Elements of Set Theory Solutions

April 30, 2017

Exercises 1

A1. This is the set of primes.

A2. P is the set of all reals such that $sin(x) = 0$, which restricts x to integer multiples of π, but this is precisely the set Q, hence set $P = Q$.

A3. $A = \{\sqrt{3}\}$

A4. The first statement is $\emptyset \subseteq A$, which means that every element that belongs to $\emptyset$ also belongs to A. Since no element belongs to the null-set, this is true trivially. The second statement is $A \subseteq A$, which means that every element that belongs to A also belongs to A. This is trivially true.

A5. The statement is $(A \subseteq B) \wedge (B \subseteq C) \Rightarrow A \subseteq C$. The first part means that every element that belongs to A also belongs to B, and the second part means that every element that belongs to B also belongs to C. Since every element that belongs to A also belongs to B also belongs to C, and every element of A also belongs to C, or $A \subseteq C$, which was the statement to be proved.

A6. Let $\mathcal{F}_1$ be the collection of all subsets of $\{1, 2, 3, 4\}$. Then $\mathcal{F}_1 = \{\emptyset, \{1\}, \{2\}, \{3\}, \{4\}, \{1, 2\}, \{1, 3\}, \{1, 4\}, \{2, 3\}, \{2, 4\}, \{3, 4\}\}$, along with $\{\{1, 2, 3\}, \{1, 2, 4\}, \{1, 3, 4\}, \{2, 3, 4\}, \{1, 2, 3, 4\}\}$.

A7. Similarly to the above, $\mathcal{F}_2 = \{\emptyset, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 2\}, \{1, 3\}, \{1, \{1, 2\}\}, \{2, 3\}\}$, along with $\{\{2, \{1, 2\}\}, \{3, \{1, 2\}\}, \{1, 2, 3\}, \{1, 2, \{1, 2\}\}, \{2, 3, \{1, 2\}\}, \{1, 2, 3, \{1, 2\}\}\}$.

A8. An element x belongs to set A if $P(x)$ is true. Since $(\forall x)[P(x) \Rightarrow Q(x)]$, if we have $P(x)$, then we have $Q(x)$. But since an element x for which $Q(x)$ is belongs to B, we have that $A \subseteq B$, which was to be proved.

A9. A set with exactly n elements contains 2^n subsets. We prove this by induction. The base case is for $n = 1$. If a set contains one element a, then the subsets are $\{\emptyset, \{a\}\}$. The number of elements in this collection is 2, and so, the base case is true since $2^1 = 2$. Assume that for $k \in \mathbb{N}$, a set with k elements has 2^k subsets is true. Suppose now we have $k+1$ elements, but for the time being, let's leave out one element. The subsets can be labeled as $S_1, S_2, S_3, \ldots, S_{2^k}$. If we add back the left-out element, we have all of our previous subsets along with all the subsets with our element added back. Thus we have $S_1, S_2, S_3, \ldots, S_{2^k}, \ldots, S_{2^{k+1}}$, which are 2^{k+1} subsets, which was to be proved from the induction hypothesis. Hence, a set with n elements has 2^n subsets.

A10. $P = \{x \in A(n),\ n = 15, 20, 6, 19, 5, 14\}$, where $A(n)$ is the n^{th} alphabet letter.

Exercises 2

A1. Um...how about...NO.

A2. Same as above.