## Problem Set 4 (for Lecture 6) Solutions

## April 13, 2017

- A1. (c).  $\neg(\forall x)[P(x) \Longrightarrow (Q(x) \lor R(x))] \iff (\exists x)[P(x) \land \neg(Q(x) \lor R(x))] \iff (\exists x)[P(x) \land \neg Q(x) \land \neg R(x)]$
- A2.  $(\forall p)(\exists q)(\exists t)[W(p,q,t)]$  means for all tennis players, there exists a player that they win a game against.
  - (a) Same.
  - (b) Not the same.
  - (c) Not the same.
  - (d) Not the same.
  - (e) Not the same.

A3.

- (a) Could be true.  $(\forall p)(\exists q)(\exists t)[W(p,q,t)]$  means for all tennis players, there exists a player that they win a game against. If there were two tennis players, then one could beat the other one time, and the other could win another time.
- (b) Not true.  $(\forall p)(\forall q)(\exists t)[W(p,q,t)]$  means for all pairs of tennis players p and q, there exists a game where p beats q. This must mean p beats themselves, which is not possible (I think?).
- (c) Could be true.  $(\forall q)(\exists p)(\exists t)[W(p,q,t)]$  means for all tennis players, there exists a player that beats them in a game.
- A4. (a).  $(\forall x)(\forall y)[(\exists z)(L(x,z) \land L(z,x)) \Longrightarrow L(y,x)]$  means for everyone who is a lover, they are loved by everyone.

A5.

- (a) True.
- (b) True.

- (c) True. One can choose one such y = x.
- (d) False. Suppose y = x, then  $\neg[(y < x) \lor (x < y)]$ .
- A6. 20 pts. Logical Correctness (4 pts) + Clarity (4 pts) + Opening (2 pts) + Stating the conclusion (4 pts) + Reasons (2 pts) + Overall valuation (4 pts).