Critical Features Tracking on Triangulated Irregular Networks by a Scale-Space Method

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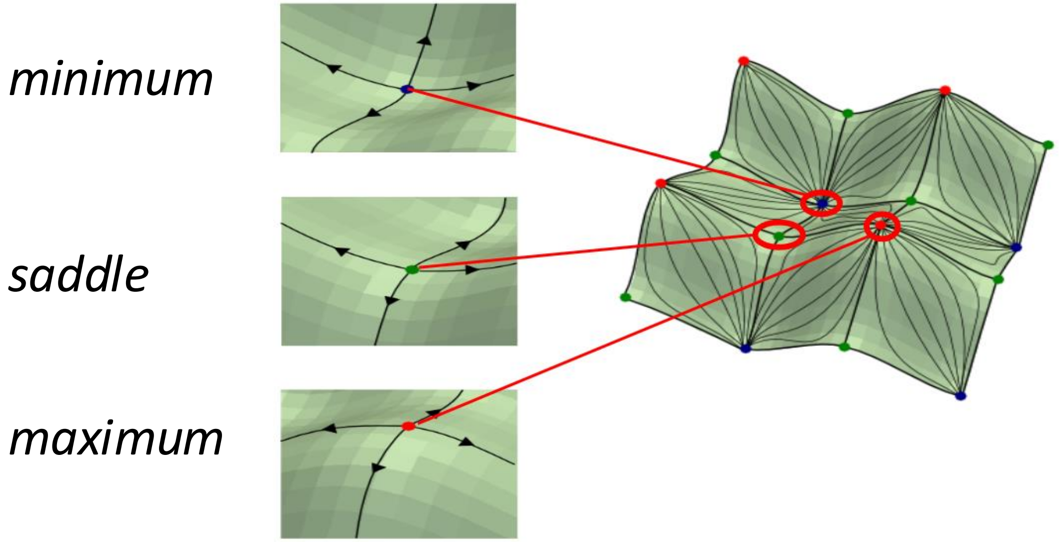
Introduction

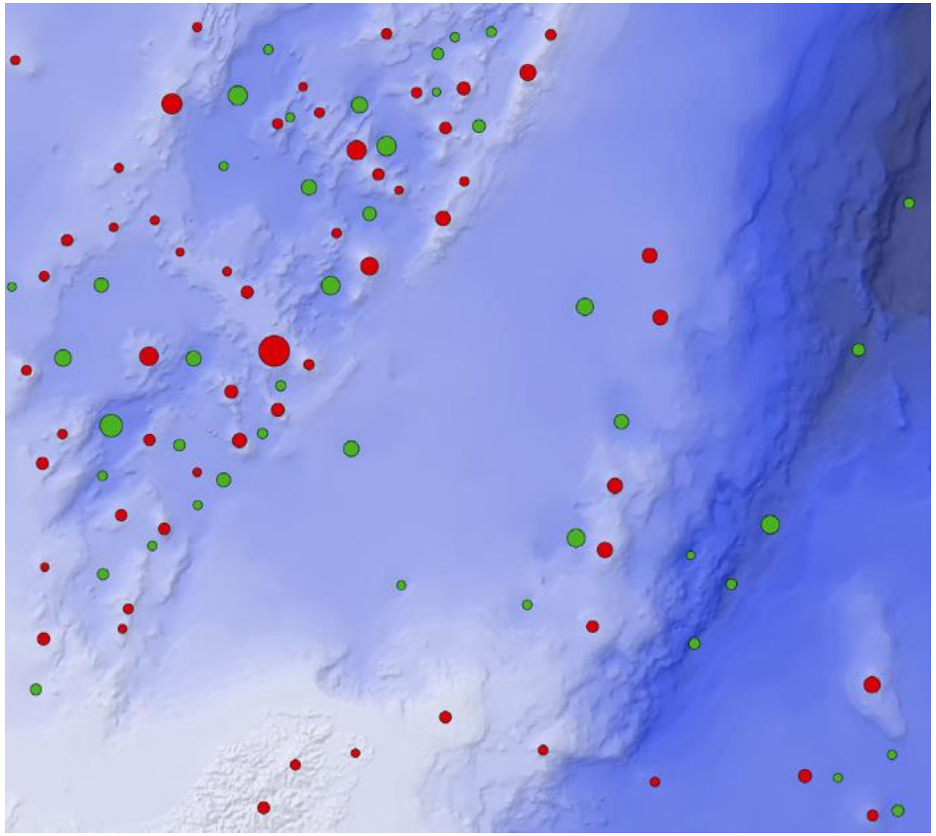
Critical features

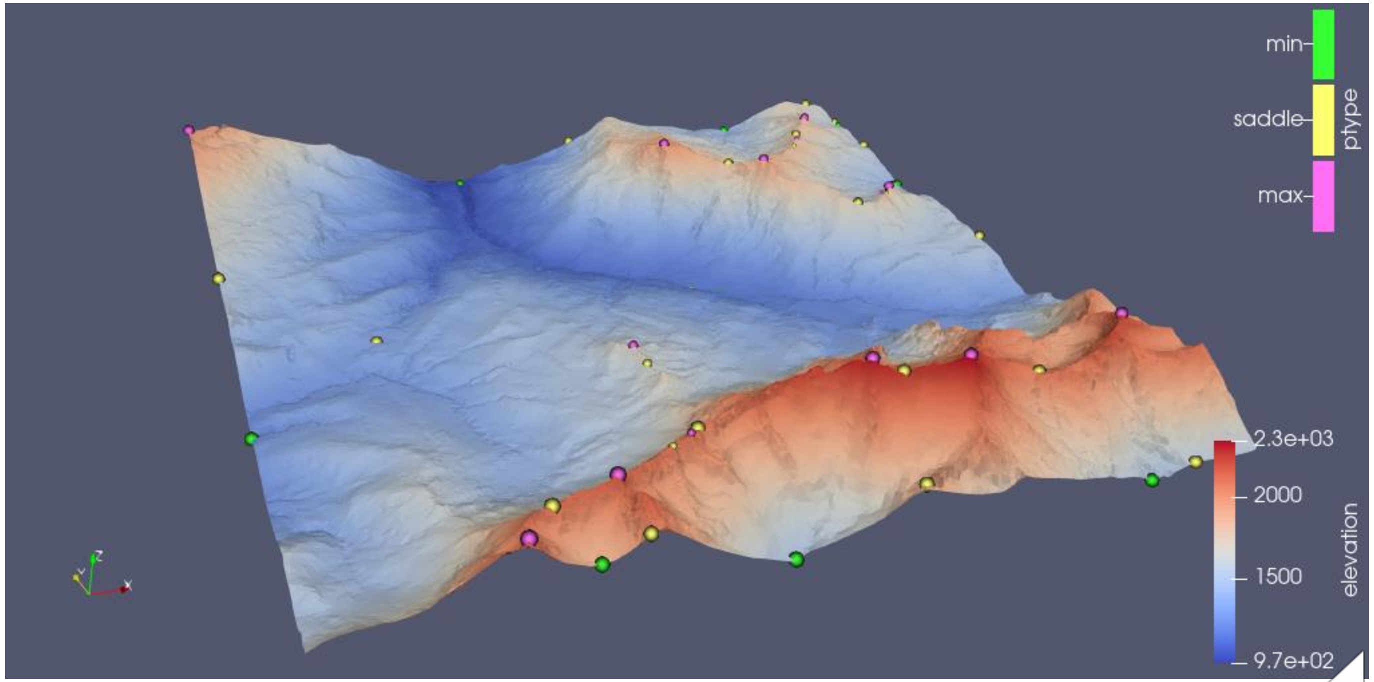
- **Critical features** consist of representative points together with their relations and regions of influence where the surface topology undergoes significant changes.
- **Critical points** of a scalar function are locations where gradient of the function vanishes – maxima (peaks), saddles, and minima (pits).



Critical features

- Understanding the topological structure of a surface is crucial in several downstream applications:
  - e.g. *cartography, land-use planning.*





The left figures from Rocca, L., Jenny, B., & Puppo, E. (2017). A continuous scale-space method for the automated placement of spot heights on maps. Computers & Geosciences, 109, 216-227.

Scale Space

- Scale space is a widely used framework in image processing and analysis.
- Similarly to the human visual system, a scale-space method processes data at different levels of details.

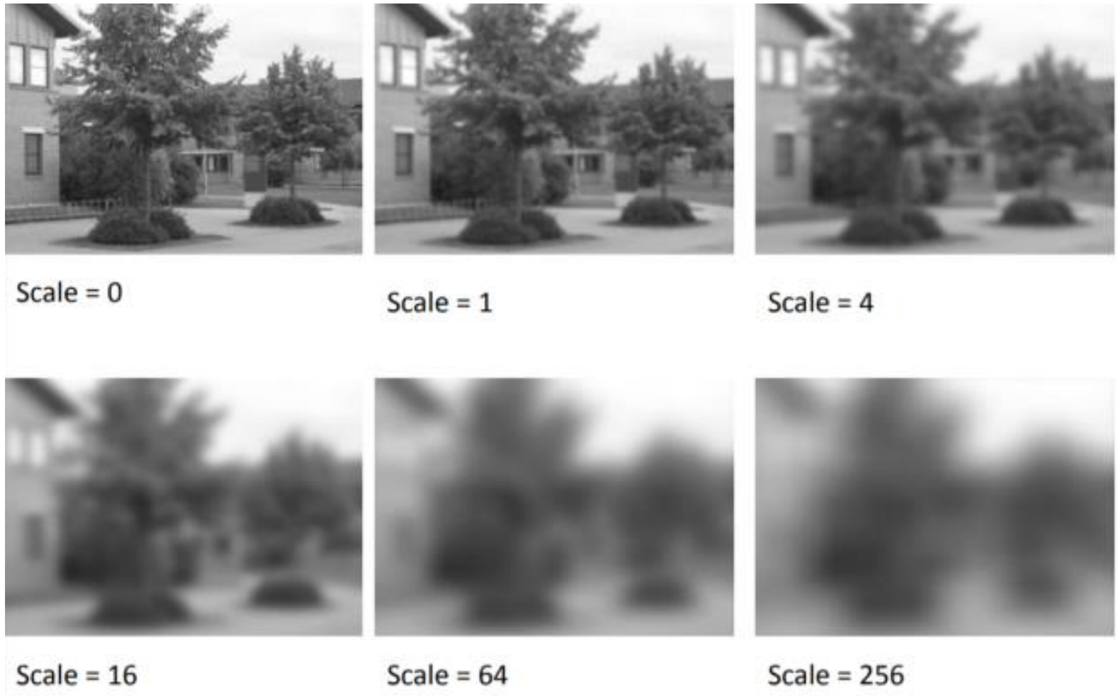


Image credit: Wikipedia of scale-space

Critical points tracking through a scale-space method

- **Deep structure of a surface**[1]: tracking critical points across scales.
  - Critical points are identified in the finest scale and tracked.
- **Life span**: number of scales at which a critical point survives
  - It indicates its significance in the data's underlying structure.

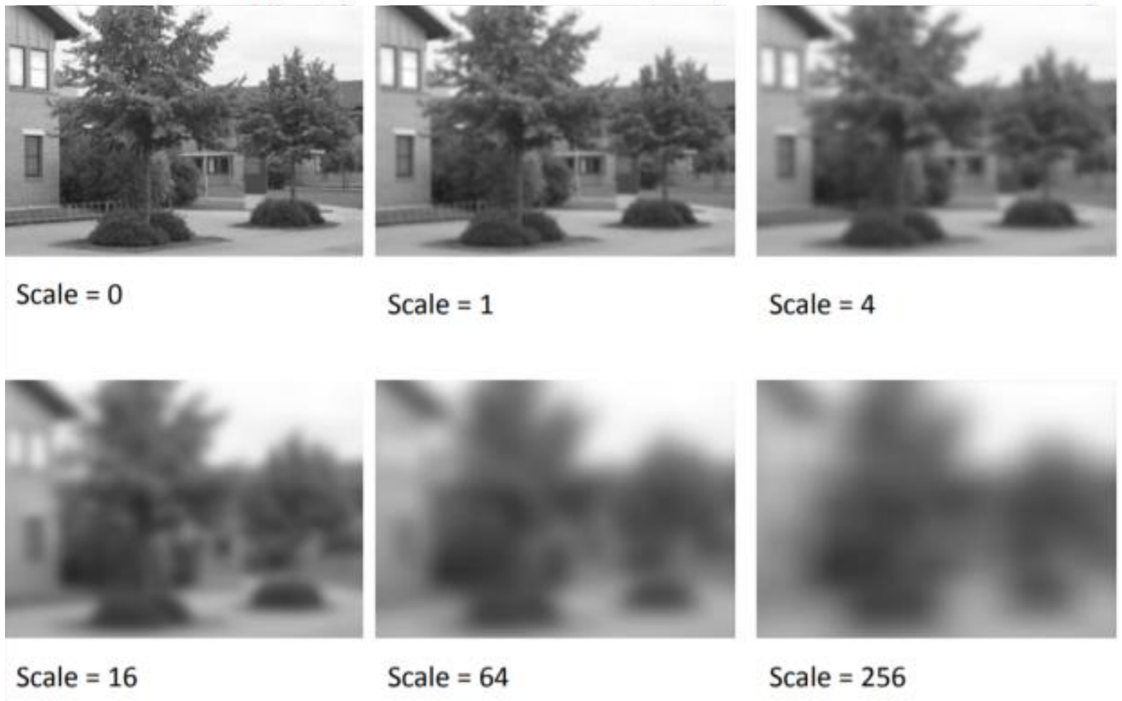
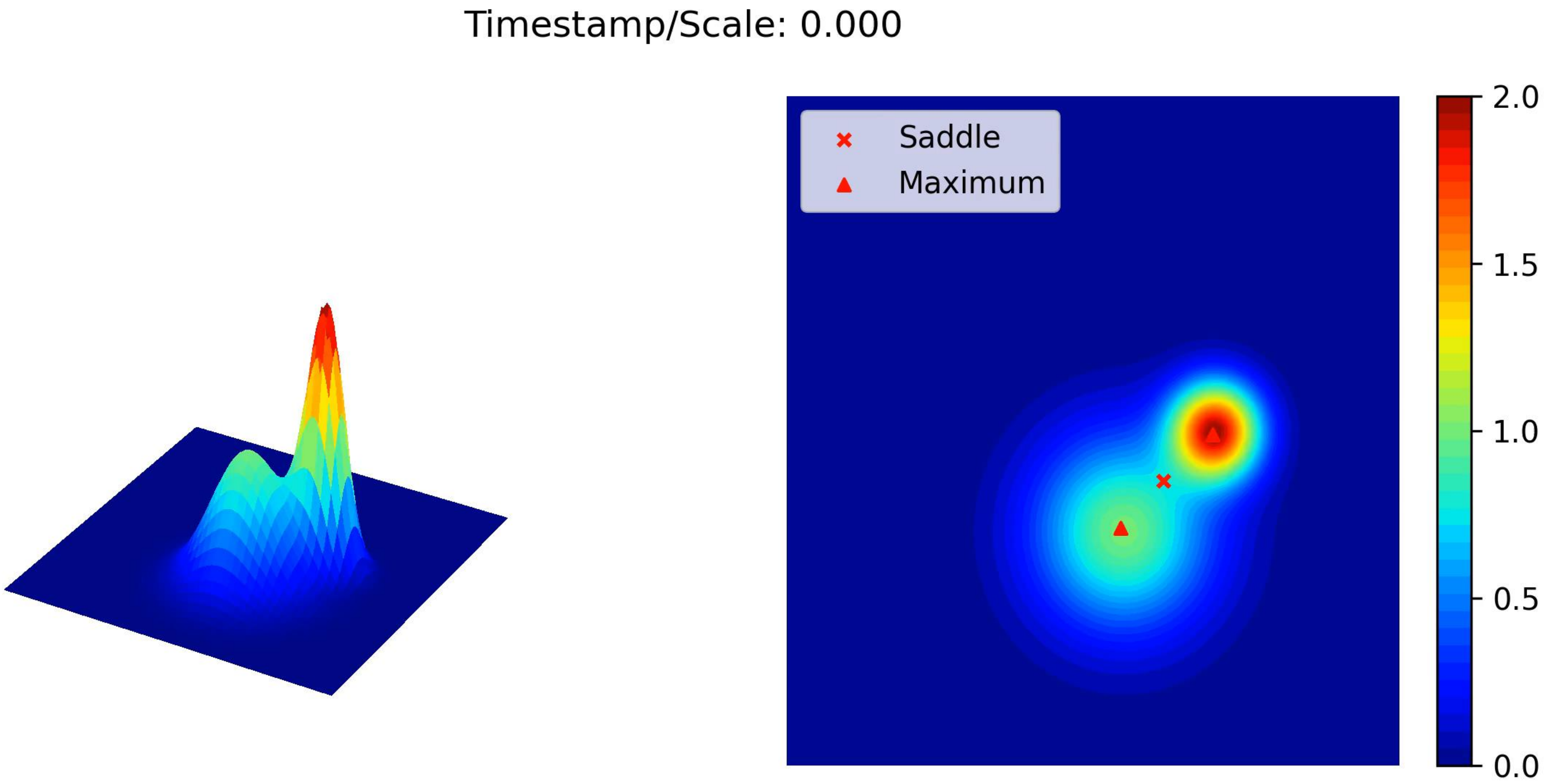


Image credit: [Wikipedia of scale-space]

[1] *Lindeberg, T. (2013). Scale-space theory in computer vision (Vol. 256). Springer Science & Business Media.*

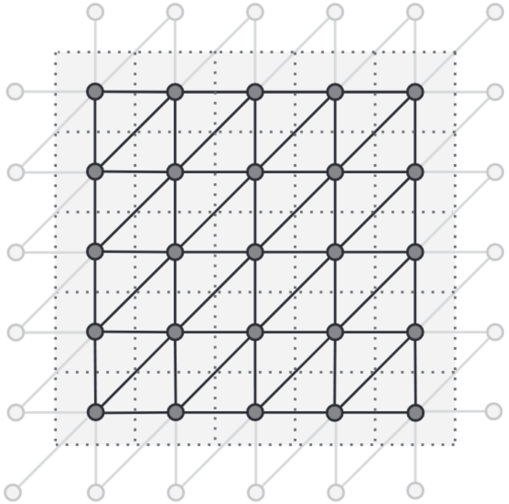
Image from Rocca, L., Jenny, B., & Puppo, E. (2017). A continuous scale-space method for the automated placement of spot heights on maps. Computers & Geosciences, 109, 216-227.

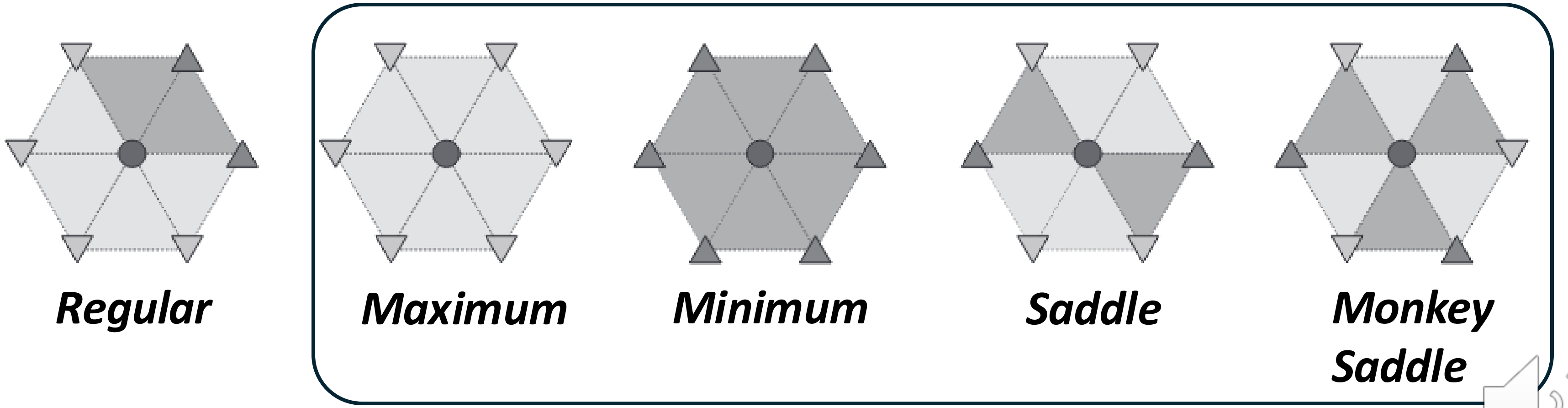
Critical points tracking through a scale-space method



A virtually continuous representation of the deep structure of the scale space[2]

- In a raster DEM, a scale space can be constructed by repeatedly applying a Gaussian smoothing operation on a regular grid.
- In [2], each regular grid cell is divided into two triangles to produce a piece-wise linear approximation of the terrain surface.
- Critical points are identified by inspecting the relative elevation to directly connected neighboring vertices on the triangle mesh.

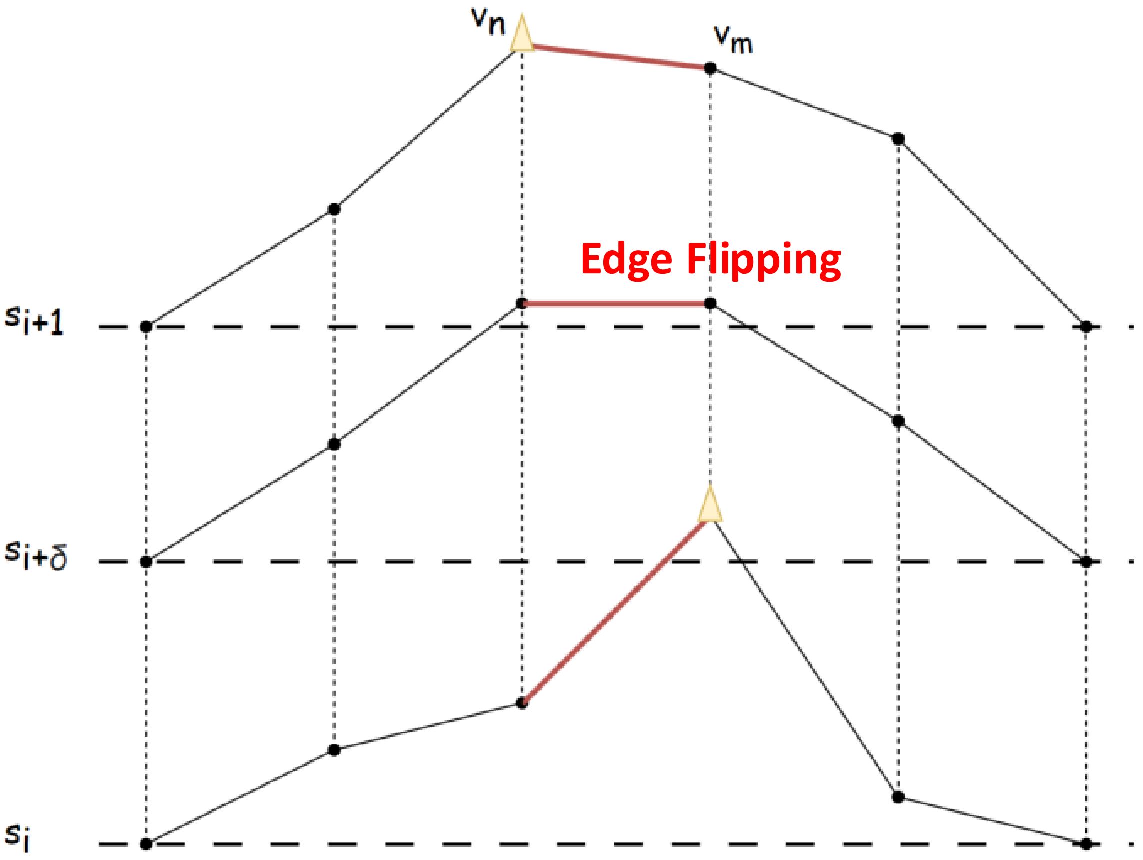




[2] Rocca, L., & Puppo, E. (2013). A virtually continuous representation of the deep structure of scale-space. In Image Analysis and Processing–ICIAP 2013: 17th International Conference, Naples, Italy, September 9-13, 2013, Proceedings, Part II 17 (pp. 522-531). Springer Berlin Heidelberg.

A virtually continuous representation of the deep structure of the scale space[2]

- Critical point tracking by
  - **Virtually continuous scale-space assumption**: vertex elevations change linearly between neighboring scale layers.
  - **Edge flipping event**: tracking critical point transition.
    - *For example, relative elevation of two vertices changes at timestamp/scale* $s_{i+\delta}$



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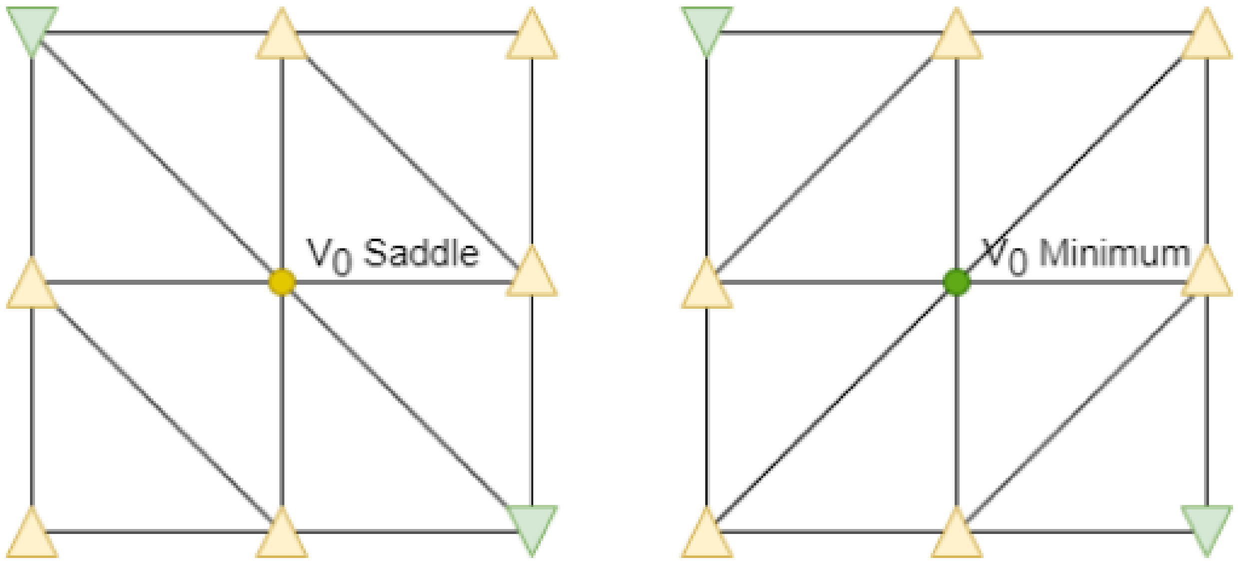
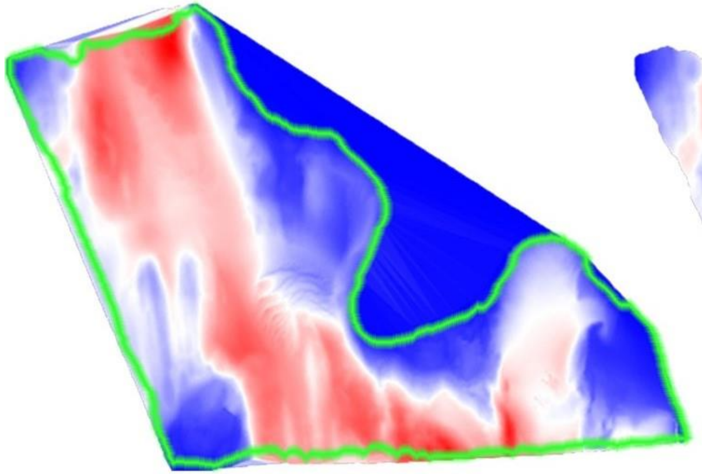
From regular grids to Triangulated Irregular Networks (TINs)

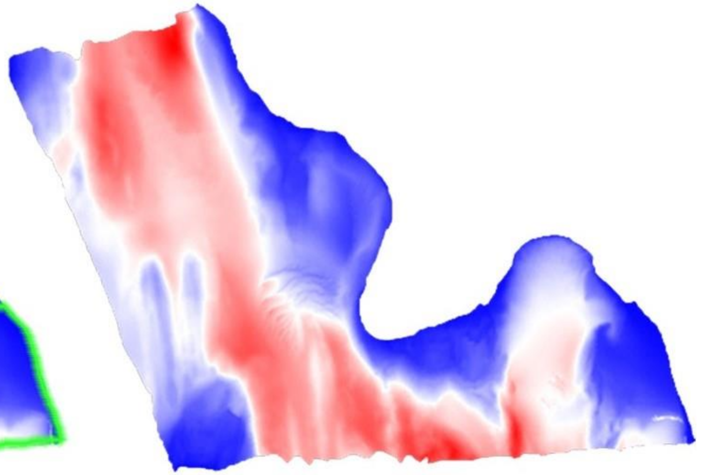
- **Wide availability of point cloud data**
  - Acquired extensively through LiDAR (Light Detection and Ranging), enabling detailed and accurate surface mapping.
- If we convert a point cloud to raster format and then triangulate the grid, the preset edge connection on a regular grid affects the classification of critical points.

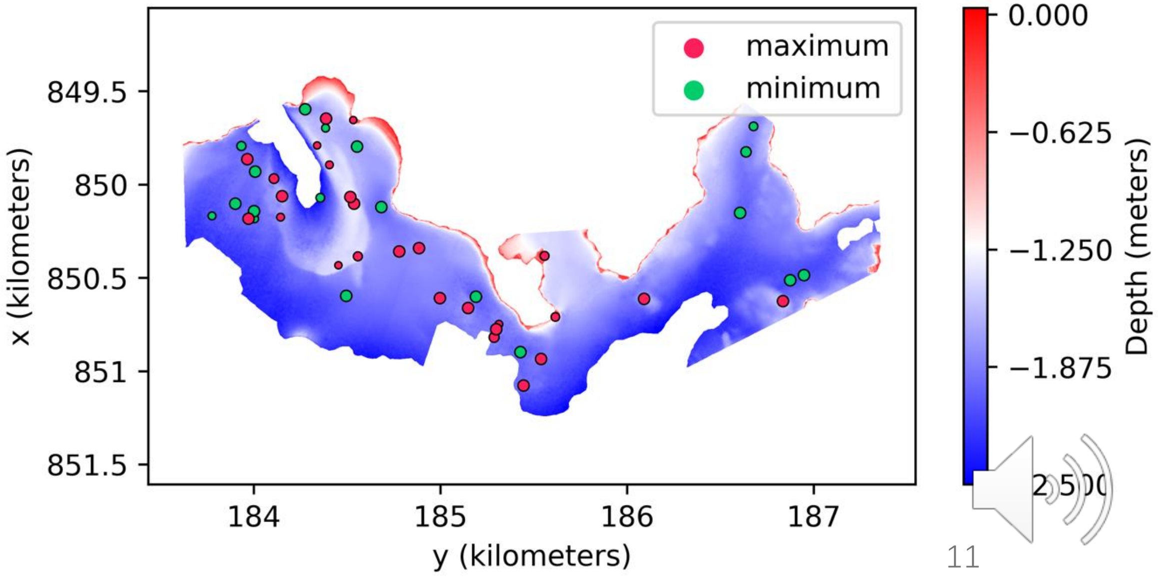


From regular grids to Triangulated Irregular Networks (TINs)

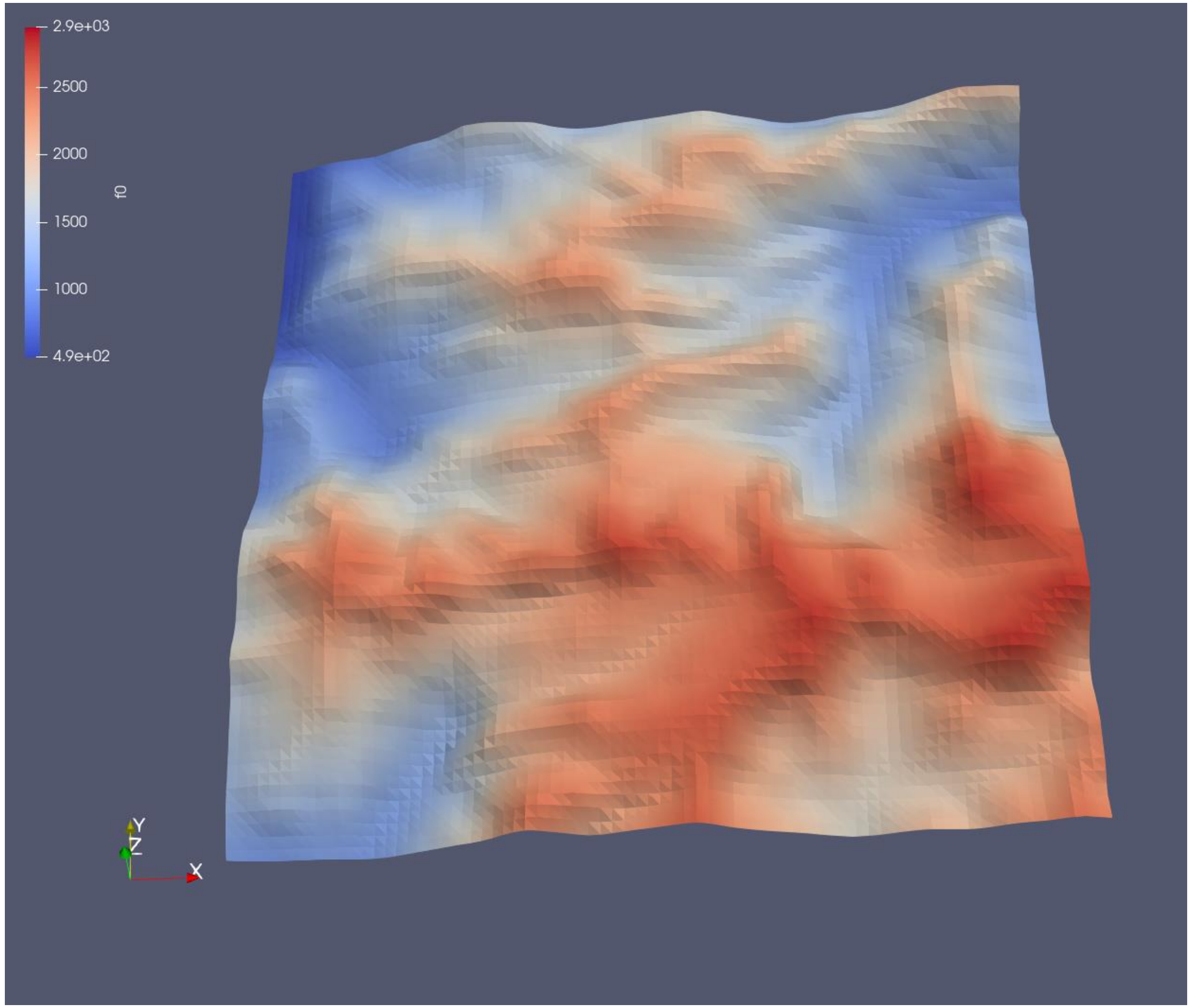
- For regular grids, trade-off between accuracy and computation burden - $O(N^2)$.
- Adaptive resolution of a TIN allows for accurate tracking with lower cost.
- TINs can naturally handle irregular boundary geometry.

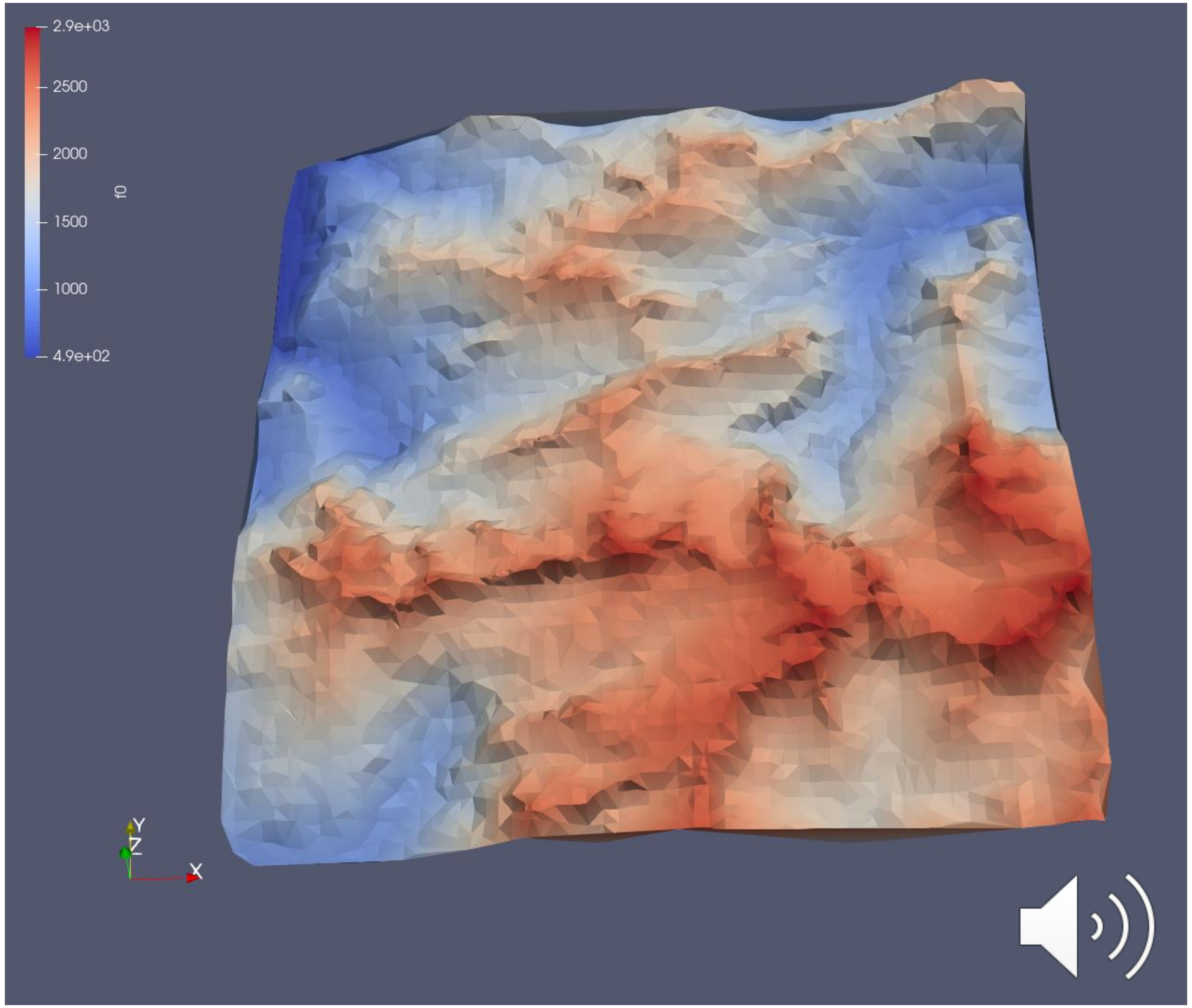




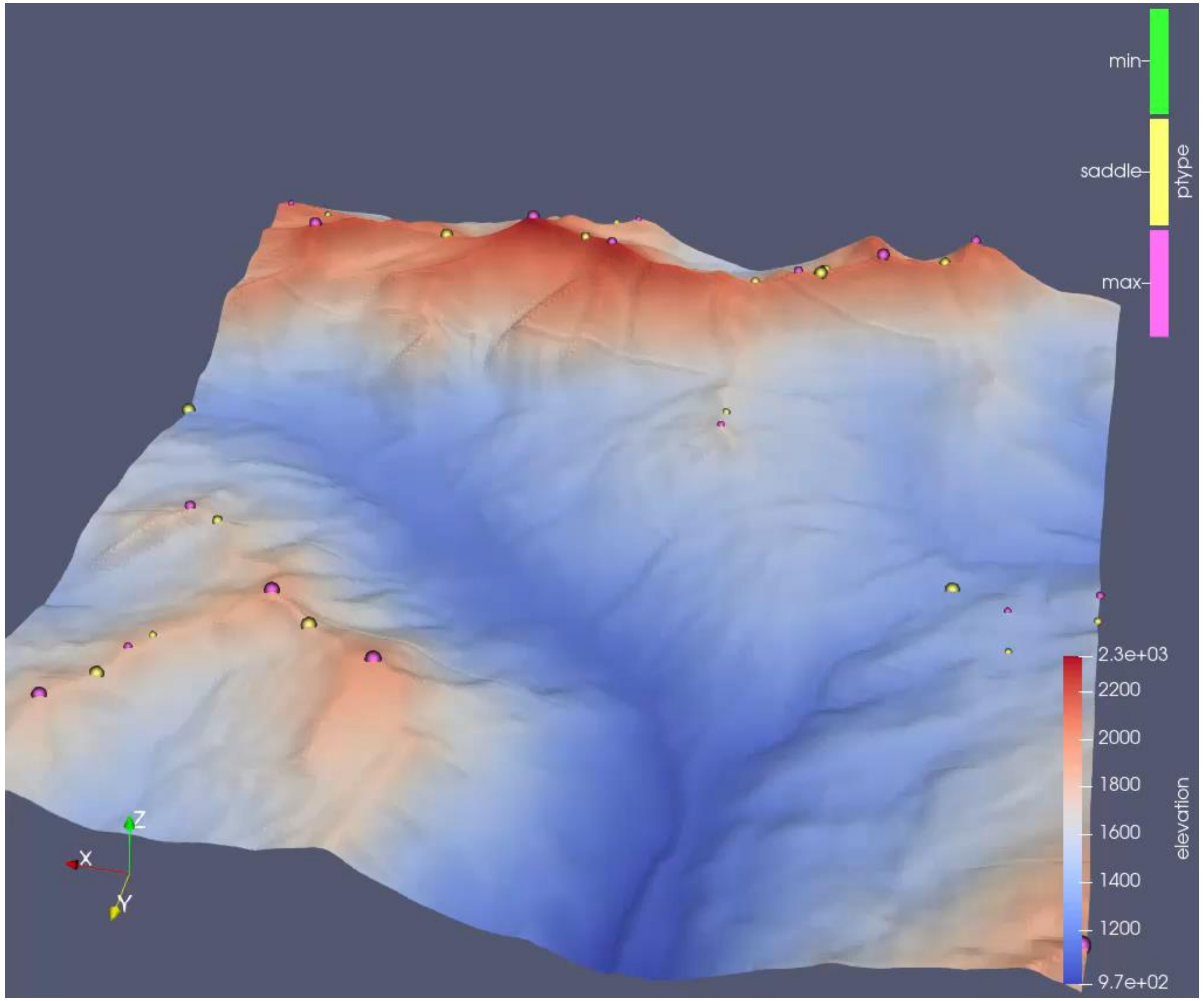


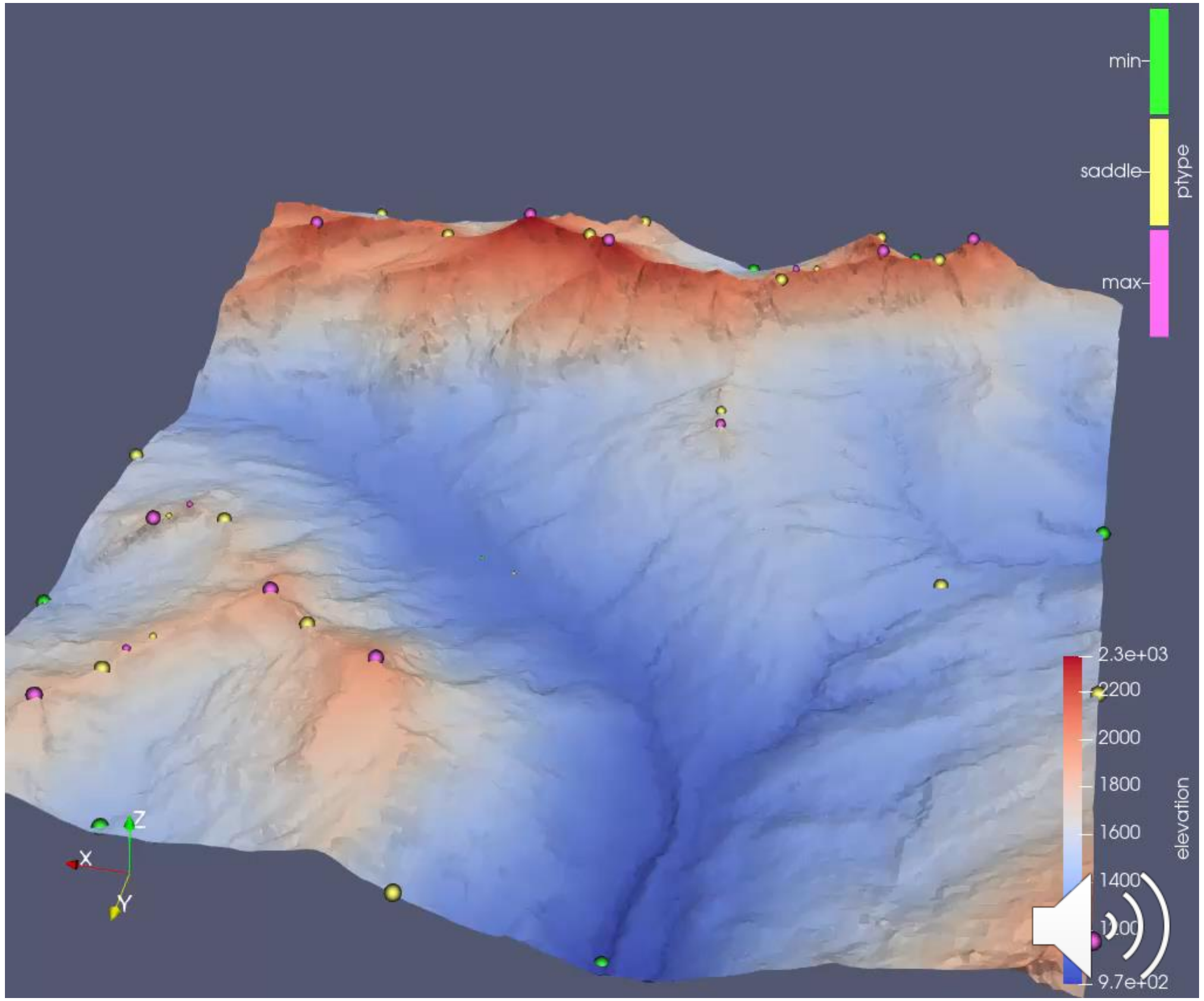
Gridded DEMs vs. TINs





Gridded DEMs vs. TINs





Methodology

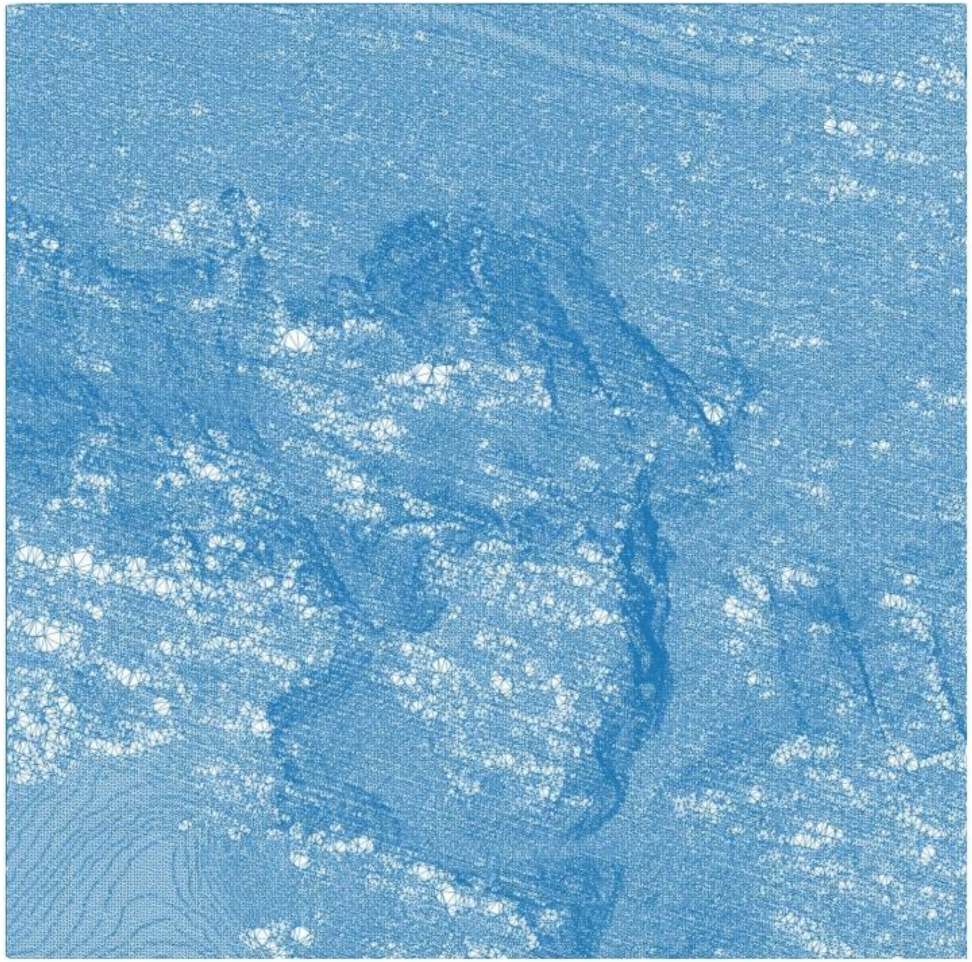
TIN generation from input point clouds

- **Furthest Point Sampling (FPS)**
  - Evenly distributed point cloud
- **Patch-based FPS (PFPS)**
  - Adaptive resolution for different local complexity levels

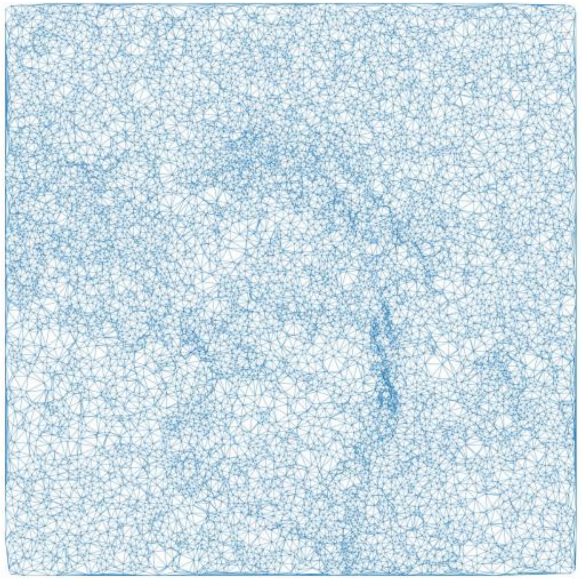
Algorithm 1: Patch-based Furthest Point Sampling

Input: Point cloud pcd, patch size s, percentage of points to keep ρ
Output: Down-sampled point cloud pcd_{down}

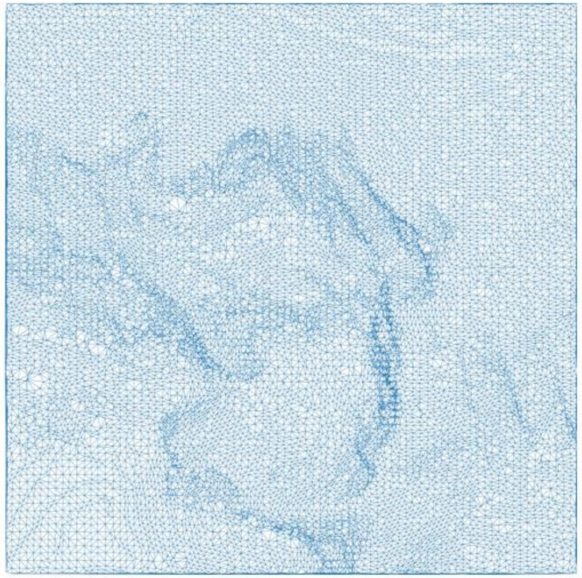
```
pcd_down ← ∅
// Divide the point cloud into patches over xy-plane
patches ← subdivide(pcd, s)
foreach patch in patches do
    // Estimate the local curvature via the covariance matrix
    covmat ← Covariance Matrix (patch)
    λ1, λ2, λ3 ← Eigenvalues (covmat)
    // λ1 ≥ λ2 ≥ λ3
    patch_curv ← λ1/(λ1 + λ2 + λ3)
end
// Normalize the estimated curvatures as sampling weights
patch_w ← patch_curv / Σ_patches patch_curv
foreach patch in patches do
    pcd_patch ← FPS (patch, ρ × patch_w)
    pcd_down ← pcd_down ∪ pcd_patch
end
return pcd_down
```



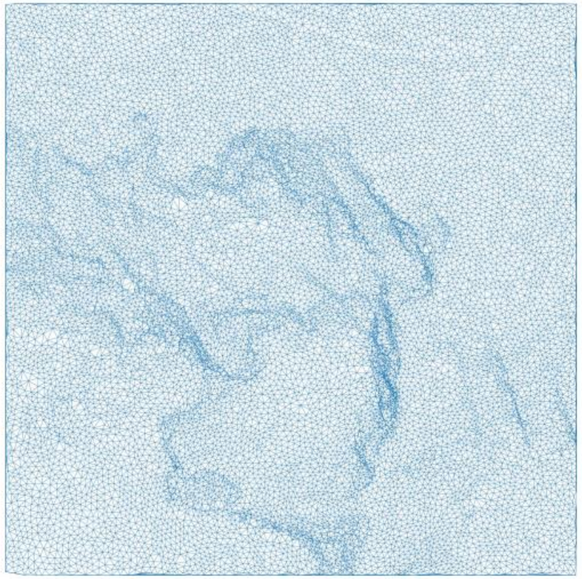
TIN from input point cloud



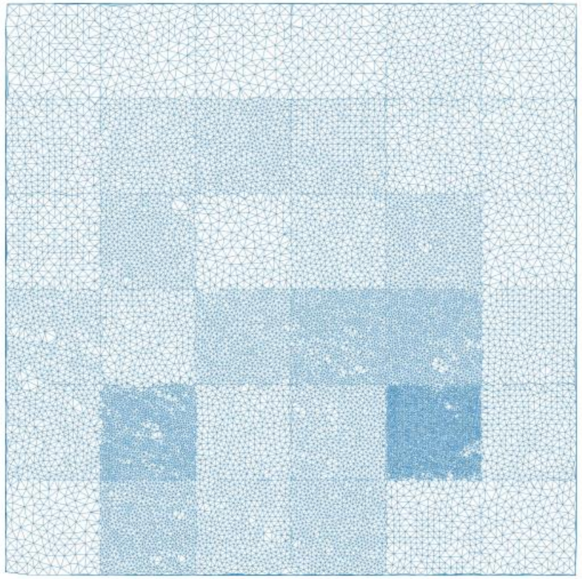
(c) TIN - Random down-sampling



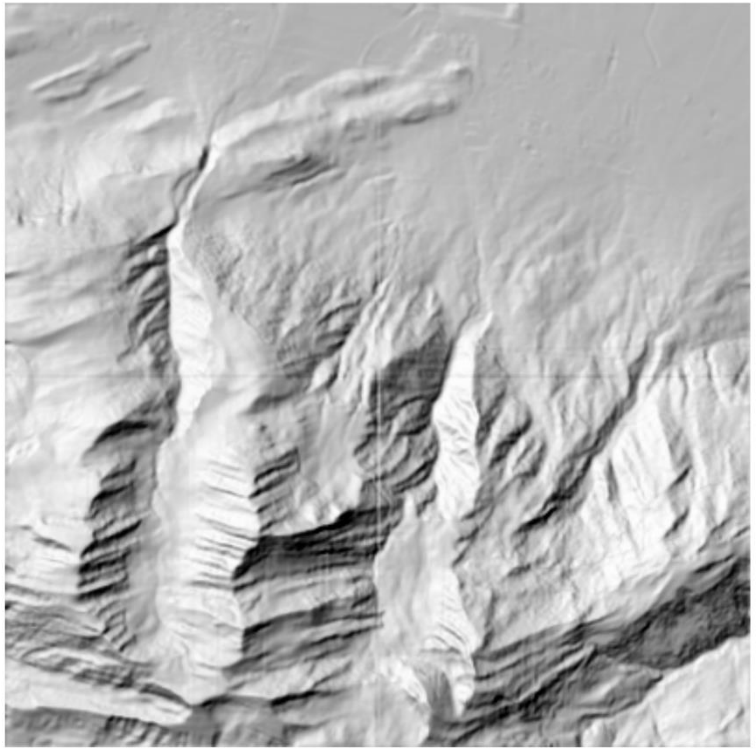
(d) TIN - Voxel down-sampling



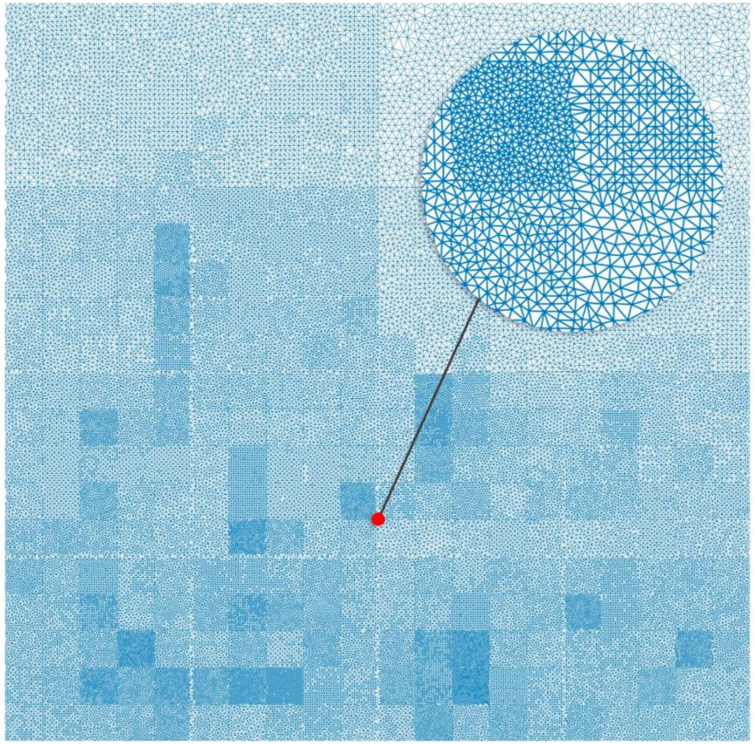
(e) TIN - FPS down-sampling



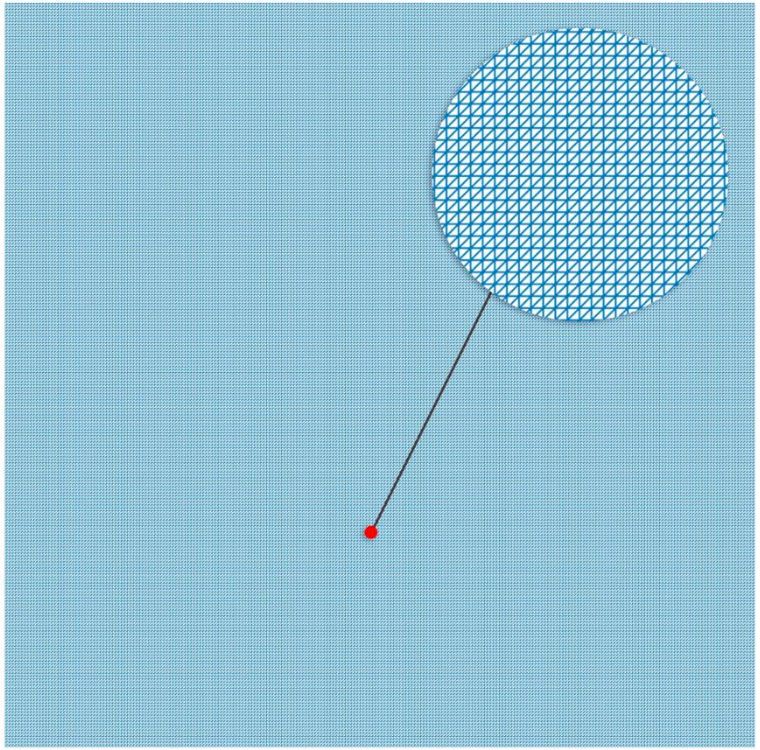
(f) TIN - PFPS down-sampling



Hillshade



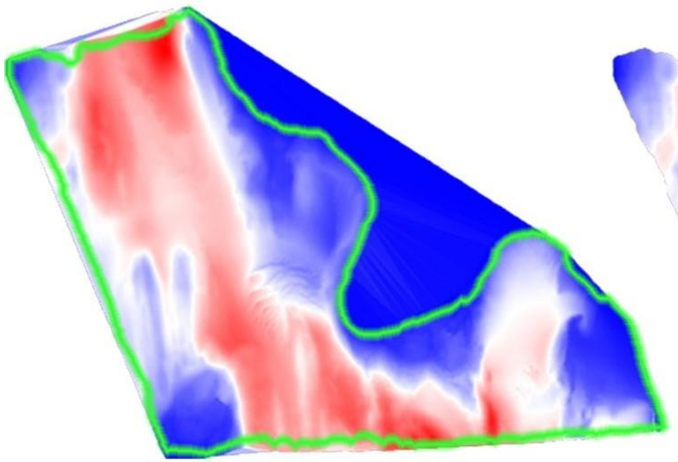
TIN wireframe

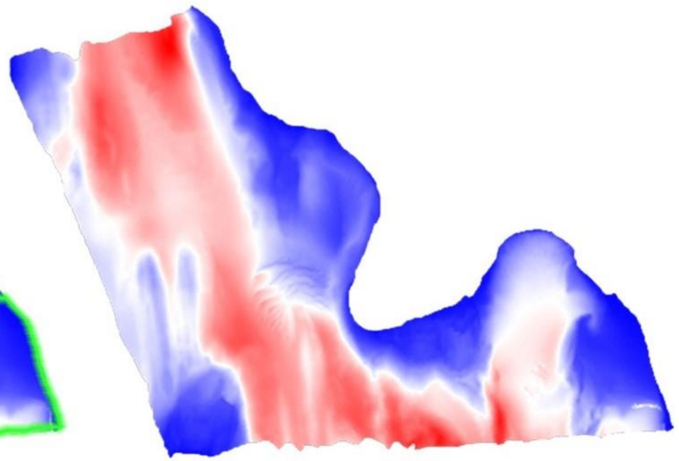


Gridded DEM wireframe

TIN generation from input point cloud

- Delaunay triangulation is applied to the down-sampled point cloud.
- Sliver triangles on the boundary are filtered out through *alpha-shape*.
- The resulting TIN is stored in a **Terrain Tree**[3] - an efficient spatial data structure for large triangulated 2½ D surfaces.

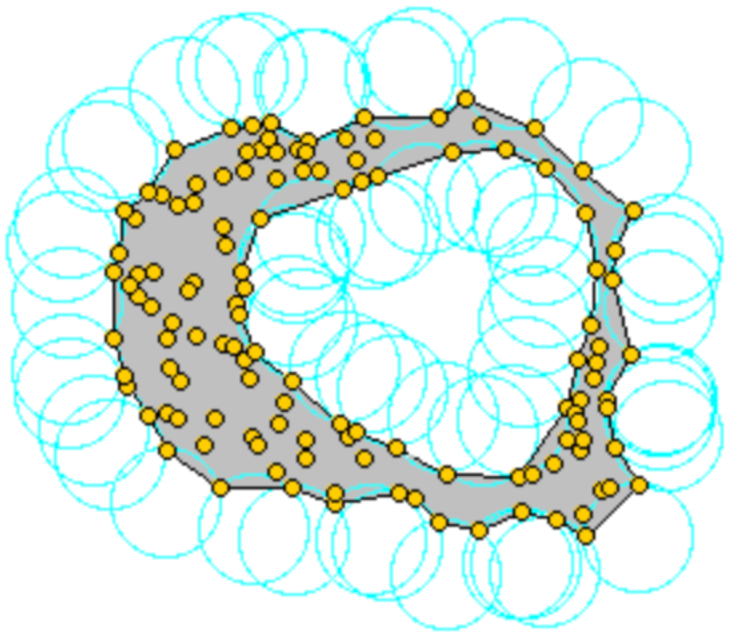


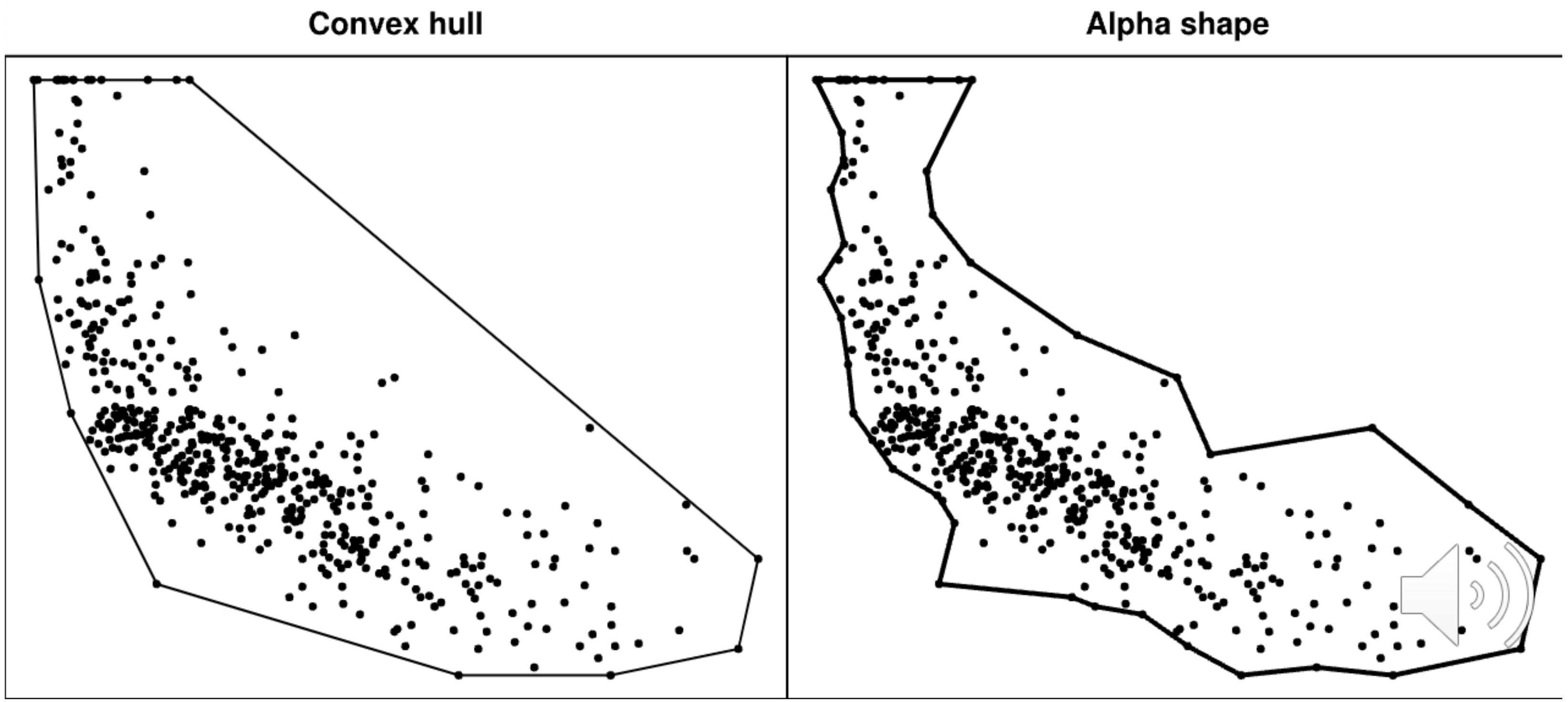


[3] *Fellegara, R., Iuricich, F., Song, Y., & Floriani, L. D. (2023). Terrain trees: a framework for representing, analyzing and visualizing triangulated terrains. GeoInformatica, 27(3), 525-564.*

TIN generation from input point cloud

- Alpha-shape from a Delaunay Triangulation result
  - For each triangle cell, its circumradius is calculated compared to a preset threshold – filtering out sliver triangles.





https://en.wikipedia.org/wiki/Alpha_shape
https://doc.cgal.org/latest/Alpha_shapes_2/index.html

Gaussian smoothing on TINs

- Smoothing operation is formulated as a *matrix-vector multiplication*.
  - For a TIN with N vertices, vertex elevations are stored in a vector $\vec{L}$. Then vertex elevations transition can be formulated as:

$$\vec{L}_{t+1} = sparse\_matvec(M_w, \vec{L}_t)$$

  - An adjacency matrix M_w is a square symmetric matrix, where each element indicates whether a pair of vertices is connected by an edge (with entries typically set to 1 for adjacent vertices and 0 otherwise).

Gaussian smoothing on TINs

- Smoothing operation is formulated as *matrix-vector multiplication*.
  - Gaussian smoothing re-weights the elements in the adjacency matrix based on edge lengths.

$$\vec{L}_{t+1} = sparse\_matvec(M_w, \vec{L}_t)$$

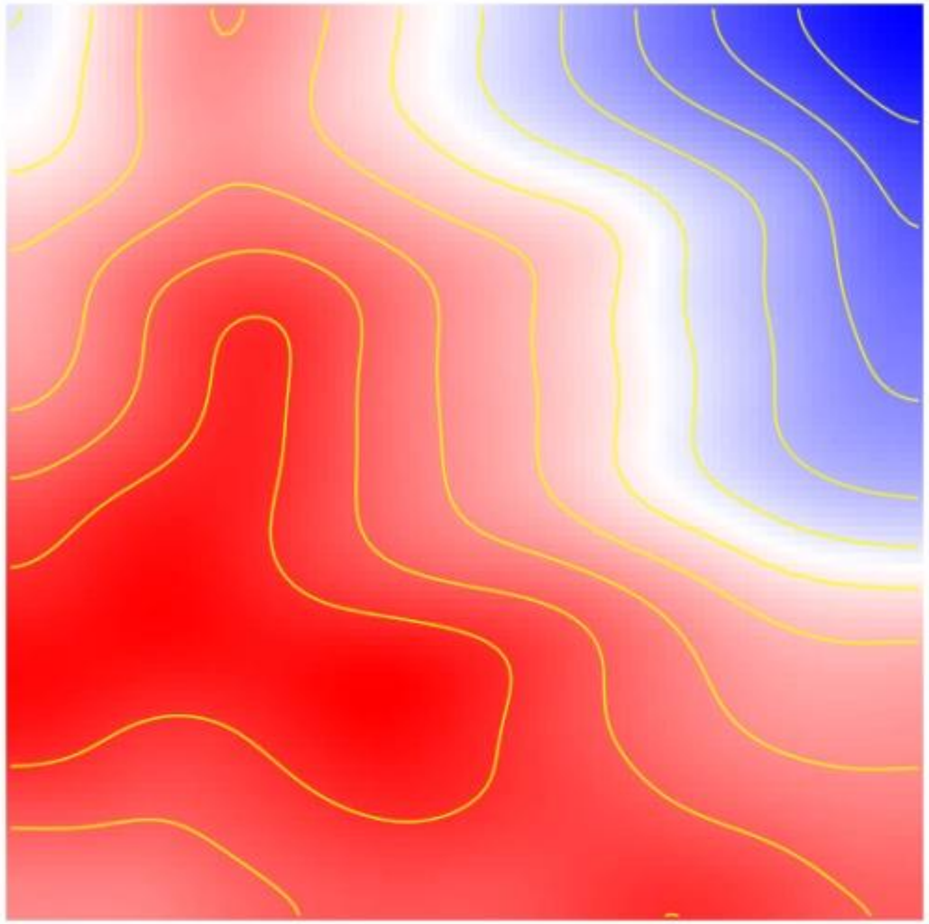
$$M_w[i,j] = \begin{cases} \frac{1}{\boxed{Z}} \exp\left(-\boxed{d(i,j)}^2 / \left(2\boxed{\sigma^2}\right)\right) & \text{if } j \in Neighbor(i) \\ 0 & \text{otherwise} \end{cases} \quad (6)$$

Euclidean distance

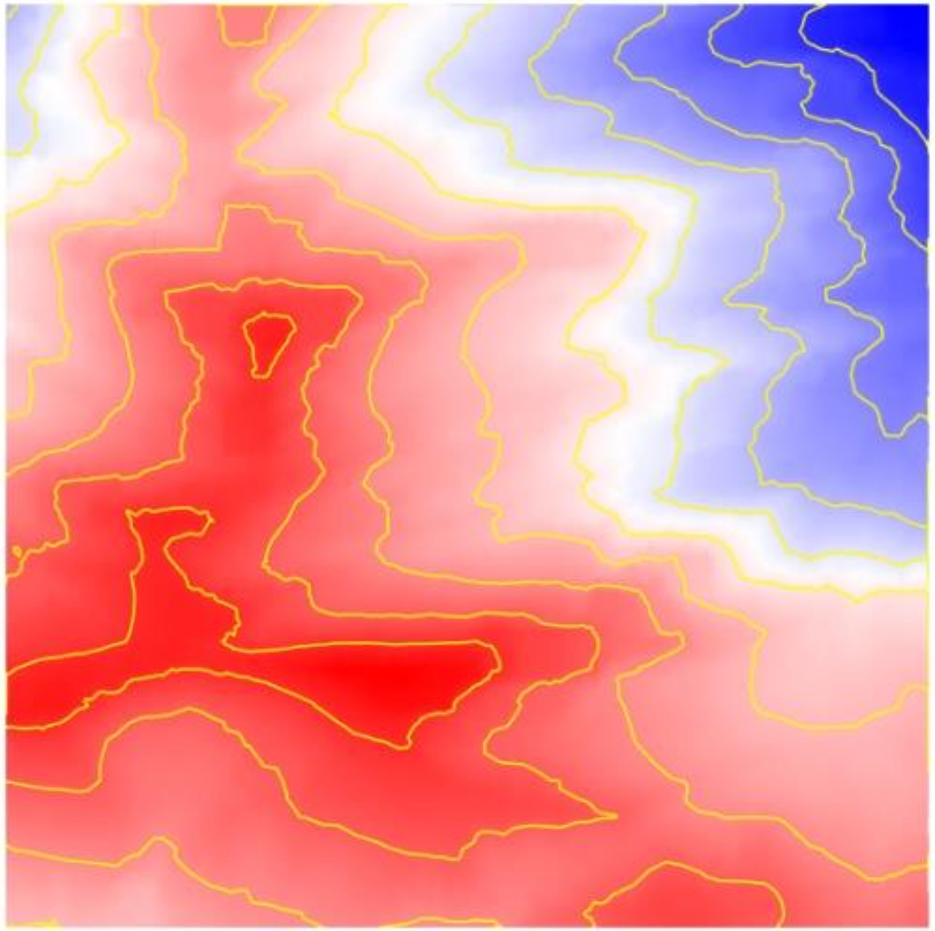
Normalization factor

Variance

However,



Gridded DEM smoothing



Naïve TIN smoothing

Gaussian smoothing on TINs

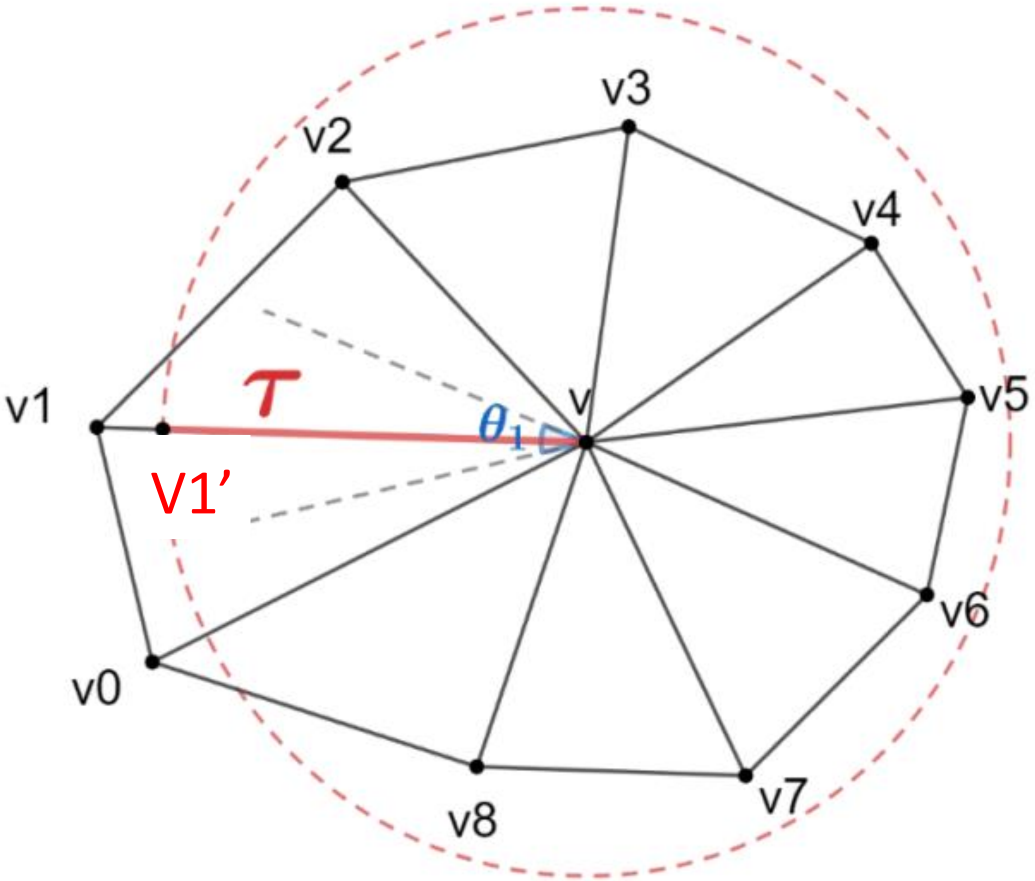
- Angle re-weighting
- Virtual neighbors

$$M_w[i,j] = \begin{cases} \frac{1}{Z}\exp\left(-d(i,j)^2/\left(2\sigma^2\right)\right) & \text{if } j \in Neighbor(i) \\ 0 & \text{otherwise} \end{cases} \quad (6)$$

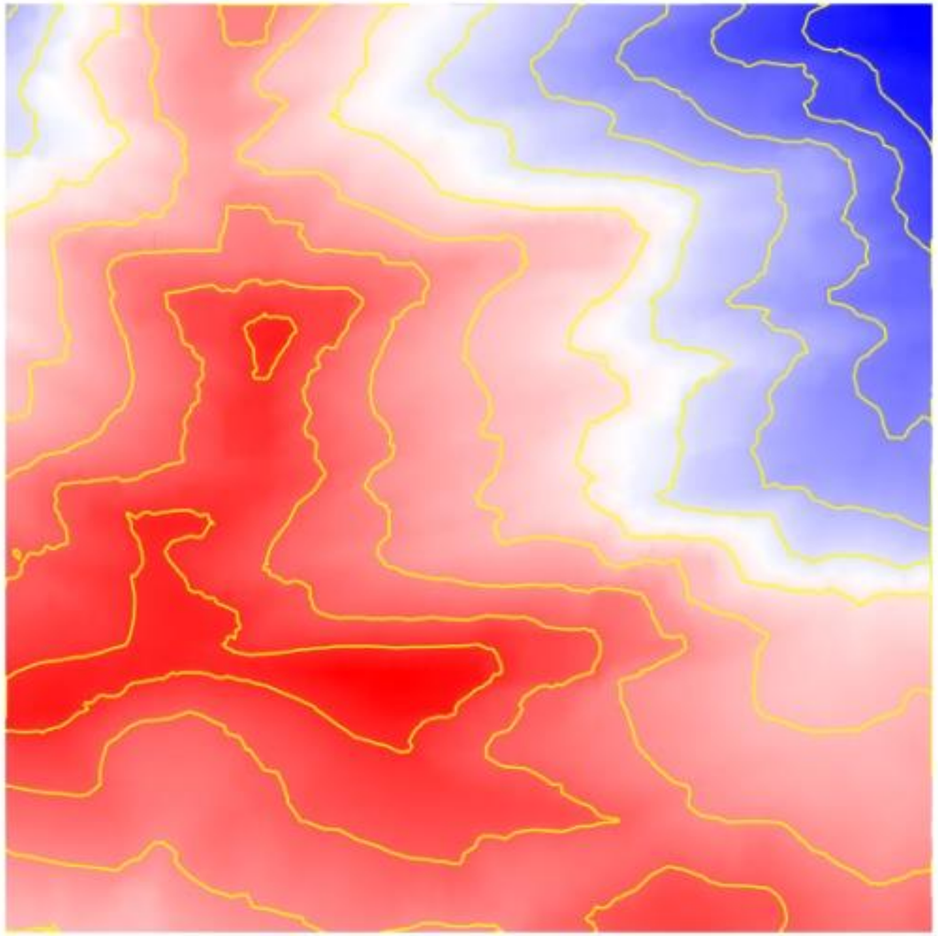
$$M'_w[i,j] = \frac{1}{Z'}\boxed{\theta_j}\exp\left(-\boxed{d'(i,j)}^2/\left(2\sigma^2\right)\right) \quad (8)$$

Euclidean distance to (virtual) neighbors

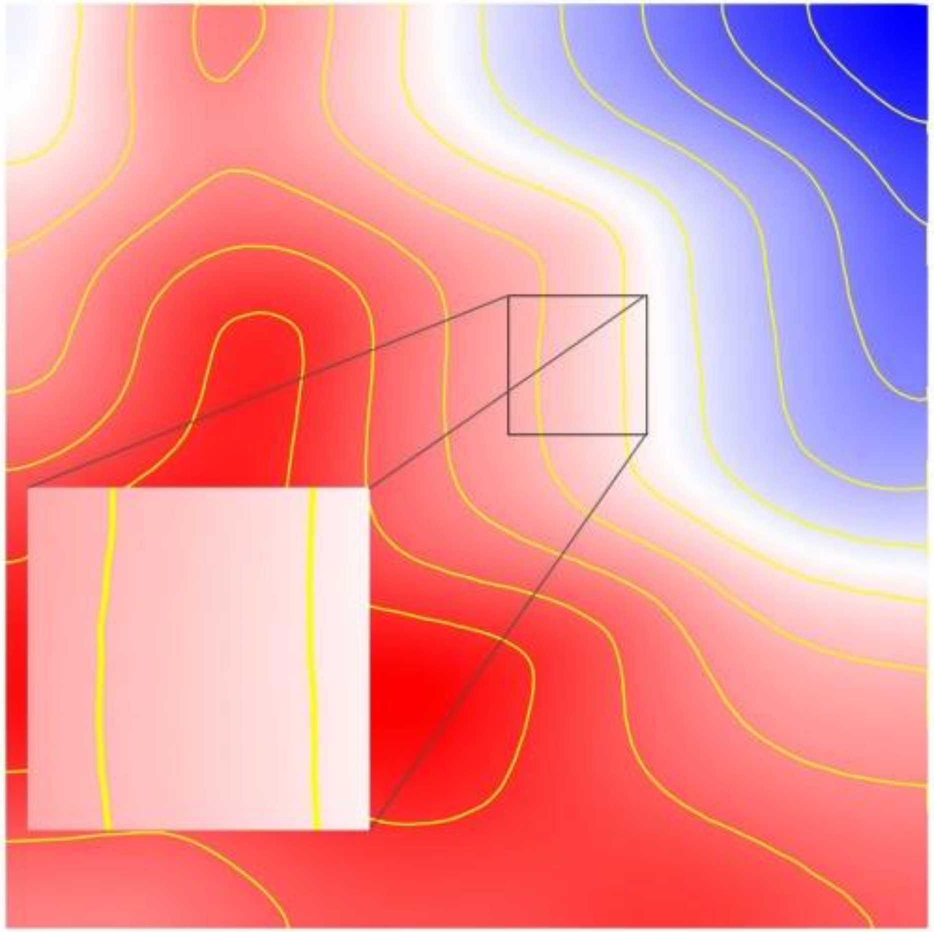
j-th neighbor angle



Now,



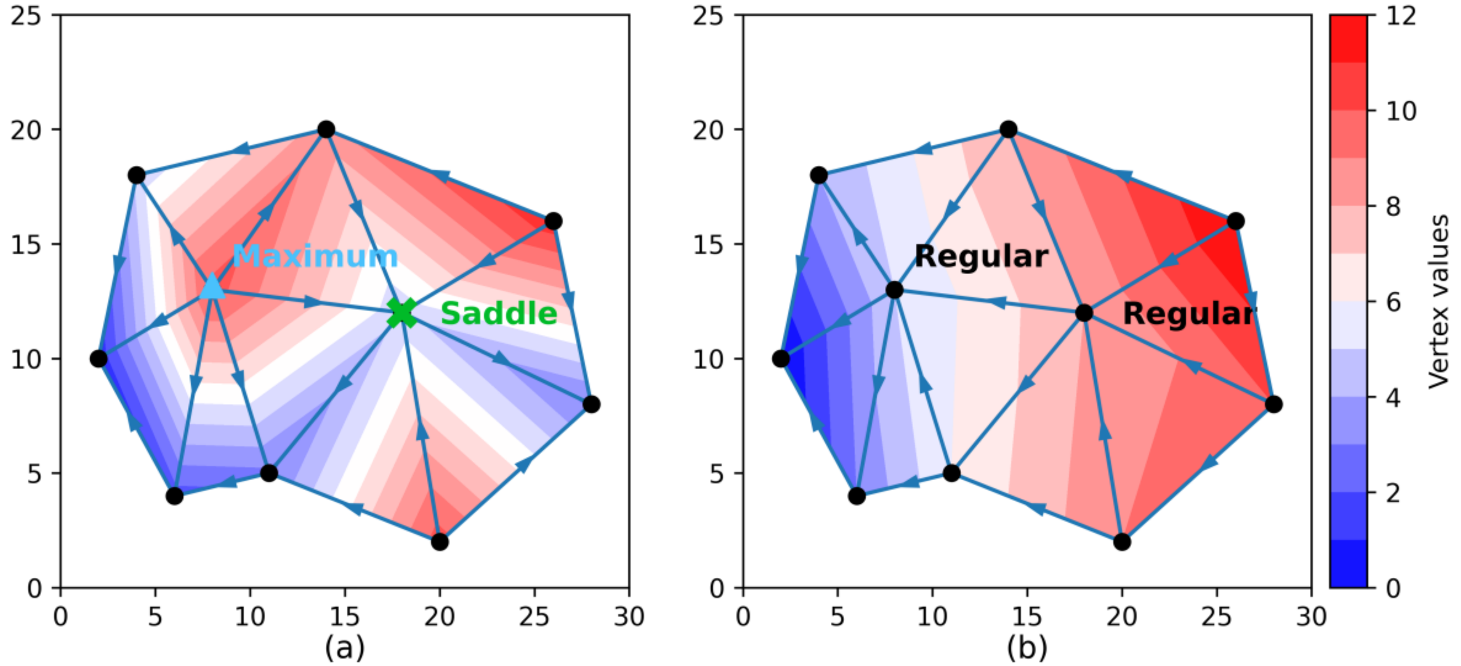
Naïve TIN smoothing



Our TIN smoothing

Critical points tracking on a TIN

- Record and sort all the edge flipping events between intermediate scale layers – track critical points from fine to coarse scale
- Critical point transitions are classified into three categories: *Displacement*, *Collapse*, and *Appearance*.

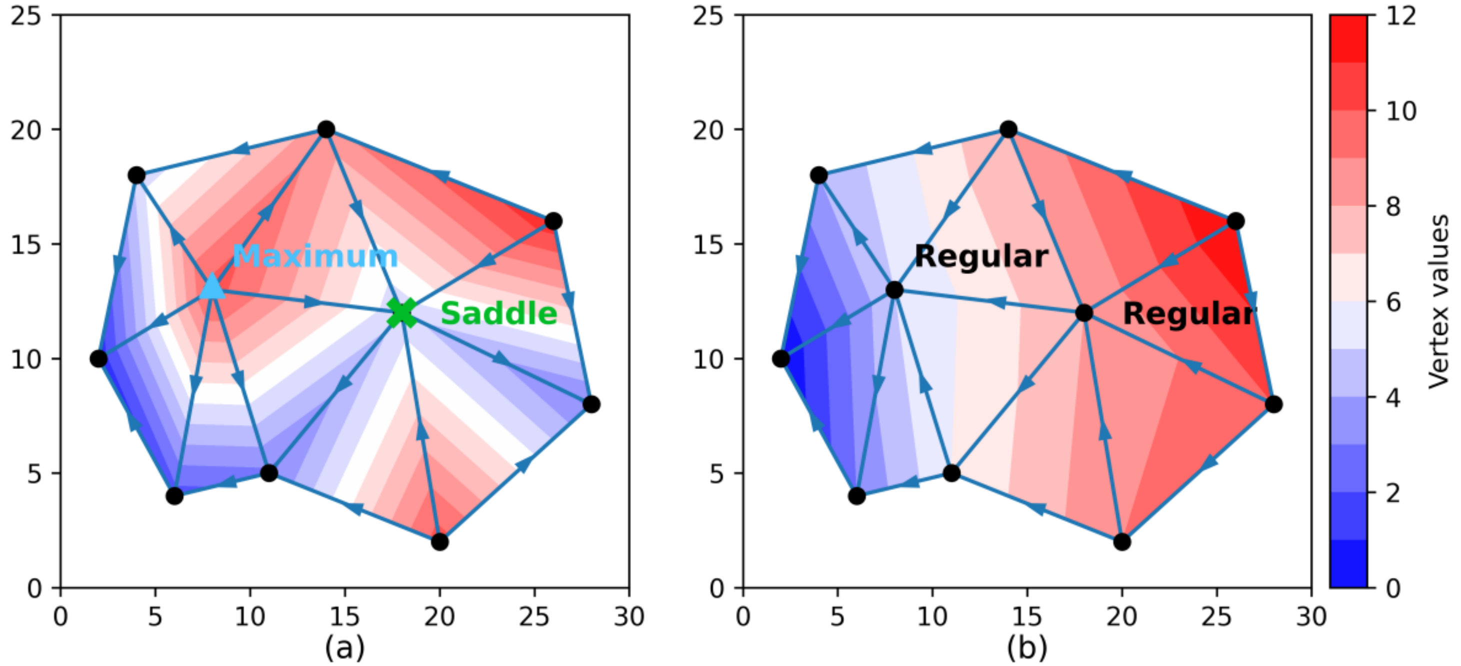


Collapse of a *(maximum, saddle)* pair to two regular points

Critical points tracking on a TIN

- By the **Poincaré-Hopf index theorem**, *Collapse* or *Appearance* can only happen to critical points in pairs of the form: *(maximum, saddle)* or *(minimum, saddle)*.

$$N_{max} + N_{min} - N_{saddle} = 2$$



Collapse of a *(maximum, saddle)* pair to two regular points

Transition table from a regular grid

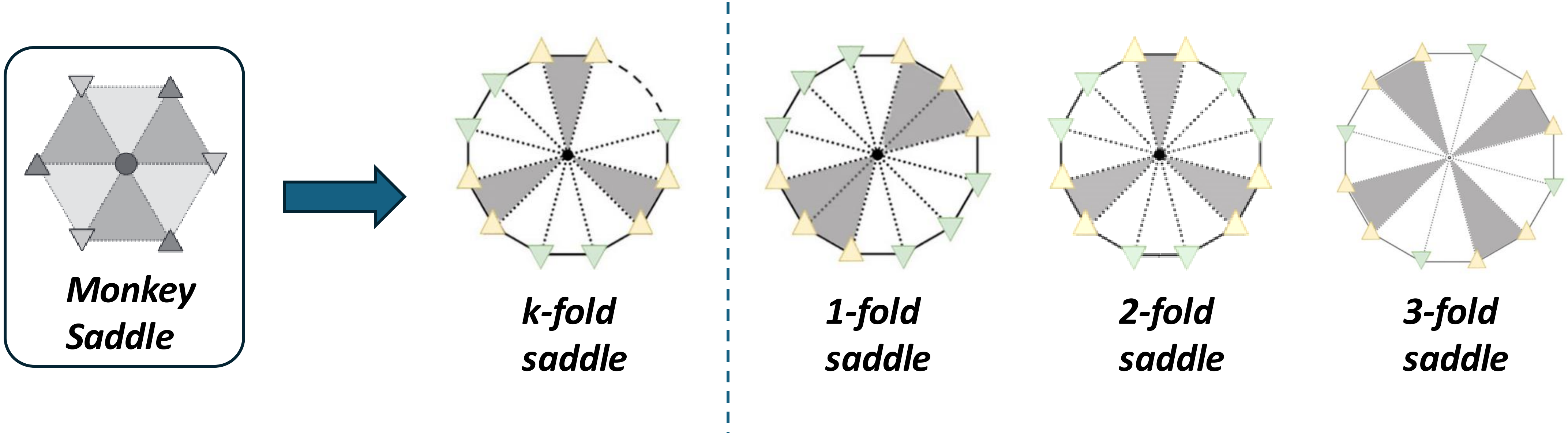
Table 1
Possible transitions in the state of a pair of vertices connected by a flipping edge, after Rocca and Puppo (2013). *r*: a sloped point; *M*: a maximum; *m*: a minimum; *s*: a saddle; *K*: a monkey saddle. Note that for every event a specular one is also possible, for a total of 32 possible events. An example would be: $(M,K)\rightarrow(r,s)$ is equivalent to $(K,M)\rightarrow(s,r)$.

Displacements			Collapses			Appearances		
(m, r)	$\rightarrow$	(r, m)	(m, s)	$\rightarrow$	(r, r)	(r, r)	$\rightarrow$	(m, s)
(M, r)	$\rightarrow$	(r, M)	(M, s)	$\rightarrow$	(r, r)	(r, r)	$\rightarrow$	(M, s)
(s, r)	$\rightarrow$	(r, s)	(m, K)	$\rightarrow$	(r, s)	(r, s)	$\rightarrow$	(m, K)
(K, r)	$\rightarrow$	(r, K)	(M, K)	$\rightarrow$	(r, s)	(r, s)	$\rightarrow$	(M, K)
(K, r)	$\rightarrow$	(s, s)						
(s, s)	$\rightarrow$	(K, r)						
(K, s)	$\rightarrow$	(s, K)						

[2] Rocca, L., & Puppo, E. (2013). A virtually continuous representation of the deep structure of scale-space. In Image Analysis and Processing–ICIAP 2013: 17th International Conference, Naples, Italy, September 9-13, 2013, Proceedings, Part II 17 (pp. 522-531). Springer Berlin Heidelberg.

Critical points tracking on a TIN

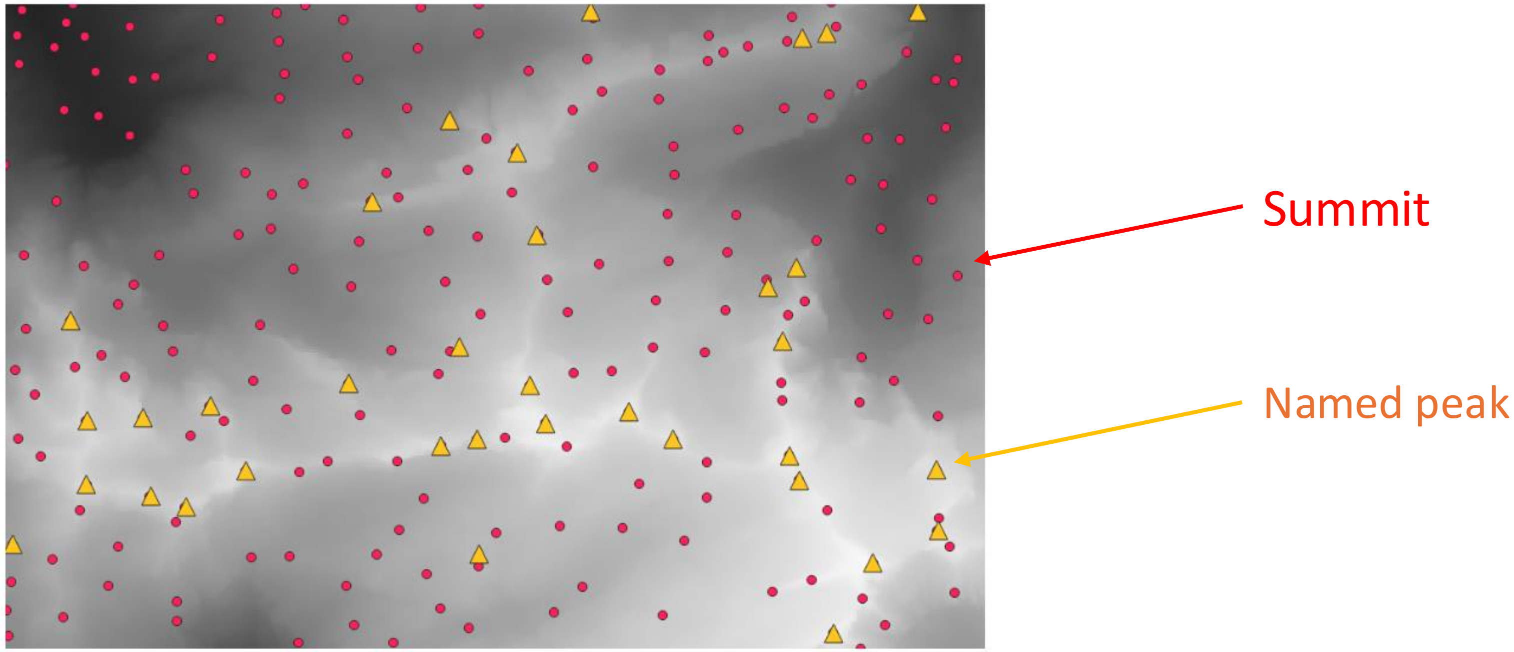
- For transition involving *k*-fold (*Monkey*) saddles
  - Each *k*-fold saddle is treated as *k* simple saddles overlapping at the location of current vertex.
  - Similarly, the critical point transition is separated into *k* transitions with each saddle treated separately.



Experimental comparison

Spot height placement

- *Spot heights are included in topographic maps to quickly and accurately ascertain the elevation values of points on a terrain surface.*
- Matching critical maxima with spot heights annotated in datasets.



Spot height placement

- Datasets from the Swiss Federal Office of Topography *
- Evaluation metrics:
  - Matching accuracy: *precision*, *recall*, F_β *score*, and *average matching distance* $dist_{avg}$.
  - Computation time and peak memory usage in scale-space pipeline.

Dataset	Dimension (km)	#points (million)	Type
Reichenburg	4×4	167	Suburban
Sörenberg	8×8	1529	Mountainous
Bannelpsee	12×12	3548	Mountainous

$$F_\beta = (1+\beta^2)\frac{precision \times recall}{\beta^2 \times precision + recall}$$

* Datasets available at: https://www.swisstopo.admin.ch/en/landscape-model-swisstlm3d
https://www.swisstopo.admin.ch/en/height-model-swisssurface3d

Spot height placement

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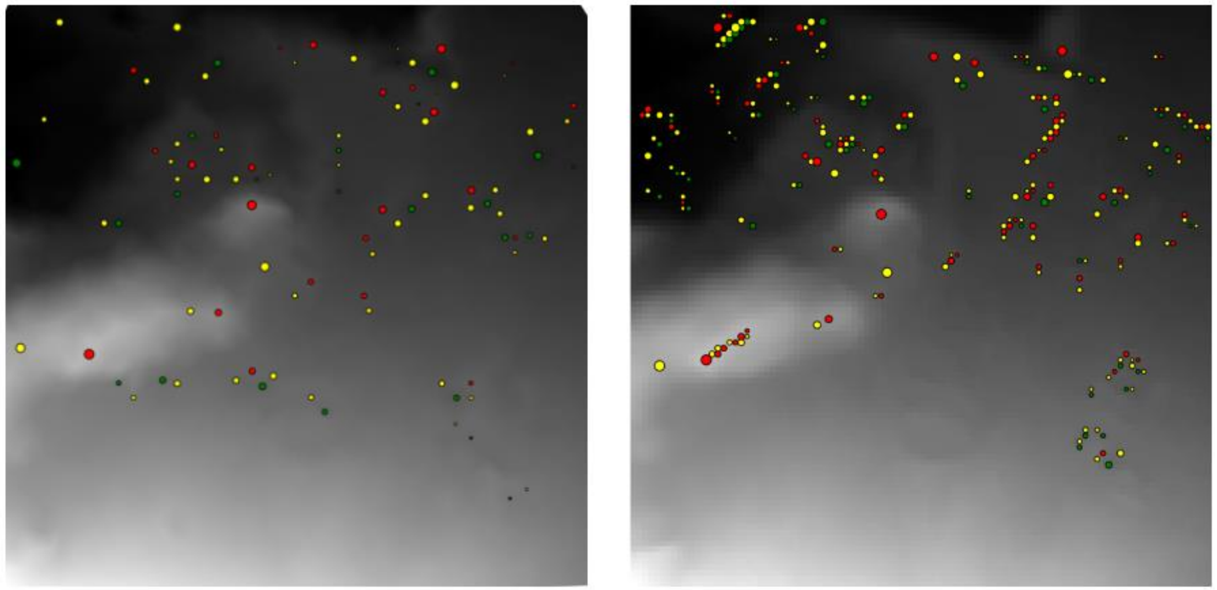
$$F_\beta = (1 + \beta^2)\frac{precision \times recall}{\beta^2 \times precision + recall}$$

Datasets	Grid cell size (m)	#edges		$dist_{avg}$ (m)		F_β *score*		t_{ss} (s)		Mem_{peak} (MBs)	
		Grid	TIN	Grid	TIN	Grid	TIN	Grid	TIN	Grid	TIN
Reichenburg	16	186,501	170,265	18.20	**4.85**	0.79	**0.92**	39.33	36.52	24.38	22.75
Sörenberg	40	119,201	112,000	30.38	**12.79**	0.85	**0.87**	26.077	27.81	17.35	17.00
Bannelpsee	40	268,801	255,945	32.18	**14.19**	0.77	**0.79**	56.88	63.43	32.65	32.04

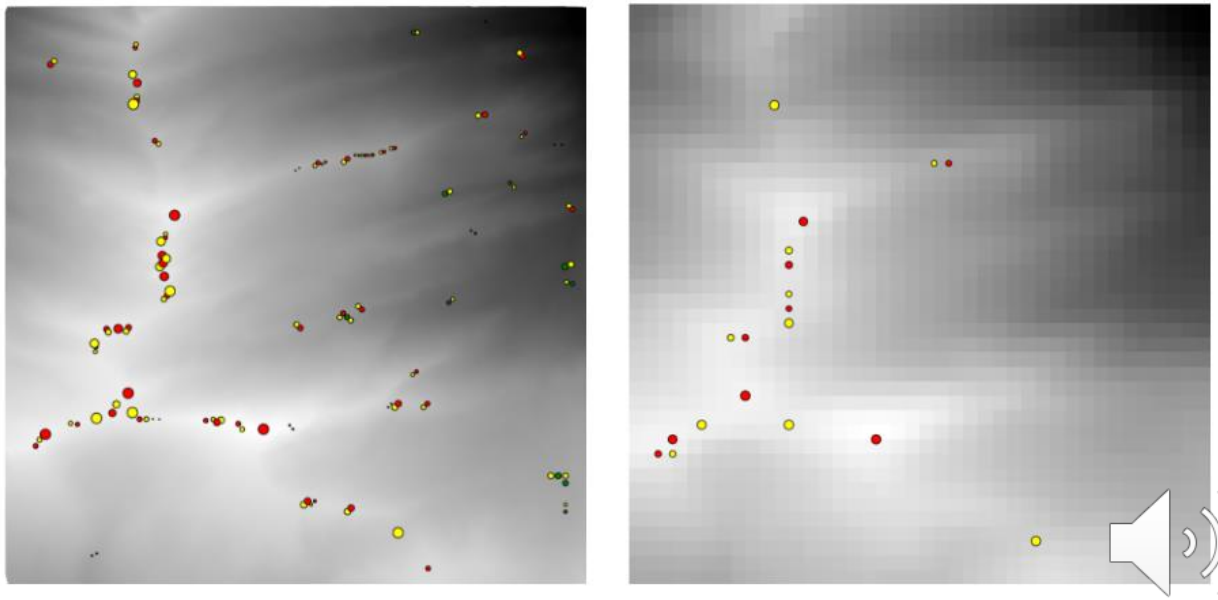
Further zoomed-in comparison of the Reichenberg dataset

Suburban plateau region



(a) TIN - 1, 126 vertices (b) Grid - 6, 400 vertices

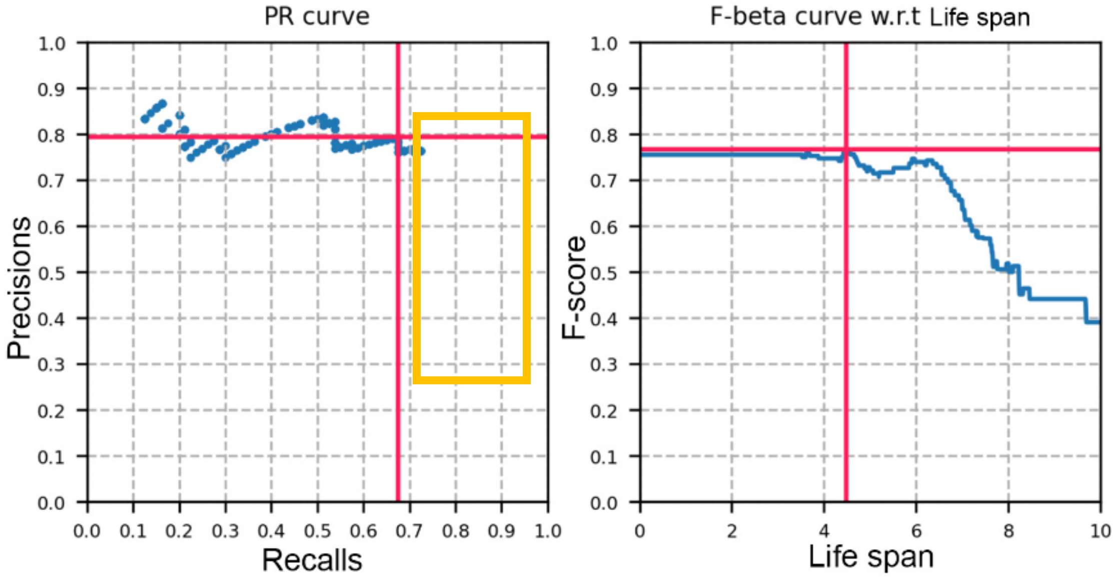
Mountain region



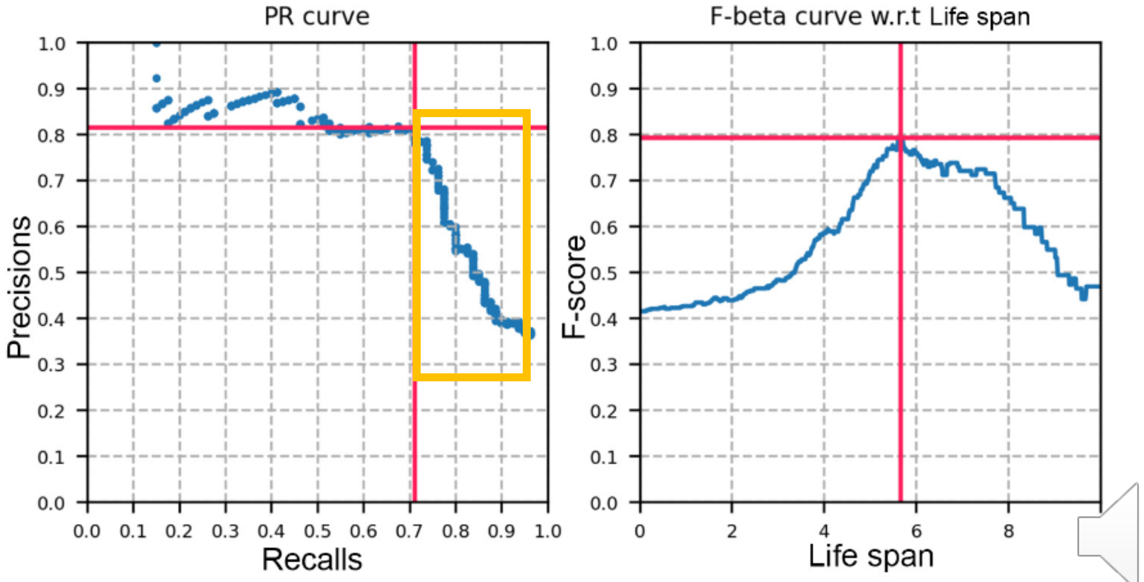
(c) TIN - 11, 447 vertices (d) Grid - 1, 600 vertices

Precision-Recall curve and F_β score graph

- Filter critical points by life spans from small to large values
- Our TIN-based matching achieves higher recall rates

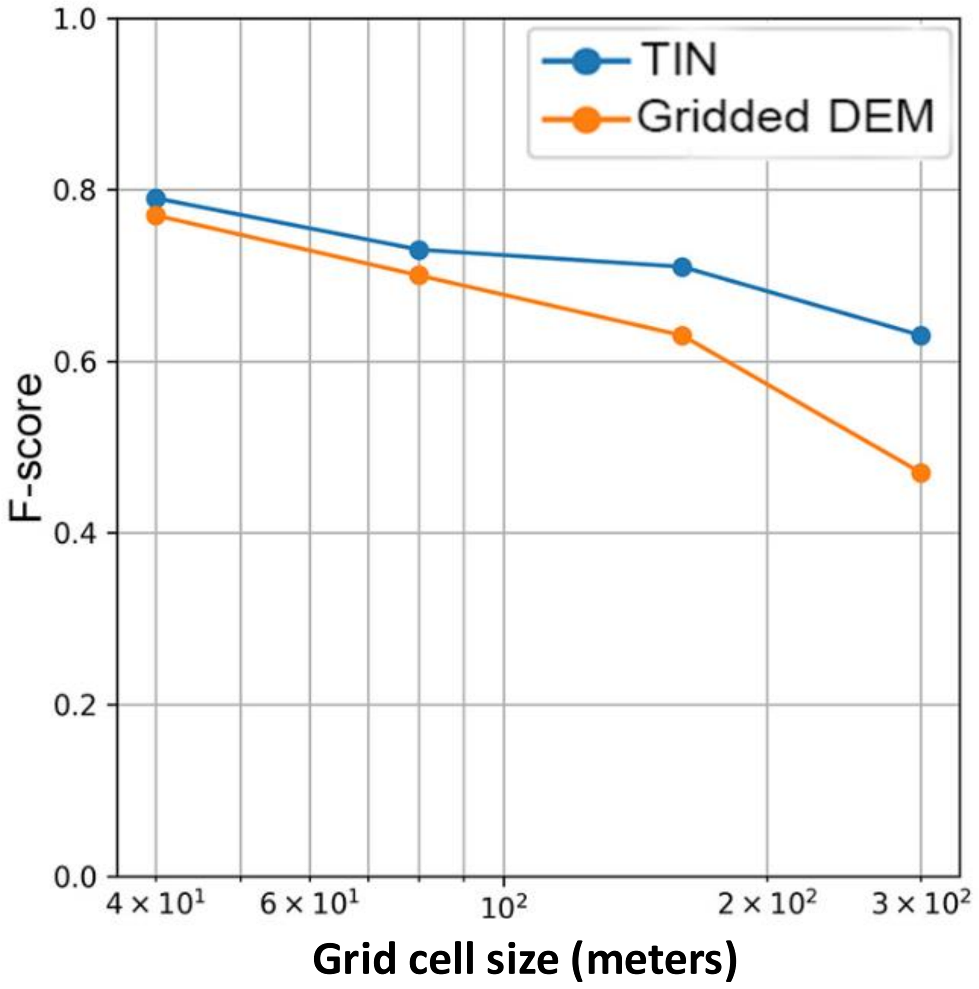


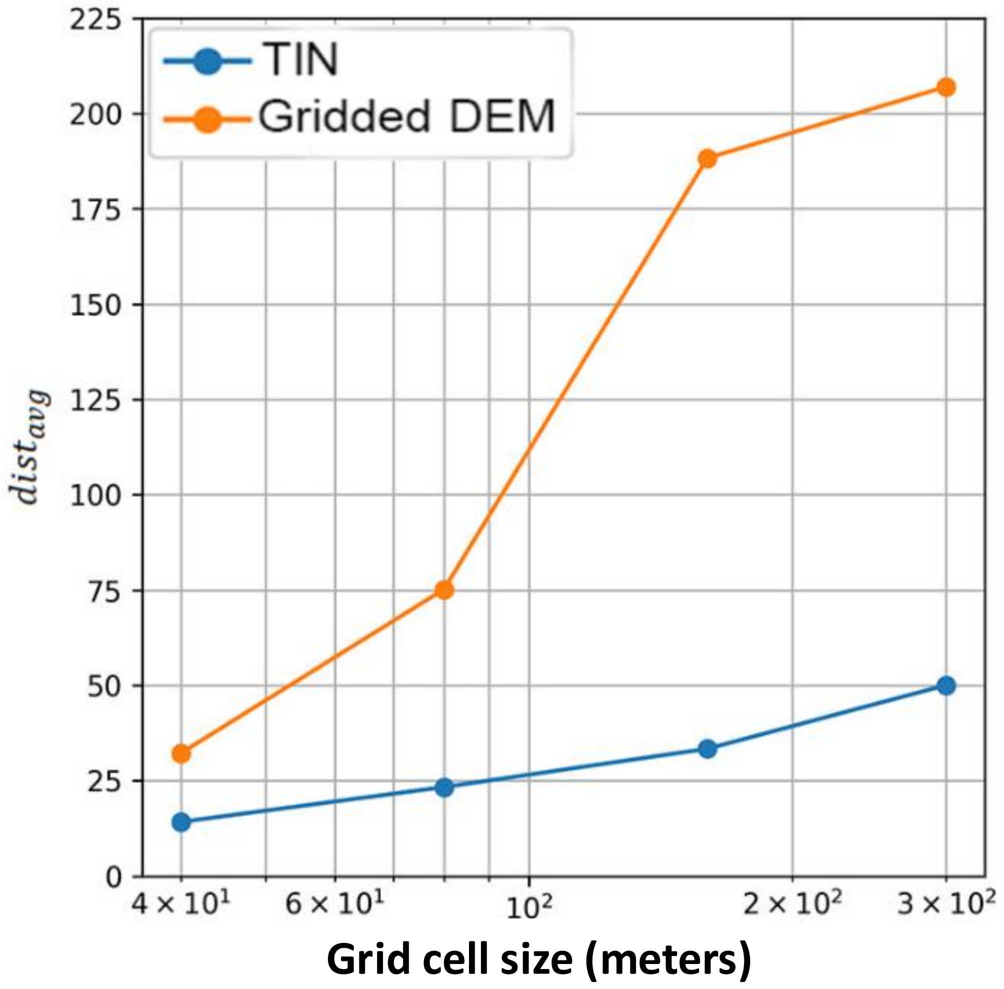
(a) Gridded DEM - best $F_\beta = 0.77$



(b) TIN - best $F_\beta = 0.79$

Resolution robustness





Conclusion

- An efficient method of critical features tracking on the scale space of a Triangulated Irregular Network (TIN).
- To take advantage of TINs' adaptive resolution, raw point clouds are down-sampled based on patch-wise curvature.
- Parallel computing of the scale space of a TIN with special customization: virtual neighbors and angle-based re-weighting.
- Compared to the previous grid-based method, working on a TIN allows more accurate localization and tracking of critical features especially when computational resources are limited.

Thank you!

Questions?

Scan me for project page

References

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