第三问：基于剪枝方法的最佳常驻点求解

Formulation:

$$\min_{O} \min_{x_{ijk}} d$$

$$\text{s.t.} \begin{cases} d = \max_{k \in K} \sum_{i \in V} \sum_{j \in V} c_{ij} x_{ijk} \\ \sum_{j \in V} \sum_{k \in K} x_{ijk} = 1, \forall i \in V - \{O\} \\ \sum_{j \in V} x_{ijk} - \sum_{j \in V} x_{jik} = 0, \forall i \in V, k \in K \\ \sum_{j \in V} x_{0jk} = 1, \forall k \in K \\ \sum_{i \in S} \sum_{j \in S} x_{ijk} \leq |S| - 1, \forall S \subset V - \{O\}, S \neq \emptyset, k \in K \\ x_{iik} = 0, \forall i \in V, k \in K \\ x_{ijk} \in \{0, 1\} \end{cases}$$

Algorithm:

Branch and cut算法是在Branch and bound算法的基础上增加切平面。回顾Branch and bound，如果子问题的下界（松弛LP问题的解）超过了规定的上界，就要剪去（此处可有图，图略）。在第一问的求解过程中，上界由子问题求解结果得到，而在LRP中，我们将规定上界为前N个常驻点求解的最小值。伪代码如下：

```
将 第一问求解的d值 赋值给 d_min
将 21个点 按一定顺序 依次赋值给 O // 一定顺序可以是根据各点距离图中心某点远近进行排序
    建立MTSP模型
    增加约束：d <= d_min
    利用Branch and cut算法求解
    解得d
    如果 d < d_min
        将 d 赋值给 d_min
解得d_min以及对应的O点
```

Code:

```python
from gurobipy import *
import pickle
import networkx as nx


# define the constants
V_NUM = 21
```

```python
K_NUM = 2
ORIGIN_IDX = 5
MAX = 1e5


# load and generate basic data
f = open("../data/distance", mode="rb")
distance = pickle.load(f)
f.close()
dist = {(i, j, k): distance[i][j] for i in range(V_NUM) for j in range(V_NUM) for k in
range(K_NUM)}
dict_linear = {(i, j, k): i*V_NUM*K_NUM + j*K_NUM + k for i in range(V_NUM) for j in
range(V_NUM) for k in range(K_NUM)}
dict_3d = mi = dict(zip(dict_linear.values(), dict_linear.keys()))


# callback - use lazy constraints to eliminate sub-tours
def mycallback(model, where):
    if where == GRB.Callback.MIPSOL:
        vals = model.cbGetSolution(model._vars[:-1])
        edges = list((i, j, k) for i, j, k in dict_linear.keys() if
vals[dict_linear[(i, j, k)]] > 0.5)
        shortest_cycle, k = cycle(edges)
        if shortest_cycle is not None:
            model.cbLazy(quicksum(x[i, j, k] for i in shortest_cycle for j in
shortest_cycle) <= len(shortest_cycle) - 1)



# find the loops
def cycle(edges):
    node_vehicle = {}
    G = nx.DiGraph()
    for e in edges:
        G.add_edge(e[0], e[1])
        node_vehicle[e[0]] = e[2]
        node_vehicle[e[1]] = e[2]
    shortest_subtour = None
    min = V_NUM
    cycle: list
    for cycle in nx.simple_cycles(G):
        if cycle.count(ORIGIN_IDX - 1) == 0:
```

```
            if len(cycle) < min:
                min = len(cycle)
                shortest_subtour = cycle
    if shortest_subtour is None:
        k = None
    else:
        k = node_vehicle[shortest_subtour[0]]
    return shortest_subtour, k


dmin = 201.24197185085075
origin_list = [2,6,1,5,4,3,7,11,10,19,9,16,20,8,21,14,17,15,18,13,12]
for o in origin_list:
    ORIGIN_IDX = o
    node_list = list(range(ORIGIN_IDX - 1)) + list(range(ORIGIN_IDX, V_NUM))

    # create a model
    MODEL = Model()
    # MODEL.setParam('OutputFlag', 0)

    # add variables
    x = MODEL.addVars(dist.keys(), obj=dist, vtype=GRB.BINARY, name='x')
    d = MODEL.addVar(name="d")
    MODEL.update()

    # set the objective
    MODEL.setObjective(d, GRB.MINIMIZE)

    # add constraints
    MODEL.addConstrs(quicksum(x[i, j, k] for j in range(V_NUM) for k in range(K_NUM))
== 1 for i in node_list)
    MODEL.addConstrs(
        quicksum(x[i, j, k] for j in range(V_NUM)) - quicksum(x[j, i, k] for j in
range(V_NUM)) == 0 for i in
        range(V_NUM) for k in range(K_NUM))
    MODEL.addConstrs(quicksum(x[ORIGIN_IDX - 1, j, k] for j in range(V_NUM)) == 1 for
k in range(K_NUM))
    MODEL.addConstrs(
        d - quicksum(distance[i][j] / 1.5 * x[i, j, k] for i in range(V_NUM) for j in
range(V_NUM)) >= 0 for k in
```

```
        range(K_NUM))
    MODEL.addConstrs(x[i, i, k] == 0 for i in range(V_NUM) for k in range(K_NUM))
    MODEL.addConstrs(x[i, j, k] + x[j, i, k] <= 1 for i in range(V_NUM) for j in
range(V_NUM) for k in range(K_NUM))
    MODEL.addConstr(d <= dmin)


    MODEL._vars = MODEL.getVars()
    MODEL.Params.lazyConstraints = 1
    MODEL.optimize(mycallback)


    try:
        d_val = MODEL.getVarByName('d').x
        if d_val < dmin:
            dmin = d_val
        print(ORIGIN_IDX, MODEL.getVarByName('d'))
    except AttributeError:
        pass
```

Result：

```
Changed value of parameter lazyConstraints to 1
   Prev: 0  Min: 0  Max: 1  Default: 0
Gurobi Optimizer version 9.1.2 build v9.1.2rc0 (mac64)
Thread count: 2 physical cores, 4 logical processors, using up to 4 threads
Optimize a model with 990 rows, 883 columns and 5168 nonzeros
Model fingerprint: 0x200cb6b7
Variable types: 1 continuous, 882 integer (882 binary)
Coefficient statistics:
  Matrix range     [1e+00, 1e+02]
  Objective range  [1e+00, 1e+02]
  Bounds range     [1e+00, 1e+00]
  RHS range        [1e+00, 1e+00]
Presolve removed 504 rows and 42 columns
Presolve time: 0.02s
Presolved: 486 rows, 841 columns, 4130 nonzeros
Variable types: 1 continuous, 840 integer (840 binary)


Root relaxation: objective 4.186460e+02, 79 iterations, 0.00 seconds
```

```
    Nodes    |    Current Node    |     Objective Bounds      |     Work
 Expl Unexpl |  Obj  Depth IntInf | Incumbent    BestBd   Gap | It/Node Time

     0     0  418.64600    0   44          -  418.64600      -     -    0s
     0     0  491.59300    0    6          -  491.59300      -     -    0s
     0     0  491.59300    0   22          -  491.59300      -     -    0s
     0     0  491.59300    0   22          -  491.59300      -     -    0s
H    0     0                     858.3580000  491.59300  42.7%     -    0s
H    0     0                     836.9446667  491.59300  41.3%     -    0s
H    0     2                     726.3273333  491.59300  32.3%     -    0s
     0     2  491.59300    0   16  726.32733  491.59300  32.3%     -    0s
H  154   179                     699.1166667  528.15400  24.5%   6.3    0s
H  450   395                     671.9060000  528.15400  21.4%   5.3    0s
H  749   613                     661.8660000  530.25000  19.9%   6.3    1s
H  756   587                     635.2026667  544.17800  14.3%   6.3    1s
H  756   558                     618.3880000  544.17800  12.0%   6.3    1s
H  758   530                     586.7400000  546.94551  6.78%   6.3    1s
H  810   526                     571.1933333  553.02531  3.18%   9.2    2s

Cutting planes:
  Gomory: 11
  MIR: 1
  Flow cover: 9
  Inf proof: 4
  Zero half: 11
  RLT: 6
  Lazy constraints: 2

Explored 1524 nodes (17168 simplex iterations) in 3.08 seconds
Thread count was 4 (of 4 available processors)

Solution count 9: 571.193 586.74 618.388 ... 858.358

Optimal solution found (tolerance 1.00e-04)
Best objective 5.711933333333e+02, best bound 5.711933333333e+02, gap 0.0000%

User-callback calls 4092, time in user-callback 0.11 sec
<gurobi.Var x[0,1,0] (value 1.0)>
<gurobi.Var x[0,17,1] (value 1.0)>
```

```
<gurobi.Var x[1,5,0] (value 1.0)>
<gurobi.Var x[2,10,0] (value 1.0)>
<gurobi.Var x[3,2,0] (value 1.0)>
<gurobi.Var x[4,0,1] (value 1.0)>
<gurobi.Var x[5,3,0] (value 1.0)>
<gurobi.Var x[6,4,1] (value 1.0)>
<gurobi.Var x[7,19,1] (value 1.0)>
<gurobi.Var x[8,9,1] (value 1.0)>
<gurobi.Var x[9,7,1] (value 1.0)>
<gurobi.Var x[10,11,0] (value 1.0)>
<gurobi.Var x[11,12,0] (value 1.0)>
<gurobi.Var x[12,13,0] (value 1.0)>
<gurobi.Var x[13,14,0] (value 1.0)>
<gurobi.Var x[14,0,0] (value 1.0)>
<gurobi.Var x[15,18,1] (value 1.0)>
<gurobi.Var x[16,15,1] (value 1.0)>
<gurobi.Var x[17,16,1] (value 1.0)>
<gurobi.Var x[18,8,1] (value 1.0)>
<gurobi.Var x[19,20,1] (value 1.0)>
<gurobi.Var x[20,6,1] (value 1.0)>
<gurobi.Var d (value 192.30266666666665)>


Process finished with exit code 0
```

寻找过程输出：

```
1   Changed value of parameter lazyConstraints to 1
2     Prev: 0  Min: 0  Max: 1  Default: 0
3   Gurobi Optimizer version 9.1.2 build v9.1.2rc0 (mac64)
4   Thread count: 2 physical cores, 4 logical processors, using up to 4 threads
5   Optimize a model with 991 rows, 883 columns and 5169 nonzeros
6   Model fingerprint: 0xca69f6c6
7   Variable types: 1 continuous, 882 integer (882 binary)
8   Coefficient statistics:
9     Matrix range     [1e+00, 1e+02]
10    Objective range  [1e+00, 1e+00]
11    Bounds range     [1e+00, 1e+00]
12    RHS range        [1e+00, 2e+02]
13  Presolve removed 505 rows and 42 columns
```

```
Presolve time: 0.01s
Presolved: 486 rows, 841 columns, 4212 nonzeros
Variable types: 1 continuous, 840 integer (840 binary)

Root relaxation: objective 1.385773e+02, 115 iterations, 0.00 seconds

    Nodes    |    Current Node    |     Objective Bounds      |     Work
 Expl Unexpl |  Obj  Depth IntInf | Incumbent    BestBd   Gap | It/Node Time

     0     0  138.57733    0   44          -  138.57733      -     -    0s
     0     0  166.97500    0   10          -  166.97500      -     -    0s
     0     0  166.97500    0   24          -  166.97500      -     -    0s
     0     0  166.97500    0   22          -  166.97500      -     -    0s
     0     2  180.36233    0   36          -  180.36233      -     -    0s
  7213  3271  infeasible   29               -  189.66997      -  11.6    5s
 16348  7249  infeasible   50               -  194.68030      -  10.5   10s
 30425 14306  200.05783   39   16          -  195.55400      -   9.8   15s
 44458 20595  197.72066   30   22          -  196.25700      -   9.5   20s
 57694 24715  198.72652   27   21          -  196.67131      -   9.3   25s
 70812 27949  197.59656   29   31          -  197.20126      -   9.2   30s
 80310 29346  200.12203   43   10          -  197.53186      -   9.2   35s
 93645 32433  199.72126   38   32          -  197.91581      -   9.2   40s
106229 33739  199.20633   54   22          -  198.27488      -   9.2   45s
118929 34684  infeasible   49               -  198.56767      -   9.2   51s
127296 34740  199.64513   45   23          -  198.80661      -   9.2   55s
139183 34573  200.57582   33   17          -  199.12167      -   9.3   60s
150929 33037  infeasible   32               -  199.41033      -   9.3   65s
162444 30785  infeasible   55               -  199.74659      -   9.4   70s
173803 27710  infeasible   40               -  200.05745      -   9.4   75s
185313 23455  infeasible   34               -  200.37912      -   9.5   80s
195243 18868  infeasible   43               -  200.68800      -   9.6   85s
203837 14017  infeasible   38               -  200.94208      -   9.6   90s
214151  5565  infeasible   53               -  201.77149      -   9.7   95s

Cutting planes:
  Gomory: 16
  Flow cover: 3
  Inf proof: 6
  Zero half: 19
```

```
  RLT: 11
  Lazy constraints: 13

Explored 220340 nodes (2146251 simplex iterations) in 97.47 seconds
Thread count was 4 (of 4 available processors)

Solution count 0

Model is infeasible
Best objective -, best bound -, gap -

User-callback calls 455334, time in user-callback 1.28 sec
Changed value of parameter lazyConstraints to 1
   Prev: 0  Min: 0  Max: 1  Default: 0
Gurobi Optimizer version 9.1.2 build v9.1.2rc0 (mac64)
Thread count: 2 physical cores, 4 logical processors, using up to 4 threads
Optimize a model with 991 rows, 883 columns and 5169 nonzeros
Model fingerprint: 0x5de9c611
Variable types: 1 continuous, 882 integer (882 binary)
Coefficient statistics:
  Matrix range     [1e+00, 1e+02]
  Objective range  [1e+00, 1e+00]
  Bounds range     [1e+00, 1e+00]
  RHS range        [1e+00, 2e+02]
Presolve removed 505 rows and 42 columns
Presolve time: 0.01s
Presolved: 486 rows, 841 columns, 4164 nonzeros
Variable types: 1 continuous, 840 integer (840 binary)

Root relaxation: objective 1.342420e+02, 107 iterations, 0.00 seconds

    Nodes    |    Current Node    |     Objective Bounds      |     Work
 Expl Unexpl |  Obj  Depth IntInf | Incumbent    BestBd   Gap | It/Node Time

     0     0  134.24200    0   44          -  134.24200      -     -    0s
     0     0  157.83683    0   34          -  157.83683      -     -    0s
     0     0  157.99700    0   14          -  157.99700      -     -    0s
     0     0  157.99700    0   20          -  157.99700      -     -    0s
     0     0  157.99700    0   20          -  157.99700      -     -    0s
```

	0	2	157.99700	0	20	-	157.99700	-	-	0s
*	5236	2983		68		200.6593333	177.96352	11.3%	12.0	4s
	6489	3594	193.05367	51	18	200.65933	179.65206	10.5%	11.9	5s
	14506	7359	cutoff	56		200.65933	184.71444	7.95%	12.2	10s
	25608	12083	cutoff	44		200.65933	186.54345	7.03%	12.0	15s
	35884	15959	cutoff	46		200.65933	187.53304	6.54%	11.9	20s
	49008	20448	cutoff	54		200.65933	188.53448	6.04%	11.7	25s
	62798	24364	191.88370	44	26	200.65933	189.25325	5.68%	11.5	30s
	77974	28206	cutoff	36		200.65933	189.98377	5.32%	11.3	35s
	91437	31114	195.58400	37	28	200.65933	190.49030	5.07%	11.3	40s
	106464	33796	cutoff	46		200.65933	191.01234	4.81%	11.2	45s
	120932	36386	194.34938	34	28	200.65933	191.40371	4.61%	11.2	50s
	134866	38616	195.99980	69	34	200.65933	191.80477	4.41%	11.1	55s
	148044	40500	195.42542	49	26	200.65933	192.10842	4.26%	11.0	60s
	155705	41333	192.57252	46	24	200.65933	192.30897	4.16%	11.1	65s
	165525	42379	cutoff	50		200.65933	192.54227	4.05%	11.1	70s
	175965	42799	infeasible	41		200.65933	192.83019	3.90%	11.2	75s
	186625	43403	200.09668	44	20	200.65933	193.08755	3.77%	11.2	80s
	196996	43924	197.43248	54	27	200.65933	193.30833	3.66%	11.2	85s
	206086	44034	cutoff	45		200.65933	193.53962	3.55%	11.2	90s
	215479	44029	199.41512	65	23	200.65933	193.74801	3.44%	11.2	95s
	225864	43900	198.28289	42	32	200.65933	193.96349	3.34%	11.2	100s
	236075	43437	199.47565	34	31	200.65933	194.17611	3.23%	11.3	105s
	246841	42705	195.57598	34	24	200.65933	194.39848	3.12%	11.2	110s
	257199	42031	199.36306	46	31	200.65933	194.61362	3.01%	11.2	115s
	266722	40933	196.76698	36	34	200.65933	194.81571	2.91%	11.2	120s
	277893	39473	199.63775	50	9	200.65933	195.07750	2.78%	11.2	125s
	287095	37916	198.48414	27	25	200.65933	195.30856	2.67%	11.2	130s
	296595	36117	198.41814	29	28	200.65933	195.53433	2.55%	11.2	135s
	306144	33932	197.19223	74	32	200.65933	195.78615	2.43%	11.2	140s
	316139	31345	198.43620	51	6	200.65933	196.04884	2.30%	11.2	145s
	325008	28605	cutoff	52		200.65933	196.32382	2.16%	11.2	150s
	330638	26714	200.11833	44	13	200.65933	196.51366	2.07%	11.2	155s
	335571	24919	infeasible	59		200.65933	196.68796	1.98%	11.2	160s
	342362	22096	cutoff	58		200.65933	196.90338	1.87%	11.3	165s
	351392	18132	198.25138	63	24	200.65933	197.30900	1.67%	11.2	170s
	359065	14386	198.76193	54	17	200.65933	197.69486	1.48%	11.2	175s
	366104	10447	cutoff	42		200.65933	198.11486	1.27%	11.2	180s
	373763	5797	cutoff	62		200.65933	198.80609	0.92%	11.2	185s

```
381403     0 infeasible   54        200.65933  200.35787  0.15%  11.1  190s

Cutting planes:
  Gomory: 13
  MIR: 1
  StrongCG: 1
  Flow cover: 7
  Inf proof: 8
  Zero half: 36
  RLT: 17
  Lazy constraints: 48

Explored 381761 nodes (4243924 simplex iterations) in 190.25 seconds
Thread count was 4 (of 4 available processors)


Solution count 1: 200.659


Optimal solution found (tolerance 1.00e-04)
Best objective 2.006593333333e+02, best bound 2.006593333333e+02, gap 0.0000%


User-callback calls 794845, time in user-callback 2.90 sec
6 <gurobi.Var d (value 200.65933333333328)>
Changed value of parameter lazyConstraints to 1
   Prev: 0  Min: 0  Max: 1  Default: 0
Gurobi Optimizer version 9.1.2 build v9.1.2rc0 (mac64)
Thread count: 2 physical cores, 4 logical processors, using up to 4 threads
Optimize a model with 991 rows, 883 columns and 5169 nonzeros
Model fingerprint: 0x46c22e9a
Variable types: 1 continuous, 882 integer (882 binary)
Coefficient statistics:
  Matrix range     [1e+00, 1e+02]
  Objective range  [1e+00, 1e+00]
  Bounds range     [1e+00, 1e+00]
  RHS range        [1e+00, 2e+02]
Presolve removed 505 rows and 42 columns
Presolve time: 0.02s
Presolved: 486 rows, 841 columns, 4417 nonzeros
Variable types: 1 continuous, 840 integer (840 binary)
```

```
Root relaxation: objective 1.395487e+02, 102 iterations, 0.00 seconds

    Nodes    |    Current Node    |     Objective Bounds      |     Work
 Expl Unexpl |  Obj  Depth IntInf | Incumbent    BestBd   Gap | It/Node Time

     0     0  139.54867    0   44          -  139.54867       -     -    0s
     0     0  163.86433    0   17          -  163.86433       -     -    0s
     0     0  163.86433    0   22          -  163.86433       -     -    0s
     0     0  163.86433    0   16          -  163.86433       -     -    0s
     0     2  176.05133    0   22          -  176.05133       -     -    0s
H  874   721                    192.3026667  181.88493  5.42%   7.3    2s
  3348   322  191.62875   36   21  192.30267  189.39933  1.51%  12.2    5s
H 4489   208                    192.3026649  190.33033  1.03%  12.0    5s
H 4517   208                    192.3026622  190.33033  1.03%  12.0    5s

Cutting planes:
  Gomory: 18
  Flow cover: 5
  Inf proof: 1
  Zero half: 22
  RLT: 10
  Lazy constraints: 3

Explored 4981 nodes (59263 simplex iterations) in 5.85 seconds
Thread count was 4 (of 4 available processors)

Solution count 3: 192.303 192.303 192.303

Optimal solution found (tolerance 1.00e-04)
Best objective 1.923026621842e+02, best bound 1.923026621842e+02, gap 0.0000%

User-callback calls 11151, time in user-callback 0.13 sec
1 <gurobi.Var d (value 192.3026621842334)>
Changed value of parameter lazyConstraints to 1
   Prev: 0  Min: 0  Max: 1  Default: 0
Gurobi Optimizer version 9.1.2 build v9.1.2rc0 (mac64)
Thread count: 2 physical cores, 4 logical processors, using up to 4 threads
Optimize a model with 991 rows, 883 columns and 5169 nonzeros
Model fingerprint: 0xd789a184
```

```
Variable types: 1 continuous, 882 integer (882 binary)
Coefficient statistics:
  Matrix range     [1e+00, 1e+02]
  Objective range  [1e+00, 1e+00]
  Bounds range     [1e+00, 1e+00]
  RHS range        [1e+00, 2e+02]
Presolve removed 505 rows and 42 columns
Presolve time: 0.02s
Presolved: 486 rows, 841 columns, 4355 nonzeros
Variable types: 1 continuous, 840 integer (840 binary)

Root relaxation: objective 1.404493e+02, 140 iterations, 0.00 seconds

    Nodes    |    Current Node    |     Objective Bounds      |     Work
 Expl Unexpl |  Obj  Depth IntInf | Incumbent    BestBd   Gap | It/Node Time

     0     0  140.44933    0   44          -  140.44933      -     -    0s
     0     0  163.89367    0   30          -  163.89367      -     -    0s
     0     0  165.37233    0   10          -  165.37233      -     -    0s
     0     0  165.37233    0   32          -  165.37233      -     -    0s
     0     0  165.37233    0   16          -  165.37233      -     -    0s
     0     2  178.68100    0   16          -  178.68100      -     -    0s
  3198  1292  190.55467   44   16          -  187.74133      -  12.9    5s
 10780  4031  189.65704   44   25          -  189.08387      -  11.4   10s
 21231  6189 infeasible   35                -  190.05995      -  10.7   15s
 28302  6503 infeasible   44                -  190.59733      -  10.5   20s
 36502  4624 infeasible   41                -  191.36832      -  10.6   25s

Cutting planes:
  Gomory: 4
  Flow cover: 9
  Zero half: 35
  RLT: 5
  Lazy constraints: 8

Explored 44639 nodes (476957 simplex iterations) in 29.88 seconds
Thread count was 4 (of 4 available processors)

Solution count 0
```

```
Model is infeasible
Best objective -, best bound -, gap -

User-callback calls 93034, time in user-callback 0.42 sec
Changed value of parameter lazyConstraints to 1
   Prev: 0  Min: 0  Max: 1  Default: 0
Gurobi Optimizer version 9.1.2 build v9.1.2rc0 (mac64)
Thread count: 2 physical cores, 4 logical processors, using up to 4 threads
Optimize a model with 991 rows, 883 columns and 5169 nonzeros
Model fingerprint: 0x55fd16cd
Variable types: 1 continuous, 882 integer (882 binary)
Coefficient statistics:
  Matrix range     [1e+00, 1e+02]
  Objective range  [1e+00, 1e+00]
  Bounds range     [1e+00, 1e+00]
  RHS range        [1e+00, 2e+02]
Presolve removed 505 rows and 42 columns
Presolve time: 0.01s
Presolved: 486 rows, 841 columns, 4230 nonzeros
Variable types: 1 continuous, 840 integer (840 binary)

Root relaxation: objective 1.353927e+02, 136 iterations, 0.00 seconds
```

Nodes		Current Node			Objective Bounds			Work	
Expl	Unexpl	Obj	Depth	IntInf	Incumbent	BestBd	Gap	It/Node	Time
0	0	135.39267	0	38	-	135.39267	-	-	0s
0	0	164.93333	0	28	-	164.93333	-	-	0s
0	0	165.79550	0	38	-	165.79550	-	-	0s
0	0	165.94000	0	39	-	165.94000	-	-	0s
0	0	165.94000	0	33	-	165.94000	-	-	0s
0	2	166.13833	0	33	-	166.13833	-	-	0s
4264	1933	187.86900	44	23	-	185.67693	-	12.7	5s
12975	3425	190.72736	55	21	-	188.39600	-	14.4	10s
22374	2356	infeasible	47		-	190.93061	-	14.6	15s

```
Cutting planes:
  Gomory: 13
```

```
   Cover: 36
   MIR: 4
   StrongCG: 1
   Flow cover: 4
   Inf proof: 1
   Zero half: 29
   RLT: 15
   Lazy constraints: 3

Explored 27185 nodes (386404 simplex iterations) in 17.75 seconds
Thread count was 4 (of 4 available processors)

Solution count 0

Model is infeasible
Best objective -, best bound -, gap -

User-callback calls 57161, time in user-callback 0.18 sec
Changed value of parameter lazyConstraints to 1
   Prev: 0  Min: 0  Max: 1  Default: 0
Gurobi Optimizer version 9.1.2 build v9.1.2rc0 (mac64)
Thread count: 2 physical cores, 4 logical processors, using up to 4 threads
Optimize a model with 991 rows, 883 columns and 5169 nonzeros
Model fingerprint: 0x09e3dbea
Variable types: 1 continuous, 882 integer (882 binary)
Coefficient statistics:
   Matrix range     [1e+00, 1e+02]
   Objective range  [1e+00, 1e+00]
   Bounds range     [1e+00, 1e+00]
   RHS range        [1e+00, 2e+02]
Presolve removed 505 rows and 42 columns
Presolve time: 0.02s
Presolved: 486 rows, 841 columns, 4390 nonzeros
Variable types: 1 continuous, 840 integer (840 binary)

Root relaxation: objective 1.380860e+02, 123 iterations, 0.00 seconds

    Nodes    |    Current Node    |     Objective Bounds      |     Work
 Expl Unexpl |  Obj  Depth IntInf | Incumbent    BestBd   Gap | It/Node Time
```

```
     0     0  138.08600    0   38          -  138.08600      -     -    0s
     0     0  167.62667    0   27          -  167.62667      -     -    0s
     0     0  168.49451    0   63          -  168.49451      -     -    0s
     0     0  168.63333    0   41          -  168.63333      -     -    0s
     0     0  168.63333    0   36          -  168.63333      -     -    0s
     0     0  168.76219    0   67          -  168.76219      -     -    0s
     0     0  168.76219    0   48          -  168.76219      -     -    0s
     0     2  169.27767    0   48          -  169.27767      -     -    0s
  5685  2512  190.95157   55   41          -  186.67605      -  12.7    5s
 14169  5298  191.02199   62   51          -  188.89858      -  12.9   10s
 24757  5933 infeasible   50                -  190.15958      -  13.4   15s
 33549  2672 infeasible   42                -  191.68755      -  13.9   20s

Cutting planes:
  Gomory: 15
  Cover: 25
  Flow cover: 3
  Inf proof: 1
  Zero half: 10
  RLT: 9
  Lazy constraints: 3

Explored 37139 nodes (508181 simplex iterations) in 21.82 seconds
Thread count was 4 (of 4 available processors)

Solution count 0

Model is infeasible
Best objective -, best bound -, gap -

User-callback calls 77349, time in user-callback 0.23 sec
Changed value of parameter lazyConstraints to 1
   Prev: 0  Min: 0  Max: 1  Default: 0
Gurobi Optimizer version 9.1.2 build v9.1.2rc0 (mac64)
Thread count: 2 physical cores, 4 logical processors, using up to 4 threads
Optimize a model with 991 rows, 883 columns and 5169 nonzeros
Model fingerprint: 0x8ffddaa9
Variable types: 1 continuous, 882 integer (882 binary)
```

```
Coefficient statistics:
  Matrix range     [1e+00, 1e+02]
  Objective range  [1e+00, 1e+00]
  Bounds range     [1e+00, 1e+00]
  RHS range        [1e+00, 2e+02]
Presolve removed 505 rows and 42 columns
Presolve time: 0.02s
Presolved: 486 rows, 841 columns, 4186 nonzeros
Variable types: 1 continuous, 840 integer (840 binary)

Root relaxation: objective 1.367647e+02, 137 iterations, 0.00 seconds

    Nodes    |    Current Node    |     Objective Bounds      |     Work
 Expl Unexpl |  Obj  Depth IntInf | Incumbent    BestBd   Gap | It/Node Time

     0     0  136.76467    0   44          -  136.76467      -     -    0s
     0     0  159.29167    0   10          -  159.29167      -     -    0s
     0     0  159.29167    0   16          -  159.29167      -     -    0s
     0     0  159.29167    0   10          -  159.29167      -     -    0s
     0     2  159.29167    0   16          -  159.29167      -     -    0s
  6250  2177 infeasible   51               -  186.13159      -  12.2    5s
 16600  3656  191.14277   50   22          -  190.63790      -   9.5   10s

Cutting planes:
  Gomory: 20
  Flow cover: 5
  GUB cover: 1
  Zero half: 10
  Mod-K: 2
  RLT: 8
  Lazy constraints: 6

Explored 23610 nodes (204138 simplex iterations) in 13.02 seconds
Thread count was 4 (of 4 available processors)

Solution count 0

Model is infeasible
Best objective -, best bound -, gap -
```

```
User-callback calls 49934, time in user-callback 0.18 sec
Changed value of parameter lazyConstraints to 1
   Prev: 0  Min: 0  Max: 1  Default: 0
Gurobi Optimizer version 9.1.2 build v9.1.2rc0 (mac64)
Thread count: 2 physical cores, 4 logical processors, using up to 4 threads
Optimize a model with 991 rows, 883 columns and 5169 nonzeros
Model fingerprint: 0x986e3d93
Variable types: 1 continuous, 882 integer (882 binary)
Coefficient statistics:
  Matrix range     [1e+00, 1e+02]
  Objective range  [1e+00, 1e+00]
  Bounds range     [1e+00, 1e+00]
  RHS range        [1e+00, 2e+02]
Presolve removed 507 rows and 42 columns
Presolve time: 0.02s
Presolved: 484 rows, 841 columns, 4634 nonzeros
Variable types: 1 continuous, 840 integer (840 binary)

Root relaxation: objective 1.432307e+02, 131 iterations, 0.00 seconds
```

Nodes		Current Node			Objective Bounds			Work	
Expl	Unexpl	Obj	Depth	IntInf	Incumbent	BestBd	Gap	It/Node	Time
0	0	143.23067	0	44	-	143.23067	-	-	0s
0	0	167.96367	0	20	-	167.96367	-	-	0s
0	0	167.96367	0	26	-	167.96367	-	-	0s
0	0	167.96367	0	12	-	167.96367	-	-	0s
0	2	168.16067	0	12	-	168.16067	-	-	0s
8366	4074	187.42123	28	22	-	186.48170	-	10.9	5s
18584	6917	infeasible	43		-	188.17500	-	10.6	10s
30813	7802	infeasible	34		-	189.63536	-	10.4	15s
41226	5839	191.67978	27	39	-	190.97094	-	10.2	20s
52386	2123	192.25175	34	30	-	192.24314	-	10.1	25s

```
Cutting planes:
  Gomory: 7
  Flow cover: 5
  Zero half: 19
```

```
  RLT: 13
  Lazy constraints: 15

Explored 54941 nodes (551851 simplex iterations) in 25.99 seconds
Thread count was 4 (of 4 available processors)

Solution count 0

Model is infeasible
Best objective -, best bound -, gap -

User-callback calls 113927, time in user-callback 0.39 sec
Changed value of parameter lazyConstraints to 1
   Prev: 0  Min: 0  Max: 1  Default: 0
Gurobi Optimizer version 9.1.2 build v9.1.2rc0 (mac64)
Thread count: 2 physical cores, 4 logical processors, using up to 4 threads
Optimize a model with 991 rows, 883 columns and 5169 nonzeros
Model fingerprint: 0xeeccc201
Variable types: 1 continuous, 882 integer (882 binary)
Coefficient statistics:
  Matrix range     [1e+00, 1e+02]
  Objective range  [1e+00, 1e+00]
  Bounds range     [1e+00, 1e+00]
  RHS range        [1e+00, 2e+02]
Presolve removed 505 rows and 42 columns
Presolve time: 0.01s
Presolved: 486 rows, 841 columns, 4296 nonzeros
Variable types: 1 continuous, 840 integer (840 binary)

Root relaxation: objective 1.372877e+02, 122 iterations, 0.00 seconds
```

Nodes		Current Node			Objective Bounds			Work	
Expl	Unexpl	Obj	Depth	IntInf	Incumbent	BestBd	Gap	It/Node	Time
0	0	137.28767	0	38	-	137.28767	-	-	0s
0	0	161.04333	0	12	-	161.04333	-	-	0s
0	0	161.04333	0	6	-	161.04333	-	-	0s
0	0	161.04333	0	12	-	161.04333	-	-	0s
0	0	161.04333	0	12	-	161.04333	-	-	0s

Nodes		Current Node			Objective Bounds			Work	
Expl	Unexpl	Obj	Depth	IntInf	Incumbent	BestBd	Gap	It/Node	Time
0	2	161.04333	0	16	-	161.04333	-	-	0s
6595	2246	infeasible	29		-	185.48500	-	12.1	5s
16129	3572	191.87051	45	30	-	190.38146	-	11.1	10s
25889	2903	infeasible	37		-	191.41138	-	10.8	15s

Cutting planes:
  Gomory: 11
  Flow cover: 9
  Inf proof: 2
  Zero half: 22
  RLT: 14
  Lazy constraints: 5

Explored 31809 nodes (341542 simplex iterations) in 17.75 seconds
Thread count was 4 (of 4 available processors)

Solution count 0

Model is infeasible
Best objective -, best bound -, gap -

User-callback calls 67134, time in user-callback 0.23 sec
Changed value of parameter lazyConstraints to 1
   Prev: 0 Min: 0 Max: 1 Default: 0
Gurobi Optimizer version 9.1.2 build v9.1.2rc0 (mac64)
Thread count: 2 physical cores, 4 logical processors, using up to 4 threads
Optimize a model with 991 rows, 883 columns and 5169 nonzeros
Model fingerprint: 0x2fea8ed1
Variable types: 1 continuous, 882 integer (882 binary)
Coefficient statistics:
  Matrix range   [1e+00, 1e+02]
  Objective range [1e+00, 1e+00]
  Bounds range   [1e+00, 1e+00]
  RHS range   [1e+00, 2e+02]
Presolve removed 505 rows and 42 columns
Presolve time: 0.01s
Presolved: 486 rows, 841 columns, 4246 nonzeros
Variable types: 1 continuous, 840 integer (840 binary)

Root relaxation: objective 1.349583e+02, 130 iterations, 0.00 seconds

```
    Nodes    |    Current Node    |     Objective Bounds      |     Work
 Expl Unexpl |  Obj  Depth IntInf | Incumbent    BestBd   Gap | It/Node Time

    0     0  134.95833    0   46          -  134.95833      -     -    0s
    0     0  157.89733    0   42          -  157.89733      -     -    0s
    0     0  161.22400    0   10          -  161.22400      -     -    0s
    0     0  161.22400    0   28          -  161.22400      -     -    0s
    0     0  161.22400    0   16          -  161.22400      -     -    0s
    0     2  161.22400    0   16          -  161.22400      -     -    0s
 5883  1395  191.22533   21   41          -  187.74362      -  12.5    5s
```

Cutting planes:
  Gomory: 15
  MIR: 2
  Flow cover: 7
  Zero half: 13
  RLT: 11
  Lazy constraints: 9

Explored 9236 nodes (115996 simplex iterations) in 7.41 seconds
Thread count was 4 (of 4 available processors)

Solution count 0

Model is infeasible
Best objective -, best bound -, gap -

User-callback calls 20159, time in user-callback 0.12 sec
Changed value of parameter lazyConstraints to 1
   Prev: 0 Min: 0 Max: 1 Default: 0
Gurobi Optimizer version 9.1.2 build v9.1.2rc0 (mac64)
Thread count: 2 physical cores, 4 logical processors, using up to 4 threads
Optimize a model with 991 rows, 883 columns and 5169 nonzeros
Model fingerprint: 0xa1bb0323
Variable types: 1 continuous, 882 integer (882 binary)
Coefficient statistics:
  Matrix range   [1e+00, 1e+02]

```
    Objective range   [1e+00, 1e+00]
    Bounds range      [1e+00, 1e+00]
    RHS range         [1e+00, 2e+02]
Presolve removed 505 rows and 42 columns
Presolve time: 0.01s
Presolved: 486 rows, 841 columns, 4268 nonzeros
Variable types: 1 continuous, 840 integer (840 binary)

Root relaxation: objective 1.310323e+02, 120 iterations, 0.00 seconds

    Nodes    |    Current Node    |     Objective Bounds      |     Work
 Expl Unexpl |  Obj  Depth IntInf | Incumbent    BestBd   Gap | It/Node Time

     0     0  131.03233    0   38          -  131.03233      -     -    0s
     0     0  157.81433    0   24          -  157.81433      -     -    0s
     0     0  159.14217    0   42          -  159.14217      -     -    0s
     0     0  160.46933    0   39          -  160.46933      -     -    0s
     0     0  162.25794    0   44          -  162.25794      -     -    0s
     0     0  162.25794    0   47          -  162.25794      -     -    0s
     0     0  163.15225    0   47          -  163.15225      -     -    0s
     0     0  163.15225    0   46          -  163.15225      -     -    0s
     0     0  164.49371    0   57          -  164.49371      -     -    0s
     0     0  164.49371    0   57          -  164.49371      -     -    0s
     0     0  166.47983    0   45          -  166.47983      -     -    0s
     0     0  169.82633    0   48          -  169.82633      -     -    0s
     0     0  169.82633    0   52          -  169.82633      -     -    0s
     0     0  169.82633    0   34          -  169.82633      -     -    0s
     0     2  170.09883    0   44          -  170.09883      -     -    0s
  5209  1241 infeasible   35                -  187.41833      -  14.9    5s

Cutting planes:
  Gomory: 14
  Cover: 1
  MIR: 5
  Flow cover: 5
  Zero half: 20
  RLT: 13
  Lazy constraints: 10
```

```
Explored 7508 nodes (110943 simplex iterations) in 6.13 seconds
Thread count was 4 (of 4 available processors)

Solution count 0

Model is infeasible
Best objective -, best bound -, gap -

User-callback calls 16361, time in user-callback 0.11 sec
Changed value of parameter lazyConstraints to 1
   Prev: 0  Min: 0  Max: 1  Default: 0
Gurobi Optimizer version 9.1.2 build v9.1.2rc0 (mac64)
Thread count: 2 physical cores, 4 logical processors, using up to 4 threads
Optimize a model with 991 rows, 883 columns and 5169 nonzeros
Model fingerprint: 0xc28448ea
Variable types: 1 continuous, 882 integer (882 binary)
Coefficient statistics:
  Matrix range     [1e+00, 1e+02]
  Objective range  [1e+00, 1e+00]
  Bounds range     [1e+00, 1e+00]
  RHS range        [1e+00, 2e+02]
Presolve removed 508 rows and 42 columns
Presolve time: 0.01s
Presolved: 483 rows, 841 columns, 4515 nonzeros
Variable types: 1 continuous, 840 integer (840 binary)

Root relaxation: objective 1.431830e+02, 121 iterations, 0.00 seconds

    Nodes    |    Current Node    |     Objective Bounds      |     Work
 Expl Unexpl |  Obj  Depth IntInf | Incumbent    BestBd   Gap | It/Node Time

     0     0  143.18300    0   44          -  143.18300      -     -    0s
     0     0  166.74033    0   22          -  166.74033      -     -    0s
     0     0  167.74433    0   53          -  167.74433      -     -    0s
     0     0  168.02867    0   28          -  168.02867      -     -    0s
     0     0  168.02867    0   28          -  168.02867      -     -    0s
     0     2  168.02867    0   34          -  168.02867      -     -    0s
  6905  3564  181.21234   27   51          -  176.84656      -  11.9    5s
 15873  4962  189.98576   50   17          -  186.48384      -  12.1   10s
```

```
  26742  3531  192.27167   34   32          -  190.91146      -  11.5   15s

Cutting planes:
   Gomory: 5
   Cover: 6
   Flow cover: 4
   Zero half: 16
   RLT: 10
   Lazy constraints: 18

Explored 31890 nodes (353264 simplex iterations) in 17.34 seconds
Thread count was 4 (of 4 available processors)

Solution count 0

Model is infeasible
Best objective -, best bound -, gap -

User-callback calls 66815, time in user-callback 0.27 sec
Changed value of parameter lazyConstraints to 1
   Prev: 0  Min: 0  Max: 1  Default: 0
Gurobi Optimizer version 9.1.2 build v9.1.2rc0 (mac64)
Thread count: 2 physical cores, 4 logical processors, using up to 4 threads
Optimize a model with 991 rows, 883 columns and 5169 nonzeros
Model fingerprint: 0x98026204
Variable types: 1 continuous, 882 integer (882 binary)
Coefficient statistics:
   Matrix range      [1e+00, 1e+02]
   Objective range   [1e+00, 1e+00]
   Bounds range      [1e+00, 1e+00]
   RHS range         [1e+00, 2e+02]
Presolve removed 505 rows and 42 columns
Presolve time: 0.01s
Presolved: 486 rows, 841 columns, 4198 nonzeros
Variable types: 1 continuous, 840 integer (840 binary)


Root relaxation: objective 1.385103e+02, 135 iterations, 0.00 seconds

    Nodes    |    Current Node    |     Objective Bounds      |     Work
```

```
 Expl Unexpl |  Obj  Depth IntInf | Incumbent    BestBd    Gap | It/Node Time


     0     0 138.51033    0   38           -  138.51033      -      -    0s
     0     0 163.68033    0   10           -  163.68033      -      -    0s
     0     0 163.68033    0   24           -  163.68033      -      -    0s
     0     0 163.68033    0   16           -  163.68033      -      -    0s
     0     2 163.87733    0   16           -  163.87733      -      -    0s
  4793    98 infeasible   30              -  190.97445      -   17.7    5s

Cutting planes:
  Gomory: 16
  Flow cover: 5
  Zero half: 19
  RLT: 3
  Lazy constraints: 7

Explored 5044 nodes (87833 simplex iterations) in 5.09 seconds
Thread count was 4 (of 4 available processors)

Solution count 0

Model is infeasible
Best objective -, best bound -, gap -

User-callback calls 11338, time in user-callback 0.08 sec
Changed value of parameter lazyConstraints to 1
   Prev: 0  Min: 0  Max: 1  Default: 0
Gurobi Optimizer version 9.1.2 build v9.1.2rc0 (mac64)
Thread count: 2 physical cores, 4 logical processors, using up to 4 threads
Optimize a model with 991 rows, 883 columns and 5169 nonzeros
Model fingerprint: 0x895eb0ed
Variable types: 1 continuous, 882 integer (882 binary)
Coefficient statistics:
  Matrix range     [1e+00, 1e+02]
  Objective range  [1e+00, 1e+00]
  Bounds range     [1e+00, 1e+00]
  RHS range        [1e+00, 2e+02]
Presolve removed 505 rows and 42 columns
Presolve time: 0.02s
```

```
Presolved: 486 rows, 841 columns, 4345 nonzeros
Variable types: 1 continuous, 840 integer (840 binary)

Root relaxation: objective 1.395757e+02, 130 iterations, 0.00 seconds

    Nodes    |    Current Node    |     Objective Bounds      |     Work
 Expl Unexpl |  Obj  Depth IntInf | Incumbent    BestBd   Gap | It/Node Time

     0     0  139.57567    0   38          -  139.57567      -     -    0s
     0     0  164.13700    0   40          -  164.13700      -     -    0s
     0     0  164.74567    0   30          -  164.74567      -     -    0s
     0     0  164.74567    0   16          -  164.74567      -     -    0s
     0     0  164.74567    0   16          -  164.74567      -     -    0s
     0     0  164.74567    0   16          -  164.74567      -     -    0s
     0     0  164.74567    0   16          -  164.74567      -     -    0s
     0     2  164.74567    0   16          -  164.74567      -     -    0s

Cutting planes:
  Gomory: 15
  Flow cover: 4
  Zero half: 6
  RLT: 9
  Lazy constraints: 2

Explored 4638 nodes (58937 simplex iterations) in 4.11 seconds
Thread count was 4 (of 4 available processors)

Solution count 0

Model is infeasible
Best objective -, best bound -, gap -

User-callback calls 10338, time in user-callback 0.06 sec
Changed value of parameter lazyConstraints to 1
   Prev: 0  Min: 0  Max: 1  Default: 0
Gurobi Optimizer version 9.1.2 build v9.1.2rc0 (mac64)
Thread count: 2 physical cores, 4 logical processors, using up to 4 threads
Optimize a model with 991 rows, 883 columns and 5169 nonzeros
Model fingerprint: 0xa4afade1
```

```
Variable types: 1 continuous, 882 integer (882 binary)
Coefficient statistics:
  Matrix range     [1e+00, 1e+02]
  Objective range  [1e+00, 1e+00]
  Bounds range     [1e+00, 1e+00]
  RHS range        [1e+00, 2e+02]
Presolve removed 507 rows and 42 columns
Presolve time: 0.01s
Presolved: 484 rows, 841 columns, 4524 nonzeros
Variable types: 1 continuous, 840 integer (840 binary)

Root relaxation: objective 1.497363e+02, 144 iterations, 0.00 seconds

    Nodes    |    Current Node    |     Objective Bounds      |     Work
 Expl Unexpl |  Obj  Depth IntInf | Incumbent    BestBd   Gap | It/Node Time

     0     0  149.73633    0   46          -  149.73633      -     -    0s
     0     0  170.63800    0   22          -  170.63800      -     -    0s
     0     0  171.92733    0   16          -  171.92733      -     -    0s
     0     0  171.92733    0   42          -  171.92733      -     -    0s
     0     0  171.92733    0   16          -  171.92733      -     -    0s
     0     2  171.92733    0   16          -  171.92733      -     -    0s

Cutting planes:
  Gomory: 8
  Cover: 2
  Zero half: 22
  RLT: 35
  Lazy constraints: 4

Explored 2158 nodes (18663 simplex iterations) in 1.14 seconds
Thread count was 4 (of 4 available processors)

Solution count 0

Model is infeasible
Best objective -, best bound -, gap -

User-callback calls 4558, time in user-callback 0.02 sec
```

```
Changed value of parameter lazyConstraints to 1
   Prev: 0  Min: 0  Max: 1  Default: 0
Gurobi Optimizer version 9.1.2 build v9.1.2rc0 (mac64)
Thread count: 2 physical cores, 4 logical processors, using up to 4 threads
Optimize a model with 991 rows, 883 columns and 5169 nonzeros
Model fingerprint: 0x8720281d
Variable types: 1 continuous, 882 integer (882 binary)
Coefficient statistics:
  Matrix range     [1e+00, 1e+02]
  Objective range  [1e+00, 1e+00]
  Bounds range     [1e+00, 1e+00]
  RHS range        [1e+00, 2e+02]
Presolve removed 505 rows and 42 columns
Presolve time: 0.01s
Presolved: 486 rows, 841 columns, 4298 nonzeros
Variable types: 1 continuous, 840 integer (840 binary)

Root relaxation: objective 1.359867e+02, 124 iterations, 0.00 seconds

    Nodes    |    Current Node    |     Objective Bounds      |     Work
 Expl Unexpl |  Obj  Depth IntInf | Incumbent    BestBd   Gap | It/Node Time

     0     0  135.98667    0   44          -  135.98667      -     -    0s
     0     0  159.83300    0   14          -  159.83300      -     -    0s
     0     0  159.83300    0   20          -  159.83300      -     -    0s
     0     0  159.83300    0   20          -  159.83300      -     -    0s
     0     2  159.83300    0   20          -  159.83300      -     -    0s
  5706  3417  180.46600   24   13          -  179.97017      -  12.2    5s
 15930  8766  191.34533   49   17          -  181.75142      -  12.4   10s
 24092 12178  185.82818   48   47          -  183.11967      -  12.6   15s
 35294 15714 infeasible   38               -  184.73477      -  12.6   20s
 46132 18513  190.52896   38   46          -  185.90689      -  12.6   25s
 57480 20607  188.69555   35   10          -  186.74650      -  12.6   30s
 68202 21664 infeasible   39               -  187.51700      -  12.7   35s
 79116 22333  191.54116   42   55          -  188.23602      -  12.6   40s
 89692 22443 infeasible   47               -  188.81470      -  12.6   45s
 97972 22139  190.31956   53   22          -  189.29066      -  12.5   50s
109619 22455 infeasible   49               -  189.75291      -  12.2   55s
120969 21415  190.45450   40   25          -  190.28246      -  12.1   60s
```

```
   132325 19617 infeasible   44                    -  190.76515      -  11.9   65s
   143767 17624 infeasible   44                    -  191.23578      -  11.7   70s
   151877 13971  191.62312   42    36              -  191.62312      -  11.6   75s
   161242  6970 infeasible   55                    -  192.39381      -  11.6   80s

Cutting planes:
  Gomory: 12
  Flow cover: 10
  Inf proof: 3
  Zero half: 30
  RLT: 19
  Lazy constraints: 34

Explored 168795 nodes (1949931 simplex iterations) in 83.02 seconds
Thread count was 4 (of 4 available processors)

Solution count 0

Model is infeasible
Best objective -, best bound -, gap -

User-callback calls 348428, time in user-callback 1.08 sec
Changed value of parameter lazyConstraints to 1
   Prev: 0  Min: 0  Max: 1  Default: 0
Gurobi Optimizer version 9.1.2 build v9.1.2rc0 (mac64)
Thread count: 2 physical cores, 4 logical processors, using up to 4 threads
Optimize a model with 991 rows, 883 columns and 5169 nonzeros
Model fingerprint: 0x9d4d15f5
Variable types: 1 continuous, 882 integer (882 binary)
Coefficient statistics:
  Matrix range     [1e+00, 1e+02]
  Objective range  [1e+00, 1e+00]
  Bounds range     [1e+00, 1e+00]
  RHS range        [1e+00, 2e+02]
Presolve removed 505 rows and 42 columns
Presolve time: 0.01s
Presolved: 486 rows, 841 columns, 4226 nonzeros
Variable types: 1 continuous, 840 integer (840 binary)
```

```
Root relaxation: objective 1.310323e+02, 124 iterations, 0.00 seconds

    Nodes    |    Current Node    |     Objective Bounds      |     Work
 Expl Unexpl |  Obj  Depth IntInf | Incumbent    BestBd   Gap | It/Node Time

     0     0  131.03233    0   38          -  131.03233      -     -    0s
     0     0  179.69667    0   38          -  179.69667      -     -    0s
     0     0  181.41467    0   28          -  181.41467      -     -    0s
     0     0  181.41467    0   34          -  181.41467      -     -    0s
     0     0  181.41467    0   34          -  181.41467      -     -    0s
     0     2  181.41467    0   34          -  181.41467      -     -    0s
  4985  1724 infeasible   43                -  187.79600      -  11.4   5s
 16061  1432 infeasible   59                -  191.62117      -  10.8  10s

Cutting planes:
  Gomory: 9
  Cover: 49
  MIR: 1
  Flow cover: 7
  Zero half: 18
  RLT: 13
  Lazy constraints: 1

Explored 18248 nodes (191476 simplex iterations) in 10.91 seconds
Thread count was 4 (of 4 available processors)

Solution count 0

Model is infeasible
Best objective -, best bound -, gap -

User-callback calls 38580, time in user-callback 0.12 sec
Changed value of parameter lazyConstraints to 1
   Prev: 0  Min: 0  Max: 1  Default: 0
Gurobi Optimizer version 9.1.2 build v9.1.2rc0 (mac64)
Thread count: 2 physical cores, 4 logical processors, using up to 4 threads
Optimize a model with 991 rows, 883 columns and 5169 nonzeros
Model fingerprint: 0x6cea7f5e
Variable types: 1 continuous, 882 integer (882 binary)
```

```
Coefficient statistics:
  Matrix range     [1e+00, 1e+02]
  Objective range  [1e+00, 1e+00]
  Bounds range     [1e+00, 1e+00]
  RHS range        [1e+00, 2e+02]
Presolve removed 509 rows and 42 columns
Presolve time: 0.02s
Presolved: 482 rows, 841 columns, 4464 nonzeros
Variable types: 1 continuous, 840 integer (840 binary)

Root relaxation: objective 1.421720e+02, 127 iterations, 0.01 seconds

    Nodes    |    Current Node    |     Objective Bounds     |     Work
 Expl Unexpl |  Obj  Depth IntInf | Incumbent    BestBd   Gap | It/Node Time

     0     0  142.17200    0   44          -  142.17200      -     -    0s
     0     0  178.39767    0   14          -  178.39767      -     -    0s
     0     0  178.39767    0   18          -  178.39767      -     -    0s
     0     0  178.39767    0   12          -  178.39767      -     -    0s
     0     0 infeasible    0               - infeasible      -     -    0s

Cutting planes:
  Gomory: 13
  Zero half: 28
  RLT: 39
  Lazy constraints: 2

Explored 1 nodes (306 simplex iterations) in 0.10 seconds
Thread count was 4 (of 4 available processors)

Solution count 0

Model is infeasible
Best objective -, best bound -, gap -

User-callback calls 116, time in user-callback 0.00 sec
Changed value of parameter lazyConstraints to 1
   Prev: 0  Min: 0  Max: 1  Default: 0
Gurobi Optimizer version 9.1.2 build v9.1.2rc0 (mac64)
```

```
Thread count: 2 physical cores, 4 logical processors, using up to 4 threads
Optimize a model with 991 rows, 883 columns and 5169 nonzeros
Model fingerprint: 0xdaced03e
Variable types: 1 continuous, 882 integer (882 binary)
Coefficient statistics:
  Matrix range     [1e+00, 1e+02]
  Objective range  [1e+00, 1e+00]
  Bounds range     [1e+00, 1e+00]
  RHS range        [1e+00, 2e+02]
Presolve removed 505 rows and 42 columns
Presolve time: 0.01s
Presolved: 486 rows, 841 columns, 4228 nonzeros
Variable types: 1 continuous, 840 integer (840 binary)

Root relaxation: objective 1.341423e+02, 146 iterations, 0.00 seconds

    Nodes    |    Current Node    |     Objective Bounds      |     Work
 Expl Unexpl |  Obj  Depth IntInf | Incumbent    BestBd   Gap | It/Node Time

     0     0  134.14233    0   38          -  134.14233      -     -    0s
     0     0  182.80667    0   41          -  182.80667      -     -    0s
     0     0  184.52467    0   28          -  184.52467      -     -    0s
     0     0  184.52467    0   28          -  184.52467      -     -    0s
     0     2  184.52467    0   28          -  184.52467      -     -    0s

Cutting planes:
  Gomory: 13
  Cover: 6
  Flow cover: 2
  Zero half: 8
  RLT: 13
  Lazy constraints: 1

Explored 3424 nodes (41558 simplex iterations) in 4.68 seconds
Thread count was 4 (of 4 available processors)

Solution count 0

Model is infeasible
```

```
Best objective -, best bound -, gap -

User-callback calls 8249, time in user-callback 0.04 sec
Changed value of parameter lazyConstraints to 1
    Prev: 0  Min: 0  Max: 1  Default: 0
Gurobi Optimizer version 9.1.2 build v9.1.2rc0 (mac64)
Thread count: 2 physical cores, 4 logical processors, using up to 4 threads
Optimize a model with 991 rows, 883 columns and 5169 nonzeros
Model fingerprint: 0x8962f751
Variable types: 1 continuous, 882 integer (882 binary)
Coefficient statistics:
  Matrix range     [1e+00, 1e+02]
  Objective range  [1e+00, 1e+00]
  Bounds range     [1e+00, 1e+00]
  RHS range        [1e+00, 2e+02]
Presolve removed 505 rows and 42 columns
Presolve time: 0.01s
Presolved: 486 rows, 841 columns, 4306 nonzeros
Variable types: 1 continuous, 840 integer (840 binary)

Root relaxation: objective 1.434620e+02, 152 iterations, 0.00 seconds

    Nodes    |    Current Node    |     Objective Bounds      |     Work
 Expl Unexpl |  Obj  Depth IntInf | Incumbent    BestBd   Gap | It/Node Time

     0     0  143.46200    0   44          -  143.46200      -     -    0s
     0     0  175.68667    0   14          -  175.68667      -     -    0s
     0     0  175.68667    0    6          -  175.68667      -     -    0s
     0     2  175.68667    0    6          -  175.68667      -     -    0s

Cutting planes:
  Gomory: 8
  Cover: 24
  Zero half: 31
  RLT: 37
  Lazy constraints: 2

Explored 1728 nodes (13429 simplex iterations) in 1.04 seconds
Thread count was 4 (of 4 available processors)
```

Solution count 0

Model is infeasible
Best objective -, best bound -, gap -

User-callback calls 3688, time in user-callback 0.01 sec
Changed value of parameter lazyConstraints to 1
   Prev: 0 Min: 0 Max: 1 Default: 0
Gurobi Optimizer version 9.1.2 build v9.1.2rc0 (mac64)
Thread count: 2 physical cores, 4 logical processors, using up to 4 threads
Optimize a model with 991 rows, 883 columns and 5169 nonzeros
Model fingerprint: 0x182ec761
Variable types: 1 continuous, 882 integer (882 binary)
Coefficient statistics:
  Matrix range   [1e+00, 1e+02]
  Objective range [1e+00, 1e+00]
  Bounds range   [1e+00, 1e+00]
  RHS range   [1e+00, 2e+02]
Presolve removed 505 rows and 42 columns
Presolve time: 0.02s
Presolved: 486 rows, 841 columns, 4219 nonzeros
Variable types: 1 continuous, 840 integer (840 binary)

Root relaxation: objective 1.496833e+02, 130 iterations, 0.01 seconds

Nodes		Current Node			Objective Bounds			Work	
Expl	Unexpl	Obj	Depth	IntInf	Incumbent	BestBd	Gap	It/Node	Time
0	0	149.68333	0	44	-	149.68333	-	-	0s
0	0	180.00833	0	27	-	180.00833	-	-	0s
0	0	180.00833	0	24	-	180.00833	-	-	0s
0	0	180.00833	0	12	-	180.00833	-	-	0s
0	0	180.00833	0	12	-	180.00833	-	-	0s
0	0	182.30522	0	38	-	182.30522	-	-	0s
0	0	infeasible	0		-	infeasible	-	-	0s

Cutting planes:
  Gomory: 5

```
    MIR: 1
    Zero half: 27
    RLT: 39
    Lazy constraints: 2

Explored 1 nodes (398 simplex iterations) in 0.15 seconds
Thread count was 4 (of 4 available processors)

Solution count 0

Model is infeasible
Best objective -, best bound -, gap -

User-callback calls 142, time in user-callback 0.01 sec

Process finished with exit code 0
```