Valuation and Risk Models

讲师：

专业来自101%的投入！

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金程教育 GOLDEN FUTURE

Valuation and Risk Models

1. Bond Valuation
2. Option Valuation
3. Market Risk Model
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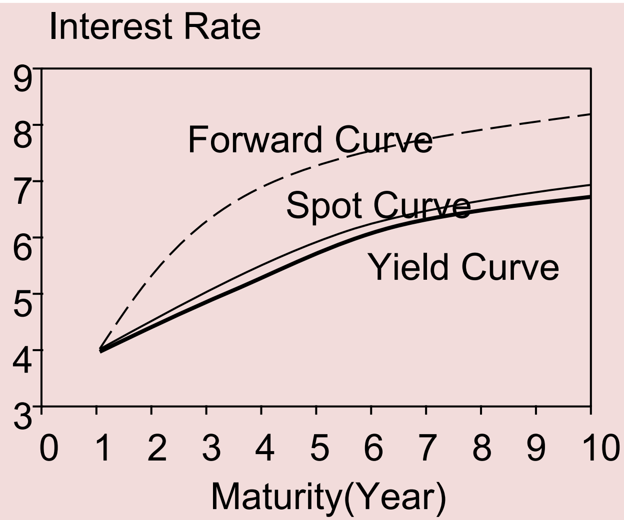
Bond Valuation

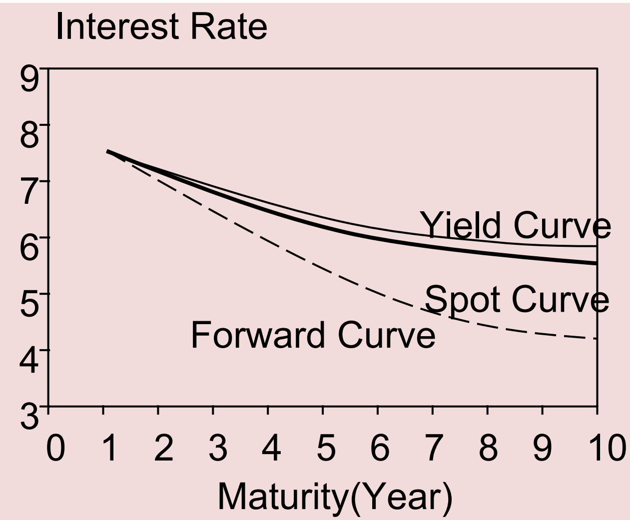
Basics	★★	性质、计算
Spot Rate	◻ A t-period spot rate, or zero rate, denoted as z_t, is the yield to maturity on a zero-coupon bond that matures in t-years.	
Forward Rate	◻ Interest rates corresponding to a future period implied by the spot curve， $f_{t1,t2}$. $$(1+R_1)^{T_1}(1+F_{1,2})^{(T_2-T_1)}=(1+R_2)^{T_2}$$ $$e^{R_1T_1}\times e^{F_{1,2}(T_2-T_1)}=e^{R_2T_2}\Rightarrow F_{1,2}=\frac{R_2T_2-R_1T_1}{T_2-T_1}$$	
Discount Factor	◻ d(t), for a term of (t) years, gives the present value of one unit of currency ($1) to be received at the end of that term. $$d(t)=\frac{1}{\left(1+\frac{z_t}{2}\right)^{2t}}$$	
Yield to Maturity	◻ Internal rate of return found by equating the present value of the cash flows to the current price of the security.	

Spot & Forward & Yield Curve	★★★	性质、计算

Spot &Forward &Yield Curve



Upward-Sloping Term Structure



Downward-Sloping Term Structure

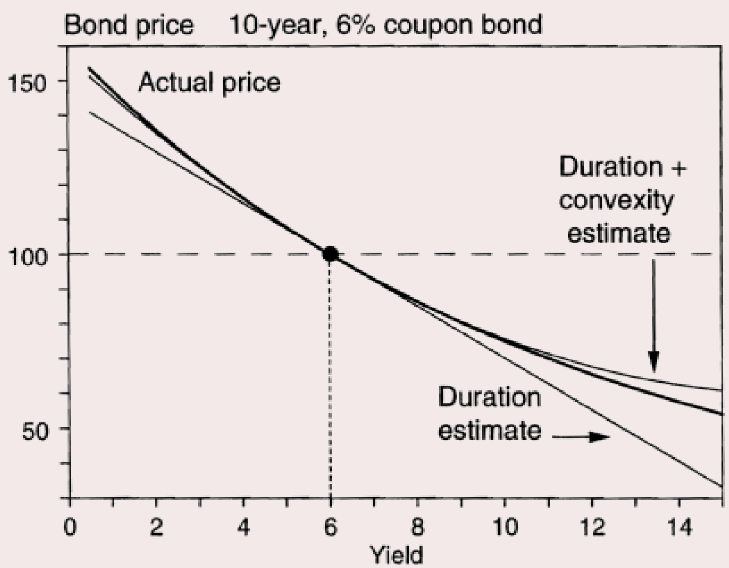
Reinvestment Risk and Interest Rate Risk	★★	性质
Reinvestment Risk	Future interest rates can be less than the yield to maturity at the time bond is purchased, known as reinvestment risk.	
Interest Rate Risk	If the bond is not held to maturity, the investor faces the risk that he may have to sell for less than the purchase price, resulting a return that is less than the yield to maturity, known as interest rate risk.	

Valuation	★★★	性质、计算
Coupon Bond	□ **Spot Rate**	$\text{Price} = \frac{CF_1}{1+z_1} + \ldots + \frac{CF_T}{(1+z_T)^T}$
	□ **Forward Rate**	$\text{Price} = \frac{CF_1}{1+f_{0,1}} + \ldots + \frac{CF_T}{(1+f_{0,1})(1+f_{1,2})\ldots(1+f_{T-1,T})}$
	□ **Discount Factor**	$\text{Price} = CF_1 \times d(1) + \ldots + CF_T \times d(T)$
	□ **Bond Replication:** Absent confounding factors, identical sets of cash flows should sell for the same price.	

Valuation	★★	性质、计算	
Annuity	$A = \frac{c}{y}\left[1-\left(\frac{1}{1+\frac{y}{2}}\right)^{2T}\right]$	**Perpetuity**	$\text{Perpetuity} = \frac{c}{y}F$

Price Approximation	
The actual, exact price	$P = f(y_0 + \Delta y)$
The duration estimate	$P = P_0 - D^* P_0 \Delta y$
The duration and convexity estimate	$P = P_0 - D^* P_0 \Delta y + \frac{1}{2} C P_0 (\Delta y)^2$

$$P(y_0 + \Delta y) = P(y_0) + \underbrace{f'(y_0)}_{\text{dollar duration}}\Delta y + \frac{1}{2}\underbrace{f''(y_0)}_{\text{dollar convexity}}(\Delta y)^2 + \cdots$$



Price Approximation

Option Valuation

Binomial Trees	★★★	性质、计算
Principles	◻ **Risk-Neutral Valuation:** All cash flows should be discounted to present using the risk-free interest rate. ◻ **Assumption:** 1) Stock price follows geometric Brownian motion; 2) u = 1/d.	
One-Step Calculation	$u = e^{\sigma\sqrt{\Delta t}}; d = e^{-\sigma\sqrt{\Delta t}}$ $p = \frac{e^{r\Delta t} - d}{u - d}$ or $p = \frac{e^{(r-q)\Delta t} - d}{u - d}$	$f = \left[pf_u + (1-p)f_d\right]e^{-r\Delta t}$
Two-Step Calculation	$f = e^{-2r\Delta t}\left[p^2 f_{uu} + 2p(1-p)f_{ud} + (1-p)^2 f_{dd}\right]$	
American Options	◻ Make sure that the option value at each node is no less than the intrinsic value.	

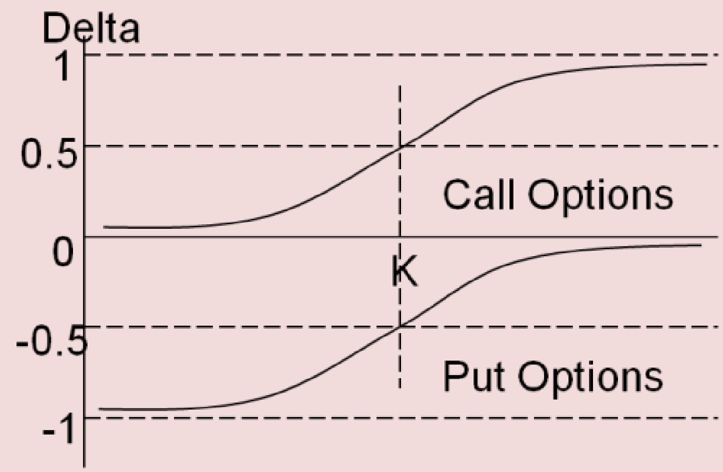
BSM Model	★★★	性质、计算
Key Assumptions	□ The price of the underlying asset follows a lognormal distribution. □ The risk-free rate is known and constant. □ The volatility of the underlying asset is known and constant. □ The option valued are European Options.	
Calculation	Call	$c = SN(d_1) - Ke^{-rT}N(d_2)$ or $c = Se^{-qT}N(d_1) - Ke^{-rT}N(d_2)$
	Put	$p = Ke^{-rT}N(-d_2) - SN(-d_1)$ or $p = Ke^{-rT}N(-d_2) - Se^{-qT}N(-d_1)$
	Where: $d_{1,2} = \frac{\ln(S/Ke^{-rt})}{\sigma\sqrt{T}} \pm \frac{1}{2}\sigma\sqrt{T}$ or $d_{1,2} = \frac{\ln(Se^{-qT}/Ke^{-rT})}{\sigma\sqrt{T}} \pm \frac{\sigma\sqrt{T}}{2}$	

BSM Model for Early Exercise of American Options	★★	性质、计算
For American call	$D_n > X\left(1-e^{-r(T-t_n)}\right)$	
For American put	$D_n < X\left(1-e^{-r(T-t_n)}\right)$	

The Greek Letters ★★★ 性质、计算

Delta

- Ratio of the price change of the option to the price change of the underlying assets.
- Call: 0 ~ 1; Put: -1~0
- Long call: >0; Short call: <0; Long put: <0; Short put: >0.
- When t→T, delta is unstable.
- For at the money call, delta = 0.5
- Portfolio Delta: Summation of the product of each option position and its delta.

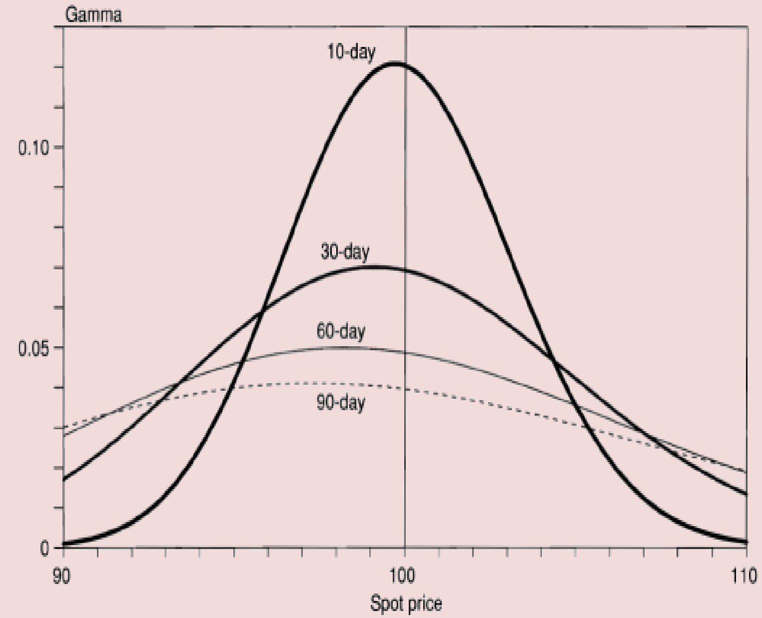


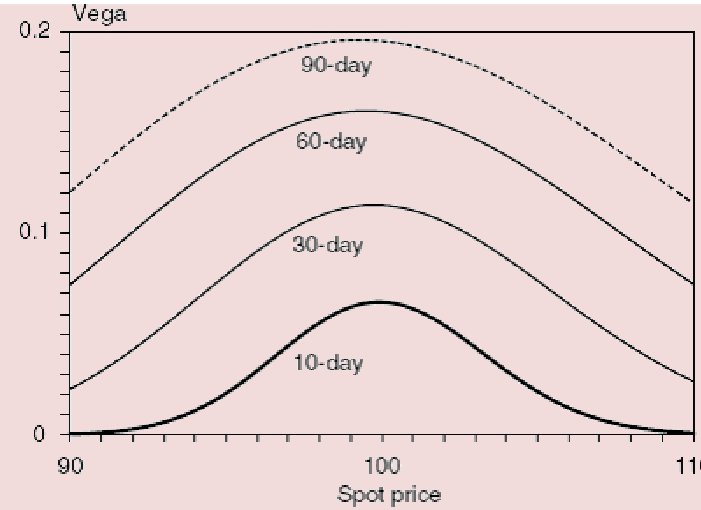
Call and Put Option Delta

$$\Delta = \frac{\partial c}{\partial S} = N(d_1) \qquad \Delta = \frac{\partial c}{\partial S} = e^{-qT}N(d_1)$$

$$\Delta = \frac{\partial p}{\partial S} = N(d_1) - 1 \qquad \Delta = \frac{\partial p}{\partial S} = e^{-qT}\left[N(d_1) - 1\right]$$

Delta Hedge: A position with a delta of zero is called a delta neutral position. To maintain a delta neutral position, the trader must re-balance the portfolio.

The Greek Letters	★★★	性质
Gamma	◻ Rate of change of the option’ s delta with respect to the price of the underlying asset. ◻ Providing added protection against large movements in the underlying asset's price. ◻ Gamma is largest when an option is at-the-money. ◻ Long call: >0; Short call: <0; Long put: >0, Short put: <0.	
	Application in Delta Hedge: If gamma is higher, rebalancing more frequently is required. On the other hand, when gamma is lower, rebalancing is required less frequently.	

The Greek Letters	★★	性质
Vega	◻ Rate of change of the value of the option with respect to the volatility of the underlying asset. ◻ Most sensitive to changes in volatility when they are at the money.	
Theta	◻ Rate of change of the value of the option with respect to the passage of time with all else remaining the same. ◻ Short-term at the money option has a greatest negative theta.	
Rho	◻ Sensitivity to the interest rates. ◻ In the money calls and puts are more sensitive to changes in rates than out-of-the-money options.	

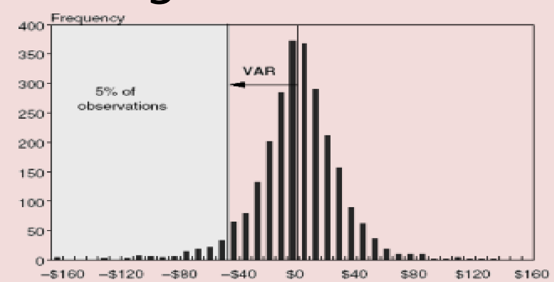
Market Risk Model

Risk Measures Properties	★★	性质

Coherent Properties	◻ Monotonicity : A random cash flow or future value R_1 that is always greater than R_2 should have a lower risk. $R_1 \ge R_2 \Rightarrow \rho(R_1) \le \rho(R_2)$
	◻ Subadditivity: The portfolio's risk should not be greater than the sum of its parts. $\rho(R_1 + R_2) \le \rho(R_1) + \rho(R_2)$
	◻ Positive Homogeneity : The risk of a position is proportional to its scale or size. $\rho(\beta R) = \beta\rho(R)$
	◻ Translation Invariance : Like adding cash for constant c, $\rho(R + c) = \rho(R) - c$

Expected Shortfall	★★★	性质、计算

◻ Conditional VaR ; Sub-additive ; Average of the worst $100\times(1-\alpha)\%$ of losses.

Value at Risk	★★★	性质、计算
Definition	VaR is the maximum loss over a target horizon and for a given confidence level. 	
Parameters	◻ Time horizon: should depend on how quickly portfolio can be unwound. Regulators in effect use 1-day for bank market risk and 1-year for credit/operational risk. ◻ Confidence level: depends on objectives. Regulators use 99% for market risk and 99.9% for credit/operational risk. ◻ Data Window	
Properties	◻ Not contain worst conditions/tail loss. ◻ Not sub-additive.	
Calculation	$\text{VaR}(X\%)=\left\|E(R)-z_{X\%}\times\sigma\right\|$ $\text{VaR}(X\%)_{J\text{-days}}=\text{VaR}(X\%)_{1\text{-days}}\times\sqrt{J}$	

Value at Risk for Derivatives	★★★	性质、计算
Delta-Normal	◻ Linear Approximation $\text{VaR}(dP) = \left\| -D^{*}P \right\| \times \text{VaR}(dy)$ $\text{VaR}(df) = \left\| \Delta \right\| \times \text{VaR}(dS)$	
Delta-Gamma	◻ Nonlinear Approximation $\text{VaR}(dP) = \left\| -D^{*}P \times \text{VaR}(dy) + \frac{1}{2}(C \times P) \times \text{VaR}(dy)^2 \right\|$ $\text{VaR}(df) = \left\| \Delta \times \text{VaR}(dS) + \frac{1}{2}\Gamma \times \text{VaR}(dS)^2 \right\|$	
Full Revaluation	◻ Full re-pricing of the portfolio under the assumption that the underlying risk factor(s) are shocked to experience a loss. ◻ Accurate but computationally burdensome. - Historical/Bootstrap Simulation - Monte Carlo Simulation	

Volatility Estimation	★★★	性质
Fat Tails	◻ **Cause:** Volatility is time-varying. ◻ **Solution:** Regime-switching volatility model	
Historical-Based Approach	◻ **Parametric Model** Assumes asset returns are normally or lognormally distributed with time-varying volatility	
	◻ **Nonparametric Model** No underlying assumptions	
	◻ **Hybrid Approach** Both parametric and nonparametric method	
Implied Volatility-Based Approach	◻ Uses derivative pricing models such as the Black-Scholes-Merton option pricing model to estimate an implied volatility based on current market data rather than historical data.	

Credit Risk Model

EL and UL	★★★	性质、计算
EL	◻ Expect to lose on average over a predetermined period of time.	$AE = OS + \alpha \times COM$ $EL = PD \times EA \times LR$
	◻ Portfolio EL is linear and additive.	$EL_P = \sum_{i=1}^{n} EL_i$
UL	◻ Risk arises from the variation in loss levels - which for credit risk is due to unexpected losses.	$UL = EA \times \sqrt{PD \times \sigma_{LR}^2 + LR^2 \times \sigma_{PD}^2}$ $\sigma_{PD}^2 = PD \times (1 - PD)$
	◻ Portfolio UL need consider the effects of diversification.	$UL_P = \sqrt{UL_1^2 + UL_2^2 + 2\rho UL_1 UL_2}$
UL Contribution	$ULC_1 = \frac{UL_1^2 + \rho UL_1 UL_2}{UL_P}$; $UL_P = \sum_{i=1}^{n} ULC_i$	

Operational Risk Model

Operational Risk	★★★	性质、计算
Classification	◻ Inadequate or failed internal processes, people, and systems or from external events.	
Loss Distribution	◻ **Loss Frequency Distribution:** The distribution of the number of losses observed during the time horizon. A common probability distribution is the Poisson Distribution. ◻ **Loss Severity Distribution:** Is the distribution of the size of a loss, given that a loss occurs. It is typically assumed that loss severity and loss frequency are independent. For the loss severity distribution, a **lognormal probability distribution** is often uses. ◻ **Loss Distribution:** Monte Carlo Simulation	

Data Issues	★★★	性质
Frequency: Internal	**Severity:** Internal & External, encourage using scenario analysis.	

Operational Risk Regulatory Capital	★★★	性质、计算
Basic Indicator Approach	$K_{operational,BIA}=\frac{\sum_{i=last\ three\ years}(GI\times\alpha)}{3}$	
Standardized Approach	Corporate Finance (18%) Payment and settlement (18%) Trading and Sales (18%) Agency Services (15%)	Commercial Banking (15%) Asset Management (12%) Retail Banking (12%) Retail Brokerage (12%)
	$K_{operational,SA}=\frac{\left\{\sum_{i=last\ three\ years}\max\left[\sum GI_{line1-8}\times\beta_{line1-8},0\right]\right\}}{3}$	
AMA Approach	☐ The capital charge for AMA is calculated as the bank’ s operational value at risk with a one-year horizon and a 99.9% confidence level.	

Features of Stress Testing	★★	性质
Advantage	**Disadvantage**	
□ Complements value at risk. □ Simple and intuitive. □ Directly examines the tails.	□ Highly subjective. □ Implausible scenarios. □ Difficult to interpret.	

Types of Stress Testing	★★	性质
Unidimensional And Multidimensional	□ Unidimensional scenarios focus ” stressing” on key one variable. The key weakness of a unidimensional analysis is that scenarios cannot account for correlation. □ Multidimensional attempts to stress multiple variables and their relationships.	

Country Risk

Sources of Country Risk	★★	性质
Sources	**Effect**	
Economic Growth Life Cycle	◻ More mature markets/companies within those markets are less risky than those firms/countries in the early stages of growth.	
Political Risk	◻ Continuous versus Discontinuous Risk (Risks in democracies vs Risks in dictatorships) ◻ Corruption ◻ Physical Violence ◻ Nationalization and Expropriation Risk.	
Legal Risk	◻ The protection of property rights and the speed with which disputes are settled affect risk.	
Economic Structure	◻ A disproportionate reliance on a single commodity or service in an economy increases a country' s risk exposure.	

Sovereign Default	★★	性质
Causes	**Foreign Currency Default** - Countries are often without the foreign currency to meet the debt obligation and are unable to print money to repay the debt. This makes up a large proportion of sovereign defaults. **Local Currency Default:** - The use of the gold standard prior to 1971 made it more difficult for some countries to print money; - Shared currencies make it impossible for countries to control their own monetary policy; - Some countries must conclude that the costs of currency debasement and potentially higher inflation are greater than the costs of default.	

Sovereign Default	★★	性质
Consequences	◻ GDP growth ◻ Sovereign ratings and borrowing costs	◻ Trade retaliation ◻ Fragile banking systems ◻ Political change
Influence Factor	◻ The country' s level of indebtedness ◻ Pension funds and social services ◻ Tax receipts ◻ Stability of tax receipts ◻ Political risk ◻ Backing from other countries/entities	
Sovereign Default Spread	**Advantage**	◻ Changes occur in real time ◻ Granularity ◻ Adjust quickly to new information
	Disadvantage	◻ Need for a risk-free security ◻ Cannot compare local currency bonds ◻ Greater volatility