

## Assignment 5 (for Lecture 5) Solutions

April 10, 2017

A1.

- (a)  $(\exists x \in \mathbb{N})[x^3 = 27]$
- (b)  $(\exists x \in \mathbb{N})[x > 1,000,000]$
- (c)  $(\exists p \in \mathbb{N})(\exists q \in \mathbb{N})[p > 1 \wedge q > 1 \wedge n = pq]$

A2.

- (a)  $(\forall x \in \mathbb{N})[(x^3 \neq 28)]$
- (b)  $(\forall x \in \mathbb{N})[0 < x]$
- (c)  $(\forall p \in \mathbb{N})(\forall q \in \mathbb{N})[(n = pq) \implies (p = 1 \vee q = 1)]$

A3. Let  $x$  and  $y$  denote two people.

- (a)  $(\forall x)(\exists y)[Loves(x, y)]$ , where  $Loves(x, y)$  denotes that  $x$  loves  $y$ .
- (b)  $(\forall x)[Tall(x) \vee Short(x)]$
- (c)  $(\forall x)[Tall(x)] \vee (\forall y)[Short(y)]$
- (d)  $(\forall x)[\neg Home(x)]$
- (e)  $Comes(John) \implies (\forall x)[Woman(x) \implies Leaves(x)]$
- (f)  $(\forall x)[Man(x) \wedge Comes(x)] \implies (\forall x)[Woman(x) \implies Leaves(x)]$

A4.

- (a)  $(\forall a \in \mathbb{R})(\exists x \in \mathbb{R})[x^2 + a = 0]$
- (b)  $(\forall a \in \mathbb{R})[(a < 0) \implies (\exists x \in \mathbb{R})(x^2 + a = 0)]$
- (c)  $(\forall x \in \mathbb{R})(\exists m \in \mathbb{N})(\exists n \in \mathbb{N})[m = nx \vee m = -nx \vee x = 0]$
- (d)  $(\exists x \in \mathbb{R})(\forall m \in \mathbb{N})(\forall n \in \mathbb{N})[m \neq nx \wedge m \neq -nx]$
- (e)  $(\forall x \in \mathbb{R})(\exists y \in \mathbb{R})[(y > x) \wedge (\forall m \in \mathbb{N})(\forall n \in \mathbb{N})(m \neq nx)]$

A5.

- (a)  $(\forall x \in C)[D(x) \implies M(x)]$

- (b)  $(\forall x \in C)[\neg D(x) \implies M(x)]$
  - (c)  $(\forall x \in C)[M(x) \implies D(x)]$
  - (d)  $(\exists x \in C)[D(x) \wedge \neg M(x)]$
  - (e)  $(\exists x \in C)[\neg D(x) \wedge M(x)]$
- A6.  $(\forall x \in \mathbb{R})(\forall y \in \mathbb{R})[(x < y) \implies (\exists z \in \mathbb{Q})(x < z < y)]$ , using  $\mathbb{Q}$  to denote the set of all rationals.
- A7.  $(\exists t)(\forall p)[Fool(p, t)] \wedge (\exists p)(\forall t)[Fool(p, t)] \wedge \neg[(\forall t)(\forall p)(Fool(p, t))]$ , where  $Fool(p, t)$  denotes that you fool person  $p$  at time  $t$ .
- A8.  $(\exists x)[A(x, t)]$
- A9.  $(\forall t)(\exists x)[A(x, t)]$