Problem Set 1 (for Lectures 1 and 2) Solutions

April 7, 2017

A1. For $(\phi \wedge \psi) \wedge \theta$ to be true, all of the symbols must be true. Thus, it is necessarily so that $\phi \wedge (\psi \wedge \theta)$ is also true, and so, the associative property holds for conjunction.

A2. For $(\phi \vee \psi) \vee \theta$ to be true, at least one of the symbols must be true. If at least one of the symbols must be true, then $\phi \vee (\psi \vee \theta)$ must also be true. Thus, the associative property holds for disjunction as well.

A3. For $\phi \wedge (\psi \vee \theta)$ to be true, ϕ must be true and at least one, ψ or θ, must be true. If this is the case, then either $\phi \wedge \psi$ or $\phi \wedge \theta$ or both are true. Thus, $(\phi \wedge \psi) \vee (\phi \wedge \theta)$ must be true.

A4. For $\phi \vee (\psi \wedge \theta)$ to be true, either ϕ or $\psi \wedge \theta$ or both are true. Suppose ϕ is true. Then we have $(\phi \vee \psi) \wedge (\phi \vee \theta)$ is true. Suppose $\psi \wedge \theta$ is true. Then both ψ and θ are true. If this is the case, then we have $(\phi \vee \psi) \wedge (\phi \vee \theta)$ is true. If both are true, then we can choose either of the cases.

A5. Showing $\neg\phi$ is true is *indeed* equivalent to showing ϕ is false.

A6. Choice (e) is the most likely because it is the least restrictive condition.

A7. Choice (a) is the most likely because it is the more expansive condition.

A8.

(a) This is false because x is strictly greater than 0.

(b) True.

(c) True.

(d) True.

(e) True.

(f) True.

(g) True, as $x \in \mathbb{R}$.

(h) True.

(i) True.

(j) True.