# Model-Based Diagnosis with Multiple Observations

Alexey Ignatiev<sup>1</sup>, Antonio Morgado<sup>1</sup>, Georg Weissenbacher<sup>2</sup>, Joao Marques-Silva<sup>1</sup>

August 14, 2019 | **IJCAI** 

<sup>1</sup>University of Lisbon, Portugal <sup>2</sup>TU Wien, Vienna, Austria Motivation

```
void foo(bool b)
                                                      b = true:
     {
 2
                                                       \Delta = \{\{3\}, \{5\}, \{6\}, \{7\}\}
           int x = 0;
 4
                                                      b = false:
          X++;
                                                       \Delta = \{\{3\}, \{5\}, \{6\}, \{9\}\}
           if (b)
                X++;
           else
                X++;
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           assert(x != 2);
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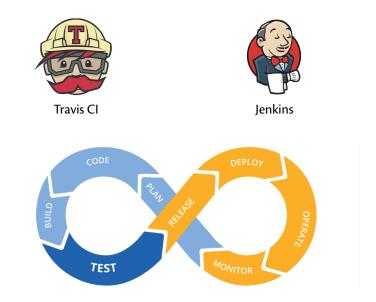
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            else
                                                           both traces:
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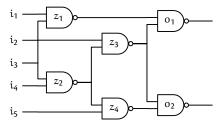
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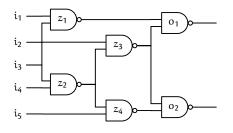
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#### Meanwhile in real life...

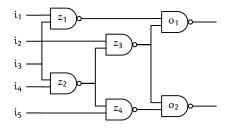


continuous integration (lots of traces!)

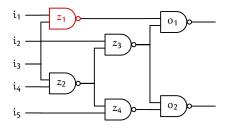




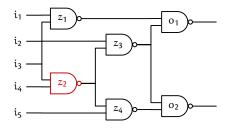
Comps 
$$\triangleq \{z_1, z_2, z_3, z_4, o_1, o_2\}$$
  
SD  $\triangleq \bigwedge_{c \in Comps} (Ab(c) \vee F_c)$ 



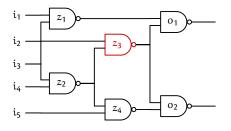
$$\begin{array}{cccc} \mathsf{Comps} & \triangleq & \{z_1, z_2, z_3, z_4, o_1, o_2\} \\ \mathsf{SD} & \triangleq & \bigwedge_{c \in \mathsf{Comps}} (\mathsf{Ab}(c) \vee \mathsf{F}_c) \\ \\ \mathsf{F}_{z_1} & \triangleq & \mathsf{CNF}(z_1 \leftrightarrow \neg(\mathsf{i}_1 \wedge \mathsf{i}_3)) \\ \mathsf{F}_{z_2} & \triangleq & \mathsf{CNF}(z_2 \leftrightarrow \neg(\mathsf{i}_3 \wedge \mathsf{i}_4)) \\ \mathsf{F}_{z_3} & \triangleq & \mathsf{CNF}(z_3 \leftrightarrow \neg(\mathsf{i}_2 \wedge z_2)) \\ \mathsf{F}_{z_4} & \triangleq & \mathsf{CNF}(z_4 \leftrightarrow \neg(z_2 \wedge \mathsf{i}_5)) \\ \mathsf{F}_{o_1} & \triangleq & \mathsf{CNF}(o_1 \leftrightarrow \neg(z_1 \wedge z_3)) \\ \mathsf{F}_{o_2} & \triangleq & \mathsf{CNF}(o_2 \leftrightarrow \neg(z_3 \wedge z_4)) \\ \end{array}$$



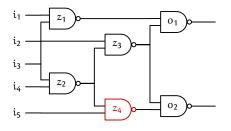
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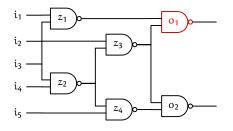
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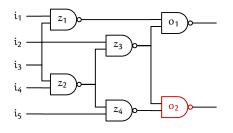
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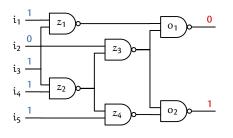
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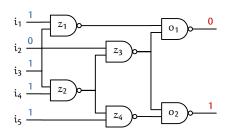


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$$Obs = \{\langle i_1, i_2, i_3, i_4, i_5 \rangle = \langle 1, 0, 1, 1, 1 \rangle, \quad \langle o_1, o_2 \rangle = \langle 0, 1 \rangle \}$$

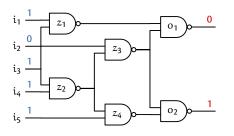


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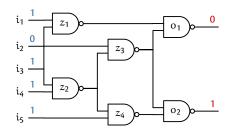
$$Obs = \{\langle i_1, i_2, i_3, i_4, i_5 \rangle = \langle 1, 0, 1, 1, 1 \rangle, \quad \langle o_1, o_2 \rangle = \langle 0, 1 \rangle \}$$



$$\mathsf{SD} \land \mathsf{Obs} \land \textstyle \bigwedge_{c \in \mathsf{Comps}} \neg \mathsf{Ab}(c) \vDash \bot$$



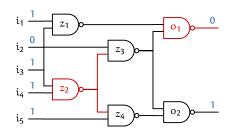
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$$\begin{array}{ccc} \mathsf{Comps} & \triangleq & \{z_1, z_2, z_3, z_4, o_1, o_2\} \\ & \mathsf{SD} & \triangleq & \bigwedge_{c \in \mathsf{Comps}} (\mathsf{Ab}(c) \vee \mathsf{F}_c) \\ & \mathsf{Obs} & \triangleq & \{i_1, \neg i_2, i_3, i_4, i_5, \neg o_1, o_2\} \\ & \mathsf{SD} \wedge \mathsf{Obs} \wedge \bigwedge_{c \in \mathsf{Comps}} \neg \mathsf{Ab}(c) \vDash \bot \end{array}$$



 $\mathsf{find}\,\Delta\subseteq\mathsf{Comps}\,\mathsf{s.t.}$   $\mathsf{SD}\,\wedge\,\mathsf{Obs}\,\wedge\,\bigwedge_{\mathsf{c}\in\Delta}\mathsf{Ab}(\mathsf{c})\,\wedge\,\bigwedge_{\mathsf{c}\in\mathsf{Comps}\setminus\Delta}\neg\mathsf{Ab}(\mathsf{c})\not\vdash\bot$ 



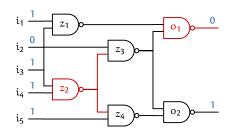
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find  $\Delta \subseteq Comps$  s.t.

$$SD \land Obs \land \bigwedge_{c \in \Delta} Ab(c) \land \bigwedge_{c \in Comps \setminus \Delta} \neg Ab(c) \not\models \bot$$

$$\Delta = \{z_2, o_1\}$$



$$\begin{array}{ccc} \mathsf{Comps} & \triangleq & \{z_1, z_2, z_3, z_4, o_1, o_2\} \\ & \mathsf{SD} & \triangleq & \bigwedge_{c \in \mathsf{Comps}} (\mathsf{Ab}(c) \vee \mathsf{F}_c) \\ & \mathsf{Obs} & \triangleq & \{i_1, \neg i_2, i_3, i_4, i_5, \neg o_1, o_2\} \\ & \mathsf{SD} \wedge \mathsf{Obs} \wedge \bigwedge_{c \in \mathsf{Comps}} \neg \mathsf{Ab}(c) \vDash \bot \end{array}$$

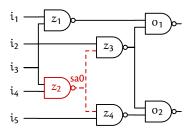


find  $\Delta \subseteq Comps s.t.$ 

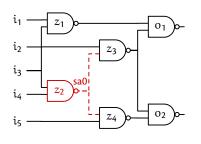
$$SD \land Obs \land \bigwedge_{c \in \Delta} Ab(c) \land \bigwedge_{c \in Comps \setminus \Delta} \neg Ab(c) \nvDash \bot$$

 $\Delta = \{z_2, o_1\}$  — minimize  $\Delta$ , e.g. with MaxSAT

Multiple observations?

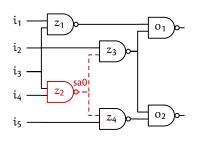


$$\begin{array}{cccc} \mathsf{Comps} & \triangleq & \{z_1, z_2, z_3, z_4, o_1, o_2\} \\ & \mathsf{SD} & \triangleq & \bigwedge_{c \in \mathsf{Comps}} (\mathsf{Ab}(c) \vee \mathsf{F}_c) \\ & \mathsf{F}_{z_1} & \triangleq & \mathsf{CNF}(z_1 \leftrightarrow \neg(i_1 \wedge i_3)) \\ & \mathsf{F}_{z_2} & \triangleq & \mathsf{CNF}(z_2 \leftrightarrow 0) \\ & \mathsf{F}_{z_3} & \triangleq & \mathsf{CNF}(z_3 \leftrightarrow \neg(i_2 \wedge z_2)) \\ & \mathsf{F}_{z_4} & \triangleq & \mathsf{CNF}(z_4 \leftrightarrow \neg(z_2 \wedge i_5)) \\ & \mathsf{F}_{o_1} & \triangleq & \mathsf{CNF}(o_1 \leftrightarrow \neg(z_1 \wedge z_3)) \\ & \mathsf{F}_{o_2} & \triangleq & \mathsf{CNF}(o_2 \leftrightarrow \neg(z_3 \wedge z_4)) \end{array}$$



$$\begin{array}{cccc} \mathsf{Comps} & \triangleq & \{z_1, z_2, z_3, z_4, o_1, o_2\} \\ \mathsf{SD} & \triangleq & \bigwedge_{c \in \mathsf{Comps}} (\mathsf{Ab}(c) \vee \mathsf{F}_c) \\ \mathsf{F}_{z_1} & \triangleq & \mathsf{CNF}(z_1 \leftrightarrow \neg(i_1 \wedge i_3)) \\ \mathsf{F}_{z_2} & \triangleq & \mathsf{CNF}(\underline{z_2} \leftrightarrow \mathbf{0}) \\ \mathsf{F}_{z_3} & \triangleq & \mathsf{CNF}(z_3 \leftrightarrow \neg(i_2 \wedge z_2)) \\ \mathsf{F}_{z_4} & \triangleq & \mathsf{CNF}(z_4 \leftrightarrow \neg(z_2 \wedge i_5)) \\ \mathsf{F}_{o_1} & \triangleq & \mathsf{CNF}(o_1 \leftrightarrow \neg(z_1 \wedge z_3)) \\ \mathsf{F}_{o_2} & \triangleq & \mathsf{CNF}(o_2 \leftrightarrow \neg(z_3 \wedge z_4)) \end{array}$$

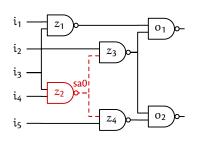
Observation	$\langle i_1,i_2,i_3,i_4,i_5,o_1,o_2\rangle$	Diagnoses
Obs <sub>1</sub>	$\langle 0, 1, 1, 0, 1, \frac{1}{1}, \frac{1}{1} \rangle$	$D_1 = \{\{z_2\}, \{z_3\}, \{z_1, z_4\}, \{z_1, o_2\}, \{z_4, o_1\}, \{o_1, o_2\}\}\}$
$Obs_2$	$\langle \mathtt{1}, \mathtt{1}, \mathtt{1}, \mathtt{0}, \mathtt{1}, \mathtt{1}, \textcolor{red}{\mathtt{1}} \rangle$	$D_2 = \{\{z_2\}, \{z_3\}, \{z_4\}, \{o_2\}\}$
$Obs_3$	$\langle 1,0,0,0,1,0,1 \rangle$	$D_3 = \{\{z_2\}, \{z_4\}, \{o_2\}, \{z_3, o_1\}\}$



$$\begin{array}{cccc} \mathsf{Comps} & \triangleq & \{z_1, z_2, z_3, z_4, o_1, o_2\} \\ \mathsf{SD} & \triangleq & \bigwedge_{c \in \mathsf{Comps}} (\mathsf{Ab}(c) \vee \mathsf{F}_c) \\ \mathsf{F}_{z_1} & \triangleq & \mathsf{CNF}(z_1 \leftrightarrow \neg(\mathsf{i}_1 \wedge \mathsf{i}_3)) \\ \mathsf{F}_{z_2} & \triangleq & \mathsf{CNF}(z_2 \leftrightarrow \mathsf{0}) \\ \mathsf{F}_{z_3} & \triangleq & \mathsf{CNF}(z_3 \leftrightarrow \neg(\mathsf{i}_2 \wedge z_2)) \\ \mathsf{F}_{z_4} & \triangleq & \mathsf{CNF}(z_4 \leftrightarrow \neg(z_2 \wedge \mathsf{i}_5)) \\ \mathsf{F}_{o_1} & \triangleq & \mathsf{CNF}(o_1 \leftrightarrow \neg(z_1 \wedge z_3)) \\ \mathsf{F}_{o_2} & \triangleq & \mathsf{CNF}(o_2 \leftrightarrow \neg(z_3 \wedge z_4)) \end{array}$$

Observation	$\langle i_1, i_2, i_3, i_4, i_5, o_1, o_2 \rangle$	Diagnoses
Obs <sub>1</sub>	$\langle 0, 1, 1, 0, 1, \frac{1}{1}, \frac{1}{1} \rangle$	$D_1 = \{\{z_2\}, \{z_3\}, \{z_1, z_4\}, \{z_1, o_2\}, \{z_4, o_1\}, \{o_1, o_2\}\}\}$
$Obs_2$	$\langle \mathtt{1}, \mathtt{1}, \mathtt{1}, \mathtt{0}, \mathtt{1}, \mathtt{1}, \textcolor{red}{\mathtt{1}} \rangle$	$D_2 = \{\{z_2\}, \{z_3\}, \{z_4\}, \{o_2\}\}$
Obs <sub>3</sub>	$\langle \texttt{1,0,0,0,1,0,1} \rangle$	$D_3 = \{\{z_2\}, \{z_4\}, \{o_2\}, \{z_3, o_1\}\}$

state-of-the-art approaches enumerate 96 aggregated diagnoses while



$$\begin{array}{cccc} \mathsf{Comps} & \triangleq & \{z_1, z_2, z_3, z_4, o_1, o_2\} \\ & \mathsf{SD} & \triangleq & \bigwedge_{c \in \mathsf{Comps}} (\mathsf{Ab}(c) \vee \mathsf{F}_c) \\ & \mathsf{F}_{z_1} & \triangleq & \mathsf{CNF}(z_1 \leftrightarrow \neg(i_1 \wedge i_3)) \\ & \mathsf{F}_{z_2} & \triangleq & \mathsf{CNF}(z_2 \leftrightarrow \mathbf{0}) \\ & \mathsf{F}_{z_3} & \triangleq & \mathsf{CNF}(z_3 \leftrightarrow \neg(i_2 \wedge z_2)) \\ & \mathsf{F}_{z_4} & \triangleq & \mathsf{CNF}(z_4 \leftrightarrow \neg(z_2 \wedge i_5)) \\ & \mathsf{F}_{o_1} & \triangleq & \mathsf{CNF}(o_1 \leftrightarrow \neg(z_1 \wedge z_3)) \\ & \mathsf{F}_{o_2} & \triangleq & \mathsf{CNF}(o_2 \leftrightarrow \neg(z_3 \wedge z_4)) \end{array}$$

Observation	$\langle i_1, i_2, i_3, i_4, i_5, o_1, o_2 \rangle$	Diagnoses
Obs <sub>1</sub>	$\langle 0,1,1,0,1,\frac{1}{1},\frac{1}{1} \rangle$	$D_1 = \{\{z_2\}, \{z_3\}, \{z_1, z_4\}, \{z_1, o_2\}, \{z_4, o_1\}, \{o_1, o_2\}\}$
$Obs_2$	$\langle \mathtt{1}, \mathtt{1}, \mathtt{1}, \mathtt{0}, \mathtt{1}, \mathtt{1}, \textcolor{red}{\mathtt{1}} \rangle$	$D_2 = \{\{z_2\}, \{z_3\}, \{z_4\}, \{o_2\}\}$
Obs <sub>3</sub>	$\langle \texttt{1,0,0,0,1,0,1} \rangle$	$D_3 = \{\{z_2\}, \{z_4\}, \{o_2\}, \{z_3, o_1\}\}$

#### state-of-the-art approaches enumerate 96 aggregated diagnoses while

$$\mathbb{D} = \{\{z_2\}, \{z_1, z_4\}, \{z_1, o_2\}, \{z_3, o_1\}, \{z_3, o_2\}, \{z_4, o_1\}, \{z_3, z_4\}, \{o_1, o_2\}\}, \text{ i.e. } |\mathbb{D}| = 8$$

# DiagCombine<sup>1</sup>:

<sup>&</sup>lt;sup>1</sup>S. Lamraoui and S. Nakajima. A formula-based approach for automatic fault localization of imperative programs. In ICFEM, pp. 251–266, 2014.

# DiagCombine<sup>1</sup>:

1. enumerate all diagnoses for each observation

<sup>&</sup>lt;sup>1</sup>S. Lamraoui and S. Nakajima. *A formula-based approach for automatic fault localization of imperative programs.* In ICFEM, pp. 251–266, 2014.

# DiagCombine<sup>1</sup>:

1. enumerate all diagnoses for each observation

2. compute all "combinations"

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# DiagCombine<sup>1</sup>:

1. enumerate all diagnoses for each observation

2. compute all "combinations"



(exponentially) many redundant diagnoses

<sup>&</sup>lt;sup>1</sup>S. Lamraoui and S. Nakajima. *A formula-based approach for automatic fault localization of imperative programs*. In ICFEM, pp. 251–266, 2014.

```
D_1 { {0}, {2} } 
 D_2 { {0}, {1,2} }
```

 $\mathbb{D}$  {

$$D_1$$
 {  $\{0\}, \{2\}\}$  }  $D_2$  {  $\{0\}, \{1,2\}\}$ 

$$\mathbb{D} \quad \{ \{ \mathbf{0} \},$$

$$D_1$$
 {  $\{0\}$ ,  $\{2\}$  }  $D_2$  {  $\{0\}$ ,  $\{1,2\}$  }

$$\mathbb{D} \{\{0\}, \{0, 1, 2\}, \}$$

$$D_1$$
 { {0}, {2} }  $D_2$  { {0}, {1,2} }

$$\mathbb{D} \quad \{ \{0\}, \{0, 1, 2\}, \{0, 2\}, \}$$

$$\begin{array}{ll} D_1 & \{ \{0\}, \{2\} \} \\ D_2 & \{ \{0\}, \{1,2\} \} \end{array}$$

$$\mathbb{D}$$
 { {0}, {0,1,2}, {0,2}, {1,2} }

$$D_1$$
 { {0}, {2} }  $D_2$ 

# redundant diagnoses!

$$\mathbb{D} \quad \{ \{0\}, \{0,1,2\}, \{0,2\}, \{1,2\} \}$$

(a) 
$$\exists_{\Delta} \forall_{D_i}$$
  $\Delta \in D_i \Rightarrow \Delta \in \mathbb{D}$ 

(a)  $\exists_{\Delta} \forall_{D_i}$   $\Delta \in D_i \Rightarrow \Delta \in \mathbb{D}$ 

(b)  $\exists_{\Delta} \forall_{D_i} \exists_{\Delta_i \in D_i} \Delta_i \subseteq \Delta \Rightarrow \Delta \in \mathbb{D}$ 

(a) 
$$\exists_{\Delta} \forall_{D_i}$$
  $\Delta \in D_i \Rightarrow \Delta \in \mathbb{D}$   
(b)  $\exists_{\Delta} \forall_{D_i} \exists_{\Delta_i \in D_i} \Delta_i \subseteq \Delta \Rightarrow \Delta \in \mathbb{D}$   
 $D_1$   $\{\{0\}, \{2\}\}\}$   
 $D_2$   $\{\{0\}, \{1,2\}\}$ 

(a) 
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  $\Delta \in D_i \Rightarrow \Delta \in \mathbb{D}$   
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$$\begin{array}{lll} \textbf{(a)} & \exists_{\Delta} \, \forall_{D_i} & \Delta \in D_i \ \Rightarrow \ \Delta \in \mathbb{D} \\ \textbf{(b)} & \exists_{\Delta} \, \forall_{D_i} \, \exists_{\Delta_i \in D_i} \ \Delta_i \subseteq \Delta \ \Rightarrow \ \Delta \in \mathbb{D} \end{array}$$

```
D_1 { \{2\}\} } D_2 { \{1,2\}\}
```

$$\begin{array}{lll} \textbf{(a)} & \exists_{\Delta} \, \forall_{D_i} & \Delta \in D_i \ \Rightarrow \ \Delta \in \mathbb{D} \\ \textbf{(b)} & \exists_{\Delta} \, \forall_{D_i} \, \exists_{\Delta_i \in D_i} \ \Delta_i \subseteq \Delta \ \Rightarrow \ \Delta \in \mathbb{D} \end{array}$$

```
D_1 \qquad \{ \qquad \{2\} \} 
D_2 \qquad \{ \qquad \}
```

$$\mathbb{D} \quad \{ \{0\}, \{0,1,2\}, \{0,2\}, \{1,2\} \}$$

Improved DiagCombine - problem..

(a) and (b) not always apply!



no silver bullet

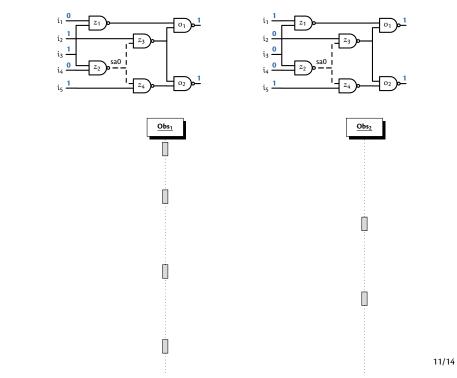
```
input
                      : SD, Obs<sub>1</sub>, ..., Obs<sub>m</sub>
     output : \mathbb{D} = \{\Delta_1, \Delta_2 ...\}, \mathbb{U} = \{\mathcal{U}_1, \mathcal{U}_2 ...\}
     (\mathcal{H}_1, \dots, \mathcal{H}_m, \mathcal{S}) \leftarrow \mathsf{Encode}(\mathsf{SD}, \mathsf{Obs}_1, \dots, \mathsf{Obs}_m)
 _{2} (\mathbb{D},\mathbb{U}) \leftarrow (\emptyset,\emptyset)
    while true:
              (st, \Delta) \leftarrow MinHS(\mathbb{U}, \mathbb{D})
                                                                                                                                                                                               # find a min HS of \mathbb{U} s.t. \mathbb{D}
              if not st:
                       break
 6
              foreach i \in \{1, ..., m\}:
 7
                       (st, \kappa) \leftarrow SAT(\mathcal{H}_i \cup (S \setminus \Delta))
 8
                       if not st:
 9
                                \mathcal{U} \leftarrow \text{Reduce}(\kappa)
                                                                                                                                                                                         # \mathcal{U} is MUS of \mathcal{H}_i \cup (\mathcal{S} \setminus \Delta)
10
                                \mathbb{U} \leftarrow \mathbb{U} \cup \{\mathcal{U}\}\
11
                                ReportExpl(\mathcal{U})
                                                                                                                                                                                                  # report min explanation
12
                                break
13
              else:
                                                                                                                                                                                              # if the loop was not broken
14
                       \mathbb{D} \leftarrow \mathbb{D} \cup \{\Delta\}
                                                                                                                                                                                                           # block diagnosis \Delta
15
                       ReportDiag(\Delta)
                                                                                                                                                                                                       # report min diagnosis
16
              foreach i \in \{1, ..., m\}:
17
                       if not SAT(\mathcal{H}_i \cup \mathbb{D}):
                                                                                                                                                                                                  # no more diagnoses exist
18
                                return
19
20 return
```

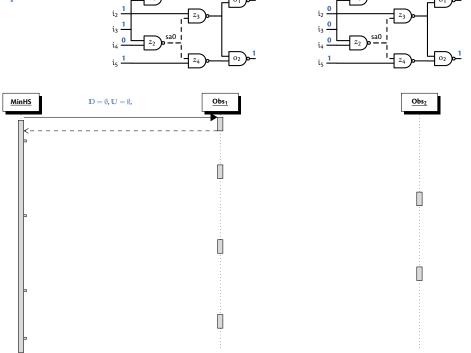
```
input
                         : SD, Obs<sub>1</sub>, ..., Obs<sub>m</sub>
                     : \mathbb{D} = {\Delta_1, \Delta_2 ...}, \mathbb{U} = {\mathcal{U}_1, \mathcal{U}_2 ...}
      output
      (\mathcal{H}_1, \dots, \mathcal{H}_m, \mathcal{S}) \leftarrow \mathsf{Encode}(\mathsf{SD}, \mathsf{Obs}_1, \dots, \mathsf{Obs}_m)
      (\mathbb{D}, \mathbb{U}) \leftarrow (\emptyset, \emptyset)
3 while true:
 5
 6
 7
 8
 9
10
11
12
                                  break
13
14
15
16
               foreach i \in \{1, ..., m\}:
17
                        if not SAT(\mathcal{H}_i \cup \mathbb{D}):
18
19
20 return
```

20 return

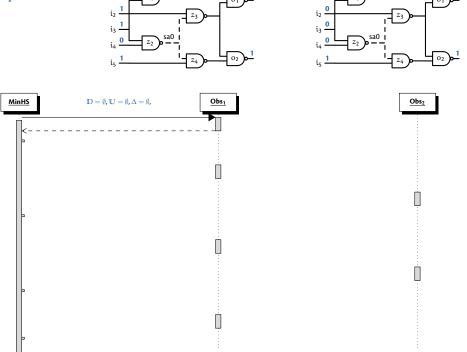
```
1 (\mathcal{H}_1, ..., \mathcal{H}_m, \mathcal{S}) \leftarrow \text{Encode}(SD, Obs_1, ..., Obs_m)
 (\mathbb{D},\mathbb{U}) \leftarrow (\emptyset,\emptyset)
     while true:
               (st, \Delta) \leftarrow MinHS(\mathbb{U}, \mathbb{D})
                                                                                                                                                                                                         # find a min HS of \mathbb{U} s.t. \mathbb{D}
 5
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                        if not st:
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                                 \mathcal{U} \leftarrow \text{Reduce}(\kappa)
                                                                                                                                                                                                   # \mathcal{U} is MUS of \mathcal{H}_i \cup (\mathcal{S} \setminus \Delta)
10
                                 \mathbb{U} \leftarrow \mathbb{U} \cup \{\mathcal{U}\}\
11
12
                                 break
13
                                                                                                                                                                                                        # if the loop was not broken
               else:
14
                        \mathbb{D} \leftarrow \mathbb{D} \cup \{\Delta\}
                                                                                                                                                                                                                      # block diagnosis \Delta
15
16
               foreach i \in \{1, ..., m\}:
17
                        if not SAT(\mathcal{H}_i \cup \mathbb{D}):
18
19
```

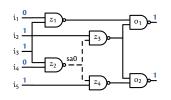
MinHS

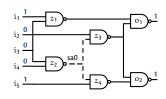


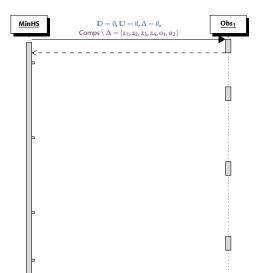


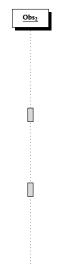
11/14

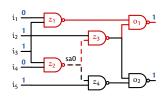


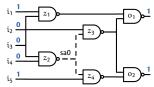


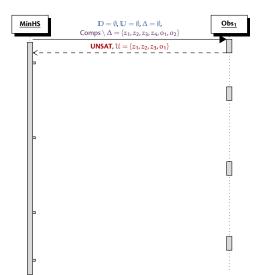


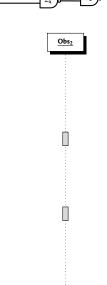


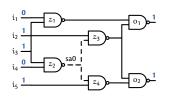


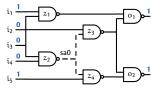


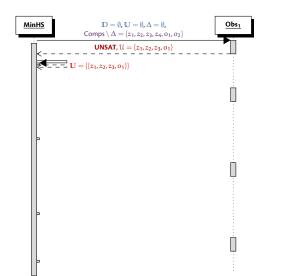


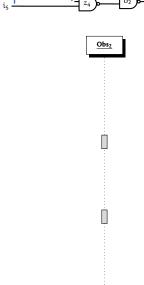


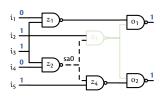


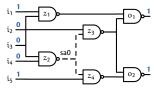


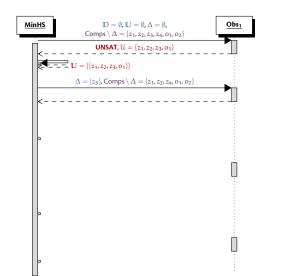




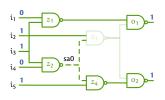


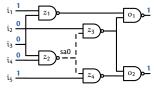


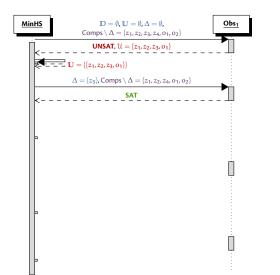


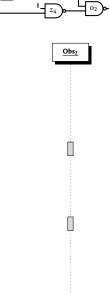


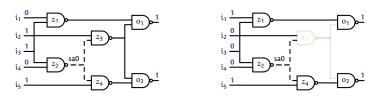


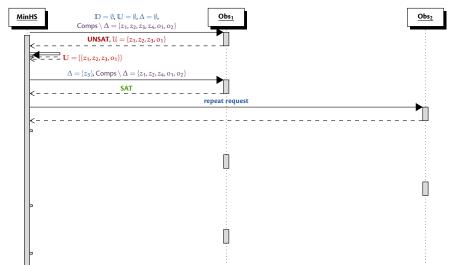


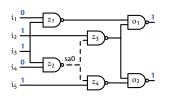


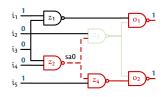


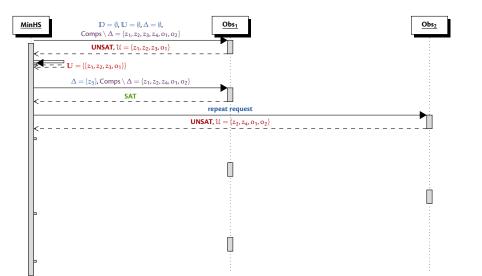


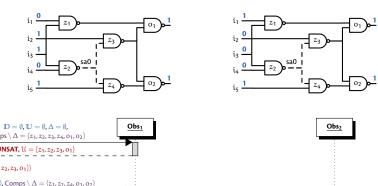


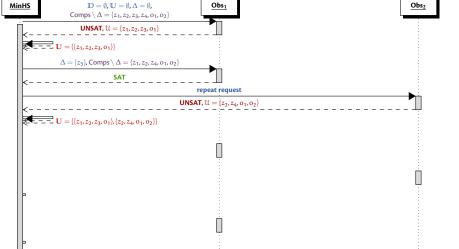


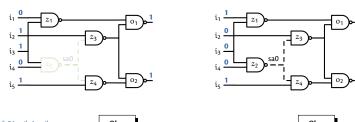


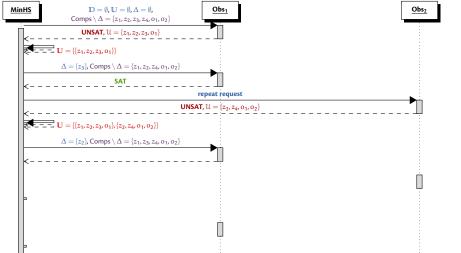


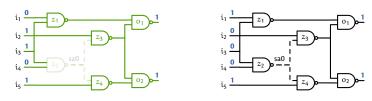


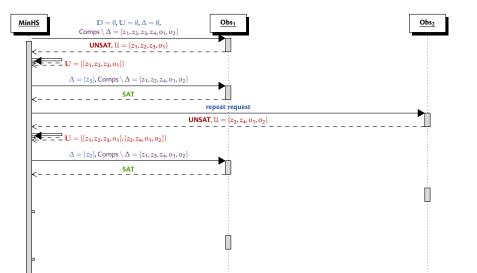


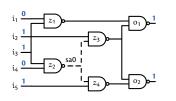


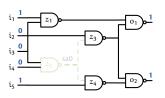


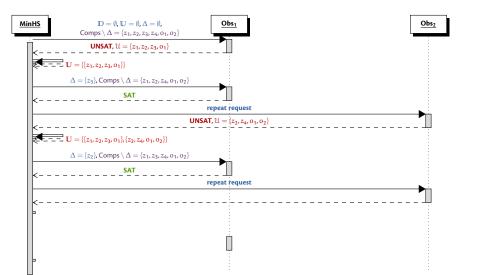


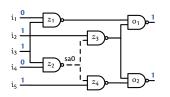


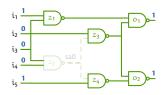


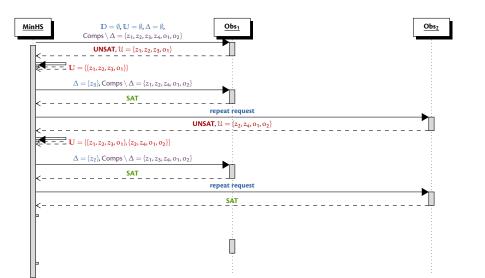






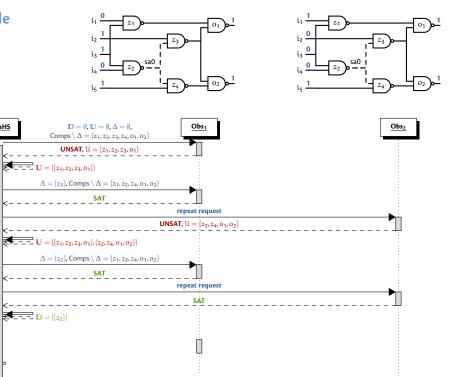






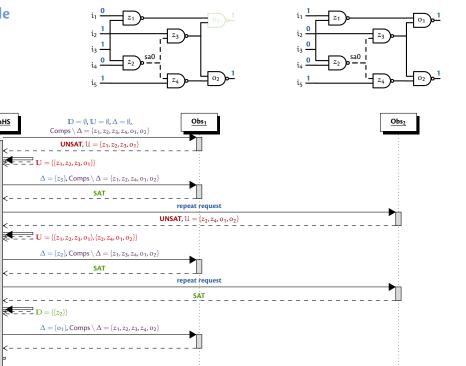
## **HSD** – example

MinHS

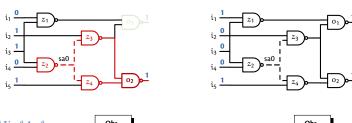


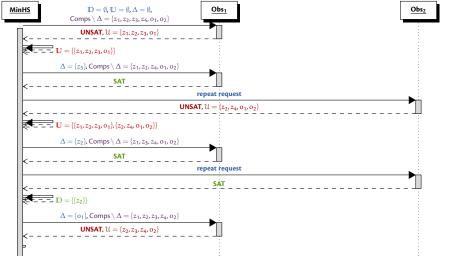
## HSD - example

MinHS



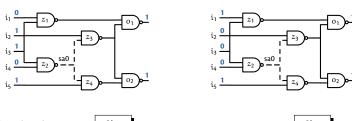
#### HSD - example

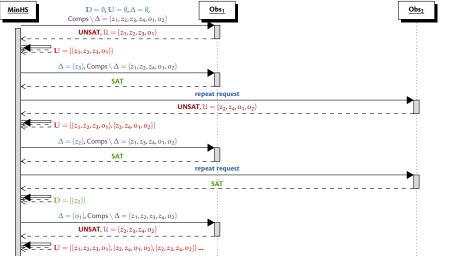




11/14

## HSD - example





11/14

• ISCAS85 circuits + single stuck-at faults

<sup>2</sup>https://github.com/alexeyignatiev/mbd-mobs

- ISCAS85 circuits + single stuck-at faults
  - 100 unique observations

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  - at most 100 aggregated diagnoses

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  - 144 benchmarks in total

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- · approaches tested:
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 $<sup>^2 \</sup>verb|https://github.com/alexeyignatiev/mbd-mobs|$ 

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Intel Xeon E5-2630 2.60GHz with 64GByte RAM

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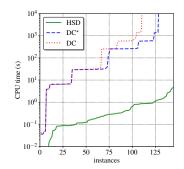
#### · approaches tested:

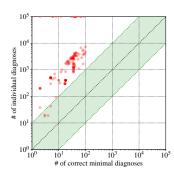
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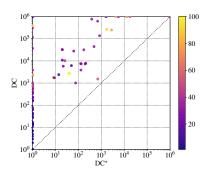
## machine configuration:

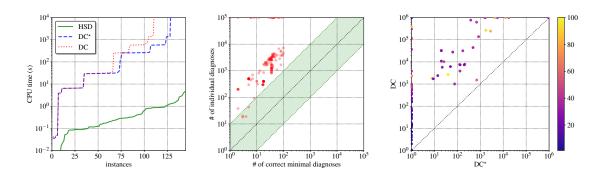
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- running Ubuntu Linux
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- 10GByte memout

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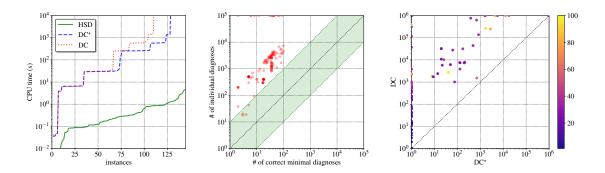




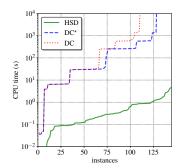


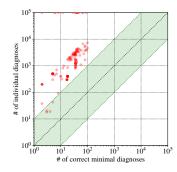


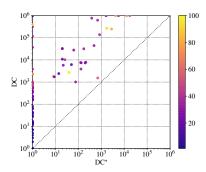
**HSD** — 2–4 orders of magnitude performance improvement



**HSD** — 2–4 orders of magnitude performance improvement  $DC + DC^* - up \text{ to } 10^5 \text{ of individual diagnoses}$ 







$$\mathbf{DC} + \mathbf{DC}^{\star}$$
 — up to  $10^5$  of individual diagnoses

MBD with multiple observations

- MBD with multiple observations
  - optimized DiagCombine

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## further improvements

- MBD with multiple observations
  - · optimized DiagCombine
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  - orders of magnitude performance improvements

- further improvements
- practical deployment

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- further improvements
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  - · software fault localization

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## further improvements

- practical deployment
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  - design debugging

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## further improvements

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- · software fault localization
- design debugging
- · spreadsheet debugging

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#### further improvements

#### practical deployment

- · software fault localization
- · design debugging
- · spreadsheet debugging
- machine learning models?
- etc...

