Quantitative Analysis

讲师：

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Quantitative Analysis

1. Probability Concepts and Probability Distributions
2. Descriptive Statistics and Inferential Statistics
3. Regression
4. Modeling and Forecasting Trend
5. Estimating Volatilities and Correlations
6. Simulations

Probability Concepts and Probability Distributions

Independent and Dependent Event	★★★	性质
Property	◻ The probability of an independent event is unaffected by the occurrence of other events, but the probability of a dependent event is changed by the occurrence of another event.	
Expression	◻ Events A and B are independent if and only if: $P(A\|B) = P(A)$, or equivalently, $P(B\|A) = P(B)$	

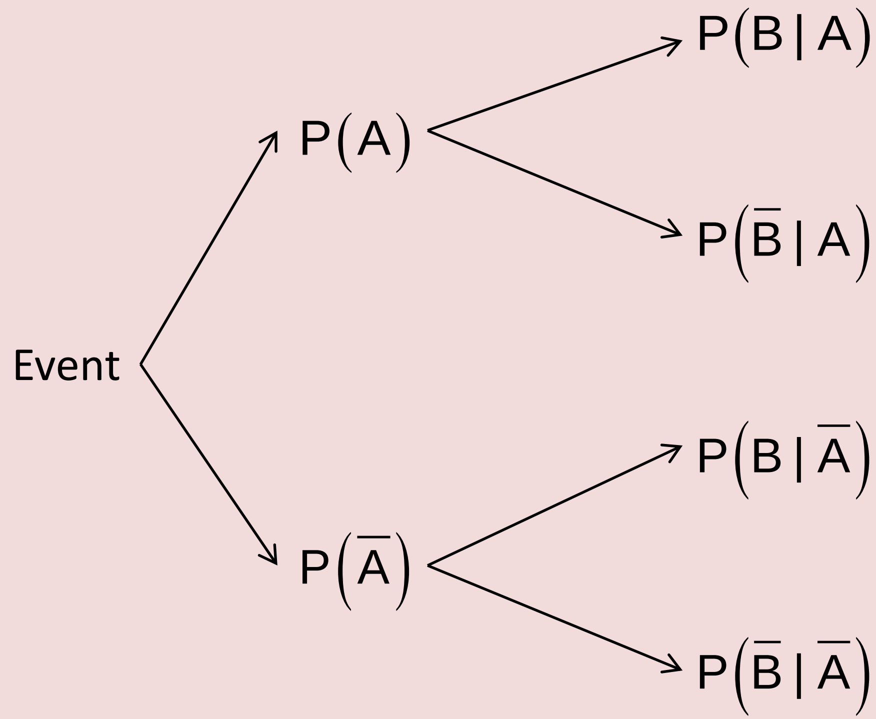
At Least One of Two will Occur	★★	计算
Expression	$P(A \text{ or } B) = P(A) + P(B) - P(AB)$ Mutually Exclusive Events: $P(A \text{ or } B) = P(A) + P(B)$, since $P(AB) = 0$	

Joint Probability	★★	性质
Property	◻ The joint probability of two events, P(AB), is the probability that they will both occur. ◻ **Unconditional probability (marginal probability)** is the probability of an event occurring. ◻ **Conditional Probability**, P(A|B), is the probability of an event A occurring given that event B has occurred.	
Expression	$P(AB) = P(A\|B) \times P(B) = P(B\|A) \times P(A)$ For independent events: $P(A\|B) = P(A)$, so that $P(AB) = P(A) \times P(B)$	

Bayes' Theorem	★★★	性质
Expression	$P(A\|B)=\frac{P(B\|A)P(A)}{P(B)}=\frac{P(B\|A)P(A)}{P(B\|A)P(A)+P(B\|\overline{A})P(\overline{A})}$	

Bayes' Theorem	★★★	性质

Graphics



$$P(A|B)=\frac{P(B|A)P(A)}{P(B)}=\frac{P(B|A)P(A)}{P(B|A)P(A)+P(B|\overline{A})P(\overline{A})}$$

Moments of Probability Distribution	★★★	性质
Basic Property	◻ The shape of a probability distribution is characterized by its raw moments and central moments. The first raw moment is the **mean** of the distribution. The second central moment is the **variance**. The third central moment divided by the cube of the standard deviation measures the **skewness** of the distribution, and the fourth central moment divided by the fourth power of the standard deviation measures the **kurtosis** of the distribution. ◻ Like mean and variance, we can generalize covariance to cross central moments. The third cross central moment is **coskewness** and the fourth cross central moment is **cokurtosis**.	
Mean	◻ Expected value. $E(R) = \mu = \sum_{i=1}^{n} p_i R_i$	
Variance	◻ Measures the dispersion of data. Variance = σ^2 = $E[(R-\mu)^2]$	

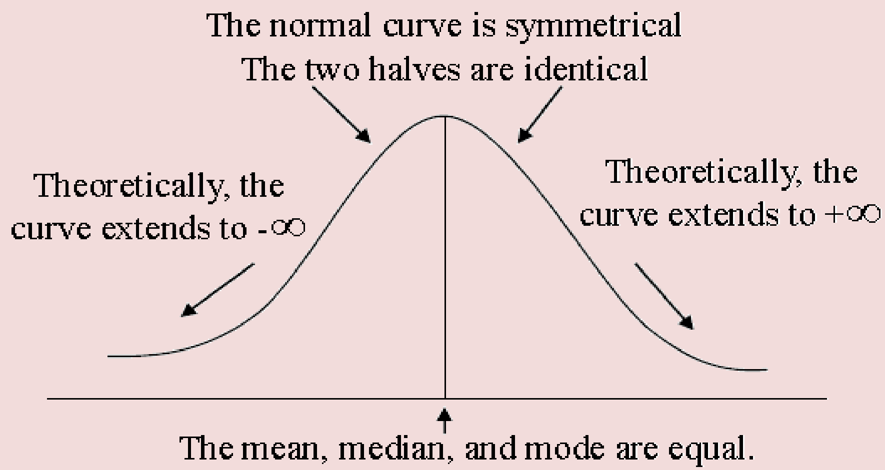
Moments of Probability Distribution	★★★	性质
Skewness	◻ Refers to the extent to which the distribution of data is not symmetric around its mean. • **The normal distribution has skewness of 0.** • **Positive skewed:** Mode < Median < Mean, having a right fat tail. • **Negative skewed:** Mode > Median > Mean, having a left fat tail.	
Kurtosis	◻ Refers to the degree of peakedness or clustering in the data distribution. • **The normal distribution has kurtosis of 3.** • **Leptokurtic:** kurtosis > 3, fat tail. • **Platykurtic:** kurtosis < 3, thin tail. ◻ Excess kurtosis = kurtosis - 3	

Discrete Probability Distribution	★★★	性质、计算
Binomial Distribution	◻ A discrete probability distribution for a random variable, X, that has one of two possible outcomes, success or failure. ◻ Expectation: np ◻ Variance: np(1 – p)	$p(x)=P(X=x)$ $=\frac{n!}{(n-x)!x!}p^x(1-p)^{n-x}$
Poisson Distribution	◻ The Poisson random variable X refers to the number of successes per unit, the parameter lambda (λ) refers to the average or expected number of successes per unit. ◻ Expectation: λ ◻ Variance: λ	$P(X=x)=\frac{\lambda^x e^{-\lambda}}{x!}$

Continuous Probability Distribution	★★★	性质、计算

Normal Distribution

- The normal distribution can be completely defined by its mean and standard deviation because the skew is always zero and kurtosis is always three.
- A linear combination of normally distributed independent random variables is also normally distributed.



- The **standard normal distribution** is the normal distribution with mean of 0 and variance of 1.
- If X ~ N(μ, σ^2), then **Z = (X – μ)/σ ~ N(0, 1)**

Reference Table: Let Z be a standard normal random variable.

z	P(Z<z)	z	P(Z<z)	z	P(Z<z)	z	P(Z<z)	z	P(Z<z)	z	P(Z<z)
-3	0.0013	-2.50	0.0062	-2.00	0.0228	-1.50	0.0668	-1.00	0.1587	-0.50	0.3085
-2.99	0.0014	-2.49	0.0064	-1.99	0.0233	-1.49	0.0681	-0.99	0.1611	-0.49	0.3121
-2.98	0.0014	-2.48	0.0066	-1.98	0.0239	-1.48	0.0694	-0.98	0.1635	-0.48	0.3156
-2.97	0.0015	-2.47	0.0068	-1.97	0.0244	-1.47	0.0708	-0.97	0.1660	-0.47	0.3192
-2.96	0.0015	-2.46	0.0069	-1.96	0.0250	-1.46	0.0721	-0.96	0.1685	-0.46	0.3228
-2.95	0.0016	-2.45	0.0071	-1.95	0.0256	-1.45	0.0735	-0.95	0.1711	-0.45	0.3264
-2.94	0.0016	-2.44	0.0073	-1.94	0.0262	-1.44	0.0749	-0.94	0.1736	-0.44	0.3300
-2.93	0.0017	-2.43	0.0075	-1.93	0.0268	-1.43	0.0764	-0.93	0.1762	-0.43	0.3336
-2.92	0.0018	-2.42	0.0078	-1.92	0.0274	-1.42	0.0778	-0.92	0.1788	-0.42	0.3372
-2.91	0.0018	-2.41	0.0080	-1.91	0.0281	-1.41	0.0793	-0.91	0.1814	-0.41	0.3409
-2.9	0.0019	-2.40	0.0082	-1.90	0.0287	-1.40	0.0808	-0.90	0.1841	-0.40	0.3446
-2.89	0.0019	-2.39	0.0084	-1.89	0.0294	-1.39	0.0823	-0.89	0.1867	-0.39	0.3483
-2.88	0.0020	-2.38	0.0087	-1.88	0.0301	-1.38	0.0838	-0.88	0.1894	-0.38	0.3520
-2.87	0.0021	-2.37	0.0089	-1.87	0.0307	-1.37	0.0853	-0.87	0.1922	-0.37	0.3557
-2.86	0.0021	-2.36	0.0091	-1.86	0.0314	-1.36	0.0869	-0.86	0.1949	-0.36	0.3594
-2.85	0.0022	-2.35	0.0094	-1.85	0.0322	-1.35	0.0885	-0.85	0.1977	-0.35	0.3632
-2.84	0.0023	-2.34	0.0096	-1.84	0.0329	-1.34	0.0901	-0.84	0.2005	-0.34	0.3669
-2.83	0.0023	-2.33	0.0099	-1.83	0.0336	-1.33	0.0918	-0.83	0.2033	-0.33	0.3707
-2.82	0.0024	-2.32	0.0102	-1.82	0.0344	-1.32	0.0934	-0.82	0.2061	-0.32	0.3745
-2.81	0.0025	-2.31	0.0104	-1.81	0.0351	-1.31	0.0951	-0.81	0.2090	-0.31	0.3783
-2.8	0.0026	-2.30	0.0107	-1.80	0.0359	-1.30	0.0968	-0.80	0.2119	-0.30	0.3821
-2.79	0.0026	-2.29	0.0110	-1.79	0.0367	-1.29	0.0985	-0.79	0.2148	-0.29	0.3859
-2.78	0.0027	-2.28	0.0113	-1.78	0.0375	-1.28	0.1003	-0.78	0.2177	-0.28	0.3897
-2.77	0.0028	-2.27	0.0116	-1.77	0.0384	-1.27	0.1020	-0.77	0.2206	-0.27	0.3936
-2.76	0.0029	-2.26	0.0119	-1.76	0.0392	-1.26	0.1038	-0.76	0.2236	-0.26	0.3974
-2.75	0.0030	-2.25	0.0122	-1.75	0.0401	-1.25	0.1056	-0.75	0.2266	-0.25	0.4013

Continuous Probability Distribution	★★★	性质
Lognormal Distribution	◻ The logarithms of lognormally distributed random variables are normally distributed. ◻ Skewed to the right. Is bounded from below by zero so that it is useful for modeling asset prices which never take negative values.	
Student' s t-Distribution	◻ A bell-shaped probability distribution that is symmetrical. ◻ The degrees of freedom (df) are equal to the number of sample observations minus 1, n – 1, for sample mean. ◻ Fatter tails than the normal distribution. ◻ As the degrees of freedom (the sample size) gets larger, the shape of the t-distribution more closely approaches a standard normal distribution. ◻ The normal is used when we know the population variance. The student' s t is used when we must rely on the sample variance.	

Continuous Probability Distribution	★★	性质
Chi-Squared Distribution	◻ It is the distribution of a squared quantity. ◻ Asymmetrical. ◻ Bounded below by zero. ◻ Approaches the normal distribution in shape as the degrees of freedom increase.	
F-Distribution	◻ Variance ratio distribution. The F ratio is the ratio of sample variance, with the greater sample variance in the numerator. ◻ Right-skewed and is truncated at zero on the left-hand side. ◻ Determined by two separate degrees of freedom, the numerator degrees of freedom and the denominator degrees of freedom. ◻ Approaches the normal distribution as the number of observations increases.	

Central Limit Theorem	★★★	性质
Principle	◻ States that for simple random samples of size n from a population with a mean μ and a finite variance σ^2, the sampling distribution of the sample mean approaches a normal probability distribution with mean μ and variance equal to σ^2/n as the sample size becomes large ($n\geq 30$).	
Important Properties	◻ If the sample size is sufficiently large ($n \geq 30$), the sampling distribution of the sample means will be approximately normal. The CLT is extremely useful because the normal distribution is relatively easy to apply to hypothesis testing and to the construction of confidence intervals. ◻ The mean of the population, μ, and the mean of the distribution of all possible sample means are equal. ◻ The variance of the distribution of sample means is the population variance divided by the sample size.	

Descriptive Statistics and Inferential Statistics

Mean and Expected Value	★★★	性质、计算
Property	◻ To compute the population mean, all the observed values in the population are summed and divided by the number of observations in the population. ◻ Expected value is the weighted average of the possible outcomes of a random variable, where the weights are the probabilities that the outcomes will occur.	
Expression	◻ The mean of a population is expressed as: $$\mu = \frac{\sum_{i=1}^{N} X_i}{N}$$ ◻ The expectation of a random variable X having possible value $x_1, \ldots, x_n$ is defined as: $$E(X) = x_1P(X = x_1) + \ldots + x_nP(X = x_n)$$ ◻ If X and Y are any random variables, then: $$E(X + Y) = E(X) + E(Y)$$	

Variance and Standard Deviation	★★★	性质、计算
Property	◻ Provide a measure of the extent of the dispersion in the values of the random variable around the mean. ◻ The square root of the variance is called the standard deviation.	
Expression	◻ Variance of a random variable is defined as: $Var(X) = E[(X - \mu)^2] = E(X^2) - [E(X)]^2$, where $\mu = E(X)$ ◻ If X and Y are independent random variables, then: $Var(X + Y) = Var(X) + Var(Y)$ $Var(X - Y) = Var(X) + Var(Y)$ ◻ If X and Y are not independent, then: $Var(X + Y) = Var(X) + Var(Y) + 2 \times Cov(X,Y)$ $Var(X - Y) = Var(X) + Var(Y) - 2 \times Cov(X,Y)$	

Covariance and Correlation	★★★	性质、计算
Property	◻ **Covariance** measures the extent to which two random variables tend to be above and below their respective means for each joint realization. If X and Y are independent random variables, their covariance is zero. ◻ **Correlation** is a standardized measure of association between two random variables; it ranges from -1 to 1. If two variables are independent, the correlation coefficient will be zero. The converse, however, is not true.	
Expression	$\mathrm{Cov}(A,B)=\sum_{i=1}^{N}P_i\left(A_i-\overline{A}\right)\left(B_i-\overline{B}\right)$ $\quad \mathrm{Corr}(A,B)=\frac{\mathrm{Cov}(A,B)}{\sigma_A\sigma_B}$ $\mathrm{Cov}(X,X)=\sigma^2(X)$ $\mathrm{Cov}(a+bX,c+dY)=bd\mathrm{Cov}(X,Y)$	

Copula	★★★	性质
Property	◻ A copula creates a joint probability distribution between two or more variables while maintaining their individual marginal distributions.	
Types	◻ A **Gaussian copula** maps the marginal distribution of each variable to the standard normal distribution. ◻ The **Student' s t-copula** maps variables to distribution of U_1 and U_2 that have a bivariate Student' s t-distribution. ◻ The Student' s t-copula is better than a Gaussian copula in describing the correlation structure of assets that historically have extreme outliers in tails at the same time.	

Best Linear Unbiased Estimator	★★	性质
BLUE	◻ If the estimator is the best available (i.e., has the minimum variance), exhibits linearity, and is unbiased, it is said to be the best linear unbiased estimator (BLUE).	
Unbiased	◻ An unbiased estimator is one for which the expected value of the estimator is equal to the parameter you are trying to estimate.	
Efficiency	◻ An unbiased estimator is also efficient if the variance of its sampling distribution is smaller than all the other unbiased estimators of the parameter you are trying to estimate.	
Consistent	◻ A consistent estimator is one for which the accuracy of the parameter estimate increases as the sample size increases.	
Linear	◻ A point estimate is a linear estimator when it can be used as a linear function of sample data.	

Standard Error of the Sample Mean	★★★	性质、计算

Definition

- The standard error of the sample mean is the standard deviation of the distribution of the sample means and is calculated as:

$$\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}}$$

Where σ, the population standard deviation, is known, and as:

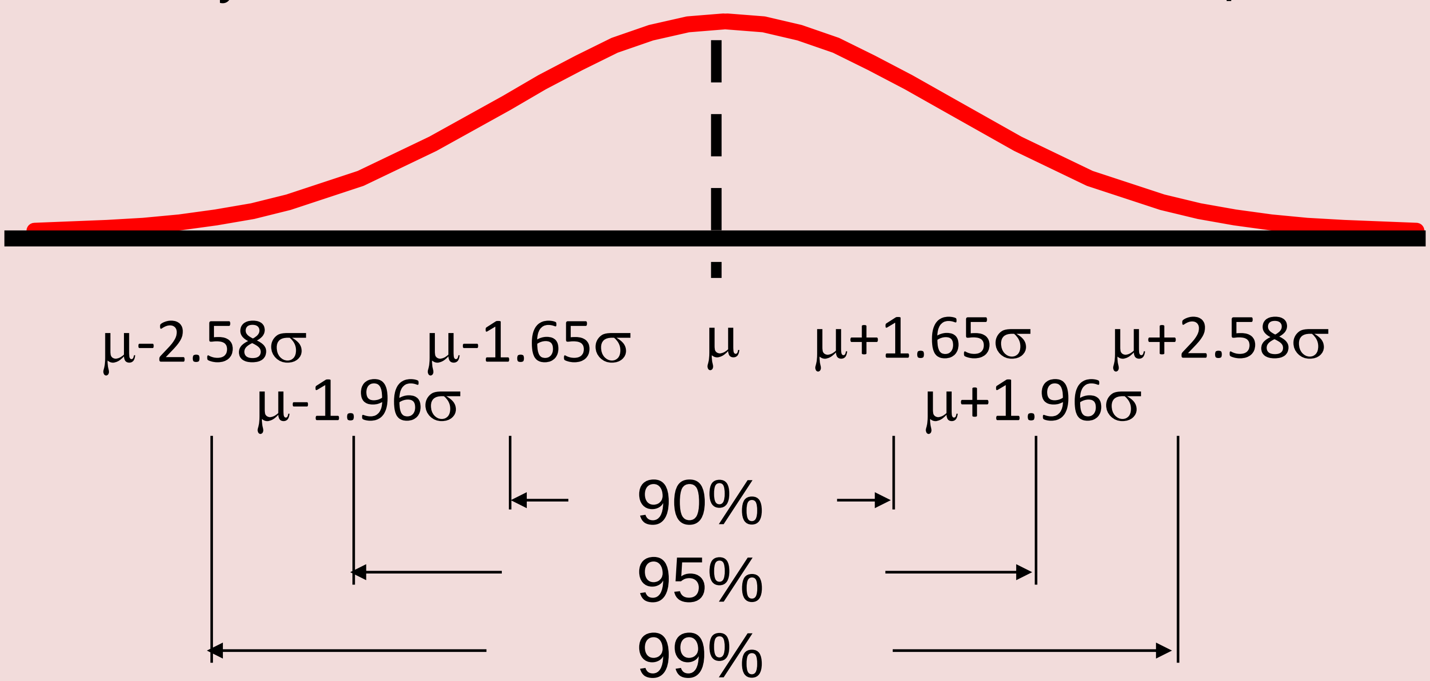
$$s_{\bar{X}} = \frac{s}{\sqrt{n}}$$

Where s, the sample standard deviation, is used because the population standard deviation is unknown.

Confidence Interval	★★★	性质、计算
Definition	◻ For a normally distributed population, a confidence interval for its mean can be constructed using a z-statistic when variance is known, and a t-statistic when the variance is unknown. The z-statistic is acceptable in the case of a normal population with an unknown variance if the sample size is large (30+).	
Expression	In general, we have: $\bar{x} \pm z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$ when the variance is known; $\bar{x} \pm t_{\alpha/2}\frac{s}{\sqrt{n}}$ when the variance is unknown	

Confidence Interval | ★★★ | 性质、计算

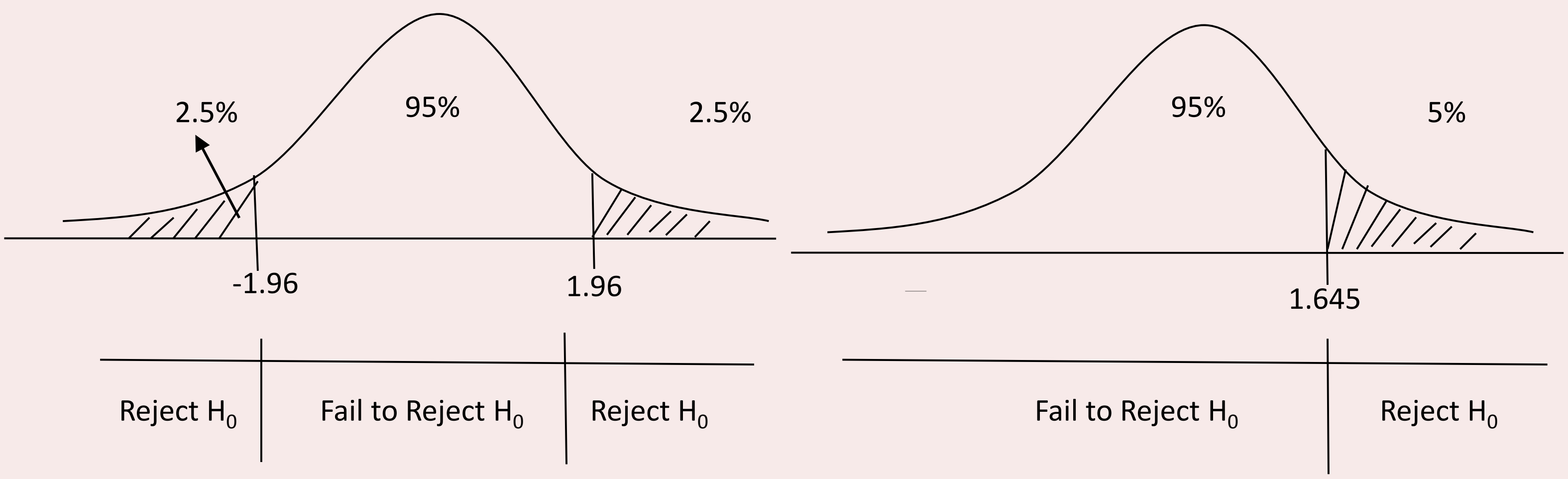
- Approximately 68% of all observations fall in the interval μ±σ
- Approximately 90% of all observations fall in the interval μ±1.65σ
- Approximately 95% of all observations fall in the interval μ±1.96σ
- Approximately 99% of all observations fall in the interval μ±2.58σ



Hypothesis Testing	★★★	性质、计算
Properties	◻ A statement of a null and an alternative hypothesis. ◻ The selection of the appropriate test statistic (z, t, χ^2, F). ◻ Specification of the significance level. ◻ A decision rule. ◻ The calculation of a sample statistic. ◻ A decision regarding the hypotheses based on the test. ◻ A decision based on the test results.	
Two-Tailed **One-Tailed**	◻ A two-tailed test results from a two-sided alternative hypothesis (e.g., H_A: $\mu \neq \mu_0$). ◻ A one-tailed test results from a one-sided alternative hypothesis (e.g., H_A: $\mu > \mu_0$, or H_A: $\mu < \mu_0$).	

Hypothesis Testing	★★★	性质、计算

Decision Rule	With critical value: □ Reject H_0 if |test statistic| > critical value. □ Fail to reject H_0 if |test statistic| < critical value. With **p-value**: □ If P-value < alpha, we reject null hypothesis.



Hypothesis Testing	★★★	性质、计算
Type I Error	□ Reject the null hypothesis when it's actually true.	
Type II Error	□ Fail to reject the null hypothesis when it's actually false.	

Decision	True Condition	
	H_0 is true	**H_0 is false**
Do not reject H_0	Correct Decision	Incorrect Decision **Type II Error**
Reject H_0	Incorrect Decision **Type I Error** Significance Level (α)	Correct Decision Power of the Test = 1 – P(Type II Error)

Mean Hypothesis Testing	★★★	性质、计算	
Normally distributed population, known population variance	$\mu = 0$	$z = \dfrac{\overline{x} - \mu_0}{\sigma / \sqrt{n}}$	N(0,1)
Normally distributed population, unknown population variance	$\mu = 0$	$t = \dfrac{\overline{x} - \mu_0}{s / \sqrt{n}}$	$t(n - 1)$

Variance Hypothesis Testing	★★★	性质、计算	
Normally distributed population	$\sigma^2 = \sigma_0^2$	$X^2 = \dfrac{(n-1)s^2}{\sigma_0^2}$	$X^2(n - 1)$
Two independent normally distributed populations	$\sigma_1^2 = \sigma_2^2$	$F = s_1^2 / s_2^2$	$F(n_1 - 1, n_2 - 1)$

Regression

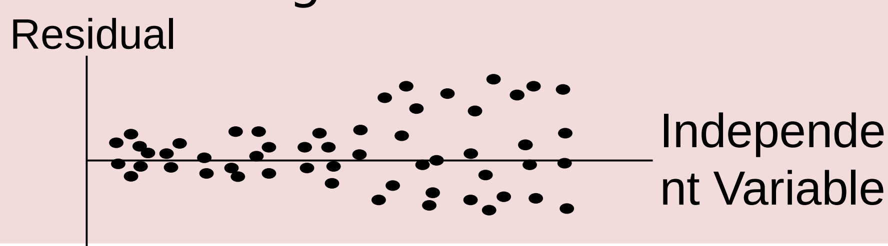
Linear Regression	★★	性质、计算
Linear Regression with One Regressor		$Y_i = B_0 + B_1X_i + \varepsilon_i$ Slope coefficient: B_1 Intercept coefficient: B_0 Error term: ε_i
Linear Regression with Multiple Regressors		$Y_i = B_0 + B_1X_{1i} + B_2X_{2i} + \ldots + B_kX_{ki} + \varepsilon_i$ B_2 is the effect on Y_i given a unit change in X_{2i} if we hold constant X_1, X_3, … and X_N

Regression Assumptions	★★★	性质
Assumptions	◻ A linear relationship exists between X and Y. ◻ X is not random, and the condition that X is uncorrelated with the error term can substitute the condition that X is not random. ◻ The expected value of the error term is zero (i.e., $E(\varepsilon_i)=0$). ◻ The variance of the error term is constant (i.e., the error terms are homoskedastic). ◻ The error term is uncorrelated across observations. ◻ The error term is normally distributed.	

Violation of Assumption	★★★	性质

Heteroskedasticity

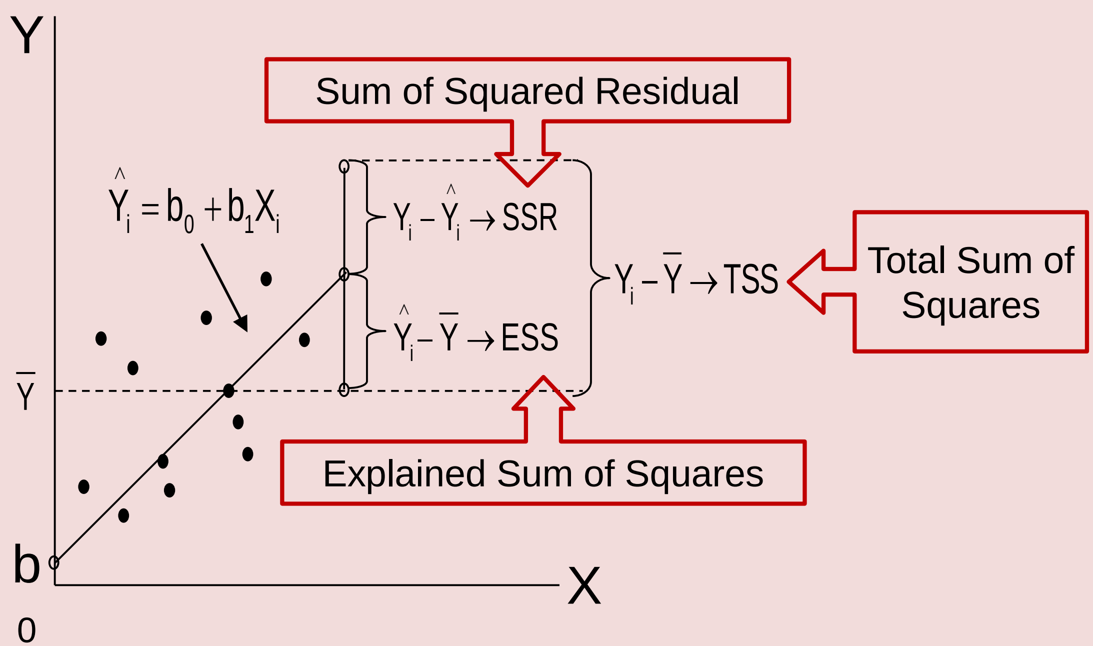
- The error term u(i) is **homoskedasticity** if the variance of the conditional distribution of u(i) given X(i) is constant for i = 1,…,n and in particular does not depend on X(i). Otherwise the error term is **heteroskedastic.** (Unconditional vs. Conditional)
- Effect
  - The standard errors are usually unreliable estimates.
  - The coefficient estimates (the b_1) aren' t affected.
  - If the standard errors are too small, but the coefficient estimates themselves are not affected, the t-statistics will be too large and the null hypothesis of no statistical significance is rejected too often. The opposite will be true if the standard errors are too large.



Violation of Assumption	★★★	性质
Autocorrelation	◻ Also known as **serial correlation**, is the cross-correlation between the values of the same variables. ◻ Effect: While it does not bias the OLS coefficient estimates, the standard errors tend to be underestimated (and the t-scores overestimated) when the auto correlations of the errors are positive.	
Multicollinearity	◻ Multicollinearity refers to the situation that two or more independent variables are highly correlated with each other. ◻ Effect: OLS estimators will be computed, but the resulting coefficients may be improperly estimated (Imperfect Multicollinearity). ◻ Detect: t-tests indicate that none of the individual coefficients is significantly different than 0, while the F-test indicates overall significance and the R^2 is high.	

Ordinary Least Squares	★★★	性质、计算
Property		OLS estimation is a process that estimates the population parameters B_i with corresponding values for b_i that minimize the squared residuals (i.e., error terms).
Calculation		minimize $\sum e_i^2 = \sum[Y_i - (\hat{b}_0 + \hat{b}_1 \times X_i)]^2$ $\hat{b}_1 = \dfrac{\sum_{i=1}^{n}(X_i - \bar{X})(Y_i - \bar{Y})}{\sum_{i=1}^{n}(X_i - \bar{X})^2} = \dfrac{\text{Cov}(X,Y)}{\text{Var}(X)}$ $\hat{b}_0 = \bar{Y} - \hat{b}_1 \bar{X}$

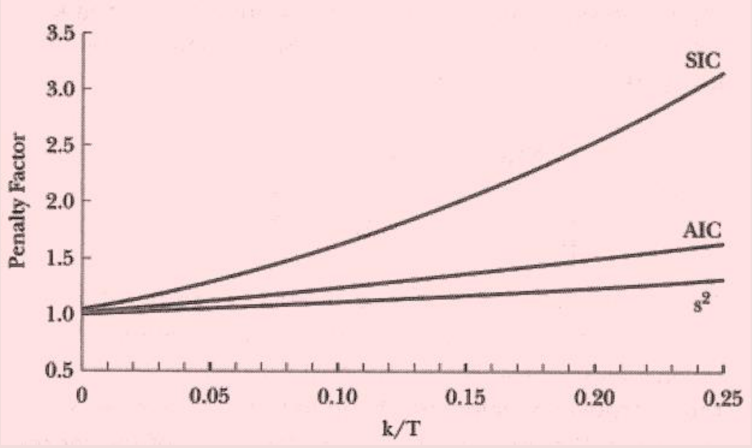
Joint Hypothesis Testing	★★	性质、计算
Property		An **F-test** is used to test whether at least one slope coefficient is significantly different from zero. $H_0: b_1 = b_2 = b_3 = \ldots = b_k = 0;$ H_a: at least one $b_j \neq 0$ ($j = 1$ to k)
Calculation		$F = \dfrac{ESS/k}{RSS/(n-k-1)}$ Decision rule: reject H_0, if F(test-statistic) > F_c(critical value)

Linear Regression Model	★★★	性质、计算
Graphics		
TSS、ESS、SSR	$\sum(Y_i-\bar{Y})^2=\sum(\hat{Y}-\bar{Y})^2+\sum(Y_i-\hat{Y})^2$ TSS = ESS + SSR	

Measures of Fit	★★★	性质、计算
R^2	$R^2 = \frac{ESS}{TSS} = 1 - \frac{SSR}{TSS}$	
r^2(correlation coefficient)	$r^2 = R^2 \rightarrow r = \pm\sqrt{R^2}$	
Adjusted R^2	$\text{Adjusted } R^2 = 1 - \frac{n-1}{n-k-1}\frac{SSR}{TSS}$	
SER	$SER = \sqrt{\frac{SSR}{n-2}} = \sqrt{\frac{e_i^2}{n-2}}$	

ANOVA Table		★★★	性质、计算
	df	SS	MSS
Regression	k	ESS	ESS/k
Residual	n – k - 1	RSS	RSS/(n – k – 1)
Total	n - 1	TSS	-

Modeling and Forecasting Trend

Model Selection	★★	性质
Mean Squared Error (MSE)	$MSE = \frac{\sum_{t=1}^{T} e_t^2}{T}$	The penalty factor for s^2, AIC, and SIC are: □ (T/T – k) □ $e^{(2k/T)}$ □ $T^{(k/T)}$ SIC has the largest penalty factor.
S^2 Measure	$s^2 = \frac{\sum_{t=1}^{T} e_t^2}{T-k}$	
Akaike Information Criterion (AIC)	$AIC = e^{\left(\frac{2k}{T}\right)} \frac{\sum_{t=1}^{T} e_t^2}{T}$	
Schwarz Information Criterion (SIC)	$SIC = T^{\left(\frac{k}{T}\right)} \frac{\sum_{t=1}^{T} e_t^2}{T}$	

White Noise	★★★	性质
Definition	◻ A time series process with a zero mean, constant variance, and no serial correlation is referred to as white noise. ◻ The lack of any correlation in a white noise process means that all autocovariances and autocorrelations are zero beyond displacement zero. The past is not correlated with the present which, in turn, is not correlated with the future.	

Box-Pierce Q-Statistic & Ljung-Box Q-Statistic	★★★	性质
Definition	◻ Q-statistics further refine the measurement of the degree to which autocorrelations vary from zero and whether white noise is present in the dataset.	

Box-Pierce Q-Statistic

$$Q_{BP} = T\sum_{\tau=1}^{m}\rho^2(\tau)$$

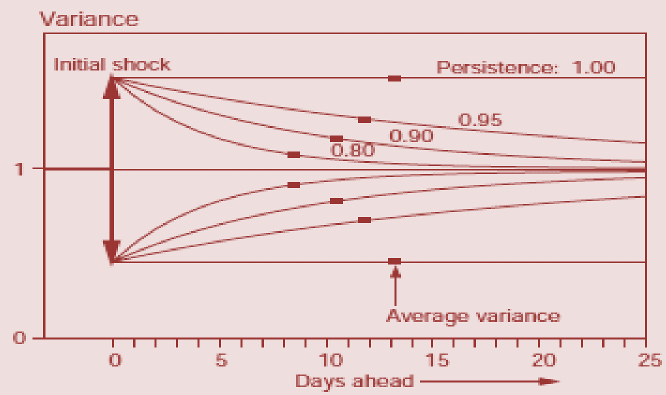
Ljung-Box Q-Statistic

$$Q_{LB} = T(T+2)\sum_{\tau=1}^{m}\rho^2(\tau)\left(\frac{1}{T-\tau}\right)$$

MA, AR, and ARMA Models	★★	性质
MA	◻ The first-order moving average process enables forecasters to consider the likely current effect on a dependent variable of current and lagged white noise error terms. $y_t = \varepsilon_t + \theta\varepsilon_{t-1} = (1+\theta L)\varepsilon_t; \varepsilon_t \sim WN(0,\sigma^2)$	
AR	◻ The AR(1) process seeks to explain the dependent variable in terms of a lagged observation of itself and an error term. $y_t = \varphi y_{t-1} + \varepsilon_t; \varepsilon_t \sim WN(0,\sigma^2)$ $(1-\varphi L)y_t = \varepsilon_t$	
ARMA	◻ The ARMA process incorporates the lagged error elements of the moving average process and the lagged observations of the dependent variable from the autoregressive process. $y_t = \varphi_1 y_{t-1} + \varphi_2 y_{t-2} + \ldots + \varphi_p y_{t-p} + \theta\varepsilon_{t-1} + \ldots + \theta_q\varepsilon_{t-q} + \varepsilon_t; \varepsilon_t \sim WN(0,\sigma^2)$	

Estimating Volatilities and Correlations

EWMA	★★★	性质、计算
Definition	◻ In an exponentially weighted moving average model, the weights assigned to the αi decline exponentially as we move back through time. ◻ The weights decline (in constant proportion, given by lambda). ◻ Special case of GARCH (1, 1).	
Basic Formulas	$\sigma_n^2 = \lambda\sigma_{n-1}^2 + (1-\lambda)\mu_{n-1}^2$ $\sigma_n^2 = (1-\lambda)\sum_{i=1}^{m}\lambda^{i-1}\mu_{n-i}^2 + \lambda^m\sigma_{n-m}^2$	$Cov_n = \lambda Cov_{n-1} + (1-\lambda)X_{n-1}Y_{n-1}$
Attractive Feature	◻ Relatively little data needs to be stored. ◻ We need only remember the current estimate of the variance rate and the most recent observation on the value of the market variable. ◻ Tracks volatility changes. The value of λ governs how responsive the estimate of the daily volatility is to the most recent daily percentage change. ◻ Risk Metrics uses λ = 0.94 for daily volatility forecasting.	

GARCH	★★★	性质、计算
Definition	◻ Is the weighted sum of a long run-variance, the most recent squared-return, and the most recent variance. ◻ The higher the persistence (α + β), the longer it will take to revert to the mean.	
Basic Formulas	$\sigma_n^2 = \omega + \alpha\mu_{n-1}^2 + \beta\sigma_{n-1}^2$ $\omega = \gamma V_L$ $\gamma + \alpha + \beta = 1 \quad \alpha + \beta < 1$	$V_L = \dfrac{\omega}{1-\alpha-\beta}$ $cov_n = \omega + \alpha x_{n-1} y_{n-1} + \beta cov_{n-1}$
Graphics	 Shocks to a GARCH Process	

Simulations

Monte Carlo Simulation	★★	性质
Basic Steps	◻ Specify the data generating process. ◻ Estimate an unknown variable. ◻ Save the estimate from step 2. ◻ Go back to step 1 and repeat this process N times.	
Reducing Standard Error $\frac{s}{\sqrt{N}}$	◻ The standard error estimate of a Monte Carlo simulation can be reduced by a factor of 10 by increasing N by a factor of 100. ◻ Variance reduction technique • Antithetic Variates • Control Variates • Random Number Re-Usage across Experiments	

Variance Reduction Technique	★★★	性质
Antithetic Variates	◻ Reduces sampling error by rerunning the simulation using a complement set of the original set of random variables.	
Control Variates	◻ Replaces a variable x that has unknown properties in a Monte Carlo simulation with a similar variable y that has known properties. The new x* variable estimate will have a smaller sampling error than the original x variable if the control statistic and statistic of interest are highly correlated.	
Random Number Re-Usage	◻ Reusing sets of random number draws across Monte Carlo experiments reduces the estimate variability across experiments.	

Random Number Generation	★★★	性质
Pseudo-Random Numbers	◻ Not truly random, they are actually generated from a formula. The choice of the initial seed value influences the properties of the random number distribution that is generated.	

Bootstrapping	★★★	性质
Difference with Simulation	◻ With simulation, the data are constructed completely artificially. ◻ Bootstrapping, on the other hand, is used to obtain a description of the properties of empirical estimators by using the sample data points themselves, and it involves sampling repeatedly with replacement from the actual data. ◻ The advantage of bootstrapping over the use of analytical results is that it allows the researcher to make inferences without making strong distributional assumptions, since the distribution employed will be that of the actual data.	