Week 20: Search III

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(based on slides from Raul Santos Rodriguez)

Outline

- A^*
- Heuristics
- Relaxation

Best of both: UCS + Greedy

Idea

A^* takes into account the cost from the root node to the current node and estimates the path cost from the current node to the goal node

$$f(n) = g(n) + h(n)$$

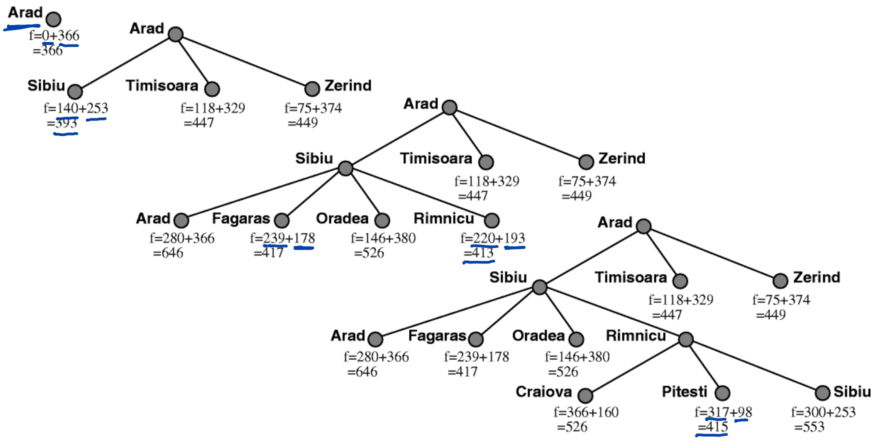
$g(n)$: path cost from the start node to node n

$h(n)$: estimated cost of the cheapest path from n to the goal

$f(n)$: estimated cost of the cheapest solution through n

$\boxed{A^* \text{ distorts costs to favour goal states}}$

A^*: example


Arad
f=0+366
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Timisoara
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Arad
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Fagaras
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Oradea
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Fagaras
f=239+178
=417
Oradea
f=146+380
=526
Rimnicu
Craiova
f=366+160
=526
Pitesti
f=317+98
=415
Sibiu
f=300+253
=553

A^*: algorithm

Algorithm: A* search [Hart/Nilsson/Raphael, 1968]

Run uniform cost search with modified edge costs:

$$Cost'(s, a) = Cost(s, a) + h(Succ(s, a)) - h(s)$$

if $h(n)$ satisfies certain conditions, is A^* search

- complete? YES
- optimal? YES

$\boxed{\text{The algorithm is identical to UCS, using } g + h \text{ instead of } g}$

A^*: conditions for optimality

Will any heuristic work? NO

Admissibility

A heuristic $h(n)$ is said to be an admissible heuristic if it never overestimates the cost to reach the goal. For every node n,

$$h(n) \leq h^*(n),$$

where $h^*(n)$ is the true cost to reach the goal state from n.

If $h(n)$ is not admissible, the method is called A.

Question

Consider the heuristic from our example (straight line distance).

Is the heuristic admissible?

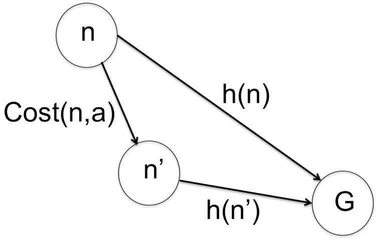
- Yes
- No

A^*: conditions for optimality

Consistency or monotonicity

A heuristic $h(n)$ is consistent if, for every node n and every successor n' of n generated by any action a, the estimated cost of reaching the goal from n is no greater than the step cost of getting to n' plus the estimated cost of reaching the goal from n':

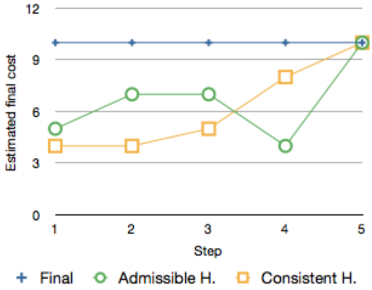
$$h(n) \leq Cost(n, a) + h(n')$$



A^*: conditions for optimality

Corollary

Every consistent heuristic is also admissible.



Comparison of an admissible but inconsistent and a consistent heuristic evaluation function.

Optimality of $A*$

1. If $h(n)$ is consistent, then the values of $f(n)$ along any path are nondecreasing.

Proof

Suppose n' is a successor of n; then $g(n') = g(n) + Cost(n, a)$ for some action a, and we have

$$f(n') = g(n') + h(n') = g(n) + Cost(n, a) + h(n') \geq g(n) + h(n) = f(n)$$

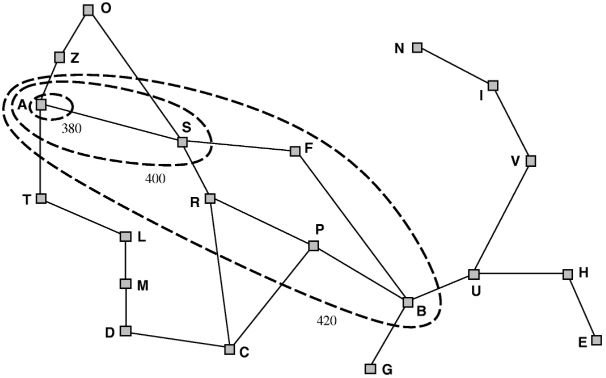
2. Whenever A^* selects a node n for expansion, the optimal path to that node has been found.

Proof

Were this not the case, there would have to be another frontier node n' on the optimal path from the start node to n, because f is nondecreasing along any path, n' would have lower f-cost than n and would have been selected first.

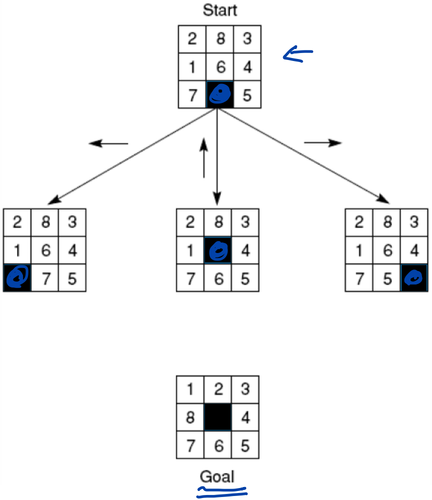
A^*: contours

f-costs are nondecreasing along any path $\rightarrow$ contours in the state space



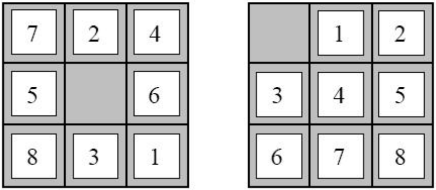
Example: 8-puzzle

Slide the tiles horizontally or vertically into the empty space until the configuration matches the goal configuration.



Example: 8-puzzle

- $h_1(n)$ = number of misplaced tiles
- $h_2(n)$ = total Manhattan distance (i.e., the number of squares from desired location of each tile)
- $h_3(n) = 2 \times$ number of direct tile reversals



Start State Goal State

- $h_1 = ?$
- $h_2 = ?$
- $h_3 = ?$

Example: 8-puzzle

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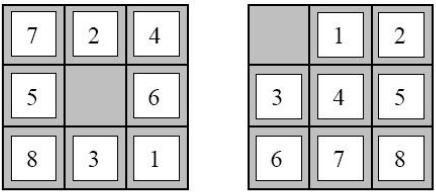


Start State Goal State

- $h_1 = 8$
- $h_2 = 3 + 1 + 2 + 2 + 2 + 3 + 3 + 2 = 18$
- $h_3 = 0$

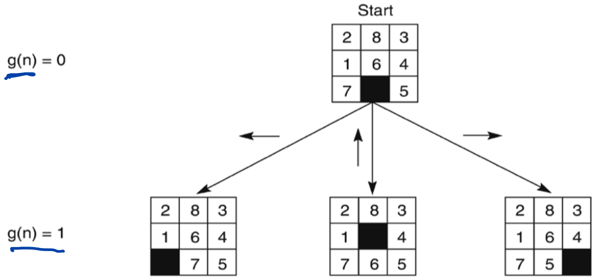
Example: 8-puzzle

2 8 3 1 6 4 7 5	**5**	**6**	**0**
2 8 3 1 4 7 6 5	**3**	**4**	**0**
2 8 3 1 6 4 7 5	**5**	**6**	**0**
	Tiles out of place	Sum of distances out of place	2 x the number of direct tile reversals

1	2	3
8		4
7	6	5

Goal

Example: 8-puzzle

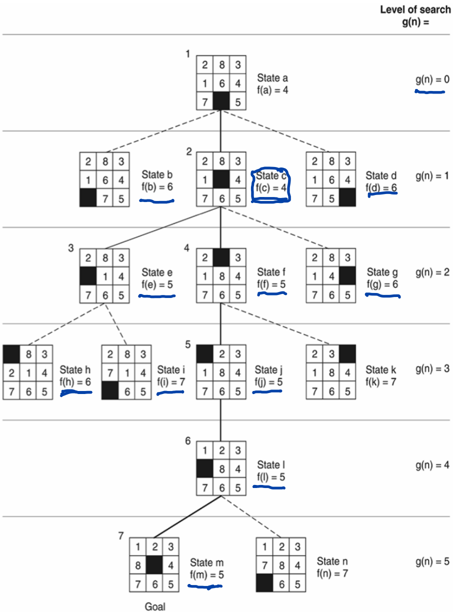


Values of f(n) for each state, **6** **4** **6**

where:

f(n) = g(n) + h(n),
g(n) = actual distance from n to the start state, and
h(n) = number of tiles out of place.

Example: 8-puzzle



Example: 8-puzzle

	Search Cost (nodes generated)			Effective Branching Factor		
d	IDS	A*(h_1)	A*(h_2)	IDS	A*(h_1)	A*(h_2)
2	10	6	6	2.45	1.79	1.79
4	112	13	12	2.87	1.48	1.45
6	680	20	18	2.73	1.34	1.30
8	6384	39	25	2.80	1.33	1.24
10	47127	93	39	2.79	1.38	1.22
12	3644035	227	73	2.78	1.42	1.24
14	–	539	113	–	1.44	1.23
16	–	1301	211	–	1.45	1.25
18	–	3056	363	–	1.46	1.26
20	–	7276	676	–	1.47	1.27
22	–	18094	1219	–	1.48	1.28
24	–	39135	1641	–	1.48	1.26

Figure 3.29 Comparison of the search costs and effective branching factors for the ITERATIVE-DEEPENING-SEARCH and A* algorithms with h_1, h_2. Data are averaged over 100 instances of the 8-puzzle for each of various solution lengths d.

[Russell and Norvig]

Dominance

Let h_1 and h_2 be two admissible heuristics. if $h_2(n) \geq h_1(n)$ for all n, then h_2 dominates h_1.

Question

Is it possible for a computer to invent such a heuristic mechanically?

h_1 and h_2 are estimates of the remaining path length for the 8-puzzle, but they are also perfectly accurate path lengths for simplified versions of the puzzle.

Relaxation

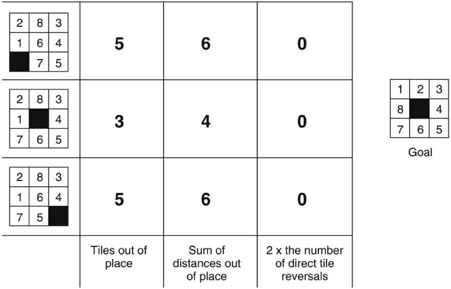
- A problem with fewer restrictions on the actions is called a relaxed problem.
- The cost of an optimal solution to a relaxed problem is an admissible heuristic for the original problem.

Relaxation

Idea: Constraints make life hard. Get rid of them.

"Due to TV network constraints, our 5 year mission has been reduced to 13 weeks, with a possible renewal."

Relaxation



- If the rules of the 8-puzzle are relaxed so that a tile can move anywhere, then $h_1(n)$ gives the exact solution!
- If the rules are relaxed so that a tile can move to any adjacent square, then $h_2(n)$ gives the exact solution!

Relaxation

If a problem is written down in a formal language, it is possible to construct heuristics automatically. Consider the following rule:

8-puzzle

A tile can move from square A to square B if A is horizontally or vertically adjacent to B and B is blank

We can generate three heuristics by removing one or both of the conditions from the above rule:

a) A tile can move from square A to square B
b) A tile can move from square A to square B if A is adjacent to B
c) A tile can move from square A to square B if B is blank

Heuristics beyond relaxation

Subproblems: relax original problem into independent subproblems

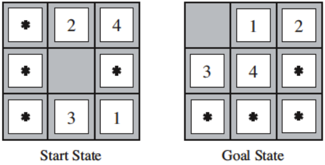


Figure 3.30 A subproblem of the 8-puzzle instance given in Figure 3.28. The task is to get tiles 1, 2, 3, and 4 into their correct positions, without worrying about what happens to the other tiles.

Learning from experience

E.g., solving lots of 8-puzzles $\rightarrow$ training data

Use ML to predict $h(n) \rightarrow \underline{h(n)} = \underline{c_1 x_1(n)} + \underline{c_2 x_2(n)}$

[Russell and Norvig]

General framework

Relaxed search problem

A relaxation P' of a search problem P has costs that satisfy:

$$Cost'(n, a) \leq Cost(n, a)$$

$\boxed{\text{Removing constraints} \rightarrow \text{Reducing edge costs}}$

Relaxed heuristic

Given a relaxed search problem P', define the relaxed heuristic $h(n) = h'^*(n)$, the minimum cost from n to a goal state using $Cost'(n, a)$.

Selecting heuristics

Question

If a collection of admissible heuristics $h_1, \ldots, h_m$ is available for a problem and none of them dominates any of the others, which should we choose?

Selecting heuristics

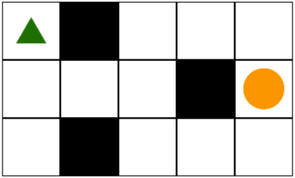
Question

If a collection of admissible heuristics $h_1, \ldots, h_m$ is available for a problem and none of them dominates any of the others, which should we choose?

$$h(n) = max\{h_1(n), \ldots, h_m(n)\}$$

Problem

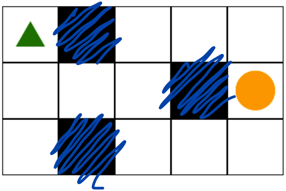
Goal: move from triangle to circle

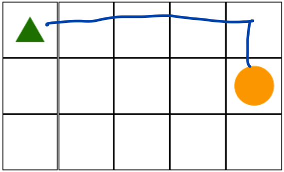


Find a good heuristic!

Problem

Goal: move from triangle to circle





$$h(n) = ManhattanDistance(n, (2, 5))$$

$$e.g., h((1, 1)) = 5$$

Summary

- Informed search: A^* expands nodes with minimal $f(n) = g(n) + h(n)$.
- Consistent and admissible heuristics.
- How to construct heuristics?
  - Relaxation
  - Subproblems
  - Learning