Maximal Falsifiability: Definitions, Algorithms, and Applications

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Experimental Evaluation of Algorithms for solving problems with combinatorial explosion

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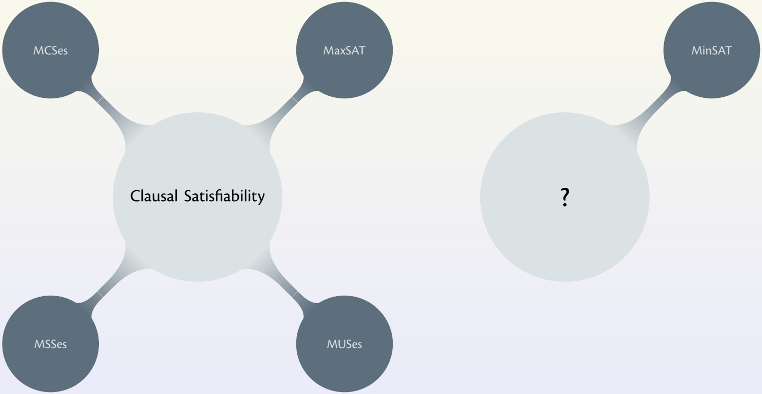
Motivation



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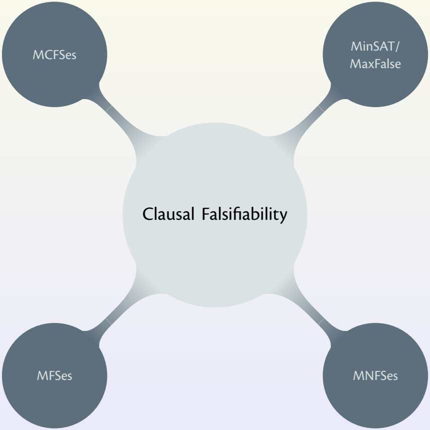
MCFSes
MinSAT/
MaxFalse
Clausal Falsifiability
MFSes
MNFSes

Maximum Falsifiability

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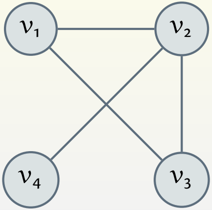
$\mathcal{F} = \mathcal{H} \cup \mathcal{R}$
$\mathcal{H} = \{ x \vee y \vee z \}$
$\mathcal{R} = \{ x, \quad y, \quad z \}$

Only one MNFS $\mathcal{N} = \mathcal{R}, \quad |\mathcal{N}| = 3.$

Connection to Maximal Independent Set



(a) Graph $\mathcal{G}$

$$\mathcal{F} = \left\{ \begin{array}{l} c_1 = x_{v_1v_2} \vee x_{v_1v_3} \\ c_2 = \neg x_{v_1v_2} \vee x_{v_2v_3} \vee x_{v_2v_4} \\ c_3 = \neg x_{v_1v_3} \vee \neg x_{v_2v_3} \\ c_4 = \neg x_{v_2v_4} \end{array} \right\}$$

(b) Set of clauses $\mathcal{F}$ for $\mathcal{G}$

Figure: From maximal independent set to maximal falsifiability

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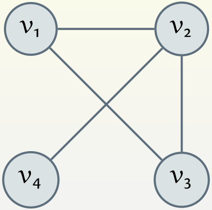

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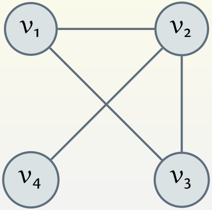

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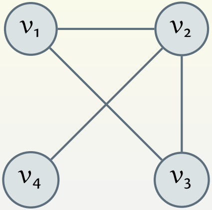

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Enumeration of MNFSes can be done for computing a lower bound on the size of any MCFS $\Rightarrow$ an upper bound for MaxFalse.

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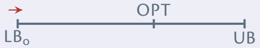

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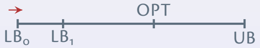

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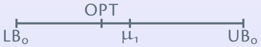

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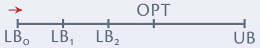

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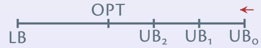
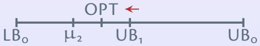


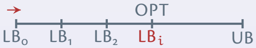
binary search



Iterative MaxFalse algorithms

- similar to iterative MaxSAT
- SAT solver is used as an oracle
- consider formula $\mathcal{F} = \mathcal{H} \cup \mathcal{R}$
- $\forall c_i \in \mathcal{R}$ modify $\mathcal{H}$ and $\mathcal{R}$:
  - $\mathcal{H} \leftarrow \mathcal{H} \cup \{\neg l_{i_j} \vee r_i\} \;\; \forall l_{i_j} \in c_i$
  - $\mathcal{R} \leftarrow \mathcal{R} \setminus \{c_i\}$
- relaxation constraints $\sum_{r_i} r_i \leqslant k$
- varying k using 3 algorithms:

linear search UNSAT/SAT



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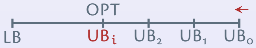
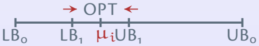
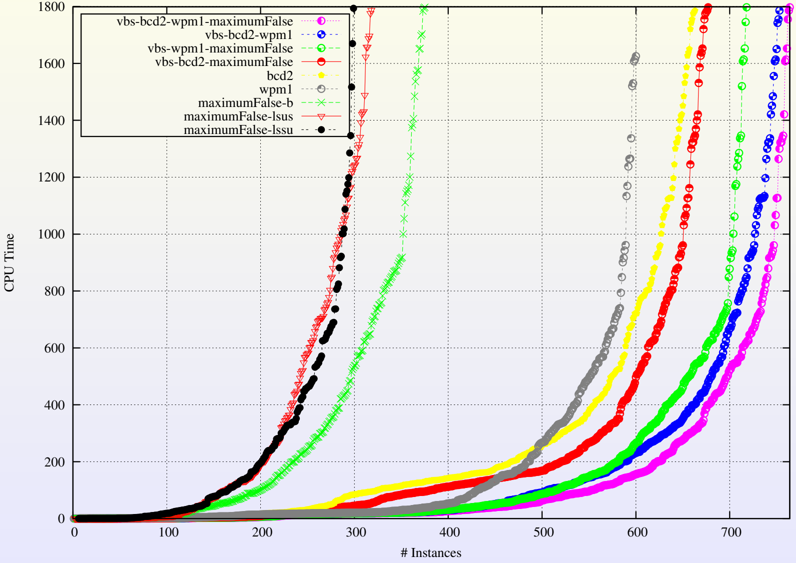


binary search



Performance comparison: MaxFalse for MaxSAT instances



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- More practical applications

Thank you for your attention!