

Debiasing item-to-item recommendations with small annotated datasets

$I \rightsquigarrow$ item inventory

$(S_1, \dots, S_N) \rightsquigarrow N$ session vectors. like / share / buy ...

$S_k : |I|$ entries. $s_{k[i]} = \text{engage}(u_k, i)$ for $i \in I$.
 full info setting \hookrightarrow user k

can also be viewed: been drawn i.i.d. directly from an overall distribution of session vector $P(S)$

$s_{k[i]} \in \{0, 1\}$, binary random variable.

Goal of item-to-item Recommendation

$$\underset{i \in I \setminus j}{\operatorname{argmax}} P(s_{i|j} = 1 \mid s_{t|j} = 1)$$

!! Remarks:

- ① "session" not necessarily means session, can also be all historical interactions
- ② can also be extended to top-K problem easily

In partial info setting:

$$s_{k[i]}^{\text{obs}} = s_{k[i]} \times d_{k[i]}$$

binary & latent observation variable, $\in \{0, 1\}$

$$d_k \in \{0, 1\}^{|I|} \sim P(D \mid X_k)$$

Assumption here: (conditionally independent)

$$P(O_{r|j}=1, O_{r|j}=1 \mid X_k) = P(O_{r|j}=1 \mid X_k) \cdot P(O_{r|j}=1 \mid X_k)$$

MLE:

$$\begin{aligned} \hat{P}^{\text{MLE}}(s_{i|j}=1 \mid s_{t|j}=1) &= \frac{\hat{P}^{\text{MLE}}(s_{i|j}=1, s_{t|j}=1)}{\hat{P}^{\text{MLE}}(s_{t|j}=1)} \\ &= \frac{\frac{1}{N} \sum_k \mathbf{1}(s_{k[i]}=1 \wedge s_{k[t]}=1)}{\frac{1}{N} \sum_k \mathbf{1}(s_{k[t]}=1)} \quad \text{assumed to be non-zero} \end{aligned}$$





remark on this:

- ① maximizing this eq. tends to find the popular item i .

thus it tends to have greater chance of co-occurrence with j .

Inverse Propensity Score

$$P_{k,i} := P(O_{k(i)}) = 1 \mid X_k) \quad \text{② still uses the full info setting } S.$$

$$\begin{aligned} \hat{P}_{(S_{[i]})=1 \mid S_{[j]}=1}^{IPS} &= \frac{\sum_k P_{ki}^{-1} P_{kj}^{-1} \mathbb{1} \left\{ S_{k(i)}^{\text{obs}} = 1 \wedge S_{k(j)}^{\text{obs}} = 1 \right\}}{\sum_k P_{kj}^{-1} \mathbb{1} \left\{ S_{k(j)}^{\text{obs}} = 1 \right\}} \xrightarrow{\text{unbiased estimator of } P(S_{[i]}=1 \wedge S_{[j]}=1)} \\ &= Z_j \sum_k P_{ki}^{-1} P_{kj}^{-1} \mathbb{1} \left\{ S_{k(i)}^{\text{obs}} = 1 \wedge S_{k(j)}^{\text{obs}} = 1 \right\} \xrightarrow{\text{unbiased estimator of } P(S_{[i]}=1)} \end{aligned}$$

W/ Slutsky's Theorem, we know that the ratio above is a consistent estimator of the true conditional prob.

Now the problem is how to estimate $P_{k,i}$ accurately.

strategy of this paper:



$P_k = f_\theta(X_k)$: choose a specific function
use a model to learn this
learning θ from small annotated dataset
↓
relevance of items.

Section 4:

Annotated dataset:

$$C_j = \left\{ (i_1, i_2) \mid \text{rel}(i_1 | j) > \text{rel}(i_2 | j) \right\}$$

↓
how relevant i is to a given item j

!! hard to get the absolute relevance. so only get pairs relative relevance relationship pairs

$$\text{rel}(i_1 | j) > \text{rel}(i_2 | j)$$

$$\Leftrightarrow P(S_{[i_1]}=1 \mid S_{[j]}=1) > P(S_{[i_2]}=1 \mid S_{[j]}=1)$$

$$\Leftrightarrow \log P_{(S_{[i_1]}=1 \mid S_{[j]}=1)}^{IPS} - \log P_{(S_{[i_2]}=1 \mid S_{[j]}=1)}^{IPS} > 0$$

⇒ Hinge Loss :

$$l(c_j; \theta) = \sum_{(i_1, i_2) \in C_j} \max \left\{ 0, -\log \hat{P}_{\theta}^{IPS}(s_{i_1} = 1 | s_{i_2} = 1) + \log \hat{P}_{\theta}^{IPS}(s_{i_2} = 1 | s_{i_1} = 1) \right\}$$

$$\Rightarrow \underset{\theta}{\operatorname{argmin}} \frac{1}{L} \sum_{j=1}^L l(c_j; \theta)$$

Exponential model:

For $p_k = f_{\theta}(x_k)$, here we set

$$p_{k,i}^{-1} = \exp \left\{ \theta^T \cdot \phi(x_k) \right\}$$

$\phi(\cdot)$: feature map: $x_k \rightarrow$ vector representation

$$p_{i,j}^{-1} = \exp \left\{ \theta^T \cdot \phi(i, j) \right\}$$

Plug this into above equations.

$$\begin{aligned} \log \hat{P}_{\theta}^{IPS}(s_{i_1} = 1 | s_{i_2} = 1) &= \log \left[\sum_j \exp \left\{ \theta^T \cdot \phi_{(k, i_1)} \right\} \exp \left\{ \theta^T \cdot \phi_{(k, i_2)} \right\} \right] \\ &\quad \cdot \mathbb{1} \left\{ S_{k(i_1)}^{obs} = 1 \wedge S_{k(i_2)}^{obs} = 1 \right\} \\ &= z_j + \theta^T \cdot \phi_{(i_1, j)} + \log \sum_k \mathbb{1} \left\{ S_{k(i_1)}^{obs} = 1 \wedge S_{k(i_2)}^{obs} = 1 \right\} \\ &\quad \hookrightarrow := \text{COUNT}(i_1, j) \end{aligned}$$

Final Loss Function:

$$l(c_j; \theta) = \sum_{(i_1, i_2) \in C_j} \max \left\{ 0, -\theta^T \phi_{(i_1, j)} + \log \text{COUNT}(i_1, j) + \theta^T \phi_{(i_2, j)} + \log \text{COUNT}(i_2, j) \right\}$$