Investigation of Simulated Annealing Cooling Schedule for Mobile Recommendations

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***Abstract*—Nowadays, mobile recommendation has become an important research topic in data science. While many researchers focus on developing new applications and designing algorithms for computational efficiency in different business areas, some significant technical problems in classical algorithms are rarely studied under these applications. Simulated annealing (SA), a key approach in solving global optimization problems, is one of them. To this end, this paper aims to investigate the performance of SA in mobile recommendation problems with a focus on identifying the optimal cooling schedule method. We also discuss the move generation, parameter estimation, and the balance between efficiency and effectiveness in SA. Specifically, our tests are based on two problems: a travelling salesman problem and a mobile route recommendation problem. The results suggest that the exponential-based method performs the best to achieve the optimal final energy, while the greedy method, constant-rated-based method, and logarithm-based method are dominant in terms of computational efficiency. Our studies would serve as a guidance of SA for mobile recommendation algorithm designs, especially for the selection of cooling schedule and related parameter estimation.**

Keywords—simulated annealing; cooling schedule; mobile recommendation; travelling salesman problem; route recommendation

I. INTRODUCTION

In this paper, we investigate the performance of simulated annealing in solving mobile recommendation problems with a focus on determining appropriate *cooling schedules*. Simulated annealing(SA) algorithm has been widely used to locate good approximations in global optimization problems. By setting a temperature variable, the algorithm allows additional attempts when peak values are reached to avoid falling into local optima. To be more specific, as shown in Fig. 1, SA explores the solution space from the near neighbors of a known solution to make sure other optimal values are located as well. As a key setting, when the temperature is reduced based on a selected cooling schedule method, the chance of accepting worse solutions decreases. It then enables the algorithm to gradually focus on an area of the search space in which a close to optimum solution can be found. As a result, the global optimum can be found after a large amount of attempts.

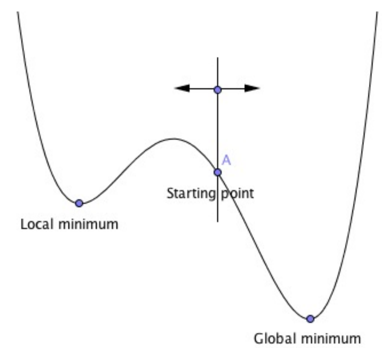

Fig. 1 Local Optimum and Global Optimum

Many business applications have been developed in order to solve different mobile recommendation problems. Mobile trajectory (driving route) recommendation is one of the most significant ones. Since the core of most of the recommendation problem is global optimization, SA can be easily implemented. However, the details in setting up SA in these problems are rarely studied and lack of guidance. This paper investigates four different cooling schedules, including greedy method, constant rate method, logarithm method and exponential method, based on a classical travelling salesman problem(TSP) and a mobile route recommendation problem. We also studied related parameter estimation for the two problems. Moreover, since simulated annealing has been widely implemented with parallel computing algorithms, we further discuss the expected performance of parallel simulated annealing in solving mobile recommendation problems.

The rest of the paper is organized as follows. In section II, we discuss the TSP and a mobile route recommendation problem. In section III, we discuss the major procedures of SA with a focus on the four cooling schedules we investigate in this paper. Then, we introduce the data generation, experimental designs, and discuss the testing results in Section IV. Related work is discussed in section V. Finally, we conclude the paper and discuss the future work in section VI.

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II. Optimization Problems in Mobile Recommendation

In this section we discuss two famous optimization problems which have been widely discussed and implemented in mobile recommendation systems: Travelling Salesman Problem (TSP) and Driving Route Recommendation problem.

A. Travelling Salesman Problem (TSP)

Given a list of cities or locations and the distances between each pair of the locations, TSP aims to find the shortest route for a person to visit each location once and return to the starting point. For city $i = 0, 1, \dots, n$, let d_{ij} to be the distance from i to j. Then TSP can be formulated as follows.

$$min \sum_{i=0}^{n} \sum_{j \neq i, j=0}^{n} d_{ij} x_{ij}$$

s.t.

$$x_{ij} = \begin{cases} 1 & \text{if the person goes from } i \text{ to } j \\ 0 & \text{otherwise} \end{cases}$$

$$\sum_{i=0, i \neq j}^{n} x_{ij} = 1$$

$$\sum_{j=0, j \neq i}^{n} x_{ij} = 1$$

$$u_i - u_j + n x_{ij} \leq n - 1$$

where $u_i \in \mathbf{Z}$ is an artificial variable; and the last constraint requires that all paths are continued.

B. Trajectory Recommendation

Many mobile recommendation problems are similar to TSP but with more points of views in business implementation. Here we introduce a driving route recommendation problem which is designed for taxi drivers [1].

Given a set of potential pick-up points $M = \{m_1, m_2, \dots m_n\}$, the probability set of successful pick-up, $P = \{P(m_1), P(m_2), \dots, P(m_n)\}$, and the current position, Pos, of a taxi driver, we aim to recommend an optimal driving route $\overrightarrow{R^L}$ with a length of L to minimize the *potential travelling distance* (PTD):

$$\min_{\overrightarrow{R_i^L} \in \vec{R}} f(Pos, \overrightarrow{R_i^L}, P_{\overrightarrow{R_i^L}}) \quad (1)$$

where $P_{\overrightarrow{R_i^L}}$ is the probabilities of successful pick-up for all points contained in $\overrightarrow{R_\iota}$. The PTD function of the route recommendation with a length of Lcan be definedas follows.

$$f\left(Pos, \overrightarrow{R_i^L}, P_{\overrightarrow{R_i^L}}\right) = D_{\overrightarrow{R^L}} \cdot P_{\overrightarrow{R^L}} \quad (2)$$

where $D_{\overrightarrow{R^L}}$ is the vector of distance from one position to the next position in driving route $\overrightarrow{R^L}$. In addition, we add a constant distance D_∞ as the last one of $D_{\overrightarrow{R^L}}$ to describe the cost of unsuccessful pick-up withthis driving route. $P_{\overrightarrow{R^L}}$ is the probability of successfullypick-up at position $j = 1,2,3, \dots L$, in the driving route, and we add the probability of unsuccessful pick-up with this driving route as the last one in $P_{\overrightarrow{R^L}}$.

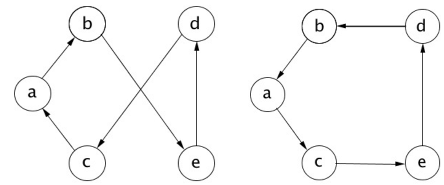


Fig. 2 Move Generation: Swapping

III. Cooling Schedule and Parameter Estimation

A. Preparing for Simulated Annealing

The major components of a simulated annealing algorithm include a probability function, move generation, cooling schedule and stopping condition. For the probability function, a widely accepted one is $e^{-\Delta E/T}$, where ΔE is the change of energy (the total travelling distance in TSP and the PTD in route recommendations) in the current step, and T is the current temperature. Regarding the move generation method, we use the swapping method in TSP problem. As shown in Fig. 2, by swapping the city b and c in the route, a-b-e-d-c-a, we get the route a-c-e-d-b-a. Given a complete route, one city is randomly selected. Then, within the rest of cities, one will be selected with an equal chance. We swap these pair of cities and get a new route, and compute the new energy. The new route will be accepted based on a probability function$e^{-\Delta E/T}$, where the temperature T is reduced based on a cooling method. Last, set a determination condition (for example, no significant energy change in 100,000 steps) to end the SA. In Trajectory Recommendation problem, the move generation combines selecting and swapping method. We randomly generate an initial Route. One pick-up point m_i is randomly selected. If m_i is not in the route, it randomly replaces one point m_j in that route. If m_i is in the route, a point m_j in the route is chosen and m_i and m_j swap. Then, we will accept the new route by a probability $e^{-\Delta E/T}$. Finally, the whole SA process is terminated based on the stopping condition (the energy remains unchanged in 100,000 steps).

Within the three major components of SA, move generation is a problem dependent strategy that can vary even for the same type of problems; and termination condition determines the convergence of the SA process; cooling schedule is the only one that can be generalized for one type of objective functions. Therefore, this paper focuses on investigating the performance of four different cooling schedule methods: greedy method, constant rate method, logarithmic method, and exponential method.

B. Cooling Schedule

Now we introduce the key idea of the four cooling schedule methods.

1) Greedy Method

The greedy method is a special case of SA that reduces the temperature to 0. Then, the system accepts the better state of the next step. It is easier for this method to converge in a fast speed but get trapped a local minimum.

2) Constant Rate Method

The constant cooling strategy, also called the Metropolis method, simply lets the temperature be a constant, T_{const}.It is originally proposed by Metropolis et al. [2]. In this way, the system can obtain the global minimum in a reasonably large number of steps.

3) Logarithmic Method

The logarithmic cooling schedule [3] has the form

$$T(t) = \frac{C}{log(1+t)} \tag{3}$$

where t is the time step and C is a parameter used to balance the need between final energy and efforts. For example: when C is larger, the initial temperature is large. It takes the system a larger number of steps to obtain a better final energy.

4) Exponential Method

The exponential cooling schedule can be described as follows. [4-6]

$$T(t) = T_0 e^{-\beta t} \tag{4}$$

where, T_0 is a given temperature, β is a positive number indicating the cooling rate and t is the time step. It can be also written as $T_{k+1} = \alpha T_k$ where α is a predetermined value [5, 6].

C. Parameter Estimation

Final energy is a major evaluation metric. The lower the final energy is, the better the performance. Therefore, the unknown parameters in cooling schedules need to be well estimated in order to result in good performance. With different parameter settings, we compare the mean final energy of 100 experiments, and choose the optimal parameters in the comparisons of the four cooling schedules.

The range that the optimal parameters are obtained is based on experimental results as well. We first set up a range for each parameter. Then, we test the performance for each parameter with different values in the range. Usually, by looking into the trend in the performance, we are able to see whether the range is suitable or not.

IV. RESULTS

In this section, we show the detailed process of optimal parameter selection for each cooling schedule. Then, by using these parameters, we compare the performance for the four cooling schedules.

A. Data Description

Since the main purpose of this study is to identify the optimal cooling schedule method in mobile recommendation problems, our experiments are based on synthetic data. Fig. 3 shows an example of the generated data for TSP. We generate 200 random numbers to locate 100 geographic points and aim to discover the shortest path connecting the red points. For mobile trajectory recommendation, we generate 10,000 geographic points and assign a random probability to each point. Given a starting point position, we find the optimal sequence with a constant length to minimize the potential travelling distance.

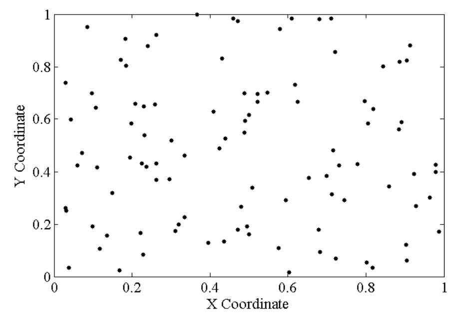


Fig. 3 An example of Synthetic Data

B. Experimental Design

In the TSP problem, the geographic points have the coordinates uniformly distributed from 0 to 1. The path needs to go through all the cities and return to its starting city.

In the mobile trajectory recommendation problem, the coordinates are generated based on a uniform distribution from 0 to 1,000. The probability for successful pick-up follows a uniform distribution from 0 to 0.4 indicating that the successful pick-up probability for each point is small and the choices of the last several pick-up points in the route still affect the PTD significantly. To set the penalty distance D_∞, the problem is modified: the taxi driver aims to go to a place where he can definitely pick up a passenger but it is far from his current location. He would like to have a route whose destination is that place and he has the best chance to pick up a passenger on his way. Therefore, the starting position is set to be (250,250) while the destination position is (750,750).

C. Result Analysis

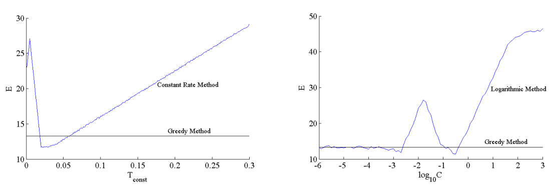
1) Travelling Salesman Problem

Identifying the optimal parameters for each cooling schedules is the first step before we are able to compare the performance of different cooling methods. In TABLE I, we report the optimal parameter estimation for the travelling salesman problem. The first row reports the parameter space where we have searched for the optimal parameters. The second row shows the number of experiments we spend in finding the optimal parameter. The last row gives us the optimal parameter one can use to obtain the best performance. The range of each testing parameter is set up. For greedy method, the temperature is 0 and the range is obvious. For constant rate method, the constant temperature T_{const} needs to be decided. Its range is between 0.001 and 0.300. For logarithmic method, the parameter Cneeds to be decided as well. Based on the experimental results, the value of Clies between 10^{-6} to 10^{3} . For the exponential method, two parameters need to be accommodated, namely, T_0 and α. By estimation, T_0 has a value between 0.1 and 1.9 and the value of α is between 0.999910 and 0.999999.

With the help of supercomputer, we are able to do all of these experiments in a reasonably small amount of time.

TABLE I. OPTIMAL PARAMETER ESTIMATIONFOR TSP

	Exponential Rate	Constant Rate	Logarithm	Greedy
Testing Para	$\alpha = 0.99991 + 0.000001 * [0:1:89]$; $T_0 = 0.1 + 0.02 * [0:1:90]$;	C=0.001:0.001:0.3	T_0= powf(10, -6+0.1*[0:1:90])	$T \equiv 0$
Number of Experiments	90*91*100	300*100	91*100	1*100
Optimal Para	α =0.999978; T_0=0.62;	C=0.027	$T_0 = 10^{-0.5}$	$T \equiv 0$



(a) (b)

Fig. 4 Parameter Estimation: Greedy, Constant Rate, and Logarithm (TSP)

In Fig. 4(a), the y axis is the final energy and the x axis is the value of the constant temperature T_{const}. It shows that the final energy first decays and then grows gradually while the constant temperature T_{const} grows within we range we estimate. According to this figure, we can see that the range we set is reasonable since the energies for the both side of the valley are much higher. Also, we are able to find out the constant temperature T_{const} that makes the system get the lowest final energy. The blue line in the figure is the final energy of the greedy method. We can see that correctly selected, one can obtain a good final energy with the specific C in constant cooling schedule.

In Fig. 4(b), the y axis is also the final energy and the x axis is the value of the parameter C in logarithm cooling schedule. As can be seen, the range is reasonably set. When the parameter C becomes smaller, the performance approaches greedy method. When C becomes larger, the energy grows significantly. Moreover, we can see that while C grows, the final energy at first has the same performance as the greedy method since the temperature is close to 0. Then, the performance gets better for some C's. This kind of dropping and growing happens twice. After that, the final energy does not decay any more. By looking into those two valleys in the figure, we are able to find out the best parameters in logarithm cooling schedule.

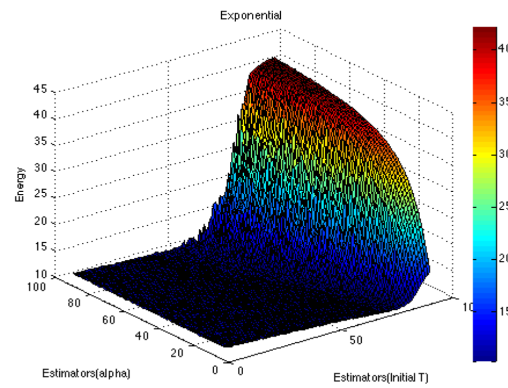


Fig. 5 Parameter Estimation: Exponential Cooling Schedule (TSP)

For the exponential cooling schedule, 3D plot is needed since there are two parameters in this schedule needs to be accommodated. Fig. 5 shows the performance for all combinations of the parameters in this schedule. The z axis is the final energy, the x axis is α and the y axis is T_0. The figure shows that while T_0 gets larger, the final energy becomes larger. We then use this figure to find out the optimal parameter for this schedule.

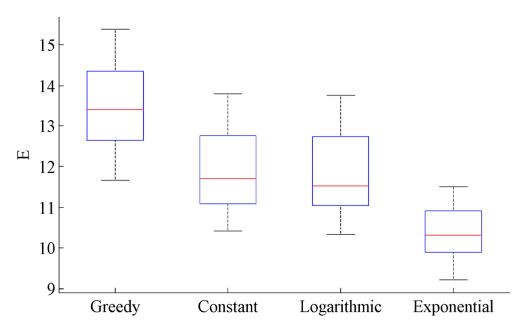
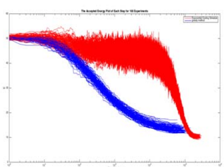


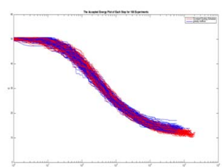
Fig. 6 Performance Comparisons of Four Cooling Schedules for TSP

Fig. 6. shows performance of the four types of cooling schedules in terms of optimal distance. It can be seen that exponential cooling schedule performs the best by achieving

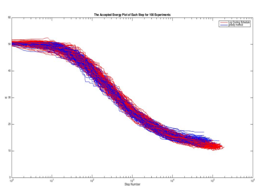
the lowest energy E. The mean final energy we obtain from exponential cooling schedule is 10.2877. When T is a non-zero constant, it works better than the Greedy method (T=0). Constant rate T and log rate T provide similar results.

We also investigate the cooling procedures. In the Fig. 7, the x-coordinate is the step number in the log scale. The y-coordinate is the energy one experiment obtains in each step number. The blue lines are from the greedy method and the red lines are from the other one. For example, in the first figure, we show how the E of the exponential cooling schedule drops when step number increases with respect to 100 experiments and compare it with the greedy method. By investigation, we find that the energy dropping procedure in exponential cooling schedule behaves differently than the others. Its energy first decreases slowly and suddenly drops in a fast speed. According to the logarithm cooling schedule and the constant cooling schedule, not only are their final energies lower, the energy dropping speeds are also faster. Besides, as for exponential cooling schedule, we can see that it takes nearly the same effort but obtain a lower final energy. That is to say, with the same amount of effort, the exponential cooling schedule still has the best performance among the four.





(a) Exponential vs. Greedy(b) Constant vs. Greedy



(c)Logarithm vs. Greedy

Fig. 7 Comparisons of Accepted Energy in Difference Cooling Schedules

2) *Mobile Route Recommendation*

Based on the same method of parameter estimation in the TSP, we identify the optimal parameters for our mobile route recommendation problem. Table II reports the important settings of the experiments and the results of the obtained optimal parameters for four cooling methods. For greedy method, the temperature is 0. For constant rate method, T_{const} is between 10^{-4} to 10^{-1}. For logarithmic method, the value of Clies between 10^{-3} to 10^{0}. For the exponential method, T_0 is between 10^{-5} to 10^{15} while α is 0.9999, 0.99999, and 0.999999. Whether the range is suitably set is based on the experimental results which will be analyzed in the following figures. However, we still have basic understandings of the range: if the initial temperature is high and cannot be lowered in a reasonable amount of time, the system keeps boiling and the PTD cannot be lowered in 100,000,000 steps. As a result, the SA process is terminated with a high PTD. On the other hand, if the temperature decreases to a number close to 0 quickly, the SA process keeps accepting only the route with better PTD. The final PTD the SA process obtains will be the same as the final PTD greedy method obtains. By this means, if the parameter is far from the optimal setting, the final PTD that the SA process obtains will be larger.

TABLE II. OPTIMAL PARAMETER ESTIMATIONFOR MOBILE ROUTE ROCOMMENDATION PROBLEMS

	Exponential Rate	Constant Rate	Logarithm	Greedy
Testing Para	α = 0.9999,0.99999, 0.999999; T_0 = pow(10, -15:1:4);	T_{const} = Pow(10,-4:0.1:-1)	T_{const} = Pow(10,-3:0.1:0)	$T \equiv 0$
Number of Experiments	3*20*100	31*100	31*100	1*100
Optimal Para	α =0.9999, $T_0 = 10^{-15}$; α =0.99999, $T_0 = 10^{-3}$; α =0.999999, $T_0 = 10^{-2}$;	$T_{const} = 10^{-1.9}$	$T_0 = 10^{-0.5}$	$T \equiv 0$

In Fig. 8, the y axis is PTD, the final energy of Trajectory Recommendation problem and the x axis is the value of the constant temperature T_{const}. For constant rate method, 100 experiments are done for each T_{const} and the mean PTD is computed. Then, for each T_{const}, we have a PTD to show its performance and it is represented as a star. 10 sets of Greedy method experiments with each set having 100 experiments are also run and the mean PTD for each set is computed. The maximum and minimum mean PTD are presented in the figure in comparison with the performance of the constant rate cooling schedule. The black lines are also shown in the following figures.

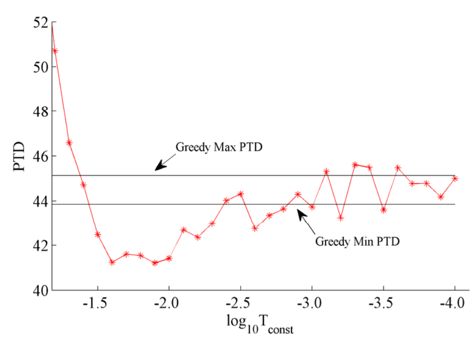


Fig. 8 Parameter Estimation: Greedy vs Constant Rate

As can be seen, the final energy first decays and then grows gradually with the similar performance of the Greedy method while the constant temperature T_{const} decreases within we range we estimate. According to this figure, the range we set is reasonable since the energies for the both side of the valley are much higher. Also, we are able to find out the constant temperature T_{const} that makes the system get the lowest final energy. We can see that parameter correctly selected, one can obtain a good final energy with the specific T_{const} in constant cooling schedule.

Fig. 9 is similar to Fig. 8. Each star shows the mean PTD of 100 experiments with each experiment given parameter C. With the decrease of the parameter C, the final energy first decays and then grows to the PTD range that Greedy method has. This also forms a valley and the best C can be selected.

Fig. 10 shows the PTD an exponential cooling schedule can achieve in comparsion with the greedy schedule. As can be seen, with different α, T_0 suitably selected, the exponential cooling schedule can obtain PTDs better than greedy schedule. In this figure, for each α, if T_0 is larger than a critical point, the SA process will achieve a good PTD. On the other hand, if we consider the effort, the smaller the T_0, the quicker the convergent speed. Therefore, the critical point for T_0 is the best point that reprsents the performance of the schedule given α .

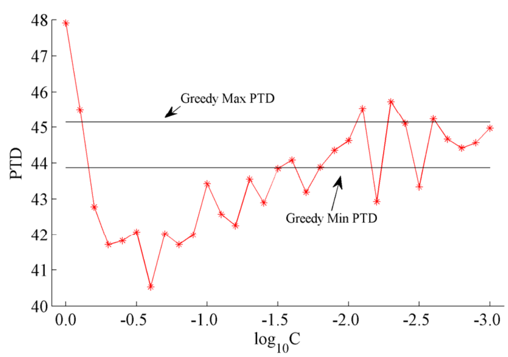


Fig. 9 Parameter Estimation: Greedy vs Logarithmic

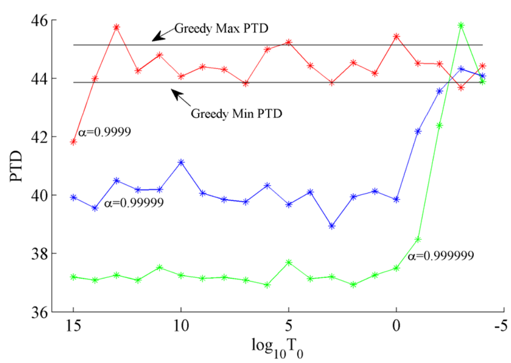


Fig. 10 Parameter Estimation: Greedy vs Exponential

In Fig. 11, the horizontal line is the amount of steps to achieve the final PTD and the vertical line shows the final PTD. In the parameter selection process, the parameter that obtains the lowest final PTD for each cooling schedule or each α in the exponential cooling schedule is chosen. Since each parameter contains 100 experiments, we present the performance of the selected parameter using crosses. The central point of the cross has the mean total step number as its x coordinates and mean PTD as its y coordinates. Also, the standard deviation for total step number and PTD among the 100 experiments is computed for the length of the cross in the x and y coordinates.

In the figure, the Greedy method can converge in a fast speed but with a higher PTD. The exponential method can obtain a better PTD with smaller amount of total step number in comparison with the constant rate method (Metropolis method) and the logarithmic method. Logarithmic method can obtain a better PTD but with a larger amount of total step number. In the exponential method, the lower the α is, the better the PTD but larger the total step number will there be.

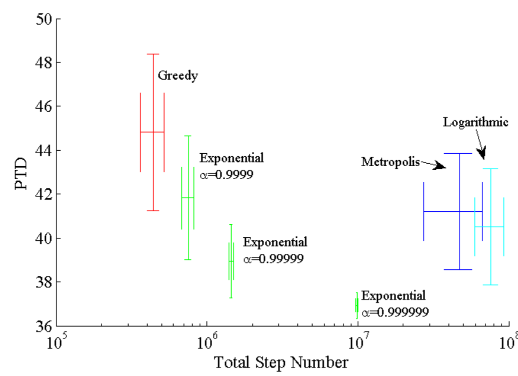


Fig. 11 Performance Comparisons of Four Cooling Schedules for Trajectory Recommendation problem

V. Related Work

A. SA Applications

Simulated annealing has been used in different areas, and has been shown that it can be easily implemented to most of optimization problems, such as multi-level lot-sizing problems [6], school timetable problem [7], and graph partition problem [8]. Regarding the cooling schedule, some research has discussed the comparisons of the constant cooling schedules, logarithm cooling schedules, and exponential cooling schedules [9]. However, they compare the performance of different cooling schedules with the same initial and final temperature with fixed iteration number regardless of the convergence of the SA process [9]. In our paper, the SA process terminates itself when it meets its stopping condition. Thus, we can say that this SA process does converge. Moreover, instead of fixing the initial and final temperature, for each cooling schedule, we decide and choose the best initial and final temperature. Only by this means can each cooling schedule achieve its best performance and the comparison is fair enough. In addition, we study the cooling schedules based on mobile recommendation problems.

B. Route Recommendation for Taxi Drivers

Optimization problems in mobile route recommendation can be found in many recent work. Ge et al. [10] introduced a taxi business intelligence system. Qu et al. [11] proposed a cost-effective system for route recommendations. Yuan et al. [12] discussed an approach for finding passengers and vacant taxi. All these problems can be considered as TSP with constraints and special conditions.

Our research focuses on discussing the implementation of simulated annealing algorithms in mobile recommendation problems. We aim to provide results from empirical studies that service as guidance for the utilization of SA in related problems.

VI. Conclusions and Future Work

In this paper, we investigate the performance of four cooling methods in simulated annealing algorithms implemented in mobile recommendation problems. We implement the simulated annealing algorithm for a classical travelling salesman problem and a mobile route recommendation problem. We design specific move generation methods for the two problems during the tests. With the optimal estimated parameters, four cooling methods (greedy method, constant rate method, exponential method, and logarithmic method) are tested. Our results suggest that the exponential method outperforms other methods in terms of the final energy and the computational efficiency.

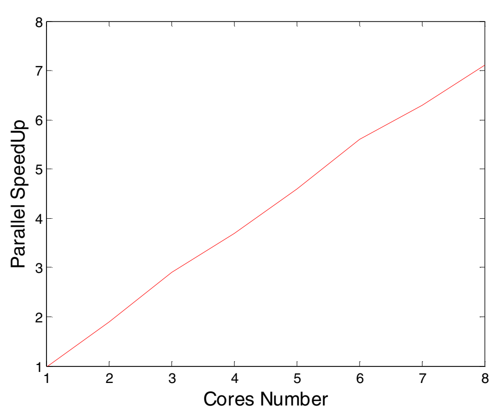


Fig. 12 Expected Parallel Speedup

In the implementation of simulated annealing algorithms, the final energy and the computation efficacy sometimes are difficult to balance, because a lower energy is resulted from a larger computational cost. For the future work, we are going to study for the judgment between the two criticisms. Moreover, we notice that, throughout the whole SA process, the acceptance rate for accepting the next step is low, which means the system keeps rejecting the new status. As a result, the system retains its status in a large number of steps. This phenomenon allows us to parallelize the SA algorithm in multiple Markov Chains. We speculate that the parallel size can be as high as 8 cores and the speedup can be 7, which is shown in Fig. 12. We are going to deeply study the problems in the parallelization such as communication cost, communication frequency, and mixing strategies.

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