

第三问：基于剪枝方法的最佳常驻点求解

Formulation:

$$\begin{aligned} & \min_{O} \min_{x_{ijk}} d \\ \text{s. t. } & \begin{cases} d = \max_{k \in K} \sum_{i \in V} \sum_{j \in V} c_{ij} x_{ijk} \\ \sum_{j \in V} \sum_{k \in K} x_{ijk} = 1, \forall i \in V - \{O\} \\ \sum_{j \in V} x_{ijk} - \sum_{j \in V} x_{jik} = 0, \forall i \in V, k \in K \\ \sum_{j \in V} x_{0jk} = 1, \forall k \in K \\ \sum_{i \in S} \sum_{j \in S} x_{ijk} \leq |S| - 1, \forall S \subset V - \{O\}, S \neq \emptyset, k \in K \\ x_{iik} = 0, \forall i \in V, k \in K \\ x_{ijk} \in \{0, 1\} \end{cases} \end{aligned}$$

Algorithm:

Branch and cut算法是在Branch and bound算法的基础上增加切平面。回顾Branch and bound，如果子问题的下界（松弛LP问题的解）超过了规定的上界，就要剪去（此处可有图，图略）。在第二问的求解过程中，上界由子问题求解结果得到，而在LRP中，我们将规定上界为前N个常驻点求解的最小值。伪代码如下：

```
将 第一问求解的d值 赋值给 d_min
将 21个点 按一定顺序 依次赋值给 0 // 一定顺序可以根据各点距离图中心某点远近进行排序
建立MTSP模型
增加约束：d <= d_min
利用Branch and cut算法求解
解得d
如果 d < d_min
    将 d 赋值给 d_min
解得d_min以及对应的0点
```

Code:

```
from gurobipy import *
import pickle
import networkx as nx

# define the constants
V_NUM = 21
```

```

K_NUM = 2
ORIGIN_IDX = 5
MAX = 1e5

# load and generate basic data
f = open("../data/distance", mode="rb")
distance = pickle.load(f)
f.close()
dist = {(i, j, k): distance[i][j] for i in range(V_NUM) for j in range(V_NUM) for k in
range(K_NUM)}
dict_linear = {(i, j, k): i*V_NUM*K_NUM + j*K_NUM + k for i in range(V_NUM) for j in
range(V_NUM) for k in range(K_NUM)}
dict_3d = mi = dict(zip(dict_linear.values(), dict_linear.keys()))

# callback - use lazy constraints to eliminate sub-tours
def mycallback(model, where):
    if where == GRB.Callback.MIPSOL:
        vals = model.cbGetSolution(model._vars[:-1])
        edges = list((i, j, k) for i, j, k in dict_linear.keys() if
vals[dict_linear[(i, j, k)]] > 0.5)
        shortest_cycle, k = cycle(edges)
        if shortest_cycle is not None:
            model.cbLazy(quicksum(x[i, j, k] for i in shortest_cycle for j in
shortest_cycle) <= len(shortest_cycle) - 1)

# find the loops
def cycle(edges):
    node_vehicle = {}
    G = nx.DiGraph()
    for e in edges:
        G.add_edge(e[0], e[1])
        node_vehicle[e[0]] = e[2]
        node_vehicle[e[1]] = e[2]
    shortest_subtour = None
    min = V_NUM
    cycle: list
    for cycle in nx.simple_cycles(G):
        if cycle.count(ORIGIN_IDX - 1) == 0:

```

```

        if len(cycle) < min:
            min = len(cycle)
            shortest_subtour = cycle
    if shortest_subtour is None:
        k = None
    else:
        k = node_vehicle[shortest_subtour[0]]
    return shortest_subtour, k

dmin = 201.24197185085075
origin_list = [2,6,1,5,4,3,7,11,10,19,9,16,20,8,21,14,17,15,18,13,12]
for o in origin_list:
    ORIGIN_IDX = o
    node_list = list(range(ORIGIN_IDX - 1)) + list(range(ORIGIN_IDX, V_NUM))

    # create a model
    MODEL = Model()
    # MODEL.setParam('OutputFlag', 0)

    # add variables
    x = MODEL.addVars(dist.keys(), obj=dist, vtype=GRB.BINARY, name='x')
    d = MODEL.addVar(name="d")
    MODEL.update()

    # set the objective
    MODEL.setObjective(d, GRB.MINIMIZE)

    # add constraints
    MODEL.addConstrs(quicksum(x[i, j, k] for j in range(V_NUM) for k in range(K_NUM))
== 1 for i in node_list)
    MODEL.addConstrs(
        quicksum(x[i, j, k] for j in range(V_NUM)) - quicksum(x[j, i, k] for j in
range(V_NUM)) == 0 for i in
        range(V_NUM) for k in range(K_NUM))
    MODEL.addConstrs(quicksum(x[ORIGIN_IDX - 1, j, k] for j in range(V_NUM)) == 1 for
k in range(K_NUM))
    MODEL.addConstrs(
        d - quicksum(distance[i][j] / 1.5 * x[i, j, k] for i in range(V_NUM) for j in
range(V_NUM)) >= 0 for k in

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```

        range(K_NUM))
MODEL.addConstrs(x[i, i, k] == 0 for i in range(V_NUM) for k in range(K_NUM))
MODEL.addConstrs(x[i, j, k] + x[j, i, k] <= 1 for i in range(V_NUM) for j in
range(V_NUM) for k in range(K_NUM))
MODEL.addConstr(d <= dmin)

MODEL._vars = MODEL.getVars()
MODEL.Params.lazyConstraints = 1
MODEL.optimize(mycallback)

try:
    d_val = MODEL.getVarByName('d').x
    if d_val < dmin:
        dmin = d_val
    print(ORIGIN_IDX, MODEL.getVarByName('d'))
except AttributeError:
    pass

```

Result:

Changed value of parameter lazyConstraints to 1

Prev: 0 Min: 0 Max: 1 Default: 0

Gurobi Optimizer version 9.1.2 build v9.1.2rc0 (mac64)

Thread count: 2 physical cores, 4 logical processors, using up to 4 threads

Optimize a model with 990 rows, 883 columns and 5168 nonzeros

Model fingerprint: 0x200cb6b7

Variable types: 1 continuous, 882 integer (882 binary)

Coefficient statistics:

Matrix range [1e+00, 1e+02]

Objective range [1e+00, 1e+02]

Bounds range [1e+00, 1e+00]

RHS range [1e+00, 1e+00]

Presolve removed 504 rows and 42 columns

Presolve time: 0.02s

Presolved: 486 rows, 841 columns, 4130 nonzeros

Variable types: 1 continuous, 840 integer (840 binary)

Root relaxation: objective 4.186460e+02, 79 iterations, 0.00 seconds

Nodes			Current Node			Objective Bounds			Work	
Expl	Unexpl		Obj	Depth	IntInf	Incumbent	BestBd	Gap	It/Node	Time
	0	0	418.64600	0	44	-	418.64600	-	-	0s
	0	0	491.59300	0	6	-	491.59300	-	-	0s
	0	0	491.59300	0	22	-	491.59300	-	-	0s
	0	0	491.59300	0	22	-	491.59300	-	-	0s
H	0	0				858.3580000	491.59300	42.7%	-	0s
H	0	0				836.9446667	491.59300	41.3%	-	0s
H	0	2				726.3273333	491.59300	32.3%	-	0s
	0	2	491.59300	0	16	726.32733	491.59300	32.3%	-	0s
H	154	179				699.1166667	528.15400	24.5%	6.3	0s
H	450	395				671.9060000	528.15400	21.4%	5.3	0s
H	749	613				661.8660000	530.25000	19.9%	6.3	1s
H	756	587				635.2026667	544.17800	14.3%	6.3	1s
H	756	558				618.3880000	544.17800	12.0%	6.3	1s
H	758	530				586.7400000	546.94551	6.78%	6.3	1s
H	810	526				571.1933333	553.02531	3.18%	9.2	2s

Cutting planes:

Gomory: 11

MIR: 1

Flow cover: 9

Inf proof: 4

Zero half: 11

RLT: 6

Lazy constraints: 2

Explored 1524 nodes (17168 simplex iterations) in 3.08 seconds

Thread count was 4 (of 4 available processors)

Solution count 9: 571.193 586.74 618.388 ... 858.358

Optimal solution found (tolerance 1.00e-04)

Best objective 5.711933333333e+02, best bound 5.711933333333e+02, gap 0.0000%

User-callback calls 4092, time in user-callback 0.11 sec

<gurobi.Var x[0,1,0] (value 1.0)>

<gurobi.Var x[0,17,1] (value 1.0)>

```

<gurobi.Var x[1,5,0] (value 1.0)>
<gurobi.Var x[2,10,0] (value 1.0)>
<gurobi.Var x[3,2,0] (value 1.0)>
<gurobi.Var x[4,0,1] (value 1.0)>
<gurobi.Var x[5,3,0] (value 1.0)>
<gurobi.Var x[6,4,1] (value 1.0)>
<gurobi.Var x[7,19,1] (value 1.0)>
<gurobi.Var x[8,9,1] (value 1.0)>
<gurobi.Var x[9,7,1] (value 1.0)>
<gurobi.Var x[10,11,0] (value 1.0)>
<gurobi.Var x[11,12,0] (value 1.0)>
<gurobi.Var x[12,13,0] (value 1.0)>
<gurobi.Var x[13,14,0] (value 1.0)>
<gurobi.Var x[14,0,0] (value 1.0)>
<gurobi.Var x[15,18,1] (value 1.0)>
<gurobi.Var x[16,15,1] (value 1.0)>
<gurobi.Var x[17,16,1] (value 1.0)>
<gurobi.Var x[18,8,1] (value 1.0)>
<gurobi.Var x[19,20,1] (value 1.0)>
<gurobi.Var x[20,6,1] (value 1.0)>
<gurobi.Var d (value 192.30266666666665)>

```

Process finished with exit code 0

寻找过程输出：

```

1 Changed value of parameter lazyConstraints to 1
2   Prev: 0  Min: 0  Max: 1  Default: 0
3 Gurobi Optimizer version 9.1.2 build v9.1.2rc0 (mac64)
4 Thread count: 2 physical cores, 4 logical processors, using up to 4 threads
5 Optimize a model with 991 rows, 883 columns and 5169 nonzeros
3 Model fingerprint: 0xca69f6c6
7 Variable types: 1 continuous, 882 integer (882 binary)
3 Coefficient statistics:
9   Matrix range      [1e+00, 1e+02]
0   Objective range   [1e+00, 1e+00]
1   Bounds range      [1e+00, 1e+00]
2   RHS range         [1e+00, 2e+02]
3 Presolve removed 505 rows and 42 columns

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4 Presolve time: 0.01s
5 Presolved: 486 rows, 841 columns, 4212 nonzeros
5 Variable types: 1 continuous, 840 integer (840 binary)
7
3 Root relaxation: objective 1.385773e+02, 115 iterations, 0.00 seconds
9
0      Nodes      |      Current Node      |      Objective Bounds      |      Work
1 Expl Unexpl | Obj Depth IntInf | Incumbent    BestBd  Gap | It/Node Time
2
3      0      0 138.57733      0 44      - 138.57733      -      -      0s
4      0      0 166.97500      0 10      - 166.97500      -      -      0s
5      0      0 166.97500      0 24      - 166.97500      -      -      0s
6      0      0 166.97500      0 22      - 166.97500      -      -      0s
7      0      2 180.36233      0 36      - 180.36233      -      -      0s
3    7213   3271 infeasible    29      - 189.66997      - 11.6    5s
9   16348   7249 infeasible    50      - 194.68030      - 10.5   10s
0   30425  14306 200.05783    39 16      - 195.55400      -  9.8   15s
1   44458  20595 197.72066    30 22      - 196.25700      -  9.5   20s
2   57694  24715 198.72652    27 21      - 196.67131      -  9.3   25s
3   70812  27949 197.59656    29 31      - 197.20126      -  9.2   30s
4   80310  29346 200.12203    43 10      - 197.53186      -  9.2   35s
5   93645  32433 199.72126    38 32      - 197.91581      -  9.2   40s
6  106229  33739 199.20633    54 22      - 198.27488      -  9.2   45s
7  118929  34684 infeasible    49      - 198.56767      -  9.2   51s
3  127296  34740 199.64513    45 23      - 198.80661      -  9.2   55s
9  139183  34573 200.57582    33 17      - 199.12167      -  9.3   60s
0  150929  33037 infeasible    32      - 199.41033      -  9.3   65s
1  162444  30785 infeasible    55      - 199.74659      -  9.4   70s
2  173803  27710 infeasible    40      - 200.05745      -  9.4   75s
3  185313  23455 infeasible    34      - 200.37912      -  9.5   80s
4  195243  18868 infeasible    43      - 200.68800      -  9.6   85s
5  203837  14017 infeasible    38      - 200.94208      -  9.6   90s
6  214151   5565 infeasible    53      - 201.77149      -  9.7   95s
7
3 Cutting planes:
9 Gomory: 16
0 Flow cover: 3
1 Inf proof: 6
2 Zero half: 19

```

```

3   RLT: 11
4   Lazy constraints: 13
5
6   Explored 220340 nodes (2146251 simplex iterations) in 97.47 seconds
7   Thread count was 4 (of 4 available processors)
8
9   Solution count 0
10
11  Model is infeasible
12  Best objective -, best bound -, gap -
13
14  User-callback calls 455334, time in user-callback 1.28 sec
15  Changed value of parameter lazyConstraints to 1
16    Prev: 0  Min: 0  Max: 1  Default: 0
17  Gurobi Optimizer version 9.1.2 build v9.1.2rc0 (mac64)
18  Thread count: 2 physical cores, 4 logical processors, using up to 4 threads
19  Optimize a model with 991 rows, 883 columns and 5169 nonzeros
20  Model fingerprint: 0x5de9c611
21  Variable types: 1 continuous, 882 integer (882 binary)
22  Coefficient statistics:
23    Matrix range      [1e+00, 1e+02]
24    Objective range   [1e+00, 1e+00]
25    Bounds range      [1e+00, 1e+00]
26    RHS range         [1e+00, 2e+02]
27  Presolve removed 505 rows and 42 columns
28  Presolve time: 0.01s
29  Presolved: 486 rows, 841 columns, 4164 nonzeros
30  Variable types: 1 continuous, 840 integer (840 binary)
31
32  Root relaxation: objective 1.342420e+02, 107 iterations, 0.00 seconds
33
34      Nodes      |      Current Node      |      Objective Bounds      |      Work
35  Expl Unexpl |  Obj  Depth IntInf | Incumbent    BestBd    Gap | It/Node Time
36
37      0      0 134.24200    0  44      - 134.24200      -    -    0s
38      0      0 157.83683    0  34      - 157.83683      -    -    0s
39      0      0 157.99700    0  14      - 157.99700      -    -    0s
40      0      0 157.99700    0  20      - 157.99700      -    -    0s
41      0      0 157.99700    0  20      - 157.99700      -    -    0s

```


2	0	2	157.99700	0	20	-	157.99700	-	-	0s
3	* 5236	2983		68		200.6593333	177.96352	11.3%	12.0	4s
4	6489	3594	193.05367	51	18	200.65933	179.65206	10.5%	11.9	5s
5	14506	7359	cutoff	56		200.65933	184.71444	7.95%	12.2	10s
3	25608	12083	cutoff	44		200.65933	186.54345	7.03%	12.0	15s
7	35884	15959	cutoff	46		200.65933	187.53304	6.54%	11.9	20s
3	49008	20448	cutoff	54		200.65933	188.53448	6.04%	11.7	25s
9	62798	24364	191.88370	44	26	200.65933	189.25325	5.68%	11.5	30s
0	77974	28206	cutoff	36		200.65933	189.98377	5.32%	11.3	35s
1	91437	31114	195.58400	37	28	200.65933	190.49030	5.07%	11.3	40s
2	106464	33796	cutoff	46		200.65933	191.01234	4.81%	11.2	45s
3	120932	36386	194.34938	34	28	200.65933	191.40371	4.61%	11.2	50s
4	134866	38616	195.99980	69	34	200.65933	191.80477	4.41%	11.1	55s
5	148044	40500	195.42542	49	26	200.65933	192.10842	4.26%	11.0	60s
3	155705	41333	192.57252	46	24	200.65933	192.30897	4.16%	11.1	65s
7	165525	42379	cutoff	50		200.65933	192.54227	4.05%	11.1	70s
3	175965	42799	infeasible	41		200.65933	192.83019	3.90%	11.2	75s
9	186625	43403	200.09668	44	20	200.65933	193.08755	3.77%	11.2	80s
0	196996	43924	197.43248	54	27	200.65933	193.30833	3.66%	11.2	85s
1	206086	44034	cutoff	45		200.65933	193.53962	3.55%	11.2	90s
2	215479	44029	199.41512	65	23	200.65933	193.74801	3.44%	11.2	95s
3	225864	43900	198.28289	42	32	200.65933	193.96349	3.34%	11.2	100s
4	236075	43437	199.47565	34	31	200.65933	194.17611	3.23%	11.3	105s
5	246841	42705	195.57598	34	24	200.65933	194.39848	3.12%	11.2	110s
3	257199	42031	199.36306	46	31	200.65933	194.61362	3.01%	11.2	115s
7	266722	40933	196.76698	36	34	200.65933	194.81571	2.91%	11.2	120s
3	277893	39473	199.63775	50	9	200.65933	195.07750	2.78%	11.2	125s
9	287095	37916	198.48414	27	25	200.65933	195.30856	2.67%	11.2	130s
0	296595	36117	198.41814	29	28	200.65933	195.53433	2.55%	11.2	135s
1	306144	33932	197.19223	74	32	200.65933	195.78615	2.43%	11.2	140s
2	316139	31345	198.43620	51	6	200.65933	196.04884	2.30%	11.2	145s
3	325008	28605	cutoff	52		200.65933	196.32382	2.16%	11.2	150s
4	330638	26714	200.11833	44	13	200.65933	196.51366	2.07%	11.2	155s
5	335571	24919	infeasible	59		200.65933	196.68796	1.98%	11.2	160s
3	342362	22096	cutoff	58		200.65933	196.90338	1.87%	11.3	165s
7	351392	18132	198.25138	63	24	200.65933	197.30900	1.67%	11.2	170s
3	359065	14386	198.76193	54	17	200.65933	197.69486	1.48%	11.2	175s
9	366104	10447	cutoff	42		200.65933	198.11486	1.27%	11.2	180s
0	373763	5797	cutoff	62		200.65933	198.80609	0.92%	11.2	185s

```

1 381403      0 infeasible   54      200.65933  200.35787  0.15%  11.1  190s
2
3 Cutting planes:
4   Gomory: 13
5   MIR: 1
6   StrongCG: 1
7   Flow cover: 7
8   Inf proof: 8
9   Zero half: 36
0   RLT: 17
1   Lazy constraints: 48
2
3 Explored 381761 nodes (4243924 simplex iterations) in 190.25 seconds
4 Thread count was 4 (of 4 available processors)
5
6 Solution count 1: 200.659
7
8 Optimal solution found (tolerance 1.00e-04)
9 Best objective 2.0065933333333e+02, best bound 2.0065933333333e+02, gap 0.0000%
0
1 User-callback calls 794845, time in user-callback 2.90 sec
2 6 <gurobi.Var d (value 200.65933333333328)>
3 Changed value of parameter lazyConstraints to 1
4   Prev: 0  Min: 0  Max: 1  Default: 0
5 Gurobi Optimizer version 9.1.2 build v9.1.2rc0 (mac64)
6 Thread count: 2 physical cores, 4 logical processors, using up to 4 threads
7 Optimize a model with 991 rows, 883 columns and 5169 nonzeros
8 Model fingerprint: 0x46c22e9a
9 Variable types: 1 continuous, 882 integer (882 binary)
0 Coefficient statistics:
1   Matrix range      [1e+00, 1e+02]
2   Objective range   [1e+00, 1e+00]
3   Bounds range      [1e+00, 1e+00]
4   RHS range         [1e+00, 2e+02]
5 Presolve removed 505 rows and 42 columns
6 Presolve time: 0.02s
7 Presolved: 486 rows, 841 columns, 4417 nonzeros
8 Variable types: 1 continuous, 840 integer (840 binary)
9

```

```

0 Root relaxation: objective 1.395487e+02, 102 iterations, 0.00 seconds
1
2      Nodes      |      Current Node      |      Objective Bounds      |      Work
3  Expl Unexpl |  Obj  Depth IntInf | Incumbent    BestBd   Gap | It/Node Time
4
5      0      0 139.54867    0  44      - 139.54867      -    -    0s
6      0      0 163.86433    0  17      - 163.86433      -    -    0s
7      0      0 163.86433    0  22      - 163.86433      -    -    0s
8      0      0 163.86433    0  16      - 163.86433      -    -    0s
9      0      2 176.05133    0  22      - 176.05133      -    -    0s
0 H  874    721                192.3026667 181.88493  5.42%   7.3    2s
1    3348    322 191.62875    36  21 192.30267 189.39933  1.51%  12.2    5s
2 H 4489    208                192.3026649 190.33033  1.03%  12.0    5s
3 H 4517    208                192.3026622 190.33033  1.03%  12.0    5s
4
5 Cutting planes:
6   Gomory: 18
7   Flow cover: 5
8   Inf proof: 1
9   Zero half: 22
0   RLT: 10
1   Lazy constraints: 3
2
3 Explored 4981 nodes (59263 simplex iterations) in 5.85 seconds
4 Thread count was 4 (of 4 available processors)
5
6 Solution count 3: 192.303 192.303 192.303
7
8 Optimal solution found (tolerance 1.00e-04)
9 Best objective 1.923026621842e+02, best bound 1.923026621842e+02, gap 0.0000%
0
1 User-callback calls 11151, time in user-callback 0.13 sec
2 1 <gurobi.Var d (value 192.3026621842334)>
3 Changed value of parameter lazyConstraints to 1
4   Prev: 0  Min: 0  Max: 1  Default: 0
5 Gurobi Optimizer version 9.1.2 build v9.1.2rc0 (mac64)
6 Thread count: 2 physical cores, 4 logical processors, using up to 4 threads
7 Optimize a model with 991 rows, 883 columns and 5169 nonzeros
8 Model fingerprint: 0xd789a184

```

```

9 Variable types: 1 continuous, 882 integer (882 binary)
0 Coefficient statistics:
1   Matrix range      [1e+00, 1e+02]
2   Objective range   [1e+00, 1e+00]
3   Bounds range      [1e+00, 1e+00]
4   RHS range         [1e+00, 2e+02]
5 Presolve removed 505 rows and 42 columns
5 Presolve time: 0.02s
7 Presolved: 486 rows, 841 columns, 4355 nonzeros
3 Variable types: 1 continuous, 840 integer (840 binary)
9
0 Root relaxation: objective 1.404493e+02, 140 iterations, 0.00 seconds
1
2   Nodes      |   Current Node   |   Objective Bounds   |   Work
3 Expl Unexpl | Obj  Depth IntInf | Incumbent    BestBd  Gap | It/Node Time
4
5      0      0 140.44933    0  44      - 140.44933    -    -    0s
6      0      0 163.89367    0  30      - 163.89367    -    -    0s
7      0      0 165.37233    0  10      - 165.37233    -    -    0s
3      0      0 165.37233    0  32      - 165.37233    -    -    0s
9      0      0 165.37233    0  16      - 165.37233    -    -    0s
0      0      2 178.68100    0  16      - 178.68100    -    -    0s
1    3198 1292 190.55467   44  16      - 187.74133    - 12.9    5s
2   10780 4031 189.65704   44  25      - 189.08387    - 11.4   10s
3   21231 6189 infeasible   35      - 190.05995    - 10.7   15s
4   28302 6503 infeasible   44      - 190.59733    - 10.5   20s
5   36502 4624 infeasible   41      - 191.36832    - 10.6   25s
6
7 Cutting planes:
3   Gomory: 4
9   Flow cover: 9
0   Zero half: 35
1   RLT: 5
2   Lazy constraints: 8
3
4 Explored 44639 nodes (476957 simplex iterations) in 29.88 seconds
5 Thread count was 4 (of 4 available processors)
6
7 Solution count 0

```

```

3
9 Model is infeasible
0 Best objective -, best bound -, gap -
1
2 User-callback calls 93034, time in user-callback 0.42 sec
3 Changed value of parameter lazyConstraints to 1
4     Prev: 0  Min: 0  Max: 1  Default: 0
5 Gurobi Optimizer version 9.1.2 build v9.1.2rc0 (mac64)
3 Thread count: 2 physical cores, 4 logical processors, using up to 4 threads
7 Optimize a model with 991 rows, 883 columns and 5169 nonzeros
3 Model fingerprint: 0x55fd16cd
9 Variable types: 1 continuous, 882 integer (882 binary)
0 Coefficient statistics:
1   Matrix range      [1e+00, 1e+02]
2   Objective range   [1e+00, 1e+00]
3   Bounds range      [1e+00, 1e+00]
4   RHS range         [1e+00, 2e+02]
5 Presolve removed 505 rows and 42 columns
3 Presolve time: 0.01s
7 Presolved: 486 rows, 841 columns, 4230 nonzeros
3 Variable types: 1 continuous, 840 integer (840 binary)
9
0 Root relaxation: objective 1.353927e+02, 136 iterations, 0.00 seconds
1
2   Nodes      |   Current Node   |   Objective Bounds   |   Work
3   Expl Unexpl |  Obj  Depth IntInf | Incumbent    BestBd  Gap | It/Node Time
4
5       0       0 135.39267   0  38       - 135.39267   -   -   0s
3       0       0 164.93333   0  28       - 164.93333   -   -   0s
7       0       0 165.79550   0  38       - 165.79550   -   -   0s
3       0       0 165.94000   0  39       - 165.94000   -   -   0s
9       0       0 165.94000   0  33       - 165.94000   -   -   0s
0       0       2 166.13833   0  33       - 166.13833   -   -   0s
1    4264    1933 187.86900  44  23       - 185.67693   - 12.7   5s
2   12975    3425 190.72736  55  21       - 188.39600   - 14.4  10s
3   22374    2356 infeasible  47       - 190.93061   - 14.6  15s
4
5 Cutting planes:
3   Gomory: 13

```

```

7   Cover: 36
3   MIR: 4
9   StrongCG: 1
0   Flow cover: 4
1   Inf proof: 1
2   Zero half: 29
3   RLT: 15
4   Lazy constraints: 3
5
3   Explored 27185 nodes (386404 simplex iterations) in 17.75 seconds
7   Thread count was 4 (of 4 available processors)
3
9   Solution count 0
0
1   Model is infeasible
2   Best objective -, best bound -, gap -
3
4   User-callback calls 57161, time in user-callback 0.18 sec
5   Changed value of parameter lazyConstraints to 1
3     Prev: 0  Min: 0  Max: 1  Default: 0
7   Gurobi Optimizer version 9.1.2 build v9.1.2rc0 (mac64)
3   Thread count: 2 physical cores, 4 logical processors, using up to 4 threads
9   Optimize a model with 991 rows, 883 columns and 5169 nonzeros
0   Model fingerprint: 0x09e3dbea
1   Variable types: 1 continuous, 882 integer (882 binary)
2   Coefficient statistics:
3     Matrix range      [1e+00, 1e+02]
4     Objective range   [1e+00, 1e+00]
5     Bounds range      [1e+00, 1e+00]
3     RHS range         [1e+00, 2e+02]
7   Presolve removed 505 rows and 42 columns
3   Presolve time: 0.02s
9   Presolved: 486 rows, 841 columns, 4390 nonzeros
0   Variable types: 1 continuous, 840 integer (840 binary)
1
2   Root relaxation: objective 1.380860e+02, 123 iterations, 0.00 seconds
3
4     Nodes    |    Current Node    |    Objective Bounds      |    Work
5   Expl Unexpl |  Obj  Depth IntInf | Incumbent    BestBd   Gap | It/Node Time

```

```

3
7      0      0 138.08600      0 38      - 138.08600      -      -      0s
3      0      0 167.62667      0 27      - 167.62667      -      -      0s
9      0      0 168.49451      0 63      - 168.49451      -      -      0s
0      0      0 168.63333      0 41      - 168.63333      -      -      0s
1      0      0 168.63333      0 36      - 168.63333      -      -      0s
2      0      0 168.76219      0 67      - 168.76219      -      -      0s
3      0      0 168.76219      0 48      - 168.76219      -      -      0s
4      0      2 169.27767      0 48      - 169.27767      -      -      0s
5    5685 2512 190.95157    55 41      - 186.67605      - 12.7      5s
3   14169 5298 191.02199    62 51      - 188.89858      - 12.9     10s
7   24757 5933 infeasible    50      - 190.15958      - 13.4     15s
3   33549 2672 infeasible    42      - 191.68755      - 13.9     20s
9
0 Cutting planes:
1   Gomory: 15
2   Cover: 25
3   Flow cover: 3
4   Inf proof: 1
5   Zero half: 10
3   RLT: 9
7   Lazy constraints: 3
3
9 Explored 37139 nodes (508181 simplex iterations) in 21.82 seconds
0 Thread count was 4 (of 4 available processors)
1
2 Solution count 0
3
4 Model is infeasible
5 Best objective -, best bound -, gap -
3
7 User-callback calls 77349, time in user-callback 0.23 sec
3 Changed value of parameter lazyConstraints to 1
9   Prev: 0 Min: 0 Max: 1 Default: 0
0 Gurobi Optimizer version 9.1.2 build v9.1.2rc0 (mac64)
1 Thread count: 2 physical cores, 4 logical processors, using up to 4 threads
2 Optimize a model with 991 rows, 883 columns and 5169 nonzeros
3 Model fingerprint: 0x8ffddaa9
4 Variable types: 1 continuous, 882 integer (882 binary)

```

```

5 Coefficient statistics:
3   Matrix range      [1e+00, 1e+02]
7   Objective range   [1e+00, 1e+00]
3   Bounds range      [1e+00, 1e+00]
3   RHS range         [1e+00, 2e+02]
0 Presolve removed 505 rows and 42 columns
1 Presolve time: 0.02s
2 Presolved: 486 rows, 841 columns, 4186 nonzeros
3 Variable types: 1 continuous, 840 integer (840 binary)
4
5 Root relaxation: objective 1.367647e+02, 137 iterations, 0.00 seconds
3
7   Nodes      |   Current Node      |   Objective Bounds      |   Work
3   Expl Unexpl |  Obj  Depth IntInf | Incumbent    BestBd   Gap | It/Node Time
3
3       0       0 136.76467    0  44       - 136.76467       -   -   0s
1       0       0 159.29167    0  10       - 159.29167       -   -   0s
2       0       0 159.29167    0  16       - 159.29167       -   -   0s
3       0       0 159.29167    0  10       - 159.29167       -   -   0s
4       0       2 159.29167    0  16       - 159.29167       -   -   0s
5     6250  2177 infeasible    51       - 186.13159       - 12.2   5s
3    16600  3656 191.14277    50  22       - 190.63790       -   9.5  10s
7
3 Cutting planes:
3   Gomory: 20
0   Flow cover: 5
1   GUB cover: 1
2   Zero half: 10
3   Mod-K: 2
4   RLT: 8
5   Lazy constraints: 6
3
7 Explored 23610 nodes (204138 simplex iterations) in 13.02 seconds
3 Thread count was 4 (of 4 available processors)
3
0 Solution count 0
1
2 Model is infeasible
3 Best objective -, best bound -, gap -

```



```

4
5 User-callback calls 49934, time in user-callback 0.18 sec
6 Changed value of parameter lazyConstraints to 1
7   Prev: 0  Min: 0  Max: 1  Default: 0
3 Gurobi Optimizer version 9.1.2 build v9.1.2rc0 (mac64)
9 Thread count: 2 physical cores, 4 logical processors, using up to 4 threads
0 Optimize a model with 991 rows, 883 columns and 5169 nonzeros
1 Model fingerprint: 0x986e3d93
2 Variable types: 1 continuous, 882 integer (882 binary)
3 Coefficient statistics:
4   Matrix range      [1e+00, 1e+02]
5   Objective range   [1e+00, 1e+00]
6   Bounds range      [1e+00, 1e+00]
7   RHS range         [1e+00, 2e+02]
3 Presolve removed 507 rows and 42 columns
9 Presolve time: 0.02s
0 Presolved: 484 rows, 841 columns, 4634 nonzeros
1 Variable types: 1 continuous, 840 integer (840 binary)
2
3 Root relaxation: objective 1.432307e+02, 131 iterations, 0.00 seconds
4
5   Nodes      |   Current Node   |   Objective Bounds   |   Work
6   Expl Unexpl |  Obj  Depth IntInf | Incumbent    BestBd  Gap | It/Node Time
7
3       0       0  143.23067    0  44        -  143.23067    -    -    0s
9       0       0  167.96367    0  20        -  167.96367    -    -    0s
0       0       0  167.96367    0  26        -  167.96367    -    -    0s
1       0       0  167.96367    0  12        -  167.96367    -    -    0s
2       0       2  168.16067    0  12        -  168.16067    -    -    0s
3     8366   4074  187.42123   28  22        -  186.48170    -  10.9    5s
4    18584   6917 infeasible   43        -  188.17500    -  10.6   10s
5    30813   7802 infeasible   34        -  189.63536    -  10.4   15s
6    41226   5839  191.67978   27  39        -  190.97094    -  10.2   20s
7    52386   2123  192.25175   34  30        -  192.24314    -  10.1   25s
3
9 Cutting planes:
0   Gomory: 7
1   Flow cover: 5
2   Zero half: 19

```

```

3   RLT: 13
4   Lazy constraints: 15
5
3   Explored 54941 nodes (551851 simplex iterations) in 25.99 seconds
7   Thread count was 4 (of 4 available processors)
3
9   Solution count 0
0
1   Model is infeasible
2   Best objective -, best bound -, gap -
3
4   User-callback calls 113927, time in user-callback 0.39 sec
5   Changed value of parameter lazyConstraints to 1
3     Prev: 0  Min: 0  Max: 1  Default: 0
7   Gurobi Optimizer version 9.1.2 build v9.1.2rc0 (mac64)
3   Thread count: 2 physical cores, 4 logical processors, using up to 4 threads
9   Optimize a model with 991 rows, 883 columns and 5169 nonzeros
0   Model fingerprint: 0xeeccc201
1   Variable types: 1 continuous, 882 integer (882 binary)
2   Coefficient statistics:
3     Matrix range      [1e+00, 1e+02]
4     Objective range   [1e+00, 1e+00]
5     Bounds range      [1e+00, 1e+00]
3     RHS range         [1e+00, 2e+02]
7   Presolve removed 505 rows and 42 columns
3   Presolve time: 0.01s
9   Presolved: 486 rows, 841 columns, 4296 nonzeros
0   Variable types: 1 continuous, 840 integer (840 binary)
1
2   Root relaxation: objective 1.372877e+02, 122 iterations, 0.00 seconds
3
4     Nodes    |    Current Node    |    Objective Bounds    |    Work
5   Expl Unexpl |  Obj  Depth IntInf | Incumbent    BestBd  Gap | It/Node Time
6
7       0      0  137.28767    0   38        -  137.28767    -    -    0s
3       0      0  161.04333    0   12        -  161.04333    -    -    0s
9       0      0  161.04333    0    6        -  161.04333    -    -    0s
0       0      0  161.04333    0   12        -  161.04333    -    -    0s
1       0      0  161.04333    0   12        -  161.04333    -    -    0s

```

```

2      0      2 161.04333      0 16      - 161.04333      -      -      0s
3 6595 2246 infeasible 29      - 185.48500      - 12.1      5s
4 16129 3572 191.87051 45 30      - 190.38146      - 11.1      10s
5 25889 2903 infeasible 37      - 191.41138      - 10.8      15s
3
7 Cutting planes:
3 Gomory: 11
9 Flow cover: 9
0 Inf proof: 2
1 Zero half: 22
2 RLT: 14
3 Lazy constraints: 5
4
5 Explored 31809 nodes (341542 simplex iterations) in 17.75 seconds
3 Thread count was 4 (of 4 available processors)
7
3 Solution count 0
9
0 Model is infeasible
1 Best objective -, best bound -, gap -
2
3 User-callback calls 67134, time in user-callback 0.23 sec
4 Changed value of parameter lazyConstraints to 1
5 Prev: 0 Min: 0 Max: 1 Default: 0
3 Gurobi Optimizer version 9.1.2 build v9.1.2rc0 (mac64)
7 Thread count: 2 physical cores, 4 logical processors, using up to 4 threads
3 Optimize a model with 991 rows, 883 columns and 5169 nonzeros
9 Model fingerprint: 0x2fea8ed1
0 Variable types: 1 continuous, 882 integer (882 binary)
1 Coefficient statistics:
2 Matrix range [1e+00, 1e+02]
3 Objective range [1e+00, 1e+00]
4 Bounds range [1e+00, 1e+00]
5 RHS range [1e+00, 2e+02]
3 Presolve removed 505 rows and 42 columns
7 Presolve time: 0.01s
3 Presolved: 486 rows, 841 columns, 4246 nonzeros
9 Variable types: 1 continuous, 840 integer (840 binary)
0

```

```

1 Root relaxation: objective 1.349583e+02, 130 iterations, 0.00 seconds
2
3      Nodes      |      Current Node      |      Objective Bounds      |      Work
4  Expl Unexpl |  Obj  Depth IntInf | Incumbent    BestBd   Gap | It/Node Time
5
6      0      0 134.95833    0  46          - 134.95833    -    -    0s
7      0      0 157.89733    0  42          - 157.89733    -    -    0s
8      0      0 161.22400    0  10          - 161.22400    -    -    0s
9      0      0 161.22400    0  28          - 161.22400    -    -    0s
0      0      0 161.22400    0  16          - 161.22400    -    -    0s
1      0      2 161.22400    0  16          - 161.22400    -    -    0s
2  5883  1395 191.22533    21  41          - 187.74362    - 12.5    5s
3
4 Cutting planes:
5   Gomory: 15
6   MIR: 2
7   Flow cover: 7
8   Zero half: 13
9   RLT: 11
0   Lazy constraints: 9
1
2 Explored 9236 nodes (115996 simplex iterations) in 7.41 seconds
3 Thread count was 4 (of 4 available processors)
4
5 Solution count 0
6
7 Model is infeasible
8 Best objective -, best bound -, gap -
9
0 User-callback calls 20159, time in user-callback 0.12 sec
1 Changed value of parameter lazyConstraints to 1
2   Prev: 0  Min: 0  Max: 1  Default: 0
3 Gurobi Optimizer version 9.1.2 build v9.1.2rc0 (mac64)
4 Thread count: 2 physical cores, 4 logical processors, using up to 4 threads
5 Optimize a model with 991 rows, 883 columns and 5169 nonzeros
6 Model fingerprint: 0xa1bb0323
7 Variable types: 1 continuous, 882 integer (882 binary)
8 Coefficient statistics:
9   Matrix range      [1e+00, 1e+02]

```

```

0   Objective range  [1e+00, 1e+00]
1   Bounds range    [1e+00, 1e+00]
2   RHS range       [1e+00, 2e+02]
3   Presolve removed 505 rows and 42 columns
4   Presolve time: 0.01s
5   Presolved: 486 rows, 841 columns, 4268 nonzeros
6   Variable types: 1 continuous, 840 integer (840 binary)
7
3   Root relaxation: objective 1.310323e+02, 120 iterations, 0.00 seconds
9
0   Nodes   |   Current Node   |   Objective Bounds   |   Work
1   Expl Unexpl | Obj  Depth IntInf | Incumbent  BestBd  Gap | It/Node Time
2
3       0      0 131.03233    0  38      - 131.03233    -   -   0s
4       0      0 157.81433    0  24      - 157.81433    -   -   0s
5       0      0 159.14217    0  42      - 159.14217    -   -   0s
6       0      0 160.46933    0  39      - 160.46933    -   -   0s
7       0      0 162.25794    0  44      - 162.25794    -   -   0s
8       0      0 162.25794    0  47      - 162.25794    -   -   0s
9       0      0 163.15225    0  47      - 163.15225    -   -   0s
0       0      0 163.15225    0  46      - 163.15225    -   -   0s
1       0      0 164.49371    0  57      - 164.49371    -   -   0s
2       0      0 164.49371    0  57      - 164.49371    -   -   0s
3       0      0 166.47983    0  45      - 166.47983    -   -   0s
4       0      0 169.82633    0  48      - 169.82633    -   -   0s
5       0      0 169.82633    0  52      - 169.82633    -   -   0s
6       0      0 169.82633    0  34      - 169.82633    -   -   0s
7       0      2 170.09883    0  44      - 170.09883    -   -   0s
8   5209  1241 infeasible    35      - 187.41833    -  14.9   5s
9
0   Cutting planes:
1   Gomory: 14
2   Cover: 1
3   MIR: 5
4   Flow cover: 5
5   Zero half: 20
6   RLT: 13
7   Lazy constraints: 10
3

```

```

9 Explored 7508 nodes (110943 simplex iterations) in 6.13 seconds
0 Thread count was 4 (of 4 available processors)
1
2 Solution count 0
3
4 Model is infeasible
5 Best objective -, best bound -, gap -
6
7 User-callback calls 16361, time in user-callback 0.11 sec
3 Changed value of parameter lazyConstraints to 1
9     Prev: 0  Min: 0  Max: 1  Default: 0
0 Gurobi Optimizer version 9.1.2 build v9.1.2rc0 (mac64)
1 Thread count: 2 physical cores, 4 logical processors, using up to 4 threads
2 Optimize a model with 991 rows, 883 columns and 5169 nonzeros
3 Model fingerprint: 0xc28448ea
4 Variable types: 1 continuous, 882 integer (882 binary)
5 Coefficient statistics:
6   Matrix range      [1e+00, 1e+02]
7   Objective range   [1e+00, 1e+00]
3   Bounds range      [1e+00, 1e+00]
9   RHS range         [1e+00, 2e+02]
0 Presolve removed 508 rows and 42 columns
1 Presolve time: 0.01s
2 Presolved: 483 rows, 841 columns, 4515 nonzeros
3 Variable types: 1 continuous, 840 integer (840 binary)
4
5 Root relaxation: objective 1.431830e+02, 121 iterations, 0.00 seconds
6
7   Nodes    |    Current Node    |    Objective Bounds    |    Work
3   Expl Unexpl |  Obj  Depth IntInf | Incumbent    BestBd  Gap | It/Node Time
9
0       0      0 143.18300    0  44        - 143.18300    -    -    0s
1       0      0 166.74033    0  22        - 166.74033    -    -    0s
2       0      0 167.74433    0  53        - 167.74433    -    -    0s
3       0      0 168.02867    0  28        - 168.02867    -    -    0s
4       0      0 168.02867    0  28        - 168.02867    -    -    0s
5       0      2 168.02867    0  34        - 168.02867    -    -    0s
3     6905   3564 181.21234   27  51        - 176.84656    -  11.9    5s
7    15873   4962 189.98576   50  17        - 186.48384    -  12.1   10s

```

```

3 26742 3531 192.27167 34 32 - 190.91146 - 11.5 15s
9
0 Cutting planes:
1 Gomory: 5
2 Cover: 6
3 Flow cover: 4
4 Zero half: 16
5 RLT: 10
3 Lazy constraints: 18
7
3 Explored 31890 nodes (353264 simplex iterations) in 17.34 seconds
9 Thread count was 4 (of 4 available processors)
0
1 Solution count 0
2
3 Model is infeasible
4 Best objective -, best bound -, gap -
5
3 User-callback calls 66815, time in user-callback 0.27 sec
7 Changed value of parameter lazyConstraints to 1
3 Prev: 0 Min: 0 Max: 1 Default: 0
9 Gurobi Optimizer version 9.1.2 build v9.1.2rc0 (mac64)
0 Thread count: 2 physical cores, 4 logical processors, using up to 4 threads
1 Optimize a model with 991 rows, 883 columns and 5169 nonzeros
2 Model fingerprint: 0x98026204
3 Variable types: 1 continuous, 882 integer (882 binary)
4 Coefficient statistics:
5 Matrix range [1e+00, 1e+02]
3 Objective range [1e+00, 1e+00]
7 Bounds range [1e+00, 1e+00]
3 RHS range [1e+00, 2e+02]
9 Presolve removed 505 rows and 42 columns
0 Presolve time: 0.01s
1 Presolved: 486 rows, 841 columns, 4198 nonzeros
2 Variable types: 1 continuous, 840 integer (840 binary)
3
4 Root relaxation: objective 1.385103e+02, 135 iterations, 0.00 seconds
5
3 Nodes | Current Node | Objective Bounds | Work

```

	Expl	Unexpl	Obj	Depth	IntInf	Incumbent	BestBd	Gap	It/Node	Time
9	0	0	138.51033	0	38	-	138.51033	-	-	0s
0	0	0	163.68033	0	10	-	163.68033	-	-	0s
1	0	0	163.68033	0	24	-	163.68033	-	-	0s
2	0	0	163.68033	0	16	-	163.68033	-	-	0s
3	0	2	163.87733	0	16	-	163.87733	-	-	0s
4	4793	98	infeasible	30		-	190.97445	-	17.7	5s

Cutting planes:

Gomory: 16
Flow cover: 5
Zero half: 19
RLT: 3
Lazy constraints: 7

Explored 5044 nodes (87833 simplex iterations) in 5.09 seconds

Thread count was 4 (of 4 available processors)

Solution count 0

Model is infeasible

Best objective -, best bound -, gap -

User-callback calls 11338, time in user-callback 0.08 sec

Changed value of parameter lazyConstraints to 1

Prev: 0 Min: 0 Max: 1 Default: 0

Gurobi Optimizer version 9.1.2 build v9.1.2rc0 (mac64)

Thread count: 2 physical cores, 4 logical processors, using up to 4 threads

Optimize a model with 991 rows, 883 columns and 5169 nonzeros

Model fingerprint: 0x895eb0ed

Variable types: 1 continuous, 882 integer (882 binary)

Coefficient statistics:

Matrix range [1e+00, 1e+02]

Objective range [1e+00, 1e+00]

Bounds range [1e+00, 1e+00]

RHS range [1e+00, 2e+02]

Presolve removed 505 rows and 42 columns

Presolve time: 0.02s


```

3 Presolved: 486 rows, 841 columns, 4345 nonzeros
7 Variable types: 1 continuous, 840 integer (840 binary)
3
9 Root relaxation: objective 1.395757e+02, 130 iterations, 0.00 seconds
0
1      Nodes      |      Current Node      |      Objective Bounds      |      Work
2  Expl Unexpl |  Obj  Depth IntInf | Incumbent    BestBd   Gap | It/Node Time
3
4      0      0 139.57567    0  38          - 139.57567    -    -    0s
5      0      0 164.13700    0  40          - 164.13700    -    -    0s
3      0      0 164.74567    0  30          - 164.74567    -    -    0s
7      0      0 164.74567    0  16          - 164.74567    -    -    0s
3      0      0 164.74567    0  16          - 164.74567    -    -    0s
9      0      0 164.74567    0  16          - 164.74567    -    -    0s
0      0      0 164.74567    0  16          - 164.74567    -    -    0s
1      0      2 164.74567    0  16          - 164.74567    -    -    0s
2
3 Cutting planes:
4   Gomory: 15
5   Flow cover: 4
3   Zero half: 6
7   RLT: 9
3   Lazy constraints: 2
9
0 Explored 4638 nodes (58937 simplex iterations) in 4.11 seconds
1 Thread count was 4 (of 4 available processors)
2
3 Solution count 0
4
5 Model is infeasible
3 Best objective -, best bound -, gap -
7
3 User-callback calls 10338, time in user-callback 0.06 sec
9 Changed value of parameter lazyConstraints to 1
0   Prev: 0  Min: 0  Max: 1  Default: 0
1 Gurobi Optimizer version 9.1.2 build v9.1.2rc0 (mac64)
2 Thread count: 2 physical cores, 4 logical processors, using up to 4 threads
3 Optimize a model with 991 rows, 883 columns and 5169 nonzeros
4 Model fingerprint: 0xa4afade1

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5 Variable types: 1 continuous, 882 integer (882 binary)
3 Coefficient statistics:
7   Matrix range      [1e+00, 1e+02]
3   Objective range   [1e+00, 1e+00]
9   Bounds range      [1e+00, 1e+00]
0   RHS range         [1e+00, 2e+02]
1 Presolve removed 507 rows and 42 columns
2 Presolve time: 0.01s
3 Presolved: 484 rows, 841 columns, 4524 nonzeros
4 Variable types: 1 continuous, 840 integer (840 binary)
5
3 Root relaxation: objective 1.497363e+02, 144 iterations, 0.00 seconds
7
3   Nodes      |   Current Node      |   Objective Bounds      |   Work
9   Expl Unexpl |  Obj  Depth IntInf | Incumbent    BestBd   Gap | It/Node Time
0
1       0       0 149.73633    0  46       - 149.73633    -    -    0s
2       0       0 170.63800    0  22       - 170.63800    -    -    0s
3       0       0 171.92733    0  16       - 171.92733    -    -    0s
4       0       0 171.92733    0  42       - 171.92733    -    -    0s
5       0       0 171.92733    0  16       - 171.92733    -    -    0s
6       0       2 171.92733    0  16       - 171.92733    -    -    0s
7
3 Cutting planes:
9   Gomory: 8
0   Cover: 2
1   Zero half: 22
2   RLT: 35
3   Lazy constraints: 4
4
5 Explored 2158 nodes (18663 simplex iterations) in 1.14 seconds
3 Thread count was 4 (of 4 available processors)
7
3 Solution count 0
9
0 Model is infeasible
1 Best objective -, best bound -, gap -
2
3 User-callback calls 4558, time in user-callback 0.02 sec

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4 Changed value of parameter lazyConstraints to 1
5   Prev: 0  Min: 0  Max: 1  Default: 0
6 Gurobi Optimizer version 9.1.2 build v9.1.2rc0 (mac64)
7 Thread count: 2 physical cores, 4 logical processors, using up to 4 threads
3 Optimize a model with 991 rows, 883 columns and 5169 nonzeros
9 Model fingerprint: 0x8720281d
0 Variable types: 1 continuous, 882 integer (882 binary)
1 Coefficient statistics:
2   Matrix range      [1e+00, 1e+02]
3   Objective range   [1e+00, 1e+00]
4   Bounds range      [1e+00, 1e+00]
5   RHS range         [1e+00, 2e+02]
6 Presolve removed 505 rows and 42 columns
7 Presolve time: 0.01s
3 Presolved: 486 rows, 841 columns, 4298 nonzeros
9 Variable types: 1 continuous, 840 integer (840 binary)
0
1 Root relaxation: objective 1.359867e+02, 124 iterations, 0.00 seconds
2
3   Nodes      |   Current Node   |   Objective Bounds   |   Work
4 Expl Unexpl | Obj Depth IntInf | Incumbent    BestBd  Gap | It/Node Time
5
6      0      0 135.98667    0  44      - 135.98667    -    -    0s
7      0      0 159.83300    0  14      - 159.83300    -    -    0s
3      0      0 159.83300    0  20      - 159.83300    -    -    0s
9      0      0 159.83300    0  20      - 159.83300    -    -    0s
0      0      2 159.83300    0  20      - 159.83300    -    -    0s
1  5706  3417 180.46600   24  13      - 179.97017    - 12.2    5s
2 15930  8766 191.34533   49  17      - 181.75142    - 12.4   10s
3 24092 12178 185.82818   48  47      - 183.11967    - 12.6   15s
4 35294 15714 infeasible   38      - 184.73477    - 12.6   20s
5 46132 18513 190.52896   38  46      - 185.90689    - 12.6   25s
6 57480 20607 188.69555   35  10      - 186.74650    - 12.6   30s
7 68202 21664 infeasible   39      - 187.51700    - 12.7   35s
3 79116 22333 191.54116   42  55      - 188.23602    - 12.6   40s
9 89692 22443 infeasible   47      - 188.81470    - 12.6   45s
0 97972 22139 190.31956   53  22      - 189.29066    - 12.5   50s
1 109619 22455 infeasible   49      - 189.75291    - 12.2   55s
2 120969 21415 190.45450   40  25      - 190.28246    - 12.1   60s

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3 132325 19617 infeasible 44 - 190.76515 - 11.9 65s
4 143767 17624 infeasible 44 - 191.23578 - 11.7 70s
5 151877 13971 191.62312 42 36 - 191.62312 - 11.6 75s
3 161242 6970 infeasible 55 - 192.39381 - 11.6 80s
7
3 Cutting planes:
9 Gomory: 12
0 Flow cover: 10
1 Inf proof: 3
2 Zero half: 30
3 RLT: 19
4 Lazy constraints: 34
5
3 Explored 168795 nodes (1949931 simplex iterations) in 83.02 seconds
7 Thread count was 4 (of 4 available processors)
3
9 Solution count 0
0
1 Model is infeasible
2 Best objective -, best bound -, gap -
3
4 User-callback calls 348428, time in user-callback 1.08 sec
5 Changed value of parameter lazyConstraints to 1
3 Prev: 0 Min: 0 Max: 1 Default: 0
7 Gurobi Optimizer version 9.1.2 build v9.1.2rc0 (mac64)
3 Thread count: 2 physical cores, 4 logical processors, using up to 4 threads
9 Optimize a model with 991 rows, 883 columns and 5169 nonzeros
0 Model fingerprint: 0x9d4d15f5
1 Variable types: 1 continuous, 882 integer (882 binary)
2 Coefficient statistics:
3 Matrix range [1e+00, 1e+02]
4 Objective range [1e+00, 1e+00]
5 Bounds range [1e+00, 1e+00]
6 RHS range [1e+00, 2e+02]
7 Presolve removed 505 rows and 42 columns
3 Presolve time: 0.01s
9 Presolved: 486 rows, 841 columns, 4226 nonzeros
0 Variable types: 1 continuous, 840 integer (840 binary)
1

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```

2 Root relaxation: objective 1.310323e+02, 124 iterations, 0.00 seconds
3
4      Nodes      |      Current Node      |      Objective Bounds      |      Work
5  Expl Unexpl |  Obj  Depth IntInf | Incumbent    BestBd   Gap | It/Node Time
6
7      0      0 131.03233    0  38          - 131.03233      -   -    0s
8
9      0      0 179.69667    0  38          - 179.69667      -   -    0s
10
11     0      0 181.41467    0  28          - 181.41467      -   -    0s
12
13     0      0 181.41467    0  34          - 181.41467      -   -    0s
14
15     0      0 181.41467    0  34          - 181.41467      -   -    0s
16
17     0      2 181.41467    0  34          - 181.41467      -   -    0s
18
19    4985   1724 infeasible    43          - 187.79600      - 11.4    5s
20
21   16061   1432 infeasible    59          - 191.62117      - 10.8   10s
22
23
24 Cutting planes:
25
26 Gomory: 9
27
28 Cover: 49
29
30 MIR: 1
31
32 Flow cover: 7
33
34 Zero half: 18
35
36 RLT: 13
37
38 Lazy constraints: 1
39
40
41 Explored 18248 nodes (191476 simplex iterations) in 10.91 seconds
42 Thread count was 4 (of 4 available processors)
43
44
45 Solution count 0
46
47
48 Model is infeasible
49
50 Best objective -, best bound -, gap -
51
52
53 User-callback calls 38580, time in user-callback 0.12 sec
54 Changed value of parameter lazyConstraints to 1
55   Prev: 0  Min: 0  Max: 1  Default: 0
56
57 Gurobi Optimizer version 9.1.2 build v9.1.2rc0 (mac64)
58
59 Thread count: 2 physical cores, 4 logical processors, using up to 4 threads
60
61 Optimize a model with 991 rows, 883 columns and 5169 nonzeros
62
63 Model fingerprint: 0x6cea7f5e
64
65 Variable types: 1 continuous, 882 integer (882 binary)

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1 Coefficient statistics:
2   Matrix range      [1e+00, 1e+02]
3   Objective range   [1e+00, 1e+00]
4   Bounds range      [1e+00, 1e+00]
5   RHS range         [1e+00, 2e+02]
3 Presolve removed 509 rows and 42 columns
7 Presolve time: 0.02s
3 Presolved: 482 rows, 841 columns, 4464 nonzeros
9 Variable types: 1 continuous, 840 integer (840 binary)
)
1 Root relaxation: objective 1.421720e+02, 127 iterations, 0.01 seconds
2
3   Nodes      |   Current Node      |   Objective Bounds      |   Work
4   Expl Unexpl |  Obj  Depth IntInf | Incumbent    BestBd   Gap | It/Node Time
5
3       0       0 142.17200    0  44       - 142.17200       -   -   0s
7       0       0 178.39767    0  14       - 178.39767       -   -   0s
3       0       0 178.39767    0  18       - 178.39767       -   -   0s
9       0       0 178.39767    0  12       - 178.39767       -   -   0s
)       0       0 infeasible    0       - infeasible       -   -   0s
1
2 Cutting planes:
3   Gomory: 13
4   Zero half: 28
5   RLT: 39
3   Lazy constraints: 2
7
3 Explored 1 nodes (306 simplex iterations) in 0.10 seconds
9 Thread count was 4 (of 4 available processors)
)
1 Solution count 0
2
3 Model is infeasible
4 Best objective -, best bound -, gap -
5
3 User-callback calls 116, time in user-callback 0.00 sec
7 Changed value of parameter lazyConstraints to 1
3   Prev: 0  Min: 0  Max: 1  Default: 0
9 Gurobi Optimizer version 9.1.2 build v9.1.2rc0 (mac64)

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0 Thread count: 2 physical cores, 4 logical processors, using up to 4 threads
1 Optimize a model with 991 rows, 883 columns and 5169 nonzeros
2 Model fingerprint: 0xdaced03e
3 Variable types: 1 continuous, 882 integer (882 binary)
4 Coefficient statistics:
5   Matrix range      [1e+00, 1e+02]
6   Objective range   [1e+00, 1e+00]
7   Bounds range      [1e+00, 1e+00]
8   RHS range         [1e+00, 2e+02]
9 Presolve removed 505 rows and 42 columns
0 Presolve time: 0.01s
1 Presolved: 486 rows, 841 columns, 4228 nonzeros
2 Variable types: 1 continuous, 840 integer (840 binary)
3
4 Root relaxation: objective 1.341423e+02, 146 iterations, 0.00 seconds
5
6   Nodes      |   Current Node   |   Objective Bounds   |   Work
7 Expl Unexpl | Obj Depth IntInf | Incumbent    BestBd  Gap | It/Node Time
8
9      0      0 134.14233    0  38      - 134.14233    -   -   0s
0      0      0 182.80667    0  41      - 182.80667    -   -   0s
1      0      0 184.52467    0  28      - 184.52467    -   -   0s
2      0      0 184.52467    0  28      - 184.52467    -   -   0s
3      0      2 184.52467    0  28      - 184.52467    -   -   0s
4
5 Cutting planes:
6   Gomory: 13
7   Cover: 6
8   Flow cover: 2
9   Zero half: 8
0   RLT: 13
1   Lazy constraints: 1
2
3 Explored 3424 nodes (41558 simplex iterations) in 4.68 seconds
4 Thread count was 4 (of 4 available processors)
5
6 Solution count 0
7
8 Model is infeasible

```

```

9 Best objective -, best bound -, gap -
0
1 User-callback calls 8249, time in user-callback 0.04 sec
2 Changed value of parameter lazyConstraints to 1
3   Prev: 0  Min: 0  Max: 1  Default: 0
4 Gurobi Optimizer version 9.1.2 build v9.1.2rc0 (mac64)
5 Thread count: 2 physical cores, 4 logical processors, using up to 4 threads
6 Optimize a model with 991 rows, 883 columns and 5169 nonzeros
7 Model fingerprint: 0x8962f751
3 Variable types: 1 continuous, 882 integer (882 binary)
9 Coefficient statistics:
0   Matrix range      [1e+00, 1e+02]
1   Objective range   [1e+00, 1e+00]
2   Bounds range      [1e+00, 1e+00]
3   RHS range         [1e+00, 2e+02]
4 Presolve removed 505 rows and 42 columns
5 Presolve time: 0.01s
6 Presolved: 486 rows, 841 columns, 4306 nonzeros
7 Variable types: 1 continuous, 840 integer (840 binary)
3
9 Root relaxation: objective 1.434620e+02, 152 iterations, 0.00 seconds
0
1   Nodes   |   Current Node   |   Objective Bounds   |   Work
2   Expl Unexpl |  Obj  Depth IntInf | Incumbent    BestBd  Gap | It/Node Time
3
4       0     0 143.46200    0  44        - 143.46200    -   -   0s
5       0     0 175.68667    0  14        - 175.68667    -   -   0s
6       0     0 175.68667    0   6        - 175.68667    -   -   0s
7       0     2 175.68667    0   6        - 175.68667    -   -   0s
3
9 Cutting planes:
0   Gomory: 8
1   Cover: 24
2   Zero half: 31
3   RLT: 37
4   Lazy constraints: 2
5
6 Explored 1728 nodes (13429 simplex iterations) in 1.04 seconds
7 Thread count was 4 (of 4 available processors)

```



```

3
9 Solution count 0
0
1 Model is infeasible
2 Best objective -, best bound -, gap -
3
4 User-callback calls 3688, time in user-callback 0.01 sec
5 Changed value of parameter lazyConstraints to 1
3   Prev: 0  Min: 0  Max: 1  Default: 0
7 Gurobi Optimizer version 9.1.2 build v9.1.2rc0 (mac64)
3 Thread count: 2 physical cores, 4 logical processors, using up to 4 threads
9 Optimize a model with 991 rows, 883 columns and 5169 nonzeros
0 Model fingerprint: 0x182ec761
1 Variable types: 1 continuous, 882 integer (882 binary)
2 Coefficient statistics:
3   Matrix range      [1e+00, 1e+02]
4   Objective range   [1e+00, 1e+00]
5   Bounds range      [1e+00, 1e+00]
3   RHS range         [1e+00, 2e+02]
7 Presolve removed 505 rows and 42 columns
3 Presolve time: 0.02s
9 Presolved: 486 rows, 841 columns, 4219 nonzeros
0 Variable types: 1 continuous, 840 integer (840 binary)
1
2 Root relaxation: objective 1.496833e+02, 130 iterations, 0.01 seconds
3
4   Nodes      |   Current Node   |   Objective Bounds   |   Work
5   Expl Unexpl |  Obj  Depth IntInf | Incumbent    BestBd   Gap | It/Node Time
3
7       0      0 149.68333    0  44        - 149.68333    -    -    0s
3       0      0 180.00833    0  27        - 180.00833    -    -    0s
9       0      0 180.00833    0  24        - 180.00833    -    -    0s
0       0      0 180.00833    0  12        - 180.00833    -    -    0s
1       0      0 180.00833    0  12        - 180.00833    -    -    0s
2       0      0 182.30522    0  38        - 182.30522    -    -    0s
3       0      0 infeasible    0          - infeasible    -    -    0s
4
5 Cutting planes:
3   Gomory: 5

```

```
7   MIR: 1
3   Zero half: 27
9   RLT: 39
0   Lazy constraints: 2
1
2   Explored 1 nodes (398 simplex iterations) in 0.15 seconds
3   Thread count was 4 (of 4 available processors)
4
5   Solution count 0
3
7   Model is infeasible
3   Best objective -, best bound -, gap -
9
0   User-callback calls 142, time in user-callback 0.01 sec
1
2   Process finished with exit code 0
```