Cross-Domain Recommendation via Tag Matrix Transfer

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***Abstract*—Data sparseness is one of the most challenging problems in collaborative filtering(CF) based recommendation systems. Exploiting social tag information is becoming a popular way to alleviate the problem and improve the performance. To this end, in recent recommendation methods the relationships between users/items and tags are often taken into consideration, however, the correlations among tags from different item-domains are always ignored. For that, in this paper we propose a novel way to exploit the rating patterns across multiple domains by transferring the tag co-occurrence matrix information, which could be used for revealing common user pattern. With extensive experiments we demonstrate the effectiveness of our approach for the cross-domain information recommendation.**

I. INTRODUCTION

Data sparseness is one of the most challenging problems in collaborative filtering(CF) based recommendation systems. Cross-domain recommendation models [1][2] have shown their advantages in alleviating the data sparsity problem, by capturing the common user rating patterns across multiple domains and domain-specific patterns among each domain. For example, some recent methods [3] [4][5] [6] [7] aim at constructing a cluster-level rating pattern shared by multiple domains to make cross-domain recommendation, CBT model [3] as well as the extended versions [5] [7] generated the user-item pattern called as *codebook* from the auxiliary data and then transferred the codebook to the target domain. Further more, [4] [6] proposed to derive common patterns shared across domains and domain-specific rating patterns distinct in each domain to improve the cross-domain recommendation performance. These methods have shown that the knowledge transferring across domains benefit the performance of recommendation. However, even rating matrix is well utilized for user-item recommendation, tag information given by users is playing more important role on learning user preference for different items. Can we take them into account of both the tags' own information as well as the correlations among tags?

In recent recommendation methods the relationships between users/items and tags are often taken into consideration, for instance, [8] [9] proposed to learn users' preferences and items' attributes by exploiting the tag information. However, the correlations among tags from different item-domains are always ignored, which can provide additional user preference information by annotating different items to improve performance. To this end, we first consider that the interrelatedness among tags is shared by cross domains, then the tags from different domains will be used to construct a tag co-occurrence matrix transfer model to depict the user behavior patterns from different rating domains, based on which we learn the specific user and item latent factors to make predictions across the domains. Here we try to give a brief example to illustrate how the correlations among the tags can help with recommendation in Figure 1. Supposing that User*1* is from a movie recommendation site, and has rated movie*1* with 5 and the tag "sci-fiction", and in another site User*2* has given ratings on movie*3* and movie*4* , which have the same two tags "sci-fiction" and "interstellar" as for the movie*1* and movie*2* in the first domain, then we may predict the rating of User*1* on movie*2* which he has never seen before based on the tag co-occurrence matrix transferring across the two related domains.

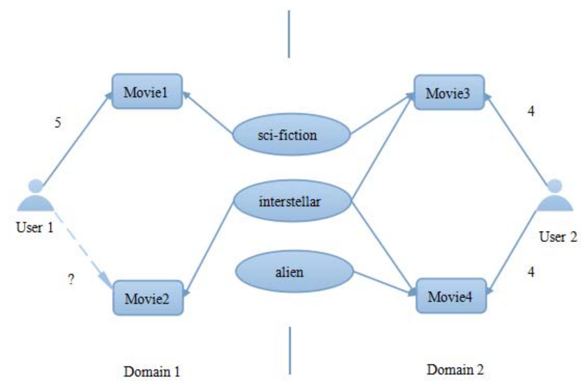

Fig. 1. An example of two movie recommendation sites with some common tags for the movies.

The rest of the paper is structured as follows. In Section 2, the background and related work are presented. In Section 3, we introduce our proposed model based on tag co-occurrence matrix transfer and then an efficient optimization algorithm is proposed to learn the latent factor model. This is followed by the present experimental study on single-domain and cross-domain cases in Section 4. In Section 5, we present conclusions and future work.

II. RELATED WORK

Our work is based on the extension to the collaborative filtering of cross-domain recommendation system as well as the tag based recommendation method.

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A. Codebook Transfer Model

CBT model[3] was proposed to predict users' ratings on items based on codebook transfer. By using the non-negative matrix factorization, a user-item rating matrix can be factorized into a product of threes factors $X = USV^T$, where the U is the cluster indicator for the users, the V is the cluster indicator for the items and the S is the cluster-level user-item rating patterns which was called codebook. In this model, the approximation can be achieved by the following matrix norm optimization:

$$\begin{aligned} &\min_{U\geq 0, V\geq 0, S\geq 0} ||X - USV^T||_F^2 \\ &s.t. \quad U^T U = I, \; V^T V = I, \end{aligned} \tag{1}$$

where $U \in_+^{n\times k}, V \in_+^{m\times l}, S \in_+^{k\times l}$ and $|| \bullet ||_F$ denotes the Frobenius norm.

To predict users' ratings on items for a sparse target domain, sharing latent common rating pattern knowledge across the related dense domain has been proved to improve performance in the target domain. We will also compare our approach with the CBT model experimentally as reported in Section 4.

B. Tag-based Recommendation

Tags help to improve the performance of recommendation in various ways, such as by integrating tags into traditional CF models [11] with the latent factor model [8], or via using the tag-based user clusters as the feature for KNN [9]. The co-occurrence distribution of Tags is able to discover from the training data and represent the relationships among tags which is beneficial for our task. In [9], every tag was represented as its co-occurrence distribution. The authors clustered tags into topics by calculating the Jensen-Shannon divergence (JSD) between tags, and their evaluation showed that this approach was an efficient way to reveal the interrelatedness between tags.

We will introduce a typical tag-based model GTagCDCF [8] and compare our model with it experimentally in Section 4. GTagCDCF model showed that the potential of tags may facilitate cross-domain collaborative filtering (CDCF). The model was based on collaborative matrix factorization (CMF), and the objective function of GTagCDCF can be formulated as follows:

$$\begin{aligned} &G(U_k, V_k, L) \\ &= \frac{1}{2}\sum_k ||W_k^R \circ \left(R_k - g\left(U_k^T V_k\right)\right)||_{Fro}^2 \\ &+ \frac{\alpha}{2}\sum_k ||W_k^{F^U} \circ \left(F_k^U - g\left(U_k^T L\right)\right)||_{Fro}^2 \\ &+ \frac{\beta}{2}\sum_k ||W_k^{F^V} \circ \left(F_k^V - g\left(V_k^T L\right)\right)||_{Fro}^2 \\ &+ \frac{\lambda}{2}\left[\sum_k \left(||U_k||_{Fro}^2 + ||V_k||_{Fro}^2\right) + ||L||_{Fro}^2\right] \end{aligned} \tag{2}$$

where U_k, V_k and L represented the latent factors of users,items and tags respectively. L was shared by all domains. W_k^R,$W_k^{F^U}$ and $W_k^{F^V}$ represented the indicator matrix. R_k meaned the rating matrix. F_k^U indicated the frequency of user using tag and F_k^V indicated the frequency of item annotated by tag. k meaned in the k-th domain. α and β were regarded as tradeoff parameters. λ was a regularization parameter that penalized the magnitude of latent features in order to alleviate over-fitting.

III. OUR MODEL

A. Model Formulation

Follow the assumption in [4], we consider that the interrelatedness among tags can be shared by cross domains, then the tags from different domains will be used to construct a tag co-occurrence matrix transfer model to capture the user behavior patterns from different rating domains, based on which we learn the specific user and item latent factors to make predictions across the domains.

For that, we first build our model in the framework of the nonnegative matrix factorization algorithm, and the rating matrix can be tri-factorized as follows:

$$\begin{aligned} \min_{U_k, V_k \geq 0,} J_k = &\sum_k ||[R_k - U_k T V_k] \circ W_k||^2 \\ &+\lambda \sum_k \left(||U_k||^2 + ||V_k||^2\right) \end{aligned} \tag{3}$$

where $U_k \in {}_+{}^{M_k\times L}$ $V_k \in {}_+{}^{L\times N_k}$ and $T \in {}_+{}^{L\times L}$.k means in the k-th domain.M_k and N_k represent the numbers of users and items respectively and L is the number of tags in all of the domain.W_k is the binary mask matrix. $U_k = \left[u_1^k, u_2^k, \cdots, u_L^k\right]$ represents the user latent factors for tags, where each u_l^k is an $M_k \times 1$ vector indicating the preference over M_k users for the l-th tag. $V_k = \left[v_1^k, v_2^k, \cdots, v_L^k\right]^T$ represents the item latent factors for tags, where each v_l^k is an $N_k \times 1$ vector indicating the probability over N_k users for the l-th tag. λ is a regularization parameter that penalizes the magnitude of latent features in order to alleviate over-fitting.

In this work we consider the case that neither the users nor the items in the multiple rating matrices are overlapping, but there is a overlap of the set of tags across multiple domains. Assuming that the implicit information of tags is shared by all domains, which may be difficult to learn accurately the correlations among tags in a sparse domain, while the related dense domain will help to learn the tag transfer model.

For all tags, we calculate their co-occurrence distribution, in which $p_z(t)$ is the co-occurrence distribution of tag. Then $p_z(t)$ is defined as follows:

$$p_z(t) = \sum_{m\in I} q(t|m) Q(m|z) \tag{4}$$

$$q(t|m) = \frac{n(m,t)}{\sum_{i\in T_m} n(m,i)} \tag{5}$$

$$Q(m|z) = \frac{n(m,z)}{\sum_{j\in I} n(j,z)} \tag{6}$$

where I represents the set of all items. T_m denotes the set of all tags applied to item m. $n(m, z)$ represents the number of times tag z is applied to m.

So the co-occurrence matrix of tags can be defined as follows:

$$T = \begin{pmatrix} p_{t_1}(t_1) & p_{t_2}(t_1) & \cdots & p_{t_L}(t_1) \\ p_{t_1}(t_2) & p_{t_2}(t_2) & \cdots & p_{t_L}(t_2) \\ \cdots & \cdots & \cdots & \cdots \\ p_{t_L}(t_L) & p_{t_2}(t_L) & \cdots & p_{t_L}(t_L) \end{pmatrix} \tag{7}$$

Through the matrix, the item latent factor for the tag which is lack of users' preference will still be taken into consideration. Note that, our proposed model is a single domain recommendation model with $k = 1$.

B. Optimization

To learn the model parameters, we need to minimize the following loss function as follows:

$$\begin{aligned} \min_{U_k, V_k \geq 0,} J_k = \sum_k ||[R_k - U_k T V_k] \circ W_k||^2 \\ +\lambda \sum_k (||U_k||^2 + ||V_k||^2) \end{aligned} \tag{8}$$

T is defined in Equation (3) (4) (5) and (6). After obtaining the tag co-occurrence matrix, the tag model can be transferred. To optimize the proposed model, we employ the alternating multiplicative updating algorithm.Then the derivative of J with respect to each variable can be computed as below:

$$\frac{\partial J}{\partial U_k} = 2((R_k \circ W_k){V_k}^T T^T - (U_k T V_k \circ W_k){V_k}^T T^T + \lambda U_k) \tag{9}$$

$$\frac{\partial J}{\partial V_k} = 2(T^T {U_k}^T (R_k \circ W_k) - T^T {U_k}^T (U_k T V_k \circ W_k) + \lambda V_k) \tag{10}$$

Using the Karush-Kuhn-Tucker complementary condition for the non-negativity of U_k and V_k and let the derivative to 0, we can get the following updating rule for each variable:

$$U_k \leftarrow U_k \sqrt{\frac{(R_k \circ W_k){V_k}^T T^T + \lambda U_k}{(U_k T V_k \circ W_k){V_k}^T T^T}} \tag{11}$$

$$V_k \leftarrow V_k \sqrt{\frac{T^T {U_k}^T (R_k \circ W_k) + \lambda V_k}{T^T {U_k}^T (U_k T V_k \circ W_k)}} \tag{12}$$

This procedure will be repeated for several iterations until convergence.

IV. EXPERIMENTS

In this section, we first describe our experimental setup and then present the evaluation results of several compared approaches followed with some discussions. In particular, we examine how our proposed method behaves on various configurations of cross-domain recommendation task.

A. Datasets

We have evaluated the models on *MovieLens* dataset, which is released in April 2015. It contains 21,000,000 ratings and 470,000 tag applications applied to 27,000 movies by 230,000 users. The dataset is a subset of MovieLens described in Table I , including users who have used both ratings and tags and items which have been rated and tagged. We divide the data into two parts D1 and D2. There is no overlap between users and no overlap between items in two parts. Each part has 500 users , 500 items and more than 400 tags. Otherwise, the density of ratings in D1 is higher than in D2.

TABLE I
THE STATISTICS OF DATASET WHICH HAS BEEN DIVIDED INTO D1 AND D2 DOMAINS.

	D1(domain1)	D2(domain2)
users	500	500
items	500	500
ratings	85887	45579
the kinds of tags	481	476
the number of tags	40601	22602

B. Experimental Setup

We examine the compared models for both cross-domain and single-domain recommendation tasks. In our experiments, we split each of all the datasets in both D1 and D2 into three sets containing different users, i.e., a training set, a validation set and a test set.

1) Experimental Protocol for Single-Domain Case: In the experiments, we compare our proposed model with several state-of-the-art single domain recommendation models.And we choose data in D1 for experiments.

- NMF model(Nonnegative Matrix Factorization)[12]: Learn the latent factors of users and items and predict the performance. The number of latent factors is set to 20.
- TagCF model [8]: A tag-induced single-domain collaborative filtering method based on model GTagCDCF when the number of domain K=1. The number of latent factors is set to 20.
- TMT model:Our proposed model

2) Experimental Protocol for Cross-Domain Case: In the experiments, we compare our proposed model with several state-of-the-art cross-domain recommendation models.And we choose data in D1 and D2 for experiments.

- CBT model [3]:A cross-domain model which transfer the common rating pattern. The number of latent factors is set to 40.
- RMGM model [5]: A cross-domain model which sharing the cluster-level rating matrix.
- GTagCDCF model [8]:A tag-induced cross-domain collaborative filtering method.The number of latent factors is set to 20.
- TMT model:our proposed model

TABLE II
RECOMMENDATION PERFORMANCES OF RMSE, MAE AND MAP FROM DIFFERENT METHODS FOR SINGLE-DOMAIN CASE.

SPARSITY	METHOD	RMSE	MAE	MAP1	MAP2	MAP3	MAP4	MAP5	MAP(TOTAL)
	NMF	0.8858	0.6756	0.1%	11.4%	54.3%	59.7%	19.6%	26.9%
4.9%	GTagCF	0.8892	0.6915	0.4%	13.4%	65.2%	53.9%	6.7%	25.7%
	TMT	**0.8462**	**0.6399**	**0.2%**	**8.7%**	**57.4%**	**64.9%**	**18.2%**	**28.5%**
	NMF	0.8964	0.6851	0.4%	10.9%	53.8%	57.9%	20.2%	26.3%
3.8%	GTagCF	0.9010	0.6997	0	11.7%	61.8%	52.8%	11.3%	25.2%
	TMT	**0.8553**	**0.6481**	**0.1%**	**9.4%**	**56.1%**	**63.2%**	**17.1%**	**27.7%**
	NMF	0.9091	0.6951	0.1%	11.3%	53.4%	57.9%	20.2%	26.2%
3.1%	GTagCF	0.9049	0.7039	0.1%	11.3%	60.4%	54.3%	9.1%	25.2%
	TMT	**0.8648**	**0.6558**	**0.1%**	**10.9%**	**55.9%**	**63.2%**	**17.0%**	**27.6%**
	NMF	0.9488	0.7282	1.1%	12.7%	50.1%	54.6%	21.3%	25.1%
2.0%	GTagCF	0.9411	0.7271	0.2%	11.3%	53.7%	49.8%	14.7%	24.8%
	TMT	**0.8984**	**0.6851**	**0.6%**	**12.4%**	**53.7%**	**59.8%**	**17.4%**	**26.5%**
	NMF	1.0091	0.7762	1.9%	16.3%	45.2%	50.9%	24.4%	23.8%
1.1%	GTagCF	0.9763	0.7716	0.2%	12.5%	58.5%	46.5%	7.3%	23.4%
	TMT	**0.9571**	**0.7399**	**1.1%**	**16.9%**	**51.9%**	**52.5%**	**18.5%**	**24.6%**

TABLE III
RECOMMENDATION PERFORMANCES OF RMSE, MAE AND MAP FROM DIFFERENT METHODS FOR CROSS-DOMAIN CASE.

SPARSITY	METHOD	RMSE	MAE	MAP1	MAP2	MAP3	MAP4	MAP5	MAP(TOTAL)
	CBT	0.8613	0.6446	0	0	52.5%	72.0%	0	28.0%
D1:4.9%	RMGM	0.8756	0.6562	0.2%	5.8%	50.0%	69.2%	7.4%	27.7%
D2:2.6%	GTagCDCF	0.8947	0.6822	0.1%	7.1%	50.9%	62.5%	18.6%	26.8%
	TMT	**0.8460**	**0.6396**	**0.1%**	**8.0%**	**56.2%**	**65.0%**	**16.4%**	**28.3%**
	CBT	0.8651	0.6559	0	0	50.9%	72.6%	0	28.0%
D1:3.8%	RMGM	0.9019	0.6857	1.3%	7.0%	48.4%	65.0%	7.3%	26.4%
D2:2.0%	GTagCDCF	0.9110	0.6982	0	6.9%	51.5%	59.5%	18.7%	26.1%
	TMT	**0.8574**	**0.6517**	**1.8%**	**12.2%**	**55.1%**	**62.5%**	**14.7%**	**27.5%**
	CBT	0.8726	0.6510	0	0	48.7%	72.4%	0	27.6%
D1:3.1%	RMGM	0.9334	0.7133	0.3%	6.8%	45.7%	63.1%	7.7%	25.5%
D2:1.7%	GTagCDCF	0.9284	0.7217	0.2%	12.0%	54.1%	55.2%	10.9%	24.7%
	TMT	**0.8796**	**0.6732**	**1.3%**	**14.7%**	**51.1%**	**60.7%**	**17.3%**	**26.7%**
	CBT	0.8751	0.6613	0	0	51.2%	69.6%	0	27.2%
D1:2.0%	RMGM	0.9866	0.7527	0.5%	4.9%	43.1%	60.6%	5.5%	24.1%
D2:1.1%	GTagCDCF	0.9798	0.7622	0.4%	11.5%	51.4%	52.1%	11.2%	23.5%
	TMT	**0.9336**	**0.7153**	**1.2%**	**15.6%**	**49.5%**	**56.2%**	**15.1%**	**25.1%**
	CBT	0.8872	0.6724	0	0	48.2%	69.7%	0	26.8%
D1:1.1%	RMGM	1.0288	0.7947	0.5%	2.7%	41.2%	57.0%	6.0%	22.8%
D2:0.6%	GTagCDCF	1.0093	0.7816	0	7.5%	44.2%	52.6%	12.9%	22.8%
	TMT	**1.0038**	**0.7785**	**1.5%**	**16.5%**	**51.3%**	**47.1%**	**16.3%**	**23.0%**

3) Evaluation Protocol: The evaluation metric we adopt are RMSE(root mean square error) MAE(mean absolute error) and MAP(mean accuracy precision) for each rating scale(from 1 to 5).

C. Experimental Results

1) Results for Single-Domain Case: The comparison results on the D1 dataset are reported in Table II. One can see that our method clearly outperforms the two baseline methods. TagCF performs slightly better than MF when the density is low, which implies that the tag data can help to improve the accuracy. The single-domain experimental results have validated that our proposed model indeed gain the useful relationship between tags and both users and items. And this relationship may be transferred cross multiple domains.

2) Results for Cross-Domain Case: The comparison results on the D1 and D2 dataset are reported in Table III. The training data is from D1 and D2 and the test data is from D2. From the results we can observe that the best performing method among all the models is our proposed TMT model. The CBT model performs well on RMSE, MAE and the total MAP, but the MAP1, MAP2 and MAP5 of it are all equal to zero. Assuming that the MAP1, MAP2 and MAP5 can represent the users' very dislikes and very likes respectively. The result shows that the prediction of CBT model hovers around the average, which means the result is undesirable and not personalized. The GTagCDCF model is slightly better than RMGM when the density is low, which implies the tag latent factor helps to improve the accuracy. Moreover, the tag based models outperforms the common rating pattern models on MAP1,MAP2 and MAP5, which shows that the tag model can indeed discover the user preference individually.

3) User-based CF and Item-based CF Experiments: We design additional user-based CF and item-based CF experiments to illustrate our model. The user latent factor U and item latent factor V are taken out from all above models as the feature of KNN. We choose k ($k = 3$) most similar neighbors by calculating the cosine distance of the features. The weighted sum of k neighbors' ratings will be regarded as the prediction. The function of user based KNN is defined as follows:

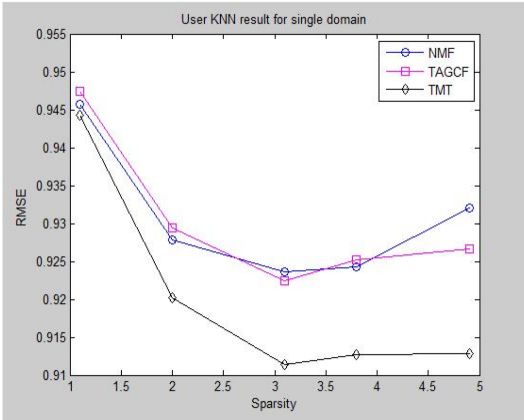

Fig. 2. User KNN result for single domain.

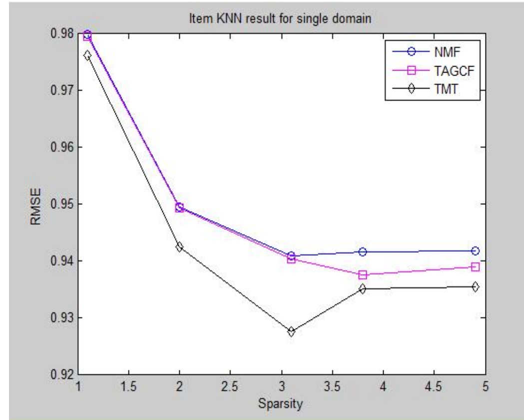

Fig. 3. Item KNN result for single domain.

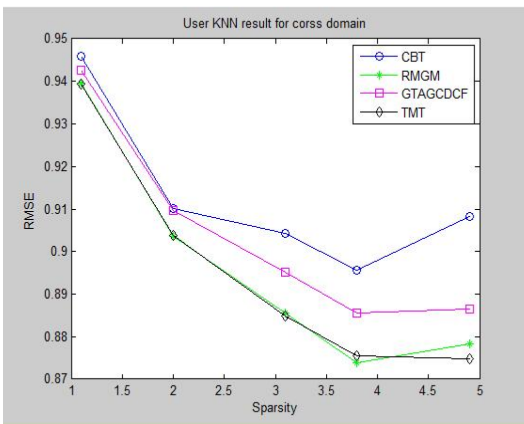

Fig. 4. User KNN result for cross domain.

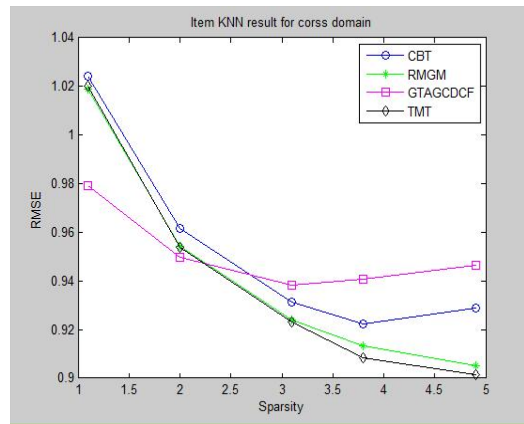

Fig. 5. Item KNN result for cross domain.

$$r_{ij} = average(i) + \frac{\sum\limits_{k \in N_i} (r_{kj} - average(i)) \times sim(k,i)}{\sum\limits_{k} sim(k,i)} \tag{13}$$

average(i) means the average rating of user i. N_i represents the neighbors of user i. $sim(k,i)$ is the cosine similarity between user i and his neighbor k. The item based KNN function is similar.

According to the results in picture 2 - 5, we can find that our proposed TMT model has the best performance. The RMSE of KNN is the lowest with the latent factor of TMT. It means that our model can discover the exact user preference and item attribute, so the most similar neighbors of user and item are selected. We may draw the conclusion that our model can learn the more accurate user and item latent factor for both single and cross domain. What's more, the KNN results of TMT for cross domain case is slight better than RMGM's, but its own results is much better than RMGM's. It perhaps illustrate that the relationship between tags is more meaningful than rating pattern.

V. CONCLUSION

In this work, we proposed a novel tag-based collaborative filtering method, named Tag Matrix Transfer (TMT) model. We investigate the usefulness of tags for recommendation task and deploy the TMT approach by expanding the CBT framework for CF to a cross-domain case. The proposed TMT model has taken into account both the relationship between tags and the user/item latent factor. The experimental results have validated that our proposed TMT model can indeed benefit from the tag matrix transfer and outperforms the state-of-art methods for both single-domain and cross-domain recommendation task.

In our future work, we will work further on exploiting the tag co-occurrence matrix, and plan to discover the relationship between tags and users/items in different ways. Moreover, our proposed TMT model should be evaluated on a large scale rating dataset for the real-world application.

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