Assignment 7 (for Lecture 7) Solutions

April 15, 2017

A1. False. An ostrich is bird that cannot fly.

A2. False. Choose $x = y = 0$. It is not the case that $(0 - 0) = 0 > 0$.

A3. Suppose there are two rationals ${}^{a}/_{b}, {}^{p}/_{q}$ such that ${}^{a}/_{b} < {}^{p}/_{q}$ $(b \neq 0 \wedge q \neq 0)$. Then there exists another rational between the two rationals given by: $\frac{aq+bp}{2bq}$. We show that this construction obeys the following relation: ${}^{a}/_{b} < \frac{aq+bp}{2bq} < {}^{p}/_{q}$. We have ${}^{a}/_{b} = \frac{aq}{bq}$, and $\frac{aq}{bq} < \frac{aq+bp}{2bq} \iff \frac{1}{2}\frac{aq}{bq} < \frac{1}{2}\frac{bp}{bq} \iff {}^{a}/_{b} < {}^{p}/_{q}$, which is true by assumption. Similarly, we have ${}^{p}/_{q} = \frac{bp}{bq}$, and $\frac{aq+bp}{2bq} < \frac{bp}{bq} \iff \frac{1}{2}\frac{aq}{bq} < \frac{1}{2}\frac{bp}{bq} \iff {}^{a}/_{b} < {}^{p}/_{q}$,which is true by assumption. Thus we have found a rational between the two rationals.

A4. If we construct a truth table for the following expression: $[(\phi \implies \psi) \wedge (\psi \implies \phi)] \implies (\phi \iff \psi)$, we will obtain a tautology.

A5. We know that $[(\neg\phi) \implies (\neg\psi)] \iff (\psi \implies \phi)$ because it is the contrapositive. Then from A4, we have that $[(\phi \implies \psi) \wedge (\neg\phi \implies \neg\psi)] \implies (\phi \iff \psi)$.

A6. The only way to ensure that each investor receives the least possible amount of money is to split the payout evenly. But if we split the payout evenly, each investor will receive $400,000. Since this is the only way to ensure each investor receives the least amount of money, we cannot do better. And so, at least one investor receives at least $400,000.

A7. Suppose that $\sqrt{3}$ is rational. Then it can be represented by a fraction written in lowest terms, ${}^{a}/_{b}$. Then we have $\sqrt{3} = {}^{a}/_{b} \iff 3 = {}^{a^2}/_{b^2} \iff 3b^2 = a^2$. This means that $3|a^2$, and $3|a$. Thus, $a = 3k$ for some $k \in \mathbb{N}$. And so we have $3b^2 = 9k^2 \iff b^2 = 3k^2$. This means that $3|b^2$, and $3|b$. Thus, $b = 3n$ for some $n \in \mathbb{N}$. But if $(a = 3k) \wedge (b = 3n)$, then a and b have a common factor, and thus ${}^{a}/_{b}$ is not in lowest terms. Our original assumption is wrong, and thus $\sqrt{3}$ is irrational.

A8.

(a) If the Yuan rises, the Dollar will fall.

(b) $(\forall x, y \in \mathbb{R})[(-y < -x) \Longrightarrow (x < y)]$

(c) If two triangles have the same area they are congruent.

(d) If $b^2 \geq 4ac$, then $ax^2 + bx + c = 0$ has a solution. $(\forall a, b, c \in \mathbb{R}$ and $x \in R \setminus \{0\})$

(e) If the opposite angles in a quadrilateral ABCD are pairwise equal, then the opposite sides of ABCD are pairwise equal.

(f) If the four angles in quadrilateral ABCD are equal, then all four sides of ABCD are equal.

(g) $(\forall n \in \mathbb{N})[3|(n^2 + 5) \Longrightarrow \neg(n|3)]$

A9.