

# Introduction to Machine Learning part 2

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- Performance metrics
- Generalisation and overfitting
- Train, validate, test

# Performance Metrics: Classification

- True Positives (TP): The class is + and the prediction is +
- True Negatives (TN): The class is - and the prediction is -
- False Positive (FP): The class is - and the prediction is +
- False Negative (FN): The class is + and the prediction is -

		Predicted	
		Dog	Cat
Actual	Dog	<u>TP</u>	<u>FN</u>
	Cat	<u>FP</u>	<u>TN</u>

# Accuracy

■  $Accuracy = \frac{TP+TN}{TP+TN+FP+FN}$

$acc = \frac{n \text{ correctly classified}}{n}$

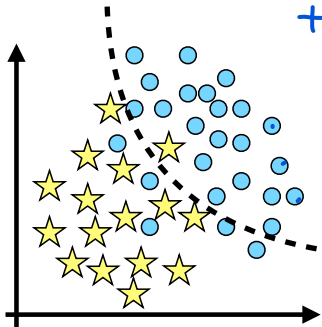
		Predicted	
		Dog	Cat
Actual	Dog	<u>TP</u>	FN
	Cat	FP	<u>TN</u>

# Precision

■  $Precision = \frac{TP}{TP+FP}$

$$\frac{21}{21 + 1} \approx \frac{21}{22}$$

Actual	Predicted	
	Dog	Cat
Dog	TP	FN
Cat	FP	TN

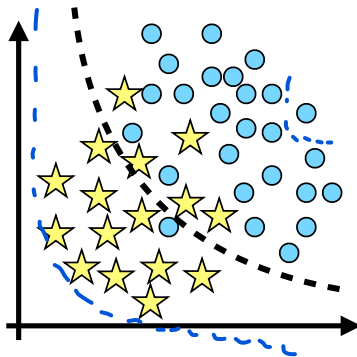


# Recall

■  $Recall = \frac{TP}{TP+FN}$

$$\frac{25}{26}$$

Actual	Predicted	
	Dog	Cat
Dog	TP	FN
Cat	FP	TN



The  $F_1$  score is the harmonic mean of precision and recall

$$\begin{aligned} F_1 &= \frac{1}{precision^{-1} + recall^{-1}} \\ &= 2 \frac{precision \cdot recall}{precision + recall} \end{aligned}$$

# Metrics for more than two classes

$$\text{Accuracy} = \frac{\sum_{i=1}^n \mathbb{1}(y_i = f(\vec{x}_i))}{n} = \frac{\text{Correct class}^n}{n}$$

- This is the same as the binary case, even though the equation looks different

	Predicted			
	Dog	Cat	Bird	
Actual	Dog	<u>10</u>	2	0
	Cat	1	<u>5</u>	1
	Bird	2	0	<u>12</u>

$$10 + 5 + 12 = 27$$

$$\text{acc} = \frac{27}{33}$$



# Metrics for more than two classes

- Macro-averaging:

$$Precision = \frac{1}{k} \sum_{j=1}^k Precision_j \quad Recall = \frac{1}{k} \sum_{j=1}^k Recall_j$$

Actual \ Predicted	Predicted		
	Dog	Cat	Bird
Dog	10	2	0
Cat	1	5	1
Bird	2	0	12

$$precision_{dog} = \frac{10}{13}$$

Actual \ Predicted	Predicted		
	Dog	Cat	Bird
Dog	10	2	0
Cat	1	5	1
Bird	2	0	12

$$Recall_{dog} = \frac{10}{12}$$

# Metrics for more than two classes

## ■ Micro-averaging

		Predicted		
		Dog	Cat	Bird
Actual	Dog	10	2	0
	Cat	1	5	1
	Bird	2	0	12

Handwritten annotations on the confusion matrix:

- TP** (True Positive) is written above the Dog-Dog cell.
- FN** (False Negative) is written to the right of the Dog-Cat and Dog-Bird cells.
- FP** (False Positive) is written below the Cat-Dog and Bird-Dog cells.
- TN** (True Negative) is written to the right of the Bird-Bird cell.
- A large blue bracket spans the bottom of the matrix, encompassing the FP and TN regions.

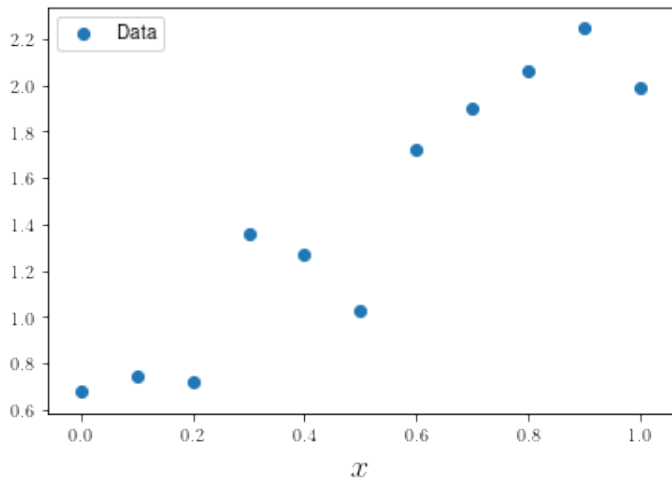
		Predicted	
		D	non D
Actual	D	10	2
	non D	3	18

		B	non B
Actual	B	12	2
	non B	1	18

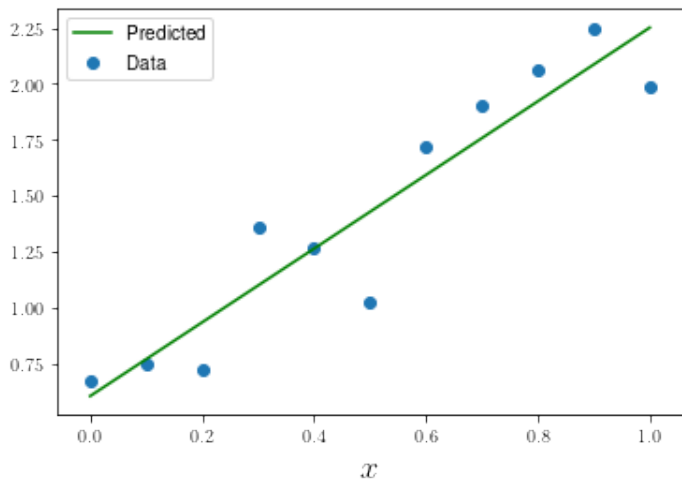
		C	non C
Actual	C	5	2
	non C	2	24

		+	-
Actual	+	27	6
	-	6	60

# Performance Metrics: Regression



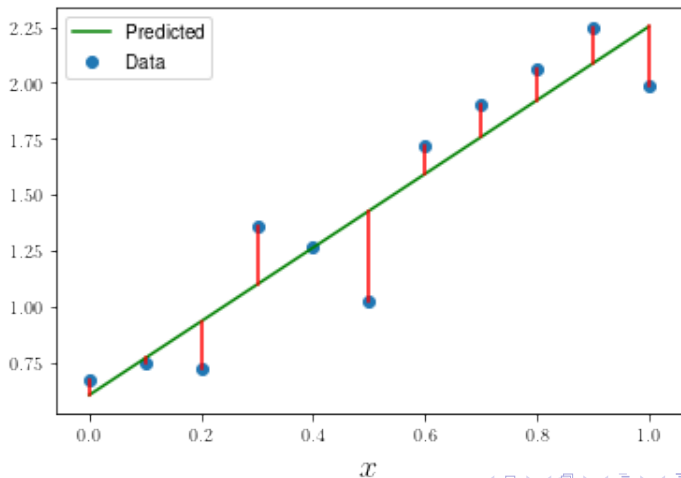
# Performance Metrics: Regression



# Performance Metrics: Regression

- Mean Squared Error (MSE):

$$\underline{E} = \frac{1}{N} \sum_{(\vec{x}, y)} (y - f(\vec{x}))^2$$

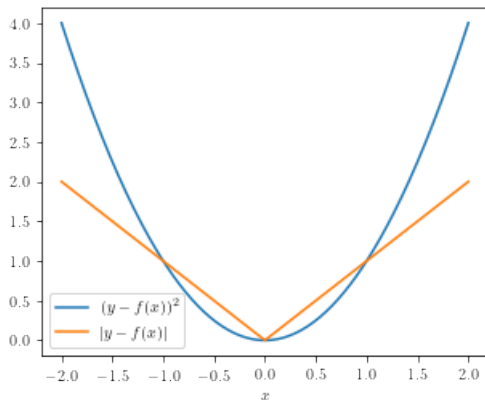


# Performance Metrics: Regression

- Root Mean Squared Error (RMSE):

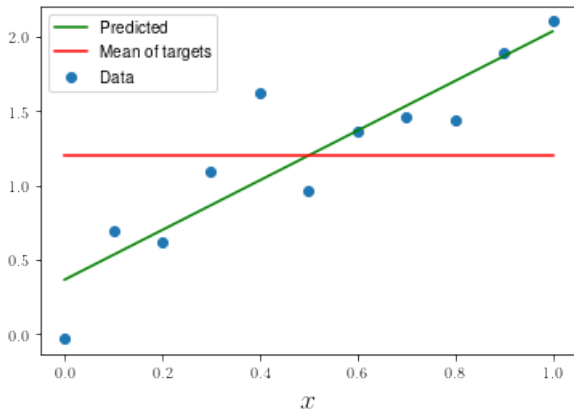
$$E = \sqrt{\frac{1}{N} \sum_{(\vec{x}, y)} (y - f(\vec{x}))^2}$$

- Mean Absolute Error (MAE):  $E = \frac{1}{N} \sum_{(\vec{x}, y)} \underline{|y - f(\vec{x})|}$



# Performance Metrics: Regression

- $R^2 = 1 - \frac{\sum_{(\vec{x}, y)} (y - f(\vec{x}))^2}{\sum_{(\vec{x}, y)} (y - \bar{y})^2}$   
where  $\bar{y}$  is the mean of the target values  $y$



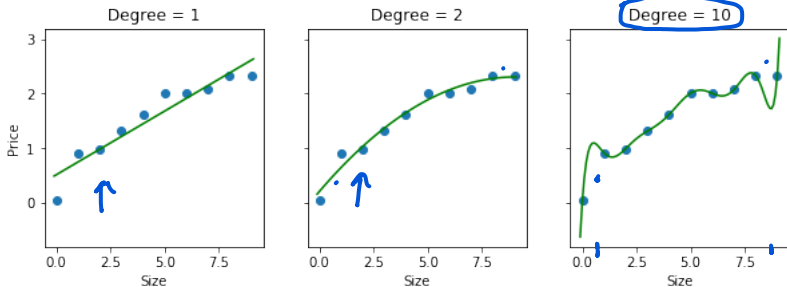
# Summary: Performance Metrics

- For classification problems, we can use metrics such as accuracy, precision, recall, or  $F_1$  score.
- These can be used for multi-class problems as well as binary problems
- Mean squared error or absolute error can be used for regression problems
- $R^2$  is often used for linear regression



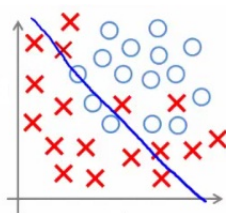
# Generalisation

- By fitting an approximation function with a high number of parameters it is possible to obtain very high accuracy for the data on which you train.
- However, this learnt approximation may not then perform well on different, previously unseen, data.
- This is called over-fitting



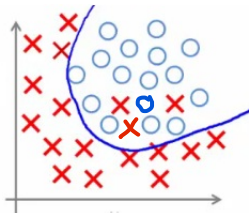
# Over-Fitting in Classification

- For classification problems we must be careful how we model the boundary between different classes in the feature space.

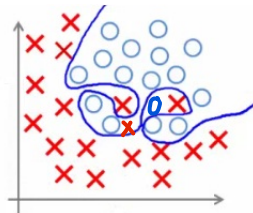


**Under-fitting**

(too simple to  
explain the  
variance)



**Appropriate-fitting**



**Over-fitting**

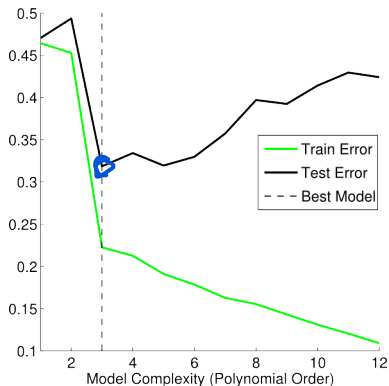
(forcefitting -- too  
good to be true)

# Summary: Generalisation and overfitting

- As well as attaining good scores on our training data, we want our algorithms to generalise well to unseen data
- Both regression and classification problems can overfit to data

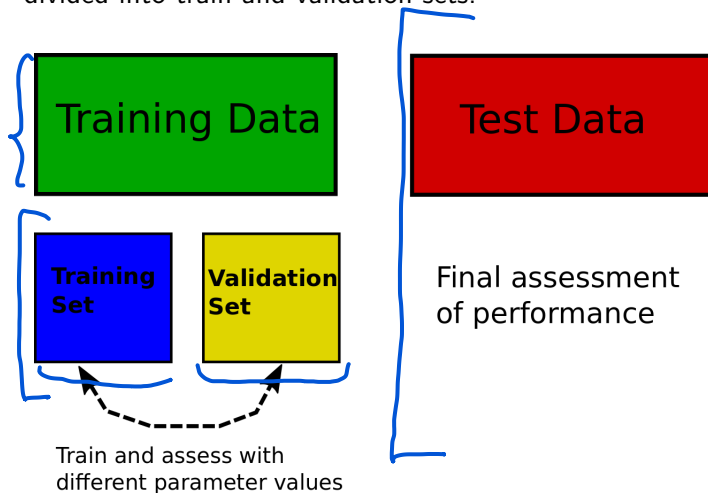
# Train and Test Data

- To evaluate the generalisation of a model available data should be divided into training and test sets.



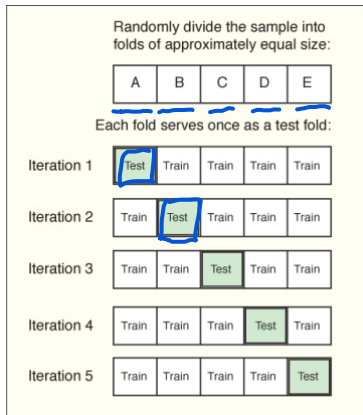
# Train, Validation and Test Data

- To fit model parameters the training data is sometimes further divided into train and validation sets.



# n-Folds

- How do you know that you have not just been lucky in your choice of training and test data?
- Answer: Repeat many times with different divisions into training and test.



Cross-validation  
can be used  
with the validation  
set.

- Splitting data into training and test sets helps ensure that our model can generalise.
- We can further split our training set into training and validation sets, to test the performance of our model on unseen data
- Cross-validation can give a more rounded view of performance

# Overall summary

We have looked at:

- Performance metrics
- Generalization and overfitting
- Training and test splits

You can practice working with these concepts in the...



- Available as pdf and also as jupyter notebook
- Covers evaluation metrics, generalisation, overfitting
- If you don't already know Python, please do work through the Introduction to Python available on BlackBoard
- You can ask questions in the lecture or in problem classes

## Simple supervised learning algorithms

- k-Nearest neighbours
- Linear regression
- Naive Bayes classifier