Measurement of Gaussian beam width using knife edge

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MEASUREMENT OF GAUSSIAN BEAM WIDTH USING KNIFE-EDGE

A Report Submitted for Summer Project

By
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DECLARATION

I hereby declare that I am the sole author of this project report for summer project at National Institute of Science Education and Research (NISER). I authorize NISER to lend this report to other institutions or individuals for the purpose of scholarly research.

Signature of the Student

Date:

The project work reported in the report entitled "Measurement of the width of a Gaussian Beam using Knife-Edge" was carried out under my supervision, in the school of Physical Sciences at NISER, Bhubaneswar, India.

Signature of the project supervisor

School:

Date:

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Introduction & Theory

The standard laser beams used in laboratory are Gaussian in nature. A Gaussian beam propagating in the z-direction may be represented as

$$U = \frac{U_0 e^{i(kz-\omega t)} e^{\frac{ikr^2}{2q}}}{q} \qquad \textit{where } q = z - z_{waist} - iz_R \tag{1}$$

Here U is the electric field amplitude of the wave, q is known as the '*complex radius*', z_{waist} is a real constant indicating the position of the beam waist and z_R is known as the '*Rayleigh range*'.

After passing through a convex lens, the radius of curvature of the wavefronts $R(z)$ obeys:

$$R(z) = z + \frac{z_R^2}{z} \tag{2}$$

The width or thickness of a Gaussian beam can be measured by using a Knife-edge. A knife edge has a thin edge which is used to penetrate the laser beam. This leads to blockage of some portion of light depending on the portion of knife inside the beam. From the intensity measured at various positions of the knife-edge perpendicular to the laser beam, the thickness of the laser can be obtained from the nature of a Gaussian curve.

The Gaussian nature can also be verified using another property. The laser beam is allowed to pass through a convex lens and again re-focussed into the EM detector. In between the beam becomes the shape of a 3-dimensional hyperbola. The thickness of the beam at regular intervals can be measured using knife-edge as described above. Then, the nature of the beam can be revealed from the graph drawn from the values of the beam-width at various positions along the hyperbolic beam.

Experimental Set-up

There are two experimental set-ups in the project.

1. Set-up 1

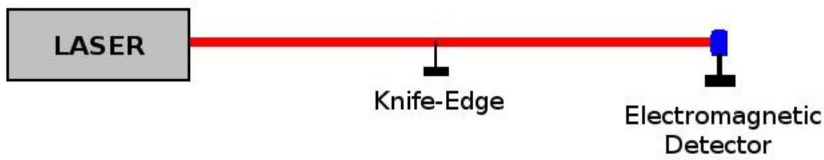


Figure 1: First experimental set-up

In the first set-up, the laser, the knife and the EM detector are all placed in the same line. The knife-edge is placed midway between the other two. The knife is placed on a one-directionally movable mount having screw-gauze for distance measurement. The detector is connected to a multimeter to take the corresponding voltage readings.Optical attenuators are used to decrease the intensity of the laser and optical wavelength filters are used to restrict the detection of only the laser, if required. All the apparatus are fixed on their mounts and bolted to the optical bench to restrict undesirable movements.

2. **Set-up 2**

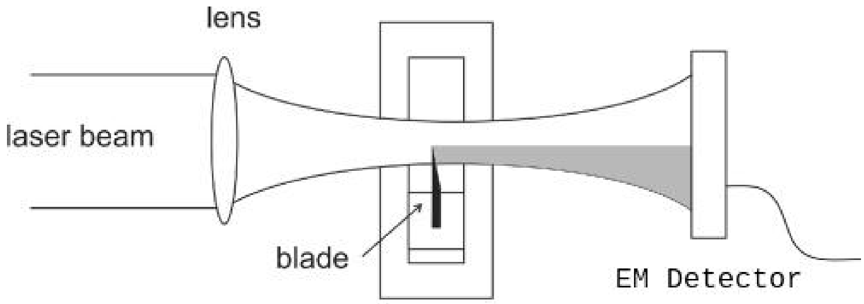


Figure 2: Second experimental set-up

In the second set-up, slight modifications are made to the first set-up. A plano-convex lens of focal-length 5 cm is placed 5 cm towards left of the knife. Another movable mount having screw-gauze is placed just below the first mount such that movement is possible along the direction of propagation of the laser. Another plano-convex lens of focal length 10 cm is placed at 7 cm towards the right of the knife and the distance of the detector from this second lens is fixed in such a way that the beam is refocussed as a point on the detector surface. As in the previous set-up the detector is connected to the multimeter to take voltage readings and optical attenuators & wavelength filters are used. All the apparatus are fixed to their mounts and are properly bolted to the optical bench to restrict undesirable movements.

Procedure

There are two parts of the experiment, one for each set-up.

1. The apparatus are arranged as shown and explained in experimental set-up 1. The laser is switched on and made ready. The EM detector and the connected voltmeter are switched on. The knife is moved forward using the screw-gauze. A point comes at which there is a change in voltage reading. From that point, the knife is moved at regular intervals and the corresponding voltage reading is noted down along with the position. This process is continued till the laser beam is completely blocked by the knife and hence the voltage reading falls to zero. The data is then fitted to a suitable function and the value of radius of the beam is determined.

2. The apparatus are arranged as shown and explained in experimental set-up 2. The knife is placed as close as possible to the lens placed on its left. The procedures for the part 1 of the experiment are repeated. Now, the second movable mount of the knife is moved such that the knife moves along the propagation of laser away from the lens on its left. A position is fixed and the procedures of part 1 are followed again. The above procedures are repeated for different positions along the direction of laser at regular intervals till the knife reaches the lens on its right (f = 10 cm). The data for the various positions is then fitted and the radius of beam at each of the positions is determined. The radius is then plotted against the position of the knife along the laser and fitted to another suitable function and the parameters are determined. It also verifies that the laser is indeed a Gaussian beam.

Measurement of Gaussian beam width using knife-edge

The data collected from both the parts of the experiment were fitted to the following function using Mathematica code :

$$f(x) = \int_x^{\infty} a e^{\frac{-(x-x_o)^2}{2\sigma^2}} dx \tag{3}$$

The function used is known as *Complementary Error Function*.
The values of a, x_o and σ were determined for each position using Mathematica.

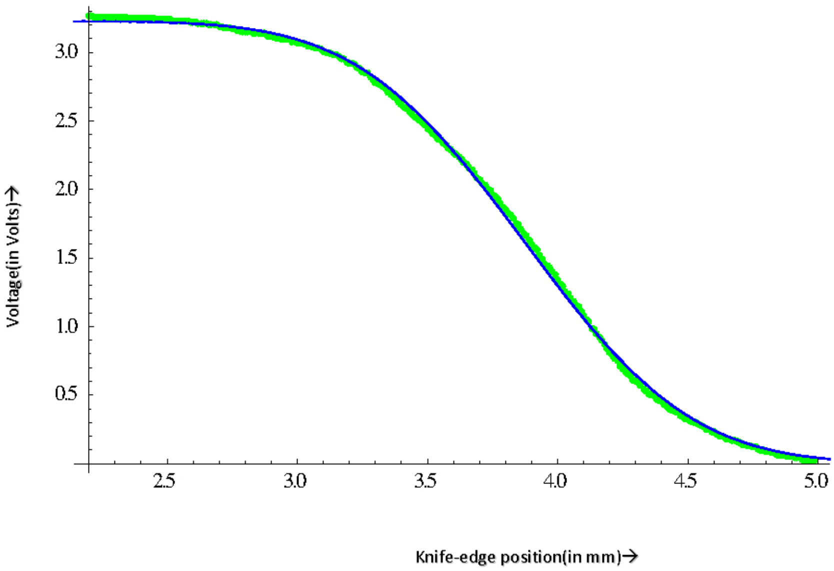
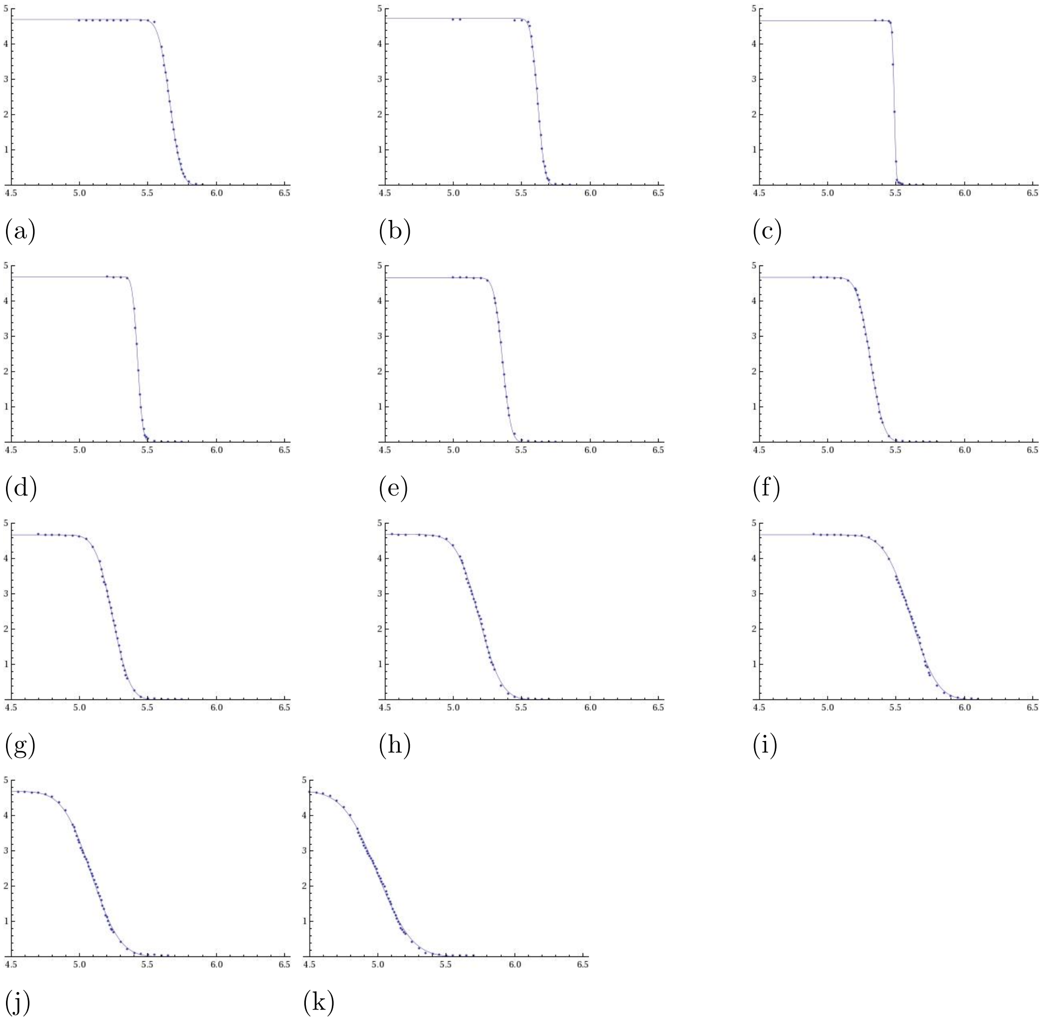


Figure 3: Data points from first set of data obtained from first part of the experiment, fitted to equation 3 using Mathematica

The following values were determined from the first set of data :
$a = 2.54904 \pm 0.00933801$
$x_o = 3.87403 \pm 0.0016013mm$
$\sigma = 0.505263 \pm 0.00210117mm$
Hence, the width of the laser beam = 2σ = 1.010526 mm

The second set of data was also fitted to equation 3 and the following results were obtained:



(a) (b) (c) (d) (e) (f) (g) (h) (i) (j) (k)

Figure 4: Data points from second set of data obtained from second part of the experiment, fitted to equation 3 using Mathematica in the order of observation and position.

The following data was obtained from the fitting :

'x' *Position* (in mm)	a	x_o (in mm)	σ (in mm)	*Beam width*(2σ) (in mm)
0	28.2491	5.66169	0.0664406	0.1328812
2	46.7554	5.61855	0.0404142	0.0808284
4	105.754	5.5473	0.0178488	0.0356976
6	152.947	5.48789	0.0121925	0.024385
8	61.7892	5.42564	0.0303278	0.0606556
10	37.5145	5.36044	0.0496412	0.0992824
12	24.0898	5.31311	0.0775234	0.1550468
14	18.6468	5.24328	0.100121	0.200242
16	14.409	5.18852	0.129819	0.259638
18	12.3302	5.60952	0.15151	0.30302
20	11.4112	5.08756	0.164073	0.328146
22	9.47445	4.99859	0.198325	0.39665

The σ values for each positions were then plotted against their respective positions.

The data points were again fitted to the following equation using Mathematica :

$$f(x) = a\left(1 + \left(\frac{x - x_1}{x_o}\right)^2\right)^{\frac{1}{2}} \tag{4}$$

The following graph was obtained after the fitting :

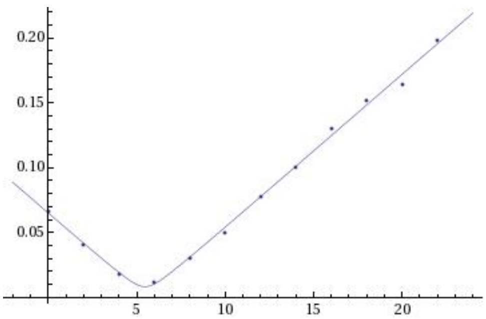


Figure 5: σ values for various positions of the knife; fitted to *equation* 4 using Mathematica

The parameters a,$x1$ and x_o were determined from the fitting as follows :

$$
\begin{aligned}
a &= 0.00832883 \pm 0.00400553 \\
x_1 &= 5.47293 \pm 0.130966\, mm \\
x_o &= 0.704925 \pm 0.341054\, mm
\end{aligned}
$$

Conclusion

The above analysis verifies the Gaussian nature of the beam and gives idea about the change in beam shape after passing through a plano-convex lens. From this experiment, we determine that the profile of the beam can be described by the following function :

$$
f(x) = \int_x^\infty 2.54904 e^{\frac{-(x-3.87403)^2}{2*(0.505263^2)}} dx \tag{5}
$$

The thickness of the beam is determined to be 1.010526 mm.
After passing through a lens, the property of the beam can be described by the following equation :

$$
f(x) = 0.00832883 \left(1 + \left(\frac{x - 5.47293}{0.704925} \right)^2 \right)^{\frac{1}{2}} \tag{6}
$$

Hence, the Gaussian nature of the beam is verified along with the width of the beam.

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