Foundations of Risk Management

FRM一级培训项目-强化班

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101% Contribution Breeds Professionalism

Framework

- Risk management: a helicopter view
- Corporate risk management: a primer
- Corporate governance and risk management
- What is ERM?
- Risk management and risk taking in banks

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Risk Management: a Helicopter View

A. Is Risk Management Useful?

- Risk management and risk taking aren't opposites, but two sides of the same coin.
  - Together they drive all our modern economies.
  - It's all about making forward-looking choices about risk in relation to reward.
- Blames for Risk Management
  - Fail to prevent market disruptions or accounting scandals.
  - Derivative markets make it easier to take on large amount of risk.
  - Sophisticated financial engineering lead to the violent implosion of firms.
  - Only transfer risks to other firms.
  - Work to the short-term benefit.

C. The Conflict of Risk and Reward

- A Tradeoff between Risks and Rewards
  - Poor risk management: powerful business leaders exaggerate returns while diminishing risks.
  - Management might leave gaps in risk measurement.
  - Risk management failure can be exacerbated by the compensation incentive schemes.
  - Trading institutions may manipulate revenues.
  - Poor risk measurement techniques: distorted risk.
  - Industry regulators can be drawn into the deception.

G. Typology of Risk Exposures

- **Market Risk:** the risk that changes in financial market prices and rates will reduce the value of a security or a portfolio.
  - Interest rate risk
  - Equity price risk
  - Foreign exchange risk
  - Commodity price risk

G. Typology of Risk Exposures

- **Credit Risk:** the risk of an economic loss from the failure of a counterparty to fulfil its contractual obligations, or from the increased risk of default during the term of the transaction.
  - Default risk
  - Bankruptcy risk(the risk of actually taking over the collateralized, or escrowed, assets of a defaulted borrower or counterparty)
  - Downgrade risk
  - Settlement risk

G. Typology of Risk Exposures

- **Liquidity Risk**
  - **Funding Liquidity Risk:** a firm's ability to raise the necessary cash to roll over its debt; to meet the cash, margin, and collateral requirements; and to satisfy capital withdrawals.
  - **Trading Liquidity Risk:** an institution will not be able to execute a transaction at the prevailing market price because there is temporarily no appetite for the deal on the other side of the market.

Corporate Risk Management: A Primer

A. Why Not to Manage Risk in Theory

- Franco Modigliani and Merton Miller (M&M):
  - The value of a firm cannot be changed merely by means of financial transactions (or risk mgmt).
  - Firms should therefore not engage in any risk reduction activity.
  - Individual investors can execute on their own.
- MM's assumption: the capital markets are perfect.
  - Highly competitive; participants are not subject to transaction costs, commissions, or taxes.

A. Why Not to Manage Risk in Theory

- CAPM: in a world with perfect capital markets, firms only base their investment decisions on systematic risks.
- Using hedging tools cannot increase the value of the firm.
- Active hedging may distract management from its core business.
- A careless risk management strategy can drag a firm down.
- Risk management strategy has compliance costs.

B. Reasons for Managing Risk in Practice

- The "perfect market" assumption does not hold.
- The high fixed costs associated with financial distress and bankruptcy.
- Managers have an interest in reducing risks ⇒ to achieve the board's objectives.
- Hedging reduces the cost of capital and increases the debt capacity of companies.
- A firm can stabilise its costs through hedging.
- Purchasing insurance is expensive.

Corporate Governance and Risk Management

B. Corporate Governance and Risk Management

- The Board Should (basic functions)
  - Look after the interests of shareholders.
  - Be sensitive to the concerns of other stakeholders.
  - Alert for any conflict between the interests of management and stakeholders.
  - Separate the role of the CEO and the chairman of the board.

B. Corporate Governance and Risk Management

- CRO (dual positions)
  - Act as a senior member of the management committee and attend board meetings regularly.
  - Have a direct reporting line to the board or its risk committees in addition to reporting to the executive team.

C. True Risk Governance

- Risk Appetite
  - The board characterises the risk appetite.
  - Be connected to a firm's overall business strategy and capital plan.
  - Clear communication of the firm's risk appetite and risk position.

C. True Risk Governance

- Four Basic Choices in Risk Management
  1. Avoid risk.
  2. Transfer risk to third parties.
  3. Mitigate risk.
  4. Accept risk.

C. True Risk Governance

- The Board Should ensure (advanced functions)
  - Economic rather than accounting performance.
  - "Soft" risks.
  - Staff are rewarded based on risk-adjusted performance.
  - Major transactions are consistent with the risk authorised.
  - Keeping healthy skepticism.

D. Committees and Risk Limits

1. Risk Management Committee

- Translate the overall risk appetite into a set of limits.
- Independently review credit, market, and liquidity risks.
- Report back to the board on a variety of items, such as all loans and credits over a specified dollar limit.

D. Committees and Risk Limits

2. Audit Committee of The Board

- Check for infringements, oversee the quality of the processes.
- But only confines to verification function.
- Audit committee members are required to be financially literate.

D. Committees and Risk Limits

3. Risk Advisory Director: is a member of the board who specialises in risk matters.

4. Compensation Committee

- Determine the compensation of top executives.
- Be aligned with the long-term interests of stakeholders, and with risk-adjusted return on capital.
- Removal of guaranteed bonuses.
- Stock-based compensation can encourage risk-taking.

H. What Is the Role of the Audit Function?

- A Key Role of the Audit Function
  - Provide an independent assessment of the bank's risk management.
  - Examine the integrity of the management information system.
  - Verify the accuracy of models through back-testing.
  - Examine the documentation relating to compliance.

What is ERM?

B. The Benefits of ERM

ERM Is All About Integration

- ERM requires an integrated risk organisation.
  - The CRO reports to the CEO and the Board in support of their risk oversight responsibilities.
- ERM requires the integration of risk transfer strategies.
  - Take a portfolio view of all types of risk within a company.
- ERM requires the integration of risk management into the business processes of a company.

B. The Benefits of ERM

Three Benefits To ERM

1. Organisational Effectiveness
   - Various functions work cohesively and efficiently.
2. Risk Reporting
   - Timely and relevant risk reporting
3. Business Performance
   - Market value improvement.
   - Lower earnings volatility …

C. The Chief Risk Officer

- A CRO Is Responsible For
  - Providing the overall leadership for enterprise risk management.
  - Establishing an integrated risk management framework for all aspects of risks.
  - Allocating economic capital to business activities.
  - Communicating the company's risk profile to key stakeholders.

C. The Chief Risk Officer

- Reporting
  - The heads of individual risk department report to the CRO.
  - The CRO reports to the CFO or CEO.
  - A dotted-line reporting relationship between the CRO and the board.

D. Components of ERM

Risk Management and Risk Taking in Banks

A. How Does Risk Management Add Value?

- The Goal of Risk Management for Banks
  - To determine the optimal level of risk the level that maximises bank value;
  - But subject to the constraints imposed by regulators, laws, and regulations.
- A Tradeoff between Return and Risk
  - Cost of taking on a single new risk < gain.
  - But risk-taking decisions must be assessed in terms of their impact on the overall risk.
  - Major challenge: the tradeoff can not be made in real time.

A. How Does Risk Management Add Value?

- Two Ways that Risk Management can Destroy Value
  - Fail to ensure that the bank has the right amount of risk.
  - Fail to exercise the right amount of flexibility.
    - ✓ Too restrictive: policemen.
    - ✓ Too flexible.
    - ✓ It is critical to strike the balance.

B. Determining a Bank's Risk Appetite

- Banks Cannot Operate with too much Risk
  - Constrained by laws and regulations.
  - Limiting a bank's ability to attract deposits.
  - Derivatives counterparties will be reluctant to deal with the bank.
  - Difficult to hire potential employees.
- Banks have to Take Some Risks
  - To create wealth for their shareholders.

B. Determining a Bank's Risk Appetite

- A Bank's Optimal Credit Rating Approaches
  - Banks targets a certain credit rating.
    - ✓ the optimal rating of a bank is generally not the highest rating
    - ✓ Banks with very different strategies, or liability and asset structures, could well end up having very different credit ratings, and different attitudes toward risk.
    - ✓ A bank with more of a deposit franchise and with more relationship lending is likely to prefer a higher rating than an institution that is engaged in more transactional activities.
  - Banks targets a certain PD depending on the bank's business model.

B. Determining a Bank's Risk Appetite

- Taking Social Costs into Account
  - To maximise value for banks' shareholders ⇒ generate systemic risk to society.
  - Regulators therefore impose restrictions (capital).
  - Banks have to choose their level of risk subject to external constraints.

C. Governance and Risk Taking

- A Good Governance
  - Make value-maximising trade-offs between risk and reward.
  - Operate within the constraints imposed by regulation.
  - Consistent with the firm's risk appetite.
- Better Risk Governance=Better Risk Mgt. Proceed ≠ Less Risk
  - Effective risk governance = taking risks rewarding for shareholders ≠ eliminating or even reducing risk-taking.
  - Optimal amount of risk for a bank ≠ optimal amount for society.

D. The Organisation of Risk Management

The Right Degree of Independence for Risk Managers

- Risk management is not an audit.
  - Auditors only have a verification function.
- If risk managers are viewed as a police ⇒ face obstacles in gathering information.
- The reporting lines of risk managers should be completely separate from the businesses.
  - Business lines have a strong commitment to managing risk ⇒ business lines collaborate with risk managers.

G. Incentives, Culture, and Risk Management

- Risk Management is not Auditing
  - Risk managers must understand the bank's businesses.
  - Risk managers also have to determine when risk appetite has to be changed.
  - Risk managers: having a dialogue with business units.

G. Incentives, Culture, and Risk Management

- Corporate Culture of Risk
  - No incentive plan can precisely tell what actions to take in every situation.
  - Not all risks can be quantified ⇒ incentive for employees to take risks that are not quantified.
  - The ability of a firm to manage risk properly depends on its corporate culture.
  - Companies where managers are viewed by their employees as trustworthy are more profitable.
  - But good governance changes.

Framework

- Financial disasters
- Deciphering the liquidity and credit crunch 2007-2008
- Getting up to speed on the financial crisis
- Risk management failures

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Financial Disasters

1. Chase Manhattan Bank/Drysdale Securities

- Incident
  - Drysdale obtained unsecured borrowing of $300 million by exploiting a flaw in computing the value of U.S. government bond collateral.
  - Drysdale lost the borrowed money in bond markets.
  - Chase Manhattan absorbed these losses because it had brokered most of Drysdale's securities borrowings.

1. Chase Manhattan Bank/Drysdale Securities

- Result
  - Severely damaged Chase’s reputation.
  - Managers in Chase were convinced they were simply acting as intermediaries.
- Lessons
  - Make the methods for computing collateral value on bond borrowings more precise.
  - New product offerings must receive prior approval from the risk control functions.

2. Kidder Peabody

- Incident
  - Kidder Peabody entered into a series of trades that were artificially inflating the firm's reported profits.
  - It had not resulted in any actual loss of cash.
  - But triggered a substantial loss of confidence in the competence of the firm's management.
- Lessons
  - Always investigate a stream of large unexpected profits.
  - Periodically review any potential violations of assumptions in models and systems.

3. Barings Bank

- Incident
  - A loss of $1.25 billion due to the unauthorised trading of a trader Nick Leeson.
  - Forced Barings into bankruptcy.
- Arose
  - Leeson disguised his speculative position and manufactured substantial reported profits for his own accounts.

3. Barings Bank

- Why not detected?
  - Allowing Leeson to function as head of trading and the back office => depriving the firm of an independent check on his activities.
- Lessons
  - The absolute necessity of an independent trading back office.

4. Allied Irish Bank

- Incident
  - John Rusnak entered into massive unauthorised trades, resulting in $691 million in losses.
- Arose
  - Rusnak invented **imaginary trades** that offset his real trades, making his trading positions appear small.
  - He persuaded back-office personnel not to check these bogus trades.
  - He booked imaginary trades with counterparties in the Asian time zone.
  - He was extremely modest in the amount of false profit he claimed.
  - He relied on arguments that costs should be cut be weakening or eliminating key controls.

4. Allied Irish Bank

- Lessons
  - This case is similar to Barings. But Rusnak had no dual role.
  - Avoid engaging in small ventures in which the firm lacks any depth of expertise.
  - Relationship between parent and overseas units needs to be clarified.
  - Strong and enforceable back-office controls are essential.

5. Union Bank of Switzerland

- Incident
  - Losses of $400 million ~ $700 million in equity derivatives during 1997.
  - Another loss of $700 million during 1998 due to a large position in LTCM.
- Result
  - The 1997 losses forced UBS into a merger with Swiss Bank Corporation.
- Arose
  - The person with senior risk management authority for the department doubled as head of quantitative analytics. His compensation tied to trading results.
- Lessons
  - Less is known about this disaster than the others.
  - The need for independent risk oversight.

6. Societe Generale

- Incident
  - In January 2008, Societe Generale reported trading losses of $7.1 billion caused by unauthorised activity of a trader, Jerome Kerviel.
- Arose
  - Kerviel took very large unauthorised positions from 2005 to 2008.
  - He concealed these unauthorised positions by entering fictitious transactions that offset the risk and P&L of his true trades.
  - He constantly replaced canceled fictitious transactions with new ones, totalling 947 transactions.

6. Societe Generale

- Why not detected?
  - No red-flagging for an unusual level of trade cancellations.
  - Kerviel’s trading assistant may have been operating in collusion with Kerviel.
    - ✓ In any case, the trading assistant appears to have accepted Kerviel’s directions without questioning.
  - No mandatory vacation.
  - No limits of Kerviel's gross positions, only his net positions.
  - Kerviel’s unusually high amount of brokerage commissions, related to his high level of gross positions
  - Kerviel was reporting trading gains in excess of levels his authorized position taking could have accounted for, but an investigation into unexpected reported trading gains was lacked.

6. Societe Generale

- Lessons
  - Flag any trader who is using an unusually high number of cancellations.
  - Don’t trust the independence of the trading assistant too much.
  - Rules for mandatory vacation should be enforced.
  - Unusually high ratios of gross to net positions are a warning sign of unauthorised activities.

7. Long-Term Capital Management

- The Crisis
  - The triggers: Russian debt default in 1998.
  - The LTCM fund's equity began to decline, and it was reluctant to cut positions in a turbulent market.
  - As competitors learned more about the actual positions, their pressure on market prices in the direction unfavorables to LTCM intensified.

7. Long-Term Capital Management

- The Fall of LTCM
  - 14 of the largest creditors contributed a fresh $3.65 billion in equity investment into the LTCM fund to allow for a substantial time period in which to close out positions.
  - By 2000, LTCM had been wound down.
- Lessons
  - LTCM failed to supplement VaR measures with a full set of stress test scenarios.
  - LTCM failed to account for the illiquidity of its largest positions.

8. Metallgesellschaft

- Background
  - MGRM: long-term contracts to supply customers with gas and oil products at fixed costs and to hedge these contracts with short-term gas and oil futures.
- The Crisis
  - In 1993, a large decrease in gas and oil prices had resulted in funding needs of around $900 million.
  - MG negotiated unwinds of these contracts at unfavourable terms.
- Lessons
  - When using shorter-term hedges against longer-term contracts, this can be successfully carried out only if proper risk controls are applied.

9. Bankers Trust

- Incident
  - Both P&G and Gibson claimed that they had suffered large losses in derivatives trades they had entered into with BT due to being misled by BT as to the nature of the positions.
- Result
  - BT was forced into an acquisition by Deutsche Bank.
- Arose
  - BT offered P&G and Gibson a probable but small reduction in funding expenses in exchange for a potentially large loss under some less probable circumstances.

9. Bankers Trust

- How detected?
  - The internal BT recordings showed that price quotes to P&G and Gibson were being manipulated to mislead them.
- Lessons
  - Banks should match the degree of complexity of trades to the degree of financial sophistication of customers.
  - Be cautious about how to use any form of communication.

10. JPMorgan, Citigroup, and Enron

- Incident
  - Enron had been engaging in dubious accounting practices to hide the size of its borrowings.
  - JPMorgan and Citigroup agreed to pay a combined fine of $286 million for "helping to commit a fraud".
- Arose
  - Enron sold oil for future delivery, getting cash, and then agreed to buy back the oil that it delivered for a fixed price. So, in effect, no oil was ever delivered. **This was in fact a loan.**
  - Enron did not have to report this as a loan.

Deciphering the Liquidity and Credit Crunch 2007-2008

A. Banking Industry Trends

- **Trend 1: Securitisation.** "Originate and distribute" model: banks repackaged loans and passed them on to various other financial investors.
  - Collateralised Debt Obligations (CDOs).
  - Tranches: senior, equity, mezzanine tranches.
  - Credit Default Swaps (CDS): buyers of these tranches can protect themselves by purchasing CDS contracts insuring against the default.

A. Banking Industry Trends

- **Trend 2: Shortening the Maturity Structure and Resulting the Maturity Mismatch**
  - Most investors prefer assets with short maturities.
    - ✓ To withdraw funds at short notice.
  - But most investment mortgages have long maturities.
  - As a result, banks increasingly financed their asset holdings with shorter maturity instruments.

A. Banking Industry Trends

- Shadow Banks
  - Raise funds by lending short-term asset-backed commercial paper.
  - The short-term assets are backed by a pool of mortgages.
  - In the case of default, owners of the asset-backed commercial paper have the power to sell the underlying collateral assets.
  - But exposes the banks to funding liquidity risk (borrow short and invest in long).

A. Banking Industry Trends

- Short-term Repurchase Agreements (repos)
  - Repo: A firm borrows funds by selling a collateral asset today and promising to repurchase it at a later date.
  - Overnight repos: banks have to roll over their funding on a daily basis.
- Conclusion
  - Leading up to the crisis, any reduction in funding liquidity could thus lead to significant stress for the financial system.

A. Banking Industry Trends

- **Why the Popularity of Structured Products rises?**
  - Lowering funding interest rates because risk is transferred.
  - Regulatory and ratings arbitrage: banks were able to reduce their capital charges.
  - Optimistic forecasts about structured finance products: low default, low correlation.
  - More favourable ratings: higher rating fees, high return.
  - Rating agencies collected **higher fees** for structured products than corporate bond.

C. Amplifying Mechanisms and Recurring Themes

Loss Spiral And Margin Spiral

- Loss Spiral
  - Decline in the value of assets ⇒ eroding the investors' net worth ⇒ selling for money ⇒ these sales depressing the price further ⇒ inducing more selling.
- Margin/haircut Spiral
  - As margins or haircuts rise ⇒ the investor has to sell more assets ⇒ a lower price ⇒ increasing margins further and forcing more sales.

C. Amplifying Mechanisms and Recurring Themes

Lending Channel

- Moral Hazard
  - Most lending is intermediated by banks.
  - The net worth of the intermediaries' stake falls ⇒ intermediaries may then reduce their monitoring effort.
- Precautionary Hoarding
  - Lenders are afraid that they might suffer from interim shocks ⇒ precautionary hoarding arises ⇒ sharp spikes in LIBOR.

C. Amplifying Mechanisms and Recurring Themes

- Runs On Financial Institutions
  - Everybody had an incentive to be the first to withdraw funds from a possibly troubled bank.
- Network Effects
  - Most financial institutions are lenders and borrowers at the same time.
  - Gridlock Risk: multilateral netting agreements can stabilise the system.

Getting up to Speed on the Financial Crisis

C. The Crisis Build-Up

- Why were Shadow Banks Growing?
  - The traditional banking model became less profitable.
  - Institutional cash pools have a demand for insured deposit alternatives.
  - And the shadow banking system rose to fill this gap.

C. The Crisis Build-Up

- Credit Boom (Foreign Factors)
  - In the US: national saving < U.S. capital investment.
  - Large and persistent capital inflows from foreigners seeking U.S. assets.
  - Institutional cash pools had to find substitutes such as repo.
- Credit Boom (Domestic Factors)
  - The increase in the production of asset-backed securities appears to be a credit boom.
  - House prices were rising.

D. The Panics

1. Asset-backed Commercial Paper

- What is ABCP?
- Why ABCP Becoming Prevalent?
  - More transparent ⇒ lowering funding costs.
  - Save on regulatory capital.
- ABCP Run
  - Lenders are unwilling to refinance CP when it comes due.
  - Programs were more likely to experience a run if they had high credit risk or high liquidity risk.

D. The Panics

2. Money Market Mutual Funds

- A Chain Effect
  - MMFs saw the values of their stakes decline when ABCP yields rose ⇒ were forced to sell their underlying assets ⇒ further downward pressure on asset classes held by many MMFs.
- The Lehman bankruptcy was a major shock to MMFs.
  - It led to run on many MMFs.
  - Investors moved to government-only MMFs.

D. The Panics

3. Repo

- Repo is the shadow-banking equivalent of a deposit market.
- Haircuts continued a steady rise throughout 2007-2008.
- Following the Lehman failure, the haircut rose by an additional 20% to 100%.

- Subprime crisis turned into the collapse of global financial institutions.

E. Policy Responses

- Liquidity support was effective during the pre-Lehman period.
- After the fall of Lehman, **capital injections** were the most effective policy.

F. Real Effects of the Financial Crisis

- The widespread loss of confidence reached the real sector of the economy.
- Intermediaries began to hoard cash and stop lending.
- Significant impacts on the real economy.

Risk Management Failures

A. A Typology of Risk Management Failures

- A large loss is not evidence of a risk management failure because a large loss can happen even if risk management is flawless.

1. Mis-measurement of Known Risks

- Risk Managers could Make a Mistake in
  - Assessing the probability of a loss.
  - Using the wrong distribution.
  - Correlations may be mis-measured.
  - A known risk may not manifest itself in the past.

A. A Typology of Risk Management Failures

2. Mis-measurement Due To Ignored Risks

- Ignored known risks
- Mistakes in information collection
  - Risk is known but did not enter the relevant risk models.
  - Losses from risks that were not accounted for (such as business risk).
- Unknown risks
  - Do not create risk management problems.

A. A Typology of Risk Management Failures

3. Communication Failures

- Risk management has to provide timely information to the board and top management.
- Right information, at the right time, to the right people.
- Hierarchical structures sometimes are filters when information was sent up.

A. A Typology of Risk Management Failures

4. Failures In Monitoring And Managing Risks

- Risks can change sharply because of derivatives positions.
- Hedges adjusted daily could create large losses.
- When liquidity dries up in the markets, many risk-mitigating options can no longer be used.
- Individuals may take risks that remain hidden for a while.
- The effectiveness of risk monitoring and control depends crucially on an institution's culture and incentives.

A. A Typology of Risk Management Failures

5. Risk Measures and Risk Management Failures

- VaR does not capture small-probability losses.
- To assess risk, firms have to look at longer horizons and have to take a comprehensive view of their risks. But VaR seems work well with shorter horizons.
- When we consider years, crises are not extremely rare events.
- During crisis periods, firms will make multiple losses.
- Crises involve complicated interactions across risks and across institutions. There is little hope for statistical risk models relying on historical data to capture such complicated situations.

Framework

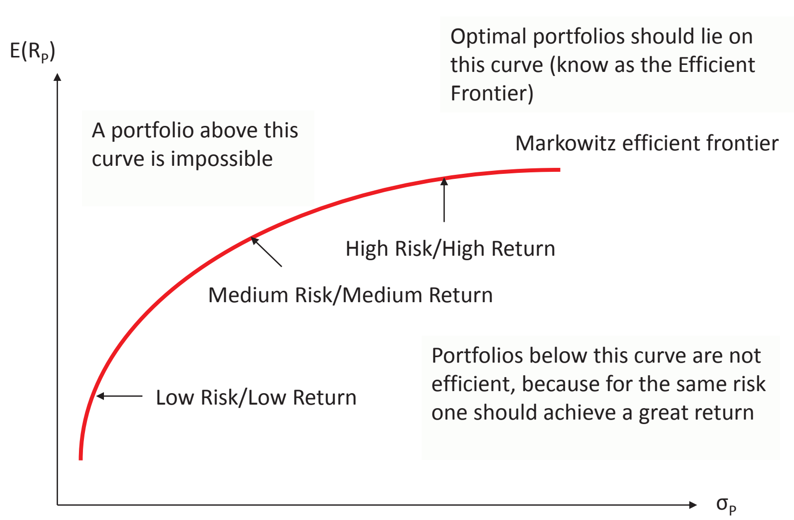
- The standard capital asset pricing model
- Applying the CAPM to performance measurement
- Arbitrage pricing theory
- Effective data aggregation and risk reporting
- GARP code of conduct

The Standard Capital Asset Pricing Model

Markowitz portfolio theory

- Assumptions about a Markowitz investor
  - No transaction costs
  - Assets are infinitely divisible
  - The absence of personal income tax
  - An individual cannot affect the price of a stock by his trading
  - Investors make decisions solely in terms of returns and standard deviation of the returns
  - Unlimited short sales are allowed
  - Unlimited lending and borrowing at the riskless rate
  - All investors have identical expectations: μ σ ρ
  - All assets are marketable

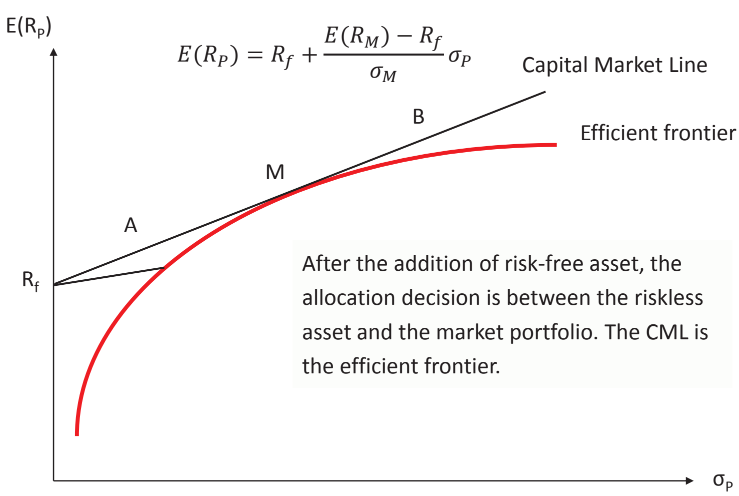
Markowitz efficient frontier



Minimum variance portfolio

- Definition: the portfolio with the smallest variance among all possible portfolios on a portfolio possibilities curve.
- The shape of the portfolio possibilities curve is best described in two pieces:
  - The portion of the portfolio possibility curve that lies above the minimum variance portfolio is concave.
  - The portion of the portfolio possibility curve that lies below the minimum variance portfolio is convex.

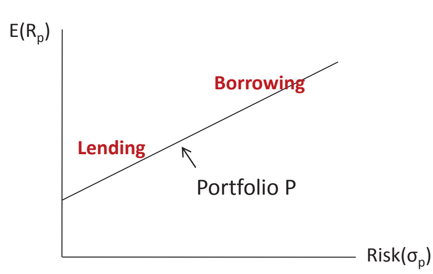
Capital Market Line (CML)



After the addition of risk-free asset, the allocation decision is between the riskless asset and the market portfolio. The CML is the efficient frontier.

Capital Market Line (CML)

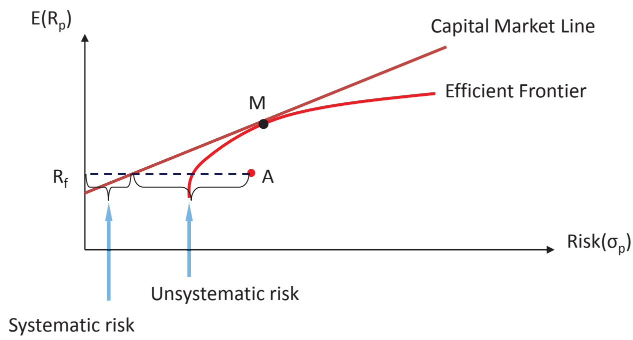
- The point of tangency – Portfolio P, is known as the market portfolio. When one can lent or borrow money use riskless rate, investor will hold a combination of the market portfolio and the risk-free asset.
- Risk-averse investors will create lower risk portfolios by lending (i.e., investing in the risk-free asset). More risk-tolerant investors will increase portfolio return by borrowing at the risk-free rate.



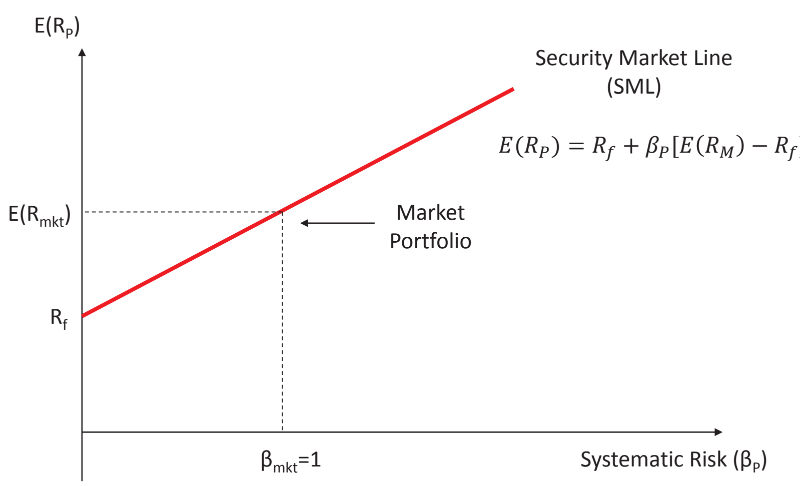
Systematic and Unsystematic Risk

Total risk= systematic risk+ unsystematic risk

Systematic risk is the only important ingredient in determining expected returns and that nonsystematic risk plays no role



Security Market Line (SML)



Capital Asset Pricing Model (CAPM)

- $E(R_P) = R_f + \beta_P[E(R_M) - R_f]$
- $\beta_P = \frac{Cov(P,M)}{\sigma_M^2} = \rho_{P,M}\frac{\sigma_P}{\sigma_M}$
  - $E(R_P)$: expected return on risky asset
  - R_f : risk-free rate
  - $E(R_M) - R_f$: market portfolio risk premium
  - β_P : systematical risk of asset P
  - $\beta_P \times [E(R_M) - R_f]$: beta-adjusted market risk premium

Applying the CAPM to Performance Measurement

Sharpe ratio

- Measures the ratio of the average rate of return $E(R_P)$, in excess of the risk-free rate R_F, to the absolute risk $\sigma(R_P)$.

$$SR = \frac{E(R_P) - R_F}{\sigma(R_P)}$$

  - Widely used for measuring portfolio performance that are not very diversified.
  - A better method for measuring historical performance.
  - Suitable for evaluating the performance of a portfolio that represents an individual's total investment.

Treynor ratio

- Treynor ratio is equal to the risk premium divided by beta (systematic risk)

$$TR = \frac{E(R_P) - R_F}{\beta_P}$$

  - More appropriate for comparing well-diversified portfolios and a more forward-looking measure.

Sortino ratio

- MAR (minimum acceptable return) is the return below which the investor does not wish to drop.
- Sortino ratio measures the ratio of the average rate of return $E(R_P)$, in excess of the risk-free rate R_F, to the semi-standard deviation, which considers only data points that represent a loss.

$$Sortino\ Ratio = \frac{E(R_P) - MAR}{\sqrt{\frac{1}{N-1}\sum_{t=1}^{N}(R_{Pt} - MAR)^2}} \qquad (R_{Pt} < MAR)$$

  - Where T is the number of observed losses.
  - The Sortino ratio is more relevant than the Sharpe ratio when the return distribution is skewed to the left.

Jensen's alpha

- Jensen's alpha is the asset's excess return over the return predicted by the CAPM.

$$E(R_P) - R_F = \alpha_P + \beta_P[E(R_M) - R_F]$$

  - Most appropriate for comparing portfolios that have the same beta and can be used to rank portfolios within peer groups.

Information ratio

- Tracking error: the standard deviation of the difference in return between the portfolio and the benchmark.

$$TE = \sigma(R_P - R_B)$$

- The information ratio measures the ratio of the residual return of the portfolio compared with its residual risk (tracking error).

$$IR = \frac{E(R_P) - E(R_B)}{\sigma(R_P - R_B)} = \frac{\alpha_P}{\sigma(\alpha_P)}$$

  - To check that the risk taken by the manager, in deviating from the benchmark, is sufficiently rewarded.

Arbitrage Pricing Theory

Single-Factor Model

- In a single-factor model, uncertainty in asset returns has two sources: a common or macroeconomic factor, and firm-specific events.
- $R_i = E(R_i) + \beta_i F + e_i$
  - F is the deviation of the common factor from its expected value;
  - β_i is the sensitivity of firm i to that factor;
  - e_i is the firm-specific disturbance;
  - $E(R_i)$ is the expected excess return on stock i;
  - The nonsystematic components of returns (e_i), are assumed to be uncorrelated among themselves and uncorrelated with the factor F.
    - ✓ The expected value of e_i for any well-diversified portfolio is zero

Multifactor models

- Multifactor Model
  - Models that allow for several factors can provide better descriptions of security returns.
  - Multifactor models of security returns can be used to measure and manage exposure to each of **many economy-wide factors such as business-cycle risk, interest or inflation rate risk, energy price risk**, and so on.
  - Factor models are tools that allow us to **describe and quantify the different factors that affect the rate of return** on a security during any time period.

Multifactor models

- The equation for multifactor model for stock i
- $R_i = E(R_i) + \beta_{i,GDP}GDP + \beta_{i,IR}IR + e_i$
  - R_i = return on stock i
  - $E(R_i)$ = expected excess rate of return for stock i
  - $\beta_{i,GDP}$ = GDP factor beta for stock i
  - $\beta_{i,IR}$ = interest rate factor beta for stock i
  - GDP = deviation of GDP factor from its expected value
  - IR = deviation of interest rate factor from its expected value
  - e_i = firm-specific return for stock i

Arbitrage Pricing Theory

- Arbitrage pricing theory (APT) is a general theory of asset pricing that holds that the expected return of a financial asset can be modeled as a linear function of various macro-economic factors or theoretical market indices, where sensitivity to changes in each factor is represented by a factor-specific beta coefficient.
- $E(R_P) = R_f + \sum \beta_i F_i$

Arbitrage Opportunities

- Arbitrage Opportunities
  - The APT assumes there are no market imperfections preventing investors from exploiting arbitrage opportunities
  - Extreme long and short positions are permitted and mispricing will disappear immediately
  - All arbitrage opportunities would be exploited and eliminated immediately

Fama-French Three-Factor Model

- The Fama-French three-factor model incorporates the following systematic factors:
- $R_{it} = \alpha_i + \beta_{iM} R_{Mt} + \beta_{i,SMB} SMB_t + \beta_{i,HML} HML_t + e_{it}$
  - SMB = Small minus big (the return of a portfolio of small stocks – return on a portfolio of large stocks)
  - HML = High minus low (the return of a portfolio of stocks with a high book-to-market ratio – return on a portfolio of stocks with a low book-to-market ratio)
- F&F have observed: firms with high ratios of book-to-market value are more likely to be in financial distress and that small stocks may be more sensitive to changes in business conditions. Thus, these variables may capture sensitivity to risk factors in the macro economy.

Effective Data Aggregation and Risk Reporting

I. Data Architecture and IT Infrastructure

- Principle 1
  - Governance - A bank's risk data aggregation capabilities and risk reporting practices should be subject to strong governance arrangements consistent with other principles and guidance established by the Basel Committee.
- Principle 2
  - Data architecture and IT infrastructure - A bank should design, build and maintain **data architecture and IT infrastructure** which fully supports its risk data aggregation capabilities and risk reporting practices not only in normal times but also during times of stress or crisis, while still meeting the other Principles.

II. Risk data aggregation capabilities

- Principle 3
  - Accuracy and Integrity – A bank should be able to generate accurate and reliable risk data to meet normal and stress/crisis reporting accuracy requirements. Data should be **aggregated on a largely automated basis** so as to minimize the probability of errors.
- Principle 4
  - Completeness – A bank should be able to capture and **aggregate all material risk data** across the banking group. Data should be available by business line, legal entity, asset type, industry, region and other groupings, as relevant for the risk in question, that permit identifying and reporting risk exposures, concentrations and emerging risks.

II. Risk data aggregation capabilities

- Principle 5
  - Timeliness – A bank should be able to generate aggregate and up-to-date risk data **in a timely manner**. The precise timing will also depend on the bank-specific frequency requirements for risk management reporting, under both normal and stress/crisis situations.
- Principle 6
  - Adaptability – A bank should be able to generate aggregate risk data to meet a broad range of **on-demand, ad hoc risk management reporting requests**, including requests during stress/crisis situations, requests due to changing internal needs and requests to meet supervisory queries.

III. Risk reporting practices

- Principle 7
  - Accuracy - Risk management reports should accurately and precisely convey aggregated risk data and reflect risk in an exact manner. Reports should be **reconciled and validated**.
- Principle 8
  - Comprehensiveness - Risk management reports should cover all material risk areas within the organization. The **depth and scope** of these reports should be consistent with the size and complexity of the bank's operations and risk profile, as well as the requirements of the recipients.

III. Risk reporting practices

- Principle 9
  - Clarity and usefulness - Risk management reports should communicate information **in a clear and concise manner**. Reports should be **easy to understand** yet comprehensive enough to facilitate informed decision-making. Reports should include meaningful information **tailored to the needs of the recipients.**
- Principle 10
  - **Frequency** – The board and senior management (or other recipients as appropriate) should set the frequency of risk management report production and distribution. The frequency of reports should be increased during times of stress/crisis.

III. Risk reporting practices

- Principle 11
  - Distribution - Risk management reports should be distributed to the relevant parties while ensuring **confidentiality** is maintained.

GARP Code of Conduct

Sample questions

➢ Junaid Manzoor has been hired as head of risk management by KDB Asset Management, a small investment firm in Pakistan. Manzoor implements a risk measurement framework to gauge portfolio risk for the firm. Unfortunately, the methodology he implements for risk measurement has changed considerably in recent years and is no longer used internationally. Neither Manzoor nor anyone else at the firm is aware of the changes to risk measurement approaches. As a GARP member, has Junaid violated the GARP Code of Conduct?

A. No, this is not a violation of the GARP Code of Conduct because neither Manzoor nor the firm is aware of the changes to risk measurement approaches.

B. No, this is not a violation as the methodology worked when Manzoor took his FRM exams.

C. This is only a violation of the GARP Code of Conduct if investment decisions are made based on Manzoor's risk reports.

D. Yes, this is a violation of the GARP Code of Conduct.

➢ Correct Answer ：D

Sample questions

➢ Which of the following is a potential consequence of violating the GARP Code of Conduct once a formal determination is made that such a violation has occurred?

A. Formal notification to the GARP Member's employer of such a violation.

B. Suspension of the GARP Member's right to work in the risk management profession.

C. Removal of the GARP Member's right to use the FRM designation.

D. Required participation in ethical training.

➢ Correct Answer：C

Sample questions

➢ GARP Members agree to uphold and implement Rules of Conduct, which includes each of the following EXCEPT for:

A. Shall exercise reasonable judgment in the provision of risk services while maintaining independence of thought and direction.

B. Shall be knowledgeable about probable future regulations in order to ensure their recommendations can endure future developments.

C. Shall be diligent about not overstating the accuracy or certainty of results or conclusions and shall clearly disclose the relevant limits of their specific knowledge and expertise concerning risk assessment, industry practices and applicable laws and regulations.

D. Shall endeavor to be mindful of cultural differences regarding ethical behavior and customs, and to avoid any actions that are, or may have the appearance of being unethical according to local customs.

➢ Correct Answer：B

It's not the end but just beginning.

By training your thoughts to concentrate on the bright side of things, you are more likely to have the incentive to follow through on your goals. You are less likely to be held back by negative ideas that might limit your performance.

试着训练自己的思想朝好的一面看，这样你就会汲取实现目标的动力，而不会因为消极沉沦停滞不前。

Quantitative Analysis

FRM一级培训讲义-强化班

讲师：周琪

101% Contribution Breeds Professionalism

Framework

1. Probability
2. Basic Statistics
3. Distributions
4. Hypothesis Tests and Confidence Intervals
5. Linear Regression
6. Elements Of Forecasting(New)
7. Simulation Modeling
8. Estimating Volatilities and Correlations

Probability

Probability

- **Properties of Probabilities**
  - If A, B, C, … are mutually exclusive events,
    - ✓ P(A + B + C + …) = P(A) + P(B) + P(C) + …
  - If A, B, C, … are mutually exclusive and collectively exhaustive set of events,
    - ✓ P(A + B + C + …) = P(A) + P(B) + P(C) + … = 1
  - Generally,
    - ✓ P(A + B) = P(A) + P(B) – P(AB)

Probability

- **Unconditional probability: P(A), P(B)**
- **Conditional probability: P(A|B)**

$$\boxed{P(A|B) = \frac{P(AB)}{P(B)};\ P(B) > 0}$$

$$\boxed{P(B|A) = \frac{P(AB)}{P(A)};\ P(A) > 0}$$

- **Joint probability: P(AB) = P(A)P(B|A) = P(B)P(A|B)**

Probability and Probability Distributions

- **Independence: The occurrence of A has <u>no influence</u> of on the occurrence of B.**
  - P(A|B) = P(A) or P(B|A) = P(B)
  - P(AB) = P(A) × P(B)
  - P(A or B) = P(A) + P(B) – P(AB)
- **Independence and Mutually Exclusive are quite different.**
  - If exclusive, must not independence;
    - ✓Cause exclusive means if A occur, B can not occur, A influents B.

Probability Distributions

- **Probability density function (p.d.f): f(x)**
  - For continuous random variable commonly
- **Cumulative probability function (c.p.f): F(x)**
  - $F(x) = P(X \le x)$
- **Properties of CDF**
  - $F(-\infty) = 0$ and $F(+\infty) = 1$
  - F(X) is a non-decreasing function such that if $x_2 > x_1$ then $F(x_2) \ge F(x_1)$.
  - $P(X \ge k) = 1 - F(k)$
  - $P(x_1 \le X \le x_2) = F(x_2) - F(x_1)$

Probability and Probability Distributions

- **Probability Matrix**
  - Summarize joint probabilities in a probability matrix.
  - Unconditional/marginal probabilities can be seen by adding across a row or down a column.

Probability and Probability Distributions

- **Total Probability Formula**
  - If an event A must result in one of the mutually exclusive events A_1, A_2, A_3, ……, An, then

$$\boxed{P(A)=P(A_1)P(A|A_1)+P(A_2)P(A|A_2)+\ldots+P(A_n)P(A|A_n)}$$

- **Bayes' Theorem**

$$\boxed{P(A|B)=\frac{P(B|A)}{P(B)}\times P(A)=\frac{P(B|A)P(A)}{P(B|A)P(A)+P(B|\bar{A})P(\bar{A})}}$$

Exercise 1

- The following is a probability matrix for X = {1, 2, 3} and Y = {1, 2, 3}.

		X		
		1	2	3
Y	1	6%	15%	9%
	2	12%	30%	18%
	3	2%	5%	3%

Each of the following is true except:

A. X and Y are independent.

B. The Covariance (X,Y) is non-zero.

C. The probability Y = 3 conditional on X = 1 is 10%.

D. The unconditional probability that X = 2 is 50%.

- Correct Answer : B

Exercise 2

- There is a unconditional probability of 20% that the Fed will initiate QE4. If the Fed announces QE4, then ABC hedge fund will outperform the market with a 70% probability. If the fed does not announce QE 4, there is only a 40% probability that ABC will outperform. If we observe that ABC outperforms the market, which is nearest to the probability that the Fed announced QE4?

A. 20%

B. 28%

C. 30%

D. 42%

- Correct Answer : C

Basic Statistics

Expected Value

- **Expected Value**
  - A measure of central tendency – the first moment

$$\boxed{E(X)=\sum P(x_i)x_i=P(x_1)x_1+P(x_2)x_2+\ldots+P(x_n)x_n}$$

- **Properties of Expected Value**

1. If b is a constant, E(b) = b.
2. If a is a constant, E(aX) = aE(X).
3. If a and b are constants, then E(aX + b) = aE(X) + E(b) = aE(X) + b.
4. E(X + Y) = E(X) + E(Y)

Variance

- **Variance**
  - A measure of dispersion – the second moment

$$\boxed{\sigma^2=E(X-\mu)^2}$$

  - Above formula is the definition of variance. To compute the variance, we use the following formula:

$$\boxed{\sigma^2=E(X^2)-[E(X)]^2=E(X^2)-\mu^2}$$

  - Measures how noisy or unpredictable that random variable is.
  - The positive square root of σ^2, σ, is known as the standard deviation, also called volatility.

Variance

Properties of Variance

1. The variance of a constant is zero.
2. If a is constant, then: $\sigma^2(aX) = a^2\sigma^2(X)$.
3. If b is a constant, then: $\sigma^2(X + b) = \sigma^2(X)$.
4. If a and b are constant, then: $\sigma^2(aX + b) = a^2\sigma^2(X)$.
5. If X and Y are two independent random variables, then:

   $\sigma^2(X + Y) = \sigma^2(X) + \sigma^2(Y)$ $\quad$ $\sigma^2(X - Y) = \sigma^2(X) + \sigma^2(Y)$
6. If X and Y are independent random variables and a and b are constants, then $\sigma^2(aX + bY) = a^2\sigma^2(X) + b^2\sigma^2(Y)$.
7. For computational convenience, we can get: $\sigma^2(X) = E(X^2) - [E(X)]^2$.

Covariance

Covariance

$$\boxed{\begin{array}{l} Cov(X,Y) = E\left[\left(X-E(X)\right)\left(Y-E(Y)\right)\right] = E(XY)-E(X)E(Y) \\ S_{XY} = \dfrac{1}{n-1}\sum_{i-1}^{n}(X_i-\overline{X})(Y_i-\overline{Y}) \end{array}}$$

- Covariance measures how one random variable moves with another random variable.
- Covariance ranges from negative infinity to positive infinity.

Covariance

Properties of Covariance

- If X and Y are independent random variables, their covariance is zero.

  $$Cov(X,\ X) = E\left[\left(X-E(X)\right)\left(X-E(X)\right)\right] = \sigma^2(X)$$
- Cov(X, Y+Z)=Cov(X, Y)+Cov(X, Z), or

  $Cov(a+bX,\ c+dY) = b\times d\times Cov(X,Y)$
- If X and Y are NOT independent, then:

  $\sigma^2(X\pm Y) = \sigma^2(X) + \sigma^2(Y) \pm 2Cov(X,Y)$

Correlation Coefficient

- **Correlation coefficient**

$$\rho_{XY} = \frac{\sigma_{xy}}{\sigma_x \sigma_y}, \sigma_{xy} = Cov(X,Y) = E[(X-\mu_X)(Y-\mu_Y)]$$

$$r_{XY} = \frac{S_{XY}}{S_X S_Y}$$

- **Properties of Correlation coefficient**
  - Correlation has no units, ranges from -1 to +1.
  - Correlation measures the linear relationship between two random variables.
  - If two variables are independent, their covariance is zero, therefore, the correlation coefficient will be zero. The converse, however, is not true. For example, $Y = X^2$.
  - Variances of correlated variables:

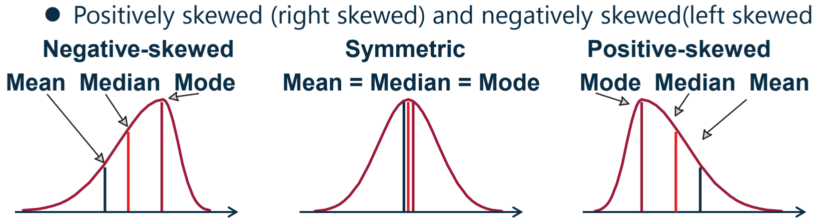
$$\sigma^2(X \pm Y) = \sigma^2(X) + \sigma^2(Y) \pm 2\rho\sigma(X)\sigma(Y)$$

Skewness

- **Skewness**
  - A measure of asymmetry of a PDF – the third moment.

$$S = \frac{E(X-\mu_x)^3}{\sigma_x^3} = \frac{\text{third moment about mean}}{\text{cube of standard deviation}}$$

- **Symmetrical and nonsymmetrical distributions**
  - Positively skewed (right skewed) and negatively skewed(left skewed)



  - Positive skewed: Mode < median < mean, having a right fat tail
  - Negative skewed: Mode > media > mean, having a left fat tail

Kurtosis

- **Kurtosis**
  - A measure of tallness or flatness of a PDF – the fourth moment.

$$K = \frac{E(X-\mu_x)^4}{\left[E(X-\mu_x)^2\right]^2} = \frac{\text{fourth moment}}{\text{square of second moment}}$$

  - For a normal distribution, the K value is 3.
  - Excess kurtosis = kurtosis – 3

	leptokurtic	mesokurtic	platykurtic
Kurtosis	> 3	= 3	< 3
Excess kurtosis	> 0	= 0	< 0
Tails (assuming same variance)	fat tail	normal	thin tail

Chebyshev's Inequality

- **Chebyshev's Inequality**
  - For any set of observations (samples or population), the proportion of the values that lie within k standard deviations of the mean is at least 1 – 1/k^2, k > 1.

$$\boxed{P\left(\left|X-\mu\right|\le k\sigma\right)\ge 1-\frac{1}{k^2},\ k>1}$$

  - This relationship applies regardless of the shape of the distribution.

Lin within	k	Standard deviations of the mean	Proportion
	2		$\ge 1-\frac{1}{2^2}=1-\frac{1}{4}=\frac{3}{4}=75\%$
	3		$\ge 1-\frac{1}{3^2}=1-\frac{1}{9}=\frac{8}{9}=89\%$
	4		$\ge 1-\frac{1}{4^2}=1-\frac{1}{16}=\frac{15}{16}=94\%$

Exercise 1

- A model of the frequency of losses (L) per day assumes the following discrete distribution: zero loss with probability of 20%; one loss with probability of 30%; two losses with probability of 30%; three losses with probability of 10%; and four losses with probability of 10%. What are, respectively, the expected number of loss events and the standard deviation of the number of loss events?
  - A. E(L) = 1.2 and σ = 1.44
  - B. E(L) = 1.6 and σ = 1.20
  - C. E(L) = 1.8 and σ = 2.33
  - D. E(L) = 2.2 and σ = 9.60

- Correct Answer : B

Exercise 2

- An analyst is concerned with the symmetry and peakedness of a distribution of returns over a period of time for a company she is examining. She does some calculations and finds that the median return is 4.2%, the mean return is 3.7%, and the mode return is 4.8%. She also finds that the measure of kurtosis is 2. Based on this information, the correct characterization of the distribution of return over time is:

	Skewness	Kurtosis
A.	Positive	Leptokurtic
B.	Positive	Platykurtic
C.	Negative	Platykurtic
D.	Negative	Leptokurtic

- Correct Answer : C

Exercise 3

- Using Chebyshev's inequality, what is the proportion of observations from a population of 250 that must lie within three standard deviations of the mean, regardless of the shape of the distribution?
  - A. 75%
  - B. 99%
  - C. 89%
  - D. 54%

- Correct Answer : C

Distributions

Binomial Distribution

- Bernoulli Distribution
- $P(X = 1) = p \quad P(X = 0) = 1 - p$

Binomial Distribution

- The probability of x successes in n trails

$$\boxed{p(x) = P(X = x) = C_n^x p^x (1-p)^{n-x} = \frac{n!}{x!(n-x)!} p^x (1-p)^{n-x}}$$

E

	Expectation	Variance
Bernoulli random variable	p	p(1 – p)
Binomial random variable	np	np(1 – p)

Poisson Distribution

- **Poisson Distribution**
  - When there are a large number of trials but a small probability of success, Binomial calculations become impractical.
  - If we substitute λ/n for p, and let n very large, the Binomial Distribution becomes the Poisson Distribution.

$$\boxed{p(k)=P(X=k)=\frac{\lambda^k}{k!}e^{-\lambda}\quad(\lambda=np)}$$

✓ X refers to the number of success per unit.

✓ λ indicates the rate of occurrence of the random events; i.e., it tells us how many events occur on average per unit of time.

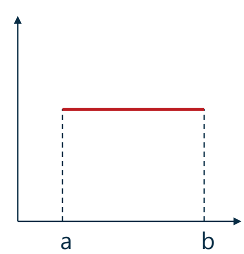
✓

Mean	Variance
λ	λ

Continuous Uniform Distribution

- **Probability density function**

$$f(x)=\begin{cases}\dfrac{1}{b-a} & a\le x\le b\\ 0 & \text{otherwise}\end{cases}$$



- **Cumulative distribution function**

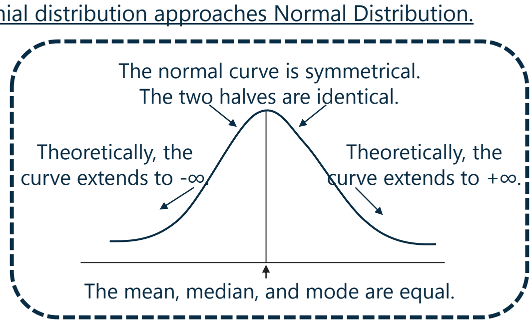
$$F(x)=\begin{cases}0 & \text{for } x\le a\\ \dfrac{x-a}{b-a} & \text{for } a<x<b\\ 1 & \text{for } x\ge b\end{cases}$$

- **Properties**
  - $E(X) = (a + b)/2$, $D(X) = (b - a)^2/12$
  - For all $a \le x_1 < x_2 \le b$, we have: $\boxed{P(x_1\le X\le x_2)=\dfrac{x_2-x_1}{b-a}}$

Normal Distribution

- **Normal Distribution**
  - As n increases, the binomial distribution approaches Normal Distribution.

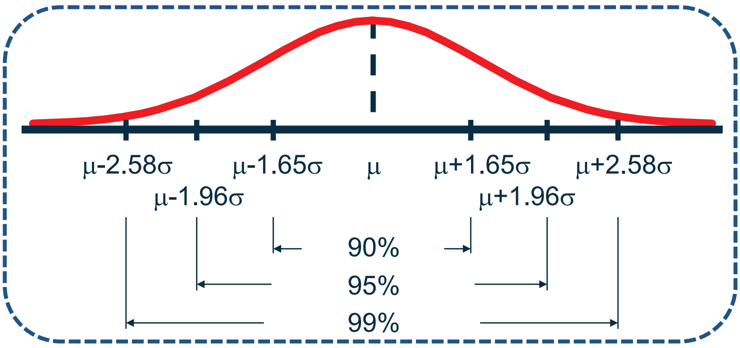
$$\boxed{f(x)=\frac{1}{\sqrt{2\pi}\sigma}\cdot e^{-\frac{1}{2\sigma^2}(x-u)^2}}$$



- **Properties**
  - $X \sim N(\mu, \sigma^2)$, fully described by its two parameters μ and σ².
  - Bell-shaped, symmetrical distribution: skewness = 0; kurtosis = 3.
  - A linear combination (function) of two (or more) normally distribution random variables is itself normally distributed.
  - The tails get thin and go to zero but extend infinitely, asymptotic.

Normal Distribution

- **The confidence intervals**
  - Approximately 68% of all observations fall in the interval $\mu \pm \sigma$
  - Approximately 90% of all observations fall in the interval $\mu \pm 1.65\sigma$
  - Approximately 95% of all observations fall in the interval $\mu \pm 1.96\sigma$
  - Approximately 99% of all observations fall in the interval $\mu \pm 2.58\sigma$



The Standard Normal Distribution

- **The standard normal distribution**
  - N(0,1) or Z
  - Standardization: if $X \sim N(\mu, \sigma^2)$, then $\boxed{Z = \dfrac{X-\mu}{\sigma} \sim N(0,1)}$
  - Z-table
- **How we use the standard normal distribution to compute various probabilities?**
  - Example: X ~ N(70, 9), compute the probability of X σ 64.12.
    - ✓ $Z = \dfrac{X-\mu}{\sigma} = \dfrac{64.12-70}{3} \approx -1.96$
    - ✓ P(Z σ -1.96) = 0.0250
  - Question 1: compute the probability of X ≥ 75.9.
  - Question 2: compute the probability of 64.12 σ X σ 75.9.

Continuous Probability Distribution

- **Student's t distribution**
  - It is similar to the normal, except <u>it exhibits slightly heavier tails.</u>

    $\boxed{t = \dfrac{\bar{X}-\mu_X}{S_x/\sqrt{n}} \sim t_{n-1}}$

    ✓ It is symmetrical.

    ✓ It has mean of zero.
  - <u>As the d.f. increase, the t-distribution converges with the standard normal distribution.</u>
  - Both the normal and student's t distribution characterize the sampling distribution of the sample mean. The difference is that <u>the normal is used when we know the population variance</u>. <u>The student's t is used when we must rely on the sample variance.</u> In practice, we don't know the population variance, so the student's t distribution is typically appropriate.

Continuous Probability Distribution

- **Lognormal Distribution**
  - If lnX is normal, then X is lognormal; if a variable is lognormal, its natural log is normal.
  - It is useful for modeling asset prices which never take negative values.
  - Right skewed.
- **Chi-Square (χ^2) Distribution**
- ***F*-Distribution**

Exercise 1

- On a multiple choice exam with four choices for each of six questions, what is the probability that a student gets less than two questions correct simply by guessing?
  - A. 0.46%
  - B. 23.73%
  - C. 35.60%
  - D. 53.39%
- Correct Answer : D

Exercise 2

- Assume random variance is normally distributed with mean of 10 and a variance of 25. Without using a calculator, what is the probability that X falls within 1.75 and 21.65?
  - A. 68%
  - B. 94%
  - C. 95%
  - D. 99%
- Correct Answer : B

Exercise 3

- The frequency of an operational risk event type – damage to physical assets – is characterized by a Poisson distribution. Over an average year, a company expects 36 of these particular loss events. During the next month, which is nearest to the probability the company will experience exactly zero of these events?

  A. 3.4%

  B. 5.0%

  C. 7.5%

  D. 9.1%

- Correct Answer : B

Exercise 4

- Which of the following statements best characterizes the relationship between the normal and lognormal distributions?

  A. The lognormal distribution is the logarithm of the normal distribution.

  B. If the natural log of the random variable X is lognormally distributed, then X is normally distributed.

  C. If X is lognormally distributed, then the natural log of X is normally distributed.

  D. The two distributions have nothing to do with one another.

- Correct Answer : C

Exercise 5

- Assume the population of hedge fund returns has an unknown distribution with mean of 8% and volatility of 10%. From a sample of 40 funds, what is the probability the sample mean return will exceed 10.6%?

  A. 5%

  B. 8%

  C. 10%

  D. 12%

- Correct Answer : A

Hypothesis Tests and Confidence Intervals

Statistical Inference: Estimation and Hypothesis Testing

- **What is Statistical Inference?**
  - Concerned with drawing conclusions about the nature or some population (e.g., the normal) on the basis of a random sample that supposedly been drawn from that population.
  - Loosely speaking, is the study of the relationship between a population and a sample drawn from that population.
- **Sampling and Estimation**

population → population parameter

sampling ↓ ↑ estimation

sample → sample statistic

Statistical Inference: Estimation and Hypothesis Testing

- **Central Limit Theorem (CLT)**
  - Properties
    - ✓ If $n \geq 30$, the sampling distribution of the sample means will be approximately normal.
    - ✓ The mean of the population μ = the mean of the distribution of all possible sample means.
    - ✓ The variance of the distribution of sample means is the population variance divided by the sample size.
  - Standard Error of the Mean $\boxed{SE(\overline{X}) = \frac{s}{\sqrt{n}}}$
    - ✓ The standard deviation of the sampling distribution of the sample means is called the standard error of the sample means.

Statistical Inference: Estimation and Hypothesis Testing

- Regardless of whether we are concerned with point estimates or confidence intervals, there are certain statistical properties that make some estimates more desirable than others. These desirable properties of an estimator are unbiasedness, efficiency, and consistency.
  - **Unbiasedness**: expected value of the estimator is equal to the parameter that are trying to estimate.
  - **Efficiency:** for all unbiased estimators, if the sampling dispersion is smaller than any other unbiased estimators, then this unbiased estimator is called efficient.
  - **Consistency**: the accuracy of the parameter estimate increases as the sample size increases.

Statistical Inference: Estimation and Hypothesis Testing

- Another property of a point estimate is linearity. A point estimate should be a linear estimator (i.e., it can be used as a linear function of the sample data). If the estimator is the best available (i.e., has the minimum variance), exhibits linearity, and is unbiased, it is said to be the best linear unbiased estimator (BLUE).

Point Estimation and Confidence Interval Estimate

- **Point Estimation**
  - Using a single numerical value to estimate the parameter of the population.

parameter

$$\boxed{\bar{X} \rightarrow \mu_X \qquad S^2 \rightarrow \sigma_X^2}$$

Estimator

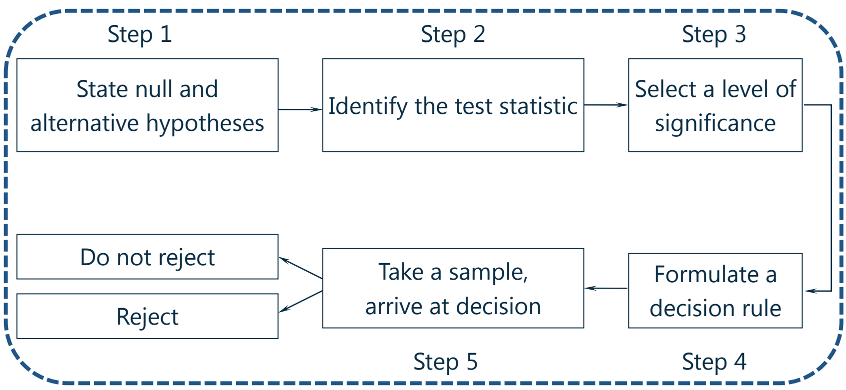
- **Confidence Interval Estimate**
  - Level of significance (alpha)
  - Degree of confidence (1 – alpha)
  - Confidence Interval = [Point Estimate +/- (reliability factor) × standard error]

Statistical Inference: Estimation and Hypothesis Testing

$$\boxed{\overline{X} \pm t_{\alpha/2}\frac{s}{\sqrt{n}}} \qquad \boxed{\overline{X} \pm z_{\alpha/2}\frac{\sigma}{\sqrt{n}}}$$

When sampling form a:	small sample (n < 30)	larger sample (n ≥ 30)
Normal distribution with known variance	z-Statistic	z-Statistic
Normal distribution with unknown variance	t-Statistic	t-Statistic or z-Statistic
Nonnormal distribution with known variance	not available	z-Statistic
Nonnormal distribution with unknown variance	not available	t-Statistic or z-Statistic

Hypothesis Testing



Hypothesis Testing

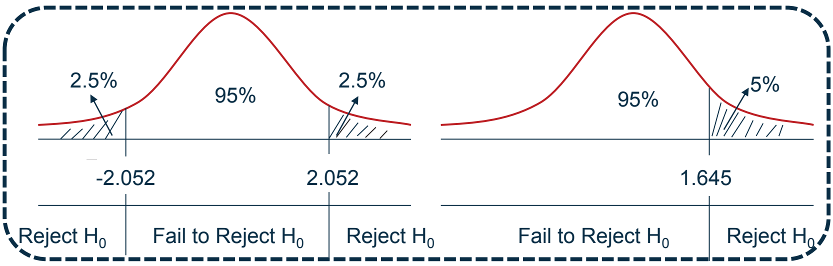
- **Hypothesis**
  - Statistical assessment of a statement or idea regarding a population parameter.
- **The null hypothesis (H_0) and alternative hypothesis (H_a)**
- **One-tailed test vs. Two-tailed test**
  - One-tailed test

    $\boxed{H_0: \mu \geq 0 \quad H_a: \mu < 0}$ $\boxed{H_0: \mu \leq 0 \quad H_a: \mu > 0}$
  - Two-tailed test

    $\boxed{H_0: \mu = 0 \quad H_a: \mu \neq 0}$
- **Critical Value**
  - The distribution of test statistic (z, t, χ^2, F)
  - Significance level (α)
  - One-tailed or two-tailed test

Decision Rule

- **Decision Rule**
  - Reject H_0 if |test statistic| > critical value.
  - Fail to reject H_0 if |test statistic| < critical value.



  - We can never say "accept" H_0.
  - State the conclusion: μ is (not) significantly different from μ_0.

Test of Single Population Mean

- **H_0: $\mu = \mu_0$**
  - z-test vs. t-test

	Normal population, n < 30	n ≥ 30
Known population variance (σ^2)	z-test	z-test
Unknown population variance	t-test	t-test or z-test

  - z-statistic $\boxed{z = \dfrac{\bar{x} - \mu_0}{\sigma / \sqrt{n}}}$
  - t-statistic $\boxed{t = \dfrac{\bar{x} - \mu_0}{s / \sqrt{n}}}$

Test of Single Population Variance

- **H_0: $\sigma^2 = \sigma_0^2$**
  - The Chi-Square test

$$\boxed{\chi^2 = \frac{(n-1)s^2}{\sigma_0^2} \qquad df = n-1}$$

Where:

n = sample size

s^2 = sample variance

σ_0^2 = hypothesized value for the population variance

Test of Variances Difference

- **H_0: $\sigma_1^2 = \sigma_2^2$**
  - The F-test

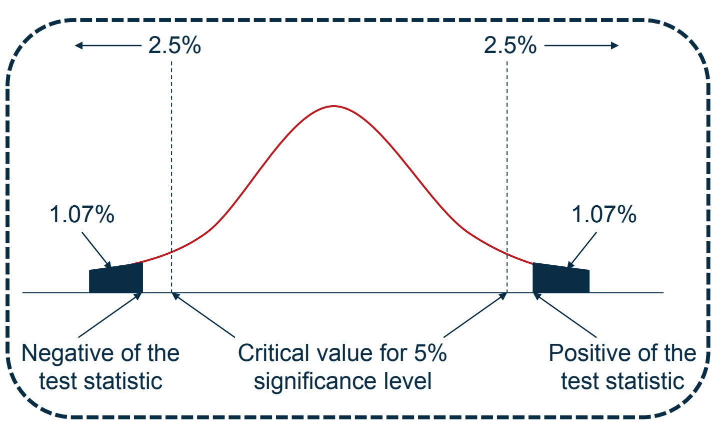
$$\boxed{F = \frac{s_1^2}{s_2^2} \quad df_1 = n_1 - 1 \quad df_2 = n_2 - 1}$$

  - Always put the larger variance in the numerator ($s_1^2 > s_2^2$).

Summary of Hypothesis Testing

Test type	Assumptions	H_0	Test-statistic	distribution
Mean hypothesis testing	Normally distributed population, known population variance	$\mu = 0$	$z = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}}$	N(0,1)
	Normally distributed population, unknown population variance	$\mu = 0$	$t = \frac{\bar{x} - \mu_0}{s / \sqrt{n}}$	t(n-1)
Variance hypothesis testing	Normally distributed population	$\sigma^2 = \sigma_0^2$	$\chi^2 = \frac{(n-1)s^2}{\sigma_0^2}$	$\chi^2(n-1)$
	Two independent normally distributed populations	$\sigma_1^2 = \sigma_2^2$	$F = s_1^2 / s_2^2$	$F(n_1 - 1,\ n_2 - 1)$

P-value testing



- If P-value < alpha, we reject null hypothesis.

Type I and Type II Errors

- **Type I error**
  - Reject the null hypothesis when it's actually true.
- **Type II error**
  - Fail to reject the null hypothesis when it's actually false.
- **Significance level (α)**
  - The probability of making a Type I error:

Significance level = P(Type I error)

- **Power of a test**
  - The probability of correctly rejecting the null hypothesis when it is false:

Power of a test = 1 – P(Type II error)

Exercise 1

- Which of the following statements regarding hypothesis testing is correct?
  A. Type II error refers to the failure to reject the H1 when it is actually false.
  B. Hypothesis testing is used to make inferences about the parameters of a given population on the basis of statistics computed for a sample that is drawn from another population.
  C. All else being equal, the decrease in the chance of making a type I error comes at the cost of increasing the probability of making a type II error.
  D. If the p-value is greater than the significance level, then the statistics falls into the reject intervals.
- Correct answer ： C

Exercise 2

- An oil industry analyst with a large international bank has constructed a sample of 1,000 individual firms on which she plans to perform statistical analyses. She considers either decreasing the level of significance used to test hypotheses from 5% to 1%, or removing 500 state-run firms from her sample. What impact will these changes have on the probability of making type I and type II errors?

	Level of Significance Decrease	Reduction in Sample Size
A.	P(type I error) increases	P(type I error) increases
B.	P(type I error) decreases	P(type II error) increases
C.	P(type II error) increases	P(type I error) decreases
D.	P(type II error) decreases	P(type II error) decreases

- Correct answer ： B

Exercise 3

- Analyst John is concerned that the average days sales outstanding has increased above its historical average of 27 days. From a large sample of 36 companies, he computes a sample mean DSO of 29 days with sample standard deviation of 7. His one-sided alternative hypothesis, stated with 95% confidence, is that DSO is greater than 27. Does she reject the null?
  - A. No, do not reject one-sided null as the t-statistic is less than 1.65.
  - B. No, do not reject one-sided null as the t-statistic is less than 1.96.
  - C. Yes, do reject one-sided null as the t-statistic is greater than 1.65.
  - D. Yes, do reject one-sided null as the t-statistic is greater than 1.96.
- Correct answer ： C

Exercise 4

- If the mean P/E of 30 stocks in a certain industrial sector is 18 and the sample standard deviation is 3.5, standard error of the mean is closest to:
  - A. 0.12
  - B. 0.34
  - C. 0.64
  - D. 1.56
- Correct answer ：C

Linear Regression

Dependent and Independent Variable

➢ **Regression analysis**

- Regression analysis is concerned with the study of the relationship between one variable called the dependent or explained variable and one or more other variables called independent or explanatory variables.

Dependent variable

$$Y = b_0 + b_1 X + \varepsilon$$

Independent variable

- The objectives of regression analysis: to predict or forecast dependent variable.

Ordinary Least Squares (OLS)

➢ **Ordinary least squares (OLS)**

- OLS estimation is a process that estimates the population parameters bi with corresponding values for bi that minimize the squared residuals (i.e., error terms).
- The OLS sample coefficients are those that:

$$\boxed{\text{minimize } \sum \varepsilon_i^2 = \sum [Y_i - (\hat{b}_0 + \hat{b}_1 \times X_1)]^2}$$

$$\boxed{\hat{b}_1 = \frac{\sum_{i=1}^{n}(X_i - \bar{X})(Y_i - \bar{Y})}{\sum_{i=1}^{n}(X_i - \bar{X})^2} = \frac{Cov(X,Y)}{Var(X)} X} \qquad \boxed{\hat{b}_0 = \bar{Y} - \hat{b}_1 \bar{X}}$$

- The estimated intercept coefficient ($\hat{b}_0$): the point ($\bar{X}$, $\bar{Y}$) is on the regression line.

The Basics of Multiple Regression

➢ **Linear Regression with Multiple Regressors**

$$\boxed{Y_i = b_0 + b_1 X_{1i} + b_2 X_{2i} + \cdots + b_k X_{ki} + \varepsilon_i}$$

- If the regressor is correlated with a variable that has been omitted from the analysis and that determines, in part, the dependent variable, then the OLS estimator will have omitted variable bias.

➢ The intercept term is the value of the dependent variable when the independent variables are all equal to zero.

➢ Each slope coefficient is the estimated change in the dependent variable for a one unit change in that independent variable, holding the other independent variables constant. That's why the slope coefficients in a multiple regression are sometimes called partial slope coefficient.

Dummy Variables

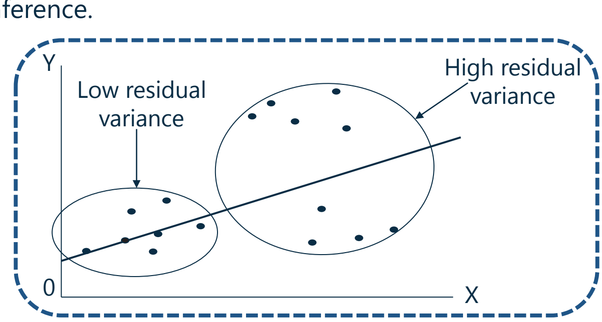
- **Dummy Variables**
  - Observations for most independent variables can take on a wide range of values. However, there are occasions when the independent variable is binary in nature - it is either "on" or "off". Independent variables that fall into this category are called **dummy variables** and are often used to quantify the impact of qualitative events.
  - Dummy variables are assigned a value of "0" or "1".
  - The coefficient on dummy variables indicates the **difference** in the dependent variable for the category represented by the dummy variable and the average due of the dependent variable for all classes except the dummy variable class.
    - ✓ For example, testing the slope coefficient for rise January dummy variable would indicate whether, and by how much, security returns are different in January as compared to the other months.

The Basics of Linear Regression

- **The assumptions of the linear regression**
  - A linear relationship exists between X and Y.
  - Independent variables are not random, or independent variables are not correlated with error term, or independent variables are not correlated(only in multiple regression).
  - The expected value of the error term is zero (i.e., $E(\varepsilon_i) = 0$).
  - The variance of the error term is constant (i.e., the error terms are homoskedastic).
  - The error term is uncorrelated across observations (i.e., $E(\varepsilon_i\varepsilon_j) = 0$ for all $i \neq j$).
  - The error term is normally distributed.

Homoskedasticity and Heteroskedasticity

- **Conditional Heteroskedasticity**
  - The heteroskedasticity is related to the level of (i.e., conditional on) the independent variable.
  - The residual variance with be larger if the values of the independent variable X is larger.
  - Conditional heteroskedasticity does create significant problems for statistical inference.



Homoskedasticity and Heteroskedasticity

- **Effect of Heteroskedaticity on Regression Analysis**
  - The standard errors are usually unreliable estimates.
  - The coefficient estimates (the b_1) aren't affected.
  - If the standard errors are too small, but the coefficient estimates themselves are not affected, the t-statistics will be too large and the null hypothesis of no statistical significance is rejected too often.
    - ✓ The opposite will be true if the standard errors are too large.

Violation of the Basic Assumption

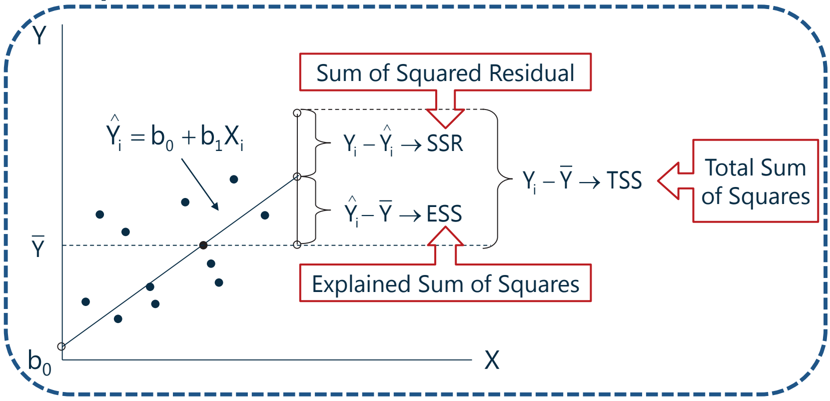
- **Autocorrelation**
  - Autocorrelation, also known as serial correlation, is the cross-correlation between the values of the same variables.
  - Impact
    - ✓ Autocorrelation violates the ordinary least squares assumption that the error terms are uncorrelated. While it does not bias the OLS coefficient estimates, <u>the standard errors tend to be underestimated (and the t-scores overestimated) when the auto correlations of the errors are positive</u>.

Violation of the Basic Assumption

- **Multicollinearity(Multiple regression only)**
  - Multicollinearity refers to the situation that two or more independent variables are highly correlated with each other.
  - Impact
    - ✓ When conducting regression analysis, we need to be cognizant of <u>imperfect multicollinearity</u> since <u>OLS estimators will be computed, but the resulting coefficients may be improperly estimated</u>.
  - Two methods to detect multicollinearity
    1. t-tests indicate that none of the individual coefficients is significantly different than zero, while the F-test indicates overall significance and the R^2 is high.
    2. The absolute value of the sample correlation between any two independent variables is greater than 0.7 (i.e., $|r| > 0.7$).
  - Methods to correct multicollinearity
    - ✓ Omit one or more of the correlated independent variables.

Analysis of Variance (ANOVA) Table

➢ **Components**



Total sum of squares = explained sum of squares + sum of squared residuals

$$\underset{TSS}{\sum (Y_i - \bar{Y})^2} = \underset{ESS}{\sum (\hat{Y} - \bar{Y})^2} + \underset{SSR}{\sum (Y_i - \hat{Y})^2}$$

Analysis of Variance (ANOVA) Table

➢ **Analysis of Variance (ANOVA) Table**

	df	**SS**	**MSS**
Regression	k = 1	ESS	ESS/k
Residual	n – 2	SSR	RSS/(n – 2)
Total	n – 1	TSS	-

Measures of Fit

➢ **SER**

- Standard error of regression (SER) estimates the standard deviation of the error term. In this way, the SER is a measure of spread of the distribution of Y around the regression line.
- The SER gauges the “fit” of the regression line. The smaller the standard error, the better the fit.
- SER will be low(relative to total variability) if the relationship is very strong, or will be high if the relationship is weak.

$$SER = \sqrt{\frac{\sum (Y_i - \hat{Y}_i)^2}{n-2}}$$

Measures of Fit

- **R^2 (the Coefficient of Determination)**
  - A measure of the "goodness of fit" of the regression. It is interpreted as a percentage of variation in the dependent variable explained by the independent variable. Its limits are $0 \le R^2 \le 1$.

$$\boxed{R^2 = \frac{ESS}{TSS} = 1 - \frac{SSR}{TSS}}$$

  - In a simple two-variable regression, the square of R^2 is the correlation coefficient(r) between X_i and Y_i:

$$\boxed{r^2 = R^2 \rightarrow r = \pm\sqrt{R^2}}$$

R^2 and Adjusted R^2

- **R^2 and Adjusted R^2**
  - In multiple regression, the R^2 increases whenever a regressor (independent variable) is added, unless the estimated efficient on the added regressor is exactly zero.
  - The adjusted R^2 is a modified version of the R^2 that does not necessarily increase with a new independent variable is added. Adjusted R^2 is given by:

$$\boxed{\text{Adjusted } R^2 = 1 - \frac{SSR/n-k-1}{TSS/n-1} = 1 - \frac{n-1}{n-k-1}\frac{SSR}{TSS}}$$

✓ Adjusted $R^2 \le R^2$; adjusted R^2 may be less than zero.

Confidence Interval

- The confidence interval for the regression coefficient, b_1, is calculated as:

$$\boxed{\hat{b}_1 \pm (t_c \times s_{\hat{b}_1}), \text{or } [\hat{b}_1 - (t_c \times s_{\hat{b}_1}) < b_1 < \hat{b}_1 + (t_c \times s_{\hat{b}_1})]}$$

  - t_c: the critical two-tailed t-value for the selected confidence level with the appropriate number of degrees of freedom, which is equal to the number of sample observations minus 2 (i.e. n – 2).
  - s_{b_1}: the standard error of the regression coefficient.
- $s_{\hat{b}_1}$ is a function of the SER: <u>as SER rises, $S_{\hat{b}_1}$ also increases</u>, and the confidence interval widens. SER measures the variability of the data about the regression line, and the more variable the data, the less confidence interval.

Hypothesis Testing

- **Regression Coefficient Hypothesis Testing**
  - The hypothesis that is tested is whether the true slope is zero ($b_1 = 0$). The appropriate test structure for the null and alternative hypotheses is:

$$H_0: b_1 = 0 \quad H_a: b_1 \neq 0$$

  - A t-test may also be used to test the hypothesis that the true slope coefficient, b_1, is equal to some hypothesized value. Letting b_1 be the point estimate for b_1, the appropriate test statistic with n – 2 degrees of freedom is:

$$\boxed{t = \frac{\hat{b}_1 - b_1}{s_{\hat{b}_1}} \sim t(n-2)}$$

  - The decision rule for tests of significance for regression coefficients is:

Reject H_0 if $t > +t_{critical}$ or $t < -t_{critical}$

Hypothesis test for a Partial Slope Coefficient

- **Hypothesis test for a Partial Slope Coefficient**
  - H_0: $b_j = 0$ (j = 1, 2, …, k)

$$\boxed{t = \frac{\hat{b}_j}{s_{\hat{b}_j}} \sim t(n-k-1)}$$

  - Regression coefficient confidence interval

$$\boxed{\hat{b}_j \pm \left(t_c \times s_{\hat{b}_j}\right)}$$

Joint Hypothesis Testing

- **Joint Hypothesis Testing**
  - An F-test is used to test whether at least one slope coefficient is significantly different from zero.

H_0: $b_1 = b_2 = b_3 = \ldots = b_k = 0$; H_a: at least one $b_j \neq 0$ (j = 1 to k)

- **F-Statistic**

$$\boxed{F = \frac{ESS/k}{SSR/(n-k-1)}}$$

  - The F-test here is always a one-tailed test.
  - The test assesses the effectiveness of the model as a whole in explaining the dependent variable.
  - Decision rule: reject H_0, if $F_{(test\text{-}statistic)} > F_{c(critical\ value)}$.

Exercise 1

- John is trying to get some insight into the relationship between the return on stock LMD ($R_{LMD,t}$) and the return on the S&P 500 index ($R_{S\&P,t}$). Using historical data he estimates the following:

Annual mean return for LMD	11%
Annual mean return for S&P 500 index	7%
Annual volatility for S&P index	18%
Covariance between the return of LMD and S&P 500 index	6%

Assuming he uses the same data to estimate the regression model given by:

$$R_{LMD,t} = \alpha + \beta R_{S\&P,t} + \varepsilon_t$$

Using the ordinary least squares technique, which of the following models will she obtain?

A. $R_{LMD,t} = -0.02 + 0.54 R_{S\&P,t} + \varepsilon_t$

B. $R_{LMD,t} = -0.02 + 1.85 R_{S\&P,t} + \varepsilon_t$

C. $R_{LMD,t} = 0.04 + 0.54 R_{S\&P,t} + \varepsilon_t$

D. $R_{LMD,t} = 0.04 + 1.85 R_{S\&P,t} + \varepsilon_t$

- Correct Answer : B

Exercise 2

- An analyst is given the data in the following table for a regression of the annual sales for Company XYZ.

Parameters	Coefficient	Standard Error of the Coefficient
Intercept	-94.88	32.97
Slope (industry sales)	0.2796	0.0363

The correlation between company and industry sales is 0.9757. Which of the following is closest to the value and reports the most likely interpretation of R^2?

A. 0.048, indicating that the variability of industry sales explains about 4.8% of the variability of company sales.

B. 0.048, indicating that the variability of company sales explains about 4.8% of the variability of industry sales.

C. 0.952, indicating that the variability of industry sales explains about 95.2% of the variability of company sales.

D. 0.952, indicating that the variability of company sales explains about 95.2% of the variability of industry sales.

- Correct Answer : C

Exercise 3

- A regression of a stock's return (in percent) on an industry index's return (in percent) provides the following results:

	Coefficient	Standard Error
Intercept	2.1	2.01
Industry Index	1.9	0.31

	Degrees of Freedom	SS
Explained	1	92.648
Residual	3	24.512
Total	4	117.160

Which of the following statements regarding the regression is incorrect?

A. The correlation coefficient between the X and Y variables is 0.889.

B. The industry index coefficient is significant at the 99% confidence interval.

C. If the return on the industry index is 4%, the stock's expected return is 9.7%.

D. The variability of industry returns explains 21% of the variation of company returns.

- Correct Answer : D

Exercise 4

- You built a linear regression model to analyze annual salaries for a developed country. You incorporated two independent variables, age and experience, into your model. Upon reading the regression results, you notice that the coefficient of experience is negative, which appears to be counterintuitive. In addition, you discover that the coefficients have low t-statistics but the regression model has a high R square. What is the most likely cause of these results?
  - A. Incorrect standard errors
  - B. Heteroskedasticity
  - C. Serial correlation
  - D. Multicollinearity
- Correct Answer : D

Forecasting Trends

Modeling and Forecasting Trend

- **Linear trend**: The trend in which appears roughly linear, meaning that it increases or decreases like **a straight line**.
- **Non-linear Trend**
  - Quadratic trend models

$$T_t = \beta_0 + \beta_1 TIME_t + \beta_2 TIME_t^2$$

  - Polynomial trend:

$$T_t = \beta_0 + \beta_1 TIME_t + \beta_2 TIME_t^2 + \beta_3 TIME_t^3 + \ldots + \beta_n TIME_t^n$$

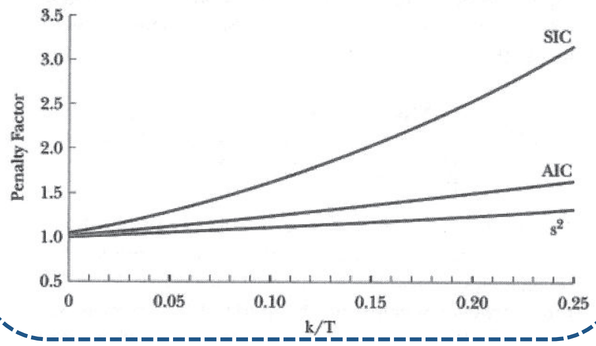
  - Exponential trend, If trend is characterized by constant growth at rate β_1, then we can write

$$T_t = \beta_0 e^{\beta_1 TIME_t}$$

Modeling and Forecasting Trend

➢ The SIC (the most consistency ceritria) generally penalizes free parameters more strongly than does the Akaike information criterion, though it depends on the size of T and relative magnitude of T and k.

Degrees-of-freedom penalties, various model selection criteria.



Modeling and Forecasting Trend

➢ We evaluate model selection criteria in terms of a key property called **consistency.** A model selection criterion is consistent if the following conditions are met:

- as the sample size gets large, the chosen measure will choose the true model correctly or with the biggest probability.
- SIC is consistency while others are not.

➢ If the no model discussed has the property of consistency. We're then led to a different optimality property, called **asymptotic efficiency.**

- as the sample size gets large, an asymptotically efficient model selection criterion chooses the model has the fastest speed to approach to the true error variance.
- The AIC, although inconsistent, is asymptotically efficient, whereas the SIC is not.

Modeling and Forecasting Seasonality

➢ **The Sources Of Seasonality**

- Any **technology** that involves the weather, such as production of agricultural commodities, is likely to be seasonal as well.
- **Preferences** may also be linked to the calendar. People want to do more vacation travel in the summer, which tends to increase both the price and quantity of summertime gasoline sales.
- **Social institutions** that are linked to the calendar, such as holidays.

➢ **A key technique for modeling seasonality is regression on seasonal dummies.**

Modeling cycle

- **What is cycle?**
  - As we mentioned the cycles, we generally considered the all-encompassing notion of cyclicality into two parts:
    - ✓Can be captured by trend or seasonal
    - ✓Cannot be captured by trend or seasonal
- **Charactering cycle**
- **Modeling cycle**

Characterizing Cycles

- **Why we need covariance stationary?**
  - If the underlying probabilistic structure of the series were changing over time, there would be no way to predict the future accurately on the basis of the past, **because the laws governing the future would differ from those governing the past**.
  - If we want to forecast a series, at minimum we require the <u>mean and covariance to be stable and finite over time</u>, which we call it **covariance stationary**.
  - In this chapter all we mentioned covariance stationarity is called **second-order stationarity** or **weak stationarity** .It means that a series whose mean and variance and covariance are stable and finite all the time but the skewness and kurtosis are not necessary.

Characterizing Cycles

- The autocovariance is just the covariance between y_t and $y_{t-\tau}$, as the series is covariance stationary, so the autocorrelation will only depend on τ, and have no relationship with t , so the function can be written as follow:

$$\gamma = (\tau) = \operatorname{cov}(y_t, y_{t-\tau}) = E(y_t - u)(y_{t-\tau} - u)$$

- The autocorrelations are just the "simple" or "regular" correlations between y_t and $y_{t-\tau}$ so the function can be written as follow:

$$\rho(\tau) = \frac{\operatorname{cov}(y_t, y_{t-\tau})}{\sqrt{\operatorname{var}(y_t)}\sqrt{\operatorname{var}(y_{t-\tau})}} = \frac{\gamma(\tau)}{\sqrt{\gamma(0)}\sqrt{\gamma(0)}} = \frac{\gamma(\tau)}{\gamma(0)}$$

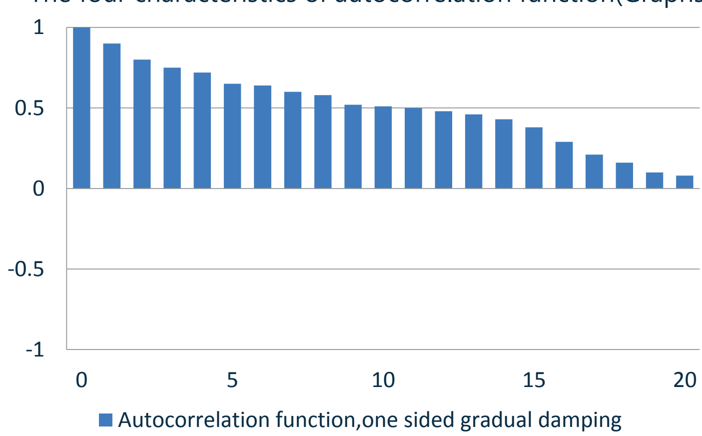
Characterizing Cycles

- The partial autocorrelations function measures the relationship between y_t and $y_{t-\tau}$ removed the effects of $y_{t-1} \cdots\cdots y_{t-\tau}$,in other words, the partial autocorrelations function measures the relationship only between y_t and $y_{t-\tau}$.

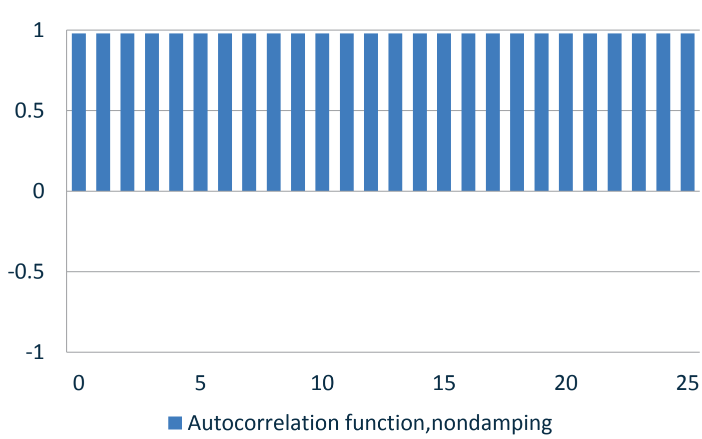
$$y_t = b_0 + b_1 y_{t-1} + b_2 y_{t-2} + \cdots + b_n y_{t-n} + \varepsilon_t$$
$$p(\tau) = b_\tau$$

Characterizing Cycles

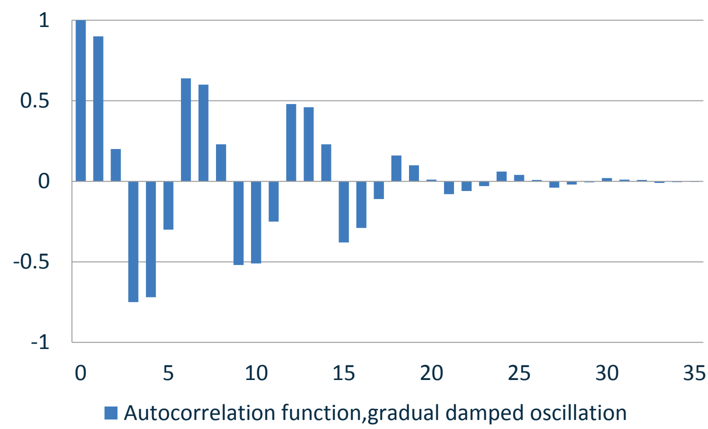
- The four characteristics of autocorrelation function(Graphs)



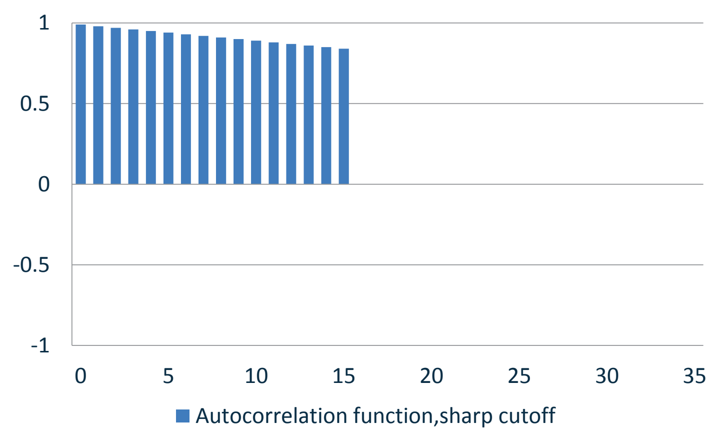
Characterizing Cycles



Characterizing Cycles



Characterizing Cycles



Characterizing Cycles

- **White noise**
  - In general, if there is a process that have zero mean, constant variance, and no serial correlation, the process is called **zero-mean white noise, or simply white noise**. Sometimes for short we write:

$$\varepsilon_t \sim WN(0,\sigma^2)$$
$$y_t = \varepsilon_t$$

hence:

$$y_t \sim WN(0,\sigma^2)$$

Characterizing Cycles

- **Wold's theorem(Wold's representation)**
  - Let $\{y_t\}$ be any zero-mean covariance-stationary process.

$$y_t = B(L)\varepsilon_t = \sum_{i=0}^{\infty} b_i \varepsilon_{t-i} \qquad \varepsilon_t \sim WN(0,\sigma^2)$$

✓ where the b_i are coefficients with $b_0 = 1$ and $\sum_{i=0}^{\infty} b_i^2 < \infty$

✓ In short, the correct "model" for any covariance stationary series is some infinite distributed lag of white noise, called the **Wold's representation.** The ε_t are often called **innovations.**

Characterizing Cycles

- Hypothesis test for white noise
  - Hypothesis testing for one autocorrelation coefficient is not enough to conclude covariance stationary, instead, a joint hypothesis test for all the autocorrelation coefficients to be zero is needed.
  - If the autocorrelation coefficients follow a normal distribution, then the sum of squared autocorrelation coefficients follow a **chi-squared distribution.**

✓ **Box-Pierce Q-Statistic & Ljung-Box Q-Statistic**

Modeling Cycle

- **MA(1) Model**

$$\boxed{y_t = \varepsilon_t + \theta\varepsilon_{t-1} = (1+\theta L)\varepsilon_t;\ \varepsilon_t \sim WN(0,\sigma^2)}$$

  - The current value of the observed series is expressed as a function of current and lagged unobservable shocks.
  - It is the very special case of Wold's representation.
- **MA(q) Model**

$$\boxed{\begin{array}{l} y_t = \varepsilon_t + \theta\varepsilon_{t-1} + \ldots + \theta_q\varepsilon_{t-q} = \Theta(L)\varepsilon_t;\ \varepsilon_t \sim WN(0,\sigma^2) \\ \Theta(L) = 1 + \theta_1 L + \ldots + \theta_q L^q \end{array}}$$

Modeling Cycle

- Characteristic of MA(1)
  - If the coefficient $|\theta|<1$, then MA(1) process is **a convergent process**.
  - The structure of the MA(1) process, in which only the first lag of the shock appears on the right, forces it to **have a very short memory**, and hence weak dynamics, regardless of the parameter value.
  - MA(1) process with parameter θ = 0.95 almost share the same persistence with the process with a parameter of θ = 0.4.
  - Autocorrelation graph appears to **have sharp cutoff**.
  - Partial Autocorrelation graph appears to have **gradual damped oscillation.**

Modeling Cycle

- **AR(1) Model**

$$\boxed{\begin{aligned} &y_t = \varphi y_{t-1} + \varepsilon_t;\ \varepsilon_t \sim WN(0,\sigma^2) \\ &(1-\varphi L)y_t = \varepsilon_t \end{aligned}}$$

- **The relationship between AR and MA model**
  - The AR model described a relationship between y_t and y_{t-i}
  - The MA model described a relationship between y_t and ε_t ,which ε_t is a white noise process.
- AR(p) Model

$$\boxed{\begin{aligned} &y_t = \varphi_1 y_{t-1} + \varphi_2 y_{t-2} + \ldots + \varphi_p y_{t-p} + \varepsilon_t;\ \varepsilon_t \sim WN(0,\sigma^2) \\ &\Phi(L)y_t = (1-\varphi_1 L - \varphi_2 L^2 + \ldots - \varphi_p L^p)y_t = \varepsilon_t \end{aligned}}$$

Modeling Cycle

- **Characteristic of AR(1)**
  - The AR(1) model is capable of capturing much more persistent dynamics than is the MA(1).
  - Autocorrelation graph appears to have **gradual damped oscillation or one sided gradual damping.**
    - ✓ |φ| < 1 is the condition for covariance stationary in the AR(1). If φ is positive, the autocorrelation decay is one-sided. If φ is negative, the decay involves back-and-forth oscillations.
  - Partial Autocorrelation appears to **have sharp cutoff.**

Modeling Cycle

- **ARMA(p, q) Model**
  - ARMA models are often both highly accurate and highly parsimonious.

$$\boxed{y_t = \varphi_1 y_{t-1} + \varphi_2 y_{t-2} + \ldots + \varphi_p y_{t-p} + \theta\varepsilon_{t-1} + \ldots + \theta_q \varepsilon_{t-q} + \varepsilon_t; \varepsilon_t \sim WN\left(0, \sigma^2\right)}$$

  - Regardless of autocorrelation or partial autocorrelation, their graphs all appear to be gradual damped.

Estimating Volatilities and Correlations

Estimating Volatilities and Correlations

- **ARCH: Add Long-Run Average Variance Rate**

$$\boxed{\sigma_n^2 = \gamma V_L + \sum_{i=1}^{m} \alpha_i u_{n-i}^2}$$

- **EWMA**
  - In EWMA, the weights decline (in constant proportion, given by lambda).

$$\boxed{\sigma_n^2 = \lambda\sigma_{n-1}^2 + (1-\lambda)u_{n-1}^2}$$

$$\boxed{\sigma_n^2 = (1-\lambda)\sum_{i=1}^{m} \lambda^{i-1} u_{n-i}^2 + \lambda^m \sigma_{n-m}^2}$$

Estimating Volatilities and Correlations

- **GARCH**
  - GARCH(1,1) is the weighted sum of a long run-variance, the most recent squared-return, and the most recent variance.

$$\boxed{\sigma_n^2 = \omega + \alpha u_{n-1}^2 + \beta\sigma_{n-1}^2}$$

$$\boxed{\begin{aligned}&\omega = \gamma V_L\\ &\gamma + \alpha + \beta = 1\\ &V_L = \frac{\omega}{1-\alpha-\beta}\end{aligned}}$$

  - EWMA is a special case of GARCH (1,1).
  - Persistence (α + β) defines the speed at which shocks to the variance revert to their long-run values. The higher the persistence, the longer it will take to revert to the mean.

Estimating Volatilities and Correlations

- **Estimating Correlations**

$$\boxed{\hat{\rho}_{XY} = \frac{Cov_n}{\sigma_{x,n}\sigma_{y,n}}}$$

  - For EWMA model

$$\boxed{Cov_n = \lambda Cov_{n-1} + (1-\lambda)x_{n-1}y_{n-1}}$$

  - For GARCH(1,1) model

$$\boxed{Cov_n = \omega + \alpha x_{n-1}y_{n-1} + \beta Cov_{n-1}}$$

Exercise 1

- Assume an exponentially weighted moving average (EWMA) model with a lambda parameter of 0.94. Yesterday's daily volatility was 1.9%. The price of the stock closed at $10 yesterday and closed up today at 11.275. What is the updated EWMA volatility estimate?
  - A. 0.1202%
  - B. 2.3750%
  - C. 3.4676%
  - D. 5.2255%

- Correct Answer : C

Exercise 2

- Given lambda of 0.94, under an infinite series, what is the weight assigned to the seventh prior daily squared return?
  - A. 4.68%
  - B. 4.40%
  - C. 4.14%
  - D. 3.89%

- Correct Answer : C

Exercise 3

- A risk manager is using the EWMA approach to estimate the volatilities of asset X and Y and their correlation. With $\lambda = 0.92$, the estimate of the covariance between assets X and Y on day $t - 1$ is 0.000243, some of the estimated returns and volatilities of the two assets are given in the following table:

	Asset X		Asset Y	
Day	Return	Volatility	Return	Volatility
t-1	0.3%		2.1%	
T	0.2%	1.44%	2.5%	2.94%

What is the estimated correlation coefficient on day t?

A. 0.18
B. 0.50
C. 0.54
D. 0.57

- Correct Answer : C

Exercise 4

- Consider the historical series of stock prices below, including the daily returns and the squared daily return. Although only the previous nine days are displayed, the actual horizon includes 55 trading days. The volatility estimates computed for the previous day, $t - 1$, are given for each of the GARCH(1, 1), EWMA. Also given are the lambda for the EWMA model and the GARCH parameters:

EWMA Parameter	
λ	0.82

GARCH(1,1) Parameters	
ω	0.00009
α	0.08
β	0.82

			Volatility Estimate	
Period	Price	Return	GARCH(1,1)	EWMA
t	$84.37	-10%		
t - 1	$93.24	-0.06%	4.3667%	6.1528%

Exercise 4

- Today's price drop contributes a dramatic -10% return to the series. Which are nearest to the updated volatility estimates given by, respectively, GARCH(1,1) and EWMA?
  - A. GARCH = 4.73% and EWMA = 4.9%
  - B. GARCH = 4.95% and EWMA = 7%
  - C. GARCH = 5.25% and EWMA = 5.25%
  - D. GARCH = 5.25% and EWMA = 5.74%

- Correct Answer : B

Exercise 5

- The parameters of a GARCH(1,1) model are ω = 0.00003, α = 0.04, and β = 0.92. These figures imply a long-run daily standard deviation of:
  - A. 1.68%
  - B. 1.55%
  - C. 1.45%
  - D. 2.74%

- Correct Answer : D

Correlations and Copulas

Copula

➢ **Drawbacks of using correlation to measure dependence.**

- Correlation is a good measure of dependence when random variables are distributed as multivariate elliptical (e.g., normal, student's).
- However, correlation is only defined if variance is finite. There might be a problem for **non-elliptical** distributions. e.g. Levy distribution can have infinite variance.
- Correlation measures the linear relationship of two variables. If risks are independent → zero correlation, however zero correlation does not imply independence.

Copula

➢ **Introduction of Copula**

- Mathematically, a "copula" is a function which allows us to combine univariate distributions to obtain a joint distribution with a particular dependence structure.
- Formally, the copula is a function of the marginal distributions F (x), plus some parameters, θ, that are specific to this function (and not to the marginals).

$$f_{12}(x_1, x_2) = f_1(x_1) \times f_2(x_2) \times c_{12}[F_1(x_1), F_2(x_2); \theta]$$

Tail Dependence

➢ **Tail dependence:**

- Tail dependence is an important issue because extreme events are often related (i.e., disasters often come in pairs or more)
- If marginal distributions are continuous, we can define a coefficient of (upper) tail dependence of X and Y as the limit, as α → 1 from below, of

$$\Pr\left[Y > F_y^{-1}(\alpha) \mid Y > F_x^{-1}(\alpha)\right] = \lambda$$

Simulation Modeling

Disadvantages of the Simulation Approach

- It might be computationally expensive.
- The results might not be precise.
- The results are often hard to replicate.
- Simulation results are experiment-specific.

Variance Reduction Techniques

- **Sampling Variation**
  - The sampling variation in a Monte Carlo study is measured by the standard error estimate, denoted S_x:

$$\boxed{S_x = \sqrt{\frac{var(x)}{N}}}$$

✓ where var(x) is the variance of the estimates of the quantity of interest over the N replications.

- **Antithetic Variates**
- **Control Variates**
- **Random Number Re-Usage across Experiments**

Bootstrapping Method

- Bootstrapping is used to obtain a description of the properties of empirical estimators by using the sample data points themselves, and it involves sampling **repeatedly with replacement from the actual data.**
- The advantage of bootstrapping over the use of analytical results is that it allows the researcher to make inferences without making strong distributional assumptions, since the distribution employed will be that of the actual data.
- Situations where the bootstrap will be ineffective:
  - Outliers in the Data
  - Non-Independent Data: Use of the bootstrap implicitly assumes that the data are independent of one another.

Random Number Generation

- Most econometrics computer packages include a random number generator.
  - The simplest class of numbers to generate are from a uniform (0,1) distribution. Computers generate continuous uniform random number draws. The initial value is called the seed.
- Computer-generated random number draws are known as **pseudo-random numbers**, since they are in fact not random at all, but entirely deterministic, since they have been derived from an exact formula.

Exercise

- The 95% confidence interval for the output of ending capital is calculated to be ($117.03, $122.97) for a simulation run with 100 scenarios. In addition, the simulation resulted in a mean ending capital amount of $120 with a standard deviation of $15. Suppose we want to improve the accuracy of this confidence interval by running a simulation of 400 scenarios. What is the new 95% confidence interval with a simulation of 400 scenarios using the same mean and standard deviations from the model with 100 scenarios?

  A. ($117.23, $122.95)

  B. ($118.52, $121.48)

  C. ($119.02, $121.99)

  D. ($119.71, $122.27)

- Correct Answer : B

It's not the end but just beginning.

By training your thoughts to concentrate on the bright side of things, you are more likely to have the incentive to follow through on your goals. You are less likely to be held back by negative ideas that might limit your performance.

试着训练自己的思想朝好的一面看，这样你就会汲取实现目标的动力，而不会因为消极沉沦停滞不前。