2020 FRM Part II

百题巅峰班

市场风险测量与管理

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1. Market Risk Measurement and Management

1.1. Key Point: Estimating VaR

1.1.1. 重要知识点

1.1.1.1. Normal VaR:

$$VaR = -(\mu - z_\alpha \sigma)$$

$$VaR = -(\mu - z_\alpha \sigma)P_{t-1}$$

1.1.1.2. Lognormal VaR:

$$VaR = 1 - e^{\mu - z_\alpha \sigma}$$

$$VaR = (1 - e^{\mu - z_\alpha \sigma})P_{t-1}$$

1.1.1.3. Expected Shortfall/ Conditional VaR (CVaR)/Tail Conditional Expectation/Conditional.

1.1.1.4. Loss/Expected Tail Loss: expected value of the loss when it exceeds VaR

1.1.2. 基础题

Q-1. The VaR at a 95% confidence level is estimated to be 1.56 from historical simulation of 1,000 observations. Which of the following statements is most likely true?

A. The parametric assumption of normal returns is correct

B. The parametric assumption of lognormal returns is correct

C. The historical distribution has fatter tails than a normal distribution.

D. The historical distribution has thinner tails than a normal distribution.

Q-2. A risk manager is comparing the use of parametric and non-parametric approaches for calculating VaR and is concerned about some of the characteristics present in the loss data. Which of the following conditions would make non-parametric approaches the favored method to use?

A. Scarcity of high magnitude loss event

B. Skewness in the distribution

C. Unusually high volatility during the data period

D. Unusually low volatility during the data period

Q-3. A fund manager owns a portfolio of options on a non-dividend paying stock TUV. The portfolio is made up of 5,000 deep in-the-money call options on TUV and 20,000 deep out-of-the-money call options on TUV. The portfolio also contains 10,000 forward contracts on TUV. Currently, TUV is trading at USD 52. Assuming 252 trading days in a year and the volatility of TUV is 12% per year, which of the following amounts would be closest to the 1-day VaR of the portfolio at the 99% confidence level?

A. USD 11,557

B. USD 12,627

C. USD 13,715

D. USD 32,000

Q-4. A portfolio consists of options on Microsoft and AT&T. The options on Microsoft have a delta of 1000, and the options on AT&T have a delta of 20000. The Microsoft share price is $120, and the AT&T share price is $30. Assuming that the daily volatility of Microsoft is 2% and the daily volatility of AT&T is 1% and the correlation between the daily changes is 0.3, the 5-day 95% VaR is

A. 26193

B. 25193

C. 27193

D. 24193

Q-5. The annual mean and volatility of a portfolio are 10% and 40%, respectively. The current value of the portfolio is GBP 1,000,000. How does the 1-year 95% VaR that is calculated using a normal distribution assumption (normal VaR) compare with the 1-year 95% VaR that is calculated using the lognormal distribution assumption (lognormal VaR)?

A. Lognormal VaR is greater than normal VaR by GBP 130,400

B. Lognormal VaR is greater than normal VaR by GBP 17,590

C. Lognormal VaR is less than normal VaR by GBP 130,400

D. Lognormal VaR is less than normal VaR by GBP 17,590

Q-6. What is a key weakness of the value at risk (VaR) measure? VaR:

A. Does not consider the severity of losses in the tail of the returns distribution.

B. Is quite difficult to compute.

C. Is subadditive.

D. has an insufficient amount of backtesting data.

Q-7. VaR is quite difficult to compute,it is not true. VaR is not subadditive.A wealth management firm has JPY 72 billion in assets. The portfolio manager computes the daily VaR at various confidence levels as follows:

Confidence Level	VaR (USD)
95%	332,760,000
95.5%	336,292,500
96.0%	340,095,000

96.5%	350,332,500
97.0%	359,107,500
97.5%	367,882,500
98.0%	378,412,500
98.5%	392,452,500
99.0%	410,880,000
99.5%	439,252,500

What is the closest estimate of the daily ES at the 97.5% confidence level?

A. JPY 398 million

B. JPY 400 million

C. JPY 405 million

D. JPY 497 million

Q-8. A large commercial bank is using VaR as its main risk measurement tool. Expected shortfall (ES) is suggested as a better alternative to use during market turmoil. What should be understood regarding VaR and ES before modifying current practices?

A. Despite being more complicated to calculate, ES is easier to backtest than VaR.

B. Relative to VaR, ES leads to more required economic capital for the same confidence level.

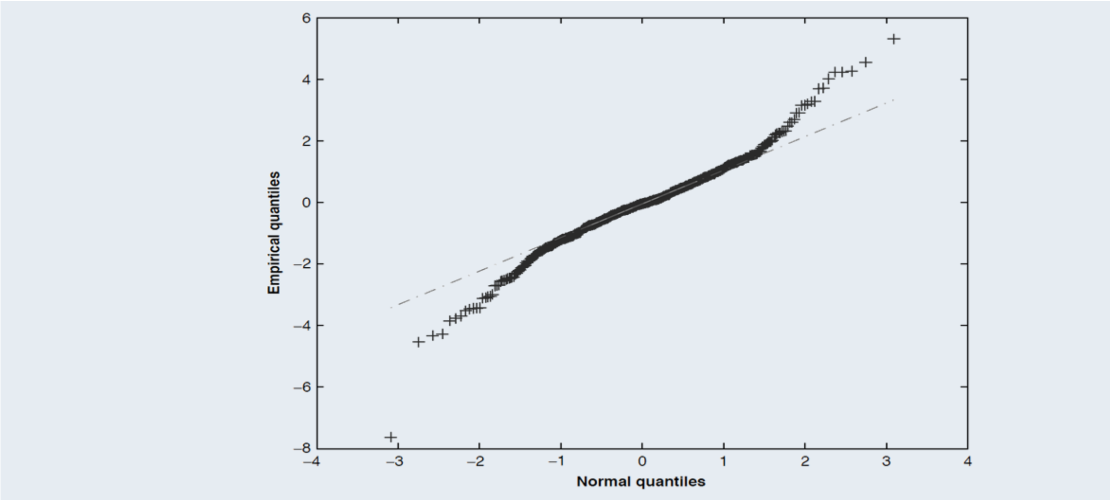
C. While VaR ensures that the estimate of portfolio risk is less than or equal to the sum of the risks of that portfolio's positions, ES does not.

D. Both VaR and ES account for the severity of losses beyond the confidence threshold.

1.2. Quantile-Quantile Plots

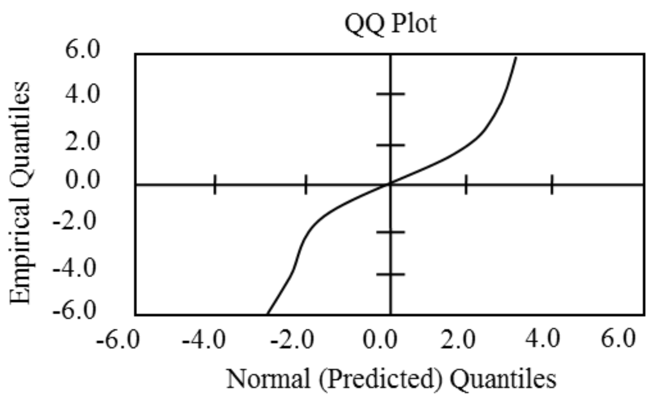
1.2.1. 重要知识点

1.2.1.1. The quantile-quantile (QQ) plot is a visual inspection of an empirical quantile relative to a hypothesized theoretical distribution. If the empirical distribution closely matches the theoretical distribution, the QQ plot would be linear.



1.2.2. 基础题

Q-9. Consider the following QQ plot:



Which is the most likely true statement about the QQ plot?

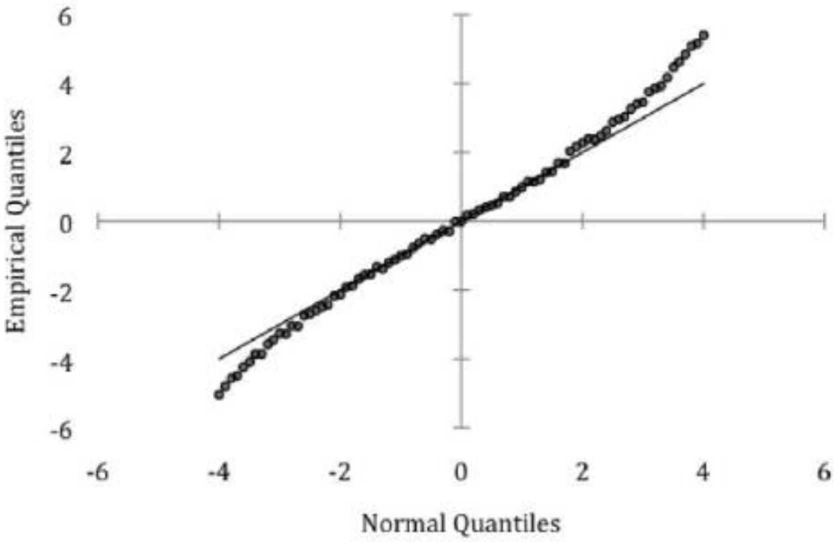
A. The empirical distribution is actually parametric.

B. The empirical distribution has positive skew.

C. The empirical distribution has leptokurtosis (Excess kurtosis>0)

D. If we perform a linear transformation of location and scale, the distribution is approximately normal.

Q-10. An analyst is examining a sample of return data. As a first step, the analyst construct a QQ plot of the data as shown below:



Based on an examination of the QQ plot, which of the following statements is correct?

A. The returns are normally distributed.

B. The return distribution has thin tails relative to the normal distribution.

C. The return distribution is negatively skewed relative to the normal distribution.

D. The return distribution has fat tails relative to the normal distribution.

1.3. Coherent Risk Measures

1.3.1. 重要知识点

1.3.1.1. Monotonicity: if $X_1 \leqslant X_2$, $\rho(X_1) \geqslant \rho(X_2)$.

- In other words, if a portfolio has systematically lower values than another (in each state of the world), it must have greater risk. Standard deviation violates the monotonicity condition.

1.3.1.2. Translation Invariance: $\rho(X+k) = \rho(X) - k$.

- In other words, adding cash k to a portfolio should reduce its risk by k. This reduces the lowest portfolio value. As with X, k is measured in dollars.

1.3.1.3. Homogeneity: $\rho(bX) = b\rho(X)$.

- In other words, increasing the size of a portfolio by a factor b should scale its risk measure by the same factor b. This property applies to the standard deviation.

1.3.1.4. Subadditivity: $\rho(X_1 + X_2) \leqslant \rho(X_1) + \rho(X_2)$.

- In other words, the risk of a portfolio must be less than the sum of separate risks. Merging portfolios cannot increase risk. VaR violates the subadditivity condition.

1.3.2. 基础题

Q-11. It is not always apparent how risk should be quantified for a given bank when there are many different possible risk measures to consider. Prior to defining specific measures, one should be aware of the general characteristics of ideal risk measures. Such measures should be intuitive, stable, easy to understand, coherent, and interpretable in economic terms. In addition, the risk decomposition process must be simple and meaningful for a given risk measure. Standard deviation, value at risk (VaR), expected shortfall (ES), and spectral and distorted risk measures are commonly used measures to calculate economic capital. However, it is not easy to select a risk measure to calculate economic capital, as each measure has its respective pros and cons. Which of the following statements pertaining to the pros and cons of these risk measures is not accurate?

A. Standard deviation does not have the property of monotonicity, and therefore, it is not coherent.

B. VaR does not have the property of subadditivity, and therefore; it is not coherent.

C. ES is not stable regardless of the loss distribution.

D. Spectral and distorted risk measures are neither intuitive nor commonly used in practice.

Q-12. Victor Stanislavsky is a recently promoted senior manager at a bank. As one of his first tasks in his new position, he is responsible for reviewing the choices for the allocation of capital within the bank and the measurement of absolute risk, respectively. Which of the following combinations of value at risk(VaR) and/or expected shortfall(ES) should Stanislavsky determine to currently be the most widely used by banks.

	Capital allocation	Absolute risk
A.	VaR	ES
B.	VaR	VaR
C.	ES	ES
D.	ES	VaR

Q-13. Consider a trader with an investment in a corporate bond with face value of \$100,000 and default probability of 0.5%. Over the next period, we can either have no default, with a return of zero, or default with a loss of \$100,000. The payoffs are thus −\$100,000 with probability of 0.5% and +\$0 with probability of 99.5%. Since the probability of getting \$0 is greater than 99%, the VaR at the 99% confidence level is \$0, without taking the mean into account. This is consistent with the definition that VaR is the smallest loss, such that the right-tail probability is at least 99%. Now, consider a portfolio invested in three bonds (A, B, C) with the same characteristics and independent payoffs. Please compute the portfolio VaR at the 99% confidence level (using loss distribution method):

A. \$0
B. \$100,000
C. \$200,000
D. \$300,000

Q-14. Which of the following statements comparing VaR with expected shortfall is true?

A. Expected shortfall is sub-additive while VaR is not.
B. Both VaR and expected shortfall measure the amount of capital an investor can expect to lose over a given time period and are, therefore, interchangeable as risk measures.
C. Both VaR and expected shortfall depend on the assumption of a normal distribution of returns.
D. VaR can vary according to the confidence level selected, but expected shortfall will not.

Q-15. Assume that an operational process has a 5% probability of creating a material loss and, otherwise, no material loss is experienced (i.e., Bernoulli). If the material loss occurs, the severity is normally distributed with a mean of \$4 million and standard deviation of \$2 million. What is the 95% expected shortfall?

A. \$0.71 million
B. \$3.29 million
C. \$4.00 million
D. \$7.29 million

Q-16. Value at risk (VaR) determines the maximum value we can lose for a given confidence level. For this reason, Kenneth Fulton is concerned that the VaR is not providing the magnitude of the actual loss. He has prepared the following table based on the assumption that returns are normally distributed and a corresponding n = 5. What is the expected shortfall using the information in the following table?

Confidence level	VaR	Difference
95%	1.6392	
96%	1.7507	0.1115
97%	1.8808	0.1301
98%	2.0537	0.1729
99%	2.3263	0.2726

A. 0.687

B. 1.930

C. 2.003

D. 2.054

Q-17. Assume position (X) contains risk of R(X) and position (Y) contains risk of R(Y). Our analysis finds that the risk of the combined portfolio R(X+Y) is greater than the sum of the individual positions risks; i.e., we find R(X+Y) > R(X) + R(Y). This illustrates a violation of which coherence property?

A. Monotonicity

B. Subadditivity

C. Positive Homogeneity

D. Translational invariance

Q-18. Which of the following is a true statement about expected shortfall (ES)?

A. ES is a coherent spectral measure which gives equal weight to the tail quantiles

B. ES is a coherent spectral measure which gives increasingly greater weight to higher tail quantiles

C. ES is a coherent spectral measure but gives decreasingly less weight to higher tail quantiles

D. ES is coherent, VaR is not coherent, and neither are spectral measures

1.4. Non-Parametric Approach and Hybrid Approach

1.4.1. 重要知识点

1.4.1.1. Bootstrap historical simulation approach: involves repeated sampling with replacement, the 5% VaR is recorded from each sample draw. The average of the VaRs from all the draws is the VaR estimate. Note: empirical analysis demonstrates that the bootstrapping technique consistently provides more precise estimates of coherent risk measures than historical simulation on raw data alone.

1.4.1.2. Weighted historical simulation approach:

- Age-weighted historical simulation:

$$\omega_{(i)} = \frac{\lambda^{i-1}(1-\lambda)}{1-\lambda^n}$$

- Volatility-weighted historical simulation:

$$\frac{r^*_{t,i}}{r_{t,i}} = \frac{\sigma_{T,i}}{\sigma_{t,i}}$$

- Correlation-weighted historical simulation: Intuitively, the historical correlation matrix needs to be adjusted to the new information environment. This is accomplished, loosely speaking, by "multiplying" the historic returns by the revised correlation matrix to yield updated correlation-adjusted returns.

- Filtered historical simulation: Combines the traditional historical simulation model with GARCH model.

1.4.2. 基础题

Q-19. Johanna Roberto has collected a data set of 1,000 daily observations on equity returns. She is concerned about the appropriateness of using parametric techniques as the data appears skewed. Ultimately, she decides to use historical simulation and bootstrapping to estimate the 5% VaR. which of the following steps is most likely to be part of the estimation procedure?

A. Filter the data to remove the obvious outliers.

B. Repeated sampling with replacement.

C. Identify the tail region from reordering the original data.

D. Apply a weighting procedure to reduce the impact of older data.

Q-20. Suppose you are using the volatility-weighted historical simulation approach to estimate value at risk(VaR) and expected shortfall(ES) for asset Y. The actual return for the asset 30 days ago was 1.5% with a daily volatility estimate of 1.0%. What is the volatility-adjusted return if the current daily volatility is 1.4%?

A. 0.9%

B. 1.6%

C. 1.8%

D. 2.1%

Q-21. Jack has collected a large data set of daily market returns for three emerging markets and he want to compute the VaR. He is concerned about the non-normal skew in the data and is considering non-parametric estimation methods. Which of the following statements about Age-weighted historical simulation approach is most accurate?

A. The age-weighted procedure incorporate estimates from GARCH model.

B. If the decay factor in the model is close to 1, there is persistence within the data set.

C. When using this approach, the weight assigned on day i is equal to:

$$\boldsymbol{\omega}_{(\mathbf{i})} = \frac{\boldsymbol{\lambda}^{\mathbf{i}-\mathbf{1}}(\mathbf{1}-\boldsymbol{\lambda})}{\mathbf{1}-\boldsymbol{\lambda}^{\mathbf{i}}}$$

D. The number of observation should at least exceed 250.

Q-22. If volatility (0) is the current (today's) volatility estimate and volatility (t) is the volatility estimate on a previous day (t), which best describes volatility-weighted historical simulation?

A. First conduct typical historical simulation (HS) on return series. Then multiply VaR by volatility(0)/volatility(t)

B. First conduct typical historical simulation (HS) on return series. Then multiply VaR by volatility(t)/volatility(0)

C. Each historical return (t) is replaced by: return (t)×volatility (0)/volatility (t). Then conduct typical historical simulation (HS) on adjusted return series.

D. Each historical return (t) is replaced by: return (t)×volatility (t)/volatility (0). Then conduct typical historical simulation (HS) on adjusted return series.

Q-23. All of the following approaches improve the traditional historical simulation approach for estimating VaR except the:

A. Volatility-weighted historical simulation.

B. Age-weighted historical simulation.

C. Market-weighted historical simulation.

D. Correlation- weighted historical simulation.

Q-24. Which of the following statements about volatility-weighting is true?

A. Historic returns are adjusted, and the VaR calculation is more complicated.

B. Historic returns are adjusted, and the VaR calculation procedure is the same.

C. Current period returns are adjusted, and VaR calculation is more complicated.

D. Current period returns are adjusted, and VaR calculation is the same.

Q-25. All of the following items are generally considered advantages of non-parametric estimation methods except:

A. Ability to accommodate skewed data.

B. Availability of data.

C. Use of historical data.

D. Little or no reliance on covariance matrices.

1.5. Key Point: Parametric Approaches (II): Extreme Value

1.5.1. 重要知识点

1.5.1.1. Generalized Extreme-Value Theory (Block Maxima)

- Consider a sample of size n drawn from F(x), and let the maximum of this sample be Mn. If n is large, we can regard Mn as an extreme value.

- Under relatively general conditions, as n gets large the distribution of extremes (i.e., Mn) converges to generalized extreme-value (GEV) distribution.
- This distribution has three parameters. μ, the location parameter of the limiting distribution, which is a measure of the central tendency of Mn, σ, the scale parameter of the limiting distribution, which is a measure of the dispersion of Mn. ξ, the tail index, gives an indication of the shape (or heaviness) of the tail of the limiting distribution. If $\xi > 0$, the GEV becomes the Frechet distribution. This case is particularly useful for financial returns because they are typically heavy-tailed.

1.5.1.2. Peaks-Over-Threshold (POT) Approach

- If x is a random i.i.d. loss with distribution function F(x), and u is a threshold value of x, we can define the distribution of excess losses over threshold u.
- The distribution of x itself can be any of the commonly used distributions and will usually be unknown to us. However, as u gets large, the distribution Fu(x) converges to a generalized Pareto distribution.
- Two parameters: a positive scale parameter, β, and a shape or tail index parameter, ξ. This latter parameter is the same as the tail index encountered already with GEV theory.

1.5.1.3. Differences

- Both are different manifestations of the same underlying EV theory.
- POT model exceedances over a high threshold while GEV theory model the maxima of a large sample.
- POT require fewer parameters.
- The block maxima approach can involve some loss of useful data, because some blocks might have more than one extreme in them.
- POT requires us to grapple with the problem of choosing the threshold

1.5.1.4. ES and VaR Estimation with POT

$$\mathrm{VaR} = u + \frac{\beta}{\xi}\left\{\left[\frac{n}{N_u}(1-\alpha)\right]^{-\xi} - 1\right\}$$

$$ES = \frac{VaR}{1-\xi} + \frac{\beta - \xi u}{1-\xi}$$

- where u is the threshold, Nu is the number of observations in excess of the threshold value, α is the confidence level.

1.5.2. 基础题

Q-26. The generalized extreme value (GEV) generally requires:

A. Fewer estimated parameters than the POT approach and does not share any parameters with the POT approach.

B. Fewer estimated parameters than the POT approach and shares one parameter with the POT.

C. More estimated parameters than the POT approach and shares one parameter with the POT.

D. More estimated parameters than the POT approach and does not share any parameters with the POT approach.

Q-27. In setting the threshold in the POT approach, which of the following statements is the most accurate? Setting the threshold relatively high makes the model:

A. Less applicable but increases the number of observations in the modeling procedure.

B. Less applicable and decreases the number of observations in the modeling procedure.

C. More applicable but decreases the number of observations in the modeling procedure.

D. More applicable but increases the number of observations in the modeling procedure.

Q-28. An investment bank with an active position in commodity futures is using the peaks-over-threshold (POT) methodology for estimating VaR and ES at the 99% confidence level. The bank's risk managers have set a threshold level to evaluate excess losses. The choice of the threshold, they argue, is suitable and consistent with the finding that 5.00% of the observations are in excess of the threshold value. The risk managers have concluded that the position's VaR using the POT measure is 4.45%. The VaR estimate is computed from the following parameters and the managers' empirical analysis is based upon the generalized Pareto distribution assumption for the excess losses.

Parameter	Symbol	Value
Loss threshold	u	3
Number of observations	N	740
Number of observations that exceed threshold	n	37
Scale	β	0.75

Shape (tail index)	ξ	0.22

Given the VaR value and the parameter assumptions, which of the following is correct?

A. Keeping all other parameters constant, increasing the value of the tail index lowers both the ES and the VaR.

B. Keeping all other parameters constant, increasing the loss threshold level increases both the ES and the VaR.

C. The value of ES is 4.57%

D. The value of ES is 5.71%

Q-29. A CRO is concerned that existing internal risk models of a firm, which are governed mainly by the central limit theorem, are not adequate in addressing potential random extreme losses of the firm. The CRO then recommends the use of extreme value theory (EVT). When applying EVT and examining distributions of losses exceeding a threshold value, which of the following is correct?

A. As the threshold value is increased, the distribution of losses over a fixed threshold value converges to a generalized Pareto distribution.

B. If the tail parameter value of the generalized extreme-value (GEV) distribution goes to infinity, then the GEV essentially becomes a normal distribution.

C. To apply EVT, the underlying loss distribution must be either normal or lognormal.

D. The number of exceedances decreases as the threshold value decreases, which causes the reliability of the parameter estimates to increase.

1.6. Backtesting VaR

1.6.1. 重要知识点

1.6.1.1. Discrete Distribution;Using Failure Rates in Model Verification N/T

- H_0: accurate model H_a: inaccurate model
- Test statistic:

  $$\mathbf{LR_{uc}} = -\mathbf{2ln}[(\mathbf{1}-\mathbf{p})^{\mathbf{T-N}}\mathbf{p^N}] + \mathbf{2ln}\{[\mathbf{1}-(\mathbf{N/T})]^{\mathbf{T-N}}(\mathbf{N/T})^{\mathbf{N}}\}$$

  P: the probability of exception, p=1-c

  N: the number of exceptions

  T: the number of samples
- If LR > 3.84, we would reject the hypothesis that the model is correct.

Basel Penalty Zones		
Zone	**Number of Exceptions**	**Multiplier(k)**
Green	0 to 4	3.00
Yellow	5	3.40

	6	3.50
	7	3.65
	8	3.75
	9	3.85
Red	10 or more	4.00

1.6.2. 基础题

Q-30. Which of the following statements regarding verification of a VaR model by examining its failure rates is false?

I. The frequency of exceptions should correspond to the confidence level used for the model.

II. According to Kupiec (1995), we should reject the hypothesis that the model is correct if the LR > 3.84.

III. Backtesting VaR models with lower confidence levels is difficult because the number of exceptions is not high enough to provide meaningful information.

IV. The range for the number of exceptions must strike a balance between the chances of rejecting an accurate model (a type 1 error) and the chance of accepting an inaccurate model (a type2 error)

A. I and IV

B. II only

C. III only

D. II and IV

Q-31. A risk manager is concerned that his value at risk (VaR) model is complicated by intraday changes as well as profit and loss factors. Which of the following backtesting techniques will most likely mitigate these issues? Backtest the VaR model using:

A. shorter time periods, such as one year as opposed to five years.

B. daily holding period returns.

C. a higher confidence level, such as 99%.

D. a lower confidence level, such as 95%.

Q-32. Basel II requires a backtest at a 99% confidence level of a bank's internal value at risk (VaR) model (IMA). Assume the bank's ten-day 99% VaR is $1 million (minimum of 99% is hard-wired per Basel). The null hypothesis is: the VaR model is accurate. Out of 1,000 observations, 25 exceptions are observed (we saw the actual loss exceed the VaR 25 out of 1000 observations).

A. We will probably call the VaR model good (accurate) but we risk a Type I error.

B. We will probably call the VaR model good (accurate) but we risk a Type II error.

C. We will probably call the model bad (inaccurate) but we risk a Type I error.

D. We will probably call the model bad (inaccurate) but we risk a Type II error.

Q-33. A bank conducted a backtest of its 95% daily value at risk (VaR) and observed 19 exceptions - i.e., the number of days where the daily P&L loss exceeded the VaR - over the last year which included 250 trading days (T = 250). If we use the normal distribution to approximate the binomial for purposes of model verification, what is our accept/reject opinion of the model under a 90% two-tailed test?

A. Accept with a test statistic of 1.25

B. Accept with a test statistic of 1.89

C. Reject with a test statistic of 1.25

D. Reject with a test statistic of 1.89

Q-34. A newly hired risk analyst is backtesting a firm's VaR model. Previously, the firm calculated a 1-day VaR at the 95% confidence level. Following the Basel framework, the risk analyst is recommending that the firm switch to a 99% VaR confidence level. Which of the following statements concerning this switch is correct?

A. The decision to accept or reject a VaR model based on backtesting results at the two-tailed 95% confidence level is less reliable with a 99% VaR model than with a 95% VaR model.

B. The 95% VaR model is less likely to be rejected using backtesting than the 99% VaR model.

C. When backtesting using a two-tailed 90% confidence level test, there is a smaller probability of incorrectly rejecting a 95% VaR model than a 99% VaR model.

D. Using a 99% VaR model will lower the probability of committing both type 1 and type 2 errors.

Q-35. Based on Basel II rules for backtesting, a penalty is given to banks that have more than four exceptions to their 1-day 99%VaR over the course of 250 trading days. The supervisor gives these penalties based on four criteria. Which of the following causes of exceptions is most likely to lead to a penalty?

A. The bank increases its intraday trading activity.

B. A large move in interest rates was combined with a small move in correlations.

C. The bank's model calculates interest rate risk based on the median duration of the bonds in the portfolio.

D. A sudden market crisis in an emerging market leads to losses in the equity positions in that country.

Q-36. A risk manager is analyzing a 1-day 99% VaR model. Assuming 225 days in a year, what is the maximum number of daily losses exceeding the 1-day 99% VaR that is acceptable in a 1-year backtest to conclude, at a 95% confidence level, that the model is calibrated correctly?

A. 3

B. 5

C. 8

D. 10

1.7. Key Point: Fundamental Review of the Trading Book

1.7.1. 重要知识点

1.7.1.1. Expected Shortfall

The FRTB is proposing a change to the measure used for determining market risk capital. Instead of VaR with a 99% confidence level, expected shortfall (ES) with a 97.5% confidence level is proposed.

1.7.2. 基础题

Q-37. Which of the following statements regarding the differences between Basel I, Basel II.5, and the Fundamental Review of the Trading Book (FRTB) for market risk capital calculations is incorrect?

A. Both Basel I and Basel II.5 require calculation of VaR with a 99% confidence interval.

B. FRTB requires the calculation of expected shortfall with a 97.5% confidence interval.

C. FRTB requires adding a stressed VaR measure to complement the expected shortfall calculation.

D. The 10-day time horizon for market risk capital proposed under Basel I incorporates a recent period of time, which typically ranges from one to four years.

Q-38. Which of the following risks is specifically recognized by the incremental risk charge (IRC)?

A. Expected shortfall risk, because it is important to understand the amount of loss potential in the tail.

B. Jump-to-default risk, as measured by 99.9% VaR, because a default could cause a significant loss for the bank.

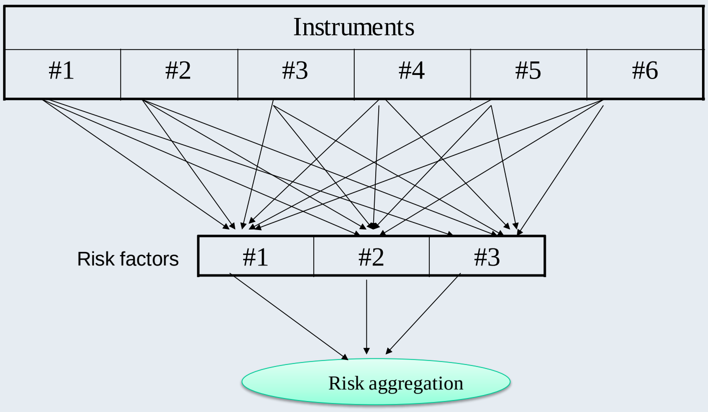
C. Equity price risk, because a change in market prices could materially impact mark-to-market accounting for risk.

D. Interest rate risk, as measured by 97.5% expected shortfall, because an increase in interest rates could cause a significant loss for the bank

1.8. VaR Mapping

1.8.1. 重要知识点

1.8.1.1. Framework



1.8.1.2. Three approaches for mapping a fixed income portfolio onto the risk factors.

- **Principal mapping.** Only the risk associated with the return of principal at the maturity of the bond is mapped. Principal mapping includes only the risk of repayment of the principal amounts.
- **Duration mapping.** The risk of the bond is mapped to a zero-coupon bond of the same duration. Duration mapping uses the duration of the portfolio to calculate the VaR.
- **Cash flow mapping.** The risk of the bond is decomposed into the risk of each of the bonds' cash flows. Cash flow mapping is the most precise method because we map the present value of the cash flows (face amount discounted at the spot rate for that maturity) onto the risk factors for zeros of the same maturities and include the inter-maturity correlations.

1.8.1.3. Mapping Approaches for Linear Derivatives

- Long currency forward contract = long foreign currency spot + long foreign currency bill + short U.S. dollar bill
- Long 6×12 FRA = long 6-month bill + short 12-month bill
- Long call option = long $N(d_1)$ asset + short $N(d_2)$ bill
- Long put option = long $N(-d_2)$ bill+ short $N(-d_1)$ asset

➢ Linear approximations may be acceptable for options with long maturities when the risk horizon is short.

1.8.2. 基础题

Q-39. An analyst is using the delta-normal method to determine the VaR of a fixed income portfolio. The portfolio contains a long position in 1-year bonds with a $1 million face value and a 6% coupon that is paid semi-annually. The interest rates on six-month and twelve-month maturity zero-coupon bonds are, respectively, 2% and 2.5%. Mapping the long position to standard positions in the six-month and twelve-month zeros, respectively, provides which of the following mapped positions?

A. $30,000 and 1,030,000

B. $29,500 and 975,610

C. $29,703 and 1,004,878

D. $30,300 and 1,035,000

Q-40. Which of the following can be considered a general risk factor?

I. Exchange rate

II. Mortgage-backed securities

III. Zero-coupon bond

IV. Interest rate

A. I only

B. II and III

C. III only

D. I and IV

Q-41. Delta-normal VaR will provide accurate estimates for option contracts when:

A. Deltas are stable

B. Options are at the money

C. The correlation matrix is available

D. The delta-normal method can never be used for option contracts

Q-42. Under these assumptions - in particular: a flat yield curve and constant yield volatility of 1.0% - why can we expect cash flow mapping to produce a lower diversified VaR than either duration and principal mapping?

A. The risk measures are non-linear.

B. Due to imperfect correlations between pairwise risk factors.

C. Fewer total cash flows will be mapped.

D. We cannot expect a lower diversified VaR.

Q-43. In fixed income portfolio mapping, when the risk factors have been selected, which of the following mapping approaches requires that one risk factor be chosen that corresponds to average portfolio maturity?

A. Principal mapping

B. Duration mapping

C. Convexity mapping

D. Cash flow mapping

Q-44. A portfolio manager is mapping a fixed-income portfolio into exposures on selected risk factors. The manager is analyzing the comparable mechanics and risk measurement outputs of principal mapping, duration mapping, and cash-flow mapping. Which of the following is correct?

A. Cash-flow mapping groups cash flows into buckets based on their size.

B. Cash-flow mapping uses the average rates in each risk group as a discount factor.

C. Principal mapping incorporates correlations among zero-coupon bonds.

D. Duration mapping replaces the portfolio with a zero-coupon bond with maturity equal to the duration of the portfolio.

Q-45. Which of these statements regarding risk factor mapping approaches is/are correct?

I. Under the cash flow mapping approach, only the risk associated with the average maturity of a fixed-income portfolio is mapped.

II. Cash flow mapping is the least precise method of risk mapping for a fixed-income portfolio.

III. Under the duration mapping approach, the risk of a bond is mapped to a zero-coupon bond of the same duration.

IV. Using more risk factors generally leads to better risk measurement but also requires more time to be devoted to the modeling process and risk computation.

A. I and II

B. I, III, and IV

C. III and IV

D. IV only

Q-46. Computing VaR on a portfolio containing a very large number of positions can be simplified by mapping these positions to a smaller number of elementary risk factors. Which of the following mappings would be adequate?

A. USD/EUR forward contracts are mapped on the USD/JPY spot exchange rate.

B. Each position in a corporate bond portfolio is mapped on the bond with the closest maturity among a set of government bonds.

C. Government bonds paying regular coupons are mapped on zero-coupon government bonds.

D. A position in the stock market index is mapped on a position in a stock within that index.

Q-47. A portfolio manager is mapping a fixed-income portfolio into exposures on selected risk factors. The manager is analyzing the comparable mechanics and risk measurement outputs of principal mapping, duration mapping, and cash-flow mapping that correspond to the average portfolio maturity. Which of the following is correct?

A. Principal mapping considers coupon and principal payments, and the portfolio VaR using principal mapping is greater than the portfolio VaR using cash-flow mapping.

B. Duration mapping does not consider intermediate cash flows and the portfolio VaR using such method is less than the portfolio VaR using principal mapping.

C. Cash-flow mapping considers the timing of the redemption cash flow payments only, and the portfolio VaR using cash flow mapping is less than the portfolio VaR using duration mapping.

D. Cash-flow mapping considers the present values of cash flows grouped into maturity buckets, and the undiversified portfolio VaR using cash-flow mapping is greater than the portfolio VaR using principal mapping.

1.9. Correlations and Copulas

1.9.1. 重要知识点

1.9.1.1. Copula:

$C_{GD}[Q_i(t), , Q_n(t)] = M_n\ [N^{-1}(Q_1(t)),, N^{-1}(Q_n(t)); \rho_M]$

$Q_i(t)$: cumulative default probability of asset i at time t.

C_{GD}: Gaussian default time copula.

M_n : the joint, n-variate cumulative standard normal distribution.

ρ_M: the n×n symmetric, positive-definite correlation matrix of the n-variate normal distribution Mn.

1.9.1.2. Mean reversion

➢ Mean reversion is present if there is a negative relationship between the change of a variable, $S_t - S_{t-1}$, and the variable S_{t-1}.

$$S_t = a(\mu_s - S_{t-1}) + S_{t-1}$$

$$\underbrace{S_t - S_{t-1}}_{Y} = \underbrace{a\,\mu_s}_{\alpha} - \underbrace{a\,S_{t-1}}_{\beta X}$$

1.9.2. 基础题

Q-48. Which of the following statements about correlation and copula are correct?

I. Copula enables the structures of correlation between variables to be calculated separately from their marginal distributions.

II. Transformation of variables does not change their correlation structure.

III. Correlation can be a useful measure of the relationship between variables drawn from a distribution without a defined variance.

IV. Correlation is a good measure of dependence when the measured variables are distributed as multivariate elliptical.

A. I and IV only

B. II, III, and IV only

C. I and III only

D. II and IV only

Q-49. A risk manager uses the past 480 months of correlation data from the Dow Jones Industrial Average (Dow) to estimate the long-run mean correlation of common stocks and the mean reversion rate. Based on historical data, the long-run mean correlation of Dow stocks was 32%, and his regression output estimates the following regression relationship: Y = 0.215 - 0.75 X. Suppose that in April 2014, the average monthly correlation for all Dow stocks was 36%. What is the expected correlation for May 2014 assuming the mean reversion rate estimated in the regression analysis?

A. 32%

B. 33%

C. 35%

D. 37%

Q-50. A risk manager uses the past 480 months of correlation data from the Dow Jones Industrial Average (Dow) to estimate the long-run mean correlation of common stocks and the mean reversion rate. Based on historical data, the long-run mean correlation of Dow stocks was 34%, and he regression output estimates the following regression relationship: Y = 0.215 - 0.77X. Suppose that in April 2014, the average monthly correlation for all Dow stocks was 33%. What is the estimated one-period autocorrelation for this time period based on the mean reversion rate estimated in the regression analysis?

A. 23%

B. 26%

C. 30%

D. 33%

Q-51. An investor purchases $1 million of Canadian bonds and is concerned about the bonds defaulting.The investor wishes to transfer this default risk to a third party, so he enters a fixed credit default swap (CDS) spread agreement with First Bank. Assume that the recovery rate is zero and that there is no accrued interest in the event that Canada defaults. Which of the following statements about the CDS between the investor and First Bank is correct?

A. A paper loss occurs for the investor if the correlation risk between First Bank and Canada increases because the value of the CDS spread will increase.

B. The fixed CDS spread is valued based on the default probability of the reference asset and the joint default correlation of First Bank and Canada.

C. The investor has wrong-way risk if there is negative correlation risk between First Bank and Canada.

D. Increasing correlation risk decreases the probability that the worst case scenario occurs.

Q-52. The dependence structure between the returns of financial assets plays an important role in risk measurement. For liquid markets, which of following statements is incorrect?

A. Correlation is a valid measure of dependence between random variables for only certain types of return distributions.

B. Even if the return distributions of two assets have a correlation of zero, the returns of these assets are not necessarily independent.

C. Copulas make it possible to model marginal distributions and the dependence structure separately.

D. With short time horizons (3 months or less), correlation estimates are typically very stable.

Q-53. A hedge fund that runs a distressed securities strategy is evaluating the solvency conditions of two potential investment targets. Currently firm RST is rated BB and firm WYZ is rated B. The hedge fund is interested in determining the joint default probability of the two firms over the next two years using the Gaussian default time copula under the assumption that a one-year Gaussian default correlation is 0.36. The fund reports that $\mathbf{X_{BB}}$ and $\mathbf{X_B}$ are abscise values of the bivariate normal distribution presented in the table below where $\mathbf{X_{BB}} = \mathbf{N^{-1}}(\mathbf{Q_{BB}}(\mathbf{t_{BB}}))$ and $\mathbf{X_B} = \mathbf{N^{-1}}(\mathbf{Q_B}(\mathbf{t_B}))$ with $\mathbf{t_{BB}}$ and $\mathbf{t_B}$ being the time-to-default of BB-rated and B-rated companies respectively; and $\mathbf{Q_{BB}}$ and $\mathbf{Q_B}$ being the cumulative distribution functions of $\mathbf{t_{BB}}$ and $\mathbf{t_B}$, respectively; and N denote the standard normal distribution:

Default Time in Year	Firm RST Default Probability	Firm RST Cumulative Default Probability $\mathbf{Q_{BB}}$（$\mathbf{t}$）	Firm RST Cumulative Standard Normal Percentiles $\mathbf{N^{-1}}(\mathbf{Q_{BB}}(\mathbf{t}))$	Firm WYZ Default Probability	Firm WYZ Cumulative Default Probability $\mathbf{Q_B}(\mathbf{t})$	Firm WYZ Cumulative Standard Normal Percentiles $\mathbf{N^{-1}}(\mathbf{Q_B}(\mathbf{t}))$
1	5.21%	5.21%	-1.625	19.06%	19.06%	-0.876
2	6.12%	11.33%	-1.209	10.63%	29.69%	-0.533
3	5.50%	16.83%	-0.961	8.24%	37.93%	-0.307
4	4.81%	21.64%	-0.784	6.10%	44.03%	-0.150
5	4.22%	25.86%	-0.648	4.03%	48.06%	-0.049

Applying the Gaussian copula, which of the following corresponds to the joint probability that firm RST and firm WYZ will both default before the end of year 2?

A. $Q(X_{BB} = 0.0612) + Q(X_B = 0.1063) – Q(X_{BB} = 0.0612)\times Q(X_B = 0.1063)$

B. $Q(X_{BB} = 0.1133) + Q(X_B = 0.2969) – Q(X_{BB} = 0.1133) \times Q(X_B = 0.2969)$

C. $Q(X_{BB} \leqslant 0.1133 \cap X_B \leqslant 0.2969)$

D. $Q(X_{BB} \leqslant -1.209 \cap X_B \leqslant -0.533)$

1.10. Empirical Approaches to Risk Metrics and Hedge

1.10.1. 重要知识点

1.10.1.1. Denoting the face amounts of the real and nominal bonds by $\mathbf{F^R}$ and $\mathbf{F^N}$and their DV01s by $\mathbf{DV01^R}$and $\mathbf{DV01^N}$, the regression-based hedge, characterized earlier as the DV01 hedge adjusted for the average change of nominal yields relative to real yields, can be written as follows:

$$\mathbf{F^R} = -\mathbf{F^N} \times \frac{\mathbf{DV01^N}}{\mathbf{DV01^R}} \times \hat{\boldsymbol{\beta}}$$

1.10.1.2. The hedge of the first equation minimizes the variance of the P&L in over the data set and used to estimate the regression parameters.

1.10.2. 基础题

Q-54. Assume that a trader is making a relative value trade, selling a U.S. Treasury bond and correspondingly purchasing a U.S. Treasury TIPS. Based on the current spread between the two securities, the trader shorts $100 million of the nominal bond and purchases $89.8 million of TIPS. The trader then starts to question the amount of the hedge due to changes in yields on TIPS in relation to nominal bonds. He runs a regression and determines from the output that the nominal yield changes by 1.0274 basis points per basis point change in the real yield. Would the trader adjust the hedge, and if so, by how much?

A. No
B. Yes, by $2.46 million (purchase additional TIPS).
C. Yes, by $2.5 million (sell a portion of the TIPS).
D. Yes, by $2.11 million (purchase additional TIPS)

Q-55. Assume that a trader wishes to set up a hedge such that he sells $100,000 of a Treasury bond and buys Treasury TIPS as a hedge. Using a historical yield regression framework, assume the DV01 on the T-bond is 0.072, the DV01 on the TIPS is 0.051, and the hedge adjustment factor (regression beta coefficient) is 1.2. What is the face value of the offsetting TIPS position needed to carry out this regression hedge?

A. $138,462
B. $169,412
C. $268,499
D. $280,067

Q-56. Assumes that bond trader,Jayme Ryan,wishes to make a relative value trade by selling a U.S. Treasury bond(T-bond) and purchasing a U.S. Treasury Inflation Protected Security(TIPS).Ryan decides to short $100 million of the nominal bond and determines that the DV01 of the TIPS is 0.088 and the DV01 of the T-bond is 0.065.Ryan then runs a least squares regression based on changes in the nominal yield and real yield and finds a yield beta of 1.03 basis points.What is the amount of TIPS that Ryan should purchase in order to hedge the short nominal bond?

A. $73.86 million
B. $76.08 million
C. $135.38 million
D. 139.45 million

1.11. Convexity Effect

1.11.1. 重要知识点

1.11.1.1. The convexity effect can be measured by applying a special case of Jensen's inequality as follows:

$$\mathbf{E}\left[\frac{\mathbf{1}}{(\mathbf{1}+\mathrm{r})}\right] > \frac{\mathbf{1}}{\mathbf{E}(\mathbf{1}+\mathbf{r})}$$

1.11.1.2. All else held equal, the value of convexity increases with maturity and volatility.

1.11.2. 基础题

Q-57. An investor expects the current 1-year rate for a zero-coupon bond to remain at 6%, the 1-year rate next year to be 8%, and the 1-year rate in two years to be 10%. What is the 3-year spot rate for zero-coupon bond with face value of $1, assuming all investor have the same expectations of future 1-year rates for zero-coupon bonds?

A. 7.888%

B. 7.988%

C. 8.000%

D. 8.088%

Q-58. Suppose an investor expects that the 1-year rate will remain at 6% for the first year for a 2-year zero-coupon bond. The investor also projects a 50% probability that the 1-year spot rate will be 8% in one year and a 50% probability that the 1-year spot rate will be 4% in one year. Which of the following inequalities most accurately reflects the convexity effect for this 2-year bond using Jensen's inequality formula?

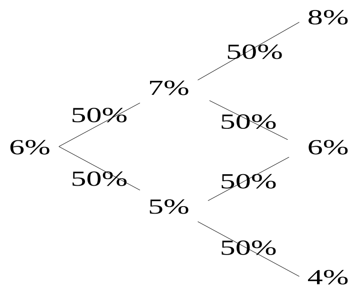
A. $0.89031 > $0.89000

B. $0.89000 > $0.80000

C. $0.94340 > $0.89031

D. $0.94373 > $0.94340

Q-59. Suppose investors have interest rate expectations as illustrated in the decision tree below where the 1-year rate is expected to be 8%, 6%, or 4% in the second year and either 7% or 5% in the first year for a zero-coupon bond.



If investors are risk-neutral, what is the price of a $1 face value 2-year zero-coupon bond today?

A. $0.88113

B. $0.88634

C. $0.89007

D. $0.89032

1.12. Term Structure Models

1.12.1. 重要知识点

1.12.1.1. Model 1: assumes no drift and that interest rates are normally distributed:

- $\mathbf{dr = \sigma dw}$
- $\mathbf{dw = \varepsilon\sqrt{dt}}$

1.12.1.2. Model 2: adds a positive drift term to Model 1 that can be interpreted as a positive risk premium associated with longer time horizons:

- $\mathrm{d}\mathbf{r} = \boldsymbol{\lambda}\mathrm{dt} + \boldsymbol{\sigma}\mathbf{dw}$

1.12.1.3. Ho-Lee Model: generalizes drift to incorporate time-dependency:

- $\mathrm{d}\mathbf{r} = \boldsymbol{\lambda}(\mathbf{t})\mathrm{dt} + \boldsymbol{\sigma}\mathbf{dw}$

1.12.1.4. Vasicek Model: assumes a mean-reverting process for short-term interest rates:

- $\mathrm{d}\mathbf{r} = \boldsymbol{\kappa}(\boldsymbol{\theta} - \mathbf{r})\mathbf{dt} + \boldsymbol{\sigma}\mathbf{dw}$
- where:
  - k = a parameter that measures the speed of reversion adjustment.
  - θ = long-run value of the short-term rate assuming risk neutrality.
  - r = current interest rate level.

1.12.1.5. Model 3: assigns a specific parameterization of time-dependent volatility:

- $\mathrm{d}\mathbf{r} = \boldsymbol{\lambda}(\mathbf{t})\mathbf{dt} + \boldsymbol{\sigma}\mathbf{e}^{-\boldsymbol{\alpha}\mathbf{t}}\mathbf{dw}$

  where:
  - σ = volatility at t = 0, which decreases exponentially to 0 for α > 0

1.12.1.6. Cox-Ingersoll-Ross (CIR) Model: mean-reverting model with constant volatility, σ, and basis-point volatility, $\boldsymbol{\sigma}\sqrt{\mathbf{r}}$, that increases at a decreasing rate:

- $\mathbf{dr = k(\theta - r)dt + \sigma\sqrt{r}dw}$

1.12.1.7. Model 4 (lognormal model): yield volatility, σ, is constant, but basis-point volatility, σr, increases with the level of the short-term rate. There are two lognormal models of importance:

- Lognormal with deterministic drift
  - $d[\ln(r)] = a(t)dt + \sigma dw$
- Lognormal with mean reversion
  - $d[\ln(r)] = k(t)[\ln\theta(t) - \ln(r)]dt + \sigma(t)dw$

1.12.2. 基础题

Q-60. Model 1 assumes zero drift and is also called a normal model. Model 2 add a term for drift. Each of the following is true about these two models except for:

A. A weakness of Model 1 is that the short-term rate can become negative.

B. Model 1 implies a term structure that is perfectly flat at the current rate for all maturities, including the long-term rates.

C. Model 2 is more capable of producing an upward-sloping term structure, which is often observed.

D. Model 2 is an equilibrium model, rather than an arbitrage-free model, because no attempt is made to match the term structure closely.

Q-61. John Jones, FRM, is discussing the appropriate usage of mean-reverting models relative to no-drift models, models that incorporate drift, and Ho-Lee models. Jones makes the following statements:

- Statement 1: Both Model 1 (no drift) and the Vasicek model assume parallel shifts from changes in the short-term rate.
- Statement 2: The Vasicek model assumes decreasing volatility of future short-term rates while Model 1 assumes constant volatility of future short-term rates.
- Statement 3: The constant drift model (Model 2) is a more flexible model than the Ho-Lee model.

How many of his statements are correct?

A. 0

B. 1

C. 2

D. 3

Q-62. Using Model 1, assume the current short-term interest rate is 5%, annual volatility is 80bps, and dω, a normally distribution random variable with mean 0 and standard deviation$\sqrt{\mathbf{dt}}$, has an expected value of zero. After one month, the realization of dω is -0.5. What is the change in the spot rate and the new spot rate?

	Change in Spot	New Spot Rate
A.	0.40%	5.40%
B.	-0.40%	4.60%
C.	0.80%	5.80%
D.	-0.80%	4.20%

Q-63. An analyst is modeling spot rate changes using short rate term structure models. The current short-term interest rate is 5% with a volatility of 80 bps. After one month passes the realization of dω, a normally distributed random variable with mean 0 and standard deviation$\sqrt{\mathbf{dt}}$, is -0.5. Assume a constant interest rate drift, λ, of 0.36%. What should the analyst compute as the new spot rate?

A. 5.37%

B. 4.63%

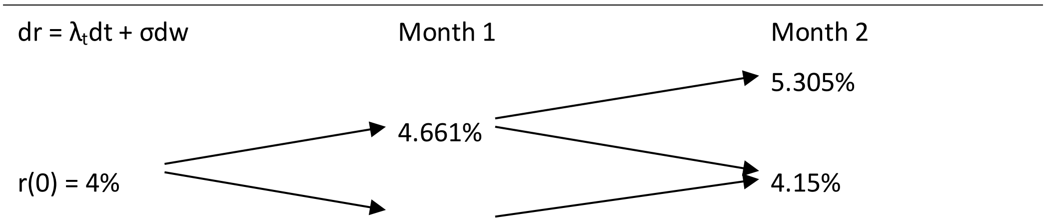
C. 5.76%

D. 4.24%

Q-64. The current short-term rate, r(0) is 4%. Under a Ho-Lee Model with time-dependent drift, the time step is monthly and the annualized drifts are as follows: +100 basis points in the first month and +80 basis points in the second month. The annual basis point volatility is 200bps.

Ho-Lee Assumptions

Month (dt)	0.0833
Annualized drift, first month, λ_1	100 bps
Annualized drift, second month, λ_2	80 bps
Volatility, annual	2%
Ho-Lee Model: Time-dependent drift	



3.506% → ?

What is the value of the missing node [2, 0] in this Ho-Lee interest rate tree?

A. 2.447%

B. 2.677%

C. 2.995%

D. 3.256%

Q-65. Which of the following statements best characterizes the differences between the Ho-Lee model with drift and the lognormal model with drift?

A. In the Ho-Lee model and the lognormal model the drift terms are multiplicative.

B. In the Ho-Lee model and the lognormal model the drift terms are additive

C. In the Ho-Lee model the drift terms are multiplicative, but in the lognormal model the drift terms are additive

D. In the Ho-Lee model the drift terms are additive, but in the lognormal model the drift terms are multiplicative.

Q-66. A CRO of a hedge fund is asking the risk team to develop a term-structure model that is appropriate for fitting interest rates for use in the fund's options pricing practice. The risk team is evaluating among several interest rate models with time-dependent drift and time-dependent volatility functions. Which of the following is a correct description of the specified model?

A. In the Ho-Lee model, the drift of the interest rate process is presumed to be constant.

B. In the Ho-Lee model, when the short-term rate is above its long-run equilibrium value, the drift is presumed to be negative.

C. In the Cox-Ingersoll-Ross model, the basis-point volatility of the short-term rate is presumed to be proportional to the square root of the rate, and short-term rates cannot be negative.

D. In the Cox-Ingersoll-Ross model, the volatility of the short-term rate is presumed to decline exponentially to a constant level.

Q-67. The trading desk at Big Bank is pricing an off-market swap. The quantitative analysis team has identified the interest rate drift in periods 1 and 2 to be 25 basis points and -10 basis points, respectively. These values were calibrated from liquid swap prices. The current short-term interest rate is 5.4% with a long-run mean reverting level of 15.1%. Additionally, the long-run true interest rate is 12.6%. The half-life of the process is 6.3

years. Using the HO-LEE Model, what is the expected short-term interest rate in two periods?

A. 5.25%

B. 5.4%

C. 5.55%

D. 5.75%

Q-68. An analyst is looking at various models used to incorporate drift into term structure models. The Ho-Lee Model:

A. Incorporates no-risk premium to the interest rate model allowing rates to vary according to their volatility.

B. Incorporates drift as a premium to interest rates that remains constant over time.

C. Allows for a risk premium to be applied to interest rates that changes over time.

D. Incorporates drift into the model following the assumption that rates revert to the long-run equilibrium value.

Q-69. A risk manager is constructing a term structure model and intends to use the Cox-Ingersoll-Ross Model. Which of the following describes this model?

A. The model presumes that the volatility of the short rate will increase at a predetermined rate.

B. The model presumes that the volatility of the short rate will decline exponentially to a constant level.

C. The model presumes that the basis-point volatility of the short rate will be proportional to the rate.

D. The model presumes that the basis-point volatility of the short rate will be proportional to the square root of the rate.

Q-70. A hedge fund risk manager is looking at various models that are flexible enough to incorporate mean reversion and risk premium into term structure modeling. Which of the following is correct about the Vasicek model?

A. It incorporates mean reversion feature and its drift is always zero.

B. It incorporates mean reversion feature and models the risk premium as a constant or changing drift.

C. It cannot incorporate risk premium and the term structure of interest rate volatility in the model is upward-sloping.

D. It cannot capture the mean reversion feature but can be used to model the time-varying risk premium.

1.13. Interest Rate Tree (Binomial) Model

1.13.1. 重要知识点

1.13.1.1. Using backward induction, the value of a bond at a given node in a binomial tree is the average of the present values of the two possible values from the next period. The appropriate discount rate is the forward rate associated with the node under analysis.There are three basic steps to valuing an option on a fixed-income instrument using a binomial tree:

- Step 1: Price the bond value at each node using the projected interest rates.
- Step 2: Calculate the intrinsic value of the derivative at each node at maturity.
- Step 3: Calculate the expected discounted value of the derivative at each node using the risk- neutral probabilities and work backward through the tree.

1.13.2. 基础题

Q-71. A risk manager is pricing a 10-year Treasuries using a successfully tested pricing model. Current interest rate volatility is high and the risk manager is concerned about the effect this may have on short-term rates when pricing the option. Which of the following actions would best address the potential for negative short-term interest rates to arise in the model?

A. The risk manager uses a normal distribution of interest rates.

B. When short-term rates are negative, the risk manager adjusts the risk-neutral probabilities.

C. When short-term rates are negative, the risk manager increases the volatility.

D. When short-term rates are negative, the risk manager sets the rate to zero.

Q-72. A market risk manager seeks to calculate the price of a 2-year zero-coupon bond. The 1-year interest rate today is 10.0%. There is a 50% probability that the 1-year interest rate will be 12.0% and a 50% probability that it will be 8.0% in 1 year. Assuming that the risk premium of duration risk is 50 bps each year, and that the bond’s face value is EUR 1,000, which of the following should be the price of the zero-coupon bond?

A. EUR 822.98

B. EUR 826.74

C. EUR 905.30

D. EUR 921.66

Q-73. A European put option has two years to expiration and a strike price of $101.00. The underlying is a 7% annual coupon bond with three years to maturity. Assume that the risk-neutral probability of an up move is 0.76 in year 1 and 0.60 in year 2. The current interest rate is 3.00%. At the end of year 1, the rate will either be 5.99% or 4.44%. If the rate in year 1 is 5.99%, it will either rise to 8.56% or rise to 6.34% in year 2. If the rate in one year is 4.44%, it will either rise to 6.34% or rise to 4.70%. The value of the put option today is closet to:

A. $1.17

B. $1.30

C. $1.49

D. $1.98

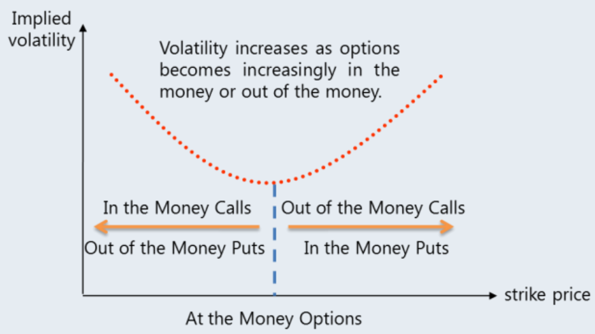
1.14. Volatility Smile

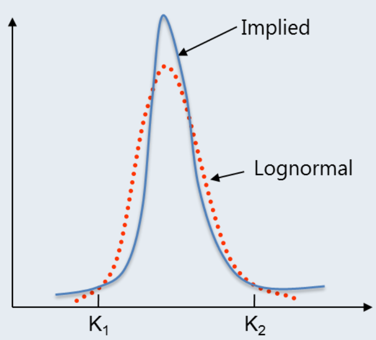
1.14.1. 重要知识点

1.14.1.1. Put-call parity indicates that the deviation between market prices and Black-Scholes-Merton prices will be equivalent for calls and puts. Hence, implied volatility will be the same for calls and puts.

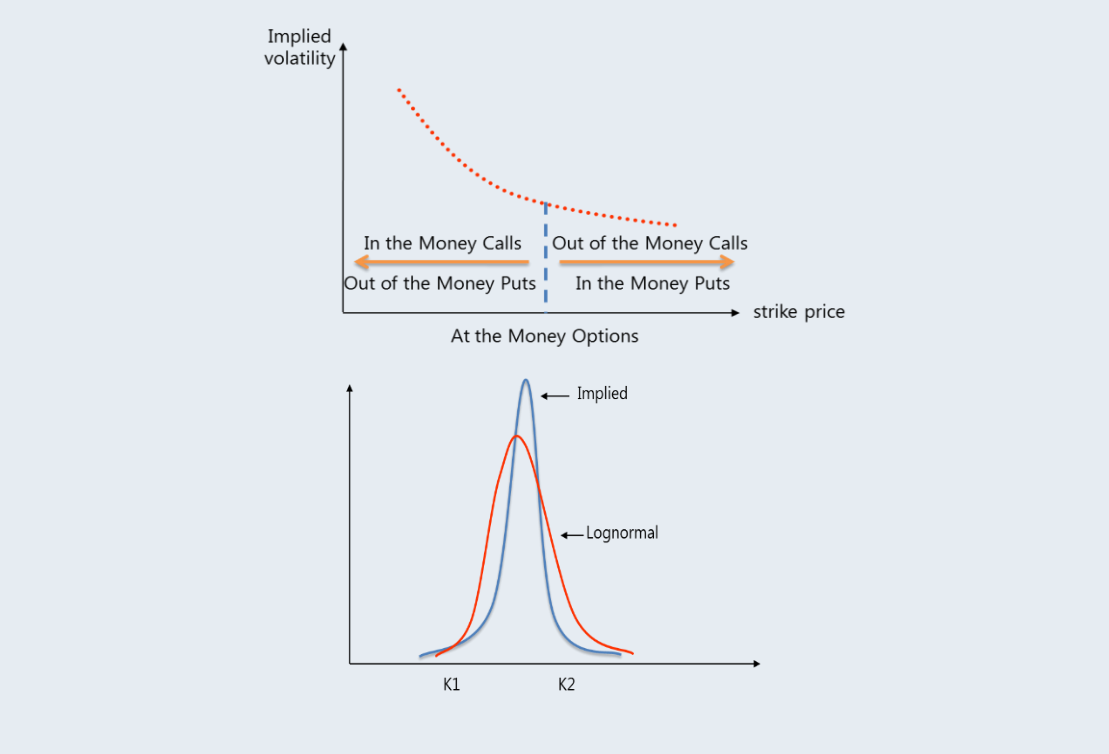
$$\mathbf{p_{BS} - p_{mkt} = c_{BS} - c_{mkt}}$$

1.14.1.2. Volatility Smile for Foreign Currency Options





1.14.1.3. Volatility Smiles (skew) for Equity Options



1.14.2. 基础题

Q-74. The Chief Risk Officer of Martingale Investments Group is planning a change in methodology for some of the risk management models used to estimate risk measures. His aim is to move from models that use the normal distribution of returns to models that use the distribution of returns implied by market prices. Martingale Group has a large long position in the German equity stock index DAX which has a volatility smile that slopes downward to the right. How will the change in methodology affect the estimate of expected shortfall (ES)?

A. ES with the updated models will be larger than the old estimate.

B. ES with the updated models will be smaller than the old estimate.

C. ES will remain unchanged.

D. Insufficient information to determine.

Q-75. With all other things being equal, a risk monitoring system that assumes constant volatility for equity returns will understate the implied volatility for which of the following positions by the largest amount:

A. Short position in an at-the-money call

B. Long position in an at-the-money call

C. Short position in a deep out-the-money call

D. Long position in a deep in-the-money call

Q-76. Which of the following regarding equity option volatility is true?

A. There is higher implied price volatility for away-from-the-money equity options.

B. “Crashophobia” suggests actual equity volatility increases when stock prices decline.

C. Compared to the lognormal distribution, traders believe the probability of large down movements in price is similar to large up movements.

D. Increasing leverage at lower equity prices suggests increasing volatility.

Q-77. You are asked to mark to market a book of plain vanilla stock options. The trader is short deep out-of-money options and long at-the-money options. There is a pronounced smile for these options. The trader’s bonus increases as the value of his book increases. Which approach should you use to mark the book?

A. Use the implied volatility of at-the-money options because the estimation of the volatility is more reliable.

B. Use the average of the implied volatilities for the traded options for which you have data because all options should have the same implied volatility with Black-Scholes and you don’t know which one is the right one.

C. For each option, use the implied volatility of the most similar option traded on the market.

D. Use the historical volatility because doing so corrects for the pricing mistakes in the option market.

Q-78. The market price of a European call is \$3.00 and its Black-Scholes price is \$3.50.The Black-Scholes price of a European put option with the same strike price and time to maturity is \$2.00.What should the market price of this option be?

A. \$1.50

B. \$2.00

C. \$1.00

D. \$0.50

Q-79. An empirical distribution of equity price derived from the price of options of such stock based on BSM that exhibits a fatter right tail than that of a lognormal distribution would indicate:

A. Equal implied volatilities across low and high strike prices.

B. Greater implied volatilities for low strike prices.

C. Greater implied volatilities for high strike prices.

D. Higher implied volatilities for mid-range strike prices.

Q-80. Compared to at-the-money currency options, out-of-the-money currency options exhibit which of the following volatility traits?

A. Lower implied volatility

B. A frown

C. A smirk

D. Higher implied volatility

Q-81. Equity options tend to exhibit a volatility "smirk" where low strike price options have a higher implied volatility than high strike price options. An explanation that has not been used for the smirk pattern is:

A. heteroscedasticity in the underlying.

B. a higher proportion of debt in the capital structure as equity prices fall.

C. less firm leverage as equity prices rise.

D. the threat of another market crash.

Q-82. Which of the following statement is incorrect regarding volatility smiles?

A. Currency options exhibit volatility smiles because the at-the-money option have higher implied volatility away-from-the-money options.

B. Volatility frowns result when jumps occur in asset prices

C. Equity options exhibit a volatility smirk because low strike price options have grater implied volatility.

D. Relative to currency traders, it appears that equity traders' expectations of extreme price movements are more asymmetric.

Q-83. A risk manager is examining a firm's equity index option price assumptions. The observed volatility skew for a particular equity index slopes downward to the right. Compared to the lognormal distribution, the distribution of option prices on this index implied by the Black-Scholes-Merton (BSM) model would have:

A. A fat left tail and a thin right tail.

B. A fat left tail and a fat right tail.

C. A thin left tail and a fat right tail.

D. A thin left tail and a thin right tail.

Q-84. A committee of risk management practitioner discusses the difference between pricing deep out-of-the-money call options on FBX stock and pricing deep out-of-the-money call options on the EUR/JPY foreign exchange rate using the Black-Scholes-Merton (BSM) model. The practitioners price these options based on two distinct probability distributions of underlying asset prices at the option expiration date:

- A lognormal probability distribution
- An implied risk-neutral probability distribution obtained from the volatility smile for options of the same maturity

Using the lognormal instead of the implied risk-neutral probability distribution will tend to:

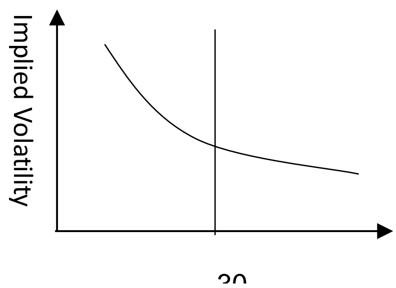
A. Price the option on FBX relatively high and price the option on EUR/JPY relatively low.

B. Price the option on FBX relatively low and price the option on EUR/JPY relatively high.

C. Price the option on FBX relatively low and price the option on EUR/JPY relatively high.

D. Price the option on FBX relatively high and price the option on EUR/JPY relatively high.

Q-85. A risk manager is in the process of valuing several European option positions on a non-dividend-paying stock XYZ that is currently priced at GBP 30.The implied volatility skew, estimated using the Black-Scholes-Merton model and the current prices of actively traded European-style options on stock XYZ at various strike prices, is



Assuming that the implied volatility at GBP 30 is used to conduct the valuation, which of the following long positions will be undervalued?

A. An out-of-the-money call

B. An in-the-money call

C. An at-the-money put

D. An in-the-money put

Solution

Q-1. Solution: D

Q-2. Solution: B

B is correct. Non-parametric approaches can accommodate fat tails, skewness, and any other non-normal features that can cause problems for parametric approaches. However, if the data period that is used in estimation includes few losses or losses with low magnitude, non-parametric methods will often produce inaccurate risk measures. Also, non-parametric approaches produce VaR and ES that are too low if the data period has unusually low volatility, and would produce VaR and ES that are too high if the data period has unusually high volatility. Hence parametric methods would be more appropriate in those situations. Therefore, A, C and D are incorrect.

Q-3. Solution: C

We need to map the portfolio to a position in the underlying stock TUV. A deep in-the-money call has a delta of approximately 1, a deep out-of-the-money call has a delta of approximately zero and

forwards have a delta of 1. The net portfolio has a delta of about 1×5,000 + 0×20,000 + 1×10,000 = 15,000 and is approximately gamma neutral.

Let:

α = 2.326 (99% confidence level)

S = price per share of stock TUV = USD 52

Dp = delta of the position = 15,000

σ = volatility of TUV = 0.12

Therefore, the 1-day VaR estimate at 99% confidence level is computed as follows:

$$2.236 \times 52 \times 15{,}000 \times 0.12 \times \sqrt{1/252} = 13{,}714.67$$

Q-4. Solution: A

VaR_{Mic} = 1.65 × 2% × 120 × 1000 = 3960

$VaR_{AT\&T}$ = 1.65 × 1% × 30 × 20000=9900

$$\mathrm{VaR}_{\mathrm{P}(5-\mathrm{day},95\%)} = \sqrt{3960^2 + 9900^2 + 2 \times 0.3 \times 3960 \times 9900} \times \sqrt{5} = 26193$$

Q-5. Solution: C

Normal VaR=|0.1−1.645×0.4|=0.558

Lognormal VaR=$1-e^{0.1-1.645\times0.4}$=0.4276

Hence, lognormal VaR is smaller than Normal VaR by 13.04% per year. With a portfolio of GBP 1,000,000, this translates to GBP 130,400.

Q-6. Solution: A

VaR does not consider losses beyond the VaR threshold level.

Q-7. Solution: C

An estimate of the expected shortfall (ES) can be obtained by taking the average of the VaRs for the various confidence levels that are greater than 97.5%. Therefore,

ES = (378,412,500+392,452,500+410,880,000+439,252,500)/4 = JPY 405,249,375.

Q-8. Solution: B

Expected shortfall is always greater than or equal to VaR for a given confidence level, since ES accounts for the severity of expected losses beyond a particular confidence level, while VaR measures the minimum expected loss at that confidence level. Therefore, ES would lead to a higher level of required economic capital than VaR for the same confidence level. In practice, however, regulators often correct for the difference between ES and VaR by lowering the required confidence level for banks using ES compared to those using VaR.

Q-9. Solution: C

Heavy tails: the steeper slope - i.e., greater than 1.0 - at the tails indicates the tails are heavier than the reference distribution.

Q-10. Solution: D

This Q-Q plot has steeper slopes at the tails of the plot, which indicate fat tails in the distribution. A normal distribution would result in a linear QQ plot. A distribution with thin tails would produce a QQ plot with less steep slopes at the tails of the plot than a linear relationship, while this one is steeper at the tails. It is not a negatively skewed distribution, as the Q-Q plot is symmetric.

Q-11. Solution: C

Expected shortfall's stability as a measure of risk depends on the loss distribution.

Q-12. Solution: B

VaR is the most widely used risk measure for both capital allocation and absolute risk calculation purposes.However,ES is increasingly being used for capital allocation purposes.

Q-13. Solution: B

State	Bonds	Probability	Payoff
No default		0.995×0.995×0.995=0.9850749	0
1 default	A,B,C	3×0.005×0.995×0.995=0.0148504	-$100,000
2 defaults	AB,AC,BC	3×0.005×0.005×0.995=0.0000746	-$200,000
3 defaults	ABC	0.005×0.005×0.005=0.0000001	-$300,000

Here, the probability of zero or one default is 0.9851 + 0.0148 = 99.99%. The portfolio VaR is therefore $100,000, which is the lowest number, such that the probability exceeds 99%. Note that the portfolio VaR is greater than the sum of individual VaRs. In this example, VaR is not sub additive. This is an undesirable property because it creates disincentives to aggregate the portfolio, since it appears to have higher risk.

Q-14. Solution: A

VaR measures the expected amount of capital one can expect to lose within a given confidence level over a given period of time. One of the problems with VaR is that it does not provide information about the expected size of the loss beyond the VaR. VaR is often complemented by the expected shortfall, which measures the expected loss conditional on the loss exceeding the VaR. Note that since expected shortfall is based on VaR, changing the confidence level may change both measures. A key difference between the two measures is that VaR is not sub-additive, meaning that the risk of two funds separately may be lower than the risk of a portfolio where the two funds are combined. Violation of the sub-additive assumption is a problem with VaR that does not exist with expected shortfall.

Q-15. Solution: C

ES = E(L | L > VaR). In this case, the 95% ES is the expected loss conditional on the loss occurring, which coincides with the mean of the normal distribution.

Q-16. Solution: C

The expected shortfall is an estimate of the tail loss by averaging the VaRs for increasing confidence levels. The tail is divided into n slices and the corresponding $\mathrm{n}-1$ VaRs are calculated. In this case, n = 5, so only the VaRs for the 96%, 97%, 98%, and 99% levels are averaged to calculate expected shortfall. $(1.7507+1.8808+2.0537+2.3263)/(5-1)=2.003$.

Q-17. Solution: B

The diversification should make the portfolio less risky, or at the very least, equally risky. But the combination should not penalize diversification in terms of the risk metric.

Q-18. Solution: A

ES is a coherent spectral measure which gives equal weight to the tail quantiles. The general class is spectral measures, which contain a weighting function. Both ES and VaR are special cases of a spectral measure (the spectral function generalized both ES & VaR). Spectral measures are coherent under conditions that are met by ES but not by VaR; “Spectral” is associated with, but does not necessarily imply, coherence.

Q-19. Solution: B

Bootstrapping from historical simulation involves repeated sampling with replacement. The 5% VaR is recorded from each sample draw. The average of the VaRs from all the draws is the VaR estimate. The bootstrapping procedure does not involve filtering the data or weighting observations. Note that the VaR from the original data set is not used in the analysis.

Q-20. Solution: D

The historical return is adjusted based on the ratio of current daily volatility to historically observed daily volatility 30 days ago.The volatility-adjusted return is calculated as follows:

$$\mathbf{r}_{\mathbf{t,i}}^{*}=\frac{\boldsymbol{\sigma}_{\mathbf{T,i}}}{\boldsymbol{\sigma}_{\mathbf{t,i}}}*\mathbf{r}_{\mathbf{t,i}}=\frac{\mathbf{0.014}}{\mathbf{0.01}}*\mathbf{0.015}=\mathbf{0.021}=\mathbf{2.1}\%$$

Once the volatility-adjusted return is computed,VAR,ES,and any other coherent risk measure can be calculated in the usual way after substituting historical returns with volatility-adjusted returns.

Q-21. Solution: B

If the intensity parameter (i.e., decay factor) is close to 1, there will be persistence (i.e., slow decay) in the estimate. The expression for the weight on day i has i in the exponent when it should be n. While a large sample size is generally preferred, some of the data may no longer be representative in a large sample.

Q-22. Solution: C

Each historical return (t) is replaced by: return(t) × volatility(0)/volatility(t). Then conduct typical historical simulation (HS) on adjusted return series

For example, if on the historical day (t), the return(t) was -2.0% and volatility(t) was 10%, while today’s volatility estimate is 20%, then the adjusted return is -2.0% × 20%/10% = - 4.0% . In this way, “Actual returns in any period t are therefore increased (or decreased), depending on whether the current forecast of volatility is greater (or less than) the estimated volatility for period t. We now calculate the HS P/L using [the adjusted returns] instead of the original data set, and then proceed to estimate HS VaRs or ESs in the traditional way (i.e., with equal weights, etc.).

Q-23. Solution: C

Market-weighted historical simulation is not discussed in this topical. Age-weighted historical simulation weights observations higher when they appear closer to the event date. Volatility-weighted historical simulation adjusts for changing volatility levels in the data. Correlation- weighted historical simulation incorporates anticipated changes in correlation between assets in the portfolio.

Q-24. Solution: B

The volatility-weighting method adjusts historic returns for current volatility. Specifically, return at time t is multiplied b (current volatility estimate/ volatility estimate at time t). However, the actual procedure for calculating VaR using a historical simulation method is unchanged; it is only the inputted data that changes.

Q-25. Solution: C

The use of historical data in non-parametric analysis is a disadvantage, not an advantage. If the estimation period was quiet (volatile) then the estimated risk measures may understate (overstate) the current risk level. Generally, the largest VaR cannot exceed the largest loss in the historical period. On the other hand, the remaining choices are all considered advantages of non-parametric methods. For instance, the non-parametric nature of the analysis can accommodate skewed data, data points are readily available, and there is no requirement for estimates of covariance matrices.

Q-26. Solution: C

The POT approach generally has fewer parameters, but GEV approaches share the tail parameter ξ.

Q-27. Solution: C

There is a trade-off in setting the threshold. It must be high enough for the appropriate theorems to hold, but if set too high; there will not be enough observations to estimate the parameters.

Q-28. Solution: B

B is correct. As can be seen from the formula below, increasing u increases both VaR and ES even if n gets lower as u increases.

A is incorrect. Increasing the tail parameter value actually increases both VaR and ES.

C and D are incorrect. According to the peaks-over-threshold (POT) risk measure, the VaR and ES (in percentage) are computed as (note: the first equation is not necessary as the value of VaR is given):

$$\mathrm{VaR} = \mathrm{u} + \left(\frac{\beta}{\xi}\right)\left\{\left[\frac{\mathrm{N}}{\mathrm{n}}(1 - \text{confidence level})\right]^{-\xi} - 1\right\}$$

$$= 3 + \left(\frac{0.75}{0.22}\right)\left\{[\frac{740}{37}(1 - 0.99)]^{-0.22} - 1\right\} = 4.45\%$$

And

$$\mathrm{ES} = \frac{4.45}{1 - 0.22} + \frac{0.75 - 0.22 \times 3}{1 - 0.22} = 5.82\%$$

Q-29. Solution: A

A key foundation of EVT Is that as the threshold value is increased, the distribution of loss exceedances converges to a generalized Pareto distribution. Assuming the threshold is high enough, excess losses can be modeled using the generalized Pareto distribution. Thus, A is correct.

B is incorrect. If the tail parameter value of the generalized extreme-value (GEV) distribution goes to zero, and not infinity, then the distribution of the original data {not the GEV) could be a light-tail distribution such as normal or log-normal. In other words, the corresponding GEV distribution is a Gumbel distribution.

C is incorrect. To apply EVT, the underlying loss distribution can be any of the commonly used distributions: normal, lognormal, t, etc.

D is incorrect. As the threshold value is decreased, the number of exceedances increases.

Q-30. Solution:C

Backtesting VaR models with higher confidence levels is difficult because the number of exceptions is not high enough to provide meaningful information.

Q-31. Solution:B

VaR models are based on static portfolios, while actual portfolios are constantly changing, and actual returns are complicated by intraday changes as well as profit and loss factors that result from commissions, fees, interest income, and bid-ask spreads. A risk manager can minimize these effects by backtesting with a relatively short time horizon such as a daily holding period.

Q-32. Solution: C

T value=(25-1000×1%)/sqrt(1000×1%×99%)=4.74

we reject the null hypothesis and decide that the model is a bad model, which implies a risk of type I error.

Q-33. Solution: D

Null hypothesis is H_0: Model is good with E[exceptions] = (1 - 95%) × 250 = 12.5 exceptions

The standard error (standard deviation) of the binomial variable

$= \sqrt{p(1-p)T} = \sqrt{5\% \times (1-5\%) \times 250} = 3.446$

The test statistic is [19 - 12.5] ÷ 3.446 = 1.89

In words, we observed 6.5 more exceptions (19 - 12.5) than expected if the model is good, which is 1.89 standard deviations away from the expected number of exceptions. Since we know that a 95% one-tailed normal confidence interval implies a 1.645 cutoff, we know that 1.645 is also the cutoff for a 90% two-tailed since the normal is symmetrical, this falls outside the acceptance region. We reject the null, assuming that luck does not explain this, and find the model faulty.

Q-34. Solution: A

A is correct. The concept tested here is the understanding of the difference between the VaR parameter for confidence (here, namely 95% vs. 99%) and the validation procedure confidence level (namely 95%), and how they interact with one another. Using a 95% VaR confidence level creates a narrower nonrejection region than using a 99% VaR confidence level by allowing a greater number of exceptions to be generated. This in turn increases the power of the backtesting process and makes for a more reliable test than using a 99% confidence level.

Q-35. Solution: C

In the case of a bank that changed positions more frequently during the day, a penalty should be considered, but it is not necessarily given. In the case of bad luck, no penalty is given, as would be the case for a bank affected by unpredictable movements in rates or markets. However, when risk models are not precise enough, a penalty is typically given since model accuracy could have easily been improved.

Q-36. Solution: B

The risk manager will reject the hypothesis that the model is correctly calibrated if the number x of losses exceeding the VaR is such that:

$$\frac{x - p \times T}{\sqrt{p(1-p)T}} > z = 1.96$$

where p represents the failure rate and is equal to 1 – 99%, or 1%; and T is the number of observations = 225. And z = 1.96 is the two-tail confidence level quantile. If:

$$\frac{x - 0.01 \times 225}{\sqrt{0.01 \times (1 - 0.01) \times 225}} > z = 1.96$$

Then, x = 5.18. So the maximum number of exceedances would be 5 to conclude that the model is calibrated correctly..

Q-37. Solution: C

The 1996 amendment to Basel I and Basel II.5 use VaR with a 99% confidence interval and the FRTB uses the expected shortfall with a 97.5% confidence interval. Basel I market risk capital requirements produced a very current result because the 10-day horizon incorporated a recent period of time. The FRTB does not require adding a stressed VaR to the expected shortfall calculation. It was Basel II.5 that required the addition of a stressed VaR.

Q-38. Solution: B

The two types of risk recognized by the incremental risk charge are: (1) credit spread risk, and (2) jump-to-default risk. Jump-to-default risk is measured by 99.9% VaR and not 97.5% expected shortfall.

Q-39. Solution: C

Standard positions in the six-month zero:

$$\frac{30{,}000}{1 + \frac{2\%}{2}} = 29{,}703$$

Standard positions in the twelve-month zero:

$$\frac{1{,}000{,}000 + 30{,}000}{1 + 2.5\%} = 1{,}004{,}878.049$$

Q-40. Solution: D

General risk factor in market risk can be classied in four categories:

1. Equity price risk
2. Interest rate risk
3. Exchange rate risk
4. Commodity price risk

Q-41. Solution: A

Delta-normal VaR methods will provide accurate estimates of VaR for options only over those ranges in which the deltas of the contracts are stable. Deltas are normally unstable near the money and close to expiration.

Q-42. Solution: B

The diversified VaR is lower due to two factors. First, risk measures are not perfectly linear with maturity. Second, correlations are below unity, which reduces risk even further.

Q-43. Solution: A

With principal mapping, one risk factor is chosen that corresponds to the average portfolio maturity. With duration mapping, one risk factor is chosen that corresponds to the portfolio duration. With cash flow mapping, the portfolio cash flows are grouped into maturity buckets. Convexity mapping is not a method of VaR mapping for fixed income portfolios.

Q-44. Solution: D

D is correct. With duration mapping, a portfolio is replaced by a zero-coupon bond with maturity equal to the duration of the portfolio.

A is incorrect. Cash-flow mapping considers the present values of the cash flows placed to correspond to the maturities for which volatilities are provided. So, in cash-flow mapping, cash flows are grouped into maturity brackets.

B is incorrect. Cash-flow mapping considers the present values of the cash flows and uses the appropriate zero-coupon rate as the discount factor.

C is incorrect. Principal mapping is a simple method that considers the timing of redemption payments only. Correlations among zero-coupon bonds with different maturities are considered in cash-flow mapping.

Q-45. Solution: C

Under the cash flow mapping approach, each payment (and not only the last one) is associated with a different risk factor, so statement I. is incorrect. Statement II. is incorrect because the CF mapping approach is more correct than duration or maturity mapping.

Q-46. Solution: C

Mapping government bonds paying regular coupons onto zero coupon government bonds is an adequate process, because both categories of bonds are government issued and therefore have a very similar sensitivity to risk factors. However, this is not a perfect mapping since the sensitivity of both classes of bonds to specific risk factors (i.e., changes in interest rates) may differ.

Q-47. Solution: B

B is correct. With duration mapping, a portfolio is replaced by a zero-coupon bond with maturity equal to the duration of the portfolio. The risk of the hypothetical zeros is less than the risk of a coupon bond of comparable maturity. Therefore, the portfolio VaR using duration mapping is less than the portfolio VaR using principal mapping.Option A is not correct. Option C is incorrect because of the reason stated for the correctness of Option B. Option D is incorrect because of the same reason for the correctness of Option A.

Q-48. Solution: A

"I" is true. Using the copula approach, we can calculate the structures of correlation between variables separately from the marginal distributions. "IV" is also true. Correlation is a good measure of dependence when the measured variables are distributed as multivariate elliptical.

"II" is false. The correlation between transformed variables will not always be the same as the correlation between those same variables before transformation. Data transformation can sometimes alter the correlation estimate. "III" is also false. Correlation is not defined unless variances are finite.

Q-49. Solution: B

There is a -4% difference from the long-run mean correlation and April 2014 correlation (32% - 36% = -4%). The inverse of the β coefficient in the regression relationship implies a mean reversion rate of 75%. Thus, the expected correlation for May 2014 is 33.0%:

$$S_t = a(\mu_s - S_{t-1}) + S_{t-1}$$

S_t = 0.75(32% - 36%) + 0.36 = 0.33

Q-50. Solution: A

The autocorrelation for a one-period lag is 23% for the same sample. The sum of the mean reversion rate (77% given the beta coefficient of -0.77) and the one-period autocorrelation rate will always equal 100%.

Q-51. Solution: B

The fixed CDS spread is valued based on the default probability of the reference asset (Canadian bond) and the joint default correlation of First Bank (counterparty) and Canada. A paper loss occurs for the investor (CDS spread buyer) if the correlation risk between First Bank and Canada increases because the value of the CDS spread will decrease. The investor has wrong-way risk if there is positive correlation risk between First Bank and Canada. Increasing correlation risk increases the probability that the worst case scenario occurs where both First Bank and Canada default.

Q-52. Solution: D

Correlation estimates tend to be very volatile when short term time horizons are considered.

Q-53. Solution: D

D is correct. The required probability is:

$P\{t_{BB} \leqslant 2 \cap t_B \leqslant 2\}$

$= P\{[N^{-1}(Q_{BB}(t_{BB})) \le N^{-1}(Q_{BB}(2))] \cap [N^{-1}(Q_B(t_B)) \le N^{-1}(Q_B(2))]\}$

$= P\{[X_{BB} \le -1.209] \cap [X_B \le -0.533]\}$

A and B are both incorrect. In fact, under copula model, both xBB and xB are continuous random variables and thus both A and B correspond to zero probability. C is also incorrect because the transformation $N^{-1}$is not properly considered in this option.

Q-54. Solution: B

The trader would need to adjust hedge as follows:

$89.8 million × 1.0274 = $92.26 million

Thus, the trader needs to purchase additional TIPS worth $2.46 million.

Q-55. Solution: B

Defining $F^R$and as F^N the face amounts of the real and nominal bonds, respectively, and their corresponding DV01 as $DV01^R$ and $DV01^N$, a DV01 hedge is adjusted by the hedge adjustment factor, or beta, as follow:

$F^R = F^N \times \frac{DV01^N}{DV01^R} \times \widehat{\beta}$=100,000×0.072/0.051×1.2=169,412

Q-56. Solution: B

Defining $F^R$and as F^N the face amounts of the real and nominal bonds, respectively, and their corresponding DV01s as $DV01^R$ and $DV01^N$, a DV01 hedge is adjusted by the hedge adjustment factor, or beta, as follow:

$F^R = F^N \times \frac{DV01^N}{DV01^R} \times \widehat{\beta}$=100M×0.065/0.088×1.03=76.08 million

This regression hedge approach suggests that for every $100 million sold in T-bonds,Ryan should buy $76.08 million in TIPS.

Q-57. Solution: B

The 3-year spot rate can be solved for using the following equation:

$$\frac{\$1}{(1+6\%)(1+8\%)(1+10\%)} = \frac{\$1}{(1+r)^3}$$

$r = 7.988\%$

Q-58. Solution: A

The left-hand side of Jensen's inequality is the expected price in one year using the 1-year spot rates of 8% and 4%.

$$\mathrm{E}\left(\frac{\$1}{1+\mathrm{r}}\right)=0.5\times\frac{\$1}{1+8\%}+0.5\times\frac{\$1}{1+4\%}=0.94373$$

The expected price in one year using an expected rate of 6% computes the right-hand side of the inequality as:

$$\frac{\$1}{0.5\times(1+8\%)+0.5\times(1+4\%)}=\frac{\$1}{1.06}=0.9434$$

Next, divide each side of the equation by 1.06 to discount 1-year zero-coupon bond price for one more year at 6%. The price of the 2-year zero-coupon bond equals $0.89031(calculated as 0.94373/1.06), which is greater than $0.89000 (the price of a 2-year zero-coupon bond discounted for two years at the expected rate of 6%). Thus, Jensen's inequality reveal that $0.89031 > $0.89000.

Q-59. Solution: C

$$\left(\frac{1}{1+7\%}\times 50\%+\frac{1}{1+5\%}\times 50\%\right)/(1+6\%)=0.89007$$

Q-60. Solution: B

Under Model 1, it is true that the middle node recombines to the same current node. But these are future short-term rates; they are not the term structure: the term structure is spot rates at all maturities. Models that take the initial term structure implied by market prices are called arbitrage-free models. A different approach, however, is to start with assumptions about the interest rate process and about the risk premium demanded by the market for bearing interest rate risk and then derive the risk-neutral process. Models of this sort do not necessarily match the initial term structure and are called equilibrium models.

Q-61. Solution: B

Only statement 2 is correct. The Vasicek model implies decreasing volatility and non-parallel shifts from changes in short-term rates. The Ho-Lee model is actually more general than Model 2 (the no drift and constant drift models are special cases of the Ho-Lee model).

Q-62. Solution: B

Model 1 has a no-drift assumption. Using this model, the change in the interest rate is predicted as:

$$\mathrm{dr}=\sigma\mathrm{dw}=0.8\%\times(-0.5)=-0.4\%$$

Since the initial rate was 5% and dr = -0.40%, the new spot rate in one month is:

5% - 0.40% = 4.60%

Q-63. Solution: B

This short rate process has an annualized drift of 0.36%, so it requires the use of Model 2 (with constant drift). The change in the spot rate is computed as:

$$dr = \lambda dt + \sigma dw = 0.36\% \times \frac{1}{12} + 0.8\% \times (-0.5) = -0.37\%$$

Since the initial short-term rate was 5% and dr is -0.37%, the new spot rate in one month is:

5% - 0.37% = 4.63%

Q-64. Solution: C

$$\text{node }[2,0] = 4\% + (1\% + 0.8\%) \times \frac{1}{12} - 2 \times 2\% \times \sqrt{\frac{1}{12}} = 2.995\%$$

Q-65. Solution: D

The Ho-Lee model with drift is very flexible, allowing the drift terms each period to vary. Hence, the cumulative effect is additive. In contrast, the lognormal model with drift allows the drift terms to vary, but the cumulative effect is multiplicative.

Q-66. Solution:C

C is correct. In the CIR model, the basis-point volatility of the short rate is not independent of the short rate as other simpler models assume. The annualized basis-point volatility equals.

Q-67. Solution:C

Because we are using the Ho-Lee model,the drift is additive from period to period.

Therefore,the expected short-term interest rate at the end of the second period (i.e., middle node) is 5.4%+0.25%-0.1%=5.55%.

Q-68. Solution: C

Choice c is correct: the Ho-Lee model incorporates a premium to each rate change that can be different at each point in time.

Q-69. Solution: D

In the CIR model, the basis-point volatility of the short rate is not independent of the short rate as other simpler models assume. The annualized basis-point volatility equals $\sigma\sqrt{r}$ and therefore increases as a function of the square root of the rate.

Q-70. Solution: B

Choice B is correct: the Vasicek model incorporates mean reversion. The flexibility of the model also allows for risk premium, which enters into the model as constant drift or a drift that changes over time.

Q-71. Solution: D

Negative short-term interest rates can arise in models for which the terminal distribution of interest rates follows a normal distribution. The existence of negative interest rates does not make much economic sense since market participants would generally not lend cash at negative interest rates when they can hold cash earn a zero return. One method that can be used to address the potential for negative interest rates when constructing interest rates trees is to set all negative interest rates to zero. This localizes the change in assumptions to points in the distribution corresponding to negative interest rates and preserves the original rate free for all other observations. In comparison, adjusting the risk neutral probabilities would alter the dynamics across the entire range of interest rates and therefore not be an optimal approach.
When a model displays the potential for negative short-term interest rates, it can still be a desirable model to use in certain situations, especially in cases where the valuation depends more on the average path of the interest rate, such as in valuing coupon bonds. Therefore, the potential for negative rates does not automatically rule out the use of the model.

Q-72. Solution: A

A is correct.
The value of the 2-year zero-coupon bond is:

$$50\% \times \frac{\frac{1}{1.125}+\frac{1}{1.085}}{1.1} \times 1000 = 822.9763$$

Q-73. Solution: A

This is the same underlying bond and interest rate tree as in the call option example from this topic. However, here we are valuing a put option.
The option value in the upper node at the end of year 1 is computed as:

$$\frac{2.44 \times 0.6 + 0.38 \times 0.4}{1.0599} = 1.52$$

The option value in the lower node at the end of year 1 is computed as:

$$\frac{0.38 \times 0.6 + 0 \times 0.4}{1.0444} = 0.22$$

The option value today is computed as:

$$\frac{1.52 \times 0.76 + 0.22 \times 0.24}{1.0300} = 1.17$$

Q-74. Solution: A

A volatility smile is a common graphical shape that results from plotting the strike price and implied volatility of a group of options with the same expiration date. Since the volatility smile is downward sloping to the right, the implied distribution has a fatter left tail compared to the lognormal distribution of returns. This means that an extreme decrease in the DAX has a higher probability of occurrence under the implied distribution than the lognormal. The ES will therefore be larger when the methodology is modified.

Q-75. Solution: D

A plot of the implied volatility of an option as a function of its strike price demonstrates a pattern known as the volatility smile or volatility skew. The implied volatility decreases as the strike price increases. Thus, all else equal, a risk monitoring system which assumes constant volatility for equity returns will understate the implied volatility for a long position in a deep-in-the-money call.

Q-76. Solution: D

There is higher implied price volatility for low strike price equity options. "Crashophobia" is based on the idea that large price declines are more likely than assumed in Black-Scholes-Merton prices, not that volatility increases when prices decline. Compared to the lognormal distribution, traders believe the probability of large down movements in price is higher than large up movements. Increasing leverage at lower equity prices suggests increasing volatility.

Q-77. Solution: C

The prices obtained with C are the right ones because they correspond to prices at which you could sell or buy the options.

Q-78. Solution: A

Based on the put-call parity, $p_{BS} + S_0 e^{-qT} = c_{BS} + Ke^{-rT}$ and $p_{mkt} + S_0 e^{-qT} = c_{mkt} + Ke^{-rT}$

We can know that:

$p_{BS} - p_{mkt} = c_{BS} - c_{mkt}$

And c_{BS}= \$3.50, c_{mkt}= \$3.00, p_{BS} = \$2.00.

So p_{mkt} = $1.50.

Therefore A is the correct answer.

Q-79. Solution: C

Explanation: An empirical distribution with a fat right tail generates a higher implied volatility for higher strike prices due to the increased probability of observing high underlying asset prices.

Q-80. Solution: D

Away-from-the-money currency options have greater implied volatility than at-the-money currency options, this pattern is a volatility smile.

Q-81. Solution: A

Leverage changes and “crashophobia” have both been offered as explanations for the inverse relation between implied volatility and asset prices for equity options. At lower equity prices, the firm’s capital structure is more leveraged, resulting in greater price volatility.

Q-82. Solution: A

Currency options exhibit volatility smiles because the at-the-money options have lower implied volatility than away-from-the-money options.

Equity traders believe that the probability of large price decrease is greater than the probability of large price increase. Currency trades’ beliefs about volatility are more symmetric as there is no large skew in the distribution of expected currency values (i.e. there is a greater chance of large price movements in either direction).

Q-83. Solution: A

A downward sloping volatility skew indicates that out of the money puts are more expensive than predicted by the Black-Scholes-Merton model and out of the money calls are cheaper than expected predicted by the Black-Scholes-Merton model. The implied distribution has fat left tails and thin right tails.

Q-84. Solution: A

The implied distribution of the underlying equity prices derived using the general volatility smile of equity options has a heavier left tail and a less heavy right tail than a lognormal distribution of underlying prices. Therefore, using the lognormal distribution of prices causes deep-out-of-the-money call options on the underlying to be priced relatively high.

The implied distribution of underling foreign currency prices derived using the general volatility smile of foreign currency options has heavier tail than a lognormal distribution of underlying prices. Therefore, using the lognormal distribution of prices causes deep-out-of-the-money call options on the underlying to be priced relatively low.

Q-85. Solution: B

An in-the-money call has a strike price below 30.Therefore, using the chart above .its implied volatility is greater than the at-the-money volatility, so using the at-the-money volatility would result in pricing an in-the-money call option lower than its fair price.