

IHE INSTITUTE FOR WATER EDUCATION
DEPARTMENT OF HYDROLOGY AND WATER RESOURCES

HYDROGEOSTATISTICAL ANALYSIS OF WATER LEVEL FOR THE UPPER BERG CATCHMENT IN SOUTH AFRICA

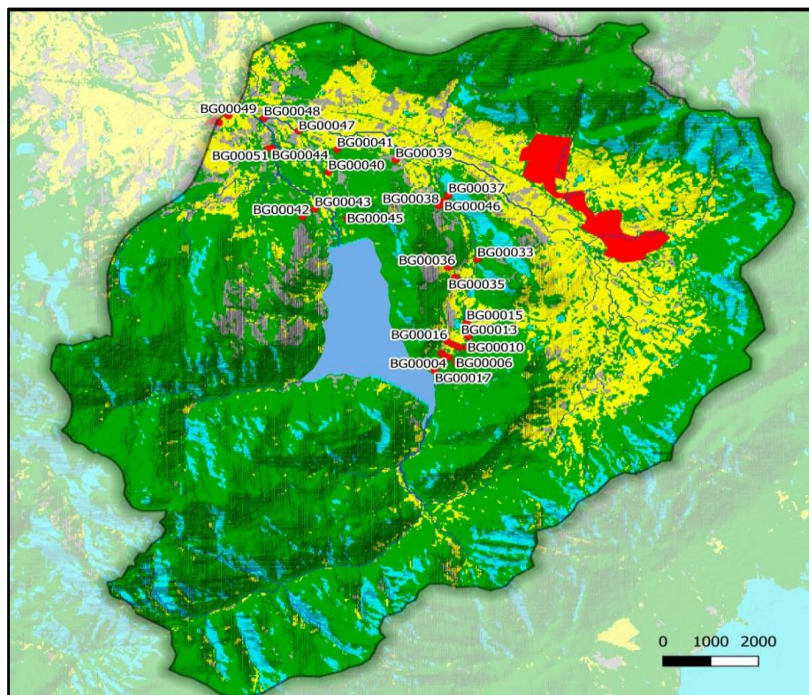
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1.0 Introduction

Semi-arid countries acknowledge groundwater to be the most important and scarce natural resource. There has thus been an objectification for quantity and quality groundwater mandated through policy. One of the main factors negatively affecting groundwater bodies is land use. Throughout history, intense land use activities (such as: damming, agriculture, urbanization, over-abstraction, etc.) climate change have resulted in clear and significant changes and negative impact on the surface and groundwater system (Albhaisi, et al., 2012).

This report aims to understand the trends, changes and possible reasons for groundwater fluctuations through the application of hydrostatistical methods. The focus area is the quaternary catchment G10A of the Berg River Catchment located in the Western Cape Province, South Africa.

1.1 Study area

The Berg River Catchment is approximately 300 km in length with an area of 7715km² - *Figure 1* approximately two thirds of the catchment is agricultural land (Greene, et al., 2011). Moreover, it supplies water for industrial use and is the primary water primary source of water for the city of Cape Town.

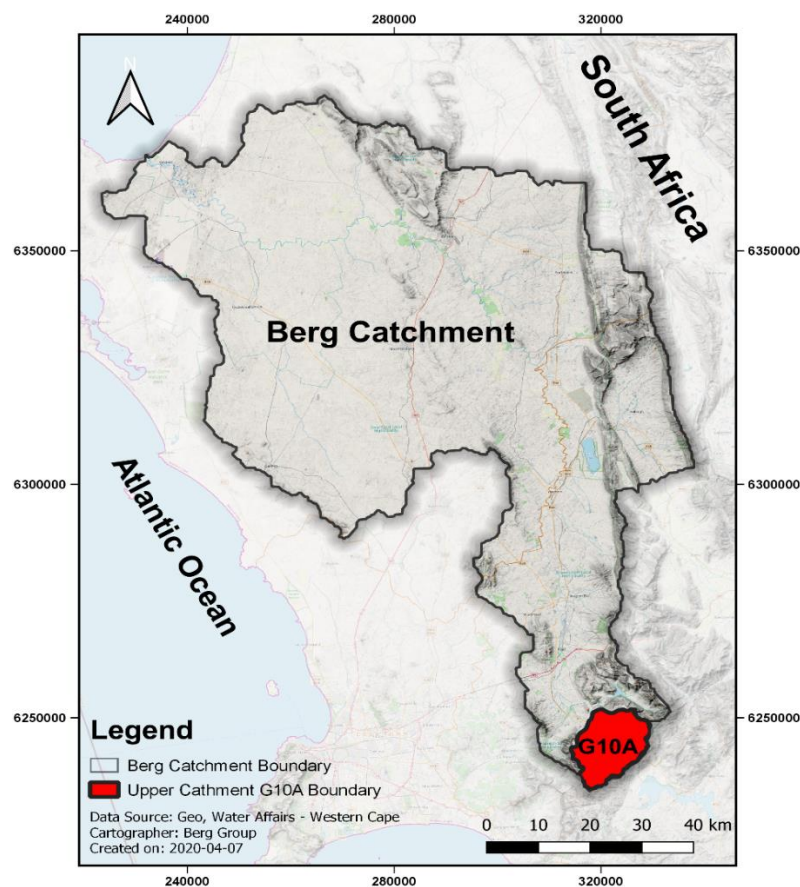


Figure 1: The Berg catchment and quaternary catchment G10A (berg Group, 2020).

The catchment is divided into 12 quaternary catchment starting from the source of the Berg River (G10A) and ending the Atlantic Ocean (G10M). Our main focus – *Figure 2* – has a surface area of 172

km² and encompasses the Berg River Dam which was commissioned in the last quarter of 2007 (Albhaisi, Brendonck and Batelaan, 2012).

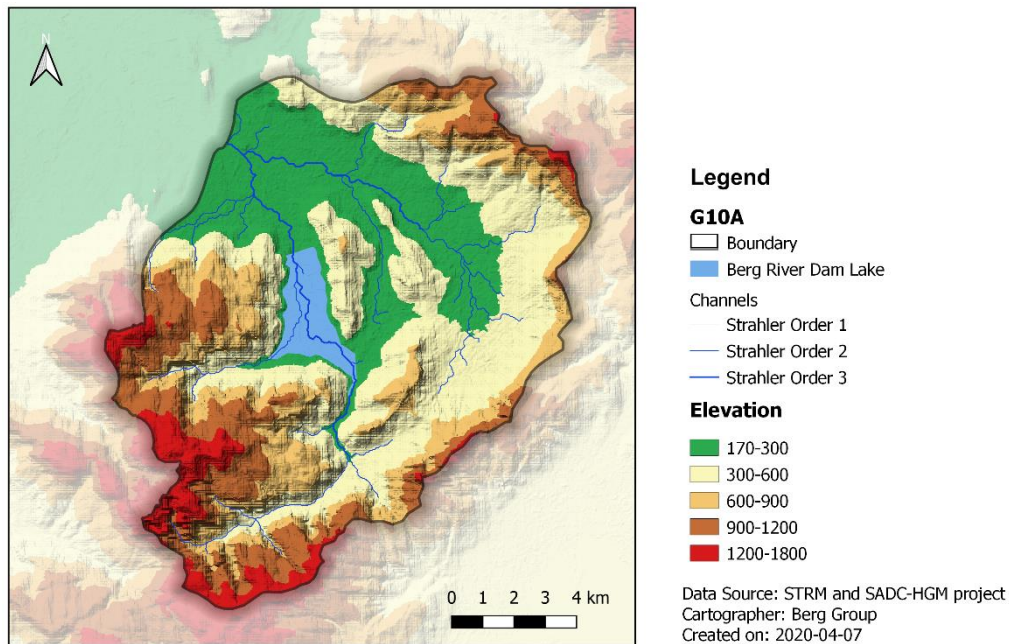


Figure 2 : The quaternary catchment G10A with its relief variation, drainage network as well as the Berg River Dam (Berg Group, 2020).

a. Climate

G10A's climate is that of Mediterranean with annual mean precipitation and potential evaporation of 1063 and 1475 mm/yr respectively. The catchment has a relatively dense drainage network with annual mean runoff of 1015 mm/yr (Albhaisi, Brendonck and Batelaan, 2012).

The area is described as a winter rainfall region with wet, cool winters and dry, warm summers. Mean winter temperature is around 7 °C and snow falls on the mountain peaks annually. The largely montane area has relief ranging from 261 to 1560m and plays a major role in the significant change in micro-climate (Albhaisi, Brendonck and Batelaan, 2012).

b. Geology

The area is comprised of with a sequence of rocks from the Malmesbury Group, Cape Granite Suite, Table Mountain Group (TMG) and younger Cenozoic sediments - Figure 3. The main rock type to outcrop is that of quartzitic sandstones from the TMG and gives the area its steep rugged topography. Moreover, to the north of the study area, are outcrops of phyllite and greywacke shale from the Malmesbury Group. This group have lenses of quartzite schists and limestone (Drakenstein Municipality, n/d).

Erosion resistant outcrops from the Cape Granite Suite moderately cover the study area while fine to medium and coarse grained Cenozoic sediments are present in the river channels of the Berg River.

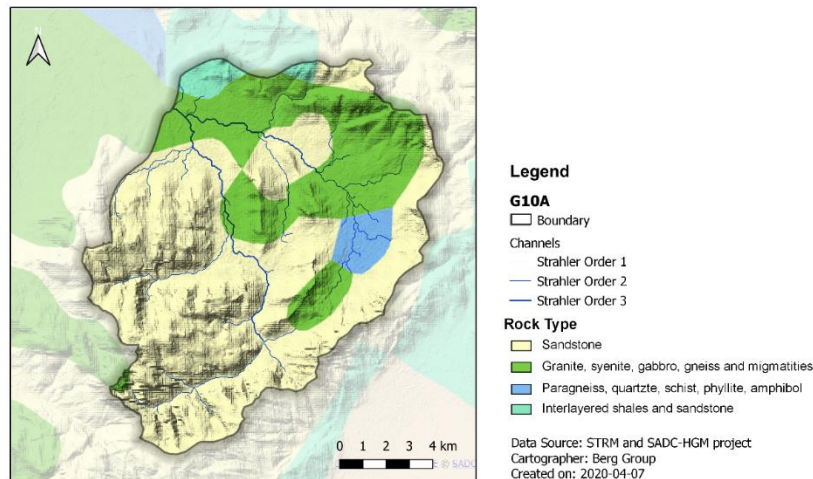


Figure 3: The lithological variation in G10A. (Berg Group, 2020).

The light yellow denotes the fractured sandstones of the Table Mountain Group, while the green and jade colours denotes the Cape Granite Suite and Malmesbury Group (shales) respectively

c. Hydrogeology

Groundwater is mainly stored in the Table Mountain Group (TMG) in G10A and plays a vital role in ensuring baseflow of rivers during dry seasons - Figure 4. It mainly hosts fractured aquifers, however, the Malmesbury Group is highly variable, thus highly variable aquifer conditions also prevail. Additionally, artesian conditions also occur due to the wide range of lithological types (DEADP WC DWA, n/d). The groundwater recharge in the Berg River catchment ranges from 5 Mm³/a up to 36 Mm³/a, with high recharge being from the Mountainous study area. This is again due to the relatively high rainfall conditions (DEADP WC DWA, n/d).

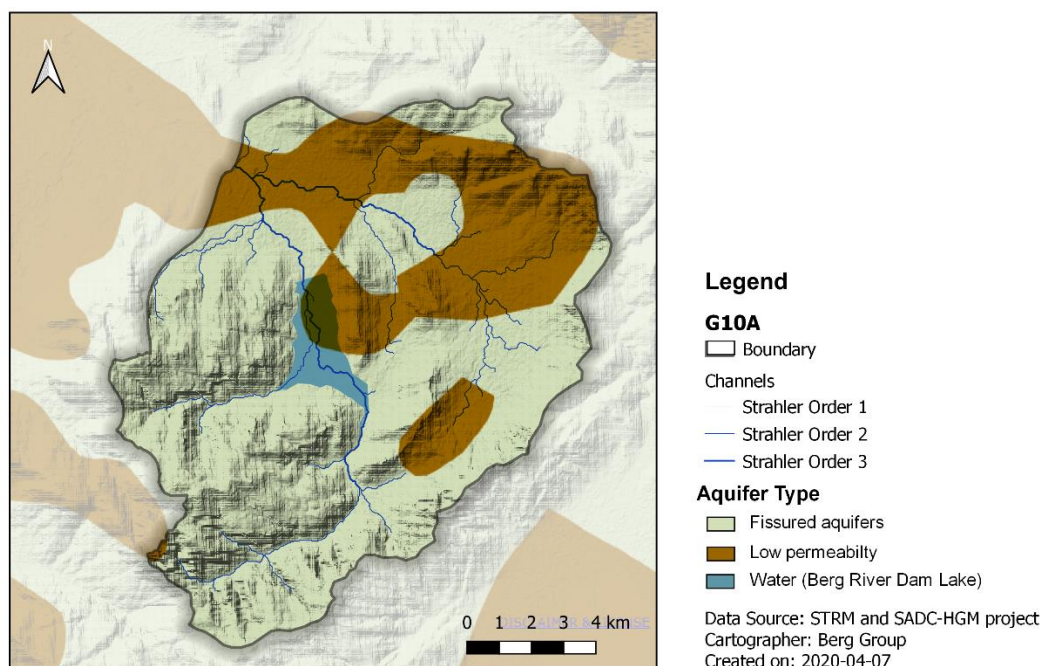


Figure 4: The aquifer types in G10A. (Berg Group, 2020).

The fissured aquifer originates from the Table Mountain Group, thus called the TMG Aquifer. While the brown colour that denotes low permeability represents the Cape Granite Suite Aquifer

1.2 Data analysed

The water level data used for the hydrogeostatistical analysis was from 24 monitoring well in the catchment area with the water level or depth data taken once every month. The data aviled to us was from the year 2004 to 2015. There were some gaps in the data in the months where water levels were not recorded.

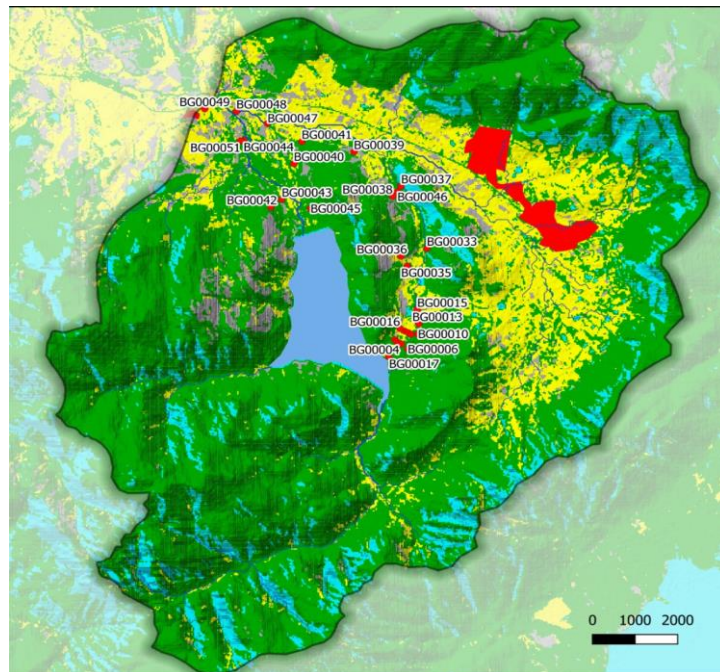


Figure 5: Spatial distribution of existing monitoring wells in the Upper Berg catchment

2. Statistical Analysis

The water depth data for one monitoring well in the recharge area and one in the discharge area were used to do the statistical analysis. The data all was from the year 2004 to 2016 sampled once a month from each monitoring point.

a. Descriptive statistics

The table below depicts the statistical characteristics for the monitoring wells at the recharge zone for well BG 00013 and the discharge zone for well BG 00041

Table 1: Statistical characteristics of the monitoring well at the recharge zone

| Characteristics | <i>Water depth (BG 00013) m</i> | <i>Water depth (BG 00041) m</i> |
|--------------------|-------------------------------------|-------------------------------------|
| Arithmetic Mean | 5.42 | 1.99 |
| Geometric mean | 5.21 | 1.90 |
| Harmonic mean | 5.01 | 1.82 |
| Standard Error | 0.15 | 0.06 |
| Median | 5.25 | 1.92 |
| Mode | 3.76 | 1.32 |
| Standard Deviation | 1.48 | 0.59 |
| Sample Variance | 2.19 | 0.35 |
| Kurtosis | -1.25 | -1.46 |
| Skewness | 0.09 | 0.24 |
| Range | 5.56 | 1.92 |
| Minimum | 2.86 | 1.19 |
| Maximum | 8.42 | 3.11 |
| Sum | 547.33 | 200.69 |
| Count | 101.00 | 101.00 |

For each of the data set in the recharge and discharge zone, arithmetic mean> geometric mean> harmonic mean. The differences between the means and the median, however, is very small and shows that the values in the samples are almost equal. This shows that the wells may be in the same aquifer system.

The standard deviation of the samples for both sets of data shows very little variation of the values about the means of the samples. The standard deviation can therefore be used as a measure of dispersion. (Zhou, 2020)

The coefficient of skewness for both sample sets is greater than zero. This shows that the distributions for the samples for the recharge and discharge zones are both positively skewed. The kurtosis coefficient for both samples are less than 3 and this indicates a flatter distribution compared to the normal distribution.

b. Frequency distribution

Table 2.2 and 2.3 show the frequency distributions for the data set from the recharge and discharge monitoring wells respectively.

Table 2: Frequency distribution of water depth for the samples of the monitoring well at the recharge zone

| <i>Class</i> | <i>Absolute Frequency</i> | <i>Relative frequency %</i> | <i>Cumulative frequency %</i> |
|--------------|-------------------------------|---------------------------------|-----------------------------------|
| 2.0 | 0 | 0% | 0% |
| 2.5 | 0 | 0% | 0% |
| 3.0 | 2 | 2% | 2% |
| 3.5 | 6 | 6% | 8% |
| 4.0 | 14 | 14% | 22% |
| 4.5 | 11 | 11% | 33% |
| 5.0 | 13 | 13% | 46% |
| 5.5 | 8 | 8% | 54% |
| 6.0 | 8 | 8% | 62% |
| 6.5 | 8 | 8% | 70% |
| 7.0 | 10 | 10% | 80% |
| 7.5 | 15 | 15% | 95% |
| 8.0 | 3 | 3% | 98% |
| 8.5 | 2 | 2% | 100% |
| More | 0 | 0% | 100% |

The absolute frequency show that the sample values are distributed in the classes. This explains why the mean and median values are almost the same.

Table 3: Frequency distribution of water depth for the samples of the monitoring well at the discharge zone

| <i>Class</i> | <i>Absolute Frequency</i> | <i>Relative frequency %</i> | <i>Cumulative frequency %</i> |
|--------------|-------------------------------|---------------------------------|-----------------------------------|
| 1.0 | 0 | 0% | 0% |
| 1.2 | 1 | 1% | 1% |
| 1.4 | 29 | 29% | 30% |
| 1.6 | 10 | 10% | 40% |
| 1.8 | 9 | 9% | 49% |
| 2.0 | 5 | 5% | 54% |
| 2.2 | 5 | 5% | 59% |
| 2.4 | 8 | 8% | 67% |
| 2.6 | 15 | 15% | 82% |
| 2.8 | 9 | 9% | 91% |
| 3.0 | 7 | 7% | 98% |
| 3.2 | 2 | 2% | 100% |
| More | 0 | 0% | 100% |

The values are distributed in the classes as shown in the table 2.3. This explains the almost equal values of the means and median.

c. Histogram and cumulative distribution

The figures 2.1 and 2.2 show the graphical representation of the frequency distribution of tables 2.2 and 2.3 respectively.

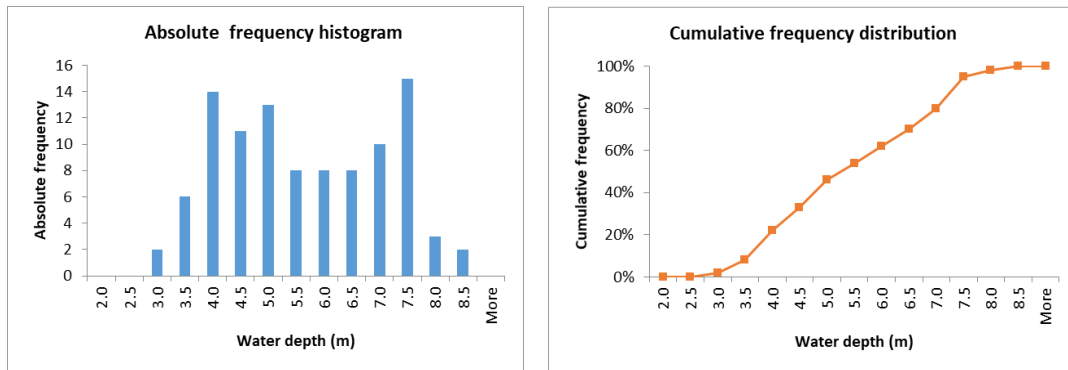


Figure 6: Histogram and cumulative distribution for monitoring well BG00013 at the recharge zone

The relative frequency histogram shows that the distribution of the values in the data set from the recharge zone monitoring well results in a multimodal distribution with several peaks as seen earlier in the value of coefficient of skewness which is 0.09. The cumulative distribution curve shows how the values are evenly distributed in all the classes.

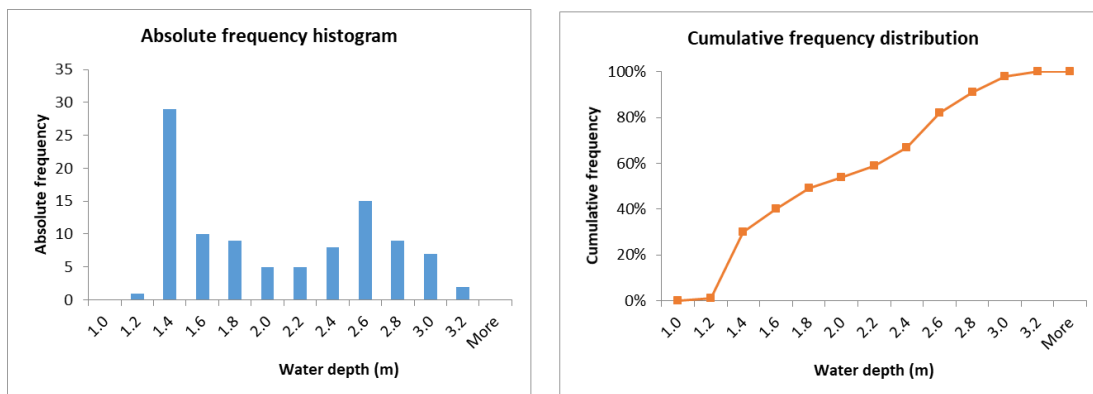


Figure 7: Histogram and cumulative distribution for monitoring well BG00041 at the discharge zone

The absolute frequency histogram shows that the frequency distribution of the sample from the monitoring well at the discharge zone results in a positively-skewed distribution with a right tail. The cumulative distribution curve shows the highest frequency occurs at the 1.4 class.

3. Regression analysis

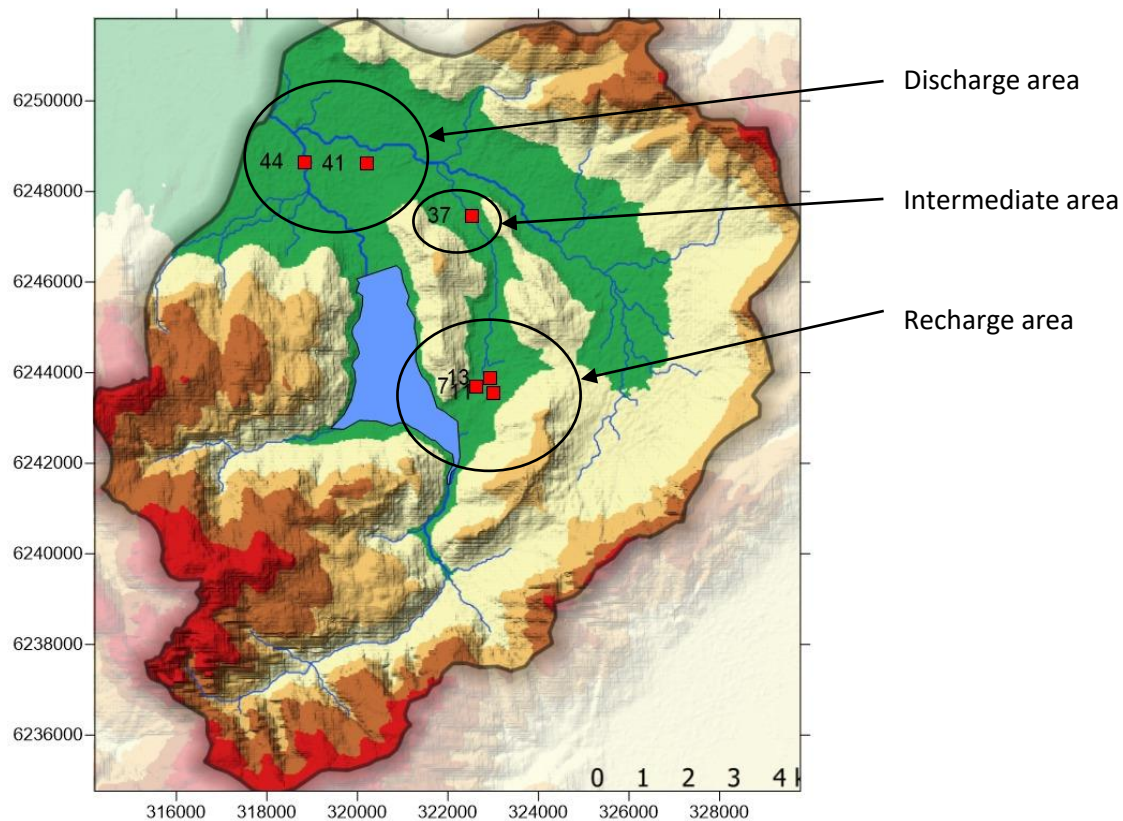
With our set of data, we have carried 4 regression analysis on the ground water level in the recharge area and the one in the discharge area.

To be able to draw consistent conclusions on the correlation between the recharge area and discharge area water levels, we have selected a set of four pairs of wells. Each pair is composed of a well in the recharge area and one in the discharge zone.

The discharge area water level are considered as the dependent variable and thus the one in the recharge area as independent.

This analysis can then be used to predict the water level in the discharge area knowing the one in the recharge area. This can be particularly useful to fill gaps in databases.

For the consistency of the analysis, the **water level** has been considered (and not the water depth). To do so, the water depths have been subtracted from the ground elevation at the well location.



a. Followed procedure

1. Constitution of the pairs: The first step of the regression analysis was to select pairs of well to carry the regression analysis. This has been done in order to determine a general correlation between the water level in the recharge area and the discharge area.
2. Organisation of the water-level data: Once the pairs have been established, the water level measurements dates have to match between the two observations boreholes to be able to

carry the analysis on the same temporal set of data. In this step, all the measurement not done the same month for both well were removed.

3. Regression analysis on excel: The regression analysis has been done using the Regression Data Analysis excel plug-in, selecting the recharge water levels as the X-axis and the discharge area water levels as the Y-axis.
4. Computation of goodness of fit: Once the regression analysis is done, the critical value $F_{\alpha}(v_1, v_2)$ is computed and compared to the $F_{statistical}$ value. The coefficient of determination and the coefficient of correlation are also taken into account to determine whether or not the linear correlation between the water levels in the recharge and discharge area is significant.
5. Result interpretation: once the correlation is confirmed, the slope and intercept of the linear function are compared between all the analysed wells to determine if a general trend between recharge and discharge area can be observed.

b. Constitution of the pairs

Table 4: Assessment and confirmation of correlation in the recharge and discharge area

| BG00007 - Recharge area | BG00037 - Discharge area | BG00013 - Recharge area | BG00041 - Discharge area | BG00011 - Recharge area | BG00041 - Discharge area | BG00011 - recharge area | BG00044 - Discharge area |
|--|--------------------------------|---|--------------------------------|---|--------------------------------|---|--------------------------------|
| Assessment of the correlation between the recharge area and an intermediate zone | | Assessment of the correlation between the recharge and the discharge area | | Confirmation of the correlation between BG00013 and BG00041 carried earlier | | Assessment of the correlation between the recharge and the discharge area | |

c. Organisation of the water-level data

An overview of the data organisation layout is given below for the regression analysis between BG00013 and BG00041:

| Measurement date | BG 00013 - Recharge area (depth) | BG 00041 - Discharge area (Depth) | BG 00013 - Recharge area (water level) | BG 00041 - Discharge area (water level) |
|------------------|--|---|--|---|
| 28/03/2006 | 7.11 | 2.92 | 248.89 | 197.08 |
| 28/04/2006 | 7.43 | 2.77 | 248.57 | 197.23 |
| 26/05/2006 | 6.32 | 1.52 | 249.68 | 198.48 |
| 28/06/2006 | 4.41 | 1.4 | 251.59 | 198.6 |
| 25/07/2006 | 3.54 | 1.35 | 252.46 | 198.65 |
| 30/08/2006 | 3.13 | 1.37 | 252.87 | 198.63 |
| 27/09/2006 | 3.76 | 1.6 | 252.24 | 198.4 |
| 31/10/2006 | 4.89 | 2.12 | 251.11 | 197.88 |
| 29/11/2006 | 4.95 | 1.94 | 251.05 | 198.06 |
| 18/12/2006 | 5.52 | 2.26 | 250.48 | 197.74 |
| 31/01/2007 | 6.26 | 2.55 | 249.74 | 197.45 |
| 30/07/2007 | 2.86 | 1.52 | 253.14 | 198.48 |
| 30/08/2007 | 3.23 | 1.57 | 252.77 | 198.43 |

| | | | | |
|------------|------|------|--------|--------|
| 27/09/2007 | 3.76 | 1.6 | 252.24 | 198.4 |
| 30/10/2007 | 4.52 | 1.75 | 251.48 | 198.25 |
| 23/11/2007 | 4.16 | 1.7 | 251.84 | 198.3 |
| 24/01/2008 | 5.35 | 2.26 | 250.65 | 197.74 |
| . | . | . | . | . |
| . | . | . | . | . |
| . | . | . | . | . |

d. Regression analysis on Excel

The data analysis plug-in is used to carry the regression analysis on Excel. The output gives the regression statistics, the NOVA table (except the critical value $F_{\alpha}(v_1, v_2)$ that has to be computed manually) and the plots of the linear function as well as the residuals.

An overview of the output for the regression analysis between BG00013 and BG00041 is given below:

Regression analysis

SUMMARY OUTPUT

| Regression Statistics | |
|-----------------------|-------------|
| Multiple R | 0.879955098 |
| R Square | 0.774320974 |
| Adjusted R Square | 0.772041388 |
| Standard Error | 0.283421679 |
| Observations | 101 |

ANOVA

| | df | SS | MS | F | p | F0.05(1;99) |
|------------|-----|----------|----------|----------|----------|-------------|
| Regression | 1 | 27.28545 | 27.28545 | 339.6761 | 9.02E-34 | 3.937116911 |
| Residual | 99 | 7.952457 | 0.080328 | | | |
| Total | 100 | 35.23791 | | | | |

| | Coefficients | Standard Error | t Stat | P-value | Lower 95% | Upper 95% | Lower 95.0% | Upper 95.0% |
|---------------------|--------------|----------------|----------|----------|-----------|-------------|-------------|-------------|
| Intercept | 109.478809 | 4.80381 | 22.78999 | 3.6E-41 | 99.94701 | 119.0106109 | 99.94701 | 119.0106 |
| BG 00013 - Recharge | 0.353315694 | 0.01917 | 18.4303 | 9.02E-34 | 0.315278 | 0.39135386 | 0.315278 | 0.391354 |

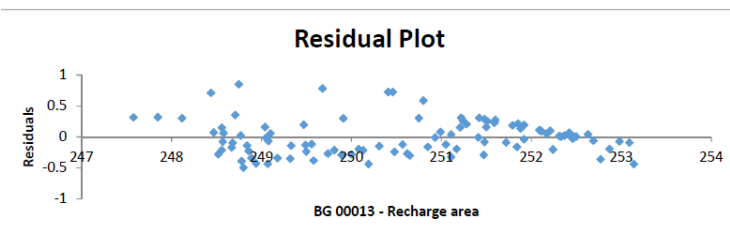
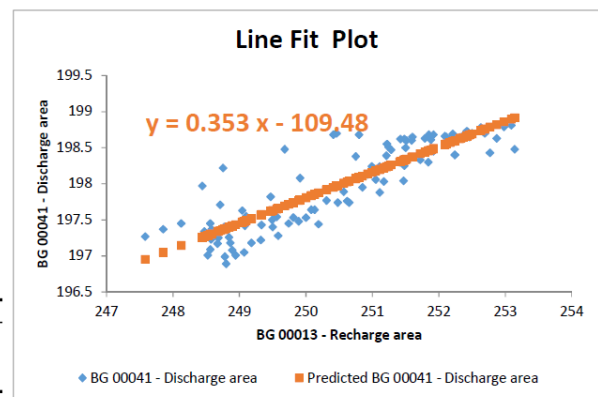


Figure 8: Regression analysis output generated through excel

e. Result interpretation

Table 5: Regression analysis results

| | X | Y | Elevation | No. of simultaneous measurement n | Correlation coefficient R | R ² Value R ² | Standard error | Fstatistic | F _{0.05(1;n-2)} | Regression model intercept b ₀ | Regression model slope b ₁ |
|--------------------------|----------|----------|-----------|--------------------------------------|------------------------------|--|----------------|------------|--------------------------|--|--|
| BG00007 - Recharge area | -33.9324 | 19.08084 | 253.9 | 83 | 0.75 | 0.57 | 0.26 | 106.6 | 3.96 | 172.964 | 0.21 |
| BG00037 - Discharge area | -33.8984 | 19.08068 | 227 | | | | | | | | |
| BG00013 - Recharge area | -33.9307 | 19.08442 | 256 | 101 | 0.88 | 0.77 | 0.28 | 339.7 | 3.94 | 109.479 | 0.353 |
| BG00041 - Discharge area | -33.8876 | 19.05572 | 200 | | | | | | | | |
| BG00011 - Recharge area | -33.9336 | 19.08505 | 263.5 | 101 | 0.88 | 0.77 | 0.28 | 340.6 | 3.94 | 166.079 | 0.125 |
| BG00041 - Discharge area | -33.8876 | 19.05572 | 200 | | | | | | | | |
| BG00011 - recharge area | -33.9336 | 19.08505 | 263.5 | 116 | 0.7 | 0.49 | 0.28 | 109.2 | 3.92 | 155.086 | 0.064 |
| BG00044 - Discharge area | -33.887 | 19.0409 | 174 | | | | | | | | |

The table above show the results of the regression analysis carried on the 4 pairs of wells. It can be seen that all of the linear regressions are significant as $F_{\alpha}(v_1, v_2) < F_{statistic}$ for all the pairs. The good correlation coefficients R (between 0.7 and 0.9) as well as quite good coefficients of determination R² (between 0.5 and 0.8) confirm this statement.

However, no explicit general trend can be noted in the correlation between the recharge area and the discharge area water levels. Indeed, the regression coefficients b₀ and b₁ are relatively scattered, considering how low the slope values are.

Therefore, no general linear function can be obtained for the whole recharge and discharge area using regression analysis. Only pairs of well or smaller areas can be correlated, which still constitute a handy analysis to predict the water level in a well in the discharge area, knowing the one in a well in the recharge area.

4. Time Series Analysis and Trend Detection

Dynamic variables of the hydrology and hydrogeology change with reference to time and space. The spatial and temporal analysis of the variables gives a clear understanding of their behavioural change. These variables, spatially, can be followed as a stochastic process in a temporal function. In other words, it is a function of time and it is the outcome of the probability trial. Thus, the groundwater time series can be represented and analysed with a trend using periodic components. In the case of our study area, the time-series analysis is performed on some collected samples at a certain time.

If the statistical properties of the time series do change with the absolute time then it is a non-stationary or evolutive time series.

The analysis of well water depths is conducted in consideration of the recharge and discharge area. The figure 9 is a location map of the time series well analysis. The trend analysis has very much influenced by human activities in the area. The construction of the dam has been an important reason for the study area which affected the well water depths. All the samples collected monthly once from 2004 to 2016.



Figure 9: Location of the wells in recharge and discharge area used for time series analysis

a. Trend analysis – Step trend

As mentioned earlier, the dam was constructed in 2007 and it is believed that it has had an important impact on the water level. The first time series analysis we carried to determine the impact of the dam is a step trend analysis.

To do so, 3 wells were selected:

- BG00017 in the upper recharge area
- BG00034 in the intermediate recharge area
- BG00041 in the discharge area

For those three wells, the data has been separated in two periods: Before the dam construction and after the dam construction. The step trend between those two periods are then analysed.

The selected wells and the dataset organisation will allow us to analyse the impact of the dam both spatially and in time.

To determine the significance of the step trend, a hypothesis test using the following t statistic is used.

$$t_{statistic} = \frac{|\bar{x}_1 - \bar{x}_2|}{2s_p/\sqrt{n}}$$

With:

\bar{x}_1, \bar{x}_2 : Mean process value for the first and second periods

s_p : Standard deviation of residual

n : Number of measurement

The $t_{statistic}$ value is then compared to the critical value $t_{\alpha/2}(n - 2)$ for a significance level $\alpha = 5\%$. If $t_{statistic} > t_{\alpha/2}(n - 2)$, the step trend hypothesis is accepted and the step trend is significant.

Results

BG00017 – Upper recharge area

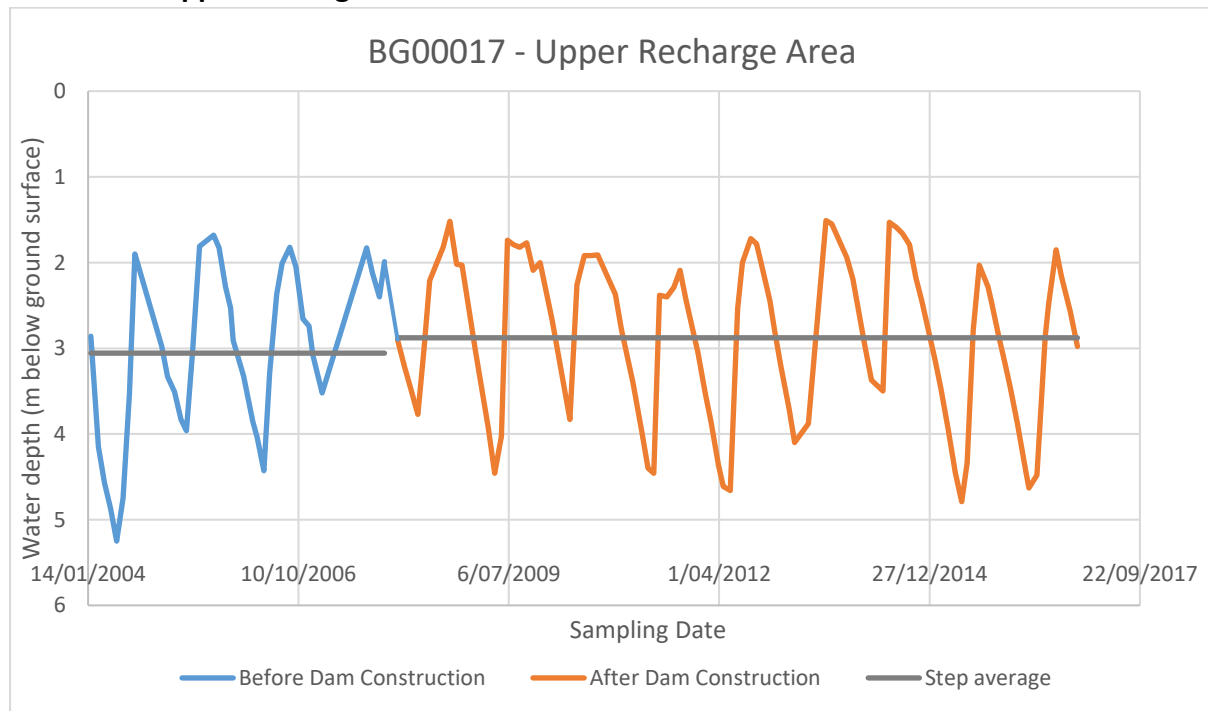


Figure 10: Step trend analysis – BG00017 – Upper Recharge area

Table 6: BG00017 hypothesis test

| Well | n | \bar{x}_1 | \bar{x}_2 | s_p | $t_{\text{statistic}}$ | $t_{0.025}(n-2)$ |
|---------|-----|-------------|-------------|-------|------------------------|------------------|
| BG00017 | 128 | 3.06 | 2.88 | 1.00 | 1.02 | 1.98 |

The first step trend analysis was carried on the well BG00017 located on the upper recharge area, upstream of the dam. The hypothesis test shows that the step trend is not significant at this location between the periods prior to and following the dam construction.

BG00034 – Intermediate recharge area

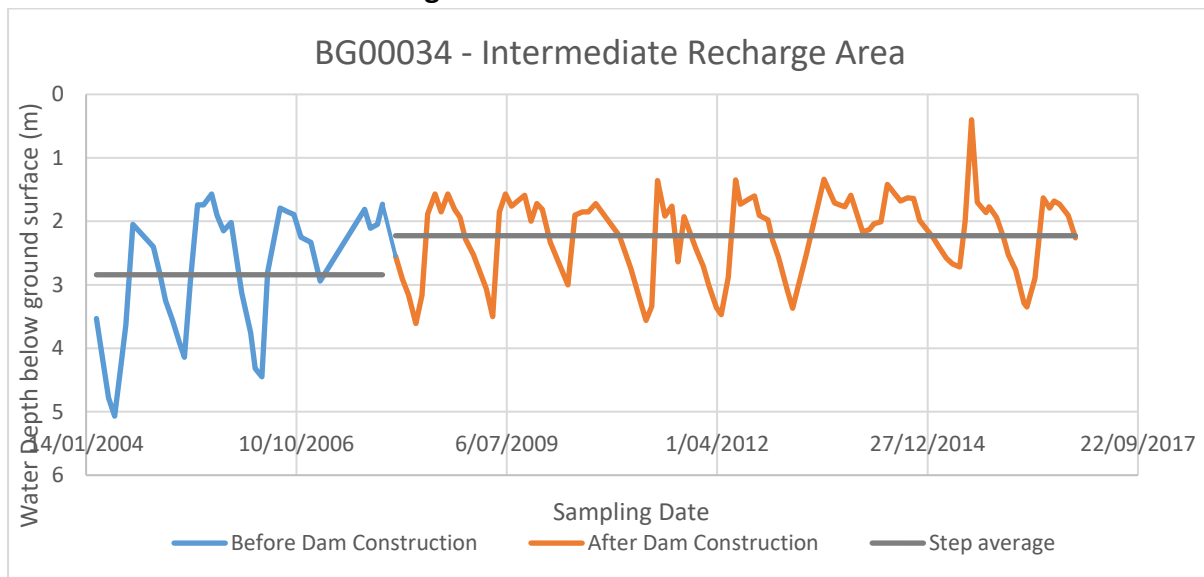


Figure 11: Step trend analysis – BG00034 – Intermediate Recharge area

Table 7: BG00034 hypothesis test

| Well | n | \bar{x}_1 | \bar{x}_2 | s_p | $t_{\text{statistic}}$ | $t_{0.025}(n-2)$ |
|---------|-----|-------------|-------------|-------|------------------------|------------------|
| BG00034 | 128 | 2.84 | 2.23 | 0.77 | 4.53 | 1.98 |

For the second analysis, the hypothesis test reveals a significant step trend as $t_{\text{statistic}} > t_{0.025}(n - 2)$. On the graph 11 the step can be seen as a decrease in the water depth – i.e. an increase in the water level – after the dam construction. This observation led to the conclusion that the intermediate area has been influenced by the dam construction and the water levels have increased in this area.

BG00041 – Discharge area

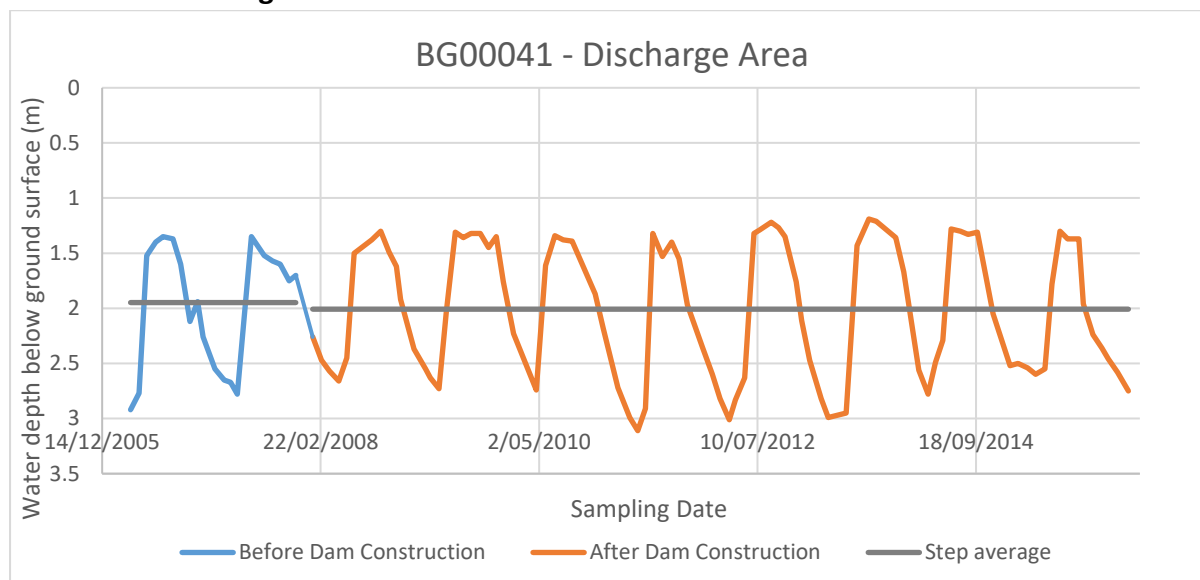


Figure 12: Step trend analysis – BG00041 – Intermediate Recharge area

Table 8: BG00041 hypothesis test

| Well | n | \bar{x}_1 | \bar{x}_2 | s_p | $t_{\text{statistic}}$ | $t_{0.025(n-2)}$ |
|---------|-----|-------------|-------------|-------|------------------------|------------------|
| BG00041 | 108 | 1.95 | 2.01 | 0.60 | 0.52 | 1.98 |

As for the well BG00017 in the upper recharge area, no significant step trend can be noticed in the discharge area between the period preceding the dam construction and the one following.

Results interpretation

Table 9: Step trend analysis results interpretation

| Well | n | \bar{x}_1 | \bar{x}_2 | s_p | $t_{\text{statistic}}$ | $t_{0.025(n-2)}$ |
|---------------------------------|-----|-------------|-------------|-------|------------------------|------------------|
| BG00017 - Upper recharge area | 128 | 3.06 | 2.88 | 1.00 | 1.02 | 1.98 |
| BG00034 - Interm. Recharge area | 128 | 2.84 | 2.23 | 0.77 | 4.53 | 1.98 |
| BG00041 - Discharge area | 108 | 1.95 | 2.01 | 0.60 | 0.52 | 1.98 |

The step trend analysis we carried on the three wells allowed us to conclude that only the intermediate recharge area, adjacent to the dam reservoir, is affected by the dam construction. The mean groundwater level in this area has increased by 0.6 m approximately. These conclusions are coherent considering that the dam has led to an increase in the surface water level and the increased gradient led to more recharge in the adjacent aquifer.

For the wells with no significant step trend, a complementary linear trend analysis has been carried to determine the groundwater depths evolution over time.

b. Trend analysis – linear trend

A series with a linear trend can be approximated by a classical linear regression model as:

$$h_t = b_0 + b_1 t$$

With h_t the groundwater depth at time t ; b_1 and b_0 the estimates of respectively the trend magnitude and the process base level.

To assess the linear trend significance, a hypothesis against the slope, using the t statistic value is carried. The manual calculation of the t statistic value is as follow:

$$t_{stat,man} = \frac{|b_1|}{\sqrt{12} \times s_l / \sqrt{n(n+1)(n-1)}}$$

With:

s_l : Standard deviation of residuals

n : Number of measurement

t statistic follows a Student t distribution with a degree of freedom of $n-2$. To use this relation, the assumption is made that the time interval is the same in the measured data.

However

The $t_{statistic}$ value is then compared to the critical value $t_{\alpha/2}(n-2)$ for a significance level $\alpha = 5\%$. If $t_{statistic} > t_{\alpha/2}(n-2)$ the non-zero slope value hypothesis can be accepted, in other words, we can say that the linear trend is significant.

The determination of the linear model as well has been done using the regression analysis function of the data analysis excel add-in. The output also contain a calculated value for the t statistic that take into account the non-consistency in the measurement dates (non-consistency in the dataset). Thus we are using this value $t_{stat,comp}$ to assess the significance of the linear relation.

For our analysis, 4 wells have been used to determine the water depth - and thus water level – evolution spatially and over time:

- BG00017 in the upper recharge area
- BG00034 in the intermediate recharge area
- BG00041 in the discharge area
- BG00044 in the discharge area

BG00017 – Upper Recharge Area

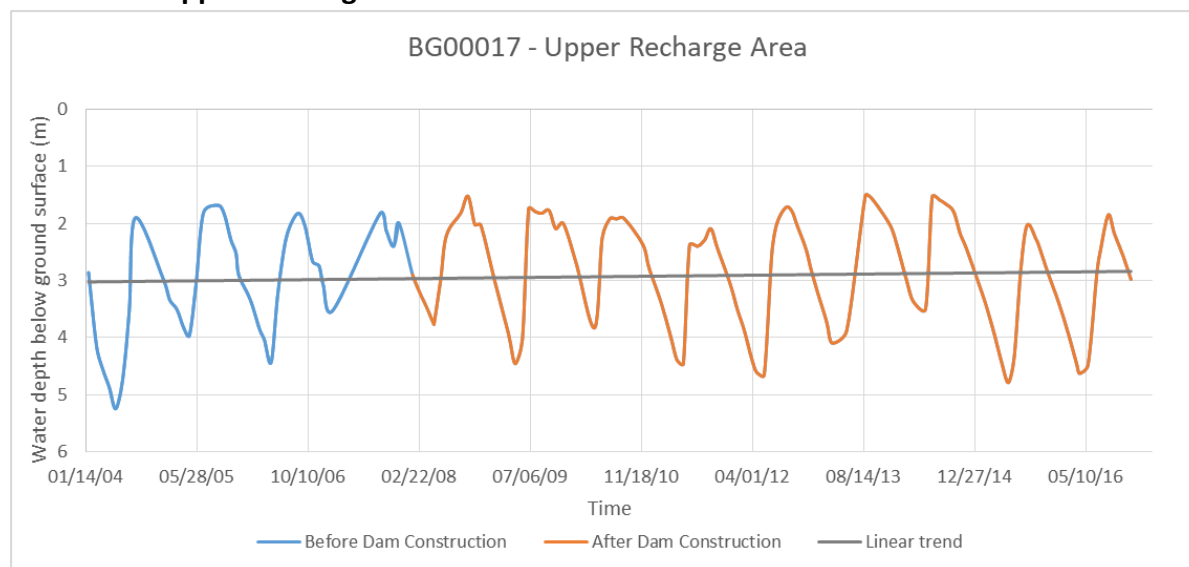


Figure 13: Linear trend analysis – BG00017 – Upper Recharge area

Table 10: BG00017 hypothesis test

| Well | n | Slope - b_1 | Intercept b_0 | s_l | $t_{\text{statistic}}$ | $t_{\text{stat, comp}}$ | $t_{0.025(n-2)}$ |
|---------|-----|---------------|-----------------|-------|------------------------|-------------------------|------------------|
| BG00017 | 129 | -3.93E-05 | 4.52 | 0.97 | 0.0172 | 0.64 | 1.98 |

For this well located in the upper part of the recharge area, the hypothesis test shows that the linear trend is not significant. However a slight decrease in the water depth (i.e. increase in the water level) can be seen from the plot, which can be attributed to the increased recharge due to the dam.

BG00034 – Intermediate Recharge Area

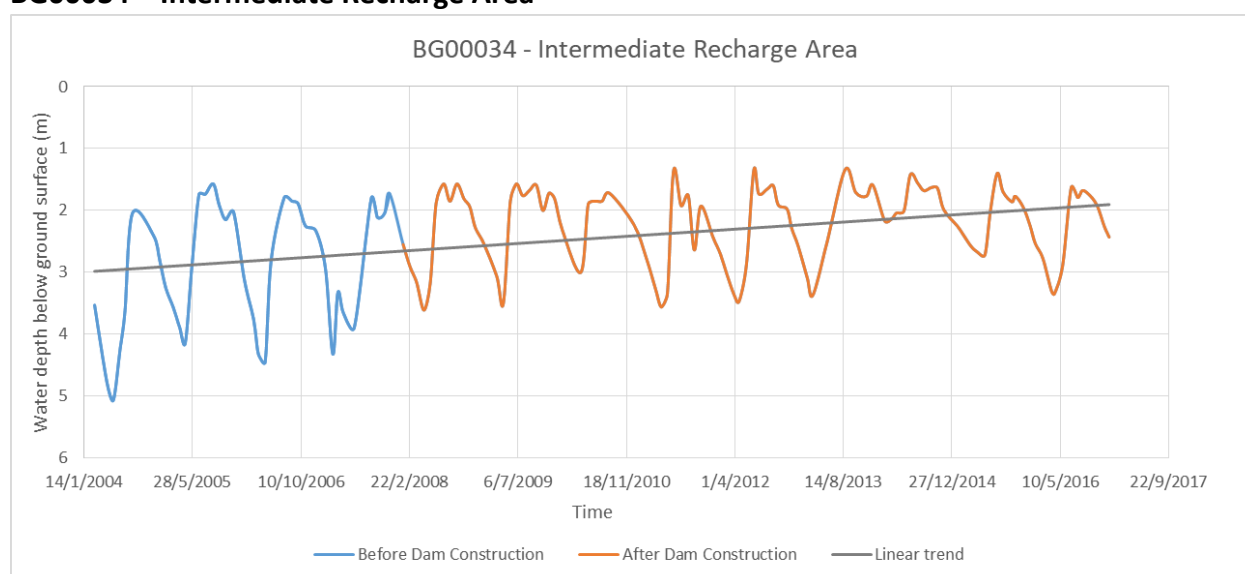


Figure 14: Linear trend analysis – BG00034 – Intermediate Recharge area

Table 11: BG00034 hypothesis test

| Well | n | Slope - b_1 | Intercept b_0 | s_l | $t_{stat, man}$ | $t_{stat, comp}$ | $t_{0025}(n-2)$ |
|---------|-----|---------------|-----------------|-------|-----------------|------------------|-----------------|
| BG00034 | 132 | -2.31E-04 | 11.76 | 0.76 | 0.1335 | 4.83 | 1.98 |

At this location which corresponds to the one most affected by the dam construction (see. previous section – Step trend analysis), the slope is steeper than for BG00017. It can be seen from the regression analysis that the linear model decrease from 3m below ground surface to less than 2 m between 2004 and 2016. The increase in water levels correlate the conclusion drew in the previous section. The hypothesis test also confirm the increasing linear trend in the groundwater level at this location as $t_{stat, comp} > t_{critical}$ standing for a significant linear trend.

BG00041 and BG00044 – Discharge area

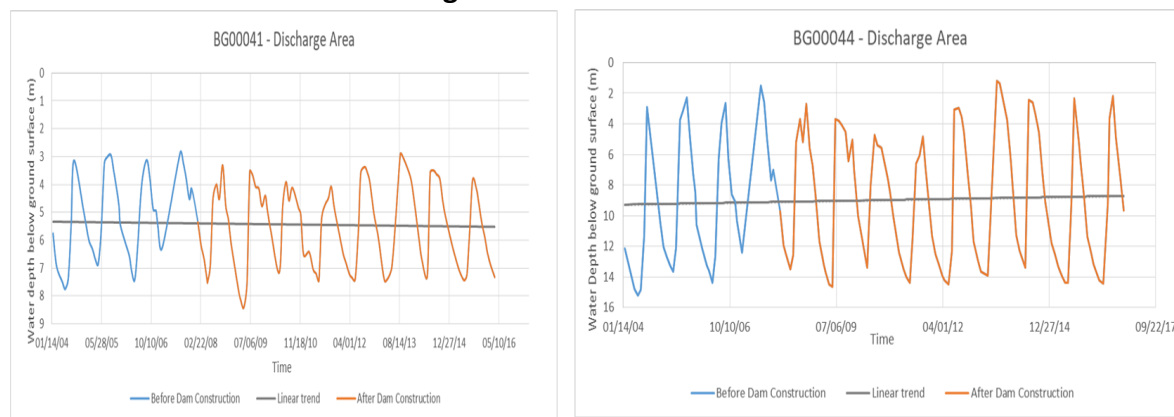


Figure 15: Linear trend analysis- BG00041 and BG00044- Discharge area

Table 12: BG00041 and BG00044 hypothesis test

| Well | n | Slope - b_1 | Intercept b_0 | s_l | $t_{statistic}$ | $t_{stat, comp}$ | $t_{0025}(n-2)$ |
|---------|-----|---------------|-----------------|-------|-----------------|------------------|-----------------|
| BG00041 | 127 | 4.14E-05 | 3.76 | 1.48 | 0.0116 | 0.41 | 1.98 |
| BG00044 | 133 | -1.23E-04 | 13.94 | 4.20 | 0.0129 | 0.45 | 1.98 |

Both locations downstream of the catchment show a really small slope obtained by regression analysis. This is confirmed by the low $t_{stat, comp}$ values, inferior to the $t_{critical}$ value. It can therefore be concluded that at the discharge location, the groundwater depth, and thus the groundwater level remains rather constant.

Results interpretation

Table 13: Linear trend analysis results interpretation

| Well | n | Slope - b_1 | Intercept b_0 | s_l | $t_{statistic}$ | $t_{stat, comp}$ | $t_{0025}(n-2)$ |
|---------|-----|---------------|-----------------|-------|-----------------|------------------|-----------------|
| BG00017 | 129 | -3.93E-05 | 4.52 | 0.97 | 0.0172 | 0.64 | 1.98 |
| BG00034 | 132 | -2.31E-04 | 11.76 | 0.76 | 0.1335 | 4.83 | 1.98 |
| BG00041 | 127 | 4.14E-05 | 3.76 | 1.48 | 0.0116 | 0.41 | 1.98 |
| BG00044 | 133 | -1.23E-04 | 13.94 | 4.20 | 0.0129 | 0.45 | 1.98 |

Overall, the linear trend analysis allows us to conclude that groundwater level remains rather constant during the whole observation period (2004-2016) with low slope and $t_{stat,comp}$ values. An exception can be made for the BG00034 observation well adjacent to the dam as this is the well for which the higher slope and $t_{stat,comp}$ values are observed leading to a significant linear trend.

Despite the increasing pressure on the groundwater in the discharge area due to population and agriculture growth, a stable trend in the water level in the discharge area is observed over the observation period. It can thus be hypothesised that the construction of the dam had a suitable effect on the groundwater levels that was expected to decrease throughout the years due to over abstraction without the dam construction.

A final remark can be made on the differences observed between the $t_{statistic}$ values calculated manually and the true $t_{stat,comp}$ value obtained via the regression analysis tool. The non-consistency in the data set lead to high error in the manual calculation compared to the computed one. This observation reflect the importance of the dataset management and consistency.

5. Variogram analysis and Kriging standard deviation

a. Fit of variograms

The plot of experimental variogram of the groundwater levels in the berg (G10A) catchment increases at the origin and, then, irregularly fluctuates with a separation distance of 500. Thus, the linear variogram model with the nugget effect was used to fit the first five observation values, as shown in fig. 5.1. According to the figure, the slope of the linear line is 0.18 and the nugget effect C is equal to 2. The Cross-validation was automatically calculated by the Surfer 11, and the mean error calculated is 0.032. Moreover, the standard deviation of the residuals is similar to that of the measurement values with 3.08 meters and 3.23 meters respectively, showing a reasonably good fit of the estimated variogram model.

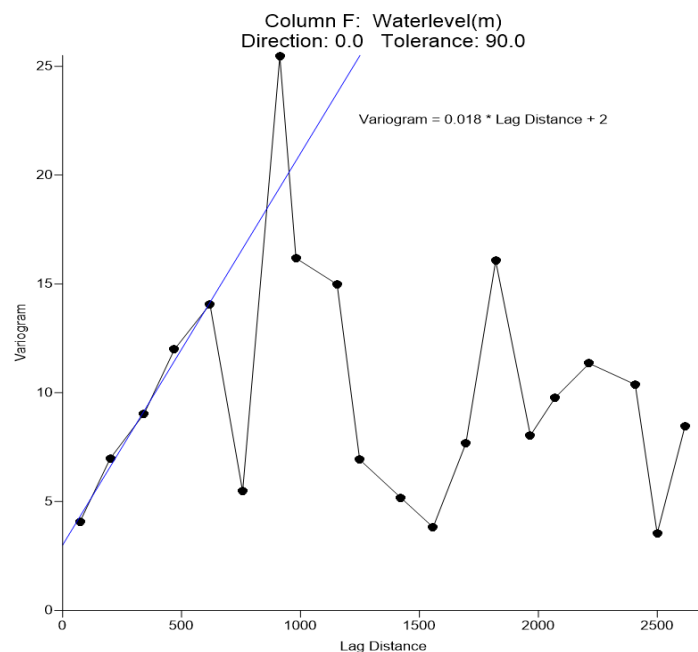


Figure 16: The fit of the variogram for the values of the groundwater level

b. Kriging estimation

The kriging estimation of the groundwater levels can be analyzed on the area where don't have the observation data, after fitting the variogram. The default shape of contour map of the estimation values in the Surfer 11 is a regular rectangular. In this case study, the contour map was blanked by the boundary of the mountain to reduce errors, as shown in fig. 5.2. The contour map shows clearly that the south and west area closed to the mountain have relatively high values of the groundwater levels (7.5 meters to 12 meters). To the north further, the groundwater levels gradually decrease to 2.5 meters.

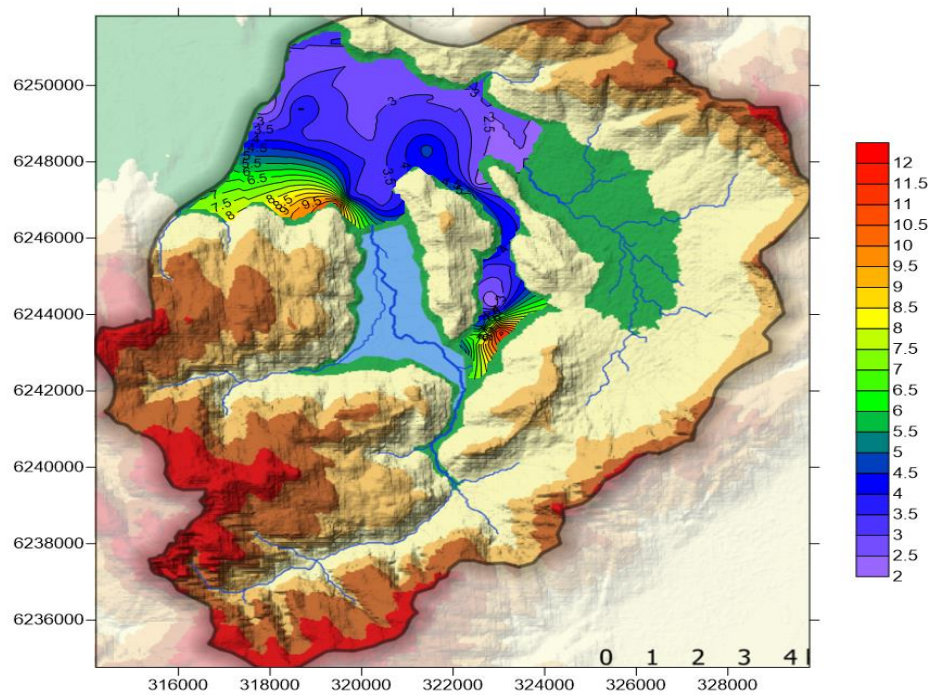


Figure 17: Contour map of the estimated groundwater levels in the Berg (G10A) area

One of the advantages of the Kriging estimation is that the variance of estimation errors can be displayed by a contour map to indicate the accuracy of the estimation. Figure 5.3 shows the contour map of estimation error variance for groundwater levels with the plot of the boreholes in the Berg (G10A) area. It is clear that the network density of the boreholes is too high in the dam area, especially the south side, and the estimation errors of west and north areas are high without any measurements.

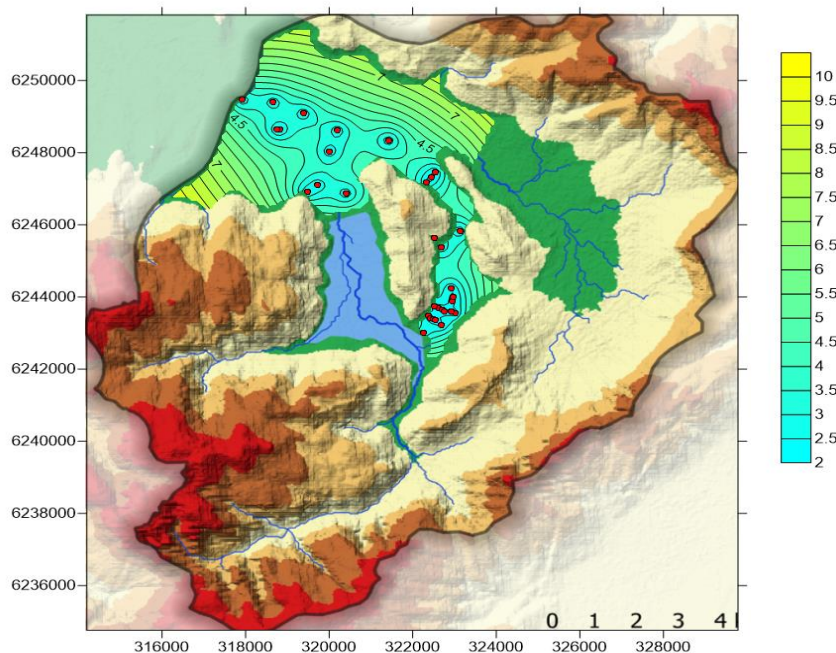


Figure 18: Contour map of the estimated errors (meter) in the Berg (G10A) area

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