Computation Tree Logic (CTL)

In CTL, there are the temporal operators of LTL, but there are also path quantifiers. These path quantifiers are used to describe the branching structure of a computation tree.

There are two path quantifiers:

- **A** means for all computation paths
- **E** means for some computation paths

These are used to describe the behaviour of the system from a particular state.

In CTL, we talk about a formula being true of a state rather than a path. (Then we check that it is true for all initial states of the system.)

CTL Syntax

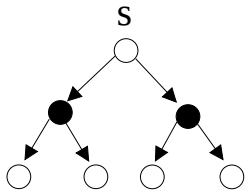
If p is an atomic proposition, and f_1 and f_2 are CTL formulae, then the set of CTL formulae consists of:

1. p
2. $\neg f_1$, $f_1 \wedge f_2$, $f_1 \vee f_2$, $f_1 \Rightarrow f_2$
3. $\mathbf{AX} f_1$, $\mathbf{EX} f_1$
4. $\mathbf{AG} f_1$, $\mathbf{EG} f_1$
5. $\mathbf{AF} f_1$, $\mathbf{EF} f_1$
6. $\mathbf{A}[f_1 \mathbf{U} f_2]$, $\mathbf{E}[f_1 \mathbf{U} f_2]$

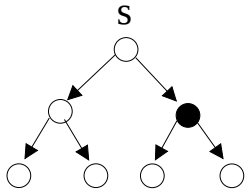
Note that the path quantifiers and temporal operators are always paired together.

Meaning of CTL Formulae

AXf if on all paths starting at state s, f holds in the next state.

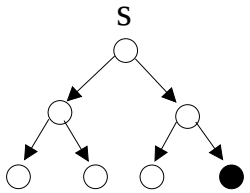


EXf if there exists a path starting at state s on which f holds at the next state.

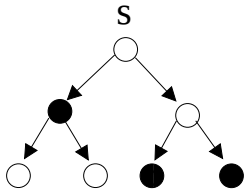


Meaning of CTL Formulae

EFf if f is reachable (i.e., if there exists a path starting at state s, on which f holds in some future state).

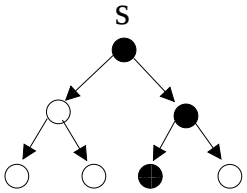


AFf if f is inevitable (i.e., if on all paths that start at state s, f holds in some future state).

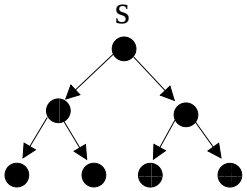


Meaning of CTL Formulae

EGf if there exists a path starting at state s, on which f holds globally.

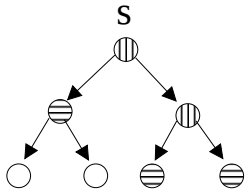


AGf if f is invariant (i.e., if on all paths that start at state s, f holds globally).

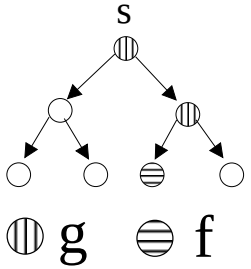


Meaning of CTL Formulae

E[g **U** f] if there exists a path starting at state s, on which g holds until f eventually holds.



A[g **U** f] if on all paths that start at state s, g holds until f eventually holds.



Examples of CTL Formulae

From Huth and Ryan [R17], p. 165:

"It is possible to get to a state where started holds, but ready does not hold."

"For any state, if a request occurs, then it will eventually be acknowledged."

"It is always the case that enabled is true infinitely often on every computation path."

Examples of CTL Formulae

"Whatever happens, the system will eventually be permanently deadlocked."

From any state it is possible to get to a state where restart is true.