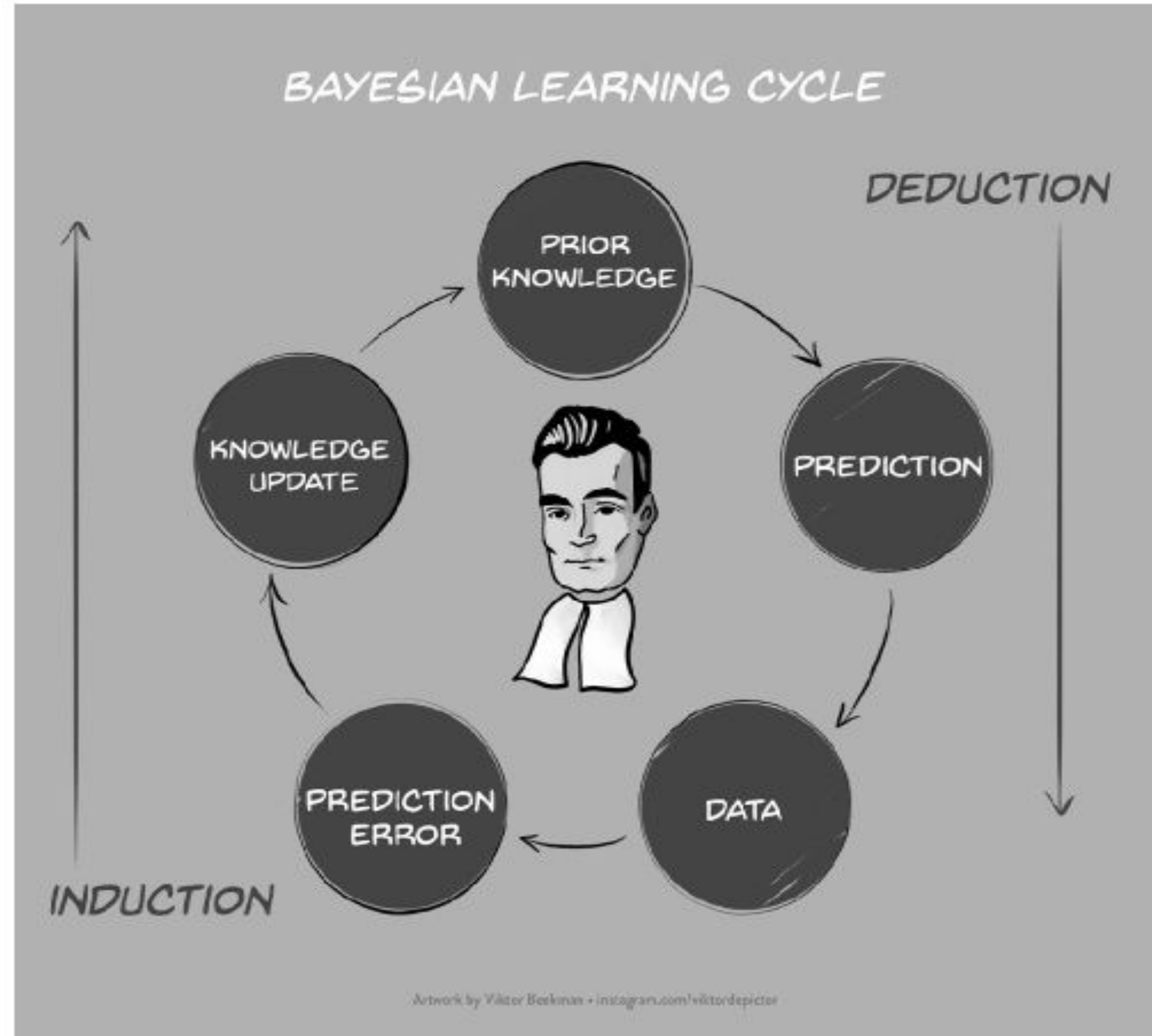
Not only Bayesian!



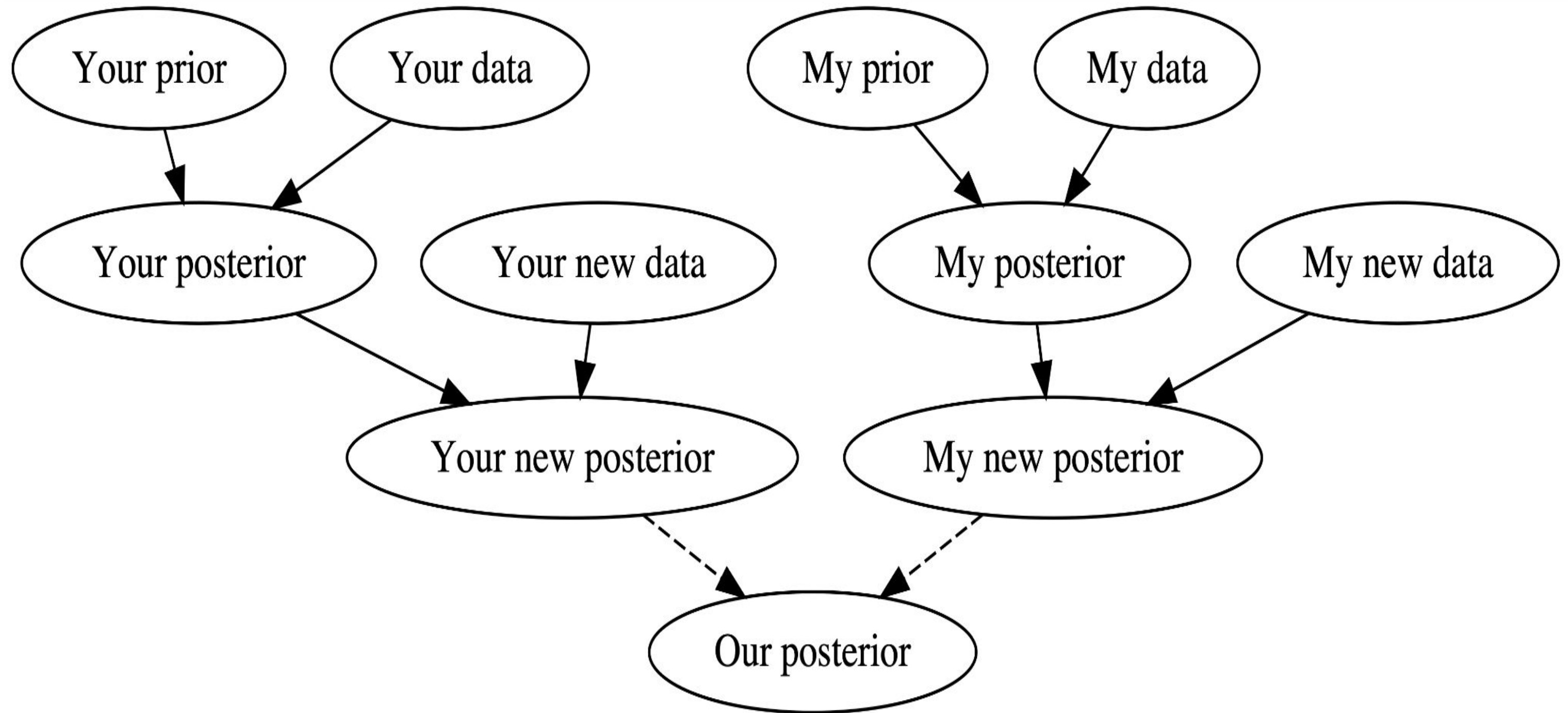
Basics of Bayesian

Terminology

Prior



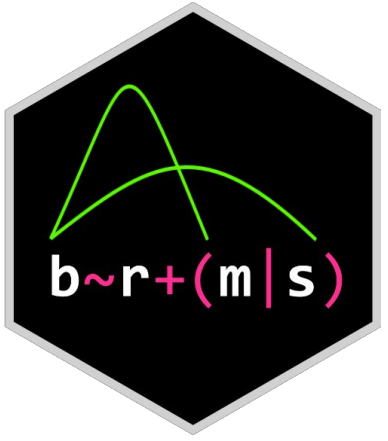
Basics of Bayesian



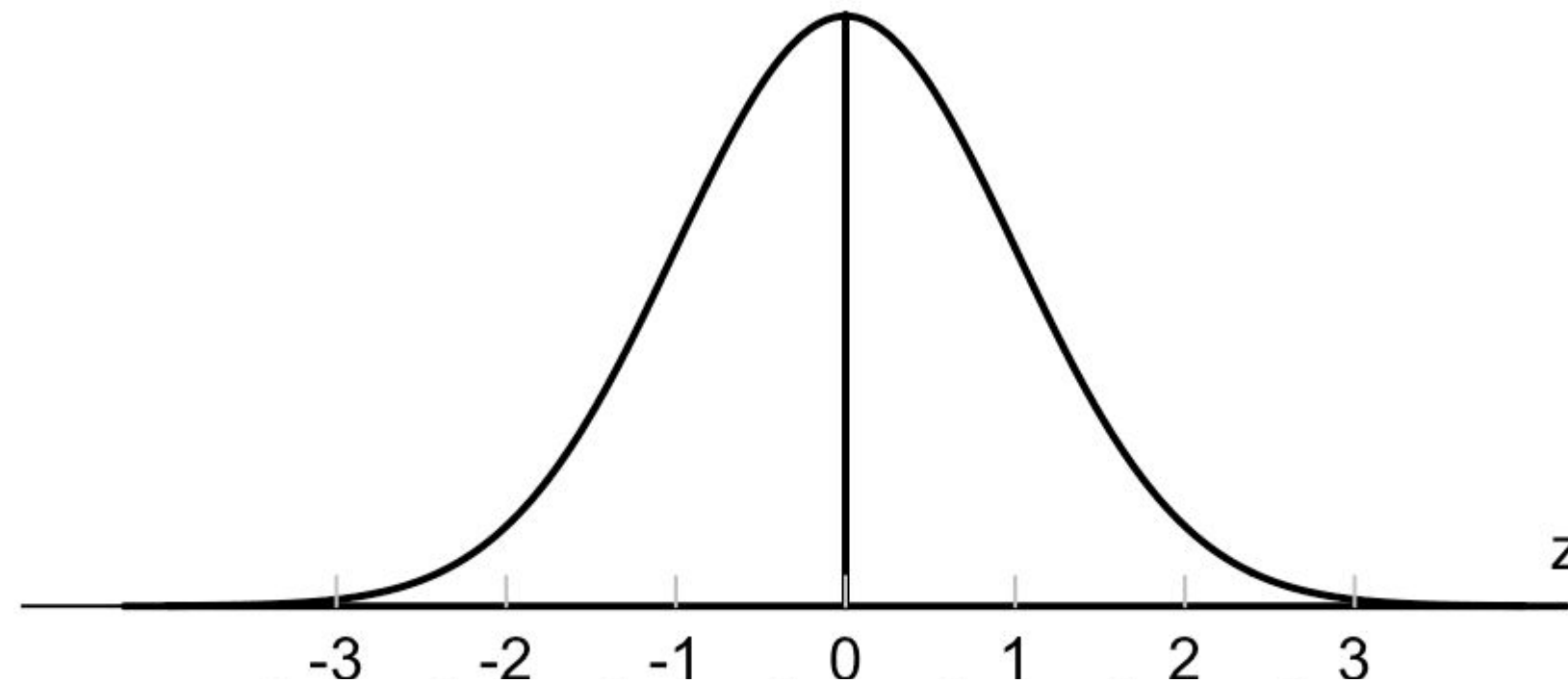


BRMS

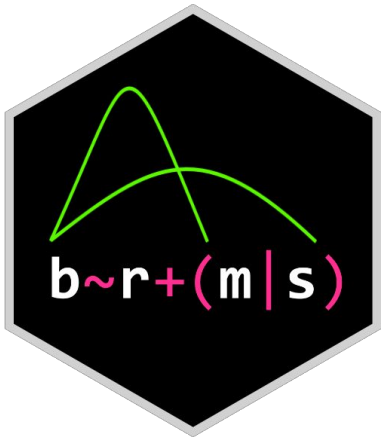
Bayesian Regression Modelling using Stan



– R package using Stan

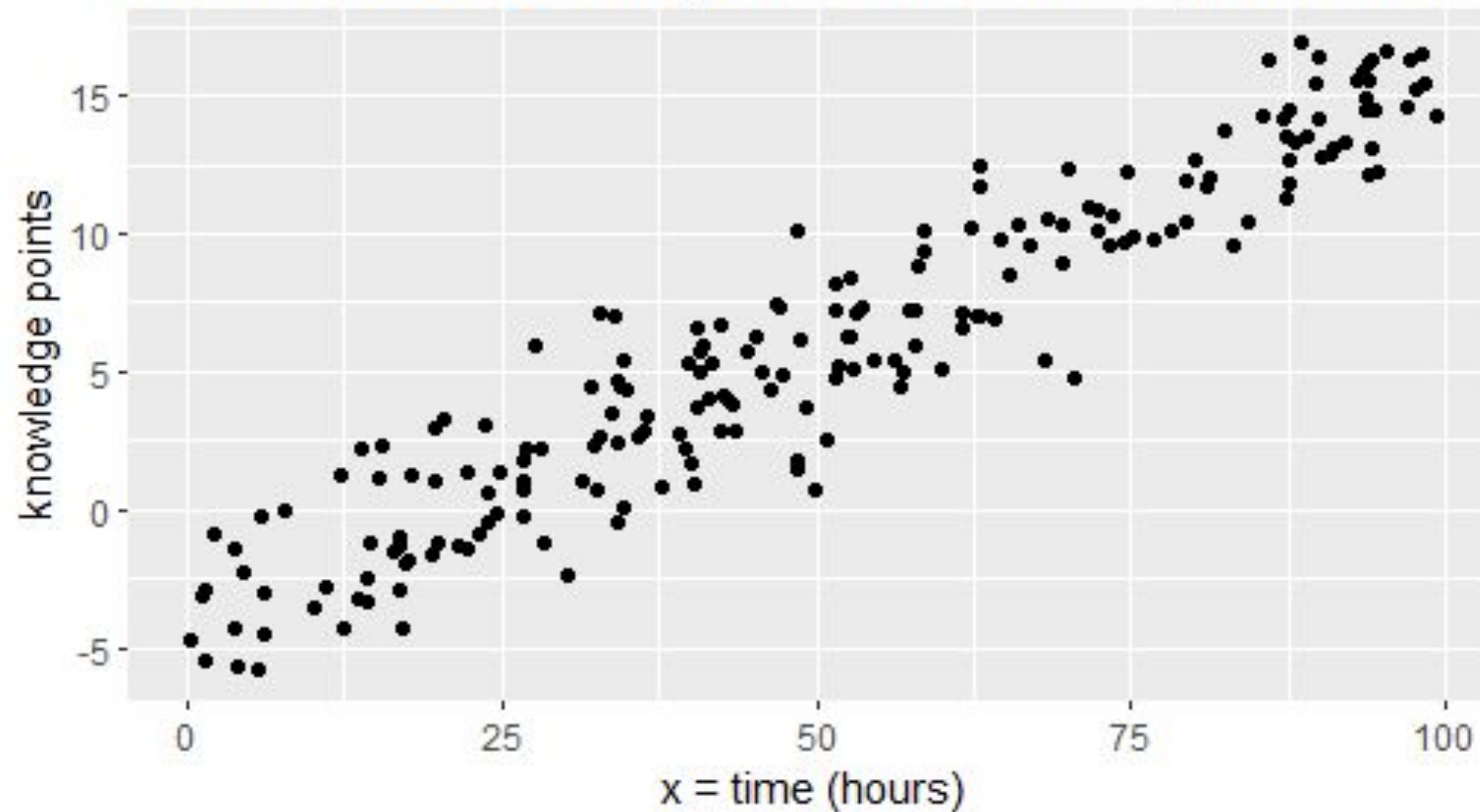


Regression Modelling

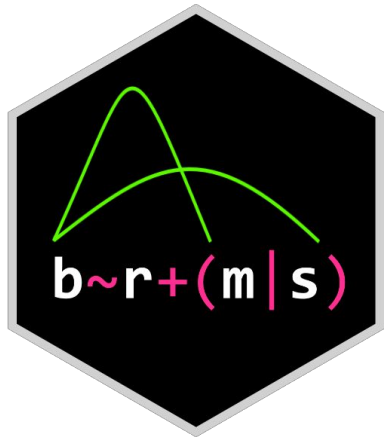


$$Y \sim a + b * X + \dots$$

it takes 10.000 hours of practice to master bayesian stats

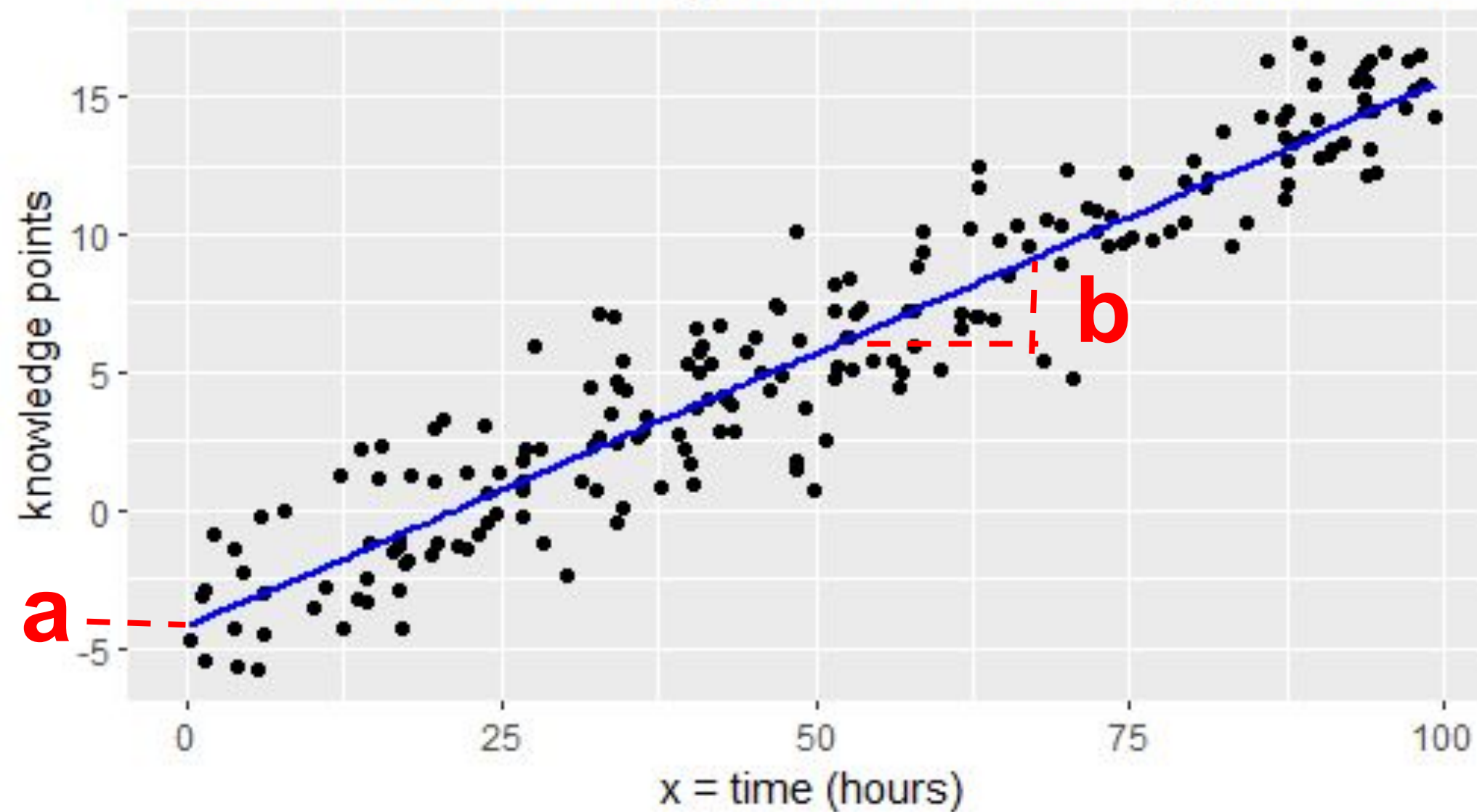


Bayesian Regression Modelling

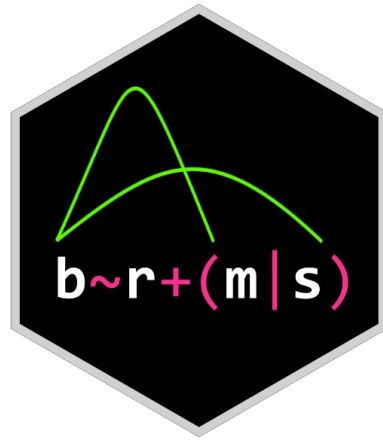


$$Y \sim a + b * X + \dots$$

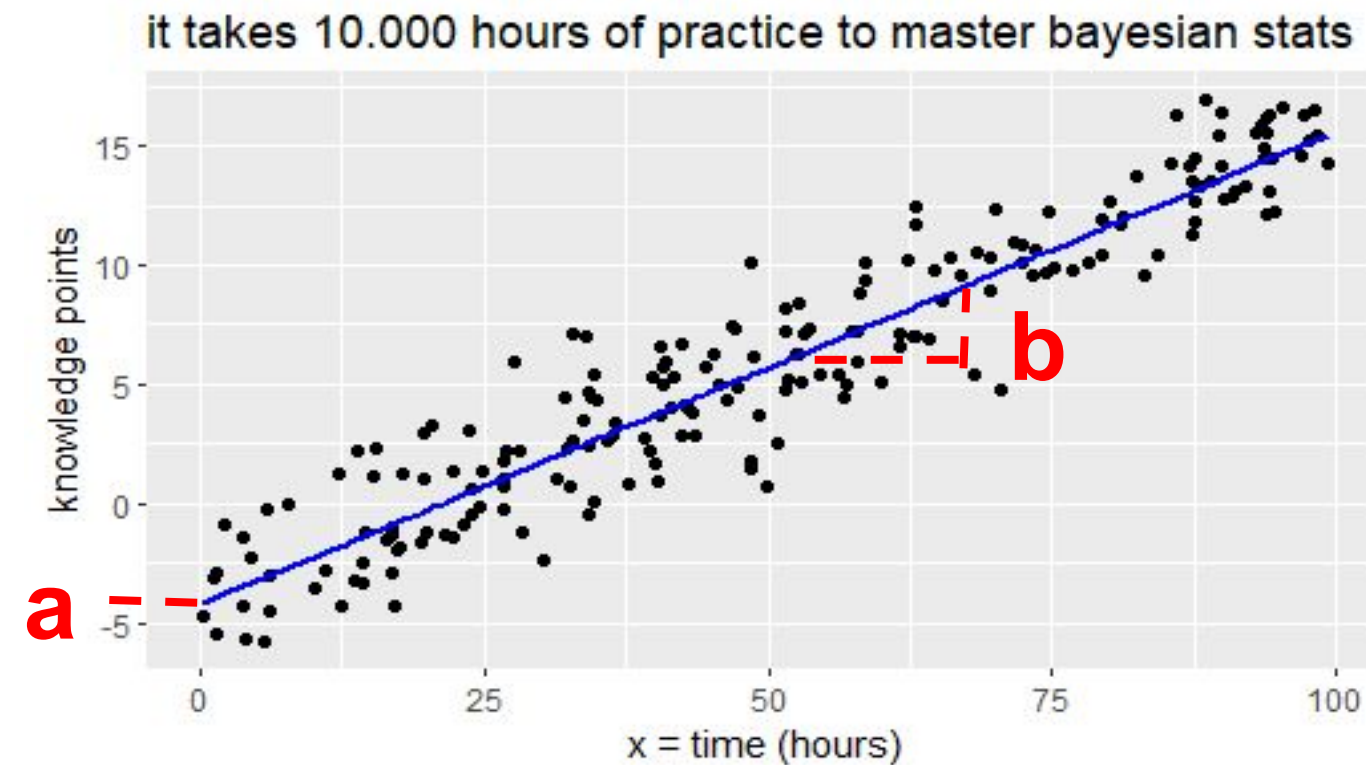
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Bayesian Regression Modelling



$$Y \sim a + b * X + \dots$$

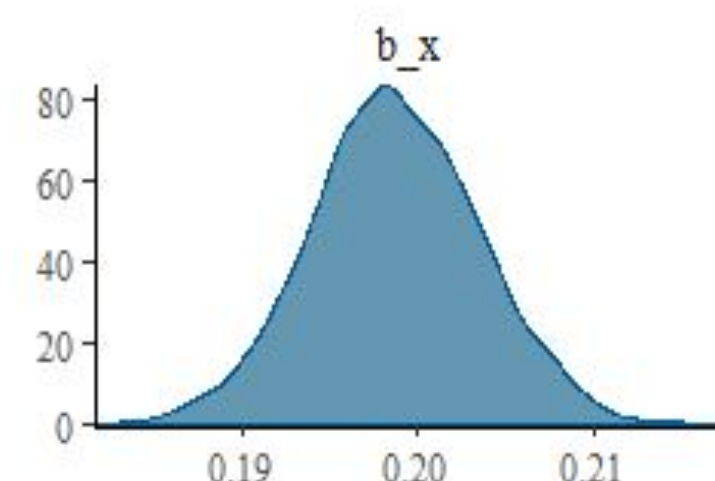
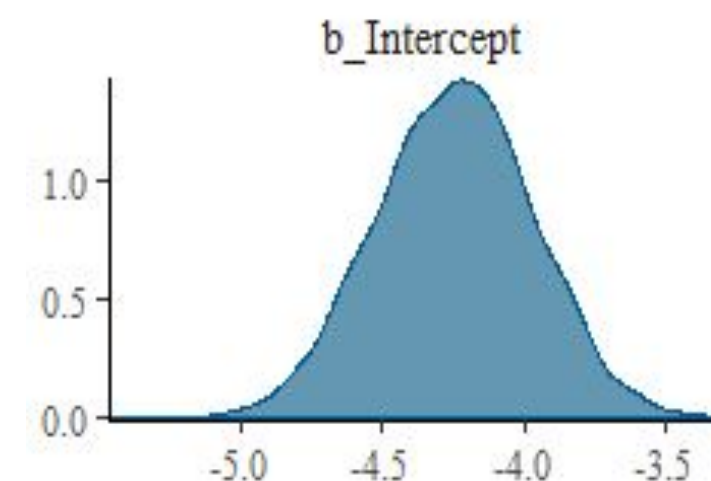


Frequentist model

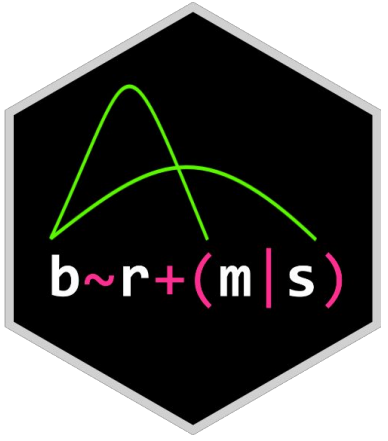
```
Call:
lm(formula = y ~ x, data = sim)
```

```
Coefficients:
(Intercept)          x
   -4.1201       0.1987
```

Bayesian model

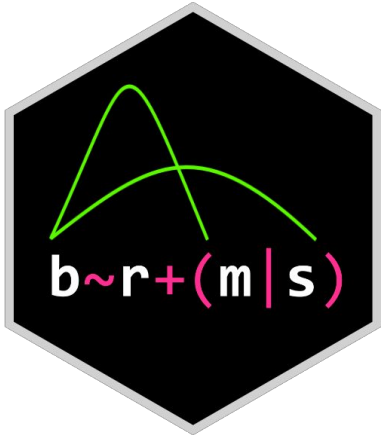


How to get a posterior distribution?



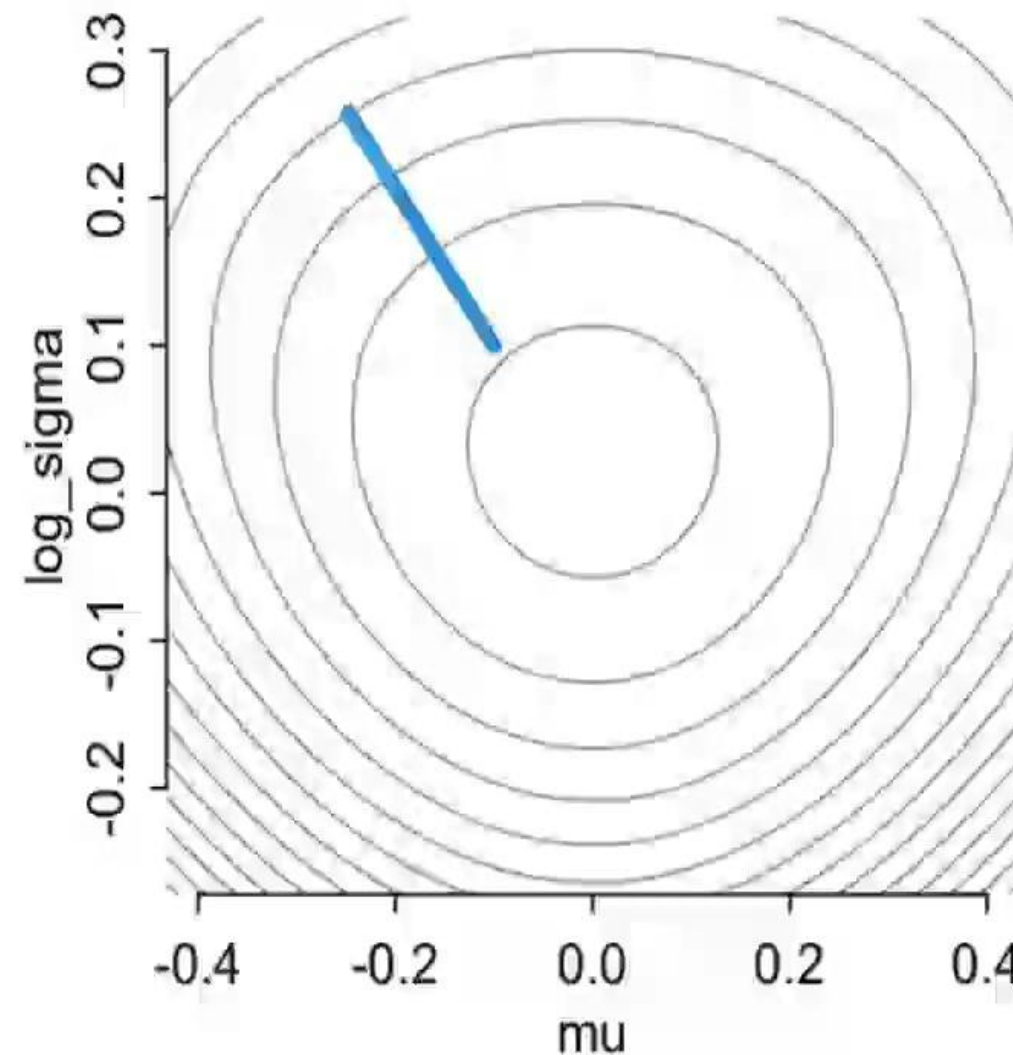
– Calculation of whole posterior distribution is hard

How to get a posterior distribution?



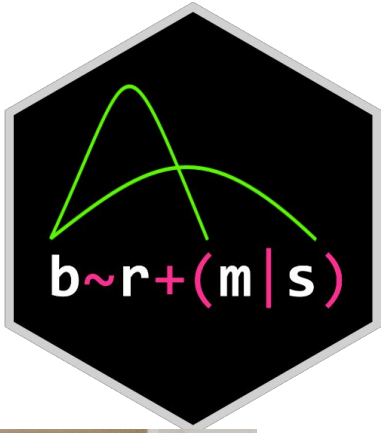
- Sample posterior distribution
- Markov Chain Monte Carlo algorithms

Basic Rosenbluth (aka Metropolis) algorithm



Richard McElreath

Example: learning Bayes

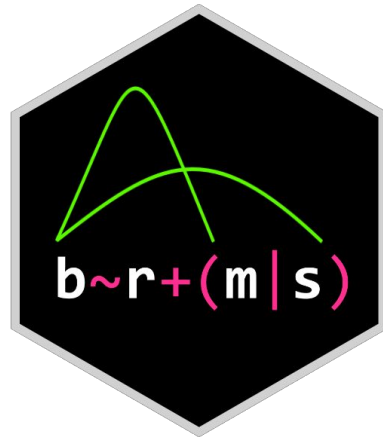
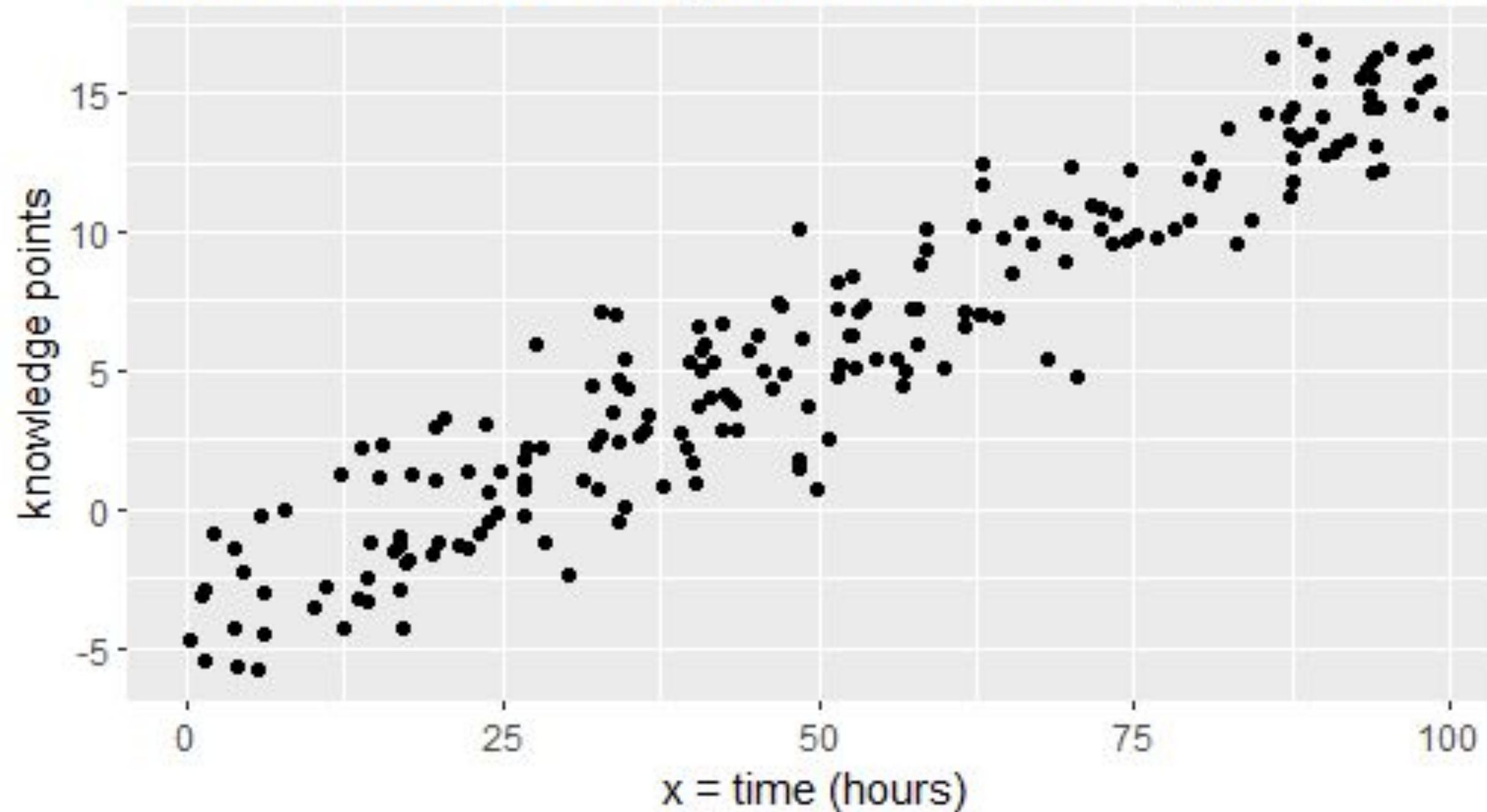


- It takes 10.000 hours to master a subject
- Simulated example

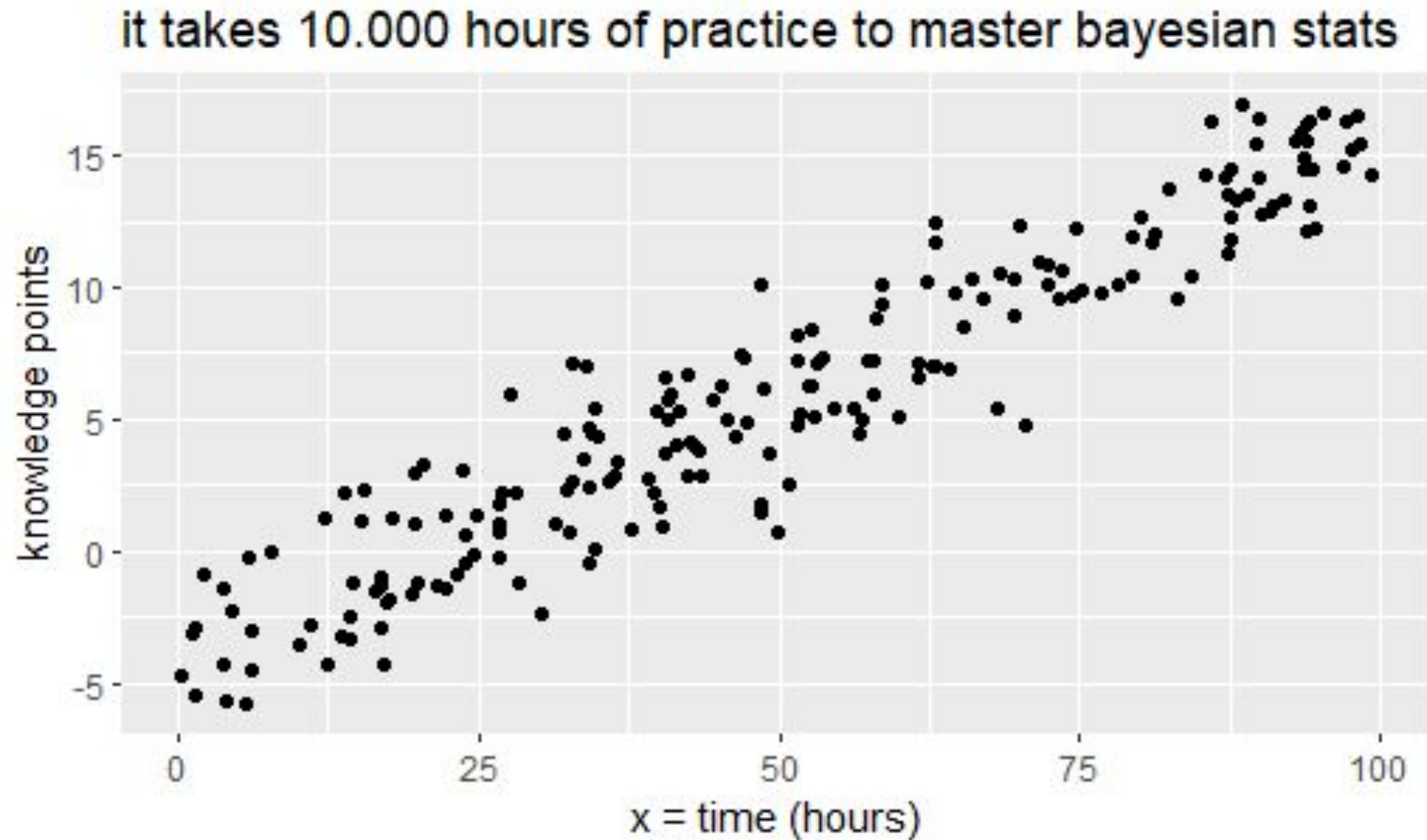
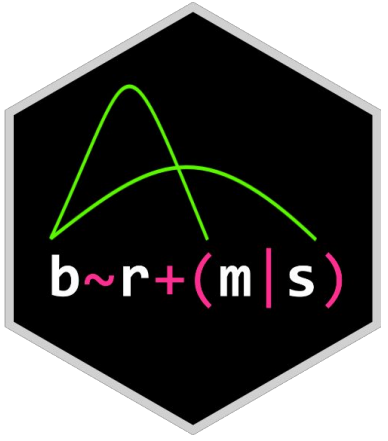


learning Bayes: data model

it takes 10.000 hours of practice to master bayesian stats



learning Bayes: data model

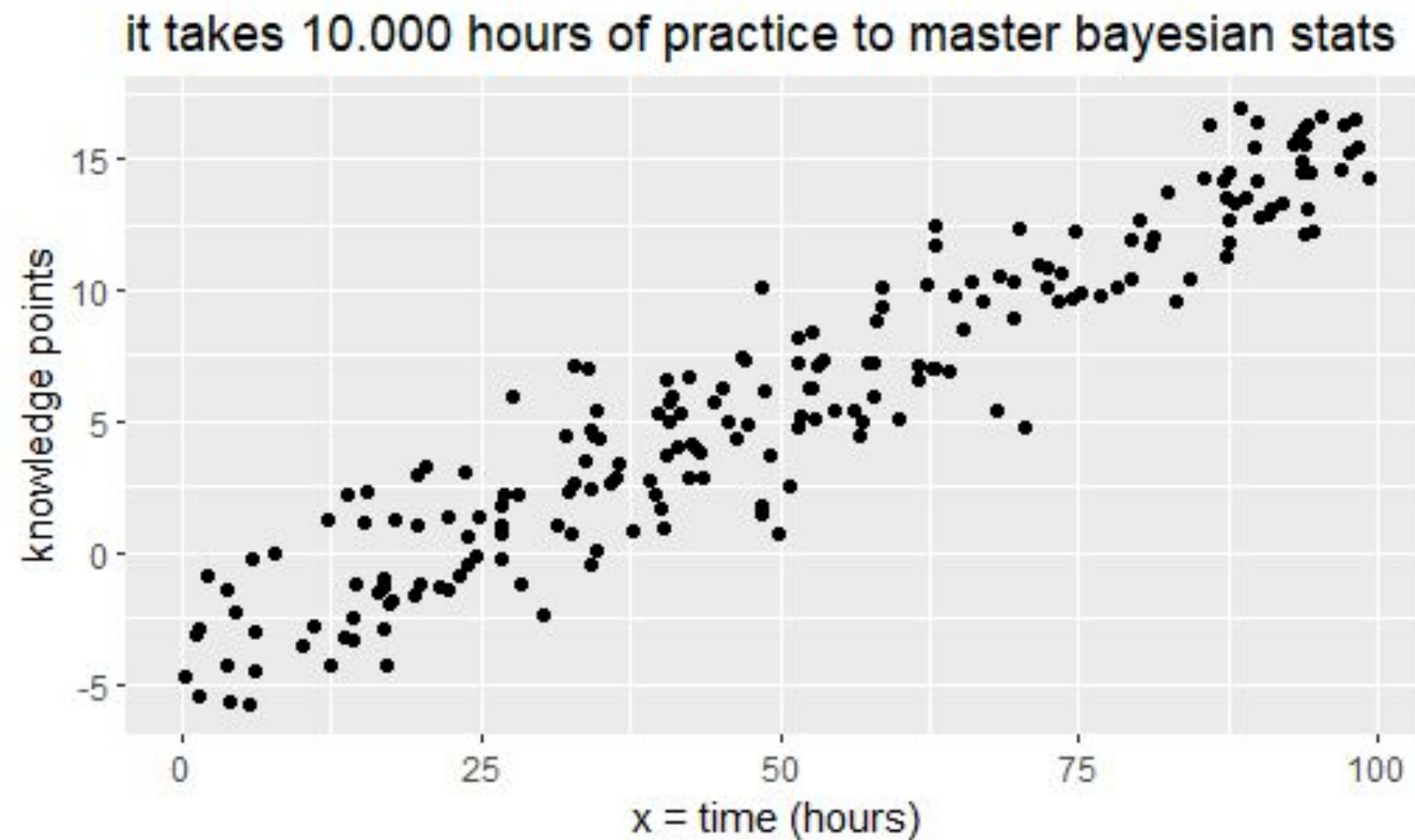
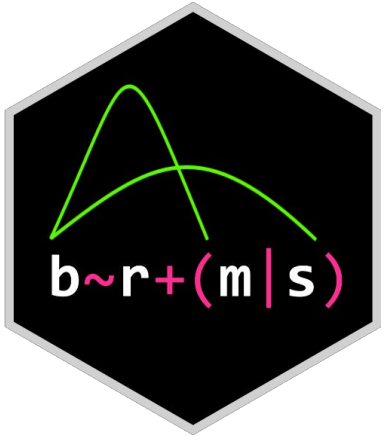


*Expected knowledge = $a + b * time$*

a = expected initial knowledge (-4)

b = expected rate of learning (0.2)

learning Bayes: data model



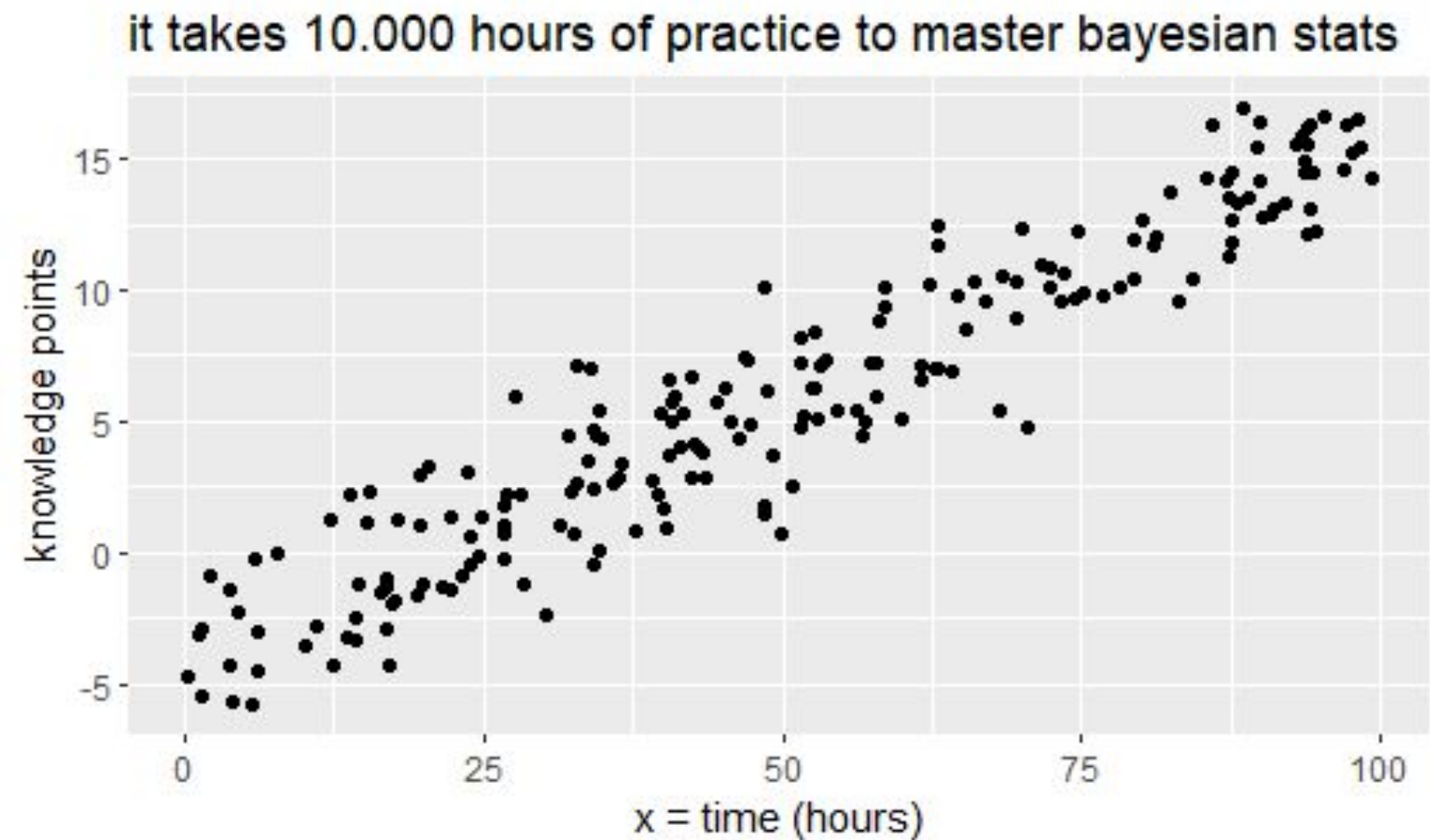
Knowledge ~ normal(mean = expected knowledge, sd = variation (2))

*expected knowledge = $a + b * \text{time}$*

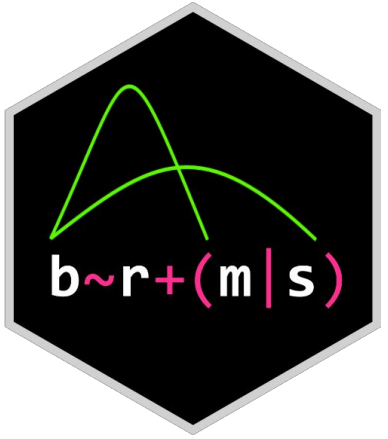
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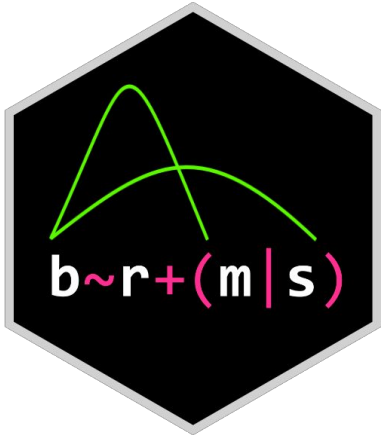
learning Bayes: formula



```
mod <- brm(formula = y ~ x, data = sim,  
          family = "gaussian"  
          prior = knowledge_prior,  
          iter = 4000, warmup = 1000,  
          chains = 2, ...)
```

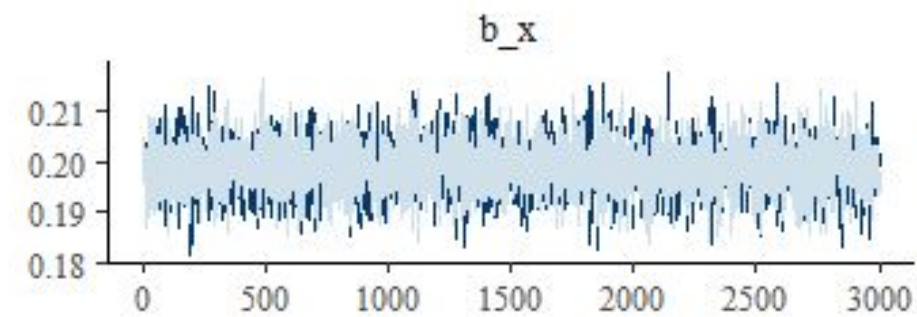
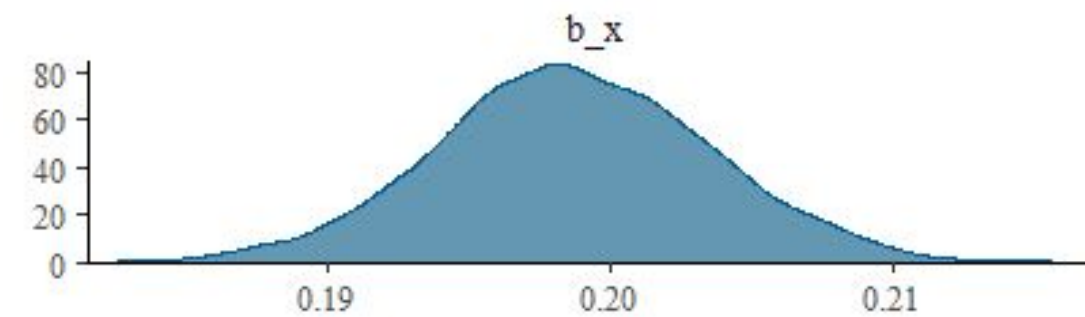
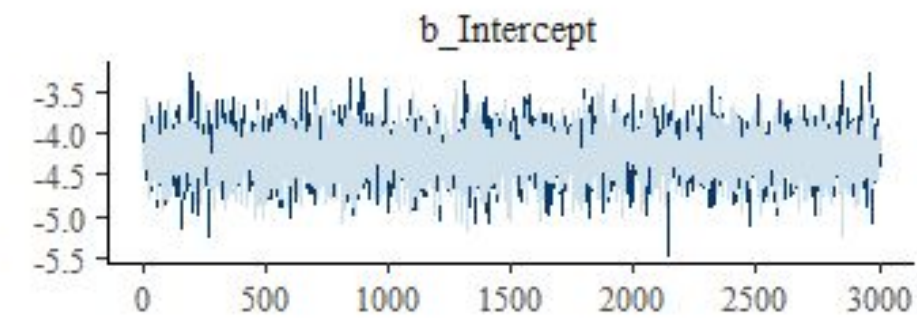
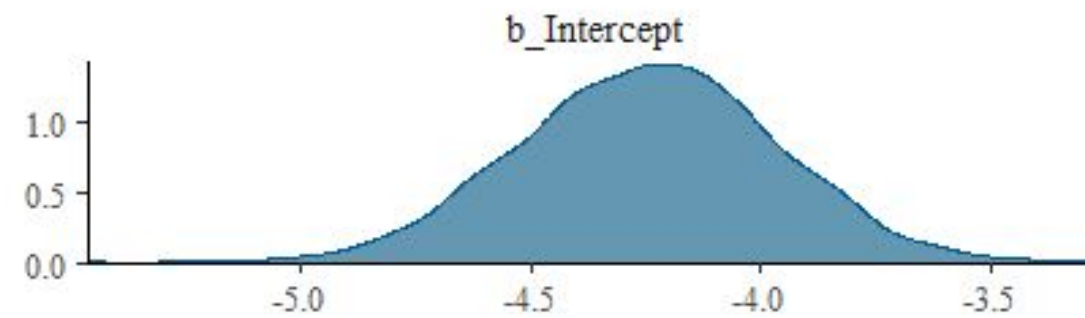


learning Bayes: posterior distributions

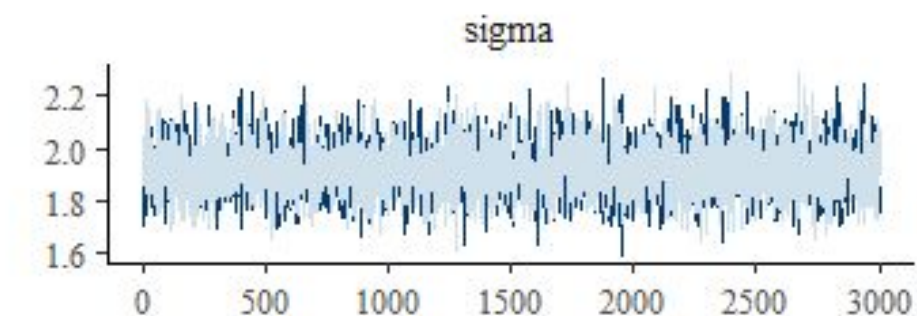
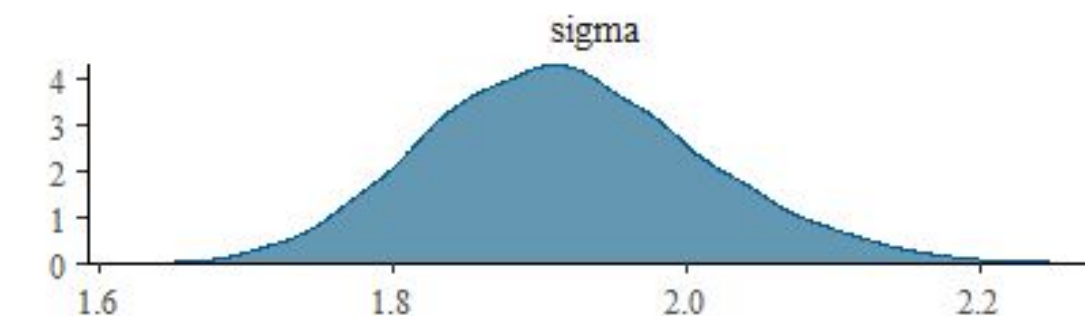


Posterior distributions

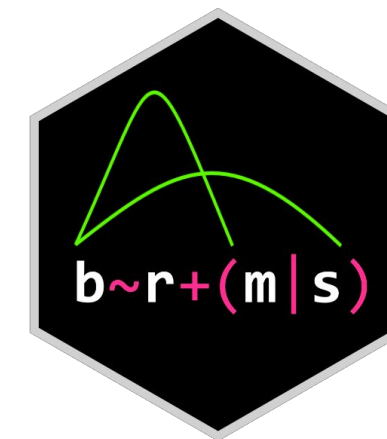
chains



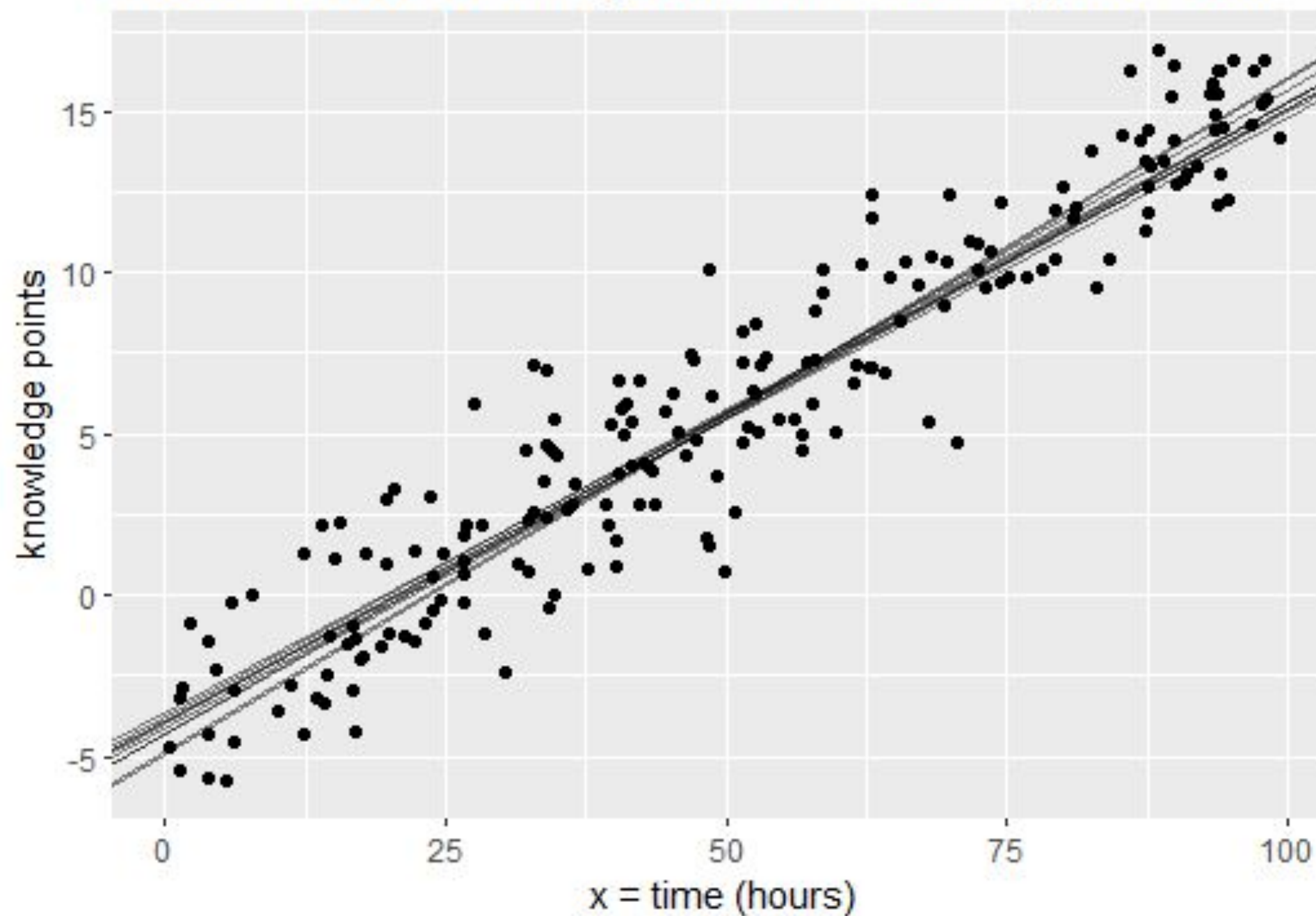
Chain
— 1
— 2



(Posterior) distribution of trend lines

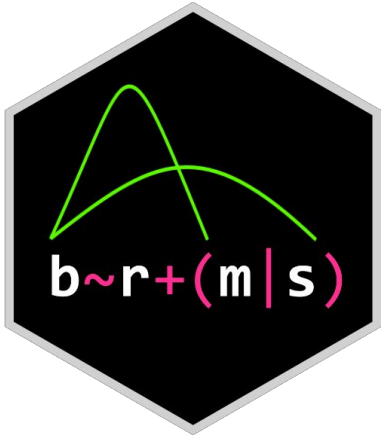
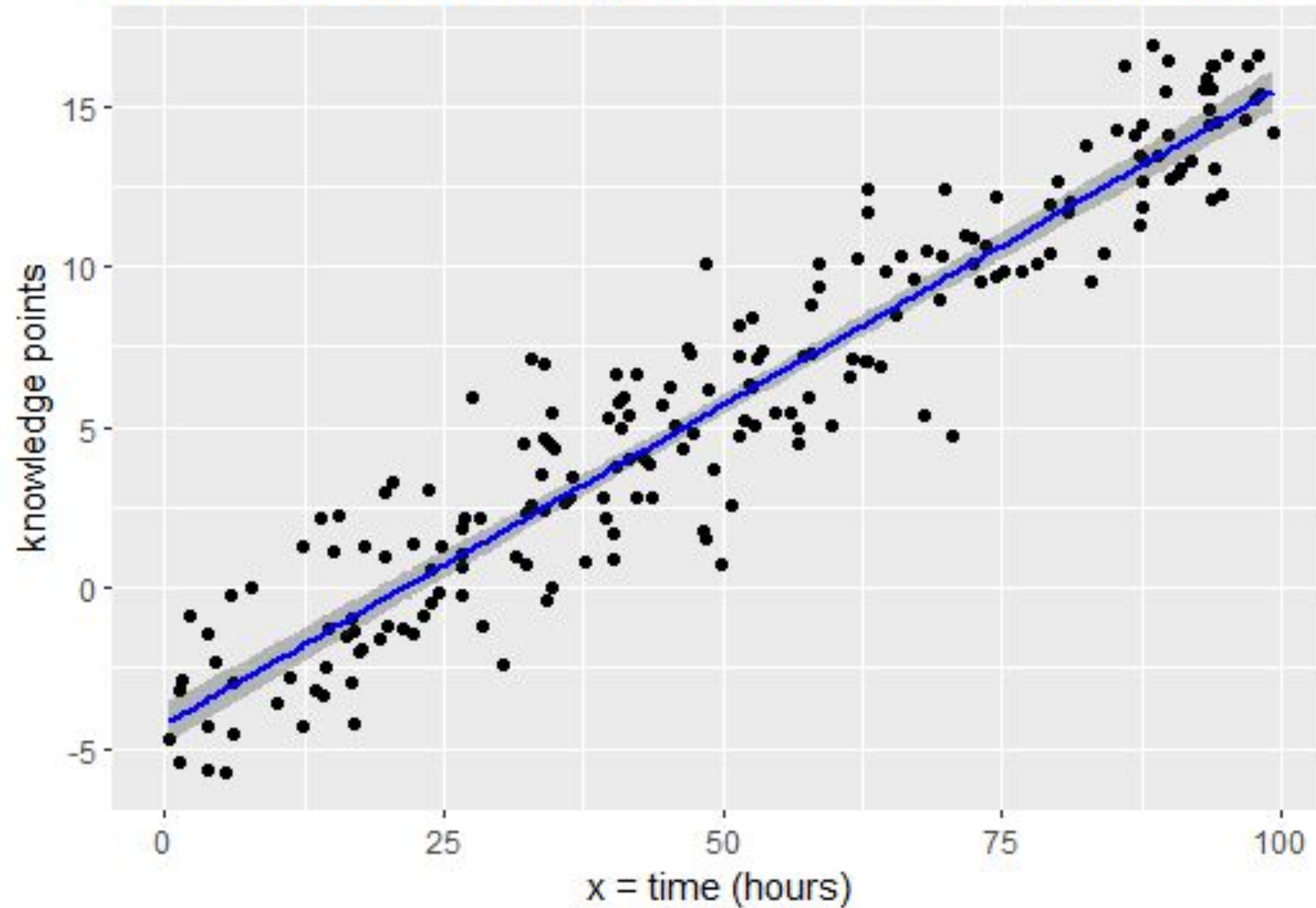


it takes 10.000 hours of practice to master bayesian stats

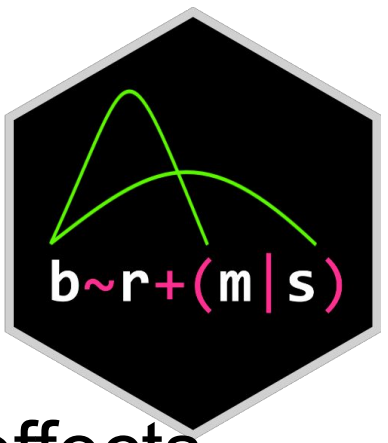


Visualize average trend

it takes 10.000 hours of practice to master bayesian stats



More than simple models



```
bf (y ~ x*z + (1 | group))
```

Interactions and variable “random” effects

```
bf (... family = binomial)
```

Any error distribution you can define

```
bf (... family =  
mixture(binomial, binomial))
```

Mixture of data distributions

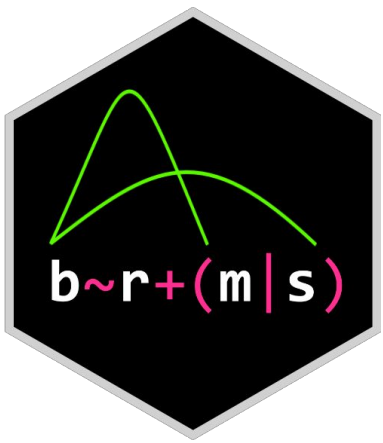
```
bf (y ~ log(rc + or),  
    rc ~ 1 + wodds), or ~ 1,  
    nl = T)
```

Non-linear models

```
bf (y ~ 1 + line:mi(z), ...)  
bf (z | mi() ~ 1)
```

Missing data / latent variable

Example: difference in cell density

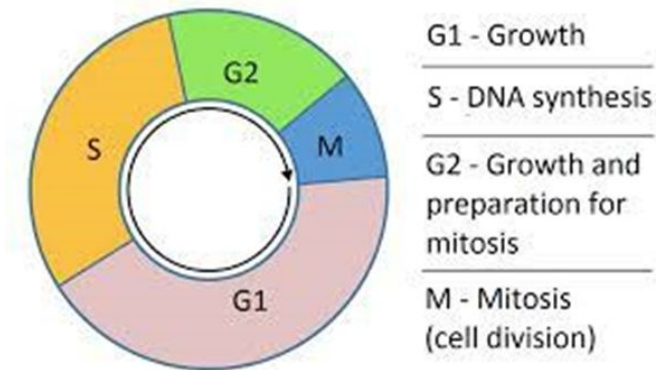


Mass of 4n, 2n

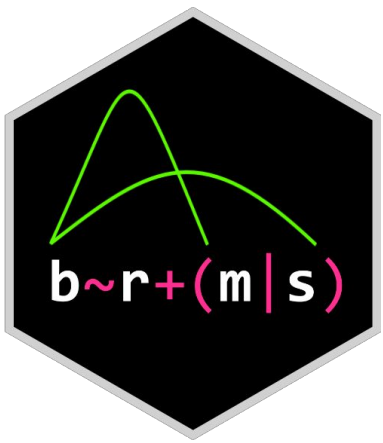
$$\frac{n_{4n}}{n_{2n}} = ?$$

Difference in cellular density

Cells in G2



Example: difference in cell density

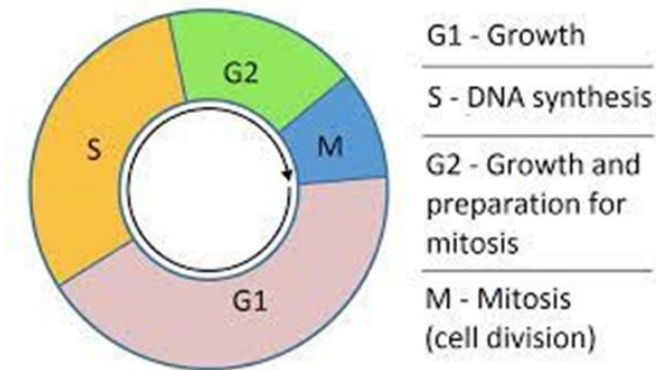


Mass of $4n$, $2n$

$$\frac{n_{4n}}{n_{2n}} = ?$$

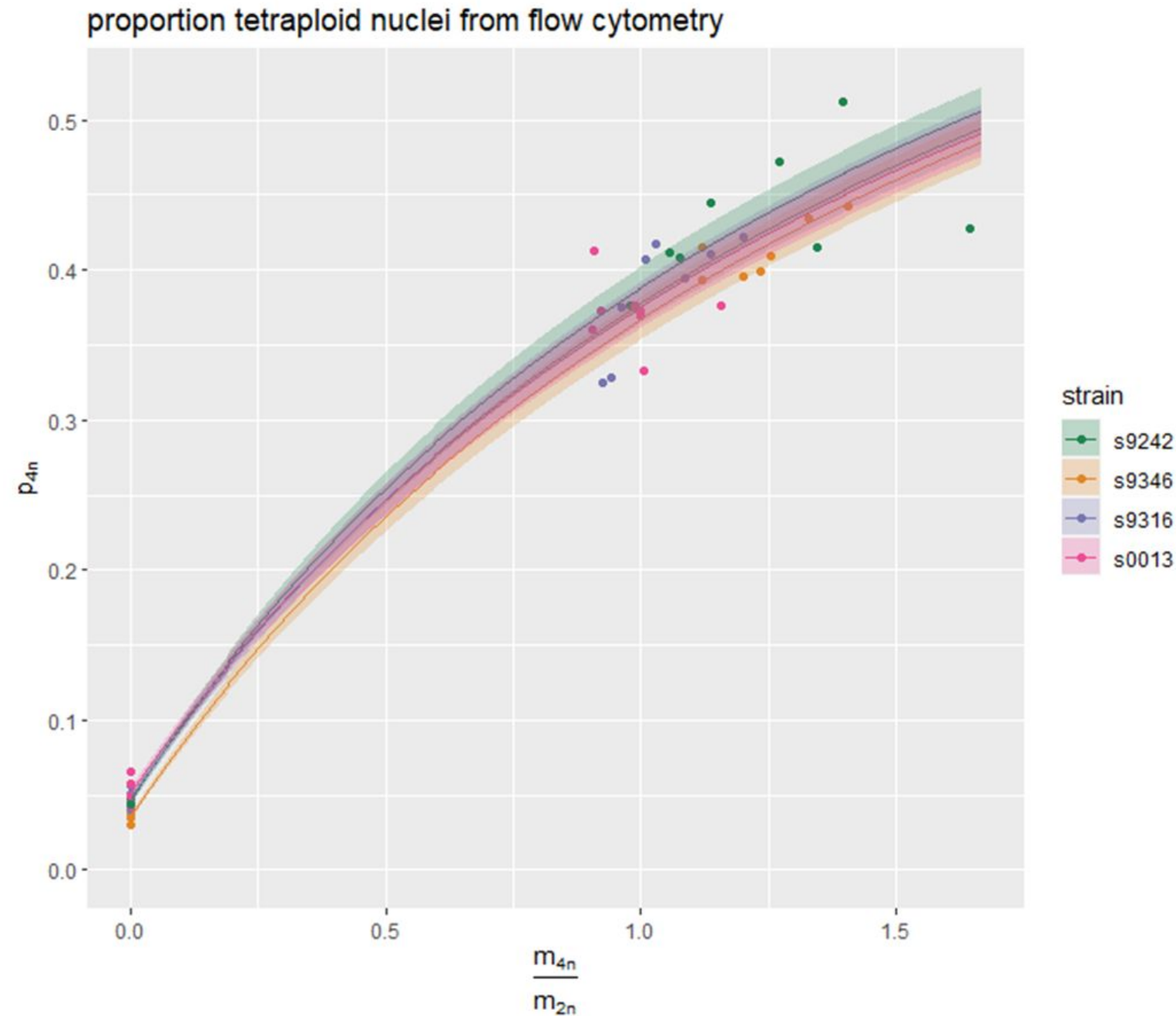
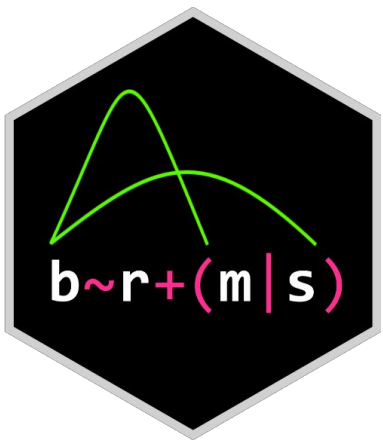
Difference in cellular density

Cells in $G2$



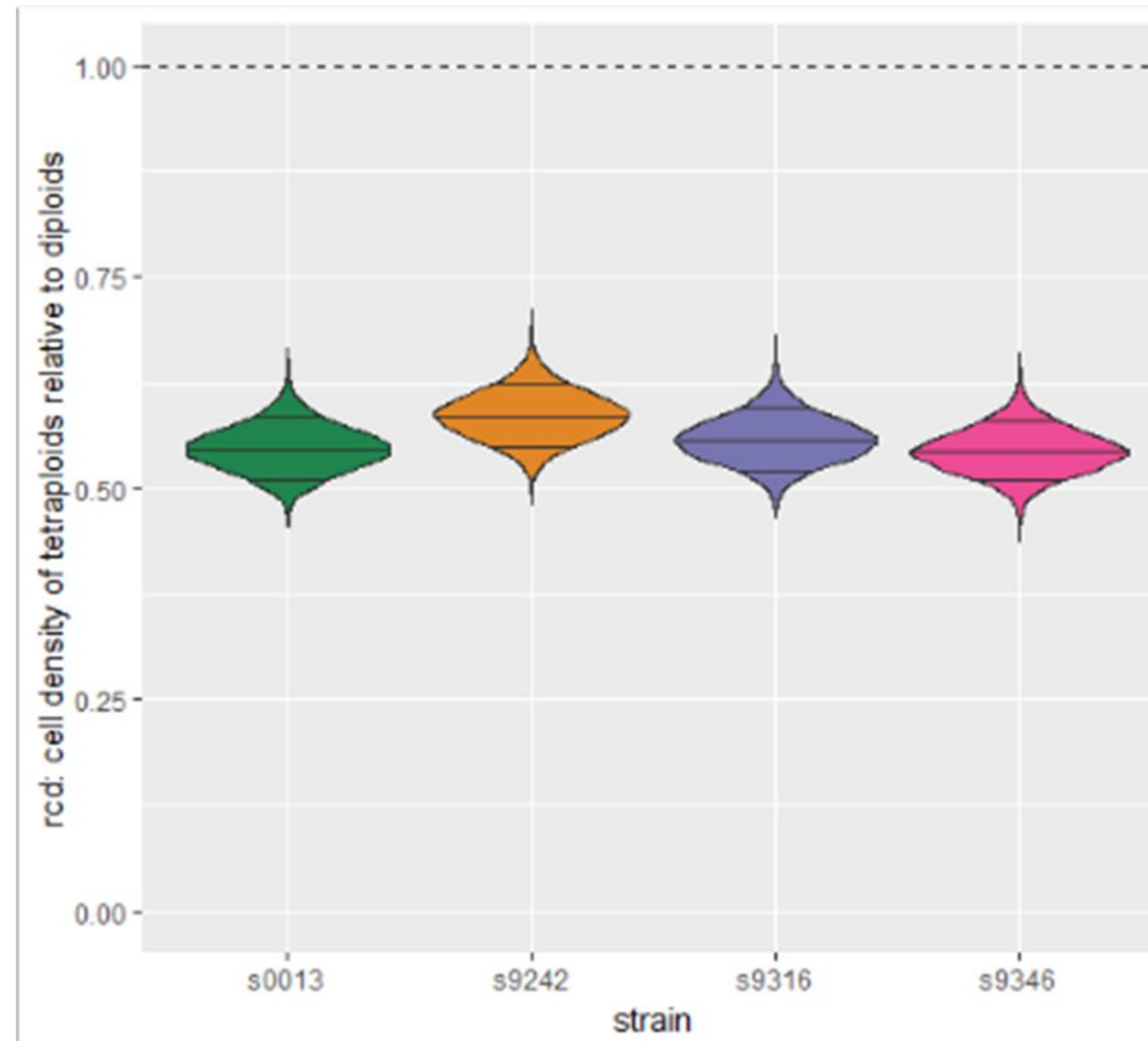
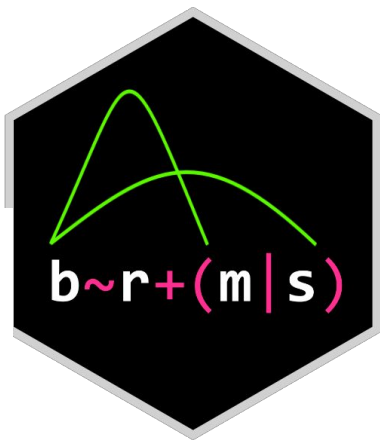
$$\text{logit}(p_{4n}) = \ln \left(\frac{m_{4n}}{m_{2n}} * \frac{\text{cell density}_{4n}}{\text{cell density}_{2n}} + \frac{p_{G2}}{1 - p_{G2}} \right)$$

Example: difference in cell density



$$\text{logit}(p_{4n}) = \ln \left(\frac{m_{4n}}{m_{2n}} * \frac{\text{cell density}_{4n}}{\text{cell density}_{2n}} + \frac{p_{G2}}{1 - p_{G2}} \right)$$

Difference in cell density estimation

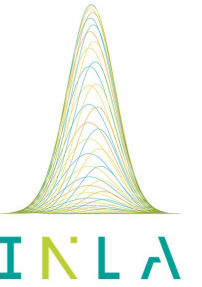


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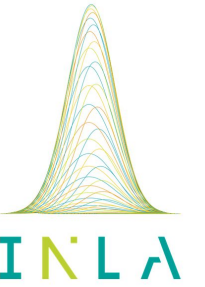
INLA

What is INLA?



- Integrated Nested Laplace Approximation
- Approximate Bayesian inference; alternative to MCMC, but ‘faster’
- ‘Restricted’ to GMRF (Gaussian Markov random field) models → posteriors of fixed effects are Gaussian
→ includes everything you are used to from frequentist ecology & more
- R-INLA project: <https://www.r-inla.org/home>

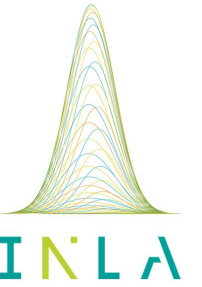
Applications of INLA



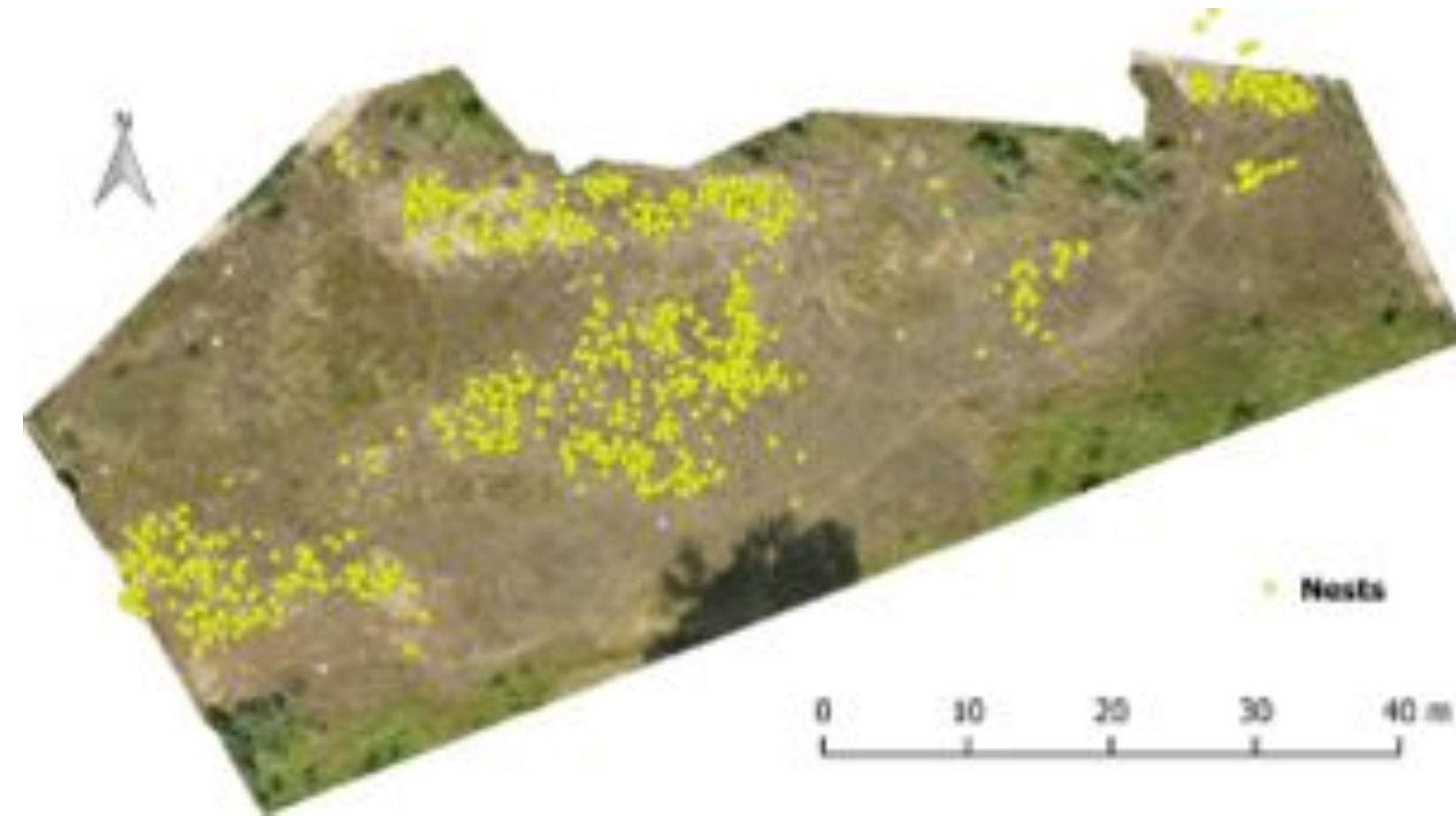
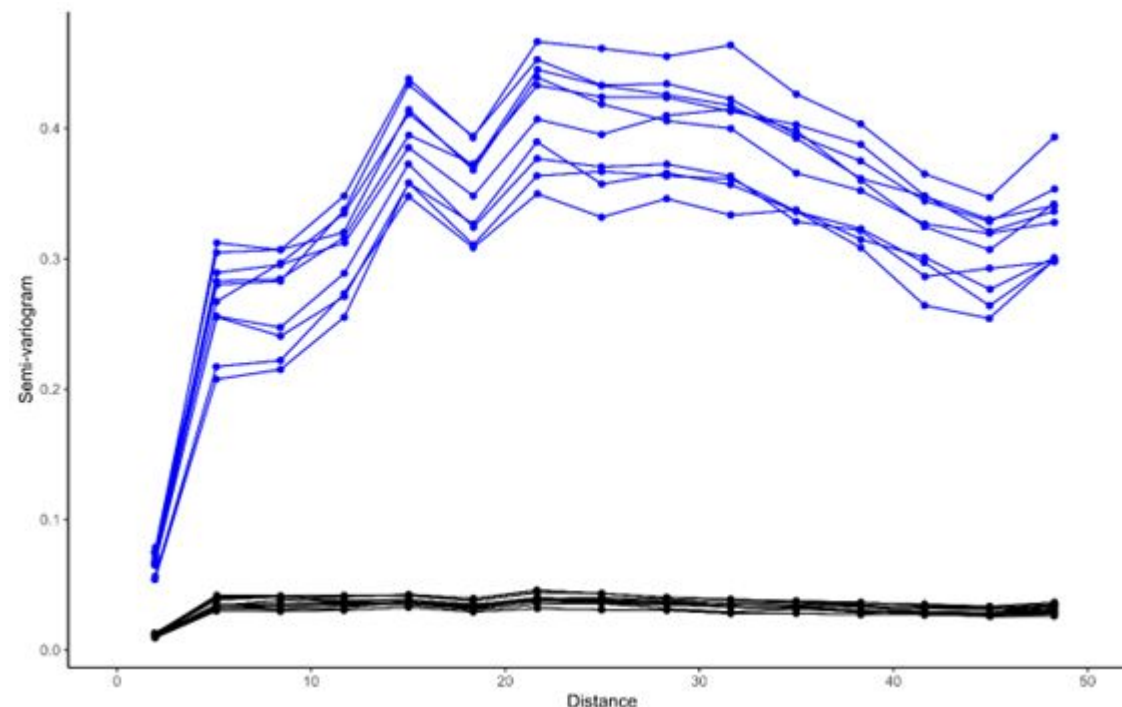
- Models correcting for spatial and/or temporal correlation
- Point processes (modelling preferential sampling)
- Modelling time series
- ...
- All your (zero-inflated) GL(M)Ms, GAM(M)s with normal, (negative) binomial, Poisson, gamma, distributions (and much more! E.g. censored/truncated Poisson)

Quick example

Batsleer, Maes & Bonte (2022) AmNat

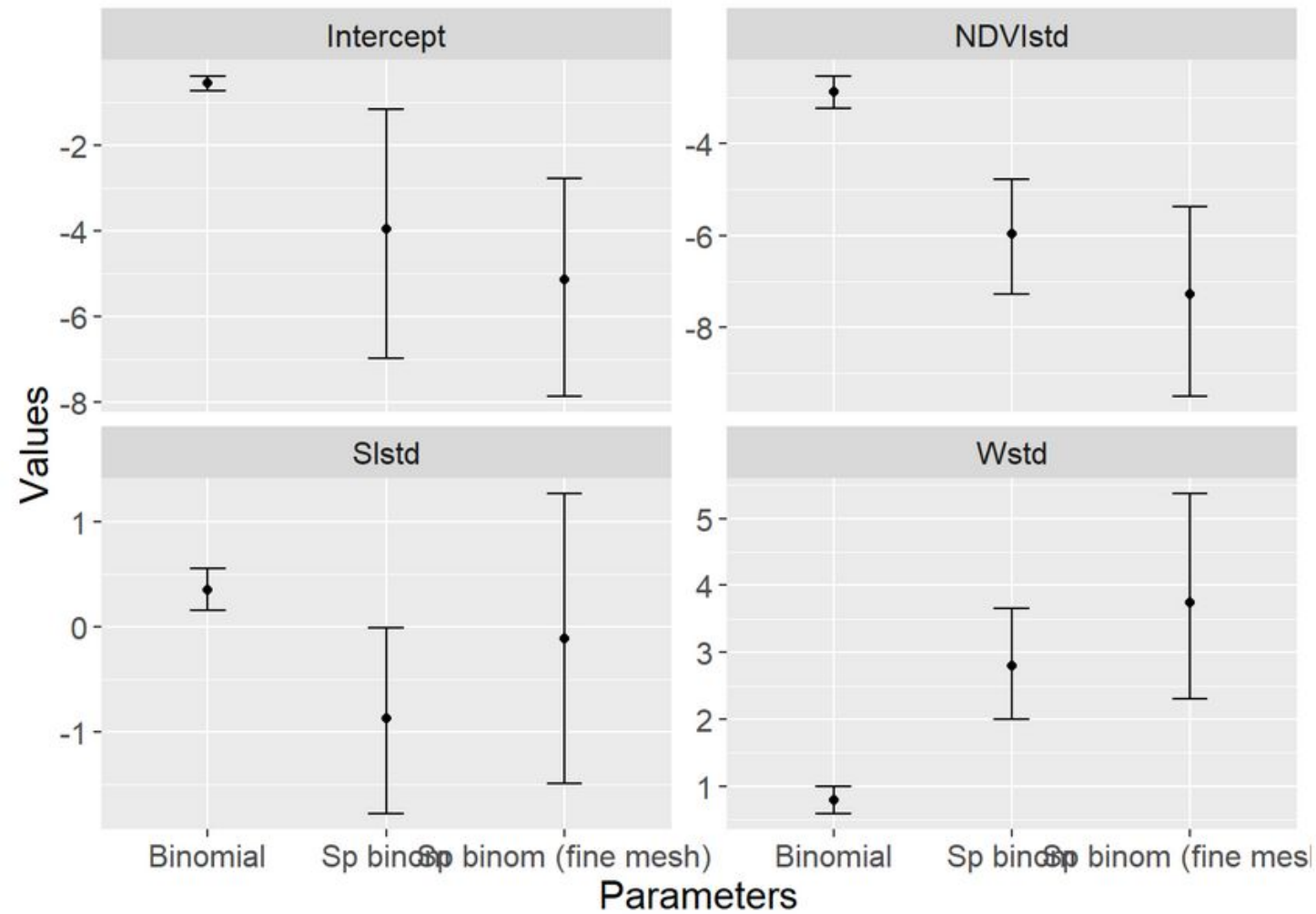
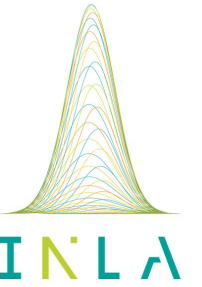


- Microhabitat suitability model for *Bembix rostrata*
- Spatial autocorrelation!
- Presence \sim NDVI + Insolation + SPDE

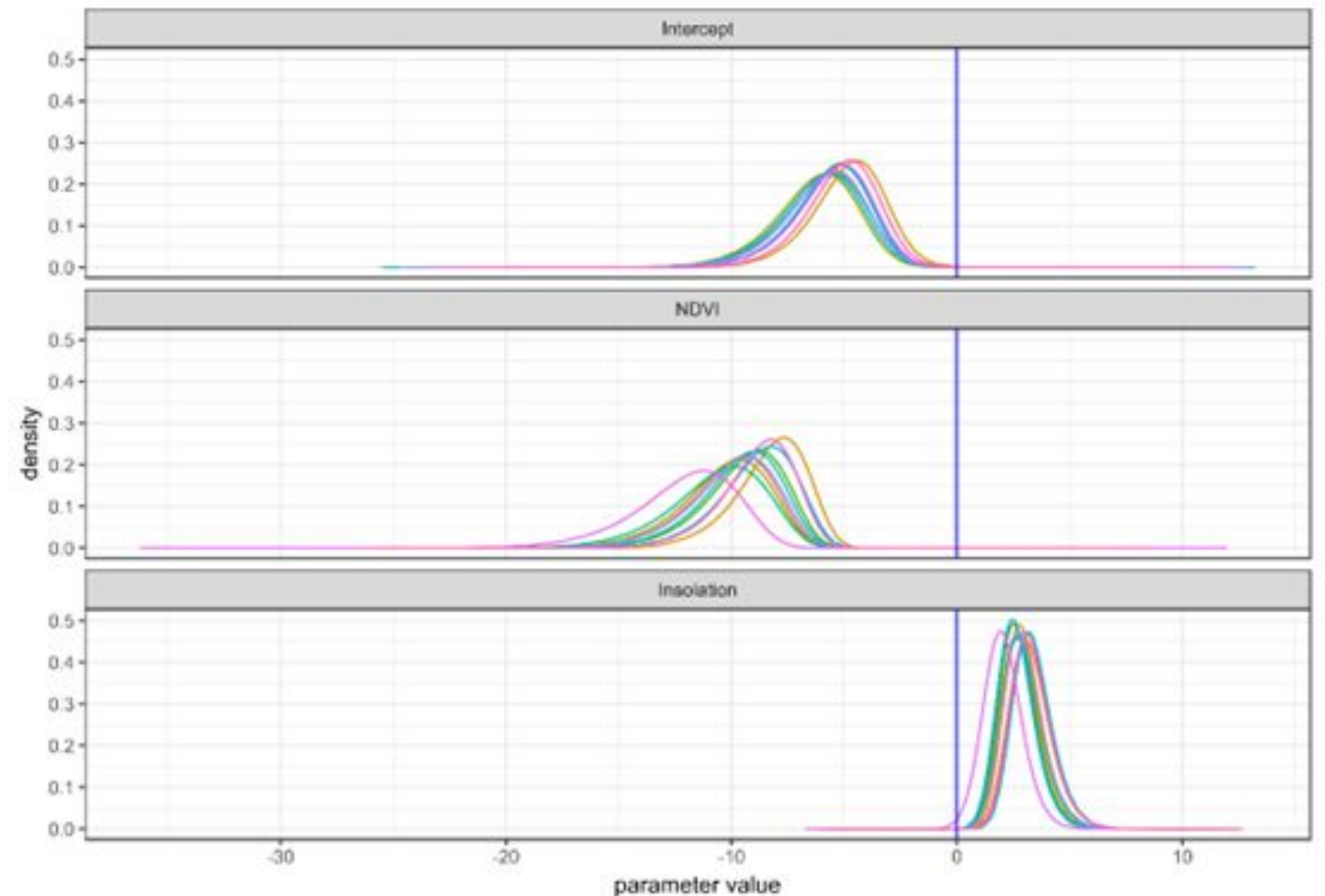


Quick example

Posterior distributions



Better estimates, but more uncertainty!

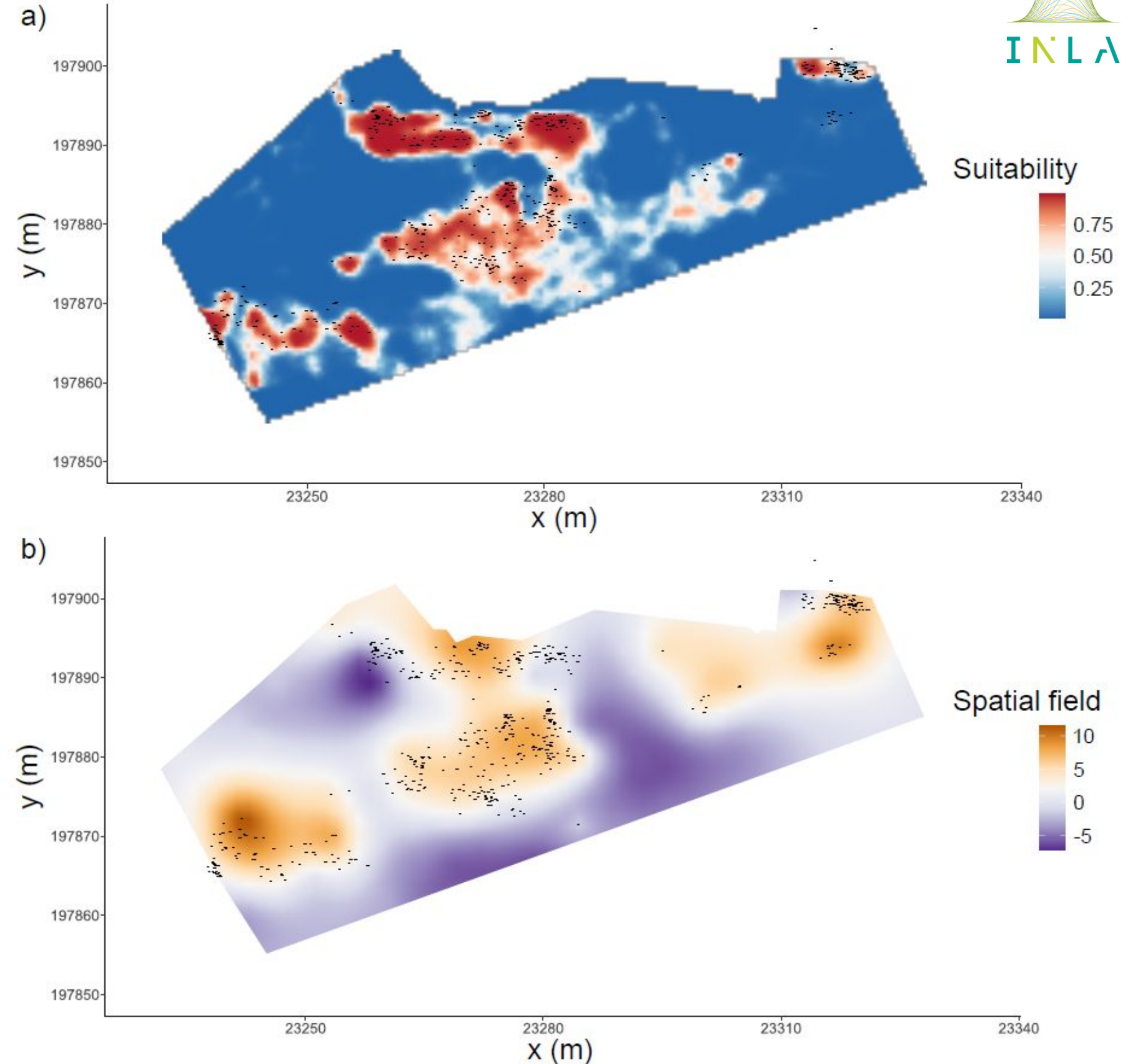


Quick example

Models were compared using the Watanabe-Akaike information criterion (WAIC; Watanabe 2010; Gelman et al. 2014)

The plotting of the spatial field, the spatial residuals that INLA corrects for, shows whether the degree of clustering was higher (hot spots) or lower (cold spots) than expected on the basis of the covariates (NDVI and insolation) in the microhabitat model

For every run, zero was excluded from the 95% credibility intervals of the effect sizes, indicating that the signs of the effect sizes were clearly determined (this is a Bayesian approach to evaluating statistical significance at a specified level).



Other applications from the lab

Linking dune morphology to spatial configuration of marram grass
Comparing mechanistic model (IBM) with GIS derived field observations

Biomorphogenic Feedbacks and the Spatial Organization of a Dominant Grass Steer Dune Development

Dries Bonte^{1*} Femke Batsleer¹ Sam Provoost² Valérie Reijers³ Martijn L. Vandegehuchte^{1,4}
Ruben Van De Walle¹ Sebastian Dan⁵ Hans Matheve¹ Pieter Rauwoens⁶ Glenn Strypsteen⁶ Tomohiro Suzuki⁵ Toon Verwaest⁵ Jasmijn Hillaert^{1,2}

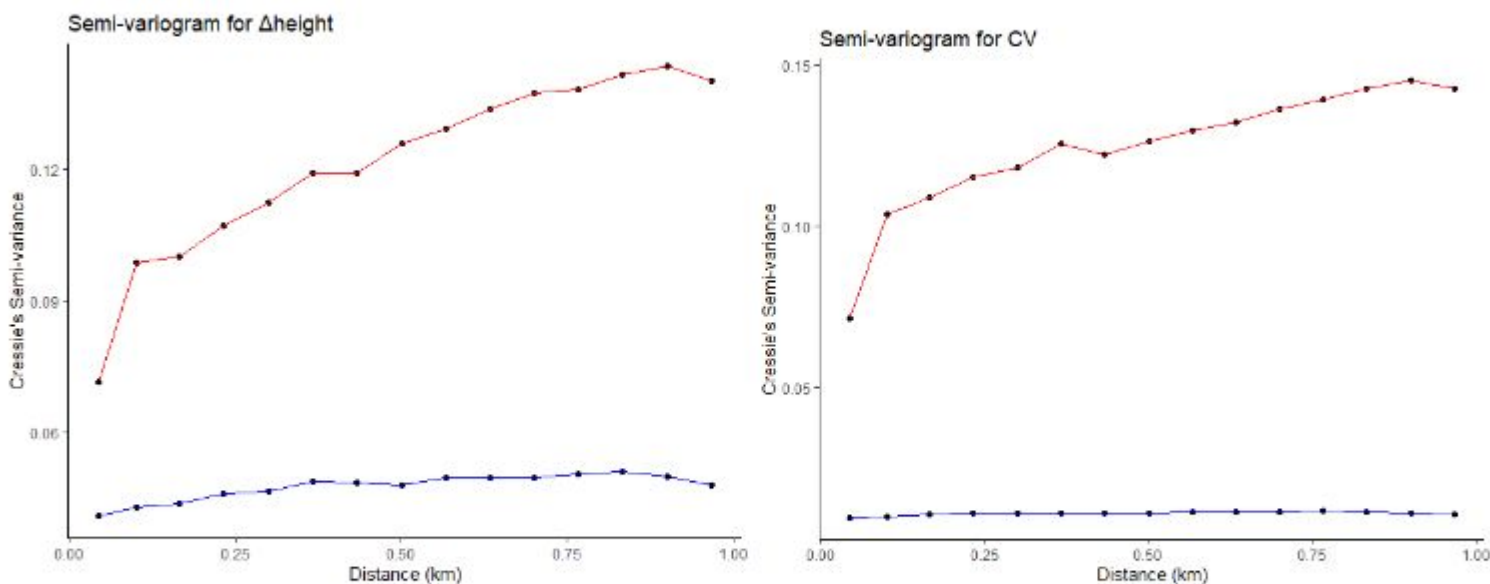
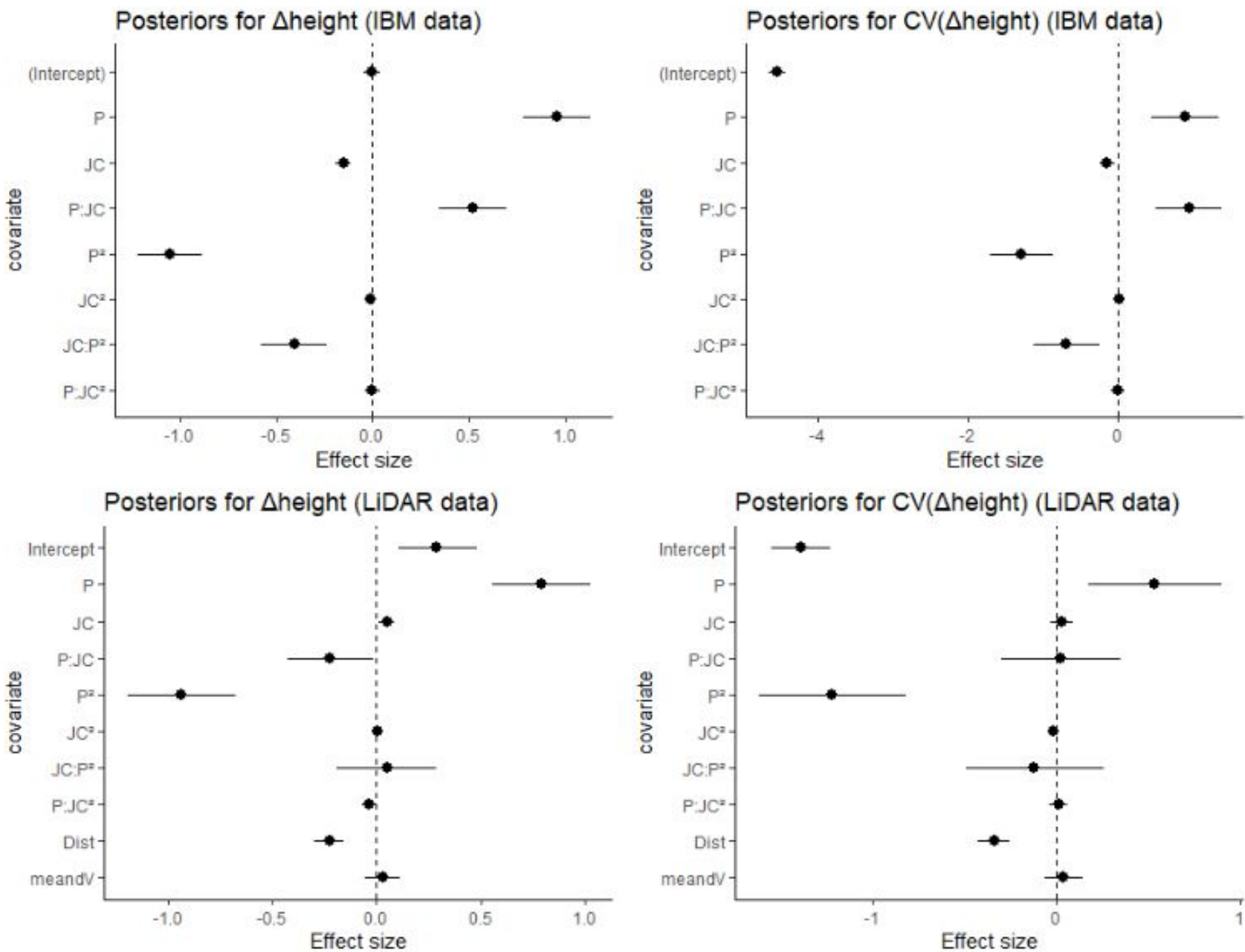


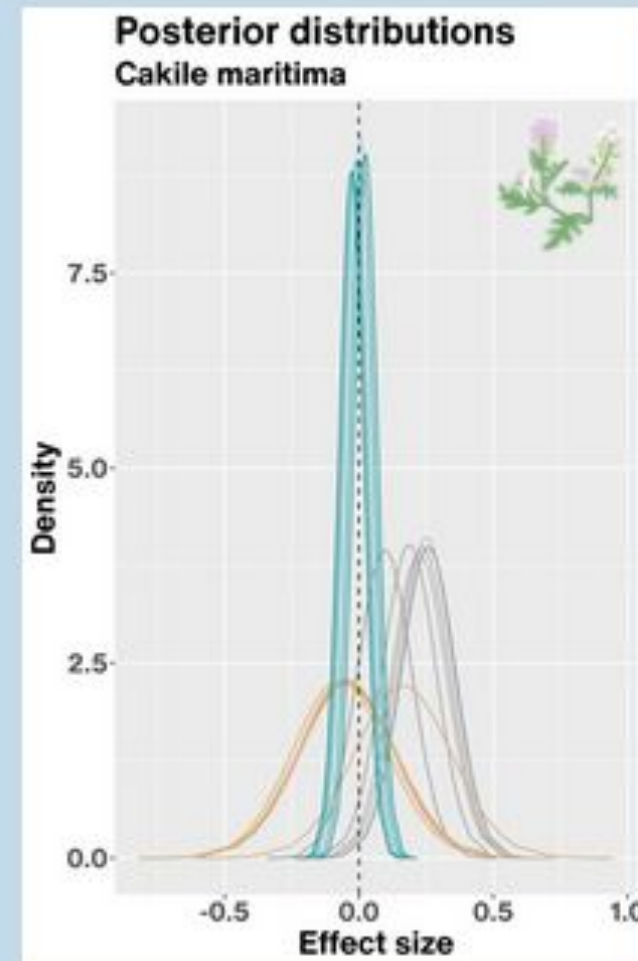
Fig S4.3: Semi-variogram of residuals for non-spatial model (red) and spatial model (blue) for Δheight (left) and CV (right).



Other applications from the lab

Charlotte Taelman

- Spatio-temporally corrected statistical models (INLA) of *C. maritima*, *E. farctus*, *H. peploides*, and *S. kali*, using annual sand dynamics and elevation on the beach relative to the mean springtide level



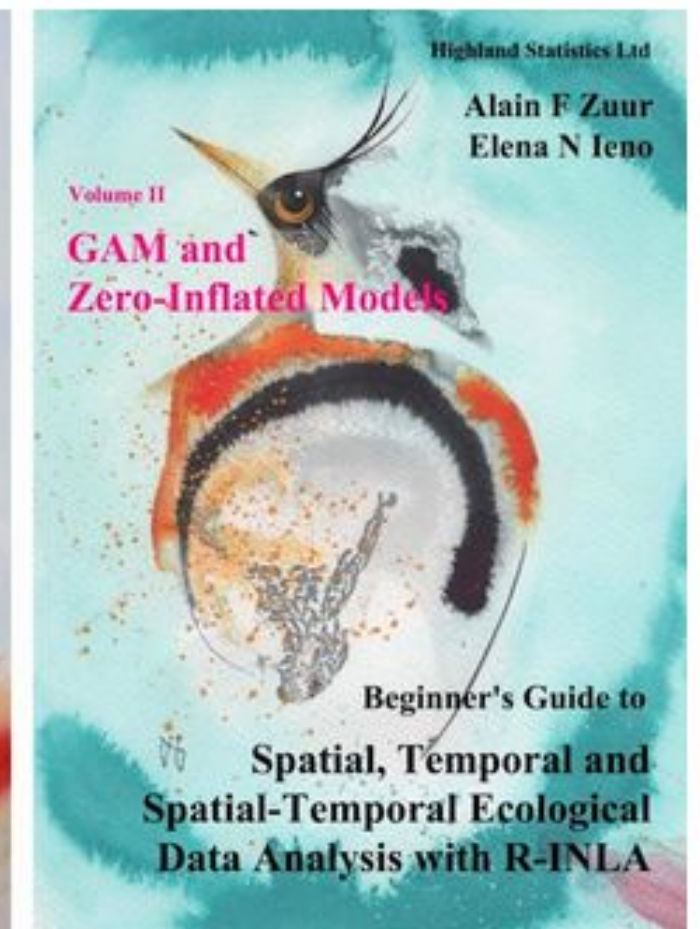
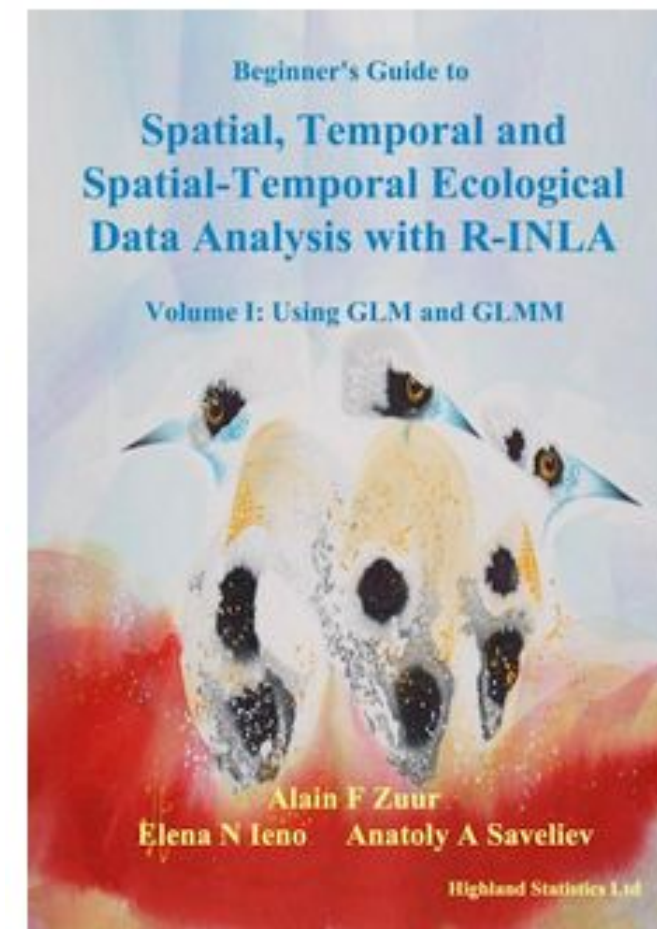
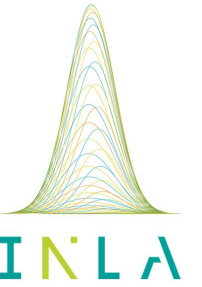
Posterior distributions of *C. maritima*: the niche variables sand dynamics and springtide elevation include zero; the plant distribution on beach-dune borders thus appears to be neutral with respect to these niche parameters. This pattern also largely holds for the other plant species *S. kali*, *H. peploides* and *E. farctus*.



Intercept
Springtide elevation
Sand dynamics

Want to learn how to implement INLA?

- Books we have at the lab
- Ask us for help!
- Coding club devoted to INLA examples?





ABC

What is ABC?



- Approximate Bayesian Computation
- Bypass the evaluation of the likelihood function
- Compare different (complex) models and estimate parameter values
- Application: evaluation of complex models in population genetics, epidemiology, ecology
- E.g.: IBM/ABM

Bypassing evaluation of the likelihood?!

$$\text{Posterior Probability } P(A|B) = \frac{\text{Prior Probability } P(A) \times \text{Likelihood } P(B|A)}{\text{Evidence } P(B)}$$

P(A|B) the probability of A happening given that B happens. Aka posterior probability.

P(B|A) the probability of B happening given that A happens. Aka likelihood.

P(A) the probability of A happening on its own. Aka prior probability.

P(B) the probability of B happening on its own. Aka evidence.

Derived from statistical model:

- Compatibility of the evidence (B) with the given hypothesis (A)
- Function of the evidence (B)

Probability of the data given the parameters

Bypassing evaluation of the likelihood?!

$$P(A|B) = \frac{P(A) P(B|A)}{P(B)}$$

Prior Probability points to $P(A)$
Likelihood points to $P(B|A)$
Posterior Probability points to $P(A|B)$
Evidence points to $P(B)$

$P(A|B)$ the probability of A happening given that B happens. Aka posterior probability.

$P(B|A)$ the probability of B happening given that A happens. Aka likelihood.

$P(A)$ the probability of A happening on its own. Aka prior probability.

$P(B)$ the probability of B happening on its own. Aka evidence.

Probability of the data given the parameters

Probability function of your model formulation

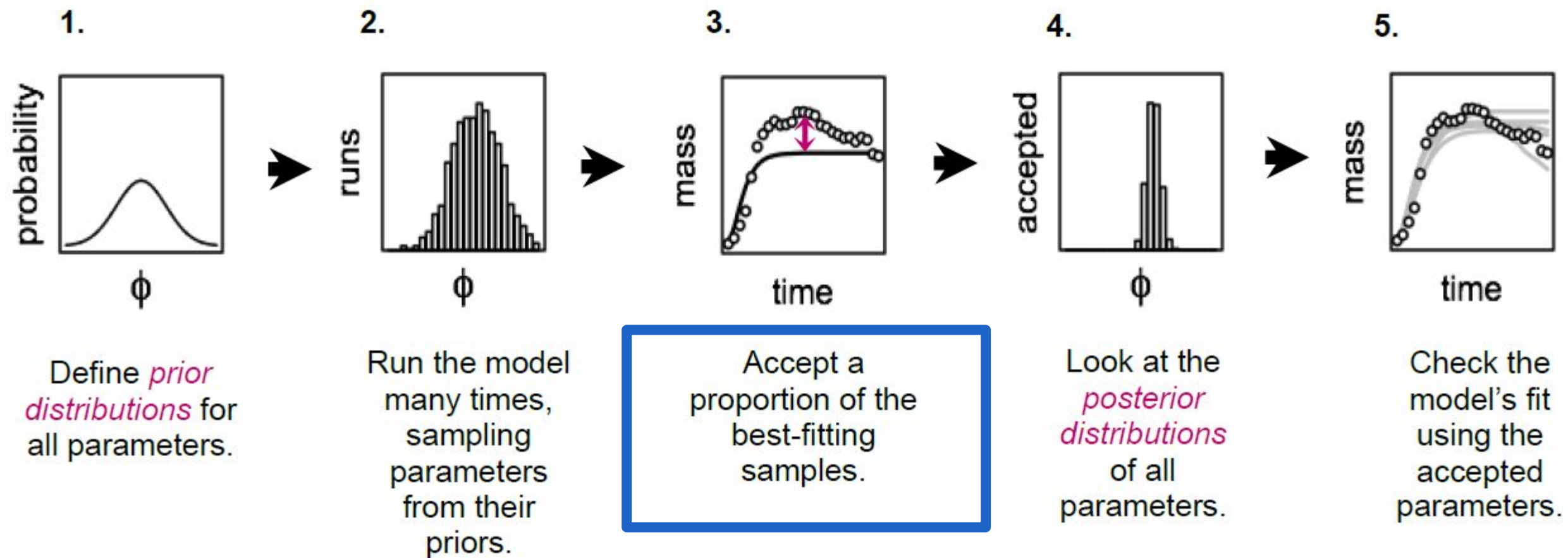
Deriving likelihood functions analytically is not possible for complex models like IBMs

→ ABC approximates likelihood using simulations

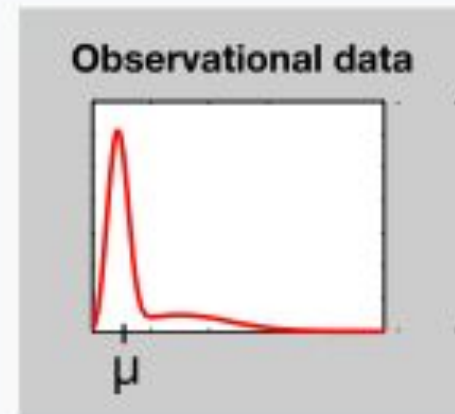
Here: approximation of likelihood
INLA: approximation of posteriors
(MCMC samples posterior)

Parameter estimations with ABC

- Where ' ϕ ' is a model parameter
- Where 'mass' [of an individual] is an example of a modelled property

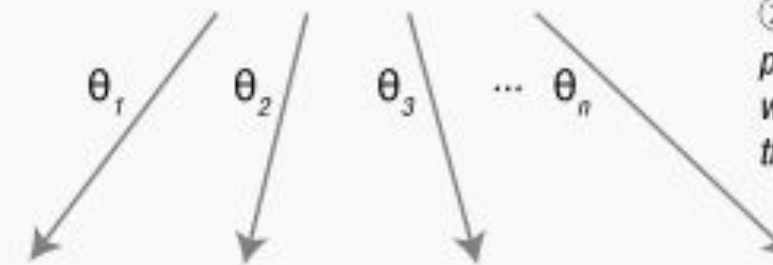


Based on summary statistics

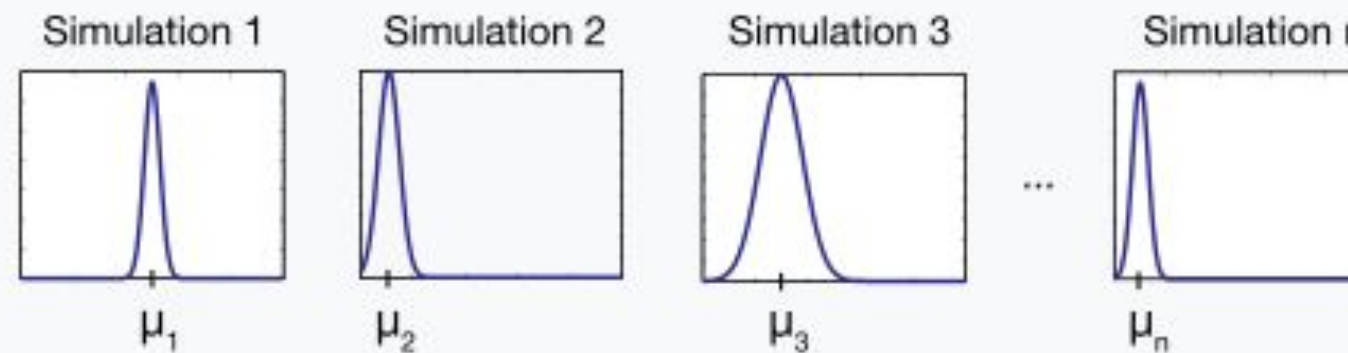


① Compute summary statistic μ from observational data

Prior distribution of model parameter θ



② Given a certain model, perform n simulations, each with a parameter drawn from the prior distribution



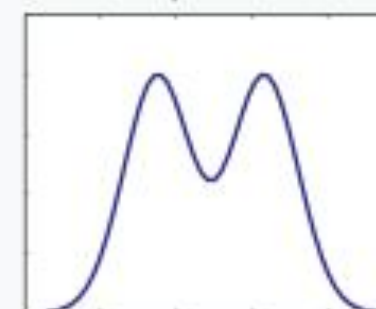
③ Compute summary statistic μ_i for each simulation

$$\rho(\mu_i, \mu) \stackrel{?}{\leq} \varepsilon$$



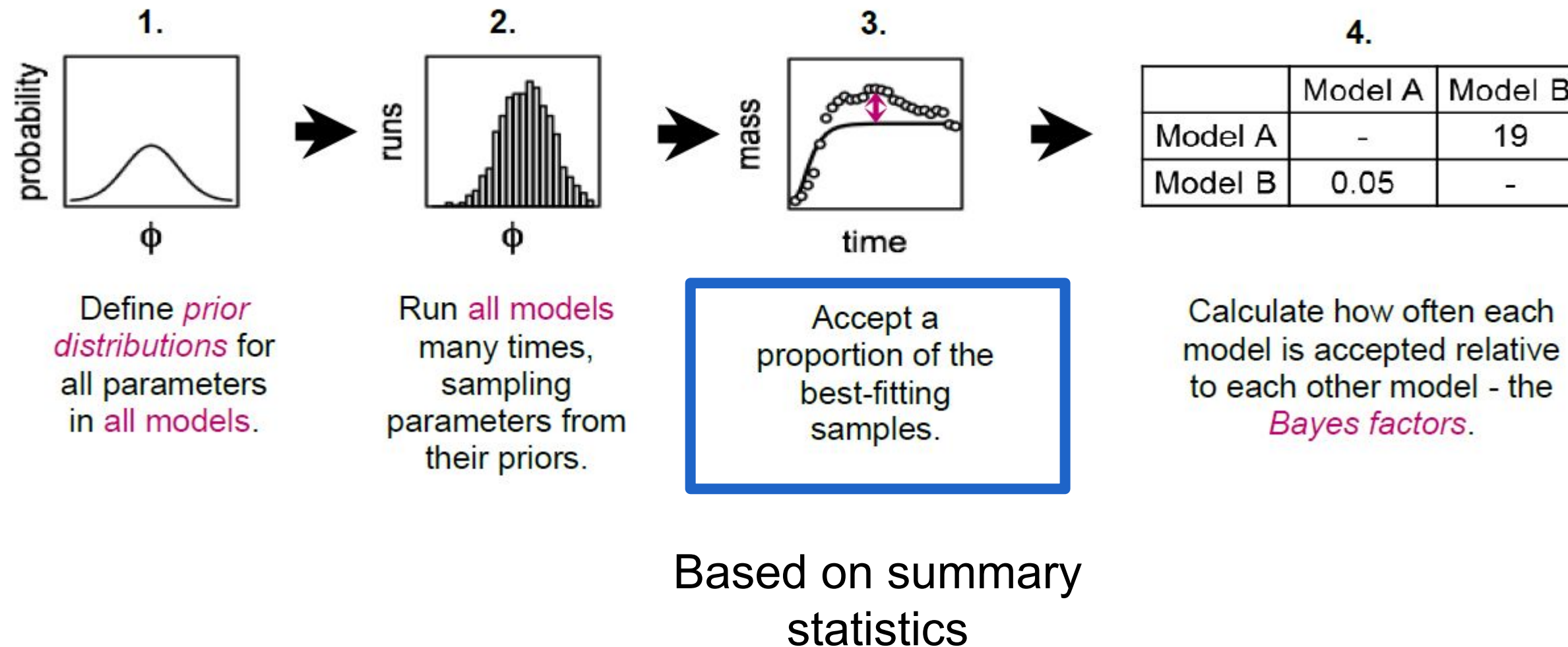
④ Based on a distance $\rho(\cdot, \cdot)$ and a tolerance ε , decide for each simulation whether its summary statistic is sufficiently close to that of the observed data.

Posterior distribution of model parameter θ



⑤ Approximate the posterior distribution of θ from the distribution of parameter values θ_i associated with accepted simulations.

Model selection with ABC

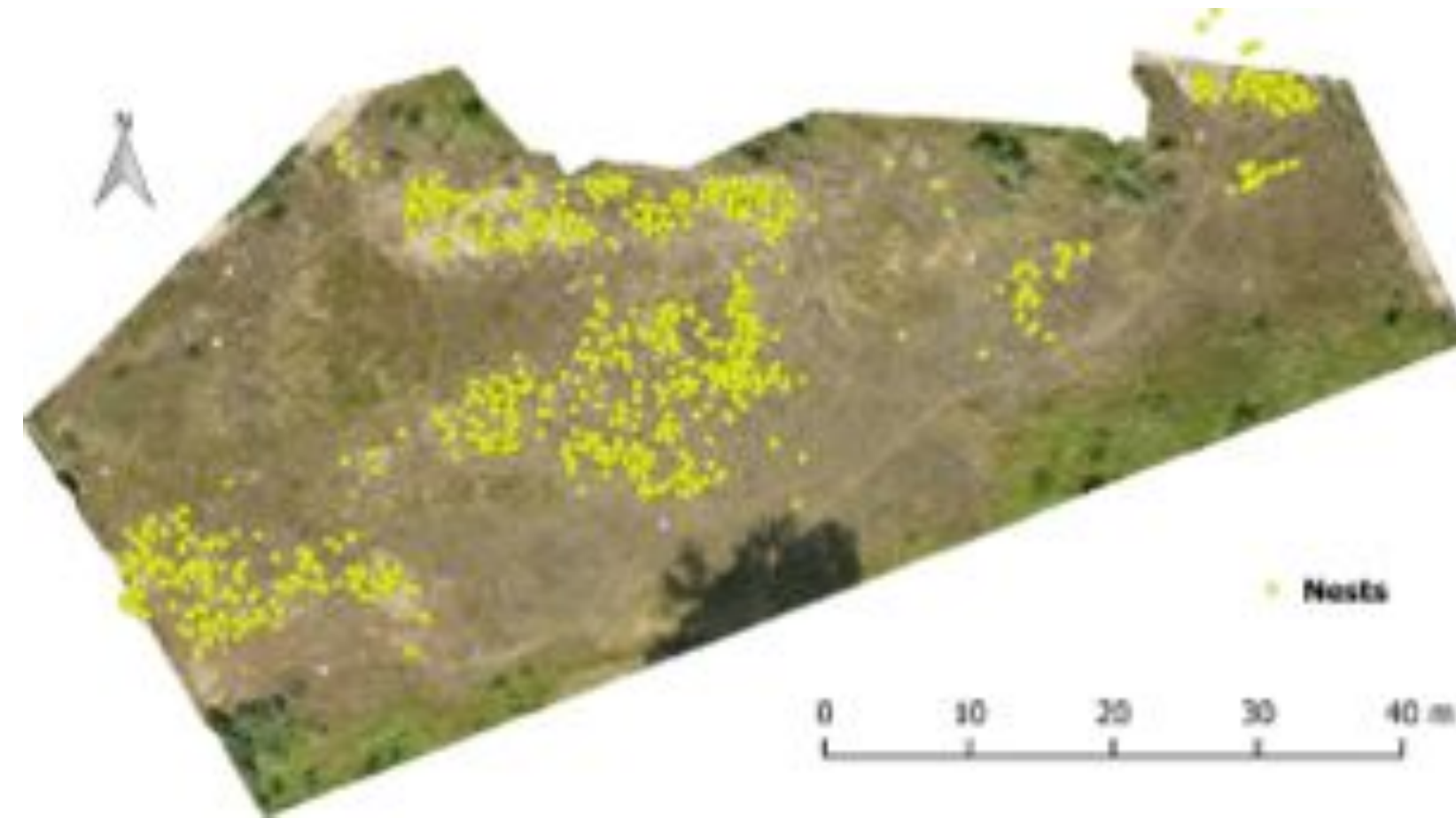


Quick example

Batsleer, Maes & Bonte (2022) AmNat



- IBM of nest spatial pattern formation of *B. rostrata*
- Evaluate with ABC4IBM:
 - spatial pattern modelled \longleftrightarrow spatial pattern in the field
- Observation in the field: spatial patterns of nests



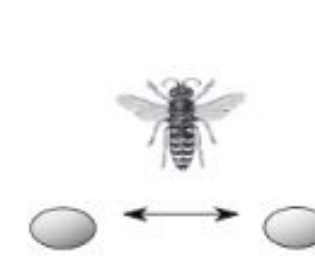
Quick example

- Model: parameters (3 strength of mechanisms + model specifics) + 3 hypotheses how these mechanism vary across the population
- Summary statistics: spatial clustering (standardized Ripley's K) + network metrics

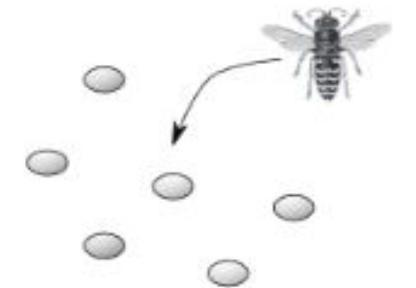
1) Microhabitat model
with *NDVI* and *Insolation*



Environment
ENV

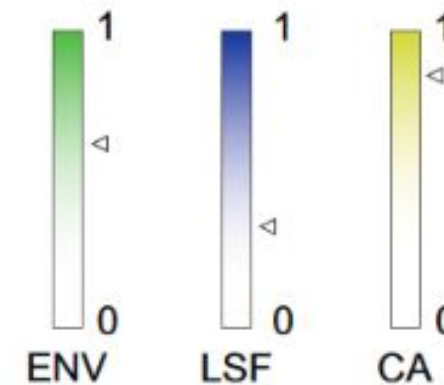


Local site fidelity
LSF



Conspecific attraction
CA

2) Individual-Based Model



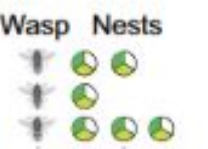
Strengths

Probability of
mechanism
present

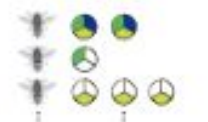
Submodels

Level of
variability of
mechanisms

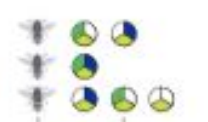
Population
mechanisms
uniformly distributed



Interindividual
mechanisms **fixed**
between individuals

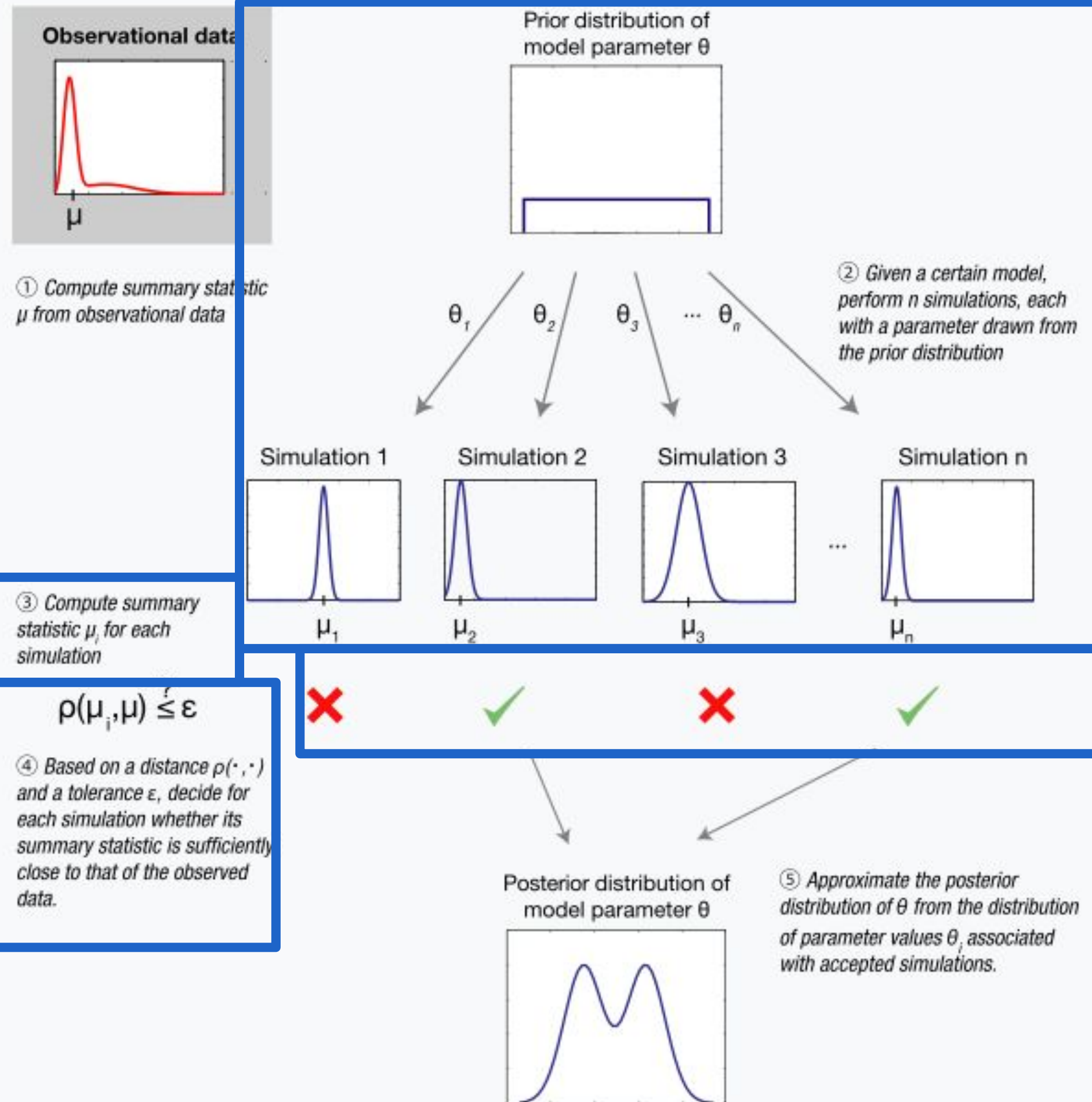


Intraindividual
mechanisms **flexible**
within individuals



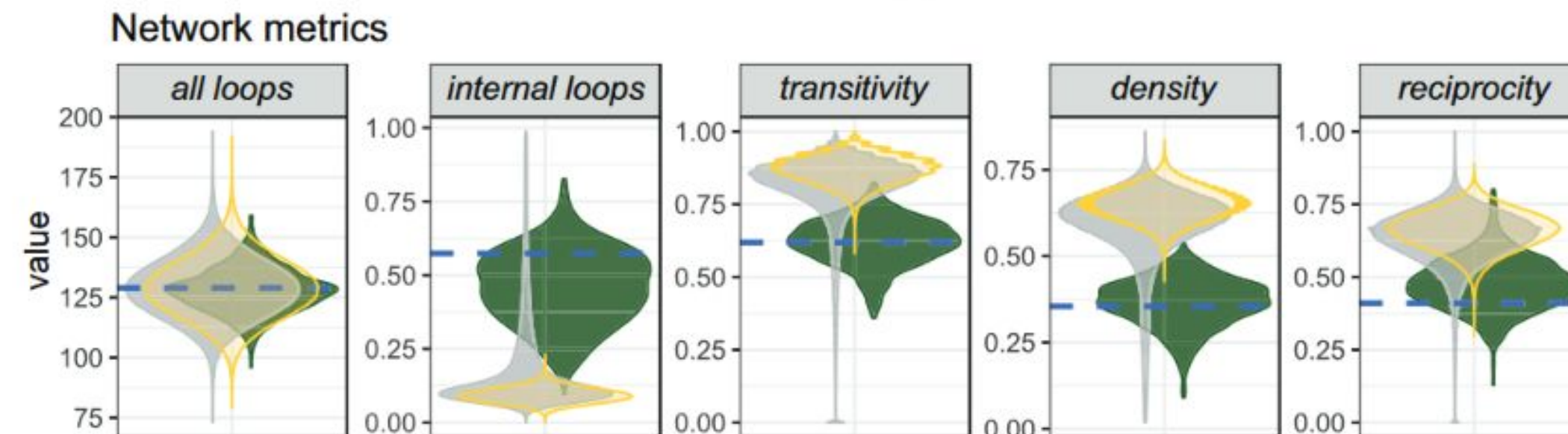
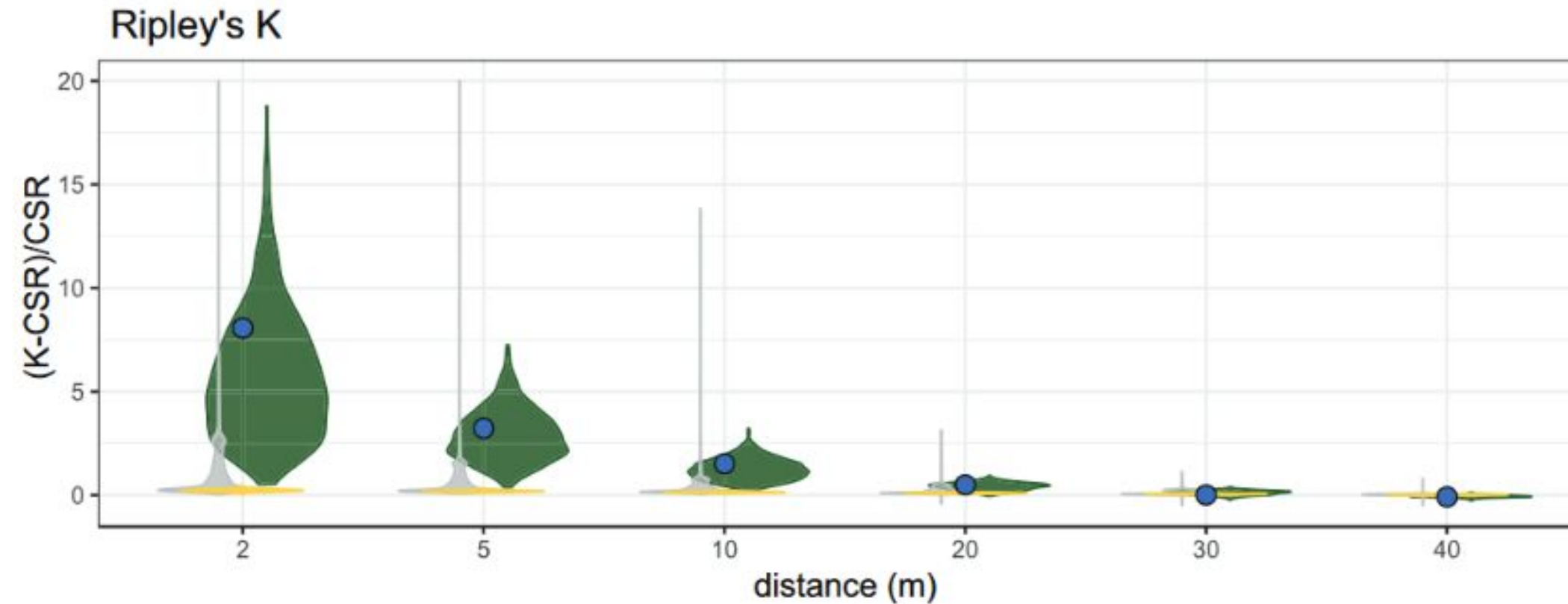
Quick example

- Drawing from prior distribution of model parameters \rightarrow run 1.000.000 simulations
- Calculate summary statistics
- Calculate distance of summary stats of each run tot the observational data
- Accepted 1000 (0.01%) best models



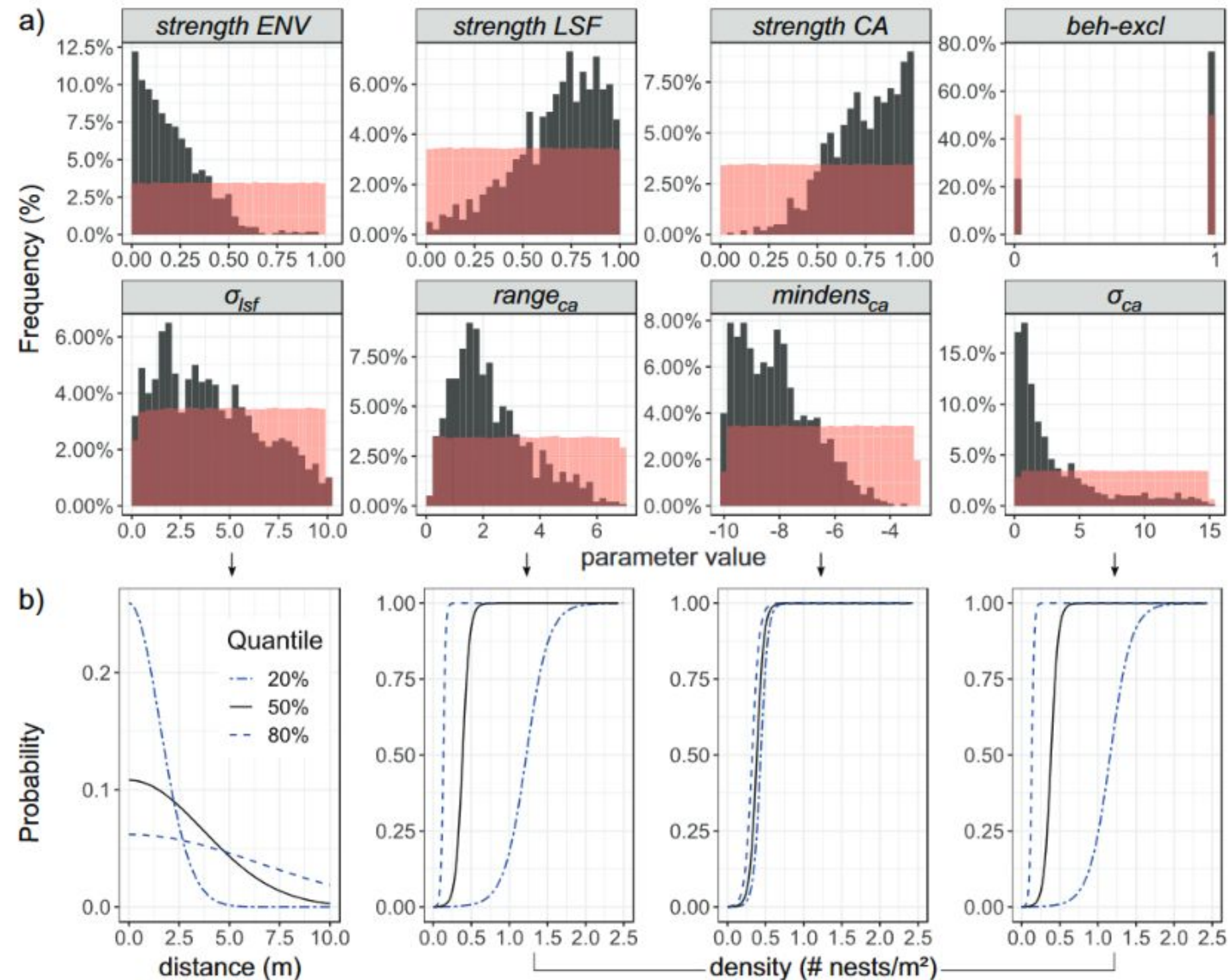
Quick example

Prior & posterior summary statistics



Quick example

Posteriors of model parameters



Quick example

Model selection

hypotheses on variation of mechanisms across population

Table 1: Bayes factors (BFs) and proportions of accepted models for model selection with approximate Bayesian computation (ABC) analysis

	Random	Population	Interindividual	Intraindividual	% accepted simulation
Random	–	.00	.00	.00	0
Population	∞	1.00	.31	.36	14.3
Interindividual	∞	3.20	1.00	1.14	45.7
Intraindividual	∞	2.80	.88	1.00	40.0

Note: The ABC analysis retained the 1,000 best simulations of 1,000,000 (0.1%). The submodels represent at which level the mechanisms can vary: population, interindividual, or intraindividual. BFs are the ratios of the posterior probabilities of two models, indicating the strength of evidence for model M_1 (rows) relative to model M_0 (columns), given the data. Evidence categories according to Kass and Raftery (1995) are as follows: $BF < 1$ indicates more evidence for M_0 than M_1 , $1 < BF < 3$ indicates weak evidence for M_1 compared with M_0 , and $3 < BF < 10$ indicates substantial evidence for M_1 compared with M_0 .

ABC conclusions



The algorithm itself is actually easy to implement!

Comes with a lot of caveats and uncertainties

- needed to perform prior predictive checks, weight summ stats,...
- a lot of simulations needed!

Needs well-balanced hypotheses formulation

Want to learn how to implement ABC?



Van der Vaart et al. (2015, 2016) Ecological modelling

Ask for help!

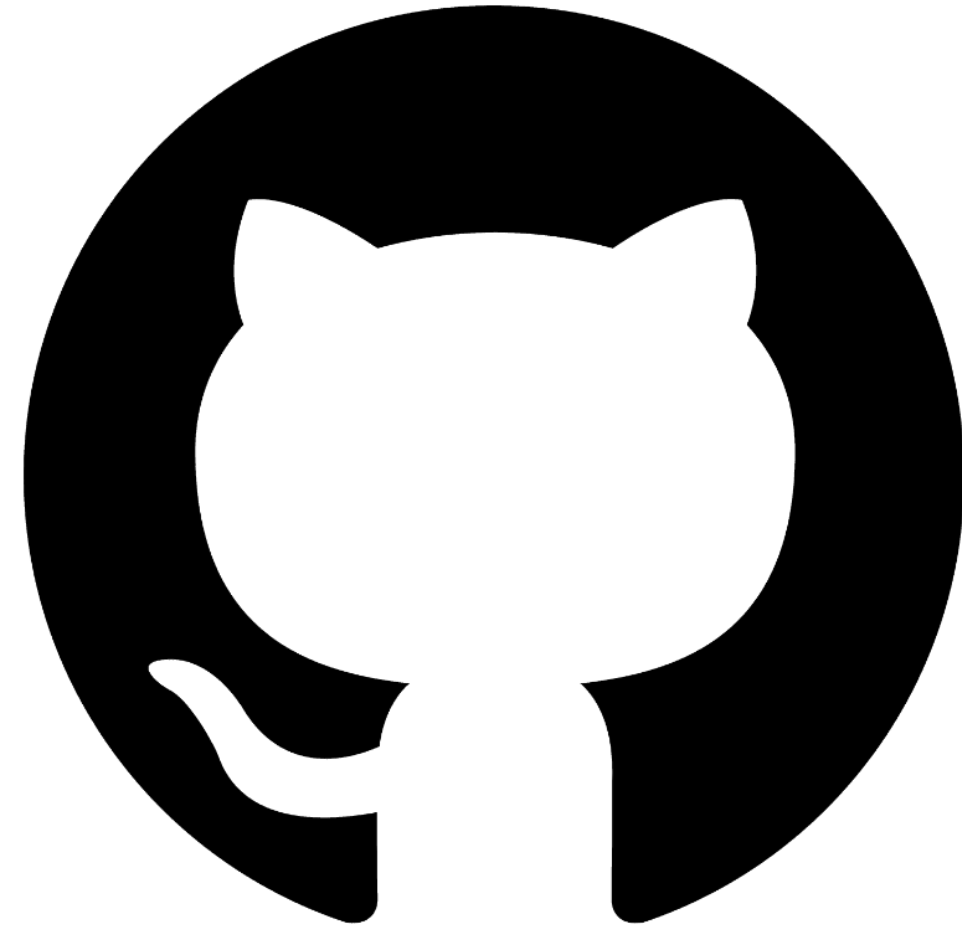
One of the next coding clubs?



VARIA

VARIA

- Next sessions: 22/4, 27/5, 17/6
- Knowledge assessment: Github?



GitHub

VARIA

- Next sessions: 22/4, 27/5, 17/6
- Knowledge assessment: Github?
- Next session theme: Github + Bayesian Deep Dive
- Other desired themes?
- Q&A
- Wrap-up



Wrap-up



Our minds = Bayesian

Posterior distributions in Brms

Spatial Autocorrelation of INLA

Evaluate complex models with ABC

REMAINING QUESTIONS ON THIS SESSION?

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