ECE/CS 498 DSU/DSG Spring 2020
In-Class Activity 5

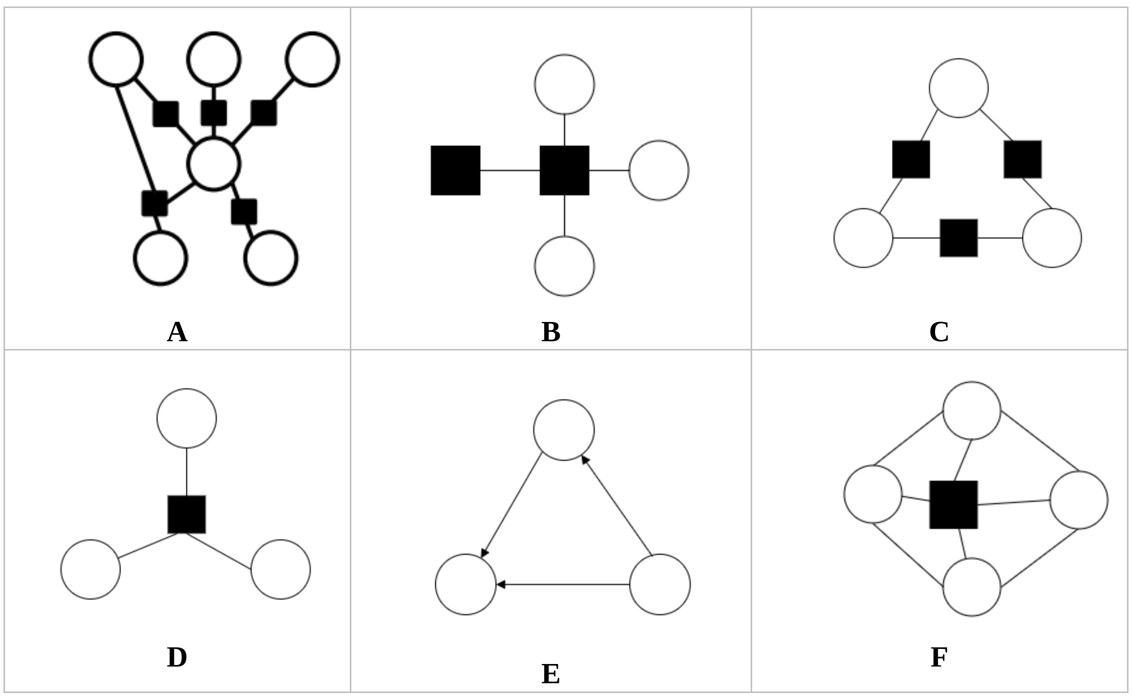
NetID: ____

The purpose of the in-class activity is for you to:

(i) Review concepts related to structure and conditional independence in factor graphs
(ii) Work out steps in belief propagation for inference on a factor graph

Factor Graphs

1. a) Which of the following graphical models are <u>invalid</u> representations of Factor Graphs? *(circle)* For invalid factor graphs, provide a short justification.



b) Consider random variables A, B connected by a factor function $f(A, B)$. Which of the following can $f(A, B)$ represent? (Mark all that apply)

i) Affinity between variables A and B
ii) Conditional relation between variables A and B – for example $P(A|B)$
iii) Joint relation between variables A and B

2. Given three binary random variables X_1, X_2, X_3 and a factor function $f(X_1, X_2, X_3)$

a) Draw and label the factor graph representing X_1, X_2, X_3 and f.

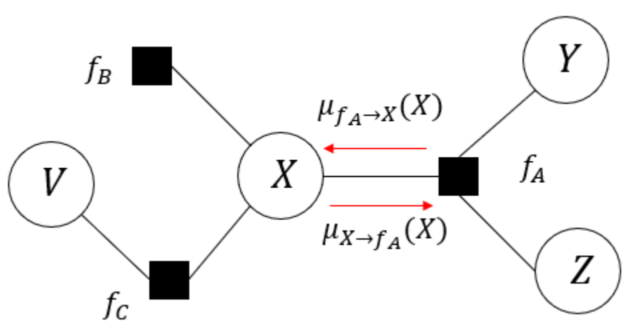
b) Assume that $f(X_1, X_2, X_3)$ factorizes to

$$f(X_1, X_2, X_3) = f_1(X_1, X_2) f_2(X_2, X_3) f_3(X_1, X_3)$$

Draw and label the factor graph.

c) Compare the two factor graph models in a) and b) in terms of model complexity, i.e., number of parameters.

3. Belief Propagation equations: For the factor graph given below, write the equations for messages between f_A and X.



$\mu_{X \to f_A}(X) =$

$\mu_{f_A \to X}(x) =$

4. Word problem on belief propagation

Engineers at Tesla built a factor graph model that captures the relationship between an AV disengagement (recall disengagement in your MP1) and the presence of a malicious attack. Such relationship is summarized in Table 3 as factor functions. In addition to Table 3, we provide Table 1 and Table 2 as the values for f_1 and f_2 respectively to model the prior information we known on disengagement and malicious attack.

- X denotes whether the AV is disengaged or not.
- Y denotes whether the AV is attacked or not.

Table 1 Based on disengagement statistics

$\boldsymbol{X}$	$\boldsymbol{f_1}$
Not Disengaged (0)	9
Disengaged (1)	1

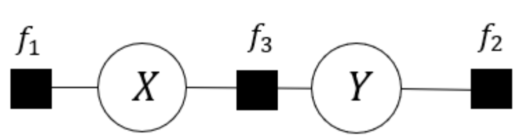
Table 2 Based on malicious attack statistics

$\boldsymbol{Y}$	$\boldsymbol{f_2}$
Not attacked (0)	6
Attacked (1)	4

Table 3 Dependency between disengagement and malicious attacks

$\boldsymbol{f_3}$	$\boldsymbol{Y} = \text{Not attacked}$	$\boldsymbol{Y} = \text{Attacked}$
$\boldsymbol{X = Not\ Disengaged}$	8	3
$\boldsymbol{X = Disengaged}$	4	7

The factor graph model that captures information presented in Tables 1, 2 and 3 is:



Use the factor graph model given above to answer the following questions.

a) Describe the joint distribution $P(X,Y)$ in terms of the factor functions. Remember to specify the equation for the normalization constant (partition function).

$P(X,Y)$=

b) Compute the marginal probabilities by enumerating all combinations of X and Y.

X	Y	Product of factor functions	$P(X,Y)$
0	0	$f_1(0) \times f_2(0) \times f_3(0,0) =$	
0	1		
1	0		
1	1		
Total		$Z =$	1

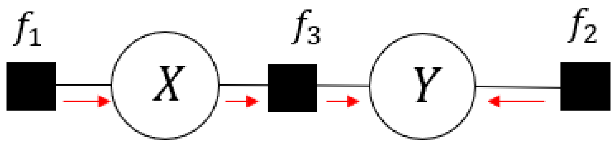
$P(X=0) \ = \ P(X=0, Y=0) \ + \ P(X=0, Y=1) \ =$

$P(X=1) \ =$

$P(Y=0) \ =$

$P(Y=1) \ =$

c) Computing marginal probability $P(Y)$ using belief propagation



i. First, calculate the message from f_3 to Y.

$$\mu_{f_1 \to X}(X) = f_1(X) = \begin{bmatrix} 9 \\ 1 \end{bmatrix}$$

$$\mu_{X \to f_3}(X) =$$

Equation for message from f_3 to Y: $\mu_{f_3 \to Y}(Y) =$

$\mu_{f_3 \to Y}(Y = 0) = f_3(0,0)\mu_{X \to f_3}(0) + \ f_3(1,0)\mu_{X \to f_3}(1) \ =$

$\mu_{f_3 \to Y}(Y = 1) \ =$

ii. Next, calculate the message from f_2 to Y.
$\mu_{f_2 \to Y}(Y) \ =$

iii. Calculate $P(Y)$.

Equation for marginal probability: $P(Y) \propto$

$$\mu_{f_3 \to Y}(0)\mu_{f_2 \to Y}(0) =$$

$$\mu_{f_3 \to Y}(1)\mu_{f_2 \to Y}(1) =$$

Exact computation taking into account the normalization:
$P(Y = 0) \ =$

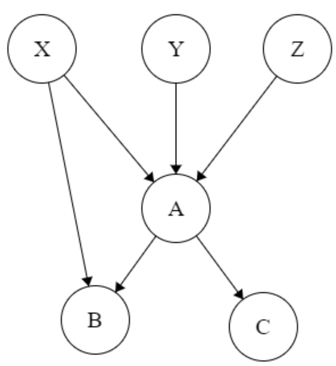
$P(Y = 1) \ =$

iv. Do your answers to parts b) and c) match for $P(Y)$? Also provide a reason why we choose to use belief propagation verses the full joint probabilities with marginalization.

v. Calculate $P(X = Disengaged \mid Y = Attacked)$ using Belief Propagation and compare the probability on disengagement given that a malicious attack was present with the probability of disengagement in general. How does the value differ? Also draw the new factor graph that reflects this knowledge. [Hint: Modify the factor graph to account for the observation of $Y = Attacked$]

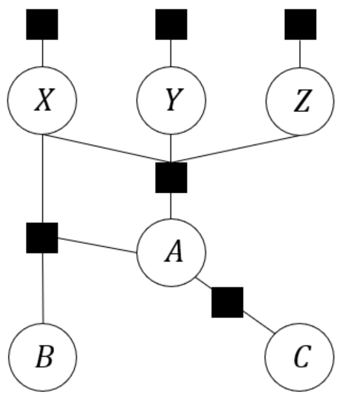
5. Bayesian Network and Factor Graph

a) Convert the Bayesian Network below into a Factor Graph. Remember that each conditional distribution in the BN becomes a factor.



b) Draw and label the Factor Graph and Bayesian Network model which represents the following joint distribution $P(X_1, \dots, X_5) = P(X_1)\ P(X_2)\ P(X_3|X_1, X_2)\ P(X_4|X_2, X_3)P(X_5|X_1, X_3)$.

6. For the factor graph given below, which of the following conditional independence relationship is true. Justify your answer.



a) $C \perp\!\!\!\perp X \mid A$

b) $B \perp\!\!\!\perp Y \mid X$

c) $B \perp\!\!\!\perp Z \mid A, X$