市场风险测量与管理

FRM Part II Program-强化班

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Topic Weightings in FRM Part II

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Framework

Market Risk Measurement and Management

- VaR and other Risk Measures
  - Estimating Market Risk Measures
  - Non-parametric Approaches
  - Extreme value
  - Backtesting VaR
  - VaR Mapping
- Risk Measurement for the Trading Book
- Modeling Dependence: Correlations And Copulas
  - Some Correlation Basics
  - Empirical Properties of Correlation
  - Financial Correlation Modeling
- Empirical Approaches to Risk Metrics and Hedges
- Term Structure Models of Interest Rates
  - The Science of Term Structure Models
  - The Evolution of Short Rates and the Shape of the Term Structure
  - The Art of Term Structure Models: Drift
  - The Art of Term Structure Models: Volatility and Distribution
- Volatility Smiles

Reading 1

VaR and other Risk Measures

Framework

➢ **VaR and other Risk Measures**

- Estimating Market Risk Measures
- Non-parametric Approaches
- Extreme value
- Backtesting VaR
- VaR Mapping

Estimating Market Risk Measures

VaR and other Risk Measures

Estimating Market Risk Measures

- **Parametric approach**
  - Normal VaR
  - Lognormal VaR
- **Non-Parametric approach(Historical Simulation)**
  - Bootstrap
  - Non-parametric density estimation
- **Hybrid approach(Semi-Parametric approach)**
  - Age-weighted(BRW)
  - Volatility-weighted(HW)
  - Correlation-weighted
  - Filtered historical simulation

Estimating VaR with Normally Distributed P/L

Normal VaR

- We assume that arithmetic returns are normally distributed with mean μ and standard deviation σ

$$VaR = -(\mu - z_\alpha \sigma) \qquad VaR = -(\mu - z_\alpha \sigma)P_{t-1}$$

Lognormal VaR

- Assume that geometric returns are normally distributed with mean μ and standard deviation σ. This assumption implies that the natural logarithm of *p* is normally distributed, or that *p* itself is lognormally distributed. Normally distributed geometric returns imply that the VaR is lognormally distributed.

$$VaR = 1 - e^{\mu - z_\alpha \sigma} \qquad VaR = (1 - e^{\mu - z_\alpha \sigma})P_{t-1}$$

Estimating Coherent Risk Measures

- **A coherent risk measure is a weighted average of the quantiles of our loss distribution.**

$$M_{\emptyset} = \int_0^1 \emptyset\,(p) q_p dp$$

- Φ(p) = weighing function specified by the user.

Estimating Expected Shortfall

- **The Conditional VaR (expected shortfall)**
  - The expected value of the loss when it exceeds VaR.
  - Measures the average of the loss conditional on the fact that it is greater than VaR.
  - CVaR indicates the potential loss if the portfolio is "hit" beyond VaR. Because CVaR is an average of the tail loss, one can show that it qualifies as a subadditive risk measure.

Estimating Expected Shortfall

- **Example:**
  - Given the following 30 ordered percentage returns of an asset,

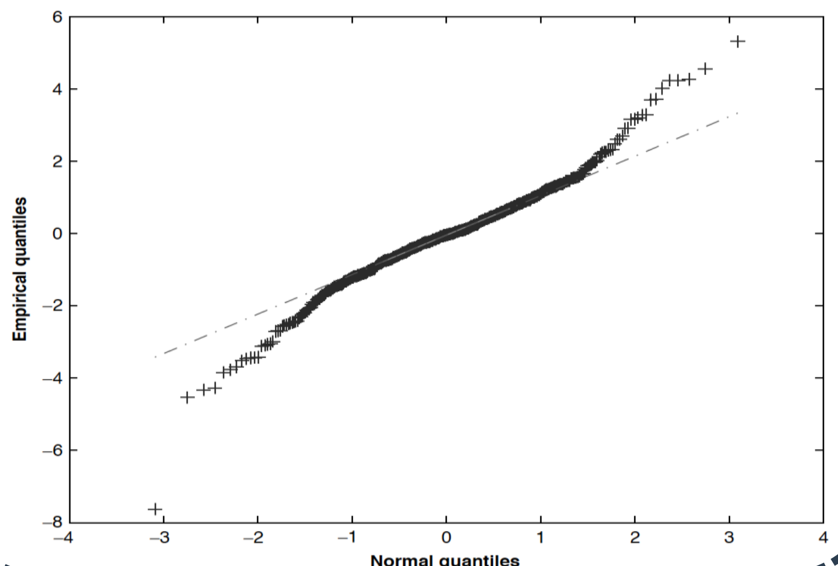
    −16, −14, −10, −7, −7, −5, −4, −4, −4, −3, −1, −1, 0, 0, 0, 1, 2, 2, 4, 6, 7, 8, 9, 11, 12, 12, 14, 18, 21, 23

    Calculate the VaR and expected shortfall at a 90% confidence level:
  - Solution:

    VaR (90%) = 7, Expected Shortfall = 13.3

Quantile-Quantile Plots

- A plot of the quantiles of the empirical distribution against those of some specified distribution. The shape of the QQ plot tells us a lot about how the empirical distribution compares to the specified one.
  - In particular, if the QQ plot is linear, then the specified distribution fits the data, and we have identified the distribution to which our data belong.



Question

- A portfolio manager owns a portfolio of options on a non-dividend paying stock RTX. The portfolio is made up of 10,000 deep in-the-money call options on RTX and 50,000 deep out-of-the money call options on RTX. The portfolio also contains 20,000 forward contracts on RTX. RTX is trading at USD 100. If the volatility of RTX is 30% per. year, which of the following amounts would be closest to the 1-day VaR of the portfolio at the 95 percent confidence level, assuming 252 trading days in a year?

  A. USD 932

  B. USD 93,263

  C. USD 111,122

  D. USD 131,892

- **Correct answer : B**

Non-parametric Approaches

VaR and other Risk Measures

Introduction

- All non-parametric approaches are based on the underlying assumption that <u>the near future will be sufficiently like the recent past.</u>
  - With non-parametric methods, there are no problems dealing with variance–covariance matrices, curses of dimensionality, etc.

Bootstrapped Historical Simulation

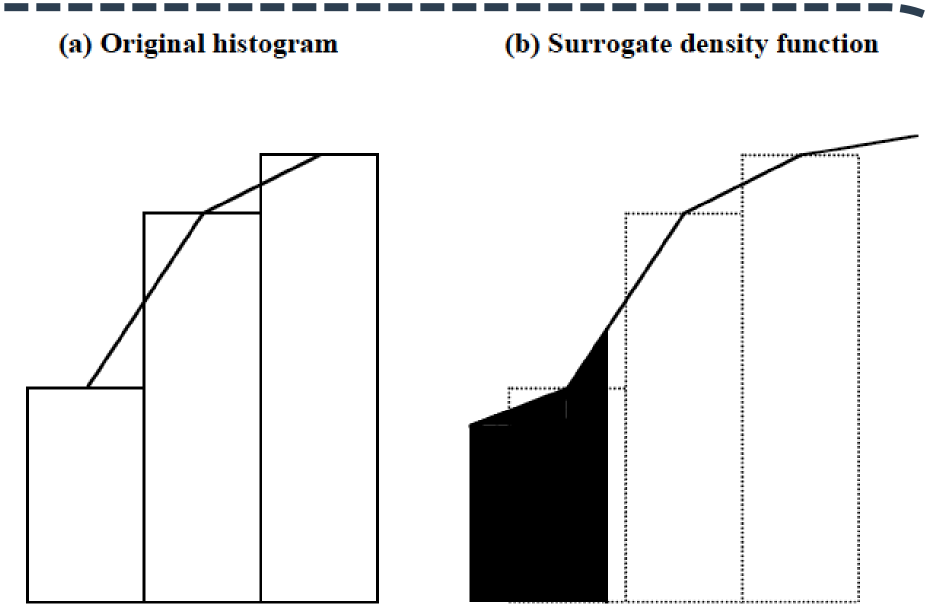
- The bootstrap is very intuitive and easy to apply.
  - We create a large number of new samples, each observation of which is obtained by drawing at random from our original sample and replacing the observation after it has been drawn.
- A bootstrapped estimate will often be **more accurate** than a 'raw' sample estimate, and bootstraps are also useful for gauging the precision of our estimates.

Drawbacks of HS

- Basic HS has the practical drawback that it only allows us to estimate VaRs at discrete confidence intervals determined by the size of our data set.
  - For instance, the VaR at the 95.1% confidence level is a problem because there is no corresponding loss observation to go with it.
  - With n observations, basic HS only allows us to estimate the VaRs associated with, at best, n different confidence levels.

Non-parametric Density Estimation

- Non-parametric density estimation offers a potential solution.
  - Draw in straight lines connecting the mid-points at the top of each histogram bar(Polygon).
  - Treating the area under the lines as a pdf then enables us to estimate VaRs at any confidence level.

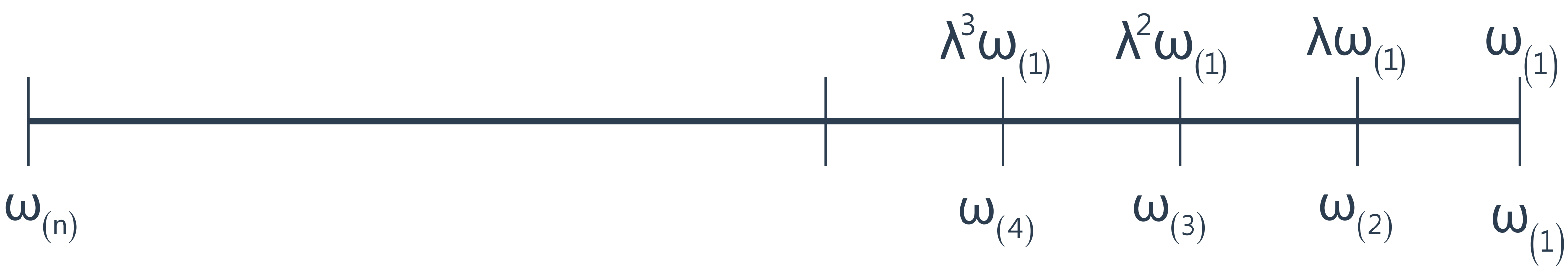


Hybrid Approach

- **Hybrid approach(Semi-Parametric approach)**
  - Age-weighted(BRW)
  - Volatility-weighted(HW)
  - Correlation-weighted
  - Filtered historical simulation

Age-weighted Historical Simulation

- **Boudoukh, Richardson and Whitelaw (BRW: 1998)**
  - $w_{(1)}$ is the probability weight given to an observation 1 day old.
  - A λ close to 1 indicates a slow rate of decay, and a λ far away from 1 indicates a high rate of decay.



$$\omega_{(1)} + \lambda\omega_{(1)} + \ldots + \lambda^{n-1}\omega_{(1)} = 1 \rightarrow$$

$$\omega_{(i)} = \frac{\lambda^{i-1}(1-\lambda)}{1-\lambda^n}$$

Age-weighted Historical Simulation

- **Major attractions**
  - It provides a nice generalization of traditional HS, because we can regard traditional HS as a special case with zero decay, or $\lambda \rightarrow 1$.
  - Helps to reduce distortions caused by events that are unlikely to recur, and helps to reduce ghost effects.
    - ✓ As an observation ages, its probability weight gradually falls and its influence diminishes gradually over time, instead of from 1/n to zero.
  - Age-weighting allows us to let our sample period grow with each new observation, so we never throw potentially valuable information away. This would improve efficiency and eliminate ghost effects, because there would no longer be any "jumps" in our sample resulting from old observations being thrown away.

Volatility-weighted Historical Simulation

- **Hull and White (HW 1998)**

  - We adjust the historical returns to reflect how volatility tomorrow is believed to have changed from its past values.

$$\frac{r_{t,i}^*}{r_{t,i}} = \frac{\sigma_{T,i}}{\sigma_{t,i}}$$

✓ $r_{t,i}$ = actual return for asset i on day t

✓ $\sigma_{t,i}$ = volatility forecast for asset i on day t

✓ $\sigma_{T,i}$ = current forecast of volatility for asset i

Volatility-weighted Historical Simulation

- **Major attractions**
  - It takes account of volatility changes in a natural and direct way.
  - It produces risk estimates that are appropriately sensitive to current volatility estimates.
  - It allows us to obtain VaR and ES estimates that can exceed the maximum loss in our historical data set.
    - ✓ In recent periods of high volatility, historical returns are scaled upwards, and the HS P/L series used in the HW procedure will have values that exceed actual historical losses.
  - Produces superior VaR estimates to the BRW one.

Correlation-weighted historical simulation

Correlation-weighted historical simulation

- Correlation-weighting is a little more involved than volatility-weighting.
- To see the principles involved, suppose for the sake of argument that we have already made any volatility-based adjustments to our HS returns along Hull-White lines, but also wish to adjust those returns to reflect changes in correlations.

Filtered historical simulation

- **Filtered historical simulation**
  - Combines the traditional historical simulation model with GARCH or AGARCH model.
  - Major attractions
    - ✓ Combine the non-parametric attractions of HS with a sophisticated (e.g., GARCH) treatment of volatility, and so take account of changing market volatility conditions
    - ✓ It is fast, even for large portfolios
    - ✓ As with the earlier HW approach, FHS allows us to get VaR and ES estimates that can exceed the maximum historical loss in our data set.
    - ✓ It maintains the correlation structure in our return
    - ✓ It can be modified to take account of autocorrelations in asset returns
    - ✓ It can be modified to produce estimates of VaR or ES confidence intervals.
    - ✓ There is evidence that FHS works well.

A/D of Non-parametric Methods

- **Advantages**
  - Intuitive and conceptually simple;
  - Do not depend on parametric assumptions;
  - Accommodate any type of position;
  - No need for covariance matrices, no curses of dimensionality;
  - Use data that are (often) readily available;
  - Are capable of considerable refinement and potential improvement if we combine them with parametric “add-ons” to make them semi-parametric.

A/D of Non-parametric Methods

- **Disadvantages**
  - Very dependent on the historical data set;
  - Subject to ghost effect;
  - If our data period was unusually quiet, non-parametric methods will often produce VaR or ES estimates that are too low, vice versa;
  - Have difficulty(act slowly) handling shifts(permanent risk change) that take place during our sample period;
  - Have difficulty handling extreme value
    - ✓ If our data set incorporates extreme losses that are unlikely to recur, these losses can dominate non-parametric risk estimates even though we don't expect them to recur;
    - ✓ Make no allowance for plausible events that might occur, but did not actually occur, in our sample period.

Problems from Long Window

- **The longer the window:**
  - The greater the problems with aged data;
  - The longer the period over which results will be distorted by unlikely-to-recur past events, and the longer we will have to wait for ghost effects to disappear;
  - The more the news in current market observations is likely to be drowned out by older observations;
  - The greater the potential for data-collection problems.

Question

- The lowest six returns for a portfolio are shown in the following table.

	Six lowest returns	Hybrid weight	Hybrid Cumulative weight
1	-4.10%	0.0125	0.0125
2	-3.80%	0.0118	0.0243
3	-3.50%	0.0077	0.0320
4	-3.20%	0.0098	0.0418
5	-3.10%	0.0062	0.0481
6	-2.90%	0.0027	0.0508

- What will the 5% VaR be under the hybrid approach?

  A. -3.10%

  B. -3.04%

  C. -2.96%

  D. -2.90%

- **Correct answer :C**

Extreme value

VaR and other Risk Measures

Extreme Value

- **Generalized Extreme-Value Theory (Block Maxima)**
  - Consider a sample of size n drawn from F(x), and let the maximum of this sample be M_n. If n is large, we can regard M_n as an extreme value.
  - Under relatively general conditions, as n gets large the distribution of extremes (i.e., M_n) converges to the following **generalized extreme-value (GEV) distribution**:

$$F(x)=\begin{cases} exp\left[-\left(1+\xi\frac{x-\mu}{\sigma}\right)^{-\frac{1}{\xi}}\right], \xi\neq 0 \\ exp\left[-exp\left(\frac{x-\mu}{\sigma}\right)\right], \xi=0 \end{cases}$$

Extreme Value

- **Generalized Extreme-Value Theory (Block Maxima) (cont'd)**
  - This distribution has **three parameters**.
    - ✓ **μ**, the **location parameter** of the limiting distribution, which is a measure of the central tendency of M_n,
    - ✓ **σ**, the **scale parameter** of the limiting distribution, which is a measure of the dispersion of M_n.
    - ✓ **ξ**, the **tail index**, gives an indication of the shape (or heaviness) of the tail of the limiting distribution. If ξ > 0, the GEV becomes the Frechet distribution. This case is particularly useful for financial returns because they are typically heavy-tailed.

Extreme Value

Peaks-Over-Threshold (POT) Approach

- If x is a random i, i, d. loss with distribution function F(x), and u is a threshold value of x, we can define the distribution of excess losses over threshold u.
- The distribution of x itself can be any of the commonly used distributions and will usually be unknown to us. However, as u gets large, the distribution $F_U(x)$ converges to a **generalized Pareto distribution**, given by:

$$G_{\xi,\beta}(x) = \begin{cases} 1-(1+\xi x/\beta)^{-1/\xi} \; if \; \xi \neq 0 \\ 1-exp(-x/\beta) \; if \; \xi = 0 \end{cases}$$

- Two parameters: a positive **scale parameter**, **β**, and a **shape or tail index** parameter, **ξ.** This latter parameter is the same as the tail index encountered already with GEV theory

Extreme Value

➤ **The expression for VaR and expected shortfall using POT parameters is given as follows:**

$$VaR = \mu + \frac{\beta}{\xi}\left(\left[\frac{n}{N_{\mu}}(1-\alpha)\right]^{-\xi} - 1\right)$$

$$ES = \frac{VaR}{1-\xi} + \frac{\beta - \xi\mu}{1-\xi}$$

- ξ = shape parameter
- β = scale parameter
- μ = threshold
- n = number of observations
- Nμ = number of observations that exceed threshold

Extreme Value

- **Peaks-Over-Threshold (POT) Approach (cont'd)**
  - **Tradeoffs Involved in Setting the Threshold Level**
    - ✓ We want a threshold u to be sufficiently high for the theorem to apply reasonably closely; but if u is too high, we won't have enough excess-threshold observations on which to make reliable estimates.
  - **Differences with GEV Theory**
    - ✓ Both are different manifestations of the same underlying EV theory.
    - ✓ POT model exceedances over a high threshold while GEV theory model the maxima of a large sample.
    - ✓ POT require fewer parameters.
    - ✓ The block maxima approach can involve some loss of useful data, because some blocks might have more than one extreme in them.
    - ✓ POT requires us to grapple with the problem of choosing the threshold.

Backtesting VaR

VaR and other Risk Measures

Model Verification Based on Failure Rates

- The failure rate: the proportion of times VaR is exceeded in a given sample.
- The number of exceptions x follows a binomial probability distribution:

$$f(x) = C_T^x p^x (1-p)^{T-x}$$

$$E(x) = pT \qquad V(x) = p(1-p)T$$

- Approximate the binomial distribution by the normal distribution:

$$z = \frac{x - pT}{\sqrt{p(1-p)T}} \approx N(0,1)$$

Example

- In 1998, daily revenue of JP Morgan fell short of the downside (95% VaR) band on 20 days, or more than 5% of the time. Nine of these 20 occurrences fell within the August to October period.

$$z = \frac{x - pT}{\sqrt{p(1-p)T}} = \frac{20 - 0.05 * 252}{\sqrt{0.05(1-0.05)252}} = 2.14 > 1.96$$

  - We reject the hypothesis that the VaR model is unbiased.

- What happens to test 99% VaR at 5% significance level and 95% VaR at 1% significance level?

Type 1 and Type 2 errors

- Users of VaR models need to balance type I errors against type II errors.
- Ideally, one would want to set a low type I error rate and then have a test that creates a very low type II error rate, in which case the test is said to be powerful.

Decision	Correct	Incorrect
Accept	OK	Type II
Reject	Type I	Power of test

Kupiec VaR Backtest

- **Backtesting Exceptions**
  - Using Failure Rates in Model Verification
    - ✓ H_0: accurate model
    - ✓ H_a: inaccurate model
    - ✓ Test statistic:

$$LR_{uc} = -2ln[(1-p)^{T-N}p^N] + 2ln\{[1-(N/T)]^{T-N}(N/T)^N\}$$

      - p: the probability of exception, p = 1-c
      - N: the number of exceptions
      - T: the number of samples
  - We would reject the null hypothesis if LR > 3.841.

Nonrejection region

Model Backtesting,95 Percent Nonrejection Test Confidence Regions

Probability Level p	VaR Confidence Level c	Nonrejection Region for Number of Failures N		
		T=252 Days	T=510 Days	T=1000 Days
0.01	99%	N<7	1<N<11	4<N<17
0.025	97.5%	2<N<12	6<N<21	15<N<36
0.05	95%	6<N<20	16<N<36	37<N<65
0.075	92.5%	11<N<28	27<N<51	59<N<92
0.10	90%	16<N<36	38<N<65	81<N<120

Nonrejection region for Kupiec VaR Backtest

LR_{uc} Values for T=255

C level	N											
	1	2	3	4	5	6	7	8	9	10	11	12
97.50%	~~7.16~~	~~4.19~~	2.27	1.04	0.33	0.02	0.06	0.39	0.98	1.81	2.84	~~4.06~~
98.00%	~~5.01~~	2.49	1.03	0.26	0	0.15	0.65	1.44	2.48	3.76	~~5.25~~	~~6.93~~
99.00%	1.24	0.13	0.08	0.71	1.86	3.42	~~5.32~~	~~7.51~~	~~9.97~~	~~12.65~~	~~15.55~~	~~18.63~~

- Conclusion:
  - The interval shrinks as the sample size extends.(Data should be larger)
  - Detection of systematic biases becomes increasingly difficult for high values of c because the exceptions in these cases are very rare events.(High confidence VaR should be avoided)

Blame on Unconditional Coverage Models

- So far the framework focuses on unconditional coverage because it ignores conditioning, or time variation in the data. The observed exceptions, however, could **cluster** or " **bunch**" closely in time, which also should invalidate the model.
- In theory, these occurrences should be evenly spread over time. If, instead, we observed that 10 of these exceptions occurred over the last 2 weeks, this should raise a red flag.
- **Christoffersen test is applied.**

Conclusions

- Ideally, one would want a framework that has very high power, or high probability of rejecting an incorrect model.
- The current framework could be improved by choosing a lower VaR confidence level or by increasing the number of data observations.
  - The horizon should be as short as possible in order to increase the number of observations and to mitigate the effect of changes in the portfolio composition.
  - The confidence level should not be too high because this decreases the effectiveness, or power, of the statistical tests.

Basel Committee Rules for Backtesting

- The Basel Committee requires that market VaR be calculated at the 99% confidence level and back testing over the past year. That is at the 99% confidence level, we would expect to have 2.5 exceptions (250×0.01) each year.
- Economic capital=VaR×(3+k)

Zone	Number of exceptions	Increase in K
Green	0-4	0
Yellow	5	0.4
	6	0.5
	7	0.65
	8	0.75
	9	0.85
Red	10+	1

Basel Committee Rules for Backtesting

- **Four categories of causes for exceptions:**
  - **Basic integrity of the model is lacking**. Exceptions occurred because of incorrect data or errors in the programming. The penalty should apply.
  - **Model accuracy needs improvement**. The exceptions occurred because the model does not describe risks precisely. The penalty should apply.
  - **Intraday trading**. Positions changed during the day. The penalty should be considered.
  - **Bad luck**. Markets were particularly volatile or correlations changed. These exceptions should be expected to occur at least some of the time.

Question

- An analyst is backtesting a daily holding period VaR model using a 97.5% confidence level over a 255-day period and is using a 3.84 test statistic. The following table shows the calculated values of a log-likelihood ratio (LR) at a 97.5% confidence level.

Number of Exceptions

1	2	3	4	5	6	7	8	9	10	11	12
7.16	4.19	2.27	1.04	0.33	0.02	0.06	0.39	0.98	1.81	2.84	4.06

- Based on the above information, which of the following statements accurately describes the VaR model that is being backtested?
  - A. If the number of exceptions is more than 3, we would not reject the model.
  - B. If the number of exceptions is more than 2 and less than 12, we may commit a Type II error.
  - C. If the number of exceptions is less than 2, we would accept the hypothesis that the model is correct.
  - D. If the number of exceptions is less than 2, we may commit a Type II error.
- **Correct answer :B**

VaR Mapping

VaR and other Risk Measures

VaR Mapping

- **Three approaches for mapping**
  - **Principal mapping**. Only the risk associated with the return of principal at the maturity of the bond is mapped.
  - **Duration mapping**. With duration mapping, one risk factor is chosen that corresponds to the portfolio duration.
  - **Cash flow mapping**. With cash-flow mapping, the portfolio cash flows are grouped into maturity buckets.

Cash Flow Mapping

- Each cash flow is represented by the present value of the cash payment, discounted at the appropriate zero-coupon rate.
  - Undiversified VaR: considering interest rate VaRs with each maturities
  - Diversified VaR: also considering interest rate correlations
- VaR(Principal) > VaR(duration) > VaR(Undiversified) > VaR(Diversified)

Mapping Linear Derivatives

- **Currency forward contracts**

$$F_t = (S_t e^{-yt})e^{rt}$$

  - Long currency forward contract = long foreign currency spot + long foreign currency bill + short U.S. dollar bill

- **Forward Rate Agreements**
  - Long 6×12 FRA = long 6-month bill + short 12-month bill

- **Interest Rate Swaps**
  - A payer swap is equivalent to buying a floating rate bond and simultaneously shorting a fixed rate bond.
  - **Or** a series of forward rate agreement with same forward rate.

- **BSM Model:**

$$c = Se^{-yt}N(d_1) - Ke^{-rt}N(d_2)$$

  - Long call option = long $N(d_1)$*asset + short $N(d_2)$ bill
  - Long put option = long $N(-d_2)$*bill+ short $N(-d_1)$*asset

Question

- An analyst is using the delta-normal method to determine the VaR of a fixed income portfolio. The portfolio contains a long position in 1-year bonds with a $1 million face value and a 6% coupon that is paid semi-annually. The interest rates on six-month and twelve-month maturity zero-coupon bonds are, respectively, 2% and 2.5%. Mapping the long position to standard positions in the six-month and twelve-month zeros, respectively, provides which of the following mapped positions?

  A. $30,000 and 1,030,000

  B. $29,500 and 975,610

  C. $29,703 and 1,004,878

  D. $30,300 and 1,035,000

- **Correct answer :C**

Reading 2

Risk Measurement for the Trading Book

VaR Implementation

- **The three categories of implementation issues reviewed are:**
  - (1) time horizon over which VaR is estimated;
  - (2) the recognition of time-varying volatility in VaR risk factors;
  - (3) VaR backtesting
    - ✓ Backtesting is not effective when the number of VaR exceptions is small. In addition, backtesting is less effective over longer time horizons due to portfolio instability.

Risk Measures

- **Value at Risk (VaR)** has become a standard risk measure in finance.
  - VaR measures only quantiles of losses, and thus disregards any loss beyond.
  - It has been criticized for its lack of subadditivity.
- **Expected Shortfall (ES)** is the most well-known risk measure following VaR.
  - **First**, ES does account for the severity of losses beyond the confidence threshold.
  - **Second**, it is always subadditive and coherent.
  - **Third**, it mitigates the impact that the particular choice of a single confidence level.
  - However, expected shortfall is more complex and computationally intensive.
- **Spectral risk measures**
  - are a promising generalization of expected shortfall .
    - ✓ favorable **smoothness properties.**
    - ✓ adapting the risk measure directly to the risk aversion of investors.
    - ✓ It requires little effort if the underlying risk model is simulations-based.

Integrated Risk Measurement

- We survey the academic literature on the implications of modelling the aggregate risks present across a bank's trading and banking books using either a
  - **Compartmentalised approach:** the sum of risks measured separately
  - **Unified approach**: considers the interaction between these risks explicitly.
- **The Basel framework** is based on a "building block" approach such that a bank's regulatory capital requirement is
  - The sum of the capital requirements for each of the defined risk categories (i.e., market, credit and operational risk), which are calculated separately within the formulas and rules that make up pillar 1.
  - Capital requirements for other risk categories are determined by the supervisory process that fits within Pillar 2;
  - This approach is therefore often referred to as a non-integrated approach to risk measurement.

FRTB

Expected Shortfall (ES)

- The FRTB is proposing a change to the measure used for determining market risk capital.
- Under the FRTB proposal, banks would forgo combining a 99% VaR with 99% Stress VaR, and instead, calculate expected shortfall (ES) with a 97.5% confidence level.
- Capital is based solely on the calculation of the expected shortfall using a 12-month stressed period(250 days).
  - ✓ Analogously to the way stressed VaR is determined for Basel II.5, banks are required to search back through time and choose a period that would be particularly difficult for the bank's current portfolio.

FRTB

- **Two approaches to calculate ES**
  - Internal models-based approach(**IMA**)
  - Revised standardized approach
- **Liquidity Horizons**
  - FRTB requires the changes to market variables (referred to as shocks) to be the changes that would take place (in stressed market conditions) over periods of time that reflect the differing liquidities of market variables. The periods of time are referred to as liquidity horizons.
  - Five different liquidity horizons are used: 10 days, 20 days, 40 days, 60 days, and 120 days.

FRTB

➢ Treatment of Credit Spread

- The FRTB provides **a modification of the IRC**. It recognizes that for instruments dependent on the credit risk of a particular company, two types of risk can be identified:
  - ✓ **Credit spread risk.** This is the risk that the company's credit spread will change, causing the mark-to-market value of the instrument to change. Still Use ten-day, 99% VaR
  - ✓ **Jump-to-default risk.** This is the risk that there will be a default by the company. Typically this leads to an immediate loss or gain to the bank. Change to one-year, 99.9% VaR.

Reading 3

Modeling Dependence: Correlations And Copulas

Framework

- **Modeling Dependence: Correlations and Copulas**
  - Some Correlation Basics
  - Empirical Properties of Correlation
  - Financial Correlation Modeling

Some Correlation Basics

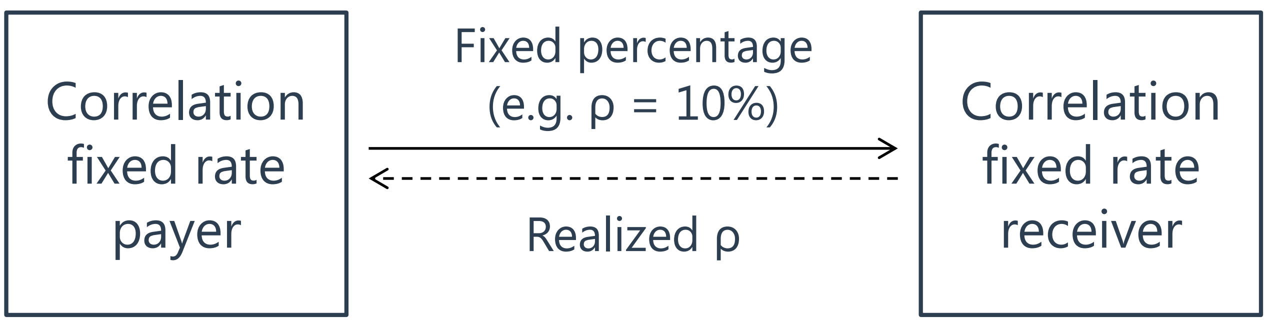
Modeling Dependence: Correlations And Copulas

Correlation Swap

- Paying a fixed rate in a **correlation swap** is also called buying correlation. The payoff = NP × (realized ρ – fixed ρ).

$$\rho_{realized} = \frac{2}{n^2 - n} \sum_{i>j} \rho_{i,j}$$

- This is because the present value of the correlation swap will increase for the correlation buyer if the realized correlation increases.
- The fixed rate receiver is selling correlation.



Correlation Swap

- What is the payoff of a correlation swap with three assets, a fixed rate of 10%, a notional amount of $1,000,000, and a 1-year maturity? Let's assume the realized pairwise correlations of the log returns at maturity are as displayed.

Pairwise Pearson Correlation Coefficient at Swap Maturity			
	$S_{j=1}$	$S_{j=2}$	$S_{j=3}$
$S_{i=1}$	1	0.5	0.1
$S_{i=2}$	0.5	1	0.3
$S_{i=3}$	0.1	0.3	1

$$\rho_{realized} = \frac{2}{3^2 - 3}\sum(0.5 + 0.3 + 0.1) = 0.3$$

- So, the payoff for the correlation fixed rate payer at swap maturity is $1,000,000 × (0.3 -0.1) = $200,000

Correlation Swap

- There is a positive relationship between correlation and volatility.
  - Therefore, if correlation between the stocks of the Dow increases, so will the implied volatility of the call on the Dow.
  - Another way of buying correlation (i.e., benefiting from an increase in correlation) is to **buy call options on an index such as the Dow Jones Industrial Average and sell call options on individual stocks of the Dow.**

Variance Swap

- A further way to buy correlation is to **pay fixed in a variance swap on an index and to receive fixed in variance swaps on individual components of the index.**
  - If correlation increases, so will the variance.
  - As a consequence, the present value for the variance swap buyer, the fixed variance swap payer, will increase.
  - This increase is expected to outperform the potential losses from the short variance swap positions on the individual components.

Credit Crisis Resulting from CDOs

- **Failure 1: Loss from the strategy that short the equity tranche of the CDO and long the mezzanine tranche with CDS.**
  - Shorting the equity tranche means receiving the (high) equity tranche contract spread.(sell CDS on equity tranche)
  - Going long the mezzanine tranche means paying the (fairly low) mezzanine tranche contract spread.(buy CDS on mezzanine tranche)
- **When the correlations of the assets in the CDO decreased, the hedge funds lost on both positions.**

Credit Crisis Resulting from CDOs

- **Failure 2: Loss from the default on equity tranche**
  - From 2007 to 2009, default correlations of the mortgages in the CDOs increased. **This actually helped equity tranche investors**.
  - **However, this increase was overcompensated by a strong increase in default probability of the mortgages**.
- As a consequence, tranche spreads increased sharply, resulting in huge losses for the equity tranche investors as well as investors in the other tranches.

Credit Crisis Resulting from CDOs

Failure 3: Loss from senior tranche

- Correlations between the tranches of the CDOs also increased during the crisis. **This had a devastating effect on the super-senior tranches**.
- In normal times, these tranches were considered extremely safe since:
  - ✓ They were AAA rated.
  - ✓ They were protected by the lower tranches.
- But with the increased tranche correlation and the generally deteriorating credit market, **these super-senior tranches were suddenly considered risky and lost up to 20% of their value**.

Credit Default Swaps

- **Failure 4: Loss from greed**
  - CDSs can also be used as **speculative instruments.**
    - ✓ For example, the CDS seller (i.e., the insurance seller) hopes that the insured event (e.g., default of a company or credit deterioration of the company) will not occur.
    - ✓ In this case the CDS seller keeps the CDS spread (i.e., the insurance premium) as income.
  - A CDS buyer who does not own the underlying asset is speculating on the credit deterioration of the underlying asset.

Correlation Risk and Other Risks

- **The default correlation within sectors is higher than between sectors.**
  - a lender is advised to have a sector-diversified loan portfolio to reduce default correlation risk.
- **Time dependency of credit risk**
  - For most investment grade bonds, the longer the time horizon, the higher the probability of adverse events.
  - For a distressed company, the longer the time horizon, the probability of default decreases.
- **Systemic risk and correlation risk are highly dependent, but not significant.**

Question

- About the role of correlation in the global financial crisis (GFC), Meissner shows firstly that several different types of correlation can be measured. For example, correlation between mortgages that underlie a collateralized debt obligation is different than correlation between CDO tranches. According to Meissner, each of the following is TRUE about the role of correlation in the crisis, EXCEPT which is false?

  A. The Gaussian copula correlation model was widely used to value collateralized debt obligations (CDOs) but the trust in the Gaussian copula was naive; for example, low default probabilities and default correlations (between CDO assets) were often assumed as model inputs.

Question

B. Due to an increase in the default correlation among mortgages in collateralized debt obligations (CDOs), ceteris paribus, equity tranche spreads increased.

C. Correlations between the tranches of the CDOs increased during the crisis, and this had a devastating effect on the super-senior tranches.

D. Credit default swaps (CDS) were used as speculative instruments in ways that did not disperse risk like we expect insurance contracts to disperse risk.

- **Correct answer :B**

Empirical Properties of Correlation

Modeling Dependence: Correlations And Copulas

Equity Correlation Behaviors

- **Correlation levels are lowest in strong economic growth times.**
  - In recessions, correlation levels typically increase.
- **Correlation volatility is lowest in an economic expansion and highest in worse economic states.**
- **Positive correlation between correlation and correlation volatility**
- **Before every recession a downturn in correlation volatility occurred.** However, the relationship between a decline in volatility and the severity of **the recession is statistically non-significant**.

Mean reversion & Autocorrelation in correlations

- **Mean reversion** is present if there is a negative relationship between the change of a variable, $S_t - S_{t-1}$, and the variable S_{t-1}.
  - $S_t = a(U_s - S_{t-1}) + S_{t-1}$
- **In that case, a is called the mean reversion coefficient, or simply, mean reversion.**

$$S_t = a(U_s - S_{t-1}) + S_{t-1} \quad \text{Or} \quad S_t - S_{t-1} = aU_s - aS_{t-1}$$

- To find the mean reversion rate a, we can run a standard regression analysis of the form

$$Y = \alpha + \beta X$$

- We are regressing S_t-S_{t-1} with respect to S_{t-1}:

$$\underbrace{S_t - S_{t-1}}_{Y} = \underbrace{a\,U_s}_{\alpha} - \underbrace{a\,S_{t-1}}_{\beta X}$$

Mean reversion & Autocorrelation in correlations

- **Autocorrelation** is the degree to which a variable is correlated to its past values.
  - In finance, positive autocorrelation is also termed **persistence**. In mutual fund or hedge fund performance analysis, an investor typically wants to know if an above-market performance of a fund has persisted for some time.
- **Autocorrelation is the opposite property of mean reversion.**

Financial Correlation Modeling

Modeling Dependence: Correlations And Copulas

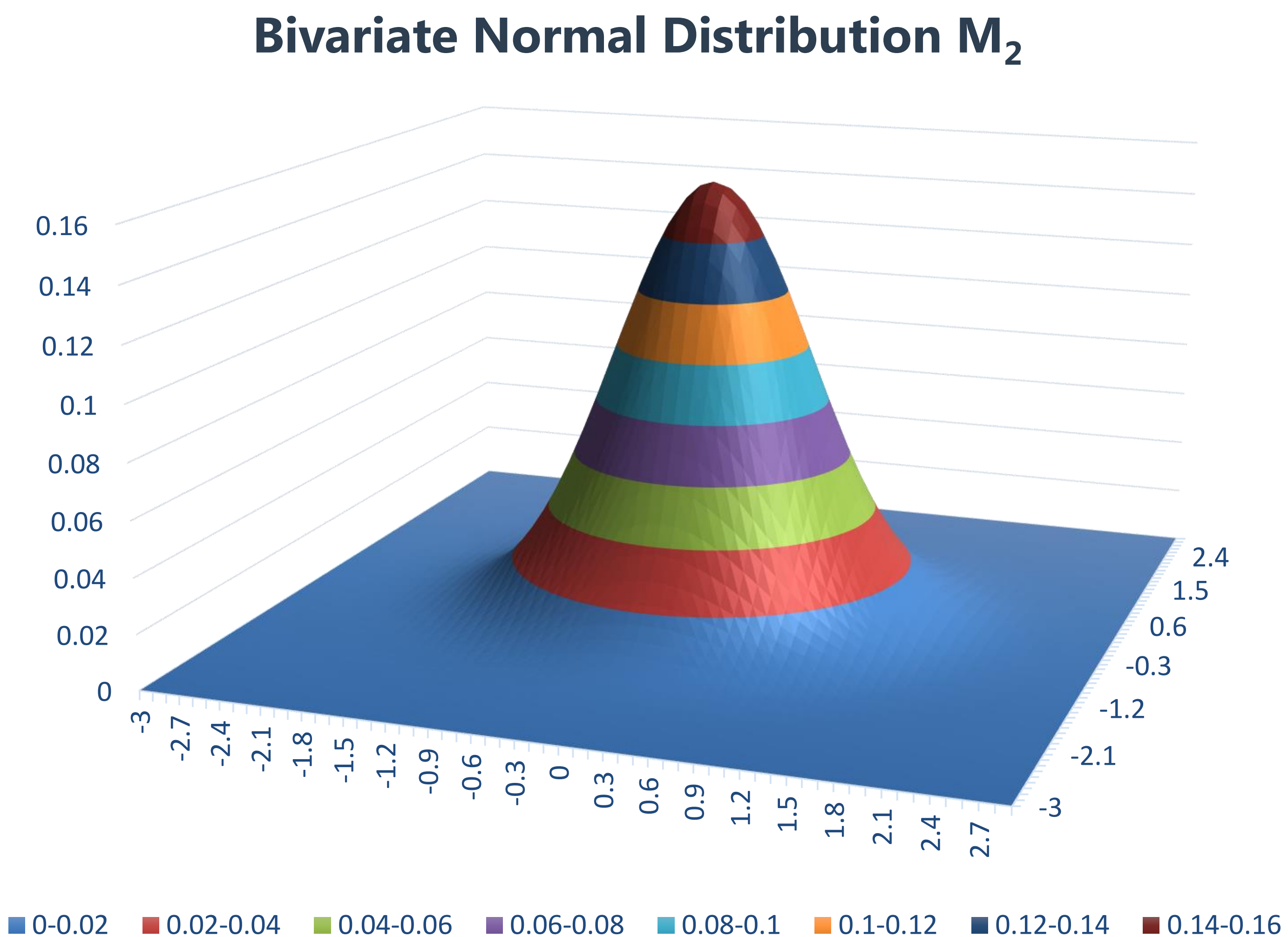
Copulas

- **Copula functions are designed to simplify statistical problems.** They allow the joining of multiple univariate distributions to a single multivariate distribution.
- **There exists a copula function C such that:**

$$C[G_1(u_1),\ldots,G_n(u_n)] = F_n[F_1^{-1}(G_1(u_1)),\ldots,F_n^{-1}(G_n(u_n));\rho_F]$$

- $G_i(u_i)$: marginal uniform distributions
- F_n: the joint cumulative distribution function
- $F_i^{-1}(G_i(u_i))$: the inverse of F_i, standard normal distribution
- ρ_F: the correlation structure of F_n

David Li's Copula



Limitations of the Gaussian Copula

- Tail Dependence: **In a crisis, correlations typically increase.** Hence it would be desirable to apply a correlation model with high co-movements in the lower tail of the joint distribution.
- Correlation smile: traders randomly alter the correlation parameters for different tranches in order to derive desired tranche spreads. **Traders increase the correlation for extreme tranches** such as the equity tranche or senior tranches.
- For certain parameter constellations, it may not be possible to imply a market CDO tranche spread for a correlation parameter between 0 and 1. **The lowest 40% and highest 20% of losses of the equity tranche cannot be explained by the model.**
- **Copula model is static** and consequently allows only limited risk management.

Question

- Which of the following statements about correlation and copula are correct?

  I. Copula enables the structures of correlation between variables to be calculated separately from their marginal distributions.

  II. Transformation of variables does not change their correlation structure.

  III. Correlation can be a useful measure of the relationship between variables drawn from a distribution without a defined variance.

  IV. Correlation is a good measure of dependence when the measured variables are distributed as multivariate elliptical.

  A. I and IV only

  B. II, III, and IV only

  C. I and III only

  D. II and IV only

- **Correct answer :A**

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Reading 4

Empirical Approaches to Risk Metrics and Hedges

Regression Hedge

- **Let Δy_t^N and Δy_t^R be the changes in the yields of the nominal and bonds.**

$$\Delta y_t^N = \alpha + \beta \Delta y_t^R + \varepsilon_t$$

  - Δy_t^N = changes in the nominal yield
  - Δy_t^R = changes in the real yield

- Least-squares estimation of α and β finds the estimates $\hat{\alpha}$ and $\hat{\beta}$ that minimize the sum of the squares of the realized error terms over the observation period.

$$\sum_t \hat{e}_t^2 = \sum_t (\Delta y_t^N - \hat{\alpha} - \widehat{\beta \Delta y_t^R})^2$$

Regression Hedge

- The trader plans to short $100 million of the (nominal) 35/8 s of August 15, 2019, and, against that, to buy some amount of the TIPS 17/8 s of July 15, 2019. The nominal yield in the data changes by 1.0189 basis points per basis-point change in the real yield. How to hedge the position?

Bond	Yield (%)	DV01
TIPS 1 7/8	1.237	0.081
T-Bond 3 5/8	3.275	0.067

$$F^R \times \frac{0.081}{100} = 100\text{mm} \times \frac{0.067}{100} \times 1.0189$$

$$F^R = \$100\text{mm} \times \frac{0.067}{0.081} \times 1.0189 = \$84.3\text{mm}$$

Regression Hedge

- The market maker in question has bought or received fixed in relatively illiquid 20-year swaps from a customer and needs to hedge the resulting interest rate exposure.
- Immediately paying fixed or **selling 20-year swaps** would sacrifice too much if not all of spread paid by the customer, **so the market maker chooses instead to sell a combination of 10- and 30-year swaps.**
- Two variable regression model to **describe the relationship between changes in 20-year swap rates and changes in 10-and 30-year swap rates:**

$$\Delta y_t^{20} = \alpha + \beta^{10}\Delta y_t^{10} + \beta^{30}\Delta y_t^{30} + \varepsilon_t$$

Principal Components Analysis

- Principal component analysis is a powerful statistical tool that can help solve the curse of dimensionality.
- PCs of rates are particularly useful because of an empirical regularity:
  - The sum of the variances of the first three PCs is usually quite close to the sum of variances of all the rates.
  - Hence, rather than describing movements in the term structure by describing the variance of each rate and all pairs of correlation, one can simply describe the structure and volatility of each of only three PCs.

Reading 5

Term Structure Models of Interest Rates

Framework

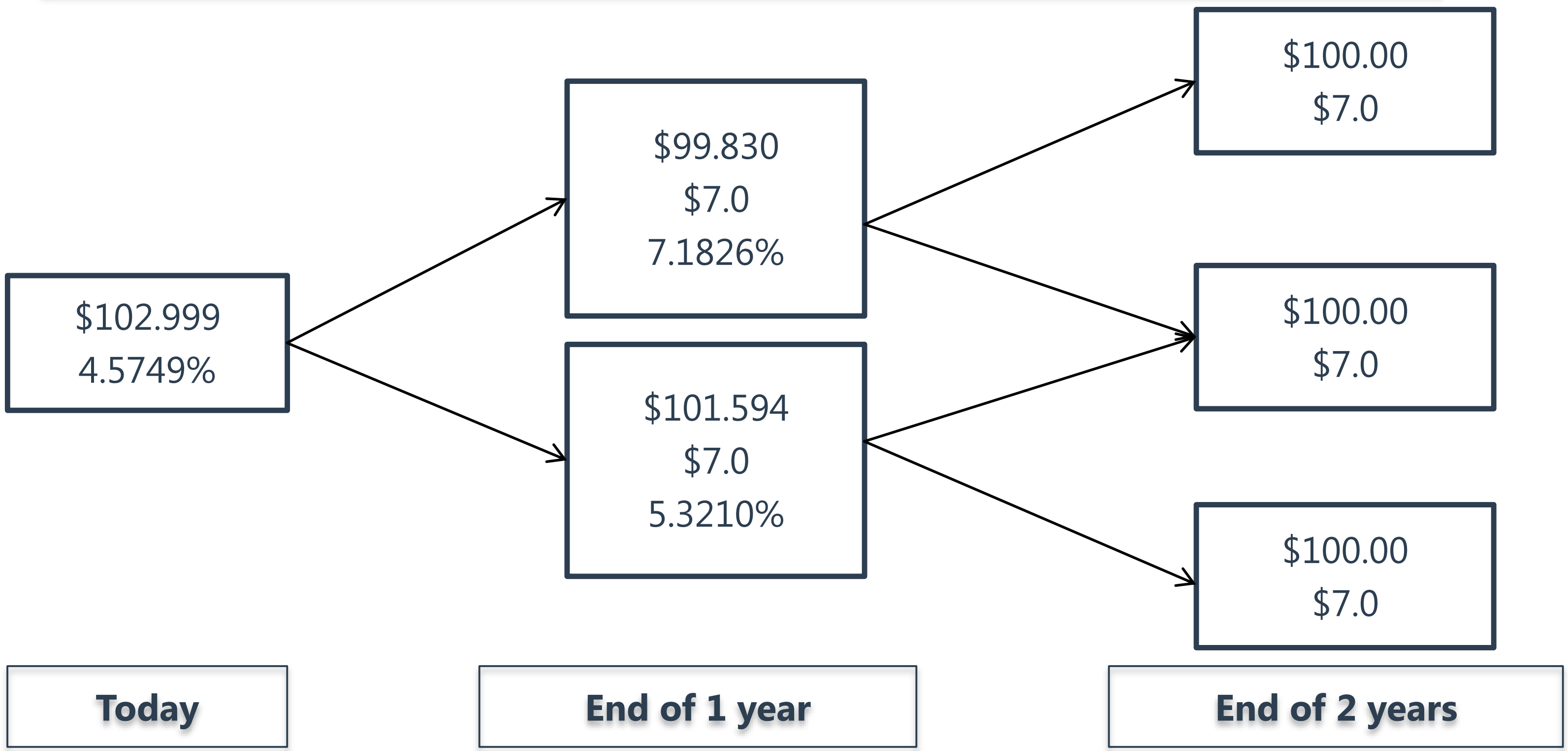
- **Term Structure Models of Interest Rates**
  - The Science of Term Structure Models
  - The Evolution of Short Rates and the Shape of the Term Structure
  - The Art of Term Structure Models: Drift, Volatility and Distribution

The Science of Term Structure Models

Term Structure Models of Interest Rates

Valuing an Option Bond with the Tree

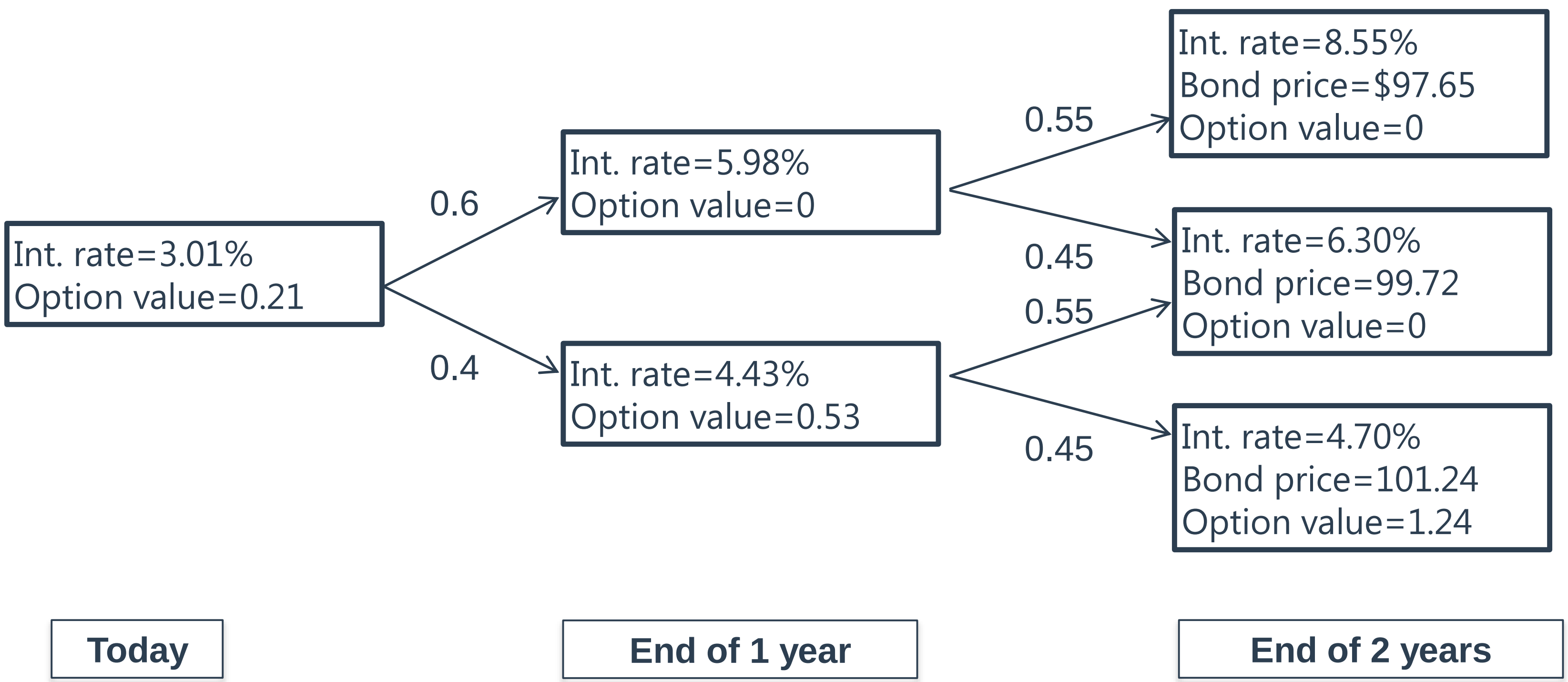
Valuing a 2-year, 7.0% Coupon, Option-free Bond



Valuing an Option on Bond with the Tree

- Value a European call option with two years to expiration and a strike price of $100.00. The underlying is a 6%, annual-coupon bond with three years to maturity. Assume that the risk-neutral probability of an up move is 0.6 in year 1 and 0.55 in year 2.

Binomial Tree for European Call Option on 3-year, 6% Bond



Constant Maturity Treasury Swap

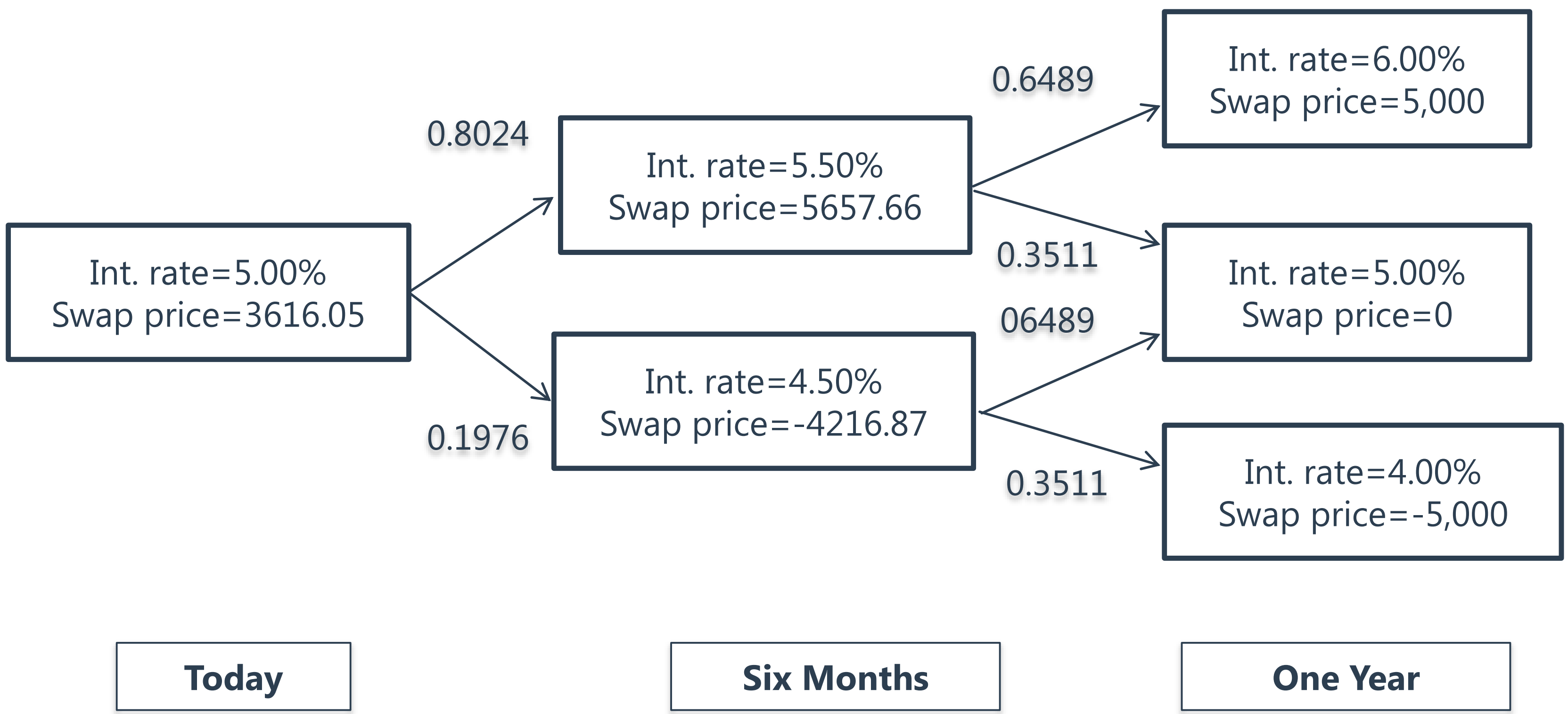
- **A CMT swap is an agreement to swap a floating rate for a Treasury rate such as the 10-year rate.**
- **Example: CMT swap**
  - The rate tree can be used to price a constant-maturity Treasury (CMT) swap. In the example, the strike (fixed rate) is 5.0% such that every six months until maturity the swap pays:

$$\left(\frac{\$1{,}000{,}000}{2}\right) \times \left(y_{CMT} - 5\%\right)$$

  - y_{CMT}: a semiannually compounded yield, of a predetermined maturity, on the payment date. The text prices a one-year CMT swap on the six month yield. In practice, CMT swaps trade most commonly on the yields of the most liquid maturities, i.e., on 2-, 5- and 10-year yields.

Constant Maturity Treasury Swap

Binomial Tree for CMT Swap



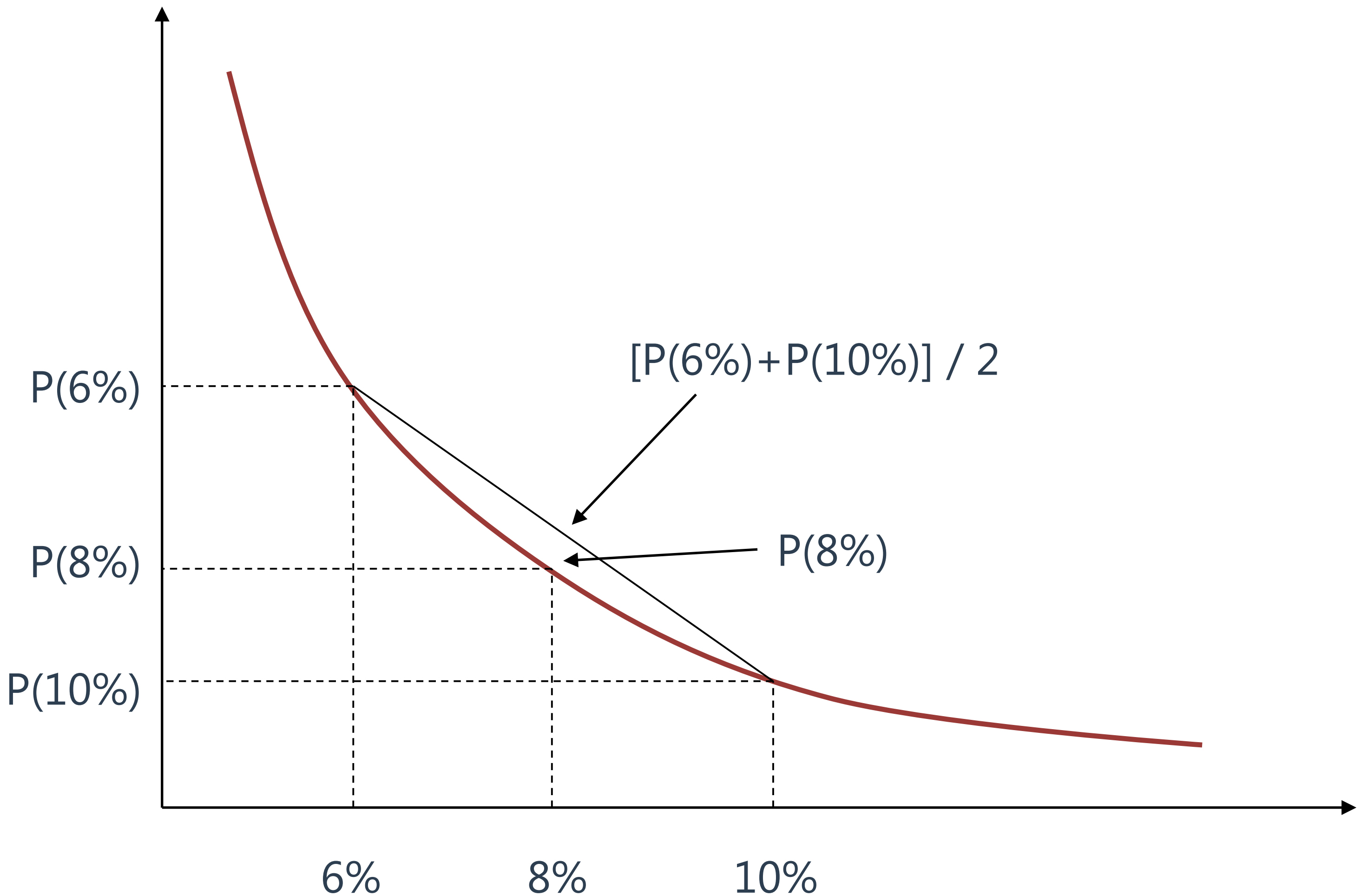
Fixed Income Securities & Black-Scholes-Merton

- Why the Black-Scholes-Merton model to value equity derivatives is not appropriate to value derivatives on fixed-income securities?
  - The price of a bond must converge to its face value at maturity while the random process describing the stock price need not be constrained in any similar way.
  - Because of the maturity constraint, the volatility of a bond's price must eventually get smaller as the bond approaches maturity. The simpler assumption that the volatility of a stock is constant is not so appropriate for bonds.
  - Since stock volatility is very large relative to short-term rate volatility, it may be relatively harmless to assume that the short-term rate is constant. By contrast, it can be difficult to defend the assumption that a bond price follows some random process while the short-term interest rate is constant.

The Evolution of Short Rates and the Shape of the Term Structure

Term Structure Models of Interest Rates

Jensen's Inequality



Jensen's Inequality

Convexity Effect

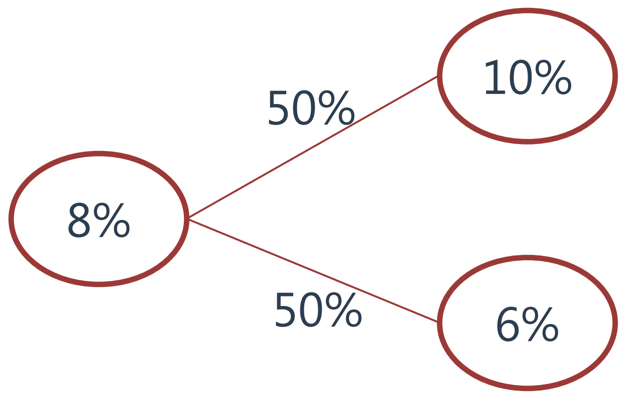
- The convexity effect arises from a special case of Jensen's Inequality:

$$E\left[\frac{1}{(1+r)}\right] > \frac{1}{E(1+r)}$$

- All else held equal, the value of convexity increases with maturity and volatility.

Risk Premium

- Calculate the price and return for the 2-year zero-coupon bond with a 20 basis point risk premium.



$$P = \frac{\left[\frac{\$1}{1.102} + \frac{\$1}{1.062}\right] \times 0.5}{1.08}$$

$$= \frac{[\$0.90744 + \$0.94162] \times 0.5}{1.08}$$

$$= \$0.85605$$

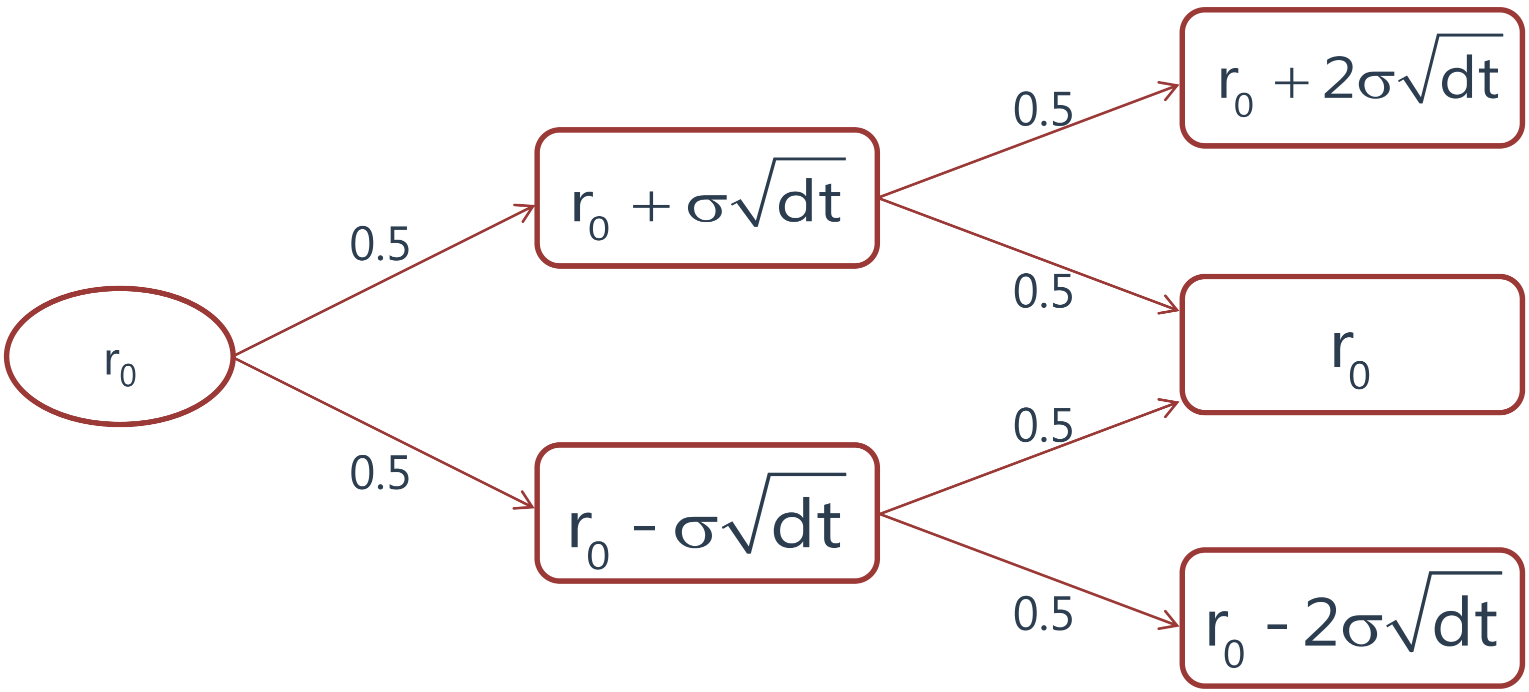
The Art of Term Structure Models: Drift & Volatility and Distribution

Term Structure Models of Interest Rates

Term Structure Model with No Drift (Model 1)

$$dr = \sigma dw \qquad dw = \varepsilon\sqrt{dt}$$

- dw = normally distributed random variable with mean 0 and standard deviation $\sqrt{dt}$.

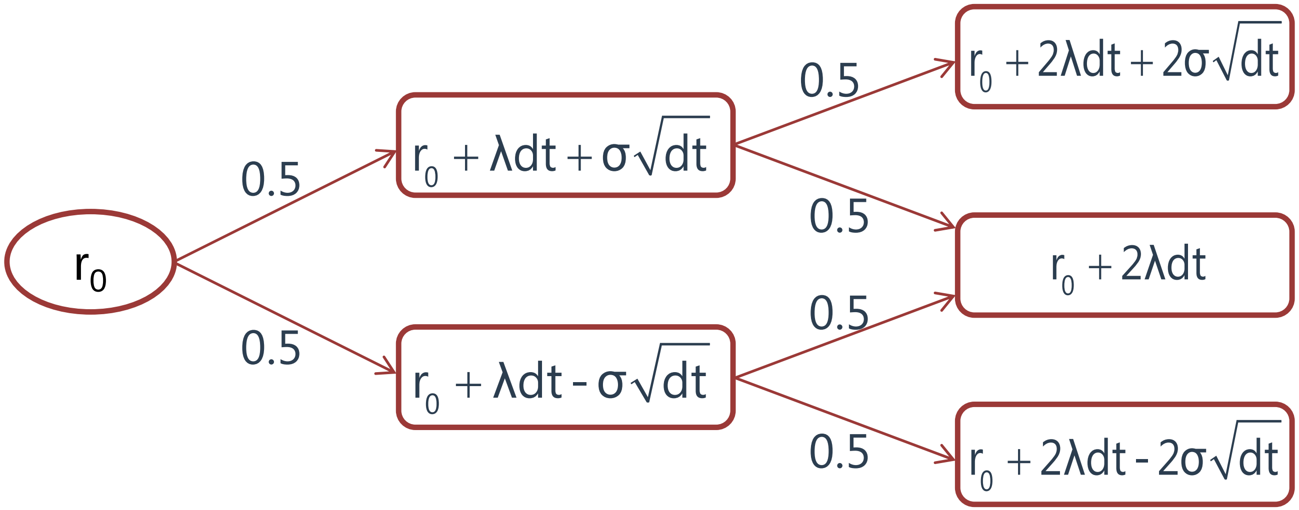


Term Structure Model with No Drift (Model 1)

- **A problem with Gaussian models is that the short-term rate can become negative.**
- **Solutions for negative interest rates**
  - **A non-normal distribution can be assumed**: For example, if we assume interest rates are lognormally distributed, then the short-term rate cannot become negative . However, building a model around a probability distribution that rules out negative rates or makes them less likely may result in volatilities that are unacceptable.
  - **Use shadow rates** (force the "adjusted" tree rates to be non-negative): Another popular method of ruling out negative rates is to construct rate trees with whatever distribution is desired, as done in this section, and then simply set all negative rates to zero. In this methodology, rates in the original tree are called the shadow rates of interest while the rates in the adjusted tree could be called the observed rates of interest.

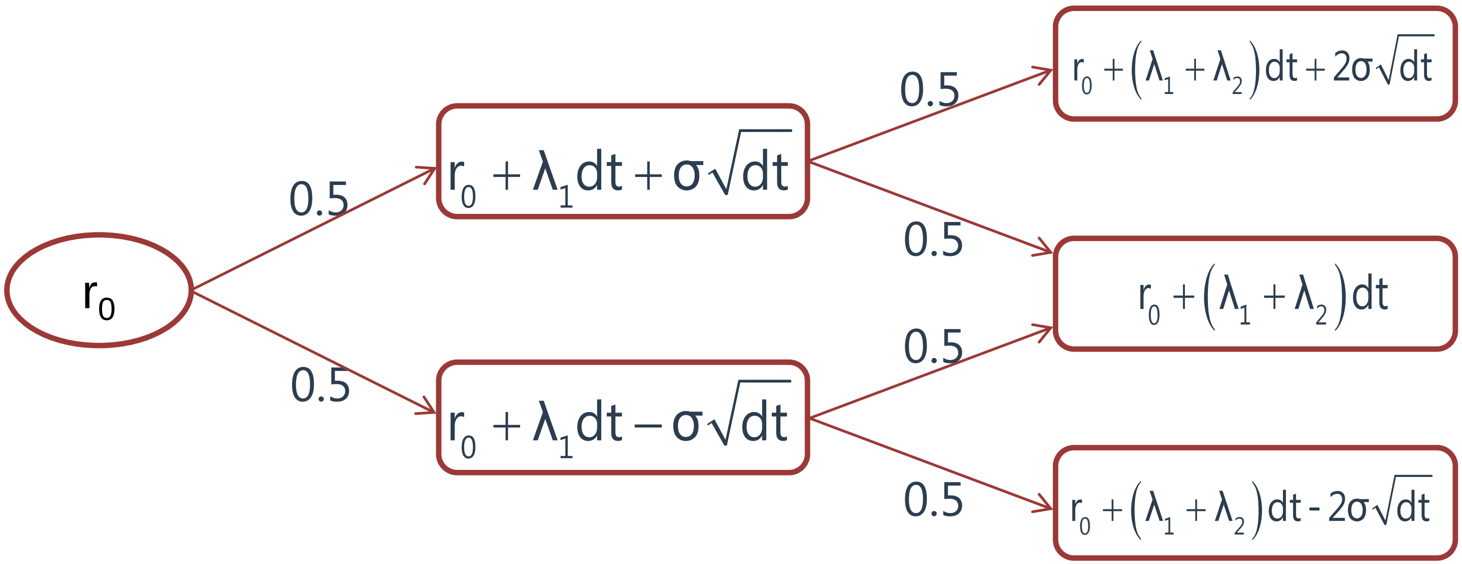
Term Structure Model with Drift (Model 2)

$$dr = \lambda dt + \sigma dw$$



Ho-lee Model with Time-dependent Drift

- It is clear that if $\lambda_1 = \lambda_2$ then Ho-Lee model reduces to Model 2.



Vasicek Model

- The Vasicek Model introduces mean reversion into the rate model, which is a common assumption for the level of interest rates. The Vasicek Model is given by:

$$dr = \kappa(\theta - r)dt + \sigma dw$$

  - k = a parameter that measures the speed of reversion adjustment
  - θ = long-run value of the short-term rate assuming risk neutrality
  - r = current interest rate level

- The greater the difference between r and θ, the greater the expected change in the short-term rate toward θ.
- **Basis point volatility** refers to the volatility of dr, which is positively correlated to the level of current interest rate.

Term Structure Model with Time-dependent Volatility (Model 3)

$$dr = \lambda(t)dt + \sigma(t)dw$$

$$dr = \lambda(t)dt + \sigma e^{-at}dw$$

- σ: volatility at t = 0, which decreases exponentially to 0.

Other Models

- **Cox-Ingersoll-Ross (CIR) Model**

$$dr = \kappa(\theta - r)dt + \sigma\sqrt{r}dw$$

- **Lognormal model (Model 4)**

$$dr = ardt + \sigma rdw$$

- **Lognormal model with deterministic drift**

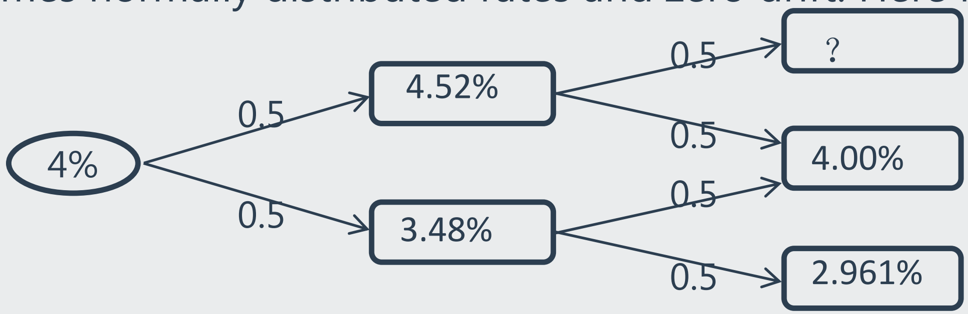
$$d[ln(r)] = a(t)dt + \sigma dw$$

- **Lognormal model with mean reversion**

$$d[ln(r)] = k(t)[ln\Theta(t) - ln(r)]dt + \sigma(t)dw$$

Question

- Analyst Greg is constructing an interest rate tree with monthly time steps; i.e., (t) =1/12. The current short-term rate is 4.0%. His term structure model assumes an annual basis point volatility of 180 basis points. He employs Tuckman's Model 1 which assumes normally distributed rates and zero drift. Here is his rate tree:



What is the undisplayed missing value at node ; i.e., the rate in the tree not the realized process?

A. 4.20%

B. 4.83%

C. 5.04%

D. 6.59%

- **Correct answer :C**

Reading 6

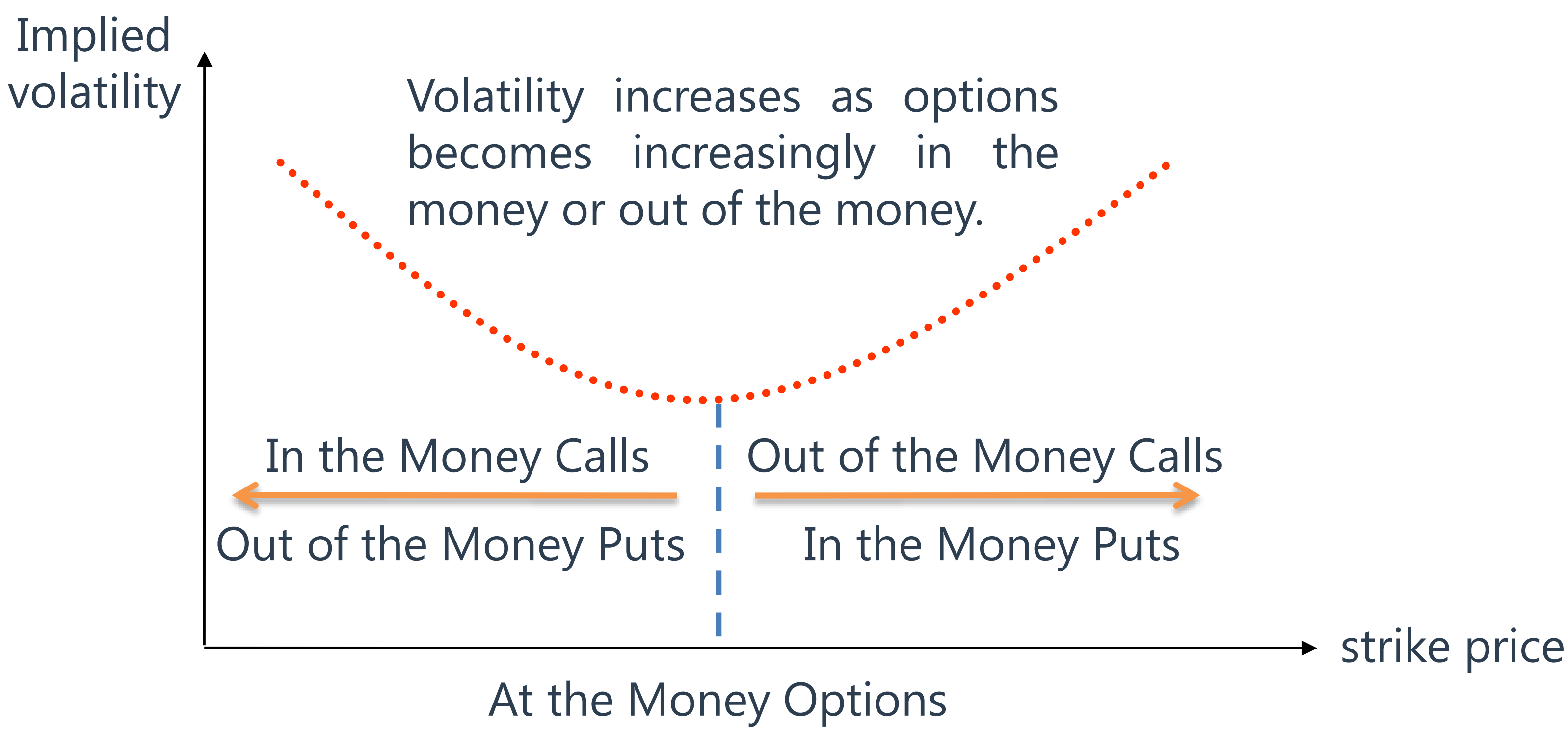
Volatility Smiles

Volatility Smiles is the Same for Calls and Puts

- **Based on the put-call parity**
  - The implied volatility of a **European call option** is always the same as the implied volatility of a **European put option** when the two have the same strike price and maturity date.

Volatility Smile for Foreign Currency Options

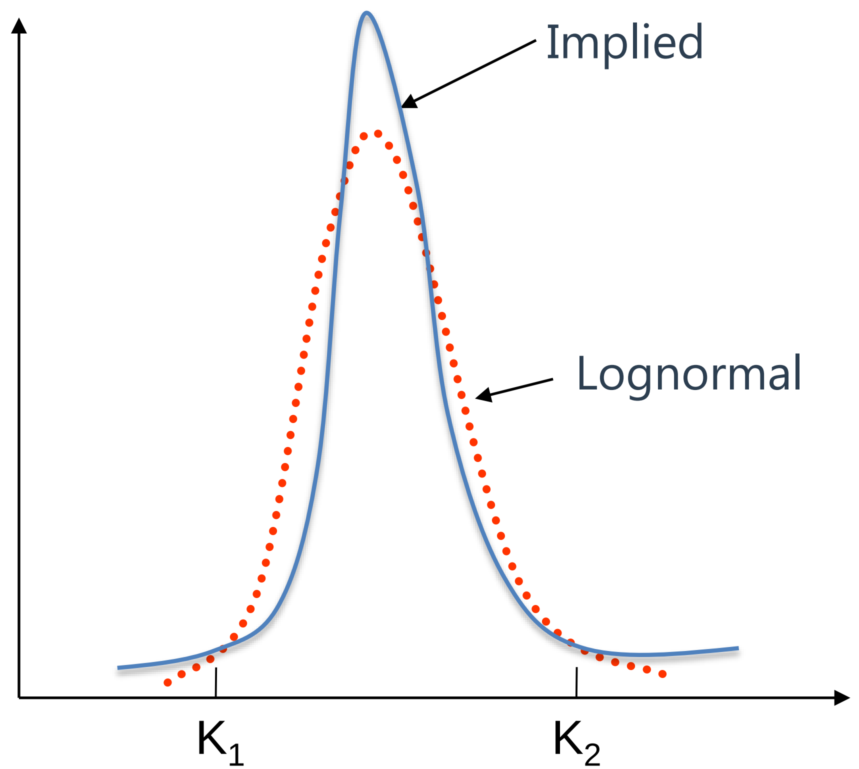
- The implied volatility is relatively low for **at-the-money** options. It becomes progressively higher as an option moves either **into the money or out of the money.**



Reasons for Smile in Foreign Currency Options

- Why are exchange rate **not lognormally distributed?** Two of the contidions for an asset price to have a lognormal distribution are:
  - **The volatility of the asset is constant.**
  - **The price of the asset changes smoothly with no jumps.**

In practice, neither of these conditions is satisfied for an exchange rate. The volatility of an exchange rate is far from constant, and exchange rates frequently exhibit jumps (sometimes the jumps are in response to the actions of **central banks**).

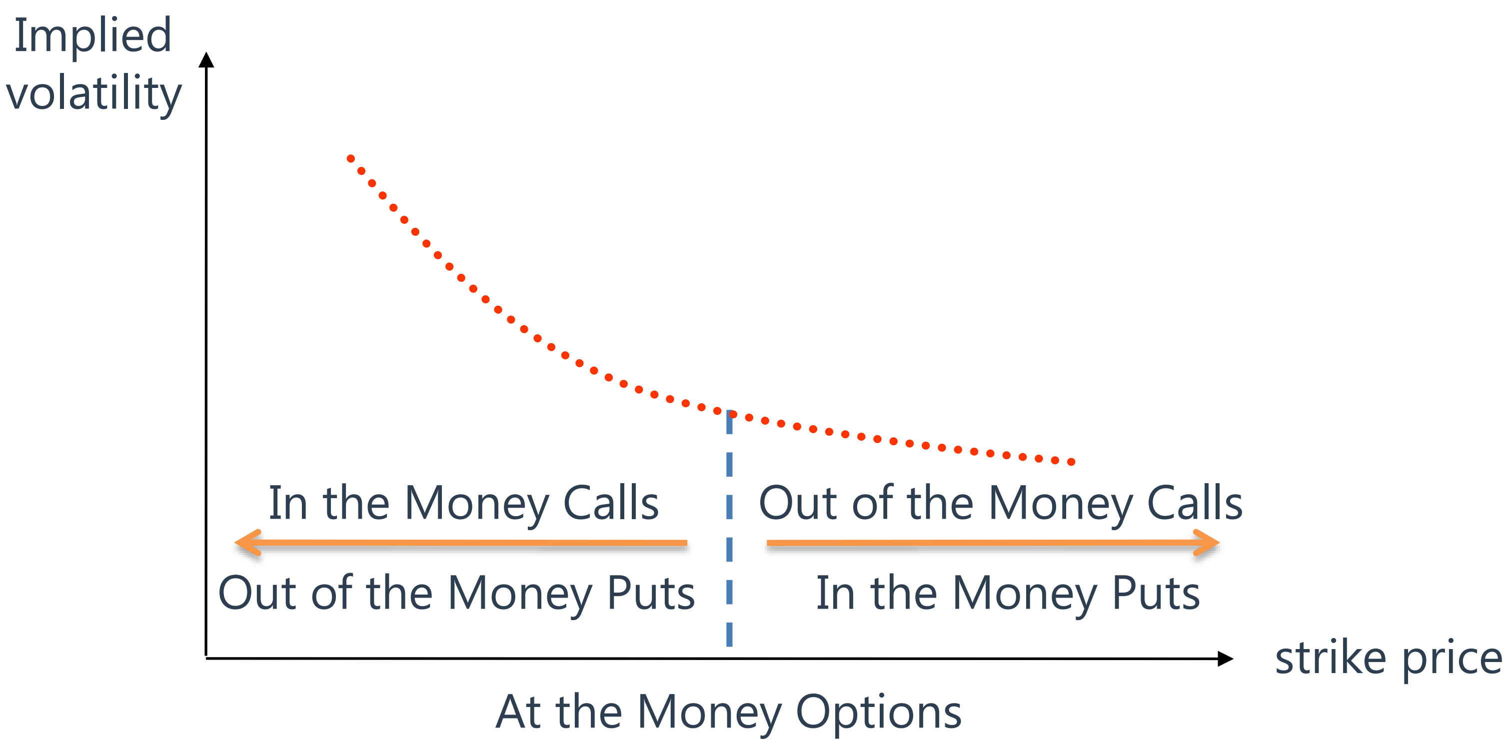


Reasons for Smiles in Foreign Currency Options

- **Alternative ways of characterizing the volatility smile**
  - The volatility smile is often calculated as the relationship between the **implied volatility** and **K/S$_0$** rather than as the relationship between the implied volatility and K.
    - ✓ A refinement of this is to calculate the volatility smile as the relationship between the **implied volatility** and **K/F$_0$**, where F_0 is the forward pirce of the asset for a contract maturing at the same time as the options that are considered.
  - Another approach to defining the volatility smile is as the relationship between **the implied volatility** and the **delta** of the option.

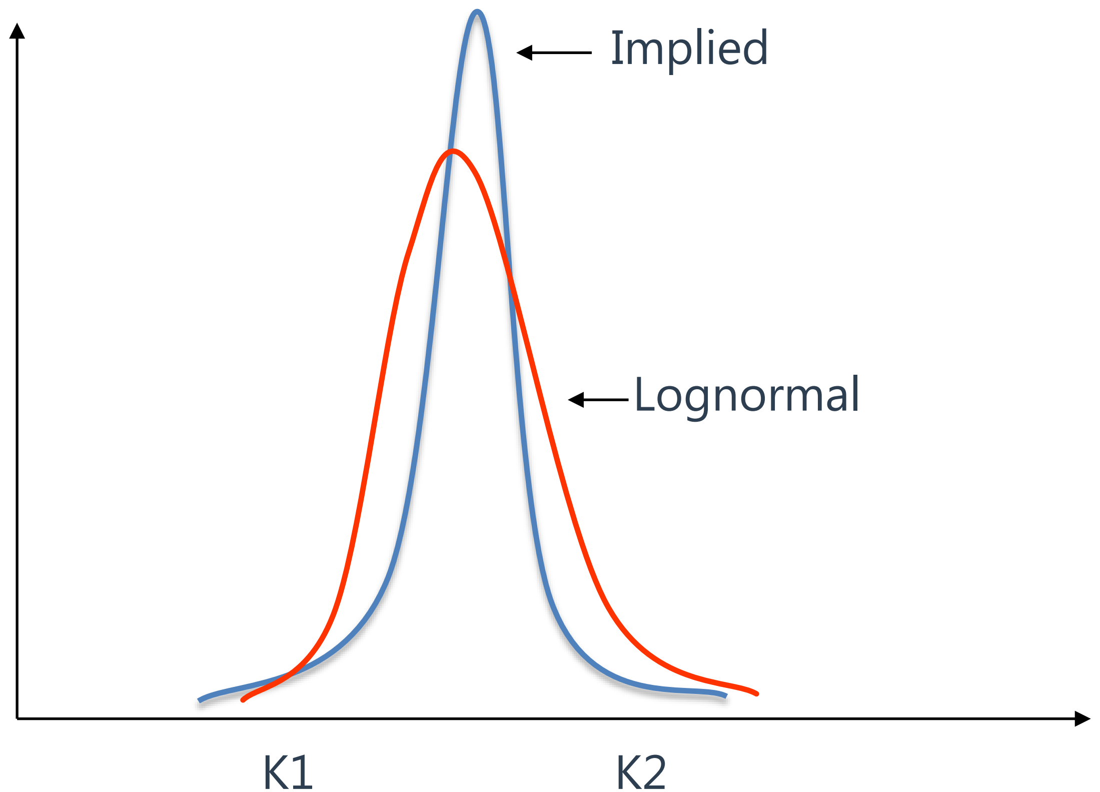
Volatility Smiles (skew) for Equity Options

- The volatility used to price a low-strike-price option (i.e., a deep out of the money put or a deep in the money call) is significantly higher than that used to price a high-strike-price option (i.e., a deep in the money put or a deep out of the money call).



Reasons for the Smile in Equity Options

- **Implied and lognormal distribution for equity options**



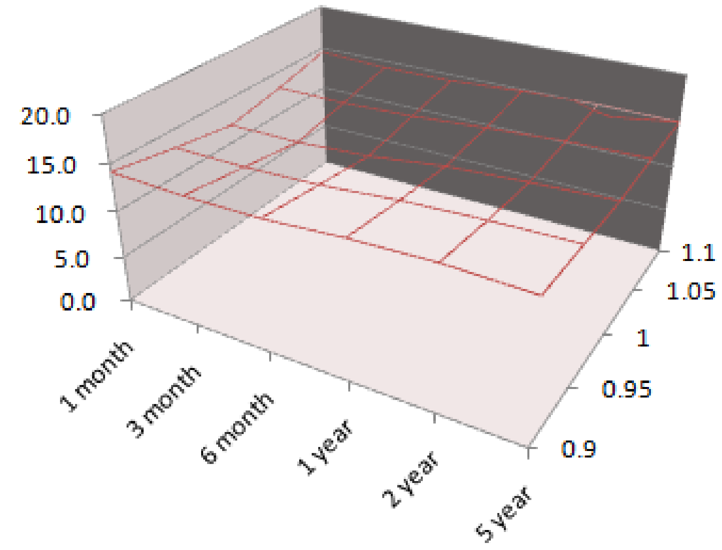
- It can be seen that the implied distribution has a heavier left tail and a less heavy right tail than the lognormal distribution.

Reasons for the Smile in Equity Options

- **Leverage**
  - As a company's equity declines in value, the company's leverage increases. This means that the equity becomes more risky and its volatility increases.
- **Volatility Feedback Effect**
  - As volatility increases (decreases) because of external factors, investors require a higher (lower) return and as a result the stock price declines (increases).
- **Crashophobia**
  - 1987 stock market crash: higher premiums for put prices when the strike prices lower.

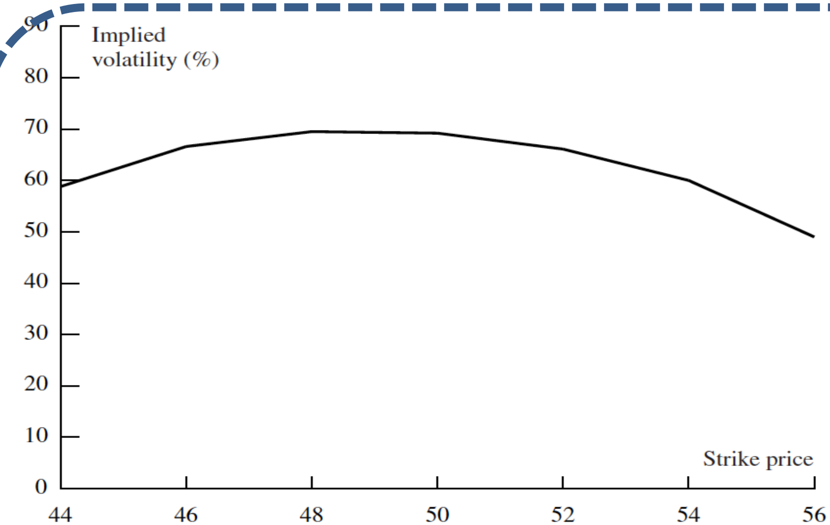
Volatility Term Structure and Volatility Surface

Volatility surface					
	K / S_0				
	0.90	0.95	1.00	1.05	1.10
1 month	14.2	13.0	12.0	13.1	14.5
3 month	14.0	13.0	12.0	13.1	14.2
6 month	14.1	13.3	12.5	13.4	14.3
1 year	14.7	14.0	13.5	14.0	14.8
2 year	15.0	14.4	14.0	14.5	15.1
5 year	14.8	14.6	14.4	14.7	15.0



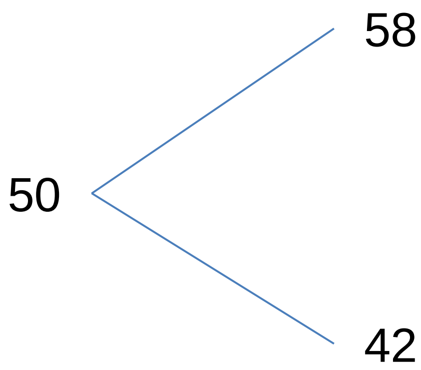
Asset Price Jumps on Volatility Smiles

- Suppose that a stock price is currently $50 and an important news announcement due in a few days is expected either to increase the stock price by $8 or to reduce it by $8.
- The probability distribution of the stock price in, say, 1 month might then consist of a mixture of two lognormal distribution, the first corresponding to favorable news, the second to unfavorable news.

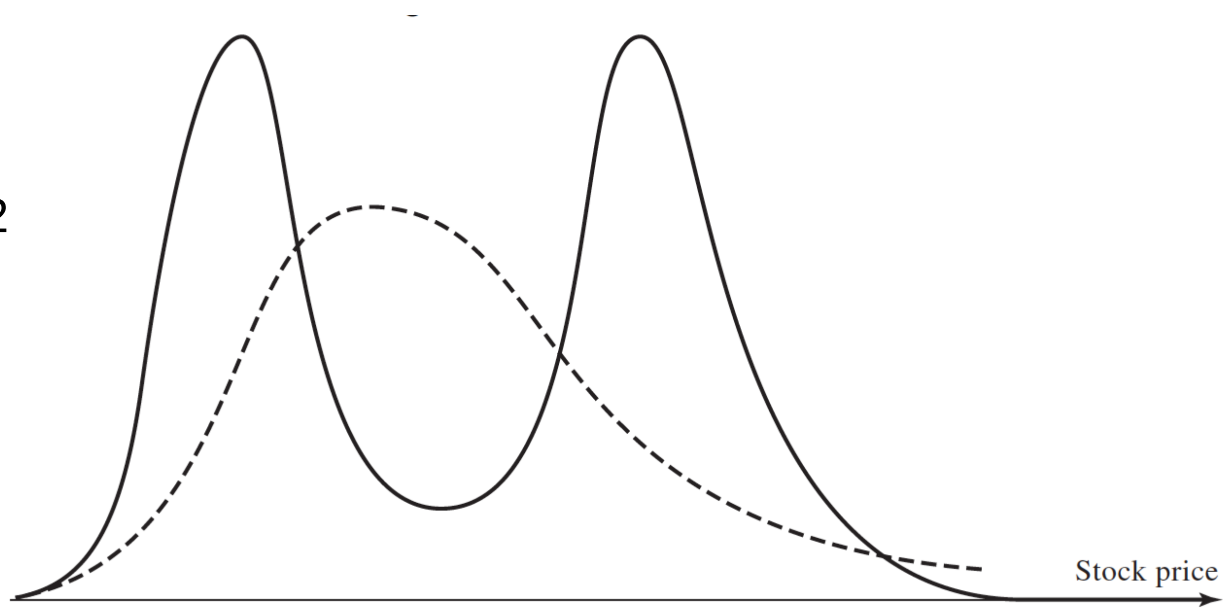


The Impact of Large Asset Price Jumps on Volatility Smiles

Change in stock price in 1 month



Effect of a single large jump. The solid line is the true distribution; the dashed line is the lognormal distribution.



Question

- The Chief Risk Officer of Martingale Investments Group is planning a change in methodology for some of the risk management models used to estimate risk measures. His aim is to move from models that use the normal distribution of returns to models that use the distribution of returns implied by market prices. Martingale Group has a large long position in the German equity stock index DAX which has a volatility smile that slopes downward to the right. How will the change in methodology affect the estimate of expected shortfall (ES)?

  A. ES with the updated models will be larger than the old estimate.

  B. ES with the updated models will be smaller than the old estimate.

  C. ES will remain unchanged.

  D. Insufficient information to determine.

- **Correct answer :A**

It’s not the end but just beginning.

Your life can be enhanced, and your happiness enriched, when you choose to change your perspective. Don't leave your future to chance, or wait for things to get better mysteriously on their own. You must go in the direction of your hopes and aspirations. Begin to build your confidence, and work through problems rather than avoid them. Remember that power is not necessarily control over situations, but the ability to deal with whatever comes your way.

一旦变换看问题的角度，你的生活会豁然开朗，幸福快乐会接踵而来。别交出掌握命运的主动权，也别指望局面会不可思议的好转。你必须与内心希望与热情步调一致。建立自信，敢于与困难短兵相接，而非绕道而行。记住，力量不是驾驭局势的法宝，无坚不摧的能力才是最重要的。

问题反馈

- 如果您认为金程**课程讲义/题库/视频**或其他资料中**存在错误，欢迎您告诉我们**，所有提交的内容我们会在最快时间内核查并给与答复。
- **如何告诉我们？**
  - 将您发现的问题通过电子邮件告知我们，具体的内容包含：
    - ✓ 您的姓名或网校账号
    - ✓ 所在班级（eg.1911FRM一级长线无忧班）
    - ✓ 问题所在科目（若未知科目，请提供章节、知识点）和页码
    - ✓ 您对问题的详细描述和您的见解
  - 请发送电子邮件至：academic.support@gfedu.net
- **非常感谢您对金程教育的支持，您的每一次反馈都是我们成长的动力。**后续我们也将开通其他问题反馈渠道（如微信等）。