Financial Markets and Products & Valuation and Risk Models

FRM一级培训讲义-强化班

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Topic Weightings in FRM Part Ⅰ

Session NO.	Content	Weightings
Study Session 1	Foundations of Risk Management	20
Study Session 2	Quantitative Analysis	20
Study Session 3	Financial Markets and Products	30
Study Session 4	Valuation and Risk Models	30

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Framework

1. Fixed-Income Products
   - Main Types of Bond Market
   - Valuation
   - Risk Metrics
2. Derivatives
   - Forward and Futures
   - Swap Market
   - Option Market
3. Financial Institutions
   - Central Counterparties
   - Banks
   - Insurance Companies
   - Mutual Funds and Hedge Funds
4. Risk Measurement and Management
   - Market Risk
   - Credit Risk
   - Operational Risk
   - Stress Testing

Fixed-Income Products

Topic 1: Main Types of Bond Market

1. Treasury Market
2. Corporate Bond Market
3. Mortgage-Backed Securities

Treasury Market

- **Treasury Bills**
  - One year or less
  - Quoted Price

$$\text{cash price} = 100(1 - \text{discount rate} \times \frac{\text{n}}{360})$$

  - **Example:** Suppose you have a 180-day T-bill with a discount rate, or quoted price, of five (i.e., the annualized rate of interest earned is 5% of face value). If face value is $100, the cash price is 97.5
- **Treasury Notes and Treasury Bonds**
  - T-Notes mature in 2 to 10 years.
  - T-Bonds mature in more than 10 years.
  - Semi-annually.
  - **Quoted Price**
  - ✓ Dollars and thirty-seconds of a dollar with face value of $100

Treasury Market

- **Quotation of Treasury Bonds**
  - **Clean Price:** The price of a coupon bond not including any accrued interest. Immediately following each coupon payment, the clean price will equal the dirty price.
  - **Dirty Price:** A bond pricing quote referring to the price of a coupon bond that includes the present value of all future cash flows, including interest accruing on the next coupon payment.

dirty price = clean price + accrued interest

  - **Accrued Interest and Day Count Conventions**
  - ✓ Treasury bonds: actual/ actual
  - ✓ Corporate and municipal bonds: 30/360
  - ✓ Money market instruments (Treasury bills): actual/360

Treasury Market

- **Treasury STRIPS**
  - Zero coupon bonds issued by the U.S. Treasury
  - STRIPS are created when someone delivers a particular bond to the Treasury and asks for it to be stripped into its principal and coupon components.
    - ✓ The coupon or interest STRIPS are called **C-STRIPS**.
    - ✓ Principal STRIPS are called **P-STRIPS**.
  - Investors like zero coupon bonds for at least two reasons
    - ✓ They can be combined or re-constructed into any required sequence of cash flows.
    - ✓ They are more sensitive to interest rate than coupon-bearing bond.

Corporate Bond Market

- **Corporate Trustee**
  - Acts in a fiduciary capacity on behalf of the investors.
  - Ensure that the bond issuer is in compliance with the covenants of the indenture at all times.
- **Zero-Coupon Corporate Bonds**
  - Eliminates reinvestment risk
  - In bankruptcy, zero-coupon bond creditor claim original offering price plus accrued and unpaid interest.
  - A zero-coupon bond's interest rate is determined by the **original issue discount (OID).** The difference between the face amount and the offering price when first issued is called the original-issue discount.
- **Credit Risk**
  - Credit Default Risk
  - Credit Spread Risk

Corporate Bond Market

- **Different Types of Corporate Bonds**
  - Mortgage Bonds
  - Collateral Trust Bonds
  - Equipment Trust Certificates
  - Debenture Bonds
  - Guaranteed Bonds
- **Corporate Bond Retirements**
  - ✓ Call provision
  - ✓ Sinking-fund provisions
  - ✓ Maintenance and replacement funds
  - ✓ Tender offers
- **High-Yield Bond**
  - Rated below investment grade.
  - Payment Features: deferred-interest, step-up, payment-in-kind

Mortgage-Backed Securities

- **Types of Mortgage Loans**
  - Agency or conforming loans
  - Non-agency or non-conforming loans (Jumbos; Alt-A; Subprime)
- **Fixed Rate Mortgage Payments**
  - Present value of monthly payments, discounted at the monthly compounded mortgage rate, equals the original amount borrowed.
  - The fixed monthly payment is often divided into its interest and principal components. The monthly interest payment over a particular period equals the mortgage rate times the outstanding principal amount at the beginning of that period. The principal component of the monthly payment is the remainder.
  - Under the fair pricing condition which assume that the term structure is flat and the interest rates have not changed, the present value of the remaining payments equals the principal outstanding.

Mortgage-Backed Securities

- **Securitization Process of MBS**
  - Bank with considerable assets in the form of mortgages sells the portfolio to get early cash to make other loans. Generally, the buyers of such mortgage portfolios are government backed organization, quasi-governmental, or private entity.
  - The buyer then groups all the similar mortgages it has already purchased.
  - It then issues securities (MBS) that represent an interest in the pool of mortgages called securitizing the pool and sells the MBS to investors in the open market.
  - When the obligors of the mortgage loans make their repayments, the bank & the MBS issuers keep a fee or spread and pass on the rest of the payment to the investors.

Mortgage-Backed Securities

- **Multiple Players Involved**
  - **Original Bank:** Banker that initially made a mortgage loan.
  - **MBS Issuer:** Original buyer of the mortgages and is responsible for the formation of pool and issuing the MBS with the underlying pool.
  - **Mortgage Servicers:** manage the flow of cash from borrowers to investors in exchange for a fee taken from those cash flows
  - **Mortgage Guarantors:** guarantee investors the payment of interest and principal against borrower defaults, also in exchange for a fee.
- **Formation of Mortgage Pools**
  - **Specific Pools:** agree to trade a particular pool of loans
  - **To Be Announced (TBAs)**
    - ✓ More liquid
    - ✓ Forward market with a delivery option

Mortgage-Backed Securities

➢ **Dollar Roll Transaction**

- A transaction similar to a repurchase transaction
- The buyer of the roll sells a TBA for one settlement month and buys the same TBA for the following settlement month.
- **Value of the Dollar Roll**

✓ Difference in proceeds between:

- Starting with a given pool and buying the roll
- Holding that pool over the month

✓ If the value of the roll is zero, the roll is said to trade at breakeven.

✓ If the forward drop is larger so that the value of the roll is positive, the roll is said to trade above carry.

Mortgage-Backed Securities

➢ **WAC, WAM and CPR for a Mortgage Pool**

- **Weighed Average Coupon (WAC):** weighted average of the mortgage loan rates paid by the borrowers.
- **Weighted Average Maturity (WAM):** weighted average of the maturities of the underlying loans of the pool.
- **Conditional Prepayment Rate (CPR)**

✓ **Prepayment Risk:** valuable when mortgage rates have fallen.

✓ **Single Monthly Mortality Rate (SMM):** percentage of principal outstanding that is prepaid during month, where prepayments do not include scheduled.

✓ SMM is annualized to a **Constant Prepayment Rate or Conditional Prepayment Rate (CPR)**

$$\text{CPR} = 1 - (1 - \text{SMM})^{12}$$

Mortgage-Backed Securities

➢ **Valuing MBS**

- **Monte Carlo methodology**

✓ Generate a large number of paths of interest rates

✓ Calculate the cash flows of the security along each path

✓ Calculate discounted value of the security's cash flows along each path.

✓ Compute the value as the average of the discounted values across paths.

- **Option-Adjusted Spread (OAS)**

✓ OAS is the most popular measure of relative value for MBS.

✓ When using simulation to derive the value of MBS, OAS is the yield spread which has to be added to a benchmark yield curve to discount MBS's payments to match its market price.

Exercise 1

- Consider the following statements about bond reinvestment risk and bond duration (interest rate risk):
  - I. Less bond reinvestment risk implies greater interest rate risk, ceteris paribus.
  - II. Due to reinvestment risk, the yield-to-maturity on a bond is unlikely to equal the bond's realized return.
  - III. Reinvestment risk is eliminated in a zero-coupon bond.

  Which of the above statements is/are true?
  - A. I only
  - B. I and II
  - C. II and III
  - D. All three

- Answer: D

Exercise 2

- Each of the following is true about the corporate trustee in a corporate bond issuance except:
  - A. The trustee is paid by bondholders.
  - B. The trustee acts in a fiduciary capacity for investors who own the bond issue.
  - C. The trustee must, at the time of issue, authenticate the bonds issued (i.e., keep track of all the bonds sole) and make sure that they do not exceed the principal amount authorized by the indenture.
  - D. If a corporate issuer fails to pay interest or principal, the trustee may declare a default and take such action as may be necessary to protect the rights of bondholders.

- Answer: A

Exercise 3-4

- All other things being equal, which of the following types of bond instruments would have the lowest interest rate?
  - A. Equipment trust certificates.
  - B. Mortgage bonds.
  - C. Junior debentures.
  - D. Senior debentures.
- Answer: A
- After five years, which is nearest to the outstanding principal balance on a 30-year fixed rate mortgage with an original balance of $200,000 and a mortgage interest rate of 3.6%?
  - A. $152,300
  - B. $165,800
  - C. $179,700
  - D. $182,500
- Answer: C

Exercise 5

- A fixed-income portfolio manager purchases a seasoned 5.5% agency mortgage-backed security with a weighted average loan age of 60 months. The current balance on the loans is USD 20 million, and the conditional prepayment rate is assumed to be constant at 0.4% per year. Which of the following is closest to the expected principal prepayment this month?
  A. USD 1,000
  B. USD 7,000
  C. USD 10,000
  D. USD 70,000

- Correct Answer: B

Exercise 6

- Suppose that the TBA prices of the Fannie Mae 6% for July 9 and August 9 settlements are $103 and $102.6, respectively. The accrued interest to be added to each of these prices is 9 actual/360 days of a month's worth of a 6% coupon, i.e., $0.15. Let the expected total principal paydown be 2% and assume short-term rate is 1%. If an investor finance a purchase of an MBS pool using dollar roll valued at $10 million. Which of the following is true?
  A. The roll trades above carry.
  B. The roll trades below carry.
  C. The roll trades at breakeven.
  D. Hard to determine.

- Answer: A

Fixed-Income Products

Topic 2: Valuation

1. Compounding Interest
2. Spot and Forward Rate
3. Spread and Yield
4. Relationship between Rates
5. Discount Factor
6. Basic Valuation Approach
7. Bond Replication
8. Decomposition of P&L

Compounding Interest

- **Compounding Interest**
  - Suppose we have an account where the simple interest is added in each year and then that money also earns interest.
  - Assuming
    - ✓ R_c is the rate of interest with continuous compounding.
    - ✓ R_m is the rate of interest with discrete compounding (m per annum)
    - ✓ n is the number of years.

$$FV = PV(1 + \frac{R_m}{m})^{mn}$$
$$FV = PVe^{R_c \times n}$$
$$PVe^{R_c n} = PV(1+\frac{R_m}{m})^{mn}$$

Spot, Forward and Par Rates

- **Spot Rate:** A spot rate is the rate on a spot loan, an agreement in which a lender gives money to the borrower at the time of the agreement to be repaid at some single, specified time in the future.
- **Forward Rate:** Interest rates corresponding to a future period implied by spot curve.

$$(1 + z_1)^{T_1}(1 + F_{1,2})^{(T_2 - T_1)} = (1 + Z_2)^{T_2}$$
$$e^{Z_1 T_1} \times e^{F_{1,2}(T_2 - T_1)} = e^{Z_2 T_2}$$
$$\Rightarrow F_{1,2} = \frac{Z_2 T_2 - Z_1 T_1}{T_2 - T_1}$$

Spot Rates and Forward Rates		
Maturity (year)	**Spot Rate**	**Forward rate**
0.5	0.94%	0.94%
1	1.37%	1.80%
1.5	1.82%	2.72%
2	2.51%	4.59%
2.5	3.08%	5.38%
3	3.87%	7.87%

- **Par Rate:** The T-year, semiannual par rate is the rate, C(T), such that a fixed-rate asset with par value of $100 that makes regular semi-annual coupon payments of C(T)/2×$100 discounts to a present value equal to the par value of $100.

Spread and Yield

- **Spread**
  - Important measures of relative value. Assume we define the term structure as a set of forward rates, then we can find a spread, s, that equates the discounted cash flows to the price of the bond.
- **YTM**
  - Single rate, when used to discount a bond's cash flow, produces the bond's market price.
  - It can be viewed as the realized return on the bond assuming all cash flows are reinvested at the YTM and the bond is held to maturity.

$$P = \frac{c_1}{(1 + y)} + \frac{c_2}{(1 + y)^2} + \cdots + \frac{c_T}{(1 + y)^T}$$
$$P = \frac{c_1}{(1 + Z_1)} + \frac{c_2}{(1 + Z_2)^2} + \cdots + \frac{c_T}{(1 + Z_T)^T}$$

Spread and Yield

➢ **Example**

- Consider a bond with $100 face value, 2-year maturity and 4% semiannual coupon. Given annualized spot rate as follow:

Maturity (Years)	Spot Rates(%)
0.5	2.5
1.0	2.6
1.5	2.7
2.0	2.9

$$P = \frac{2}{(1+0.025/2)} + \frac{2}{(1+0.026/2)^2} + \frac{2}{(1+0.027/2)^3} + \frac{102}{(1+0.029/2)^4} = \$102.14$$

N=4;PV=-102.14;PMT=2;FV=100,CPT→I/Y=1.4455

Yield=2.89%

Relationship between Rates

➢ **Relationship between spot rates and YTM**

- There are several spot rates and a single yield (YTM). YTM is a summary of all the spot rates that enter into the bond pricing equation.
  - ✓ A flat term structure of sport rates: the yield must equal the one-year spot rate level as well.
  - ✓ A term structure where spot rates are upward sloping over a two-year period: the two-year bond yield is below the two-year spot rate.
  - ✓ A term structure where spot rates are downward sloping over a two-year period: the two-year yield is above the two-year spot rate.

➢ **Relationship between spot rates and forward rates**

- Spot rates are an average of forward rates.
- If spot rates are increasing with term while forward rates are greater than spot rates

Discount Factor

➢ **Discount Factor**

- For a term of t years, gives the present value of one unit of currency to be received at the end of that term.

$$d(t) = \frac{1}{(1+\frac{z(t)}{2})^{2t}}$$

STRIPS Prices and Discount Factors			
Maturity	STRIPS Price	Discount Factor	Spot Rate
0.5	99.5322	0.995322	0.94%
1	98.6439	0.986439	1.37%
1.5	97.3189	0.973189	1.82%
2	95.1336	0.951336	2.51%
2.5	92.6433	0.926433	3.08%
3	89.1374	0.891374	3.87%

- **Example:** consider a 7.875% bond due in 6 months with a market price of 101.40. We can extract the d(0.5) and the implied spot rate:

$$101.4 = (100 + 100 \times 7.875\%/2)d(0.5) \qquad Z_{0.5} = 2[(1/d(0.5)) - 1]$$

Basic Valuation Approach

➢ **Characteristics of Bonds**

- Coupon Rate
- Face Value
- Maturity
- Interest Rate

➢ **Pricing Bond using Discount Factors, Spot Rates, or Forward Rates**

- Given a 1-year treasury bond that pays a 8% semi-annual coupon

Maturity	Spot Rate	Discount Factor	6 Month Forward Rate
0.50	0.94%	0.995322	0.94%
1.00	1.37%	0.986439	1.80%
1.50	1.82%	0.973189	2.72%
2.00	2.51%	0.951336	4.59%
2.50	3.08%	0.926433	5.38%

Price = ($4×0.995322) + ($104×0.986439) = $106.57

$$\text{Price} = \frac{\$4}{(1+\frac{0.94\%}{2})} + \frac{\$104}{(1+\frac{1.37\%}{2})^2} = \frac{\$4}{(1+\frac{0.94\%}{2})} + \frac{\$104}{(1+\frac{0.94\%}{2})\times(1+\frac{1.80\%}{2})} = \$106.57$$

Basic Valuation Approach

➢ **Price of an Annuity and a Perpetuity**

- **Annuity**

✓ An annuity with semiannual payments is a security that makes a payment c/2 every six months for T years but never makes a final "principal" payment (i.e., FV=0). The price of an annuity, A(T), is given by:

$$A = \frac{CF}{y}\left(1 - \left(\frac{1}{1+\frac{y}{2}}\right)^{2T}\right)$$

- **Perpetuity**

✓ A perpetuity bond is a bond that pays coupons forever. The price of a perpetuity is simply the coupon divided by the yield.

$$\text{Perpetuity} = \frac{CF}{y}$$

Bond Replication

➢ **Replication Approach**

- **Law of One Price:** Absent confounding factors (e.g., liquidity, financing, taxes, credit risk), identical sets of cash flows should sell for same price.
- Example: three bond yields and prices are shown below.

	Maturity	YTM	Coupon	Price (% of par)
1	1 year	5%	0%	95.238
2	2 years	6%	0%	89.00
3	2 years	6%	6%	100

- The 2-year spot rate is 6.03%. Is there an arbitrage opportunity using these three bonds? If so, describe the trades necessary to exploit the arbitrage opportunity?

Bond Replication

- **Replication Approach (cont'd)**

Time = 0		1 year		2 years	
-1,000,000.00	(cost of 2-year, 6% coupon bonds)	+60,000	(coupon)	+1,060,000	(coupon)
+57,142.86	(proceeds 1-year, 0% coupon bonds)	-60,000	(maturity)		
+943,396.20	(proceed 2-year, 0% coupon bonds)			-1,060,000	
+539.06	Net	0		0	(maturity)

Decomposition of P&L

	Start period	2010-1-1	2011-1-1	2012-1-1	price	P&L
Pricing date: 2010-1-1; annual coupon=1						
Initial forwards	Term structure	2%	3%	4%	93.0229	
	spreads	0.5%	0.5%	0.5%		
Pricing date: 2011-1-1; annual coupon=1						
Carry-roll-down	Term structure		3%	4%	94.3485	+1.3256
	spreads		0.5%	0.5%		
Rate change	Term structure		2%	3%	96.1800	+1.8315
	spreads		0.5%	0.5%		
Spread change	Term structure		2%	3%	95.2577	-0.9223
	spreads		1%	1%		

Exercise 1

- The following discount function contains semi-annual discount factors out to two years: d(0.5) = 0.9970, d(1.0) = 0.9911, d(1.5) = 0.9809, d(2.0) = 0.9706. What is the implied 1.5 year zero rate?
  - A. 0.600%
  - B. 1.176%
  - C. 1.290%
  - D. 1.505%

- Answer: C

Exercise 2

- The price of a six-month zero-coupon bond is $99.90 and the price of a one-year zero-coupon bond is $98.56. What is the implied six-month forward rate, under semi-annual compounding?
  A. 1.30%
  B. 2.95%
  C. 2.73%
  D. 3.08%

- Answer: C

Exercise 3

- Given the following information:

Maturity	YTM	Coupon	Price
1 year	4%	0%	96.154
2 years	6.5%	0%	88.166
3 years	9.5%	0%	76.165

  - 1-year forward rate one year from today = 9.7%
  - 1-year forward rate two years from today = 11.3%
  - 2-year forward rate one year from today = 12.27%

  Which of the following statements about the forward rates, based on the bond prices, is true?
  A. The 1-year forward rate one year from today is too low.
  B. The 2-year forward rate one year from today is too high.
  C. The 1-year forward rate two years from today is too low.
  D. There's no opportunities for arbitrage.

- Answer: C

Exercise 4

- A 3-year bond with a current price of $105.91 pays a semi-annual coupon with a coupon rate of 5%. What is the bond's YTM?
  A. 1.97%
  B. 2.25%
  C. 2.93%
  D. 3.56%

- Answer: C

Exercise 5

- There are two U.S. Treasury bond. The first has a price of 99.98, matures in six months, and pays a semi-annual coupon at a rate of 3%. The second has a price of 101.11, matures in one year, and pays a semi-annual coupon at a rate of 4%. What are, respectively, the six-month and one-year discount factor?
  A. d(0.5) = 0.9790, d(1.0) = 0.9830
  B. d(0.5) = 0.9850, d(1.0) = 0.9720
  C. d(0.5) = 1.0020, d(1.0) = 0.9830
  D. d(0.5) = 0.9650, d(1.0) = 1.0340

- Answer: B

Exercise 6

- Three different U.S. Treasury notes pay semi-annual coupons and mature in exactly one year; i.e., each pays the next coupon in six months and matures six months subsequently. The price of Bond A with a coupon rate of 2.0% per annum is $99.02 and the price of Bond C with a coupon rate of 7.0% per annum is $103.91. If Bond B has a coupon rate of 4.0% per annum, what is the price of Bond B?
  A. $99.12
  B. $100.56
  C. $100.98
  D. $101.12

- Answer: C

Fixed-Income Products

Topic 3: Risk Metrics

1. One-Factor Risk Metrics
2. Multi-Factor Risk Metrics

One-Factor Risk Metrics

- **Duration**
  - **Macaulay Duration**
    - ✓ For a zero coupon bond, the Macaulay duration equals to its maturity.
    - ✓ For a plain bond, the Macaulay duration is less than or equal to its maturity.

$$\text{Mac. D} = \sum_{t=1}^{T}\left[\frac{PV(C_t)}{P}\times t\right] = \sum_{t=1}^{T}(\omega_t \times t)$$

  - **Modified Duration and Dollar Duration**
    - ✓ All else being equal, duration increase for longer maturities, lower coupons, and lower the yields.

$$MD = -\frac{\Delta P/P}{\Delta y} = \frac{\text{Mac. D}}{1+y} \qquad DD = -\frac{\Delta P}{\Delta y} = MD \times P$$

One-Factor Risk Metrics

- **Duration (cont'd)**
  - **DV01**

$$DV01 = MD \times P \times 0.0001$$

  - **Effective Duration**

$$D^E = \frac{P_- - P_+}{2P_0\Delta y}$$

- **Convexity**
  - Non-linear relationship
  - All else being equal, convexity increase for longer maturities, lower coupons, and lower the yields.
  - **Effective Convexity**

$$C^E = \frac{DD_- - DD_+}{P_0\Delta y} = \frac{P_- + P_+ - 2P_0}{P_0\Delta y^2}$$

$$\Delta P = -DP_0\Delta y + \frac{1}{2}CP_0(\Delta y)^2$$

One-Factor Risk Metrics

- **Convexity (cont'd)**
  - **Example:** Suppose there is a 10-year option-free noncallable bond with an annual coupon of 7% trading at par. If interest rates rise by 30 bps, estimated price is 97.922. If interest rates fall by 30 bps, estimated price is 102.137. Calculate the convexity of this bond.

$$\text{Convexity} = \frac{102.137 + 97.922 - 2\times 100}{100\times(0.003)^2} = 65.56$$

  - **Example:** Estimate the effect of a 100 bps increase and decrease on a 10-year, 5%, option-free bond currently trading at par, using the duration/convexity approach with a duration of 7 and a convexity of 90.

$$\text{percentage bond price change} \approx \text{duration effect} + \text{convexity effect}$$
$$\Delta B_{+\Delta y} \approx [-7\times 100\times 0.01] + [(1/2)\times 90\times 100\times 0.01^2] = -6.55$$
$$\Delta B_{-\Delta y} \approx [-7\times 100\times(-0.01)] + [(1/2)\times 90\times 100\times(-0.01)^2] = 7.45$$

One-Factor Risk Metrics

- **Negative Convexity**
  - Most mortgage bonds are negatively convex, and callable bonds usually exhibit negative convexity at lower yields.
  - For example, with a callable bond, as interest rate fall, the incentive for the issuer to call the bond at par increases; therefore, its price will not rise as quickly as the price of a non-callable bond. This is why the shape of callable bond's curve of price with respect to yield is concave or negatively convex.
- **Portfolio Duration and Convexity**
  - In regard to both modified (effective) duration and convexity, portfolio duration and convexity equal the weighted sum of individual, respectively, durations and convexities where each component's weight is its value as a percentage of portfolio value.

Multi-Factor Risk Metrics

- **Key Rate Shifts**
  - An approach take nonparallel shifts into account.
  - Allowing that changes in all rates can be determined by changes from selected key rates.
  - A few rates along the term-structure are picked which are representative of the curve.
  - Shifts in the key-rates are decline linearly.
  - The rate of a given maturity is affected solely by its closest key-rate.
- **Key Rate Duration**
  - Which is the key rate equivalent of durations.
- **Key Rate 01s**
  - Which is the key rate equivalent of DV01.

Exercise 1

- MTGE4. MTGE7. MTGE10 are mortgage-backed securities (MBS) that pay 4%. 7%. and 10% coupons, respectively prevailing mortgage rates are 10% Assuming these securities have the same maturity and coupon frequency, which of the following is correct?
  - A. In most cases, convexity is sufficient to approximate MBS price changes resulting from yield changes for purpose of estimating VaR.
  - B. In most cases, duration is sufficient to approximate MBS price changes resulting from yield changes for purpose of estimating VaR.
  - C. The Optionality embedded in a MBS makes the implementation of the duration-convexity method less appropriate the purpose of estimating VaR.
  - D. As rates fall, MTGE10 price change approximations using the duration-convexity method are likely to be better than MTGE4 price change approximations.
- Answer: C

Exercise 2

- The following table provides the initial price of a C-STRIP and its present value after application of a one basis shift in three key rates.

	Value
Initial value	24.122
2-year shift	24.126
5-year shift	24.127
10-year shift	24.073

① What is the key rate '01 for a 10-year shift?

A. 0.014 B. 0.016 C. 0.303 D. 0.049

② What is the key-rate duration for a 10-year shift?

A. 20.31 B. 23.450 C. 19.60 D. 36.14

- Correct Answer

① D ② A

Exercise 3

- An underlying exposure has a 5-year key-rate '01 of +$23,970. If this key rate exposure can be hedged by trading five-year bond that itself has a 5-year key rate '01 of $0.048 per 100 face amount, what is the hedge trade?

A. Buy $499.375 in face amount

B. But $49.94 million in face amount

C. Sell $499.375 in face amount

D. Sell $49.94 million in face amount

- Answer: D

Derivatives

Topic 1: Forward and Futures

1. Forward Market Characteristics
2. Futures Market Characteristics
3. Forward and Futures Prices
4. Interest Rate Futures
5. Hedging Strategies using Futures

Forward Market Characteristics

- **Forward Contract**
  - Agreement to buy/sell asset at future time for certain price.
  - Over-the-counter
- **Commodity Forward Contract**
  - **Commodity Terminology**
    - ✓ Storage Costs
    - ✓ Lease Rate
    - ✓ Convenience Yields
  - **Commodity Spread**
    - ✓ If we can take a long position in one commodity that is an input (e.g., oil) into another commodity that is an output (e.g., gas or heating oil), then we can take a short position in the output commodity and the difference is the commodity spread.
    - ✓ Can also formed by futures contract

Forward Market Characteristics

- **Commodity Forward Contract (cont'd)**
  - **Commodity Spread (cont'd)**
    - ✓ **Example:** A refiner in June 2010 planning for July production could have purchased July oil for $72.86/barrel and sold August gasoline and heating oil for $2.0279/gallon and $2.0252/gallon. The 3-2-1 crack spread is the gross margin from buying 3 gallons of oil and selling 2 gallons of gasoline and 1 of heating oil.
    - ✓ The 3-2-1 spread tells us the amount of profit that can be locked in:

the spread is

$$=2\times\$2.0279+\$2.0252-3\times\$72.86/42=0.8767$$

or $\$0.8767/3=\$0.29224/\text{gallon}$

Forward Market Characteristics

- **Financial Forward Contract**
  - **Forward Rate Agreement**
    - ✓ The long benefits from an increase in rates, and the short benefits from a fall in rates.
    - ✓ An FRA settles in one month on three-month LIBOR is called 1×4.
    - ✓ **Valuation**

$$V_{\text{receive fixed } R_K}=L(R_K-R_F)(T_2-T_1)e^{-R_2T_2}$$
$$V_{\text{pay fixed } R_K}=L(R_F-R_K)(T_2-T_1)e^{-R_2T_2}$$

Futures Market Characteristics

- **Futures Contract**
  - Like forward, agreement to buy/sell asset at certain price & time.
  - Standardized, trades on an exchange.
  - Require a daily settlement of gains and losses.
- **Trading Manner of a Futures Contract**
  - Close Out
  - Physical Delivery
  - Cash Settlement
  - Exchange for Physicals
- **Specifications of Futures Contract**
  - Contract Size

Treasury bond futures has a face value of $100,000;
S&P 500 Futures contract is index×$250 (multiplier of 250)
Eurodollar futures contract has a face value of $1 million

Futures Market Characteristics

- **Market Quotes**
  - **Opening Price:** prices at which contracts were trading immediately after the start of trading.
  - **Highest Price during the Day and Lowest Price during the Day**
  - **Settlement Price:** Price at which the contract traded immediately before the end of a day's trading session. It is the price used for calculating daily gains and losses and margin requirements.
  - **Trading Volume:** number of contracts traded in a day.
  - **Open Interest:** number of contracts outstanding, that is, the number of long positions or, equivalently, the number of short positions. If there is a large amount of trading by day traders the volume of trading in a day can be greater than either the beginning-of-day or end-of-day open interest.

Futures Market Characteristics

- **Margin Requirement**
  - **Initial Margin**
  - ✓ Must be deposited when contract is initiated.
  - **Maintenance Margin**
  - ✓ Investor can withdraw funds in the margin account in excess of the initial margin. When the balance in the margin account falls below the maintenance margin, broker executes a **margin call**. The next day, the investor needs to "top up" the margin account back to the initial margin level.
  - **Variation Margin**
  - ✓ Extra funds deposited by the investor after receiving a margin call.
  - ✓ Variation margin = initial margin – margin account balance

Futures Market Characteristics

- **Margin Requirement (cont'd)**
  - **Example:** A manager long a gold futures contract at $983.60. Each contract controls 100 troy ounces for a current market value of $98,360. Assume that the initial margin is $2,500, the maintenance margin is $2,000, and the futures price drops to $981 at the end of the first day and $973 at the end of the second day. Compute the amount in the margin account at the end of each day for the long position and any variation margin needed.
  - ✓ At the end of the first day, the loss is computed as -$260. The manager's margin account balance is now $2,240.
  - ✓ At the end of the second day, the daily loss is -$800, and the manager's margin account balance is reduced to $1,440. The manager will get a margin call since the margin account balance is less than the maintenance margin. The variation margin is $1060.

Futures Market Characteristics

- **Trading Order**
  - **Market Order:** at the best price available in the market
  - **Limit Order:** at this price or at one more favorable
  - **Stop Order/Stop-Loss Order:** at the best available price once a bid or offer is made at that particular price or a less-favorable price
  - **Stop-Limit Order:** a combination of a stop order and a limit order
  - **Market-if-Touch Order:** at the best available after a trade occurs at a specified price/at a price more favorable than the specified price
  - **Discretionary Order/Market-not-Held Order:** execution may be delayed at the broker's discretion in an attempt to get a better price
  - **Fill-or-Kill:** must be executed immediately on receipt or not at all

Forward and Futures Prices

- **Cost of Carry Model**
  - This model sets a futures price as a function of the spot price: the futures price equals the spot price compounded at the interest rate plus the storage cost of the asset less any income earned on the asset.

$$F_0 = S_0 e^{rT}$$

$$F_0 = S_0 e^{(r+u-q-y)T}$$
$$F_0 = (S_0 + U - I)e^{(r-y)T}$$

$$F_0 = S_0 e^{(r-q)T}$$
$$F_0 = (S_0 - I)e^{rT}$$

- **Interest Rate Parity**
  - A no-arbitrage condition representing an equilibrium state under which investors will be indifferent to interest rates available on deposits in two countries.

$$\text{Given } S_0 \text{ A/B}, F_0 = S_0 \left(\frac{1+r_A}{1+r_B}\right)^T \text{ or } F_0 = S_0 e^{(r_A - r_B)T}$$

Forward and Futures Prices

- **Normal and Inverted Futures Market**
  - If the forward price is higher than the spot price (or the distant forward price is higher than the near forward price) the Futures curve is said to be **normal**, or in **Contango**.
  - If the forward price is less than the spot price (or the distant forward price is less than the near forward price), the Futures curve is said to be **inverted**, or in **Backwardation**.
- **Valuing the Forward Contract**
  - If today appears to be the day when the contract is first negotiated, the delivery price K is set equal to the forward price F_0 and the value of the contract f is 0.

$$f = (F_0 - K)e^{-rT} = S_0 - Ke^{-rT}$$

Interest Rate Futures

- **T-Bond Futures**
  - In Treasury bonds futures contract, any government bond that has between 15 and 25 years to maturity on the first day of the delivery month can be delivered.
  - CBOT has created conversion factors since the deliverable bonds have very different market values.
  - Specially, the cash received by the short position is:

    Cash received = (QFP×CF) + AI
  - **Cheapest-to-Deliver Bond**

$$\text{Cost} = \text{QBP} - (\text{QFP} \times \text{CF})$$

Interest Rate Futures

- **Eurodollar Futures**
  - Three-Month
  - 1 million

$$P = 10{,}000 \times [100 - 0.25(100 - FQ_t)] = 1{,}000{,}000 \times (1 - 0.25F_t)$$

  - 1 basis point up move in the futures quote corresponds to a gain of $25 per contract for long position.
- **Eurodollar Futures vs. FRA**
  - **Difference between Forward and Futures Price**
    - ✓ ρ(S, r) > 0, Futures are more attractive by two reasons: daily settlement and reinvest profit. Futures price is higher than forward price.
    - ✓ ρ(S, r) < 0, Futures price is lower than forward price.
    - ✓ For short maturities, the differences are small enough to be ignored.
  - **Convexity Adjustment**

$$\text{Forward Rate} = \text{Futures rate} - \frac{1}{2}\sigma^2 T_1 T_2$$

Hedging Strategies using Futures

- **Short Hedge and Long Hedge**
  - **Short hedge** occurs when hedger shorts (sells) futures contract to hedge against a price decrease in the existing long position.
  - **Long hedge** occurs when hedger buys futures contract to hedge against an increase in the value of the asset that underlies a short position.
- **Strip Hedge and Stack Hedge**
  - **Strip Hedge:** Hedge a stream of obligations by offsetting each individual obligation with futures contract matching maturity and quantity of the obligation.
  - **Stack Hedge:** Hedge using futures with single maturity to offset changes in the present value of the future obligations.
- **Minimum Variance Hedge Ratio**

$$h=\rho_{S,F}\frac{\sigma_S}{\sigma_F}$$

- **Effectiveness of the Hedge**
  - Coefficient of Determination (R^2)/Correlation coefficient (ρ^2)

Hedging Strategies using Futures

- **Hedging with Stock Index Futures**

$$\text{number of contracts} = (\beta^* - \beta) \times \frac{\text{portfolio value}}{\text{value of futures contract}}$$
$$= (\beta^* - \beta) \times \frac{\text{portfolio value}}{\text{futures price} \times \text{contract multiplier}}$$

- **Hedging with Interest Rate Futures**

$$N^* = \frac{PD_P}{FD_F}$$

- **Tailing the Hedge**
  - To correct for the possibility of over-hedging, a hedger can implement a tailing the hedge strategy. It is to multiply the hedge ratio by the daily spot price to futures price ratio.

Hedging Strategies using Futures

- **Basis Risk**
  - **Basis:** Difference between the price of the futures contract and the spot price of the underlying asset.

Basis = spot price – futures price

  - **Long the Basis**
  - ✓ A short futures position and a long cash position
  - ✓ Position that are long the basis benefit when the basis is strengthening.
  - **Short the Basis**
  - ✓ A long futures position and a short cash position
  - ✓ Positions that are short the basis benefit when the basis is weakening.
  - **Basis risk**
  - ✓ Risk (to the hedger) created by the uncertainty in the basis
  - Different asset (Cross Hedging)
  - Different maturity

Exercise 1

- Assume that the 3-month and 6-month LIBOR spot rates are 4% and 5% respectively (continuously compounded). An investor enters into an FRA in which she will receive 8% (quarterly compounding) on a principal of $5,000,000 between months 3 and 6. Calculate the value of an FRA.
  A. $23,773
  B. $24,773
  C. $25,773
  D. $26,773

- Answer: A

Exercise 2

- The December oil futures price is $100 per barrel ($2.38 per gallon). The December unleaded gasoline futures price is $2.70 per gallon, and the December heating oil futures price is $2.80 per gallon. What is the gross margin that can be locked-in with a"3-2-1" crack spread trade?
  A. Zero
  B. $0.56
  C. $1.06
  D. $2.40

- Answer: C

Exercise 3

- The spot exchange rate EUR/CHF is CHF 1.20. Interest rates are flat at 2.0% in the Eurozone (EUR) and 5.0% in Switzerland (CHF). According to interest rate parity (IRP), what should be the 12-month EUR/CHF forward exchange rate if we assume annual (discrete) compounding?
  A. EUR/CHF 1.14
  B. EUR/CHF 1.20
  C. EUR/CHF 1.24
  D. EUR/CHF 1.28

- Answer: C

Exercise 4

- A European firms needs to hedge the Mexican pesos in six months, but peso futures are not liquid. So the firm decided to hedge its exposure by buying futures contract on USD. The standard deviation of pesos against the Euros over a six-month period is 18%, while the standard deviation of USD/EUR futures price over a six-month period is 10%. If the correlation coefficient between pesos and dollars is 0.65, calculate the optimal hedge ratio.
  A. 0.15
  B. 0.36
  C. 1.17
  D. 2.77
- Answer: C

Derivatives

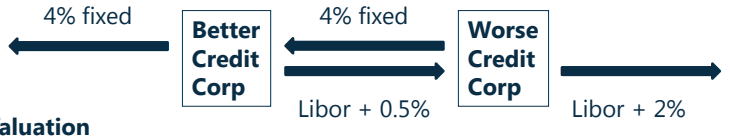
Topic 2: Swap Market

1. Interest Rate Swap
2. Currency Swap

Interest Rate Swap

- **Comparative Advantage Argument**

	Fixed	Floating	Comparative Advantage
BetterCreditCorp	4%	Libor +1%	Fixed Market
WorseCreditCorp	6%	Libor + 2%	Floating Market



- **Valuation**

$V_{swap} = B_{Float} - B_{Fixed}$ $V_{swap} = B_{Fixed} - B_{Float}$

  - Notes: **Value of floating rate bond** will be equal to the notional amount at periodic settlement dates or it can be treated as a single cash flow: the sum of the next coupon plus a bond that must trade at par immediately after paying the coupon.

Currency Swap

- **Comparative Advantage Argument**

	USD	AUD	Comparative Advantage
General Electric	5%	7.6%	USD Market
Qantas Airways	7%	8%	AUD Market

- **Valuation**

$$V_{Swap} = B_D - S_0 B_F$$
$$V_{swap} = S_0 B_F - B_D$$

Exercise 1

- ABC corp. entered into a swap agreement over a 2-year period on August 9, 2014, with which it received a 4% fixed rate and paid LIBOR plus 1.2% on a notional amount of USD 6.5 million. Payments were to be made every 6 months. The table below displays the actual annual 6-month LIBOR rates over the 2-year period:

Date	6-month LIBOR
Aug 9, 2014	3.11%
Feb 9, 2015	1.76%
Aug 9, 2015	0.84%
Feb 9, 2016	0.39%
Aug 9, 2016	0.58%

Assuming no default, how much did corp. receive on August 9, 2016?

A. USD 72,150
B. USD 78,325
C. USD 117,325
D. USD 156,650

- Answer: B

Exercise 2

- interest rate swap with a notional of $100 million has a remaining life of nine (9) months. Under the swap, 6-month LIBOR is exchanged for 3.0% per annum (compounded semiannually). Three months ago (t - 0.25 years) the 6-month LIBOR rate was 2.0%. Currently, the swap rate curve is flat at 1.0% for all maturities; e.g., the three- and nine-month LIBOR rates are 1.0% per annum (compounded continuously). What is the current value of the swap to the counterparty who is paying fixed?

A. - $2.99 million
B. - $1.49 million
C. + $2.99 million
D. +$1.49 million

- Answer: B

Derivatives

Topic 3: Option Market

1. Properties of Stock Options
2. Trading Strategies involving Options
3. Exotic Options
4. Valuation
5. Greek Letters

Properties of Stock Options

- **Basics**
  - Call and Put Options
  - European and American Option
  - ✓ It is never optimal to execute an early exercise on an American call option on a non-dividend paying stock.
- **Moneyness**
  - **In the money:** Immediate exercise would generate a positive payoff
  - **At the money:** Immediate exercise would generate no payoff
  - **Out of the money:** Immediate exercise would result in a loss
- **Intrinsic Value and Time Value**
  - **Intrinsic Value:** The amount that it is in the money, and zero otherwise
  - ✓ Intrinsic value of call option: C=max [S – X, 0]
  - ✓ Intrinsic value of put option: P=max [X – S, 0]
  - **Time Value:** The difference between the price of an option (called its premium) and its intrinsic value is due to its time value.

Properties of Stock Options

- **Upper and Lower Bounds for Option Prices**

Option	Proxy	Min Value	Max Value
European call	c	max (0, S_0— Ke^{-rT})	S_0
American call	C	max (0, S_0— Ke^{-rT})	S_0
European put	p	max (0, Ke^{-rT} —S_0)	Ke^{-rT}
American put	P	max (0 , K—S_0)	K

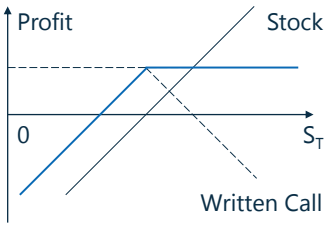
- **Put-Call Parity**

$$C + Ke^{-rT} = p + S_0 \qquad p + S_0 = c + D + Ke^{-rT}$$

$$S - K \le C - P \le S - Ke^{-rT} \qquad S_0 - K - D \le C - P \le S_0 - Ke^{-rT}$$

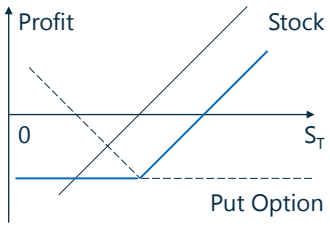
Trading Strategies involving Options

- **Simple Strategies**



Covered Call = -C + S

- Income Strategy
- Outlook is neutral to bullish

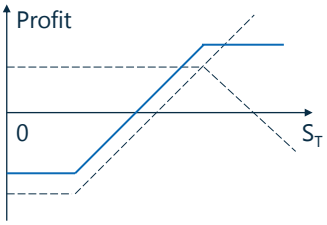


Protective Put = S + P

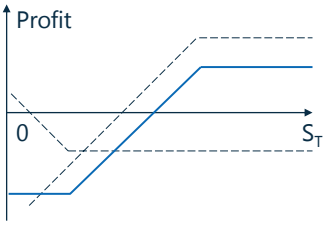
- Insurance Strategy

Trading Strategies involving Options

- **Spread Strategies**



Bull Call Spread

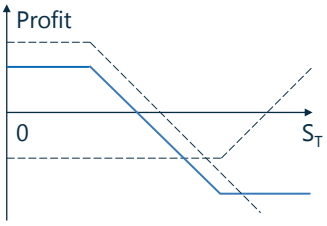


Bull Put Spread

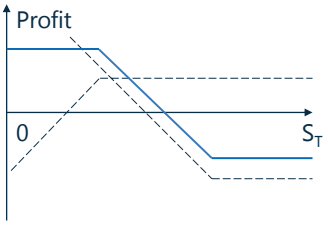
- Vertical spread
- Outlook is bullish

Trading Strategies involving Options

- **Spread Strategies (cont'd)**



Bear Call Spread

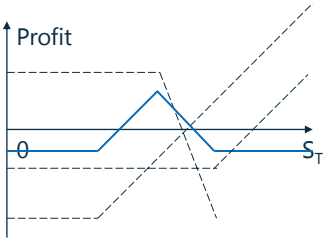


Bear Put Spread

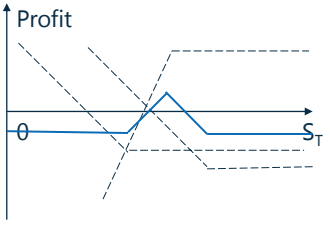
- Vertical spread
- Outlook is bearish

Trading Strategies involving Options

➢ **Spread Strategies (cont'd)**



Butterfly Spread



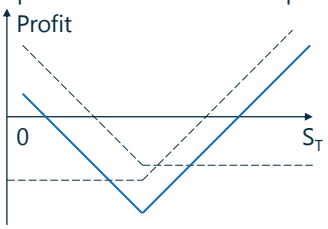
Butterfly Spread

- Expects low volatility
- Capped risk

Trading Strategies involving Options

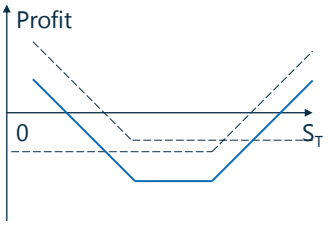
➢ **Combination Strategies**

- A Combination is an option trading strategy that involves taking a position in both calls and puts on the same stock.



Straddle

- A call and a put
- Same strike price
- Direction neutral
- Wants volatility

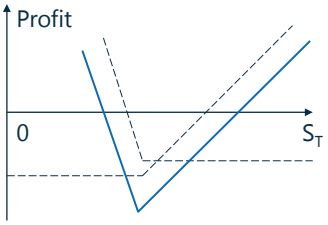


Strangle

- A call and a put
- Different strike price
- Like straddle, but cheaper

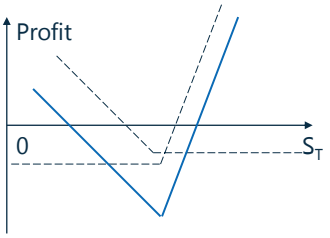
Trading Strategies involving Options

➢ **Combination Strategies (cont'd)**



Strip

- Two puts and one call
- Bet on volatility
- More bearish



Strap

- Two calls and one put
- Bet on volatility
- More bullish

Exotic Options

- **Compound Options**
  - Options on options
  - A call on a call, a put on a call, a call on a put, and a put on a put
  - Large leverage and they are cheaper than straight options
  - If both options are exercised, the total premium will be more than the premium on a single option
- **Chooser Option**
  - After a specified period of time, the holder can choose whether the option is a call or a put.

$$\max(c,p) = \max\left[c, c + Ke^{-r(T_2-T_1)} - S_1\right] = c + \max\left[Ke^{-r(T_2-T_1)} - S_1, 0\right]$$
$$= \max\left[p, p + S_1 - Ke^{-r(T_2-T_1)}\right] = p + \max\left[S_1 - Ke^{-r(T_2-T_1)}, 0\right]$$

Exotic Options

- **Barrier Options**
  - Payoffs and existence depend on whether the underlying's asset price reaches a certain barrier level over the life of the option.
  - less expensive
  - A **knock-out** option ceases to exist when the underlying asset price reaches a certain barrier while a **knock-in** option comes into existence only when the underlying asset price reaches a barrier.
  - **In-out parity**
  - ✓ If we combine one "in" option and one "out" barrier option with the same strikes and expirations, we get the price of a vanilla option. Note that this argument only works for European options.
  - Path-dependent

Exotic Options

- **Binary Options/Digital Option**
  - **Cash-or-Nothing**
  - ✓ Pays some fixed amount of cash if the option expires in-the-money.

$$\text{Call: } Qe^{-rT}N(d_2) \quad \text{Put: } Qe^{-rT}N(-d_2)$$

  - **Asset-or-Nothing**
  - ✓ Pays the value of the underlying security.

$$\text{Call: } S_0e^{-qT}N(d_1) \quad \text{Put: } Se^{-qT}N(-d_1)$$

  - A regular European call option is equivalent to a long position in an asset-or-nothing call and a short position in a cash-or-nothing call where the cash payoff in the cash-or-nothing call equals the strike price.
  - Similarly, a regular European put option is equivalent to a long position in a cash-or-nothing put and a short position in an asset-or-nothing put where the cash payoff on the cash-or-nothing put equals the strike price.

Exotic Options

- **Lookback Options**
  - Payoffs depend on maximum or minimum price of the underlying asset
  - With **floating strike** and with **fixed strike**.

$$\text{Call}_{\text{floating strike}} = \text{Max}(S_T - S_{\min}, 0) \quad \text{Put}_{\text{floating strike}} = \text{Max}(S_{\max} - S_T, 0)$$
$$\text{Call}_{\text{fixed strike}} = \text{Max}(S_{\max} - k, 0) \quad \text{Put}_{\text{fixed strike}} = \text{Max}(K - S_{\min}, 0)$$

- **Asian Options**
  - Payoff depends on arithmetic average of the underlying asset price
  - **Average price option** and **average strike option**.

$$\text{Call}_{\text{average price}} = \text{Max}(S_{\text{avg}} - K, 0) \quad \text{Put}_{\text{average price}} = \text{Max}(K - S_{\text{avg}}, 0)$$
$$\text{Call}_{\text{average strike}} = \text{Max}(S_T - S_{\text{avg}}, 0) \quad \text{Put}_{\text{average strike}} = (S_{\text{avg}} - S_T, 0)$$

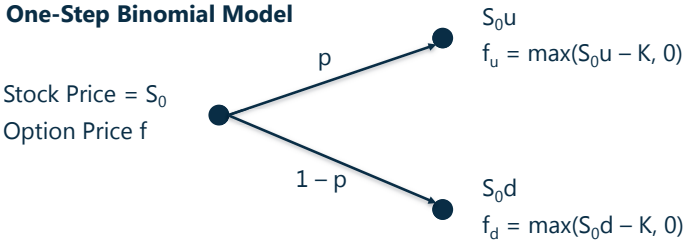
- **Shout Options**
  - Option holder receives either the usual payoff from a European option or the intrinsic value at the time of the shout, whichever is greater.

Exotic Options

- **Volatility and Variance Swap**
  - **Volatility Swap**
    - ✓ Exchanging of volatility based on a national principal
    - ✓ Payments base on pre-specified volatility and historical volatility.
  - **Variance Swap**
    - ✓ Exchanging pre-specified fixed variance rate for realized variance rate
    - ✓ Easier to price and hedge than volatility swap since they can be replicated using a collection of regular calls and puts.
- **Static Options Replication**
  - This technique involves searching for a portfolio of actively traded options (regular options) that approximately replicates the exotic option. Shorting this position provides the hedge.

Valuation

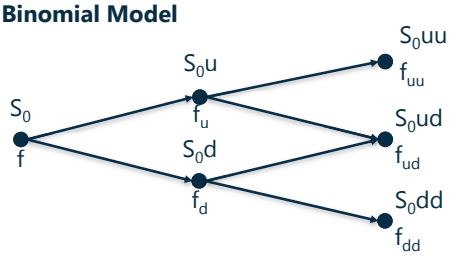
- **Binomial Trees**
  - **Assumption**
    - ✓ The stock price follows geometric Brownian motion
    - ✓ Risk neutral;
    - ✓ u=1/d;
  - **One-Step Binomial Model**



$$p = \frac{e^{r\Delta t} - d}{u - d} \text{ or } p = \frac{e^{(r-q)\Delta t} - d}{u - d} \quad f = [pf_u + (1-p)f_d]e^{-r\Delta t} \quad u = e^{\sigma\sqrt{\Delta t}}; d = e^{-\sigma\sqrt{\Delta t}}$$

Valuation

- Binomial Trees (cont'd)
  - Two-Step Binomial Model



$$p = \frac{e^{r\Delta t} - d}{u - d}$$

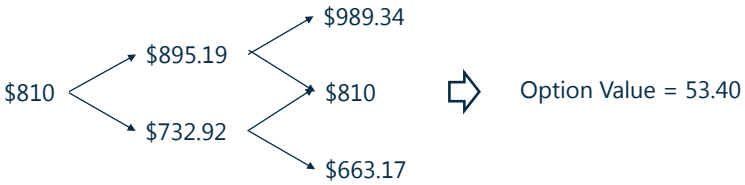
$$f = e^{-2r\Delta t}[p^2 f_{uu} + 2p(1-p)f_{ud} + (1-p)^2 f_{dd}]$$

Valuation

- Binomial Trees (cont'd)
  - Example: 2-step European Call, Up and Down Informed by Volatility

Asset	Strike	Time	Volatility	Riskless	Div. Yield
$810	$800	0.5	20%	5%	2%

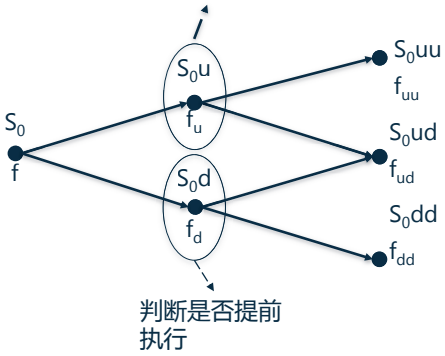
u	d	p
1.1052	0.9048	0.5126



Valuation

- Binomial Trees (cont'd)
  - American Options

判断是否提前执行



判断是否提前执行

Make sure that the option value at each node is no less than the intrinsic value.

Valuation

- **Binomial Trees (cont'd)**
  - **Example: American Put Option with Price Jump of +/-20%**

Asset	Strike	Time	Riskless	Div. Yield
$50	$52	2	5%	0%

⇩

u	d	p
1.2	0.8	0.6282

Valuation

- **Black-Scholes-Merton Model**
  - **Assumptions**
    - ✓ The stock price follows a lognormal distribution and is continuous. Jump is not considered.
    - ✓ The risk-free rate and volatility of the stock is known and constant.
    - ✓ There are no transaction costs or taxes.
    - ✓ There are no cash flows on the stock.
    - ✓ European options
  - **Valuation**

$$c = Se^{-qT}N(d_1) - Ke^{-rT}N(d_2)$$
$$p = Ke^{-rT}N(-d_2) - Se^{-qT}N(-d_1)$$

$$d_{1,2} = \frac{\ln(Se^{-qT}/Ke^{-rT})}{\sigma\sqrt{T}} \pm \frac{\sigma\sqrt{T}}{2}$$

  - **The early exercise of American options**

American call: $D_n > X\left(1 - e^{-r(T-t_n)}\right)$ American put: $D_n < X\left(1 - e^{-r(T-t_n)}\right)$

Valuation

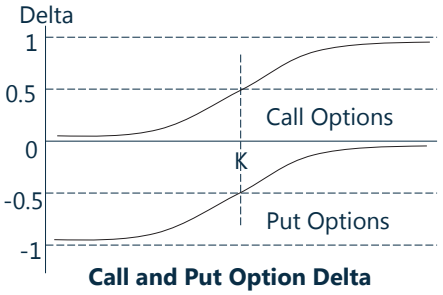
- Warrants and employee stock options are different from regular call options in that exercise leads to the company issuing more shares and then selling them to the option holder for the strike price.
- As the strike price is less than the market price, this dilutes the interest of the existing shareholders.
- Suppose the company has N shares worth S_0 each and the number of new options contemplated! is M, with each option giving the holder the right to buy one share for K. The value of each option is the value of

$$\frac{N}{N+M}$$

- Regular call options on the company's stock. Therefore the total cost of the options is M times this.

Greek Letters

- **Impact of Underlying Asset Price – Delta**
  - The delta of an option, Δ, is defined as the ratio of change in option price to change in underlying asset price.
  - Call option Δ range from 0 to 1
  - Put option Δ range from -1 to 0
  - When t→T, delta is unstable



Call and Put Option Delta

Long Call	Δ > 0
Short Call	Δ < 0
Long Put	Δ < 0
Short Put	Δ > 0

Greek Letters

- **Impact of Underlying Asset Price – Delta (cont'd)**
  - **According to the BSM Model, option delta is as follow**

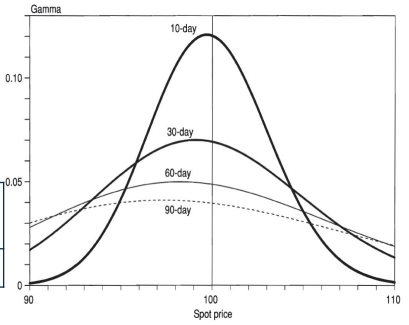
$$\Delta = \frac{\partial c}{\partial S} = e^{-qT}N(d_1) \quad \Delta = \frac{\partial p}{\partial S} = e^{-qT}[N(d_1) - 1]$$

  - **Portfolio Delta:** summation of product of each position and its delta.
- **Delta Hedge**
  - A position with a delta of zero is called a delta neutral position.
  - To maintain a delta neutral position, trader must re-balance the portfolio. If gamma is higher (such as the case when options are at-the-money), rebalancing more frequently is required. On the other hand, when gamma is lower (such is the case when options are deeply in the money/deeply out of the money), rebalancing is required less frequently.
  - Hedge against small changes in asset price.

Greek Letters

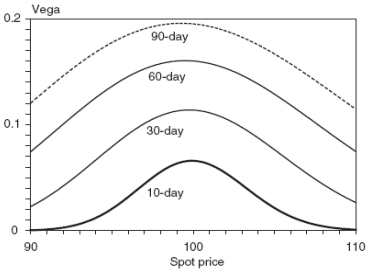
- **Impact of Underlying Asset Price – Gamma**
  - Rate of delta change with respect to price change of underlying asset.
  - Providing added protection against large movements.
  - If gamma is large, delta is very sensitive to the price change of the underlying asset.
  - Largest when is at-the-money.
  - Same for call and put options.
- **Gamma Hedge**
  - Hedge against larger changes

Long Call	Short Call	Long Put	Short Put
γ > 0	γ < 0	γ > 0	γ < 0



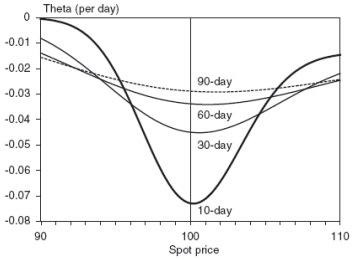
Greek Letters

- **Delta and Gamma Hedge**
  - **Example:** Suppose that a portfolio is delta neutral and has a gamma of -3,000. The delta and gamma of a particular traded call option are 0.62 and 1.50, respectively. Create a gamma-neutral position.
    - ✓ Buy 3,000/1.5 = 2,000 options
    - ✓ Sold 2,000 ×0.62 = 1,240 shares of the underlying position
  - You can also create a gamma needed position, e.g., gamma of -6,000.
- **Impact of Volatility – Vega**
  - Rate of change of the value of the option with respect to the volatility of the underlying asset.
  - Largest when is at-the-money.
  - Same for call and put options.



Greek Letters

- **Impact of Maturity – Theta**
  - Rate of change of the value of option with respect to the passage of time.
  - Time decay. As time to maturity decreases, option tends to become less valuable, so theta is usually negative for an long position, means option lose value as time goes by.
  - Short-term at the money option has a greatest negative theta.



- **Impact of Interest Rate – Rho**
  - Sensitivity to the interest rate
  - In the money calls and puts are more sensitive to changes in rates than out-of-the-money options.

Exercise 1

- Vega is the sensitivity of an option's price to changes in volatility. Increases in an underlying instrument's volatility will usual increase the value of options since increases in volatility produce a greater probability that an option will find its way into the money. Of the four options listed below, which investment has the potential to produce a negative Vega measure?
  - A. Shout Option
  - B. Call Option
  - C. Put Option
  - D. Barrier Option

- Answer: D

Exercise 2

- A cash-or-nothing call (also known as a digital call) pays a fixed amount to the buyer if the asset finishes above the strike price. Assume that at the end of a 1-year investment horizon, the stock is equal to $50, the fixed payment amount is equal to $45, and $N(d_1)$ and $N(d_2)$ from the Black-Scholes-Merton model are equal to 0.9767 and 0.9732, respectively. The value of this cash-or-nothing call when the risk-free rate equals 3% is closest to:
  A. $5
  B. $42
  C. $44
  D. $47

- Answer: B

Exercise 3

- A stock is currently trading at USD 45, and its annual price volatility is 30%. The risk-tree rate is 1.5% per year. A risk manager is developing a 1-step binomial tree for a 2-year horizon. What is the risk-neutral probability that the stock will move down?
  A. 30%
  B. 43%
  C. 57%
  D. 70%

- Answer: C

Exercise 4

- GF Corporation is a non-dividend-paying stock that is currently priced at $48. An analyst has determined that the annual standard deviation of returns on GF stock is 8% and that the annual risk-free interest rate on a continuously compounded basis is 6%. Calculate the value of a 12-month American call option on GF stock with a strike price of $50 using a two-period binomial model.
  A. $1.25
  B. $2.01
  C. $2
  D. $2.21

- Answer: B

Exercise 5

- Stock XYZ trades for $50 and has 1-year call and put options written on it with an exercise price of $50. The annual standard deviation estimate is 10%, and the continuously compounded risk-free rate is 5%. The value of both the call and put using the BSM option pricing model are closest to:

	Call	Put
A.	$4.12	$1.11
B.	$3.4	$0.96
C.	$3.4	$1.11
D.	$4.12	$0.96

z	0.04	0.05	0.06
0.4	0.6700	0.6736	0.6772
0.5	0.7054	0.7088	0.7123

- Answer: B

Exercise 6

- An investor holds an American call option on a dividend paying stock with the following characteristics
  - Current stock price ,S=USD 50
  - Strike price, K=USD 50
  - Time to expiration ,T=2 months

  A divided, D, of USD 1 per share has just been announced ,with an ex-dividend date, t, of one month from now, Assuming the risk-free rate, r, is 1.5% and the option stays at-the-money, is it optimal to exercise the option right before the ex-dividend date?
  A. Yes, because $S < K*e^{(-r(T-t))} + D$
  B. Yes, because $D > K*(1-e^{(-r(T-t))})$
  C. No, because the call option is at-the-money ,and early exercise is only optimal when it is deep in-the-money
  D. No, because unlike an American put option, it is never optimal to exercise an American call option early.
- Answer: B

Exercise 7-8

- Which of the following is true regarding options' Greeks?
  A. Theta tends to be large and positive when buying at-the-money options.
  B. Gamma is greatest for in-the-money options with long maturities.
  C. Vega is greatest for at-the-money options with long maturities.
  D. Delta of deep in-the-money put options tends towards +1
- Answer: C

- How can a trader produce a short Vega, long gamma position?
  A. Buy short-maturity options, sell long-maturity options.
  B. Buy long-maturity options, sell short-maturity options.
  C. Buy and sell options of long maturity.
  D. Buy and sell options of short maturity.
- Answer: A

Exercise 9

- A delta-neutral position exhibits a gamma of -5000. An existing option with a delta equal to 0.6 exhibits a gamma of 1.5. Which of the following will generate a gamma-neutral position for the existing portfolio?
  A. Buy 3,333 of the available options.
  B. Sell 3,333 of the available options.
  C. Buy 3,014 of the available options.
  D. Sell 3,014 of the available options.

- Answer: A

Financial Institutions

Topic 1: Central Counterparties

1. Exchanges and OTC Market
2. Central Counterparty

Exchanges and OTC Market

- **Exchanges Market**
  - **Clearing Function**
    - ✓ Reconciling and resolving of contracts between counterparties
    - ✓ Becomes counterparty to all transactions
    - ✓ Complete clearing
- **OTC Market**
  - **5 Broad Classes**
    - ✓ Interest rate, foreign exchange, equity, commodity, credit derivatives.
  - **Main Risk**
    - ✓ Counterparty Risk
    - ✓ Systemic Risk
  - **Ultimate Solution to Risk Management**
    - ✓ Have the means in place to manage periodic failures in a controlled manner, which is one role of a CCP.

Exchanges and OTC Market

- **Bilateral Clearing in OTC Market**
  - Those OTC transactions that are not cleared through CCPs are cleared bilaterally. In the bilaterally cleared OTC market, two companies usually enter into a master agreement covering all their trades.
  - The master agreement usually includes an annex, referred to as the credit support annex or CSA, requiring the counterparties to provide collateral. The collateral is similar to the margin required by exchange clearing houses or CCPs from their member.
  - From 2016, regulations require both initial margin and variation margin to be provided for bilaterally cleared transactions between financial institutions.

Central Counterparty

- **Membership**
  - General Clearing Member (GCM)
  - Individual Clearing Member (ICM)
  - Non-Clearing Member (NCM)
- **Major Role of CCP**
  - Reduce the interconnectedness within financial markets
  - Provide more transparency
  - Counterparty risk mitigation
- **Risk Management Function**
  - Novation
  - Netting
  - Loss Sharing Model/Default Funds
  - Margin Requirement
  - Auction Process

Central Counterparty

- **Margin Requirement**
  - CCPs usually set margin levels solely on the risks of the transactions held in each member's portfolio.
  - **Variation Margin**
  - ✓ Dynamic
  - ✓ Daily mark-to-market valuation
  - ✓ Relating to daily payment of profits and losses
  - ✓ Has to be transferred on a daily or even intra-daily basis, and must usually be in cash
  - **Initial Margin**
  - ✓ Potential close out cost of positions that a CCP could experience when a member defaulted.

Central Counterparty

- **Types of Risks Faced by CCPs**
  - **Default Risk**
  - ✓ Default of a clearing member ✓ Possible associated or knock-on effects
  - **Liquidity Risk**
  - ✓ Due to large quantities of cash that flow through them due to variation margin payments and other cash flows.
  - ✓ In the event of a default, the CCP must continue to fulfil its obligations to surviving members in a timely manner.
  - **Model Risk**
  - ✓ Valuation models are required to mark-to-market products for **variation margin** purposes. For **initial margin** modelling, problems could arise from misspecification with respect to volatility, tail risk, dependencies and wrong-way risk.

Central Counterparty

- **Types of Risks Faced by CCPs (cont'd)**
  - **Operational Risk**
  - ✓ Such as systems failures and fraud
  - **Legal Risk**
  - ✓ Aspects such as segregation and the movement of margin and positions can be subject to legal risk from laws in different jurisdictions.
  - **Investment Risk**
  - ✓ Losses from investments of cash and securities held as margin
  - **Other Risks**
  - ✓ Settlement and payment risk, foreign exchange risk, custody risk, concentration risk, sovereign risk, and wrong-way risk.

Central Counterparty

- **Lessons Learned from Prior CCP Failures**
  - **Margin Requirement**
  - ✓ **Variation margins** should be recalculated frequently and collected promptly (intradaily in volatile markets).
  - ✓ **Initial margin** and **default funds** should be resilient to large negative asset shocks or gaps in market variables and to extreme dependency.
  - **Large Positions Management**
  - ✓ A CCP should carefully monitor positions, penalize concentration and act quickly in the case of excessively large positions.
  - ✓ Operational risk must be controlled as much as possible.
  - **Liquidity Risk Management**
  - ✓ A CCP should have availability to external liquidity sources since it could otherwise default due to being illiquid but not insolvent.

Central Counterparty

- **Disadvantages of OTC Central Clearing**
  - CCPs will not so much reduce counterparty risk but rather distribute it and convert it into different forms.
  - CCPs also concentrate these risks in a single place and therefore magnify the systemic risk linked to their own potential failure.
  - OTC markets have proved that they are a good source of financial innovation and can continue to offer cost-effective and well-tailored risk reduction products. There is a risk that mandatory central clearing has a negative impact on the positive role that OTC derivatives play.
  - Moral hazard
  - Adverse selection
  - Bifurcations
  - Procyclicality

Financial Institutions

Topic 2: Banks

1. Service Features
2. Essentials of Management

Service Features

- **Investment Banking**
  - **Private Placement**: large institutional investors
  - **Public Offering**: general public
  - ✓ **Best Efforts**
  - Does as well as it can ; Paid a fee depends on its success.
  - ✓ **Firm Commitment**:
  - Agrees to buy the securities and then attempts to sell them
  - **Initial Public Offering (IPO)**:
  - ✓ Issues shares is note publicly traded
  - ✓ **Dutch auction approach:** Shares are first issued to the highest bidder, then to the next highest bidder, and so on, until all the shares have been sold. The price paid by all successful bidders is the lowest bid that leads to a share allocation.

Service Features

- **Originate-to-Distribute Model**
  - Originating but not keeping loans. Portfolios of loans are packaged into tranches which are then sold to investors. Also termed **securitization**.
  - **Benefits**
  - ✓ Gets loans off the balance sheet; Frees up funds; Frees up capital
  - ✓ Earn a further fee if it services the loan after it has been sold
  - **Drawbacks**
  - ✓ Transfers the prepayment risk from the bank to investors
  - ✓ When mortgages are guaranteed by GNMA, FNMA, or FHLMC credit risk may be avoided but in many cases where there is no such guarantee, investors bear the credit risk.
  - ✓ May relax the mortgage lending standards. Credit quality of the instruments being originated may decline sharply.

Essentials of Management

- **Conflicts of Interest Problem**
  - **Main Problems**
  - ✓ **Between Security Trading and Investment Banking**
  - Tempted to recommend securities that the investment banking part is trying to sell or stuff difficult-to-sell securities into the fiduciary account
  - Tempted to recommend a company's share as a "buy" to please the company and obtain investment banking business
  - ✓ **Between Commercial Banking and Investment Banking**
  - Tempted to pass confidential information to mergers and acquisitions arm to help it provide advice on potential takeover opportunities
  - Tempted to ask the investment bank to arrange a bond issue for the company in order to replace its loan with a loan made by investors who were less well-informed
  - **Recommend Solutions - Internal Barriers (Chinese Walls)**
  - ✓ Prohibit the transfer of information from one part of the bank to another

Essentials of Management

- **Banking Book and Trading Book**
  - **Banking Book**
  - ✓ Includes loans made to corporations and individuals
  - ✓ Not marked to market
  - Principal and interest payment on time: principal + accrued interest
  - More than 90 days past due (non-performing loan): principal
  - Principal will not be repaid (loan loss): charged against reserves
  - **Trading Book**
  - ✓ Includes all assets & liabilities bank has as a result of trading operations
  - ✓ Value are marked to market daily
- **Capital Management**
  - **Regulatory Capital**
  - ✓ Requirement for central bank regulators
  - **Economic Capital**
  - ✓ Own management requirement

Essentials of Management

- **Deposit Insurance**
  - Guaranty programs introduced by government regulators
  - Insure depositors against losses up to a certain level
  - **Moral Hazard Problem**
  - ✓ The possibility that the existence of insurance changes the behavior of the insured party
  - ✓ Deposit insurance allowed banks to follow risky strategies that would not otherwise be feasible.
  - ✓ The introduction of risk-based deposit insurance premiums has reduced moral hazard to some extent.

Financial Institutions

Topic 3: Insurance Companies

1. Service Features
2. Essentials of Management

Service Features

- **Life Insurance**
  - **Valuation**
  - ✓ **Mortality tables**
  - ✓ **Break-even Premium**: calculated by finding the value of premium by equating PV of expected premium to PV of the expected payout.

	Male		
Age (Years)	Probability of Death within 1 Year	Survival Probability	Life Expectancy
0	0.006990	1.00000	75.90
1	0.000447	0.99301	75.43
2	0.000301	0.99257	74.46
3	0.000233	0.99227	73.48
…	…	…	…

  - **Basic Risks**
  - ✓ **Mortality Risk**: living not as long as expected
  - Adversely affects most types of life insurance contracts
  - Increase profitability of annuity contracts
  - ✓ **Longevity Risk**: living longer
  - Increases the profitability of most life insurance contracts
  - Adversely affects the profitability of most types of annuity contracts
  - ✓ **Hedging**: reinsurance, longevity derivative contract (longevity bond)

Service Features

- **Property-Casualty Insurance**
  - **Performance Indicator**
  - ✓ **Loss Ratio**
  - Payouts/Premiums
  - ✓ **Expense Ratio**
  - Expenses/Premiums
  - ✓ **Combined Ratio**
  - Loss Ratio + Expense Ratio
  - ✓ **Combined Ratio after Dividends**
  - Combined Ratio + Dividend Yield
  - ✓ **Operating Ratio**
  - Combined Ratio after Dividend – Investment Income

Service Features

- **Pension Plan**
  - **Defined Benefit Plan**
  - ✓ Pension that employee will receive is defined by the plan.
  - ✓ All contributions are pooled and to retirees are made out of the pool.
  - ✓ Significant risks on employers because they are ultimately responsible for paying the promised benefits.
  - **Defined Contribution Plan**
  - ✓ Contributions are invested on behalf of the employee.
  - ✓ An account is set up for each employee and the pension is calculated only from the funds contributed to that account.
  - ✓ If the performance of the plan's investment is less than anticipated, the employee bears the cost.

Essentials of Management

- **Risks facing Insurance Companies**
  - **Moral Hazard**
  - ✓ Risk that existence of insurance will cause the policyholder to behave differently than he or she would without the insurance.
  - ✓ **Solution:** Aligning interest of policyholders more closely with those of insurance company: Deductible; Co-Insurance Provision; Policy Limit.
  - **Adverse Selection**
  - ✓ Risk arises when company cannot distinguish between good and bad risks and offers the same price. This will inadvertently attracts more of the bad risks.
  - ✓ **Solution:** Try to find out as much as possible about the policyholder before committing itself.

Essentials of Management

- **Guaranty System**
  - **Comparison between Banks and Insurance Companies**
  - ✓ For banks, a permanent fund is created from premiums paid by banks to the FDIC to protect depositors.
  - ✓ For insurance companies, no permanent fund is created but the companies make contributions after an insolvency has occurred.
- **Regulatory Requirement**
  - **In United States**
  - ✓ Regulated at the state level
  - **In Europe**
  - ✓ Regulated centrally
  - ✓ Solvency II

Exercise 1

- A term insurance contract of $2,000,000 is being proposed for a 40-year-old male. Given that
  - The payouts occur halfway throughout the year.
  - The premiums are paid annually at the beginning of the year.
  - The semiannual compounding interest rate at present is 2%.

  Using the mortality tables below, the breakeven premium for a two-year term is closest to:

Age	Male		
	Probability of Death within 1 Year	Survival Probability	Life Expectancy
40	0.002092	0.95908	38.53
41	0.002240	0.95708	37.61

A. $4,246 B. $4,287 C. $4,332 D. $8,482

- Answer: B

Exercise 2

- In its annual report, a property casualty insurance company presents a summary of key ratios as follow:
  - Loss ratio: 75%
  - Expense ratio: 30%

  Each of the following statements is true except which is false?

  A. The expense ratio includes loss adjustment expenses.
  B. The expense ratio includes marketing expenses and commissions paid to brokers.
  C. Because its combined ratio is greater than 100%, It is not a profitable business.
  D. For each $1 in premiums received, it pays out about $0.75 in claims to its customers.

- Answer: C

Financial Institutions

Topic 4: Mutual Funds and Hedge Funds

1. Mutual Funds Market
2. Hedge Funds Market
3. Hedge Funds Strategies

Mutual Funds Market

- **Different Types of Mutual Funds**
  - **Open-End Funds:** Shares in fund can be bought/sold back at any time.
    - ✓ **NAV**
    - market value of the portfolio/Number of Shares Outstanding
  - **Closed-End Funds:** Have a fixed number of shares outstanding.
    - ✓ **NAV**
    - Price at which the shares of the fund are trading
    - Market value of the fund's portfolio/Number of Shares Outstanding
  - **Exchange-Traded Funds**
    - ✓ Investors can give up shares they hold and receive the block of securities or they can deposit new block of securities and receive new shares.
    - ✓ Some or all of the shares in the ETF are then traded on a stock exchange.
    - ✓ No appreciable difference between trading price and fair market value

Hedge Funds Market

- **Mutual Funds vs Hedge Funds**
  - **Mutual Funds**
    - ✓ Relatively small investors
    - ✓ Are required to explain their investment policies in a prospectus that is available to potential investors.
  - **Hedge Funds**
    - ✓ Wealthy individuals and large investors such as pension funds.
    - ✓ Less regulation and free to use a wider range of trading strategies
    - ✓ Fees are relatively higher and dependent on performance
- **Fee Structure**
  - Management Fee
  - Incentive Fee
    - ✓ Hurdle Rate ✓ High-Water Mark Clause ✓ Clawback Clause

Hedge Funds Strategies

- **Fixed Income Arbitrage**
  - **Relative Value Strategy**
  - ✓ Buy bonds that the zero-coupon yield curve indicates are undervalued by the market and sell bonds that it indicates are overvalued.
  - **Market-Neutral Strategy**
  - ✓ Tries to ensure that fund has no exposure to interest rate movements.
  - **Directional Strategy**
  - ✓ Based on a belief that a certain spread between interest rates, or interest rates themselves, will move in a certain direction.
- **Convertible Arbitrage**
  - Buy convertible bonds and then hedge the risks by shorting the stock and nonconvertible bonds.

Hedge Funds Strategies

- **Long/Short Equity**
  - Takes a long position in undervalued set of stocks and a short position in overvalued set of stocks.
  - **Equity-Market-Neutral Fund**
  - ✓ Dollar-Neutral
  - ✓ Beta-Neutral
  - ✓ Sector-Neutral
  - ✓ Factor-Neutral
- **Dedicated Short**
  - Look exclusively for overvalued companies and sell them short.

Hedge Funds Strategies

- **Distressed Securities**
  - Searching for debt undervalued by market when in financial difficulty.
  - Passive investors buy it when the price is below its fair value and wait. Active investors purchase a sufficiently large position in outstanding debt claims so that they have the right to influence a reorganization proposal.
- **Merger Arbitrage**
  - Involves trading after a merger or acquisition is announced in the hope that the announced deal will take place.
  - ✓ **Cash Deals**
  - ✓ **Share-for-Share Exchanges**
  - The principal risk is usually deal risk, should the deal fail to close.

Hedge Funds Strategies

- **Global Macro**
  - Carry out trades that reflect global macroeconomic trends.
- **Managed Futures**
  - Created and maintained by a commodity trading advisor (CTA) and invests in commodity futures contracts.
- **Emerging Market**
  - Specialize in investments associated with developing countries.
  - Some focus on equity investments
    - ✓ Local Exchange
    - ✓ American Depository Receipts
  - Some focus on debt investment
    - ✓ Local currency bonds
    - ✓ Eurobonds

Exercise 1

- John manages a hedge fund for a mid-sized money management firm. The fund frequently changes styles according to identified profit opportunities. At the beginning of the year, the fund took a long position in 15-year subordinated 10% coupon debt issued by a firm expected to undergo reorganization under Chapter 11. John felt that analysts had been paying too little attention to the issuer. Six months later, the fund completed a second transaction involving a long position in USD and a short position in Japanese Yen based on forecasted movements in interest rates in the two countries. What two hedge fund strategies are most likely being employed?
  - A. Distressed securities strategy and equity long/short strategy.
  - B. Fixed-income arbitrage and global macro strategy.
  - C. Distressed securities strategy and global macro strategy.
  - D. Fixed-income arbitrage and equity long/short strategy.
- Answer: C

Exercise 2

- A major acquisition has just been announced, targeting company B. The bid from Company A is an exchange offer with a ratio of 2. Just after the announcement, the prices of A and B are $50 and $90, respectively. A hedge fund takes a long position in company B hedged with A's stock. After the acquisition goes through, the prices move to $120 and $60. For each share of B, the gain is
  - A. $30
  - B. $20
  - C. $10
  - D. $0 since the acquisition is successful
- Answer: C

Risk Measurement and Management

Topic 1: Market Risk

1. Measures of Financial Risk
2. Quantifying Volatility in VaR Models
3. Foreign Exchange Risk

Measures of Financial Risk

- **Coherent Risk Measure**
  - **Monotonicity**
  - ✓ A random cash flow or future value R_1 that is always greater than R_2 should have a lower risk.

$$R_1 \geq R_2, \text{then } \rho(R_1) \leq \rho(R_2)$$

  - **Subadditivity**
  - ✓ The portfolio's risk should not be greater than the sum of its parts.

$$\rho(R_1 + R_2) \leq \rho(R_1) + \rho(R_2)$$

  - **Positive Homogeneity**
  - ✓ The risk of a position is proportional to its scale or size.

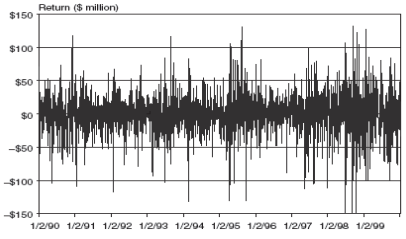
$$\beta > 0, \rho(\beta R) = \beta\rho(R)$$

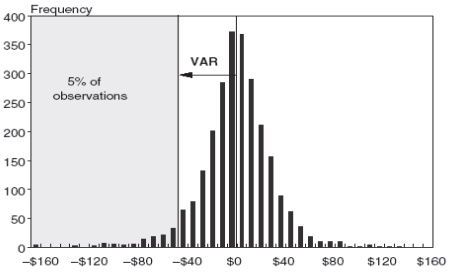
  - **Translation Invariance**
  - ✓ Like adding cash

$$\text{for constant } c, \rho(R + c) = \rho(R) - c$$

Measures of Financial Risk

- **Mean-Variance Framework**
- **Value at Risk**
  - VaR is the maximum loss over a target horizon and for a given confidence level.





  - **Disadvantages of VaR**
  - ✓ Did not contain worst conditions, did not describe tail loss.
  - ✓ Not sub-additive.

Measures of Financial Risk

- **Value at Risk (cont'd)**
  - If an expected return other than zero:

$$\text{VaR}(X\%) = |\mu - z_{X\%} \times \sigma|$$

  - **Example:** Given that for a $1,000,000 portfolio, the expected 1-year portfolio return and standard deviation are 0.00124 and 0.0321. Calculate the 1-year VaR at 1% significance level.
  - **Square Root Rule**
  - ✓ If there are no serial correlations or other dependencies, volatility increases with the square root of the unit of time.

$$\text{VaR}(X\%)_{J-\text{days}} = \text{VaR}(X\%)_{1-\text{days}} \times \sqrt{J}$$

  - ✓ With trends→positive correlation→VaR increase
  - ✓ With mean reversion→negative correlation→VaR decrease

Measures of Financial Risk

- **Value at Risk (cont'd)**
  - **Delta-Normal Approach**
  - ✓ Linear approximation
  - ✓ Underlying factor is assumed to follow a normal distribution
  - ✓ Not good for derivatives with extreme nonlinearities (MBS, Fixed-income securities with embedded option).

$$\text{VaR}(dP) = \text{MD} \cdot P \cdot \text{VaR}(dy)$$
$$\text{VaR}(df) = |\Delta| \cdot \text{VaR}(dS)$$

  - ✓ **Example:** The Big Pharma Inc's stock is trading at USD 23 and the stock has a daily volatility of 1.5%. Using the delta-normal method, what is the VaR at the 95% confidence level of a long position in an at-the-money put on this stock with a delta of -0.5 over a 1-day holding period?

Measures of Financial Risk

- **Value at Risk (cont'd)**
  - **Delta-Gamma Approach**
  - ✓ Nonlinear approximation

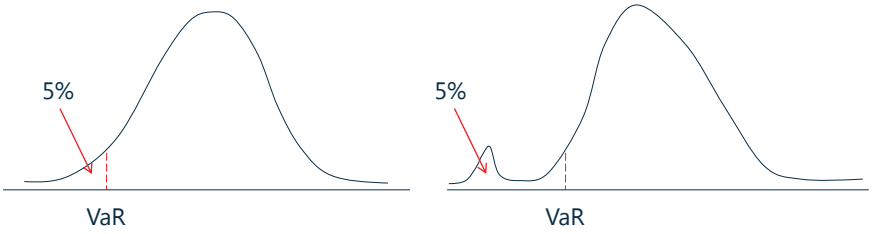
VaR of Long Position:

$$\text{VaR}(df) = |\Delta| \cdot \text{VaR}(dS) - \frac{1}{2} \cdot \Gamma \cdot \text{VaR}(dS)^2$$

  - **Full Revaluation Method**
  - ✓ Full re-pricing under the assumption that the underlying risk factor(s) are shocked to experience a loss. i.e., what is the worst expected change in the risk factor, given some confidence and time horizon. Then, full revaluation prices the portfolio under changed risk factors.
  - ✓ Accurate but computationally burdensome.

Measures of Financial Risk

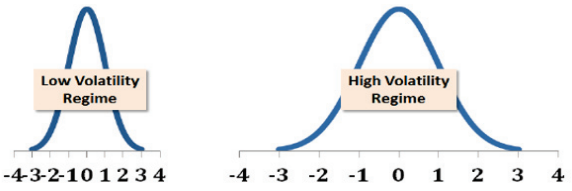
- **Expected Shortfall/Conditional VaR**
  - Average of the worst 100×(1-α)% of losses.



  - **Example:** Given the following 30 ordered percentage returns of an asset, calculate the VaR and expected shortfall at a 90% confidence level: -16, -14, -10, -7, -7, -5, -4, -4, -3, -1, -1, 0, 0, 0, 1, 2, 2, 4, 6, 7, 8, 9, 11, 12, 12, 14, 18, 21, 23.
- **Spectral Risk Measures**

Quantifying Volatility in VaR Models

- **Potential Reasons for the Existence of Fat Tails**
  - When an assets returns are normally distributed, they are then defined by constant mean and volatility parameters. One of the possible explanation for fat tails is that the volatility is time-varying.
  - Different market or economic conditions may cause the mean and variance of the return distribution to change over time. In such cases, the return distribution is referred to as a conditional distribution.
  - A **regime-switching volatility model** assumes different market regimes exist with high or low volatility. The model captures conditional normality and may resolve the fat-tail problem.

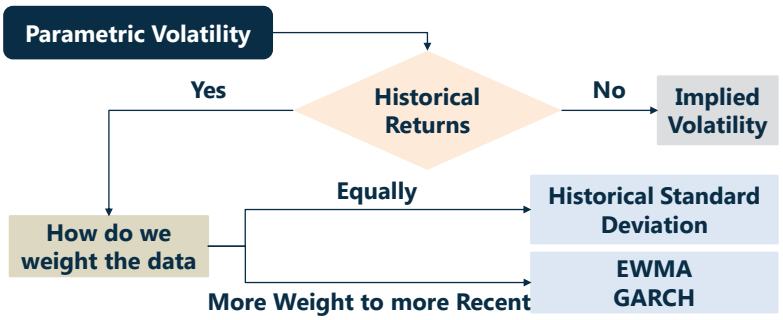


Quantifying Volatility in VaR Models

- **Various Approaches for Estimating VaR**
  - **Historical-Based Approach:** use historical time series data
  - ✓ **Parametric Approach**
  - Imposes a specific distribution assumption on asset returns
  - ✓ **Nonparametric Approach**
  - No specific distribution assumptions.
  - Historical simulation is the simplest and most prominent representative
  - ✓ **Hybrid Approach**
  - Combined approach. For example, Filtered historical simulation updates the volatility by fitting a model such as GARCH to the time-series.
  - **Implied Volatility Based Approach:** uses derivative pricing models and current derivative prices in order to impute an implied volatility without having to resort to historical data. The use of implied volatility obtained from BSM option pricing model is the most prominent example.

Quantifying Volatility in VaR Models

- **Parametric Approach**
  - **Historical Standard Deviation**
  - **EWMA**
  - **GARCH**



Quantifying Volatility in VaR Models

- **Parametric Approach (cont'd)**
  - **Historical Standard Deviation Approach**
  - ✓ Assuming equally weighted
  - ✓ Each day, the forecast is updated by adding the most recent day and dropping the furthest day.
  - ✓ Raw returns are used instead of returns around the mean (i.e., the expected mean is assumed zero).

$$\sigma_n^2 = \left(\frac{1}{M}\right)\sum_{i=1}^{M} u_{n-i}^2$$

  - **Exponential Smoothing Method**
  - ✓ Exponential smoothing places exponentially declining weights on historical data, placing more weight on more recent information and less weight on past information.
  - ✓ Two models, **EWMA** and **GARCH** employ exponential smoothing.

Quantifying Volatility in VaR Models

- **Parametric Approach (cont'd)**
  - **EWMA**

$$\sigma_n^2 = \lambda\sigma_{n-1}^2 + (1-\lambda)u_{n-1}^2$$

  - **GARCH**

$$\sigma_n^2 = \omega + \alpha u_{n-1}^2 + \beta\sigma_{n-1}^2$$

$$V_L = \frac{\omega}{1-\alpha-\beta}$$

  - ✓ In GARCH (1,1), the sum of the alpha (α) and beta (β) parameters is called persistence.
  - ✓ GARCH (1, 1) is unstable if the persistence > 1. A persistence of 1.0 implies no mean reversion. A persistence of less than 1.0 implies "reversion to the mean," where a lower persistence implies greater reversion to the mean.
  - ✓ EWMA is a special case of GARCH.

Quantifying Volatility in VaR Models

- **Nonparametric Methods**
  - **Historical Simulation**
  - ✓ No parameter estimates are required
  - ✓ Once the window length is determined, we order returns in descending order, and go directly to the tail of this ordered vector.
  - ✓ The model is not subject to estimation error related to correlations and the problem of higher correlations in downward markets.
  - **Multivariate Density Estimation**
  - ✓ The weights are not a constant function of time.
  - ✓ Allows for weights to vary based on how relevant the data is to the current market environment.
  - ✓ The current state – as parameterized by a state vector – is compared to the historical state: the more similar the states (current versus historical period), the greater the assigned weight.

Quantifying Volatility in VaR Models

- **Hybrid Approach**
  - It combines historical simulation (by estimating the percentiles of returns directly) and EWMA (by using exponentially declining weights on past data). The following three steps are required to implement the hybrid approach.
  - ✓ Denote by $u_{t-1,1}$ the realized return from t - 1 to t. To each of the most recent K returns $u_{t-1,1}$, $u_{t-2,t-1}$,... $u_{t-k,t-k-1}$ assign a weight:

$$\frac{1-\lambda}{1-\lambda^k}, \frac{1-\lambda}{1-\lambda^k}\cdot\lambda, \ldots, \frac{1-\lambda}{1-\lambda^k}\cdot\lambda^{k-1}$$

  - ✓ Order the returns in ascending order.
  - ✓ In order to obtain the x percent VaR of the portfolio, start from the lowest return and keep accumulating the weights until x percent is reached.

Quantifying Volatility in VaR Models

- **Hybrid Approach**
  - **Example:** calculating 5% VaR using hybrid approach. The most recent observations is K = 100, and λ = 0.98.

Order	Return	Periods Ago	hybrid weight	hybrid cumulative weight
1	-3.30%	3	0.0221	0.0221
2	-2.90%	2	0.0226	0.0447
3	-2.70%	65	0.0063	0.0510
4	-2.50%	45	0.0095	0.0605
5	-2.40%	5	0.0213	0.0818
6	-2.30%	30	0.0128	0.0946

Foreign Exchange Risk

- **Foreign Exchange Risk**
  - The risk that an investor will have to close out a long or short position in a foreign currency at a loss due to an adverse movement in exchange rates. Also known as "currency risk" or "exchange-rate risk".
  - A **positive net exposure position** means that we are net long in a currency; A **negative net exposure position** means that we are net short in a currency.
  - **On-Balance-Sheet Hedging** is achieved when a financial institution has a matched maturity and currency foreign asset-liability book. Rather than matching foreign assets with foreign liabilities, we may choose to remain un-hedged on the balance sheet, and hedge off-balance-sheet by taking a position in the forward market.

Exercise 1

- Your colleague reports a 95.0% one-day value-at-risk (VaR) of $1.4 million for a equities portfolio. If we assume 250 trading days in a year, each of the following is a valid conclusion except which of the following is false (cannot be concluded from the statement)?
  - A. If the VaR is accurate, we expect a daily loss in excess of $1.4 million to occur on about 12 or 13 days (12.5) during year
  - B. If the return distribution is normal, then we can assume the VaR is sub-additive
  - C. This is a parametric VaR and therefore cannot characterize a heavy-tailed distribution
  - D. If the returns are i.i.d. normal, we can scale to a 10-day VaR with $1.4*SQRT(10) = $4.3 million 95% 10-day VaR

- Answer: C

Exercise 2

- A bond with a face value of $10.0 million has a one-year probability of default (PD) of 1.0% and an expected recovery rate of 35.0%. What is the bond's one-year 99.0% expected shortfall (ES; aka, CVaR)?
  - A. $3.25 million
  - B. $6.5 million
  - C. $9.1 million
  - D. Not enough information: need the tail distribution

- Answer: B

Exercise 3

- An investor has a short position in put option (i.e., has written a put) on an underlying asset with value of $100,000. The delta of the put option is -0.40. The 95.0% value at risk (VaR) of the underlying asset is 12.0%. Which of the following statements is correct when second-order terms (quadratic VaR) are considered?
  A. VaR of the short option position is slightly less than $4,800
  B. VaR of the short option position is slightly more than $4,800
  C. VaR of the short option position is slightly less than $12,000
  D. VaR of the short option position is slightly more than $12,000

- Answer: B

Exercise 4

- A stock with a current price of $200.00 has an annual volatility of 30.0%. An at-the-money (ATM) call option has a delta, $N(d_1)$, of 0.6180 and gamma of 0.00950. If we assume 250 trading days, what is the one-day 95.0% confident delta-gamma value at risk (VaR) of a long position in the call option?
  A. $1.99
  B. $3.67
  C. $3.85
  D. $14.72

- Answer: B

Exercise 5

- Which of the following best summarizes the key difference(s) between these two approaches to VaR estimation: exponentially weighted moving average (EWMA; aka, RiskMetrics) versus the hybrid approach?
  A. Hybrid approach does not utilize exponentially declining weights
  B. Hybrid is parametric and EWMA is non-parametric
  C. Hybrid estimates the VaR as a quantile (percentile) of ordered (but weighted) historical returns, but EWMA does not sort returns
  D. There is no difference: EWMA is the hybrid approach

- Answer: C

Exercise 6

- A risk manager is considering switching from using historical volatility to using implied volatility in VaR calculations. Which of the following statements about implied volatility are correct?
  - A. Implied volatility estimates are model dependent and a mis-specified model can result in erroneous forecasts.
  - B. Implied volatility estimates require that historical returns are indicative of future returns.
  - C. Implied volatility estimates tend to underestimate future volatility as a result of mean reversion
  - D. Implied volatility estimates are generally accurate even if there is only one trade in the option used to calculate an estimate.
- Answer: A

Risk Measurement and Management

Topic 2: Credit Risk

1. Expected and Unexpected Loss
2. External and Internal Ratings
3. Country Risk

Expected and Unexpected Loss

- **Credit Risk**
  - Credit risk is the risk that arises from any nonpayment or rescheduling of any promised payments (i.e., default-related events) or from (unexpected) credit migrations (i.e., events that are related to changes in the credit quality of a borrower) of a loan and that gives rise to an economic loss to the bank.
- **Three Drivers**
  - **Probability of Default**: Probability that a borrower will default before the end of a predetermined period of time or at any time before the maturity of the loan.
  - The **exposure amount** of the loan at the time of default.
  - The **loss rate**, that is, the fraction of the exposure amount that is lost in the event of default, meaning the amount that is not recovered after the sale of the collateral.

Expected and Unexpected Loss

➢ **Expected Losses**

- A bank can expect to lose, on average, a certain amount of money over a predetermined period of time when extending credits to its customers.
- A prudent bank should set aside a certain amount of money (often called loan loss reserves) to cover these losses that occur during the normal course of their credit business.

$$\begin{aligned} EL &= EA - E(EA) \\ &= EA - \left[(1 - PD) \times EA + PD \times \left(EA \times (1 - LR)\right)\right] \\ &= PD \times EA \times LR \end{aligned}$$

$$\text{adjusted exposure} = OS + \alpha \times COM$$

Expected and Unexpected Loss

➢ **Unexpected Loss**

- By definition, EL does not itself constitute risk. If losses always equaled their expected levels, there would be no uncertainty, and there would be no economic rationale to hold capital against credit risk. Risk arises from the variation in loss levels – which for credit risk is due to unexpected losses (UL).
- Unexpected loss is the standard deviation of credit losses.
- ULs cannot be anticipated and hence cannot be adequately priced for in a loan's interest rate. They require a cushion of economic capital.

$$UL = EA \times \sqrt{PD \times \sigma_{LR}^2 + LR^2 \times \sigma_{PD}^2}$$

Where
σ_{LR} = standard deviation of the loss rate
σ_{PD} = standard deviation of the default probability
$\sigma_{PD}^2 = PD(1 - PD)$

Expected and Unexpected Loss

➢ **Portfolio Credit Risk**

- The expected loss of a portfolio of credits is straightforward to calculate because EL is linear and additive.

$$EL_P = \sum_{i=1}^{n} EL_i = \sum_{i=1}^{n} EA_i \times PD_i \times LR_i$$

where
EL_P = Expected loss of a portfolio of n credits

- When measuring unexpected loss at the portfolio level, we need to consider the effects of diversification because – as always in portfolio theory – only the contribution of an asset to the overall portfolio risk matters in a portfolio context.

$$UL_P = \sqrt{\Sigma_{i=1}^{n} \Sigma_{j=1}^{n} \omega_i \omega_j \rho_{ij} UL_i UL_j}$$

Expected and Unexpected Loss

- Portfolio Credit Risk (cont'd)
  - Unexpected Loss Contribution

$$\begin{aligned}\mathrm{ULMC_i} &= \frac{\partial \mathrm{UL_P}}{\partial \mathrm{UL_i}} = \frac{\partial \left(\mathrm{UL_P^2}\right)^{\frac{1}{2}}}{\partial \mathrm{UL_i}} \\ &= \frac{1}{2}\left(\mathrm{UL_P^2}\right)^{-\frac{1}{2}} \\ &\times \frac{\partial \left(\mathrm{UL_P^2}\right)}{\partial \mathrm{UL_i}} \\ &= \frac{\left(\Sigma_{j=1}^{n} \mathrm{UL_j}\rho_{ij}\right)}{\mathrm{UL_P}}\end{aligned}$$

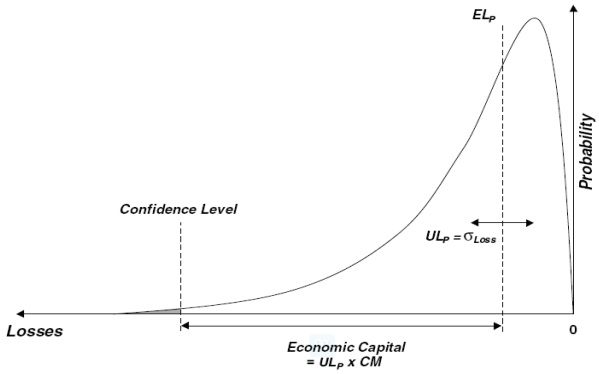
  - Total Contribution to the Portfolio's UL

$$\mathrm{ULC_i} = \mathrm{ULMC_i} \times \mathrm{UL_i} = \frac{\Sigma_{i-1}^{n} \mathrm{UL_j}\rho_{ij}}{\mathrm{UL_P}} \times \mathrm{UL_i}$$

$$\mathrm{UL_P} = \Sigma_{i=1}^{n} \mathrm{ULMC_i} \times \mathrm{UL_i}$$

Expected and Unexpected Loss

- Economic Capital



$$\text{Economic Capital}_{\mathrm{P}} = \mathrm{UL_P} \times \mathrm{CM} \quad \text{Economic Capital}_{\mathrm{i}} = \mathrm{ULC_i} \times \mathrm{CM}$$

External and Internal Ratings

Investment Grade Ratings			
Interpretation (abridged)	S&P	Moody's	Interpretation (abridged)
Highest rating. Extremely strong capacity to meet obligations	AAA	Aaa	Highest quality, with minimal credit risk
	AA+	Aa1	
Capacity to meet its financial obligation is very strong	AA	Aa2	High quality and subject to very low credit risk
	AA-	Aa3	
	A+	A1	
Capacity to meet obligation still strong but susceptible to adverse changes in economic conditions	A	A2	Considered upper-medium grade and subject to low credit risk
	A-	A3	
	BBB+	Baa1	
Exhibits adequate "protection parameters"	BBB	Baa2	Subject to moderate credit risk
	BBB-	Baa3	

External and Internal Ratings

Speculative Grade Ratings Interpretation (abridged)	S&P	Moody's	
	BB+	Ba1	
Less vulnerable to nonpayment than other speculative issues but faces "major ongoing uncertainties"	BB	Ba2	Judged to have speculative elements and are subject to substantial credit risk
	BB-	Ba3	
	B+	B1	
More vulnerable to nonpayment than 'BB' but has current capacity to meet financial obligation	B	B2	Considered speculative and are subject to high credit risk
	B-	B3	
	CCC+	Caa1	
Vulnerable to nonpayment	CCC	Caa2	In poor standing
	CCC-	Caa3	
Highly vulnerable to nonpayment.	CC		
Highly vulnerable to nonpayment	C	Ca	Highly speculative and likely to default
In payment default	**D**	**C**	**Lowest rated bonds—typically in default, with little prospect for recovery**

External and Internal Ratings

- **The rating process will differ according to the type of instrument being rated. The rating process for industrial bonds (following the example of S&P) focuses on the following areas:**
  - Business Risk
  - Competitive Positioning
  - Financial Risk
  - Financial Policies
  - Capitalization
  - Financial Flexibility
  - Industry Characteristics
  - Management
  - Financial Characteristics
  - Profitability
  - Cash flow protection
- **Internationally, the sovereign rating will be the ceiling for the rating of an issuer within that country. For sovereigns, there are additional factors to consider such as:**
  - Political stability
  - Social and economic coherence.
  - Integration into global economic system.

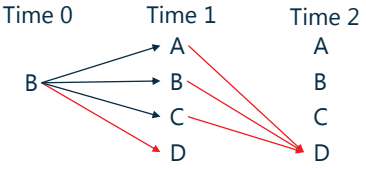
External and Internal Ratings

- **Rating and Default**
  - Ratings communicate an opinion about the creditworthiness of an issuer or an obligation. They should indicate the likelihood and severity of default. Characteristics of ratings performance for corporate bonds:
  - ✓ Ratings and corporate default rates are inversely related. This inverse relationship holds for all time periods following the ratings, such as one year, five years, ten years, etc.
  - ✓ Yield spreads over treasury bonds correlate highly with ratings (i.e., the greater the spread, the lower the credit rating).
  - ✓ Default rates for investment grade issues (Baa or better) are substantially lower than default rates for speculative grade issues (Ba or worse).

External and Internal Ratings

- **Rating and Default (cont'd)**
  - Agencies publish cumulative default rates categorized by rating (i.e., the cumulative default rate per rating category) and transition matrices. Transition matrices plot the frequency of rating migrations over time;

Rating From	Rating To			
	A	B	C	D
A	97%	3%	0%	0%
B	2%	93%	2%	3%
C	1%	12%	64%	23%
D	0%	0%	0%	100%



External and Internal Ratings

- **At-the-Point Approach and Through-the-Cycle**
  - **At-the-Point-in-Time**: assesses credit quality over the near term; i.e., a few months or one year.
  - **Through-the-Cycle**: rating agencies try to incorporate business cycles. Ratings are therefore typically considered "through-the-cycle". Through-the-cycle ratings try to "filter out" cycle fluctuations. Because they incorporate an average, when economic conditions vary from the average, through-the-cycle may over- and under-estimate credit quality.
- **Impact of Rating Changes on Corporate Security Price**
  - If ratings bring information about the credit quality of firms, a change in rating should lead to changes in the prices of corporate securities such as bonds issued by firm.
    - ✓ An upgrade is likely to have a positive impact on bond prices while a downgrade is likely to have a negative impact on bond price.
    - ✓ The result is statistically stronger for downgrades.

Country Risk

- **Sources of Country Risk**
  - Economic growth life cycle
  - Political risk
  - Legal system in a country
  - Disproportionate dependence on a particular product or service
- **Composite Measures of Country Risk**
  - Political Risk Services (PRS)
  - Euromoney
  - The Economist
  - The World Bank

Country Risk

- **Sovereign Default**
  - **Foreign Currency Defaults**
  - ✓ Many governments have been dependent on debt borrowed from other countries, usually denominated in a foreign currency. A large proportion of sovereign defaults have occurred with this type of sovereign borrowing, as the borrowing country finds it's short of the foreign currency to meet its obligations, without the option of being able to print money in that currency.
  - **Local Currency Defaults**
  - ✓ Countries should be able to print more of the local currency to meet their obligations and should never default. There are three reasons why local currency default occurs. Gold Standard; Shared Currency; The Alternative of Printing more Currency to Pay Debt Obligations has Costs.
  - ✓ Some countries may choose to default in local currency debt because they have foreign currency debt funding local currency assets.

Country Risk

- **Sovereign Default (cont'd)**
  - **Factors Determining Sovereign Default Risk**
  - ✓ Degree of Indebtedness
  - ✓ Pensions/Social Service Commitments
  - ✓ Revenues/Inflows to Government
  - ✓ Stability of Revenues
  - ✓ Political Risk
  - ✓ Implicit Backing from other Entities
  - **Problems with Sovereign Default Measurement by Rating Agencies**
  - ✓ Ratings are upward biased
  - ✓ There is herd behavior
  - ✓ Too little, too late
  - ✓ Vicious Cycle
  - ✓ Ratings Failures

Country Risk

- **Sovereign Default (cont'd)**
  - **Using Sovereign Default Spread as a Predictor of Defaults**
  - ✓ **Sovereign Default Spread**
  - When a government issues bonds, denominated in a foreign currency, the interest rate on the bond can be compared to a rate on a riskless investment in that currency.
  - ✓ **Advantage**
  - Market differentiation for risk is more granular than the ratings agencies
  - Market-based spreads are more dynamic than ratings
  - ✓ **Disadvantages**
  - Tend to be more volatile than ratings
  - Can be affected by variables that have nothing to do with default. Liquidity and investor demand can cause shifts in spreads that have little or nothing to do with default risk.

Exercise 1

- A bank uses a 4-grade scale for its internal credit model. The 1-year rating transition probabilities for this model are given. If a newly issued bond is rated "A" by this model, what is the probability that it will be rated "B" or lower two years from now?

Rating Form	Rating To			
	A	B	C	D
A	90%	10%	0%	0%
B	10%	81%	8%	1%
C	0%	5%	80%	15%
D	0%	0%	0%	100%

A. 9%
B. 10%
C. 18%
D. 19%

- Answer: C

Exercise 2

- GF Bank has contractually agreed to a $50,000,000 credit facility with ABC Corp., of which $32,000,000 is currently outstanding. ABC has very little collateral, so GF Bank estimates a one-year probability of default of 3%. The collateral is unique to its industry with limited resale opportunities, so GF Bank assigns an 90% loss rate. The expected loss (EL) for GF Bank is closest to:

A. $78,000
B. $96,000
C. $788,000
D. $864,000

- Answer: D

Exercise 3

- GF Bank has two assets outstanding. The features of the loans are summarized in the following table. Assuming a correlation of 0.3 between the assets, what is the value of the unexpected loss of the portfolio (ULP)?

	Asset A	Asset B
Exposure	$6,000,000	$4,000,000
PD	2%	3%
LR	50%	50%
σ_{PD}	2%	5%
σ_{LR}	25%	20%

A. Less than $500,000
B. Between $500,000 and $2,000,000
C. Between $2,000,000 and $3,000,000
D. Greater than $3,000,000

- Answer: A

Exercise 4

- A bank has extended two loans to customers in the same industry. Both loans are have an exposure amount (EA) of $50.0 million, default probability (PD) of 2.0%, loss rate (LR) of 50.0%, and standard deviation of loss rate of 60.0% such that each loan has an expected loss of $500,000 and an unexpected loss of $5.5 million. In this way, the bank's credit portfolio consists of these two credit assets; and the default correlation between the two loans is 0.28. Which is nearest to the risk contribution of each asset to the portfolio's unexpected loss?
  A. $3.33 million
  B. $4.40 million
  C. $5.37 million
  D. $5.50 million

- Answer: B

Risk Measurement and Management

Topic 3: Operational Risk

1. Operational Risk
2. Loss Distribution Approach
3. Regulatory Capital Requirement
4. Risk Control Self-Assessment

Operational Risk

- **Operational Risk**
  - The risk of loss resulting from inadequate or failed internal processes, people, and systems or from external events.
- **Basel's Seven Categories of Operational Risk**
  - Internal Fraud
  - External Fraud
  - Employment Practices and Workplace Safety
  - Clients, Products, and Business Practices
  - Damage to Physical Assets
  - Business Disruption and System Failures
  - Execution, Delivery, and Process Management

Loss Distribution Approach

- **Loss Frequency Distribution**
  - Models number of losses
  - Common probability distribution: Poisson Distribution
- **Loss Severity Distribution**
  - Models size of a loss
  - Common probability distribution: Lognormal Distribution
- **Loss Distribution**
  - Assume that loss severity and loss frequency are independent
  - The frequency and severity distributions must be combined; Monte Carlo simulation can be used for this purpose.
    - ✓ Sample from frequency distribution to determine number of losses
    - ✓ Sample n times from the loss severity distribution to determine the loss experienced for each loss events
    - ✓ Determine the total loss experienced

Loss Distribution Approach

- **Data Issues**
  - **Loss frequency distribution:** bank's own data as far as possible.
  - In regard to the **loss severity data**, regulators encourage banks to use their own data in conjunction with external data. There are two sources of external data: data obtained through sharing arrangements between banks; and publicly available data collected by third-party vendors.
  - Relevant historical data is difficult to obtain, so regulators encourage banks to use scenario analysis, in addition to internal and external loss data. This involves managerial judgment to generate scenarios where large losses occur.

Regulatory Capital Requirement

- **Basic Indicator Approach:** through a simple calculation of the average gross revenue for the past 3 years, multiplied by 15%.

$$\mathrm{ORC}^{\mathrm{BIA}} = \left[\left(\mathrm{GI}_{1,\ldots,n} \times \alpha\right)\right]/n,\ \alpha = 15\%$$

- **Standardized Approach:** The total capital charge is calculated as the three-year average of the simple summation of the regulatory capital charges across each of the business lines in each year.

$$\mathrm{ORC}^{\mathrm{TSA}} = \frac{\Sigma_{\mathrm{year}\ 1-3} \max\left(\Sigma\left(\mathrm{GI}_{1-8} \times \beta_{1-8}\right), 0\right)}{3}$$

Corporate Finance	18%	Angency and custody services	15%
Trading and sales	18%	Asset management	12%
Settlement and payment activities	18%	Retail brokerage	12%
Commercial banking	15%	Retail banking	12%

Regulatory Capital Requirement

- **Advanced Measurement Approach (AMA)**
  - Allows a bank to design its own model for calculating operational risk capital. Three main requirements:
  - ✓ Must hold capital for a 1-year horizon at 99.9% confidence level.

$$\text{ORC}^{\text{AMA}} = \text{UL}(1-\text{year}, 99.9\% \text{ confidence})$$

  - ✓ All four elements of the framework must be included in the model: internal loss data, external loss data, scenario analysis, and business environment internal control factors.
  - ✓ There must be an appropriate method for allocating the capital to the businesses to incent good behavior.

Risk Control Self-Assessment

- **RCSA**
  - Involves asking business unit managers to identify their operational risks. Sometimes questionnaires designed by senior managers are used.
  - First identify and assess inherent risks by making no inferences about controls embedded in the process: controls are assumed to be absent.
  - Once these inherent risks are understood, controls will be added in the RCSA framework. The effectiveness of these controls are then assessed. The residual risk is also calculated, which is the risk that is left after inherent risks are controlled.
  - Risk metrics such as **key risk indicators (KRIs)** would contribute to the risk identification process. These indicators/factors are mostly quantitative and are used as a proxy for the quality of the control environment of a business. Examples of key risk indicators are staff turnover and number of failed transactions.

Exercise 1

- Which of the following statements concerning the measurement of operational risk is correct?
  - A. Economic capital should be sufficient to cover both expected and worst-case operational risk losses.
  - B. Loss severity and loss frequency tend to be modeled with lognormal distributions.
  - C. Operational loss data available from data vendors tend to be biased towards small losses.
  - D. The standardized approach used by banks in calculating operational risk capital allows for different beta factors to be assigned to different business lines.
- Answer: D

Exercise 2

- In the Basel II Standardized Approach for operational risk, the beta factor serves as a proxy for the industry-wide relationship between the operational risk loss experience for a given business line and the aggregate level of gross income for that business line. Which of the following lines has the highest beta factor?
  A. Corporate finance
  B. Retail banking
  C. Commercial banking
  D. Asset management.

- Answer: A

Risk Measurement and Management

Topic 4: Stress Testing

1. Governance over Stress Testing
2. Stress Testing and Other Risk Management Tools
3. Principles for Sound Stress Testing

Governance over Stress Testing

- **Key Elements of Effective Governance over Stress Testing**
  - Governance Structure
  - Policies, Procedures and Documentation
  - Internal Audit
  - Validation and Independent Review
- **Governance Structure**
  - **Board of Directors**
    - ✓ The board should evaluate and discuss information received from senior management about stress testing to ensure that the stress testing activities are in line with the institution's risk appetite.
    - ✓ Need to review with a critical eye, challenging key assumption.
    - ✓ The board should be able to take action based on its review of stress test results including changing capital levels, altering strategies or withdrawing from certain activities. Stress-testing activities can serve as a useful "early-warning" mechanism for the board.

Governance over Stress Testing

- **Governance Structure (cont'd)**
  - **Senior Management**
    - ✓ Duties: Establishing policies and procedures and ensuring their compliance. Allocating resources and assigning skilled staff. Overseeing stress-test development and implementation. Evaluating stress-test results. Reviewing findings related to the functioning of stress test processes and taking necessary remedial action.
    - ✓ Responsible for reporting to the board.
    - ✓ Ensure there is appropriate buy-in at different levels of the institution.
    - ✓ Benchmarking or other comparative analysis should be used to evaluate the stress-testing results.

Governance over Stress Testing

- **Governance Structure (cont'd)**
  - **Senior Management (cont'd)**
    - ✓ Should use stress testing to supplement other information it develops and provides to the board, such as other risk metrics or measures of capital and liquidity adequacy.
    - ✓ Should ensure that staff involved in stress testing operate under the proper incentives.
    - ✓ Should ensure that there is a regular assessment of stress testing activities across the institution by an independent, unbiased party.
    - ✓ Ensures that stress-testing activities are updated in light of new risks, better understanding of the institution's exposures and activities, new stress-testing techniques, updated data sources and changes in its operating structure and its internal and external environment.

Governance over Stress Testing

- **Internal Audit**
  - Provides independent evaluation of the ongoing performance, integrity and reliability of stress-testing activities. It is not expected that internal audit will have knowledge of all stress-test details, or will have to independently assess each stress test used.
  - Serves the valuable task of assessing the full suite of stress-testing activities across the institution on a regular basis to evaluate whether, as a whole, such activities are functioning as intended, in adherence with policies and procedures and serving the institution properly.

Governance over Stress Testing

- **Policies, Procedures and Documentation**
  - **Stress Testing Policies should:**
  - ✓ Describe the overall purpose of stress-testing activities.
  - ✓ Indicate stress-testing roles and responsibilities.
  - ✓ Outline the process for choosing stressful conditions for tests.
  - ✓ Include information about validation and independent review.
  - ✓ Provide transparency to third parties for their understanding of an institution's stress testing activities.
  - ✓ Be reviewed and updated as necessary to ensure that stress testing practices remain appropriate and keep up to date with changes.
  - An institution should ensure that its stress tests are **documented properly**, including a description of the types of stress tests and methodologies used, test results, key assumptions, limitations and uncertainties, and suggested actions.

Governance over Stress Testing

- **Validation and Independent Review**
  - Ensuring that there is appropriate independence and effective challenge in the validation and review process;
  - Including validation and independent review of the qualitative or judgmental aspects of a stress test – such aspects can be an integral component of a stress test and should be reviewed, even if they cannot be tested in a quantitative/statistical sense;
  - Ensuring that stress tests are subject to development standards, including a clear statement of purpose, proper theory and design, sound methodologies and processing components, and developmental testing (including testing of assumptions);
  - Acknowledging limitations in stress-testing methodologies, even if they represent best practices;

Governance over Stress Testing

- **Validation and Independent Review (cont'd)**
  - Recognizing any data limitations or weaknesses in data quality;
  - Ensuring that stress tests are implemented in a rigorous manner that is appropriate for the stated use, and accounting for any changes to the developed stress test that occur during implementation;
  - Monitoring performance on an ongoing basis and assessing any degradation in performance (where possible);
  - Expressing stress-test uncertainty and inaccuracy, including in the form of confidence bands around estimates and/or factors not observable or not fully incorporated;
  - Ensuring that vendor or other third-party models are sufficiently validated, including their implementation, to ensure they function as intended and are appropriate for the institution's use.

Stress Testing and Other Risk Management Tools

- **Relationship between Stress Testing and Other Risk Measure**
  - In practice, **stress tests** usually focus on a few scenarios, whereas **VaR measures** commonly utilize a very large number of scenarios.
  - In practice, the loss estimates are often defined differently between stress tests and economic capital/VaR method:
    - ✓ **Measurement units (accounting vs. market):** Stress test losses measure accounting-based implications.
    - ✓ **Time horizon:** Enterprise-wide stress tests often examine a long period such as losses over 9 quarters in the Dodd–Frank stress tests in the US.
    - ✓ **Role of Probabilities:** usually assign ordinal rank assignments
    - ✓ **Approach:** rather than unconditional scenarios typically generated in VaR-type metrics. Especially in the case of regulatory stress tests, the scenario-generation process looks at the present period as the starting point and generates two or three hypothetical scenarios from that point.

Principles for Sound Stress Testing

- **Stress Testing Principles for Banks**
  - Stress testing should form an integral part of the overall governance and risk management culture of the bank.
  - Should operate a stress testing program that promotes risk identification and control; provides a complementary risk perspective to other risk management tools; improves capital and liquidity management; and enhances internal and external communication.
  - Should take account of views from across the organization and should cover a range of perspectives and techniques.
  - Should have written policies and procedures governing the stress testing program.
  - Should have a suitably robust infrastructure in place, which is sufficiently flexible to accommodate different and possibly changing stress tests at an appropriate level of granularity.
  - Should regularly maintain and update its stress testing framework.

Principles for Sound Stress Testing

- **Stress Testing and Scenario Selection**
  - Stress tests should cover a range of risks and business areas, including at the firm-wide level.
  - Stress testing programs should cover a range of scenarios, including forward-looking scenarios, and aim to take into account system-wide interactions and feedback effects.
  - Stress tests should feature a range of severities, including events capable of generating the most damage whether through size of loss or through loss of reputation.
  - Reverse stress tests start from a known stress test outcome (such as breaching regulatory capital ratios, illiquidity or insolvency) and then asking what events could lead to such an outcome for the bank.
  - As part of an overall stress testing program, a bank should aim to take account of simultaneous pressures in funding and asset markets, and the impact of a reduction in market liquidity on exposure valuation.

Principles for Sound Stress Testing

- **Stress Testing and Scenario Selection (cont'd)**
  - A bank should enhance its stress testing practices by considering important interrelations between various factors.
  - Effectiveness of risk mitigation techniques should be systematically challenged.
  - The stress testing program should explicitly cover complex and bespoke products such as securitized exposures.
  - The stress testing program should cover pipeline and warehousing risks. A bank should include such exposures in its stress tests regardless of their probability of being securitized.
  - A bank should enhance its stress testing methodologies to capture the effect of reputational risk.
  - A bank should enhance its stress testing approaches for highly leveraged counterparties and in assessing potential wrong-way risk related to risk mitigating techniques.