Assignment 6 (for Lecture 6) Solutions

April 12, 2017

A1. $\neg[\exists x A(x)]$ means that it is not the case that there exists an x such that $A(x)$ is true. This is the same as saying no such x exists such that $A(x)$ is true. In symbols, this is the same as $\forall x[\neg A(x)]$.

A2. Suppose there exists an even prime p. By the definition of an even number, $2|p$, or $p = 2n$ for some $n \in \mathbb{N}$. But if p is to be prime, it cannot have any factors other than 1 and itself. Since this is not true, our original assumption must have been false, and thus we can conclude that there is no even prime.

A3.

(a) $(\forall p)[Student(p) \implies LikesPizza(p)]$

(b) $(\exists p)[Friend(p) \implies \neg Car(p)]$

(c) $(\exists a)[Elephant(a) \implies \neg LikesMuffins(a)]$

(d) $(\forall g_f)[Triangle(g_f) \implies Isosceles(g_f)]$

(e) $(\exists s)[StudentInRoster(s) \implies \neg PresentToday(s)]$

(f) $(\forall p_1)[(\exists p_2)Loves(p_1, p_2)]$

(g) $\neg(\exists p_1)[(\forall p_2)Loves(p_1, p_2)]$

(h) $(\forall p)[(Man(p) \wedge Comes(p)) \implies (\forall p)(Woman(p) \implies Leaves(p))]$

(i) $(\forall p)[Tall(p) \vee Short(p)]$

(j) $(\forall p)[Tall(p)] \vee (\forall p)[Short(p)]$

(k) $\neg(\forall s)[Precious(s) \implies Beautiful(s)]$

(l) $(\forall p)[\neg LovesMe(p)]$

(m) $(\exists s)[American(s) \implies Poisonous(s)]$

(n) $(\exists s)[(Snake(s) \wedge American(s) \implies Poisonous(s)]$

A4.

(a) $(\exists p)[Student(p) \wedge \neg LikesPizza(p)]$. There exists a who does not like pizza.

(b) $(\forall p)[Friend(p) \wedge Car(p)]$. All my friends have a car.

(c) $(\forall a)[Elephant(a) \wedge LikesMuffins(a)]$. All elephants like muffins.

(d) $(\exists g_f)[Triangle(g_f) \wedge \neg Isosceles(g_f)]$. There is a triangle that is not isosceles.

(e) $(\forall s)[StudentInRoster(s) \wedge PresentToday(s)]$. All the students in the class are here today.

(f) $(\exists p_1)[(\forall p_2) \neg Loves(p_1, p_2)]$. There is someone is loves no one.

(g) $(\exists p_1)[(\forall p_2) Loves(p_1, p_2)]$. There is someone who loves everyone.

(h) $(\exists p)[(Man(p) \wedge Comes(p)) \wedge (\exists p)(Woman(p) \wedge \neg Leaves(p))]$. There is a man who comes and a woman who doesn't leave.

(i) $(\exists p)[\neg Tall(p) \wedge \neg Short(p)]$. There's someone who is neither tall nor short.

(j) $(\exists p)[\neg Tall(p)] \wedge (\exists p)[\neg Short(p)]$. There's someone who isn't tall and there's someone who isn't short.

(k) $(\forall s)[Precious(s) \implies Beautiful(s)]$. All precious stones are beautiful.

(l) $(\exists p)[LovesMe(p)]$. There's someone who loves me.

(m) $(\forall s)[American(s) \wedge \neg Poisonous(s)]$. All American snakes are not poisonous.

(n) $(\forall s)[((\neg Snake(s) \vee \neg American(s)) \wedge \neg Poisonous(s)]$. All animals are not poisonous, and either not snakes or not American.

A5.

(a) False. ($x = \frac{2}{3} \notin \mathbb{N}$)

(b) False. ($x = \sqrt{2} \notin \mathbb{Q}$)

(c) True. (The formula is given)

(d) True. (The formula is given)

(e) False. (If $x = 0$, it is trivially true. If $x \neq 0$, then $y = z$, but cannot find one y such that $y = z$, for all z)

(f) False. (We can simplify to $y = z$, but cannot find one prime y such that $y = z$, for all prime z)

(g) False. (Choose $x = -1$)

(h) True. (The antecedent is always false)

A6.

(a) $(\forall x \in \mathbb{N})[2x + 3 \neq 5x + 1]$

(b) $(\forall x \in \mathbb{Q})[x^2 \neq 2]$

(c) $(\exists x \in \mathbb{R})(\forall y \in \mathbb{R})[y \neq x^2]$

(d) $(\exists x \in \mathbb{N})(\forall y \in \mathbb{N})[y \neq x^2]$

(e) $(\exists x \in \mathbb{R})(\forall y \in \mathbb{R})(\exists z \in \mathbb{R})[xy \neq xz]$

(f) $(\exists x \in \mathbb{P})(\forall y \in \mathbb{P})(\exists z \in \mathbb{P})[xy \neq xz]$

(g) $(\exists x \in \mathbb{R})[(x < 0) \wedge (\forall y \in \mathbb{R})(y^2 \neq x)]$

(h) $(\exists x \in \mathbb{R}_+)[(x < 0) \wedge (\forall y \in \mathbb{R}_+)(y^2 \neq x)]$

A7.

(a) $(\exists x \in \mathbb{N})(\forall y \in \mathbb{N})[x + y \neq 1]$

(b) $(\exists x > 0)(\forall y < 0)[x + y \neq 0]$

(c) $(\forall x \in \mathbb{R})(\exists \epsilon > 0)[(x \leq -\epsilon) \vee (x \geq \epsilon)]$

(d) $(\exists x \in \mathbb{N})(\exists y \in \mathbb{N})(\forall z \in \mathbb{N})[x + y \neq z^2]$

A8. $(\forall t)(\exists p)[\neg Fool(p,t)] \vee (\forall p)(\exists t)[Fool(p,t)] \vee (\forall p)(\forall t)[Fool(p,t)]$. You cannot fool some people all the time or you can fool everyone some of the time or you can fool everyone all the time.

A9. $(\exists \epsilon > 0)(\forall \delta > 0)(\exists x)[(|x - a| < \delta) \wedge (|f(x) - f(a)| \geq \epsilon)]$. There exists an $\epsilon > 0$ for all $\delta > 0$ such that one may find an x such that $|x - a| < \delta$ and $|f(x) - f(a)| \geq \epsilon$.