Problem Set 5 (for Lecture 7) Solutions

April 16, 2017

A1. No. One must prove the converse is also true to prove equivalence.

A2. (a)

A3. True. This is equivalent to saying $(\phi \implies \psi) \vee (\psi \implies \phi)$, which can be simplified to $\neg\phi \vee \psi \vee \neg\psi \vee \phi$, which is a tautology.

A4. Yes. $\neg(\phi \implies \psi) \iff \neg(\neg\phi \vee \psi) \iff \phi \wedge \neg\psi$.

A5. Yes. $[(\phi \vee \psi) \implies \theta] \iff [\neg(\phi \vee \psi) \vee \theta] \iff [(\neg\phi \wedge \neg\psi) \vee \theta] \iff [(\neg\phi \vee \theta) \wedge (\neg\psi \vee \theta)] \iff [(\phi \implies \theta) \wedge (\psi \implies \theta)]$.

A6. True. Proof by contradiction. Suppose there are a finite number of natural numbers such that $\sqrt{n}$ is rational, and you can list them in a set in increasing order $\{a_1, a_2, a_3, \ldots, a_n\}$. Take another natural number formed using the last element a_n, and square it to obtain a_n^2. We have that $a_n^2 > a_n$, $\forall a_n > 1$, but $\sqrt{a_n^2} = a_n \in \mathbb{Q}$. Hence, our list was incomplete and thus we have a contradiction. Our assumption that there are a finite number of natural numbers such that $\sqrt{n}$ is rational is false, and thus there are an infinite number of naturals numbers such that $\sqrt{n}$ is rational.

A7. The error is at the step where the square root is taken. In general, it is not the case that $\sqrt{x^2} = x$ because inside the square root, the squared number will never be negative even if the number is negative. The correct result of the square root is $\sqrt{x^2} = |x|$. I would give this student a grade of a 14. Logical Correctness (0 pts) + Clarity (4 pts) + Opening (2 pts) + Stating the conclusion (4 pts) + Reasons (4 pts) + Overall valuation (0 pts).