Problem Set 6 (for Lecture 8) Solutions

April 18, 2017

A1. Not valid. The base case is *not* true as $2^1 \not> 2 \cdot 1$ (it's also not true for $n = 2$ in fact!).

A2. Yes, valid.

A3. True. The proof is by contradiction. Assume that $\sqrt{p}$ is rational. Then it can be written as a ratio, in lowest terms, as a/b. If we square, we obtain $p = a^2/b^2 \iff b^2p = a^2$, or that $p|a^2$ and thus $p|a$ by Euclid's Lemma. We can write $a = np$, and substitute to obtain $b^2p = n^2p^2 \iff b^2 = n^2p$. By similar reasoning, $p|b^2$ and thus $p|b$ by Euclid's Lemma. But if p divides both a and b, a and b must share a factor. But this contradicts the original assumption that $\sqrt{p}$ was written in lowest terms. Thus, $\sqrt{p}$ must be irrational.

A4. 16 pts. Logical Correctness (4 pts) + Clarity (2 pts) + Opening (4 pts) + Stating the conclusion (4 pts) + Reasons (0 pts) + Overall valuation (2 pts). This is a good proof, but it needs to be more verbose, especially when making assumptions based on the induction hypothesis.

A5. Step 13 need not be true. Suppose $n = 2$ i.e. there are two Americans in a group that have the same age. It is not given that there exists another American, that is not one of the two Americans in the group, which is not true for $n = 2$.