Assignment 9 (for Lecture 9) Solutions

April 29, 2017

A1. $b|a$ is a true or false statement. It's equivalent to saying $b|a \iff (\exists n \in \mathbb{Z})[a = nb]$. a/b is more like an operation, and the result is an element of the rationals.

A2.

(a) False, because $b = 0$.

(b) True, $9|0$ because $9 \cdot 0 = 0$.

(c) False, because $b = 0$.

(d) True, because $1 \cdot 1 = 1$.

(e) False, because $44 = 6 \cdot 7 + 2$.

(f) True, because $7 \cdot (-6) = -42$.

(g) True, because $(-7) \cdot (-7) = 49$.

(h) True, because $(-7) \cdot 8 = -56$.

(i) True, because $1 \cdot n = n$.

(j) True, because $n \cdot 0 = 0$.

(k) False, because one n will equal 0.

A3.

(a) $a|0 \iff (\exists n \in \mathbb{Z})[a \cdot n = 0]$, choose $n = 0$. $a|a \iff (\exists n \in \mathbb{Z})[a \cdot n = a]$, choose $n = 1$.

(b) We prove the reverse direction first. If $a = \pm 1$, then $a|1$ since $1 \cdot 1 = 1$ and $(-1) \cdot (-1) = 1$ respectively. Now, the other direction. $(a|1) \iff (\exists n \in \mathbb{Z})[a \cdot n = 1]$, but the only solutions to this equation in the integers are when $a = 1,\ n = 1$ and $a = -1,\ n = -1$.

(c) $(a|b) \iff (\exists n \in \mathbb{Z})[a \cdot n = b]$ and $(c|d) \iff (\exists m \in \mathbb{Z})[c \cdot m = d]$. If we multiply b and d, we have $bd = a \cdot n \cdot c \cdot m = (nm) \cdot (ac)$. Since we have that $nm \cdot ac = bd$, $ac|bd$.

(d) $(a|b) \iff (\exists n \in \mathbb{Z})[a \cdot n = b]$ and $(b|c) \iff (\exists m \in \mathbb{Z})[b \cdot m = c]$. If we rewrite c in terms of a, we have $c = bm = (an)m = anm$. Since we have that $a \cdot nm = c$, $a|c$ are required.

(e) By (f), we have $|a| \leq |b| \wedge |a| \geq |b|$, which implies that $|a| = |b|$, which is true whenever $a = \pm b$.

(f) $(a|b) \iff (\exists n \in \mathbb{Z})[a \cdot n = b]$. If we take the absolute value of the equation we obtain $|a| \cdot |n| = |b|$. Since $|n| \geq 1$ by $a \neq 0 \implies b \neq 0$, and $|b| \geq 1$, and thus we have $|a| \leq |b|$.

(g) $(a|b) \iff (\exists n \in \mathbb{Z})[a \cdot n = b]$ and $(a|c) \iff (\exists m \in \mathbb{Z})[a \cdot m = c]$. If we add b and c together, we obtain $b+c = an+am = a(n+m) = a \cdot p$, thus $a|(b+c)$. In fact, even if we introduce a constant factor to each dividend, we obtain $bx + cy = anx + amy = a(nx+my) = ak$, and so $a|(bx + cy)$ also.

OPTIONAL PROBLEMS

A1. Not even sure what the converse of the statement is... Maybe later.

A2. Have a little clue, but not up for it now. Maybe later.