EE117/217
Winter 2026

Homework 3

Problem 1: Basics of PRBS Signaling (20+10 points)

a) (10 points) Write and submit Python code to generate the bit sequence for PRBS7, PRBS127, PRBS511, and PRBS1023. For convenience, a handy table of generator polynomial coefficients is:

Bits (n)	Feedback polynomial	Taps	Taps (hex)	Period ($2^n - 1$)
2	$x^2 + x + 1$	11	0x3	3
3	$x^3 + x^2 + 1$	110	0x6	7
4	$x^4 + x^3 + 1$	1100	0xC	15
5	$x^5 + x^3 + 1$	10100	0x14	31
6	$x^6 + x^5 + 1$	110000	0x30	63
7	$x^7 + x^6 + 1$	1100000	0x60	127
8	$x^8 + x^6 + x^5 + x^4 + 1$	10111000	0xB8	255
9	$x^9 + x^5 + 1$	100010000	0x110	511
10	$x^{10} + x^7 + 1$	1001000000	0x240	1,023
11	$x^{11} + x^9 + 1$	10100000000	0x500	2,047
12	$x^{12} + x^{11} + x^{10} + x^4 + 1$	111000001000	0xE08	4,095
13	$x^{13} + x^{12} + x^{11} + x^8 + 1$	1110010000000	0x1C80	8,191
14	$x^{14} + x^{13} + x^{12} + x^2 + 1$	11100000000010	0x3802	16,383
15	$x^{15} + x^{14} + 1$	110000000000000	0x6000	32,767
16	$x^{16} + x^{15} + x^{13} + x^4 + 1$	1101000000001000	0xD008	65,535
17	$x^{17} + x^{14} + 1$	10010000000000000	0x12000	131,071
18	$x^{18} + x^{11} + 1$	100000010000000000	0x20400	262,143
19	$x^{19} + x^{18} + x^{17} + x^{14} + 1$	1110010000000000000	0x72000	524,287
20	$x^{20} + x^{17} + 1$	10010000000000000000	0x90000	1,048,575
21	$x^{21} + x^{19} + 1$	101000000000000000000	0x140000	2,097,151
22	$x^{22} + x^{21} + 1$	1100000000000000000000	0x300000	4,194,303
23	$x^{23} + x^{18} + 1$	10000100000000000000000	0x420000	8,388,607
24	$x^{24} + x^{23} + x^{22} + x^{17} + 1$	111000010000000000000000	0xE10000	16,777,215

b) (10 points) Augment your code to generate a plot of the autocorrelation curve for PRBS511 (yes, it should be a spike of height 511 at zero offset, and -1 everywhere else, assuming you map the bits from 0/1 to +1/-1). This is to make sure you got the PRBS511 sequence right (using polynomial 0x110 from the table above).

If you take a subsequence of length 255 from the PRBS511 sequence, does it remain pseudorandom? ie, is the autocorrelation curve of that subsequence a spike of height 255 at zero offset and -1 everywhere else? In other words, is a 255-bit subsequence of

PRBS511 actually a PRBS255 sequence? Why or why not? Does it depend on which 255-bit subsequence you choose? Plot the autocorrelation curve for the best possible 255-bit subsequence, "best" meaning the subsequence with the smallest maximum off-peak autocorrelation.

c) (10 points) Extra credit: Another possible generator polynomial for a PRBS511 sequence is given as: $x^9 + x^4 + 1$ (polynomial is now 0x108)

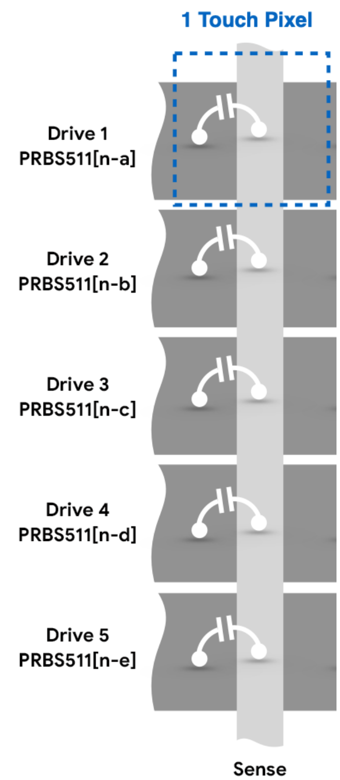
Call the PRBS511 sequence you generated in 3(a) as seqA, and this new generator polynomial seqB. Is seqA equal to seqB? What is the maximal value of the cross-correlation between seqA and seqB? Plot the cross correlation between seqA and seqB as a function of time offset, i.e.:

$$\text{xcor}(n) = \sum_{k=0}^{N-1} seqA[k]seqB[k-n]$$

(where the (k-n) is taken modulo N, since this has to be circular)

<u>Problem 2:</u> CDMA-Based Touch Sensors (Or, Hopefully This Helps You with Project 2) (20+10 points total)

From the touch lecture, one method of speeding up touch sensors is to drive all of the drive lines in parallel, at the same time. The obvious problem is that if the same waveform were used on all of the drive lines, you would not be able to address each pixel, since an entire sense row would have the same signal no matter where the touch occurred. By introducing a PRBS modulation, it turns out you can separately measure the signal from each drive line into the sense line. For this problem, we construct this as a one-dimensional touch sensor, since there are 5 drive lines and 1 sense line.

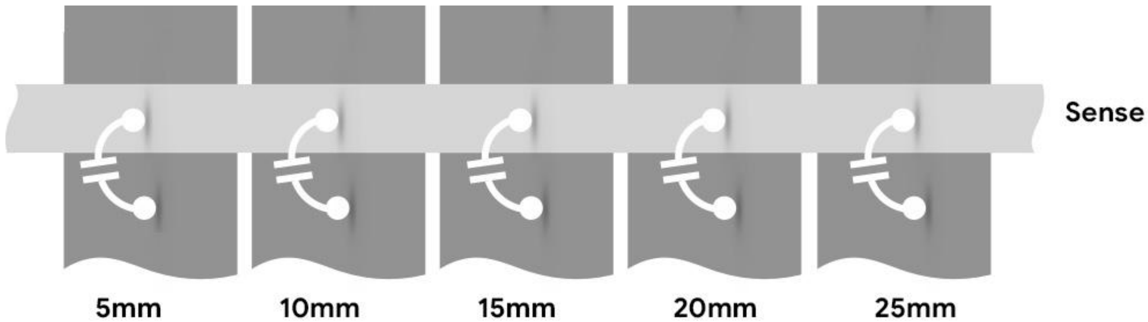


As shown in the figure, for the 5 drive lines, different phases of PRBS511 signal were used (the same PRBS511 signal from problem 2, polynomial 0x110). A simulated ADC output waveform from the sense line has been placed up on the class Canvas site (creatively titled HW3.Pr3.notouch.txt, which is synchronized to the bits in the PRBS sequence (and the PRBS sequence is driven as +/- X volts as square pulses into the drive lines. For the purposes of this problem, the exact amplitude X is unimportant, but obviously equal on the positive and negative sides). Gaussian noise in the front-end has been added. The starting state of the linear shift register is (integer) 257, 100000001 in binary. When doing autocorrelations, use the form in the slides. The last two facts are important, otherwise your synchronization in the PRBS sequence will be incorrect in the provided files.

a) (10 points) Build code in Python to take in the sense waveform data, and then correlate against the PRBS511 sequence to find the waveform for the 5 correlation peaks, each one corresponding to each drive line. The output of the correlator at each peak corresponds to the relative mutual drive->sense capacitance for each drive line. Hint: the peaks are NOT equally spaced in the PRBS sequence, just to keep you on your toes.

Assuming the start of the data file is T=0, what is the phase offset (in samples) for each correlation peak, and what is the (relative) value of the capacitance for each drive/sense pair?

b) (10 points) In the even-more-creatively named file HW3.Pr3.touch.txt, there is a touch signal modulating the capacitances. Using the HW3.Pr3.notouch.txt waveform as the notouch baseline of the sensor, where is the touch located? The physical coordinates (in millimeters) of the drive/sense pairs is given below. The signal with the smallest delay corresponds to the drive/sense pair on the left (5 mm), the signal with the second smallest corresponds to 10 mm etc. You can assume the finger sensor response is a Gaussian in space.



c) (10 points) Extra credit: How much noise is there in each sample in the HW3.Pr3.notouch.txt (before the correlation is done?)

Problem 3: Some DFT/FFT Exercises (and I *really, really* hope this is a review problem) (30 points total)

a) (10 points) A linear time-invariant system with frequency response H(ω) is excited with the periodic input x(n):

$$x(n) = \sum_{k=-\infty}^{\infty} \delta(n - kN)$$

resulting in an output signal y(n). Suppose that we compute the N-point DFT Y(k) of the samples y(n), 0 <= n <= N – 1 of the output sequence. How is Y(k) related to H(ω)?

b) (10 points) Let x(t) be an analog signal with bandwidth B = 4kHz. We wish to use a $N = 2^m$-point DFT to compute the spectrum of the signal with a resolution less than or equal to 50 Hz. Determine the minimum sampling rate, the minimum number of required samples, and the minimum length of the analog signal record (in time).

c) (10 points) The signal $x(n) = a^{|n|}$, $-1 < a < 1$ has a (discrete-time) Fourier transform given as:

$$X(\omega) = \frac{1 - a^2}{1 - 2a\cos(\omega) + a^2}$$

Plot $X(\omega)$ for $0 <= \omega <= 2\pi$, a = 0.8. Now take N samples of $X(\omega)$ in the frequency domain, denoted as $X(2\pi k/N)$, $0 < k < N - 1$. Using Python, or any appropriate signal processing tool, generate the time-domain signal using an inverse FFT on these N samples, for N = 20 and N = 100. What is happening in the time-domain when N=20?