2021-03-10.

2.3(a) $r''(t)+4r'(t)+3r(t)=0$

$\lambda^2+4\lambda+3=0 \Rightarrow (\lambda+3)(\lambda+1)=0$

$\begin{matrix} 1 & & +3 \\ & \times & \\ 1 & & +1 \end{matrix} \Rightarrow r(t)=Ae^{-3t}+Be^{-t}$

$r(0_-)=r(0_+)=0$
$r'(0_-)=r'(0_+)=2$ $\Rightarrow \begin{cases} A+B=0 \\ -3A-B=2 \end{cases}$

$\Rightarrow \begin{cases} A=-1 \\ B=1 \end{cases} \Rightarrow r_{zi}(t)=-e^{-3t}+e^{-t} \quad (t\geq 0)$

(b) $r''(t)+2r'(t)+r(t)=0$

$\lambda^2+2\lambda+1=0 \Rightarrow (\lambda+1)^2=0$

$\Rightarrow \lambda=-1$ (二重)

$\Rightarrow r(t)=(A+Bt)e^{-t}, \quad r'(t)=Be^{-t}-(A+Bt)e^{-t}$

$\begin{cases} A=0 \\ B-A=2 \end{cases} \Rightarrow \begin{cases} A=0 \\ B=2 \end{cases} \Rightarrow r_{zi}(t)=2te^{-t} \quad (t\geq 0)$

2.8 $\lambda^2+6\lambda+5=0$

$(\lambda+1)(\lambda+5)=0 \Rightarrow$ 齐次通解:

$\lambda=-1$ 或 $-5$. $\quad A_1e^{-t}+A_2e^{-5t}$

当 $t>0$, $e(t)=2e^{-2t}$.

设特解为 $Be^{-2t}$, 代入得 $B=-6$

结合 $r(0_+)=-4$, $r'(0^+)=6 \Rightarrow A_1=A_2=1$

$\Rightarrow$ 当 $t>0$, $r(t)=e^{-t}+e^{-5t}-6e^{-2t}$

当$t<0$，$e(t)=e^{2t}$. 设特解为$Be^{2t}$，代入得：

$4B+6\cdot 2B+5B=4-2\cdot 2+1=1 \Rightarrow B=\frac{1}{21}$

$\Rightarrow$ 特解为 $r(t)=\frac{1}{21}e^{2t}$.

求$r(0_-)$与$r'(0_-)$

$e(t)=e^{2t}\varepsilon(-t)+2e^{-2t}\varepsilon(t)$

$e'(t)=2e^{2t}\varepsilon(-t)-e^{2t}\delta(t)+(-4)e^{-2t}\varepsilon(t)+2e^{-2t}\delta(t)$

$=2e^{2t}\varepsilon(-t)+\delta(t)-4e^{-2t}\varepsilon(t)$.

$e''(t)=4e^{2t}\varepsilon(-t)-2e^{2t}\delta(t)+\delta'(t)+8e^{-2t}\varepsilon(t)-4e^{-2t}\delta(t)$

$=\delta'(t)-6\delta(t)\ldots$

右端：$\delta'(t)$ $\delta(t)\ldots$

设 $r''(t)=A\delta'(t)+B\delta(t)+Cu(t)$ ①

$r'(t)=A\delta(t)+Bu(t)$ ②

$r(t)=Au(t)$ ③

②积分 $\Rightarrow r(0_+)-r(0_-)=A=1 \Rightarrow r(0_-)=-5$.

①积分 $\Rightarrow r'(0_+)-r'(0_-)=B$

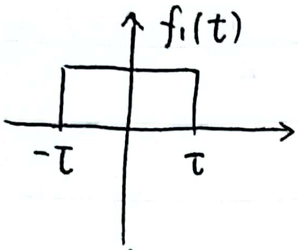
由$\delta(t)$配平：$B+6A=-8 \Rightarrow B=-14$

$\Rightarrow r'(0_-)=20$，$r(0_-)=-5$ ⑤

代入原方程，结合⑤的初始条件，

得 当$t<0$时，$r(t)=-\frac{4}{3}e^{-t}-\frac{26}{7}e^{-5t}+\frac{1}{21}e^{2t}$

2.17(b) $f_1(t)=f_2(t)=\varepsilon(t+\tau)-\varepsilon(t-\tau)$

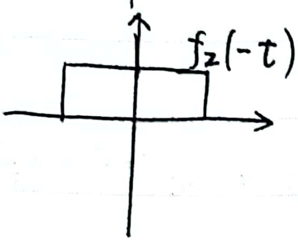


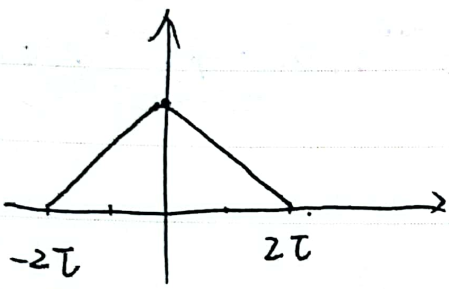

$\Rightarrow$ 当$|t|<2\tau$时，卷积值为不为0

$f_1(t)*f_2(t)=f_1'(t)*\int f_2(t)d\tau$

$=R(t+2\tau)-2R(t)+R(t-2\tau)$

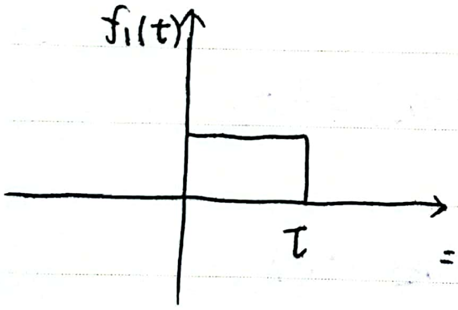
$(|t|<2\tau)$




改写：$ans=\begin{cases}-|t|+2\tau, & |t|<2\tau\\ 0, & |t|>2\tau\end{cases}$

2.17(c) $f_1(t)=f_2(t)=\varepsilon(t)-\varepsilon(t-\tau)$

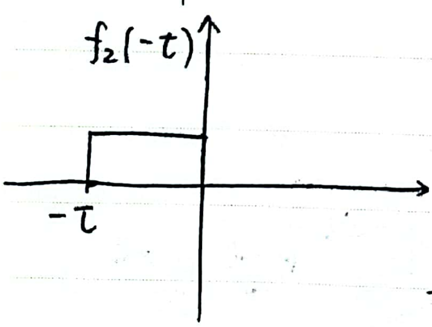


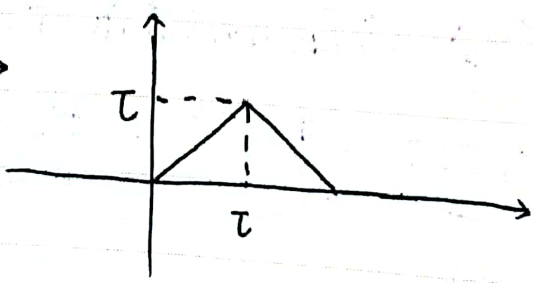

$\Rightarrow$ 当 $0<t<2\tau$时，卷积 $\neq 0$.

$f_1(t)*f_2(t)$

$=f_1'(t)*\int f_2(t)dt=[\delta(t)-\delta(t-\tau)]*[R(t)-R(t-\tau)]$

$=R(t)-R(t-\tau)-R(t-\tau)+R(t-2\tau)$

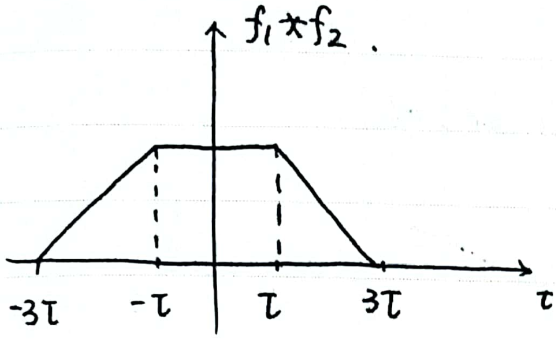



$\Rightarrow ans=\begin{cases}R(t)-2R(t-\tau)+R(t-2\tau), & 0\le t\le 2\tau\\ 0, & else\end{cases}$

(d) $f_1 * f_2 = [\delta(t+\tau) - \delta(t-\tau)] * [R(t+2\tau) - R(t-2\tau)]$

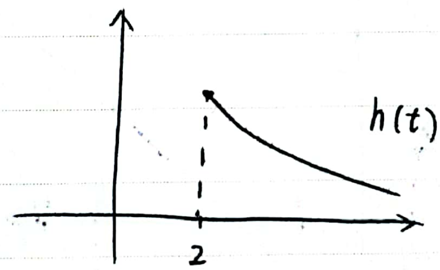
$= R(t+3\tau) - R(t-\tau) - R(t+\tau) + R(t-3\tau)$




2.18 (a) $r(t) = \int_{-\infty}^{t} e^{-(t-\tau)} e(\tau-2) d\tau$

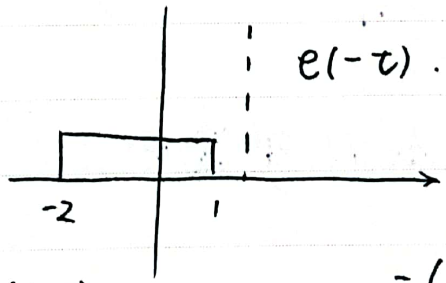
当 $e(t) = \delta(t)$ 时，$h(t) = \int_{-\infty}^{t} e^{-(t-\tau)} \delta(\tau-2) d\tau$

$= e^{-(t-2)} \varepsilon(t-2)$




(b) $e(t) = \varepsilon(t+1) - \varepsilon(t-2)$

$r(t) = e(t) * h(t)$




当 $1 \leq t \leq 4$ 时，$r(t) = \int_{2}^{1+t} e^{-(t-2)} dt = 1 - e^{-(t-1)}$

当 $4 < t < +\infty$ 时，$r(t) = \int_{-2+t}^{1+t} e^{-(t-2)} dt = e^{-(t-4)} - e^{-(t-1)}$

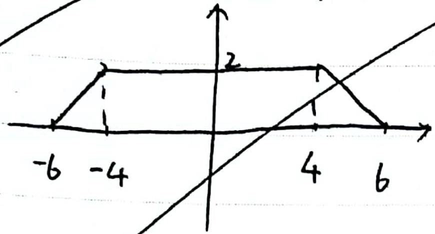
当 $t < 1$，$r(t) = 0$

21. $f_1(t) = \varepsilon(t+1) - \varepsilon(t-1)$

$f_2(t) = \delta(t+5) + \delta(t-5)$

$f_3(t) = \delta(t+\frac{1}{2}) + \delta(t-\frac{1}{2})$

(a) $f_1 * f_2 = \varepsilon(t+6) - \varepsilon(t+4) - \varepsilon(t-4) + \varepsilon(t-6)$

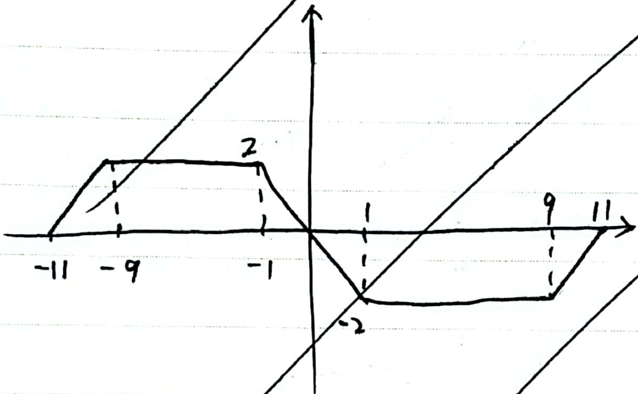



(b) $f_1 * f_2 * f_2 = f_1 * (f_2 * f_2)$

$= f_1 * [\delta(t+5) - \delta(t-5)] * [\delta(t+5) - \delta(t-5)]$

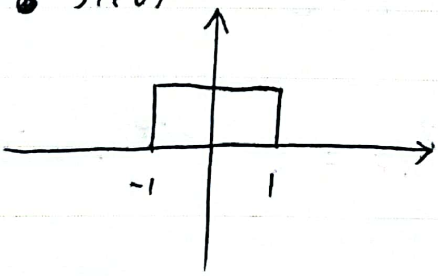
$= f_1 * [\delta(t+10) - 2\delta(t) \cancel{-\delta(t)} + \delta(t-10)]$

$= \varepsilon(t+11) - \varepsilon(t+9) - 2\varepsilon(t+1) + 2\varepsilon(t-1) + \varepsilon(t-9) - \varepsilon(t-11)$

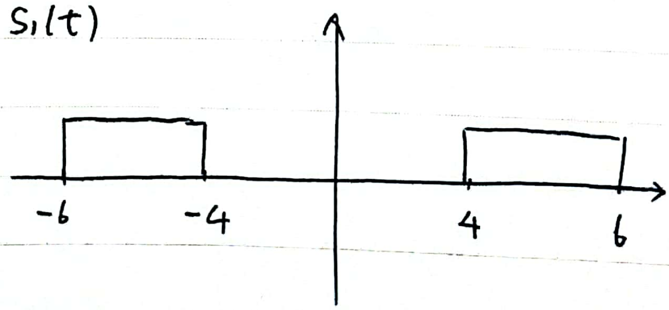



(c) 先对 $s_1(t)$ 取 10 的窗口，

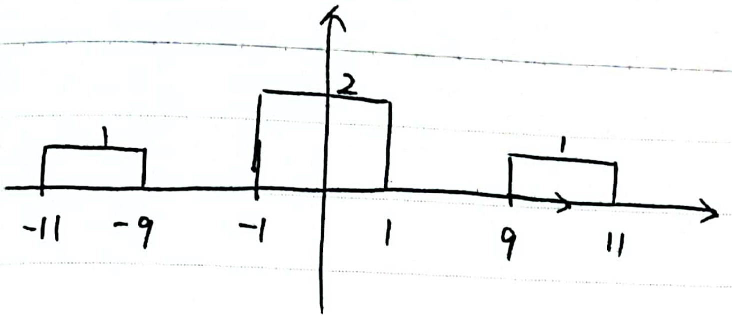
(a) $f_1(t)$



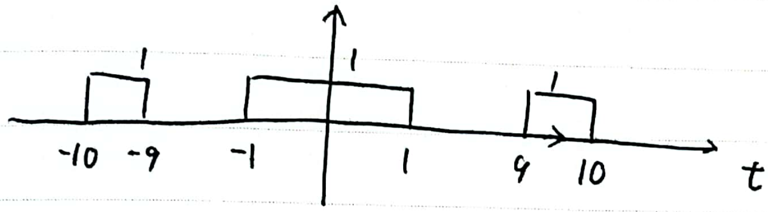

$\Rightarrow s_1(t)$



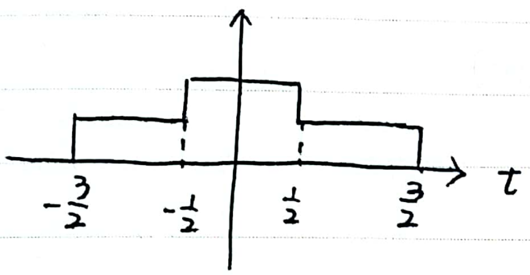
(b)




(c)




(d)




2.26 $r_1(t) = \delta(t)$

$r_2(t) = \delta(t) * h_a(t) = \delta(t-1)$

$r_3(t) = [\delta(t) * h_a(t)] * h_b(t)$

$= \delta(t-1) * [\varepsilon(t) - \varepsilon(t-3)] = \varepsilon(t-1) - \varepsilon(t-4).$

$\Rightarrow h_1(t) = \varepsilon(t) - \varepsilon(t-3)$

$h_2(t) = \varepsilon(t-1) - \varepsilon(t-4)$

$h_3(t) = [\varepsilon(t) - \varepsilon(t-3)] * [\varepsilon(t-1) - \varepsilon(t-4)]$

$= [\delta(t) - \delta(t-3)] * [R(t-1) - R(t-4)]$

$= R(t-1) - R(t-4) - R(t-4) + R(t-7).$

$\Rightarrow h(t) = u(t) - u(t-3) + u(t-1) - u(t-4) +$
$R(t-1) - 2R(t-4) + R(t-7)$

2.29 $\cancel{r_{zs}=}$ $e(t) = \sin t \cdot u(t)$

$e'(t) = \cos t\, u(t) + \sin t \cdot \delta(t)$

$= \cos t \cdot u(t)$

$e''(t) = -\sin t \cdot u(t) + \cos t \cdot \delta(t)$

$= -\sin t \cdot u(t) + \delta(t)$.

$= -e(t) + \delta(t)$

$\Rightarrow \quad e'' + e = \delta(t)$.

$r_{zs} = e(t) * h(t)$

$r''_{zs} = e''(t) * h(t) \quad = (\delta(t) - e(t)) * h(t)$

$= h(t) - r_{zs}$

$\Rightarrow h(t) = r_{zs} + r''_{zs}$

$r_{zs} = R(t) - 2R(t-1) + R(t-2)$

$\Rightarrow h(t) = R(t) - 2R(t-1) + R(t-2) + \delta(t) - 2\delta(t-1) + \delta(t-2)$