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# HW2: Adder

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❖  $1101 + 0110 = ?$

$$\begin{array}{r} 1101 \\ + 0110 \\ \hline \end{array}$$

---

# Binary Adder

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- ❖  $1101 + 0110 = ?$
- ❖ Calculation by hand...

$$\begin{array}{r} 1101 \\ + 0110 \\ \hline 10011 \end{array}$$

# Half Adder

- ❖ What is a half adder?
- ❖ Two 1-bit numbers are being added
  - ❖ Two inputs A and B
  - ❖ Two outputs S(um) and C(arry out)
- ❖ No carry in
- ❖  $S = ?$
- ❖  $C = ?$
- ❖ Please write down your answer

A	B
0	0
0	1
1	0
1	1

# Half Adder

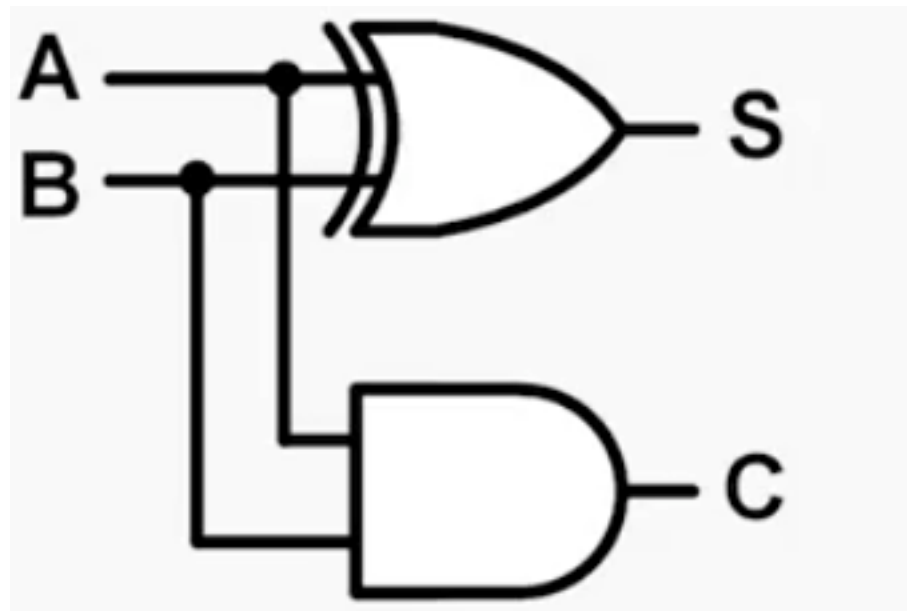
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- ❖  $C = ?$
- ❖ Please write down your answer

A	B
0	0
0	1
1	0
1	1

C	S
0	0
0	1
0	1
1	0

# Half Adder

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- ❖ Two 1-bit numbers are being added
  - ❖ Two inputs A and B
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- ❖  $S = ?$
- ❖  $C = ?$
- ❖ Please write down your answer



A	B
0	0
0	1
1	0
1	1

C	S
0	0
0	1
0	1
1	0

---

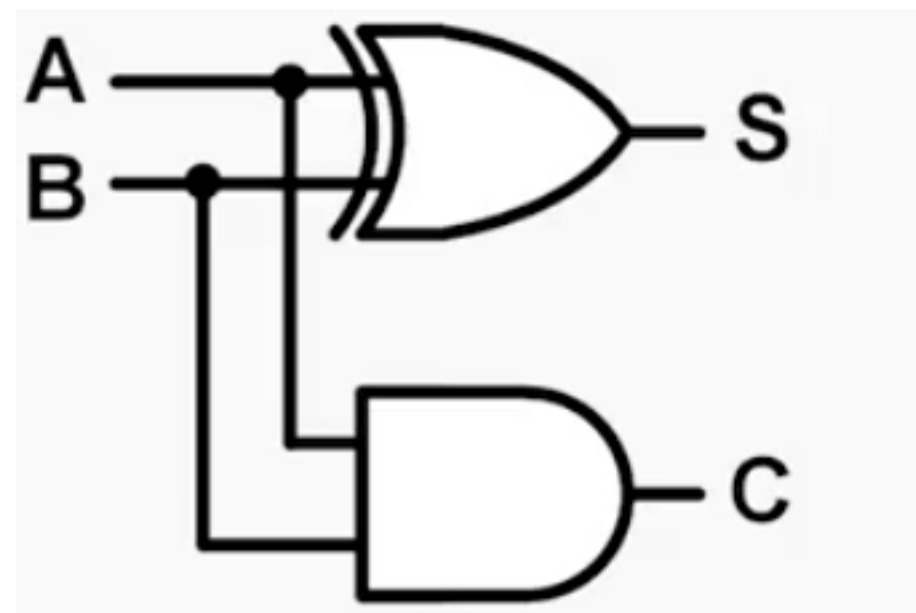
# Half Adder

---

- ❖ What is a half adder?
- ❖ Two 1-bit numbers are being added
  - ❖ Two inputs A and B
  - ❖ Two outputs S(um) and C(arry out)
- ❖ No carry in
- ❖  $S = A \text{ XOR } B$
- ❖  $C = A \text{ AND } B$

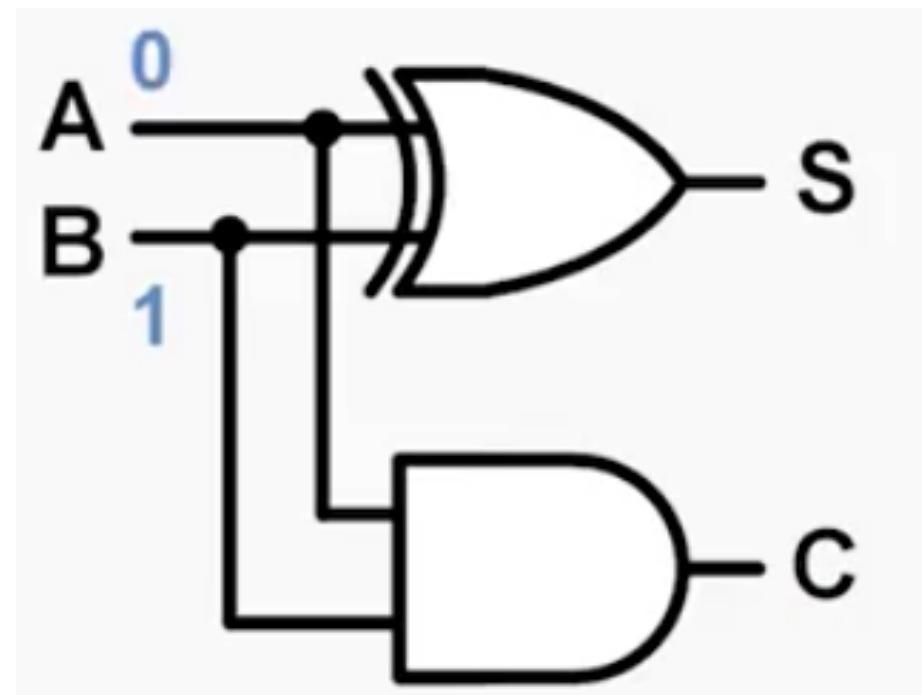
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# Half Adder

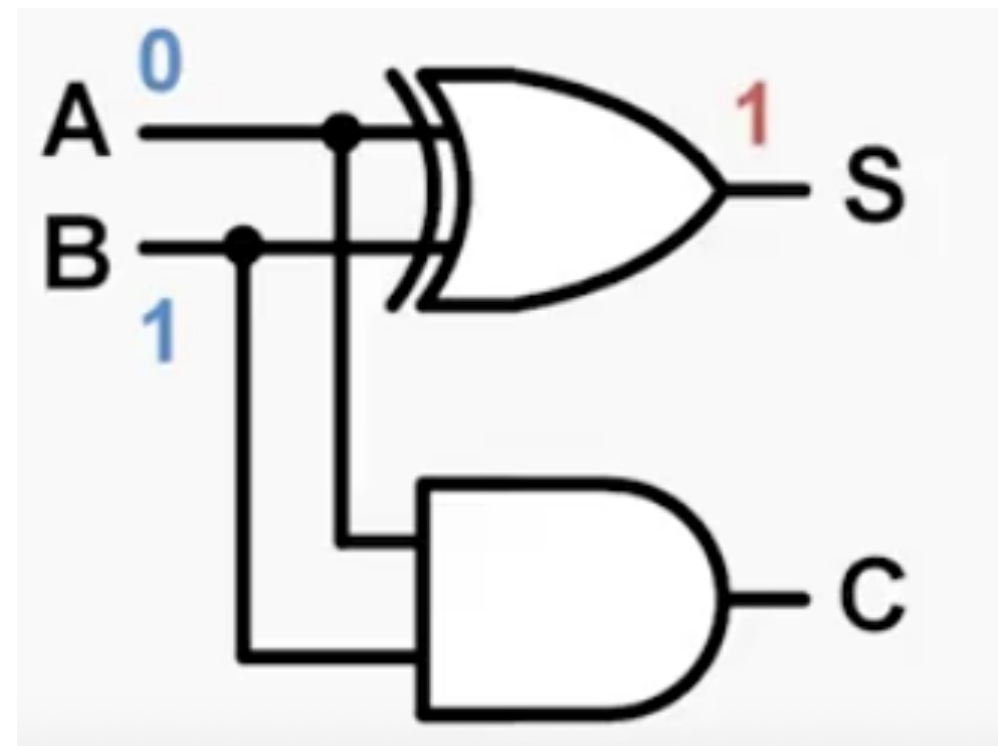
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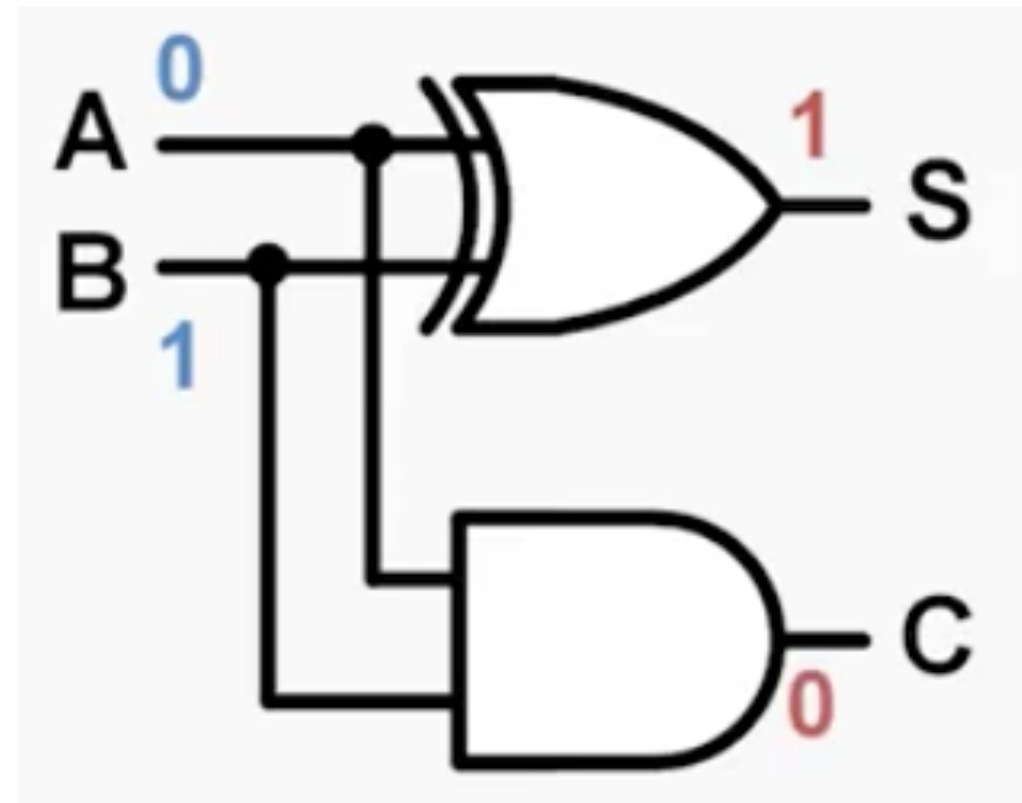
# Half Adder

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# Half Adder

- ❖ What is a half adder?
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  - ❖ Two outputs S(um) and C(arry out)
- ❖ No carry in
- ❖  $S = A \text{ XOR } B$
- ❖  $C = A \text{ AND } B$



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# Full Adder

---

❖ What is a full adder?

---

# Full Adder

---

- ❖ What is a full adder?
  - ❖ Has 3-bit input

---

# Full Adder

---

- ❖ What is a full adder?
  - ❖ Has 3-bit input
  - ❖ Has a Cin (carry in) bit

---

# Full Adder

---

- ❖ What is a full adder?
  - ❖ Has 3-bit input
  - ❖ Has a  $C_{in}$  (carry in) bit
  - ❖ Has two outputs:  $S_{um}$  and  $C_{out}$

---

# Full Adder

---

- ❖ What is a full adder?
  - ❖ Has 3-bit input
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# Full Adder

- ❖ What is a full adder?
  - ❖ Has 3-bit input
  - ❖ Has a  $C_{in}$  (carry in) bit
  - ❖ Has two outputs: S(um) and  $C_{out}$

A	B	$C_{in}$
0	0	0
0	1	0
1	0	0
1	1	0
0	0	1
0	1	1
1	0	1
1	1	1



# Full Adder

- ❖ What is a full adder?
  - ❖ Has 3-bit input
  - ❖ Has a  $C_{in}$  (carry in) bit
  - ❖ Has two outputs: S(um) and  $C_{out}$

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0	0	0
0	1	0
1	0	0
1	1	0
0	0	1
0	1	1
1	0	1
1	1	1

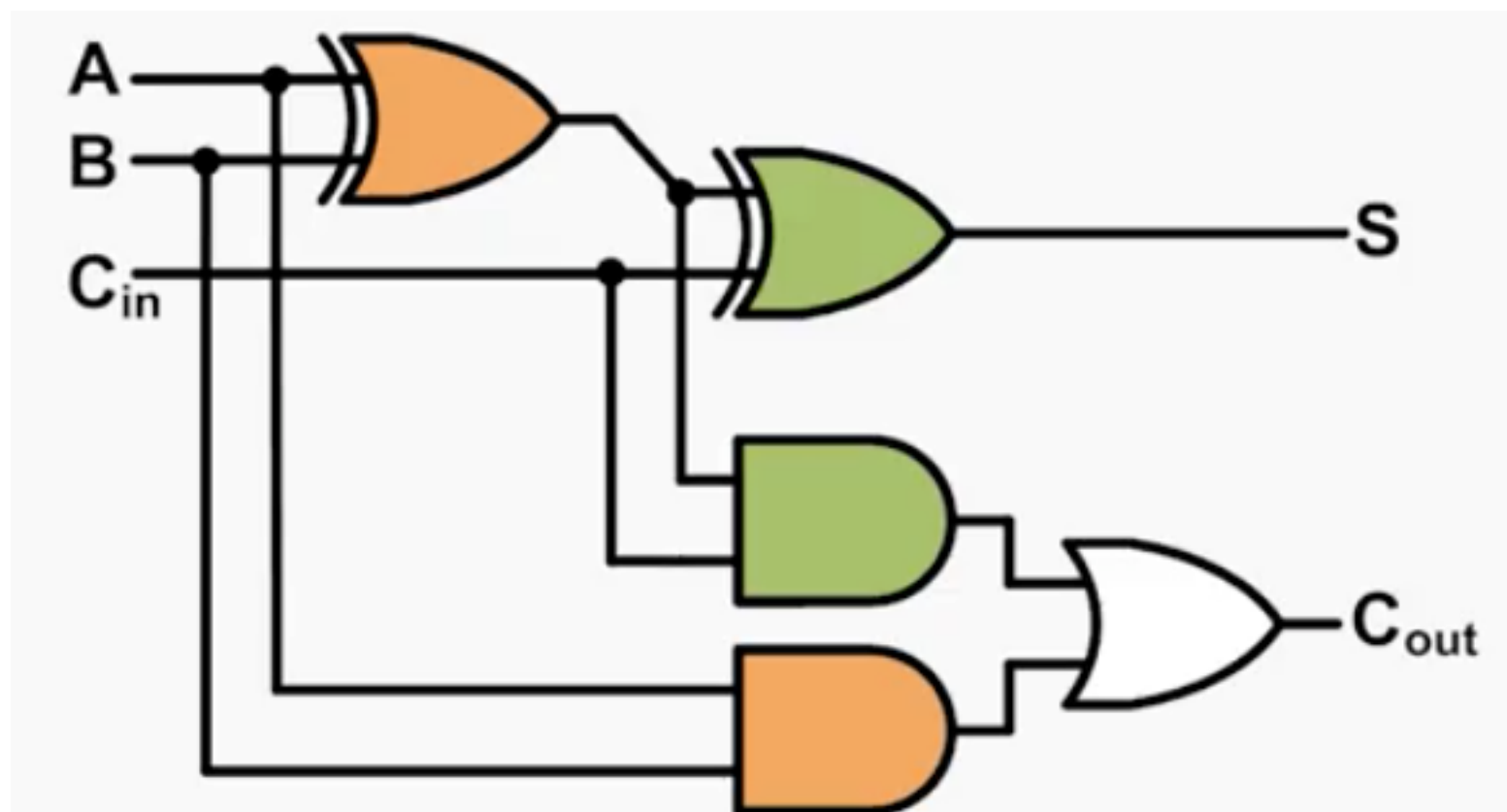
- ❖ Write down the truth table for  $C_{out}$  and S

# Full Adder

- ❖ What is a full adder?
  - ❖ Has 3-bit input
  - ❖ Has a  $C_{in}$  (carry in) bit
  - ❖ Has two outputs:  $S$ (um) and  $C_{out}$
- ❖ Write down the truth table for (

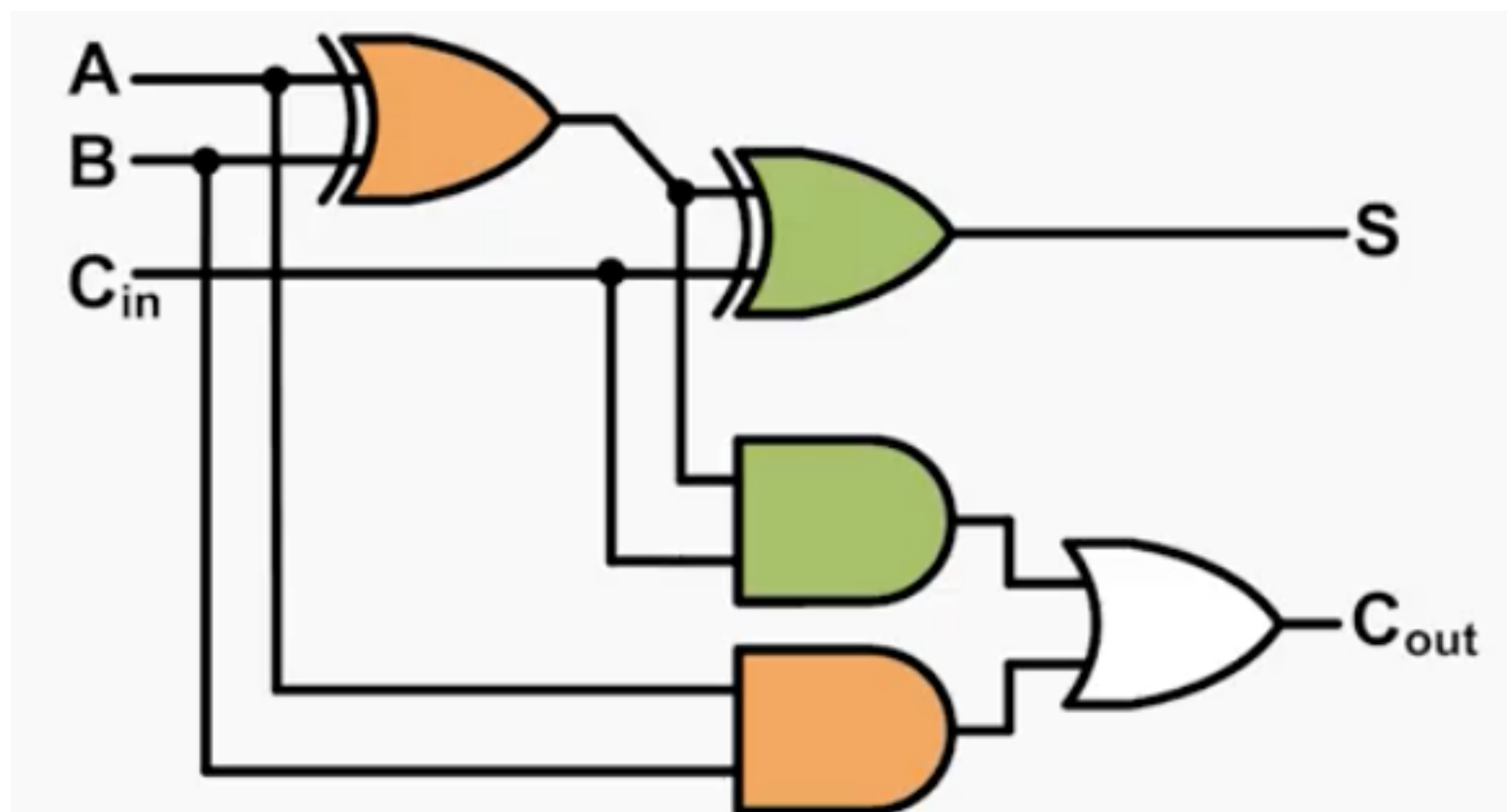
A	B	$C_{in}$	$C_{out}$	S
0	0	0	0	0
0	1	0	0	1
1	0	0	0	1
1	1	0	1	0
0	0	1	0	1
0	1	1	1	0
1	0	1	1	0
1	1	1	1	1

# Full Adder



# Full Adder

- ❖ Composition
  - ❖ Two half-adders: colored differently!



---

# 4-bit Full Adder

---

❖  $A=1101$

❖  $B=0110$

# 4-bit Full Adder

❖ A=1101

❖ B=0110

A hand-drawn style illustration of a 4-bit full adder calculation. It shows the binary numbers 1101 (orange) and 0110 (blue) being added. A blue plus sign is to the left of the second number. A horizontal blue line separates the addends from the sum. The sum is 10011 (red), which is a 5-bit result. The digits are arranged in columns: the first column has a carry-in of 1 (red) and a carry-out of 1 (red); the second column has 1 (orange) and 0 (blue) summing to 0 (red); the third column has 1 (orange) and 1 (blue) summing to 0 (red); the fourth column has 0 (orange) and 1 (blue) summing to 1 (red); and the fifth column has 1 (orange) and no input, summing to 1 (red).

$$\begin{array}{r} 1101 \\ + 0110 \\ \hline 10011 \end{array}$$

# 4-bit Full Adder

❖ A=1101

❖ B=0110

A hand-drawn style illustration of a 4-bit full adder calculation. It shows the binary numbers 1101 (orange) and 0110 (blue) being added. A blue plus sign is to the left of the second number. A horizontal blue line separates the addends from the sum. The sum is 10011 (red), which is a 5-bit result. The digits are arranged in columns: the first column has a carry-out of 1; the second column has 1+0=1; the third column has 1+1=0 with a carry-out of 1; the fourth column has 0+1=1 with a carry-in of 1 from the third column; and the fifth column has 1+0=1 with a carry-in of 1 from the fourth column.

$$\begin{array}{r} 1101 \\ + 0110 \\ \hline 10011 \end{array}$$

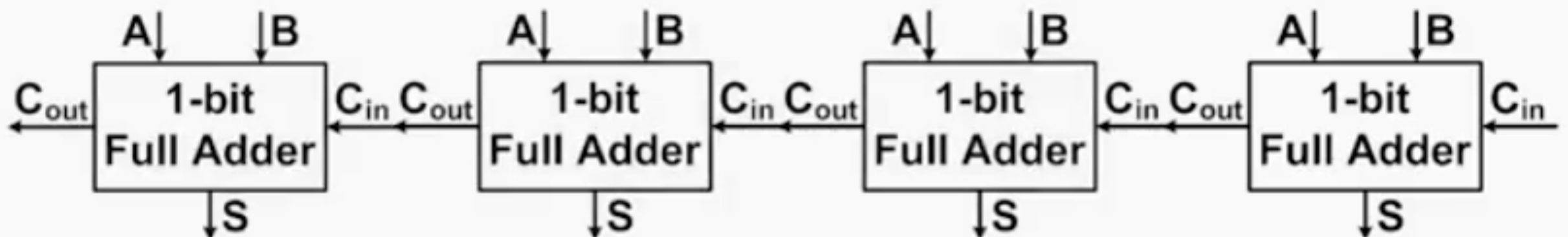
❖ How to build a 4-bit full adder?

# 4-bit Full Adder

- ❖  $A=1101$
- ❖  $B=0110$

$$\begin{array}{r} 1101 \\ + 0110 \\ \hline 10011 \end{array}$$

- ❖ How to build a 4-bit full adder?





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# 4-bit Full Adder

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❖ How it works?

# 4-bit Full Adder

❖ How it works?

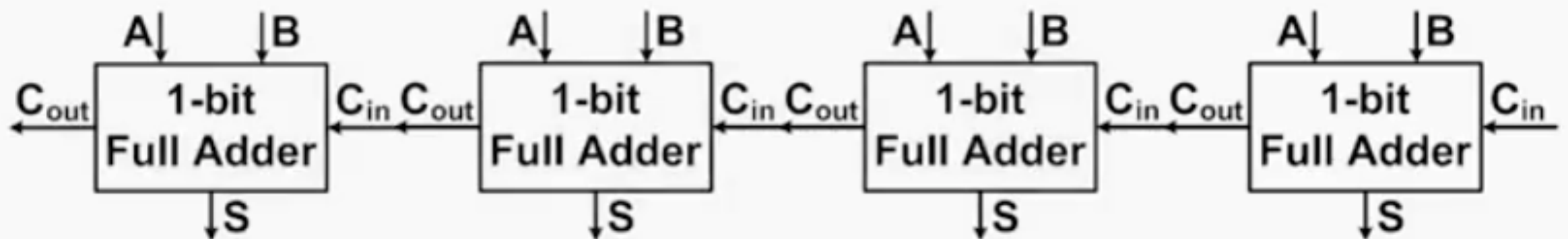
A diagram illustrating a 4-bit full adder. It shows two 4-bit numbers being added: 1101 (orange) and 0110 (blue). The result is a 5-bit sum, 10011 (red), which includes a carry-out of 1. The addition is performed bit-by-bit from right to left, with carries being propagated to the next higher bit.

$$\begin{array}{r} 1101 \\ + 0110 \\ \hline 10011 \end{array}$$

# 4-bit Full Adder

❖ How it works?

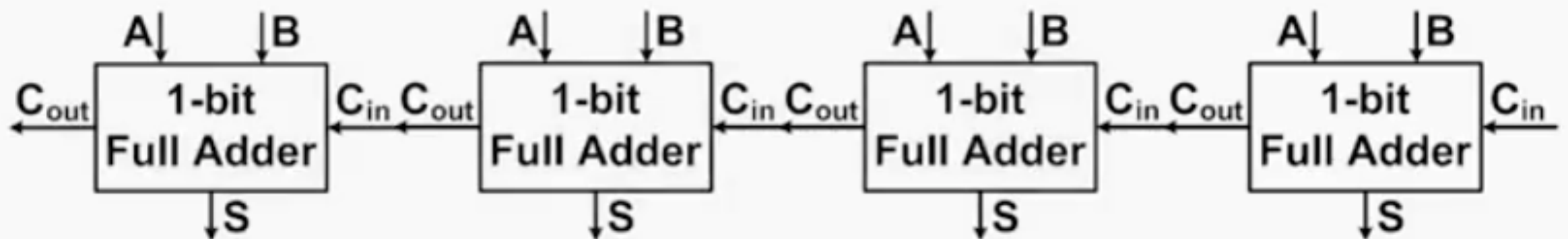
$$\begin{array}{r} 1101 \\ + 0110 \\ \hline 10011 \end{array}$$



# 4-bit Full Adder

- ❖ How it works?
- ❖ Each bit assigned!

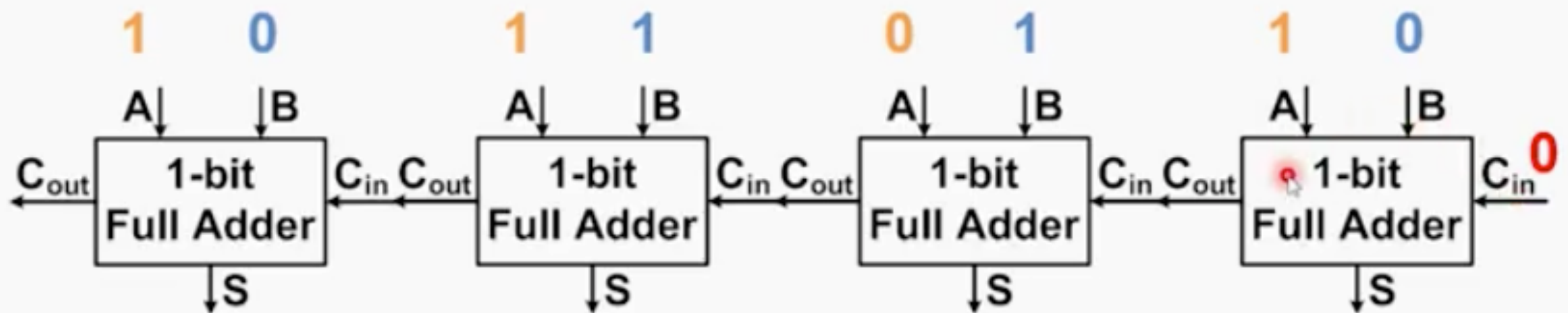
$$\begin{array}{r} 1101 \\ + 0110 \\ \hline 10011 \end{array}$$



# 4-bit Full Adder

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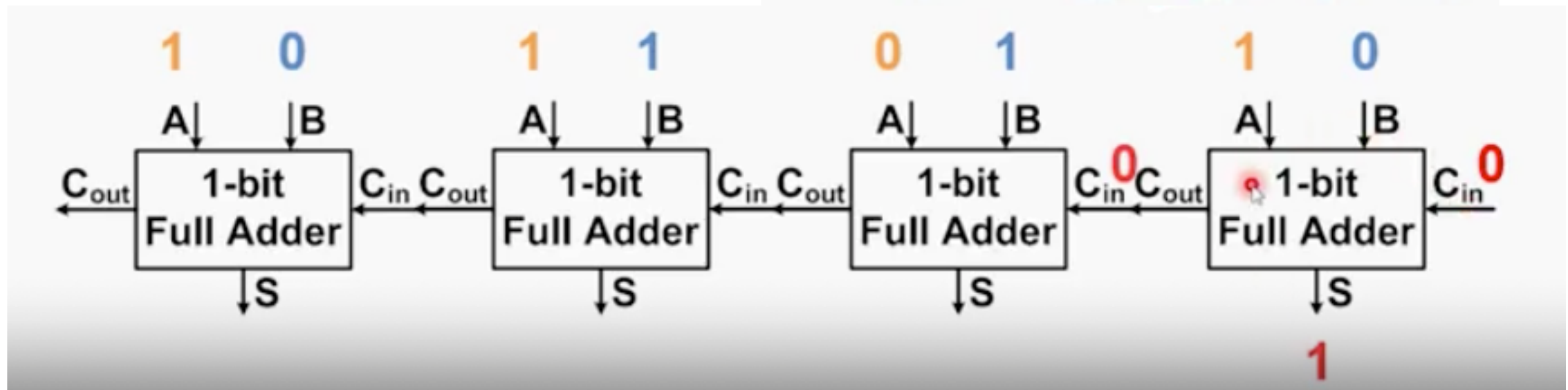
$$\begin{array}{r} 1101 \\ + 0110 \\ \hline 10011 \end{array}$$



# 4-bit Full Adder

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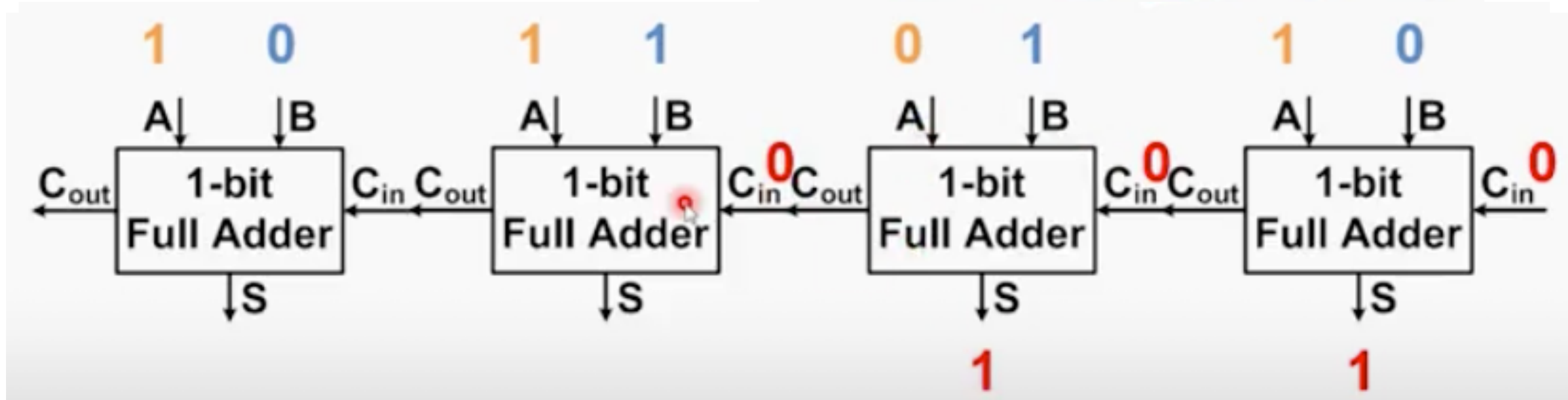
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# 4-bit Full Adder

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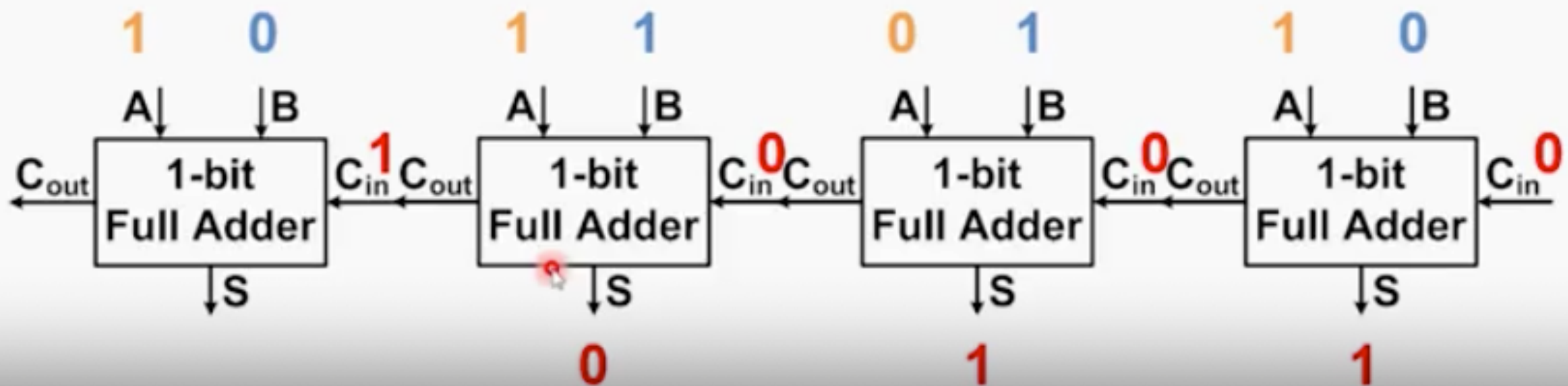
$$\begin{array}{r} 1101 \\ + 0110 \\ \hline 10011 \end{array}$$



# 4-bit Full Adder

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$$\begin{array}{r} 1101 \\ + 0110 \\ \hline 10011 \end{array}$$

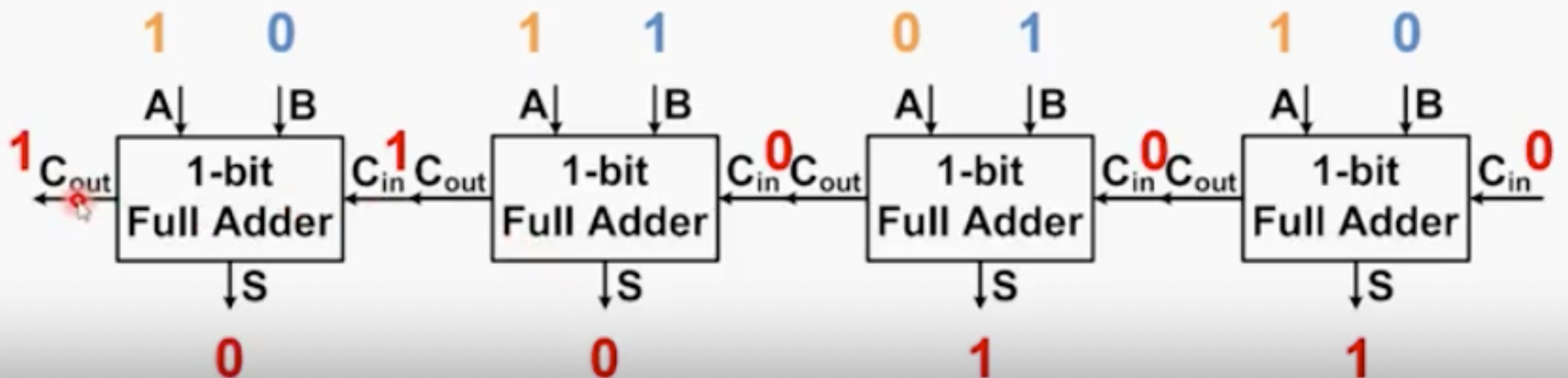




# 4-bit Full Adder

- ❖ How it works?
- ❖ Each bit assigned!

$$\begin{array}{r} 1101 \\ + 0110 \\ \hline 10011 \end{array}$$



---

# More bits Added

---

- ❖ 32-bit full adder

---

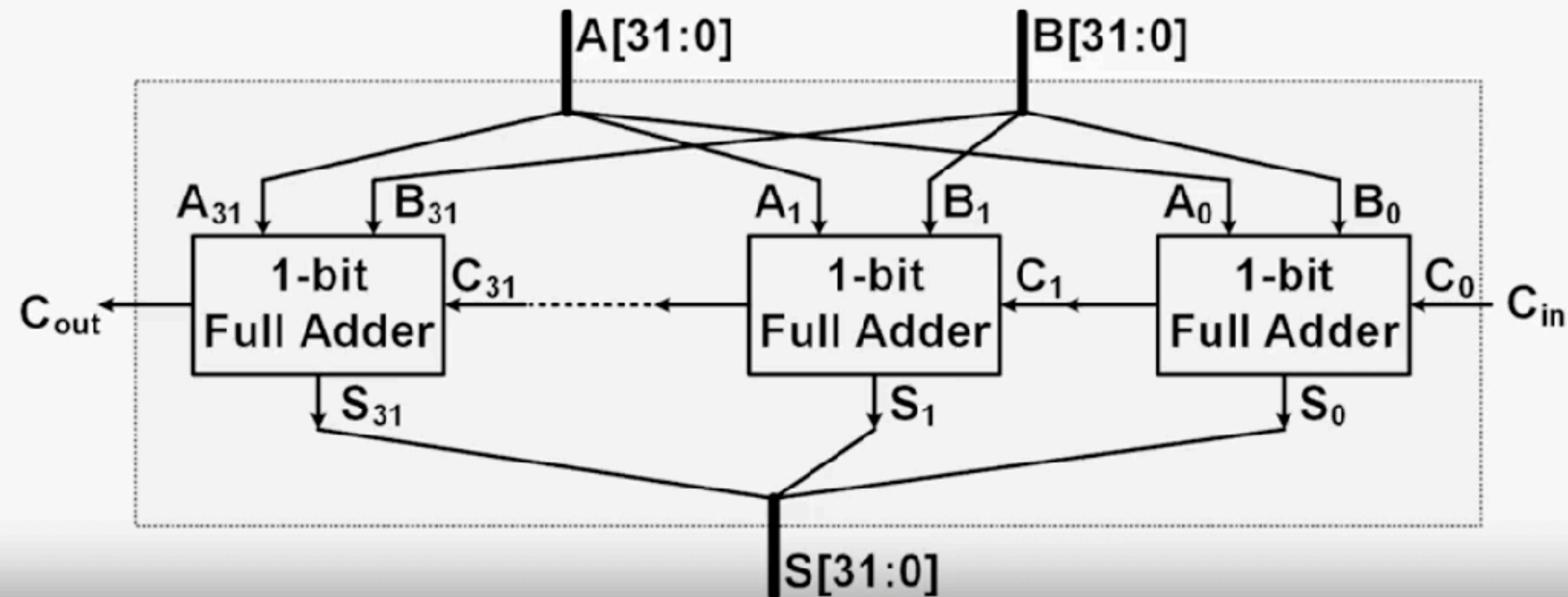
# More bits Added

---

- ❖ 32-bit full adder
  - ❖ How to design?

# More bits Added

- ❖ 32-bit full adder
  - ❖ How to design?



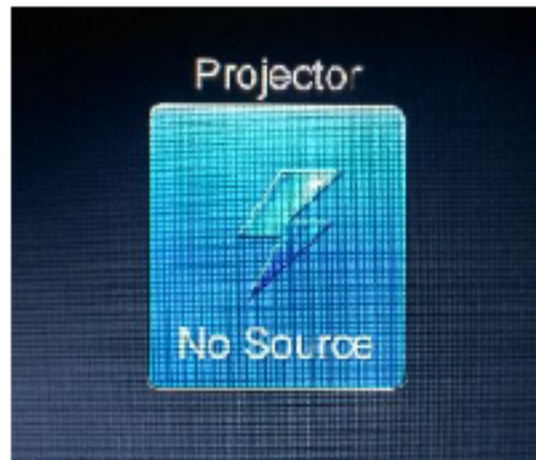
# EECE 2322: Fundamentals of Digital Design and Computer Organization

## Lecture 3\_3: MUX, Decoder, and Encoder

Xiaolin Xu

Department of ECE  
Northeastern University

# Real-World Digital Applications



**Select Input Source:**  
Press the Projector  
Source button



**Select Input Source:**  
Press the desired source button



**To Use Computer:**  
Button will show  
Computer



**To Use Doc Cam:**  
Button will show  
Doc Cam



**To Use Laptop VGA:**  
Button will show  
Laptop VGA



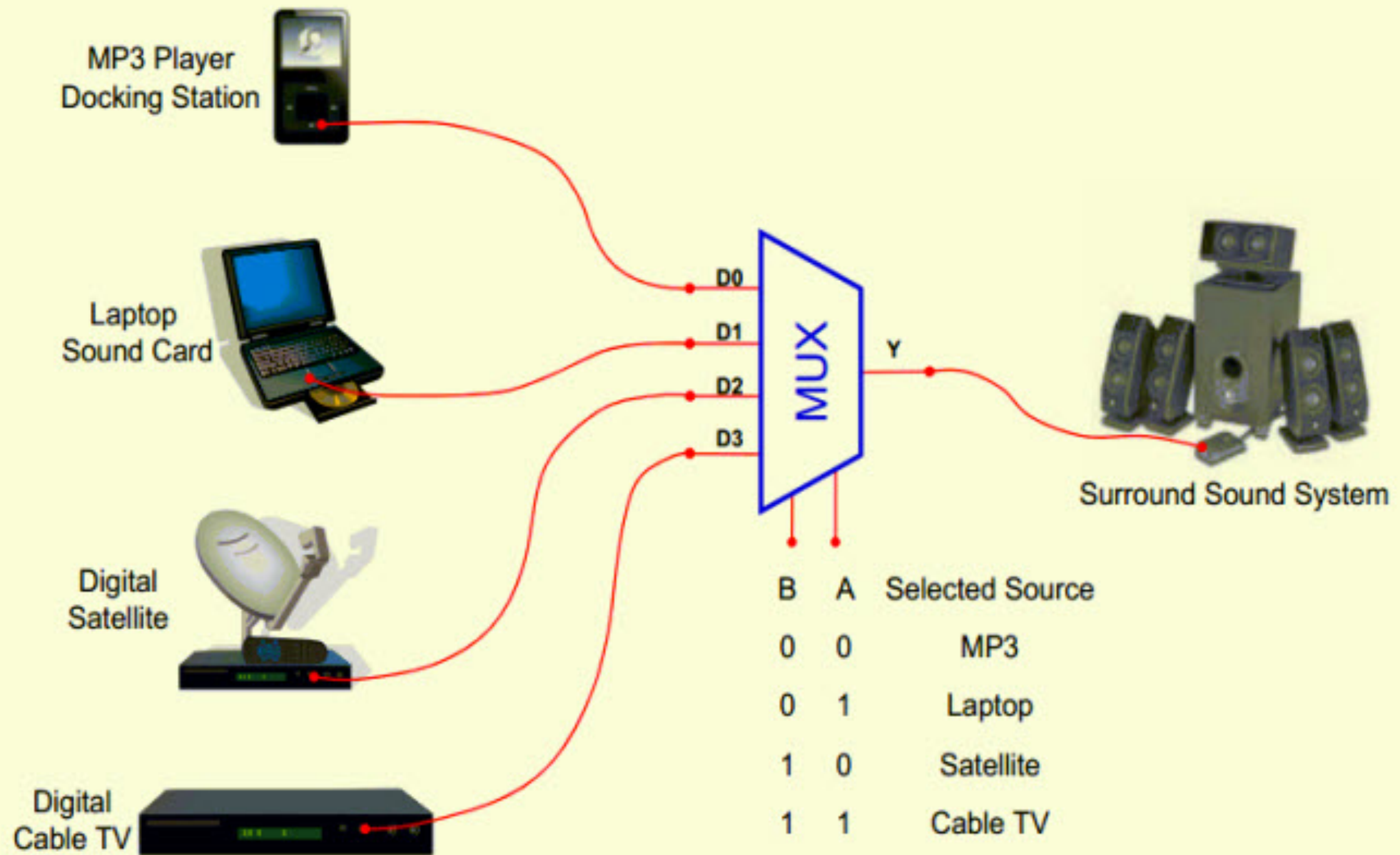
**To Use Laptop HDMI:**  
Button will show  
Laptop HDMI



**To Use AirMedia:**  
Button will show  
AirMedia



# Real-World Digital Applications



---

# Revisit Multiplexers (MUX)

---

- ❖ Data selector.
- ❖ Selects binary information from one of many input lines to a single output line.
- ❖ Selection inputs control which particular input line to select.
- ❖ Normally, there are  $2^n$  input lines,  $n$  selection inputs, and one output



---

# MUX

---

- ❖ A simple multiplexer has a single bit output
- ❖ Common Multiplexers are
  - ❖ 2-to-1    2 inputs      ? select lines
  - ❖ 4-to-1    4 inputs      ? select lines
  - ❖ 8-to-1    8 inputs      ? select lines
  - ❖  $2^n$ -to-1     $2^n$  inputs      ? select lines

---

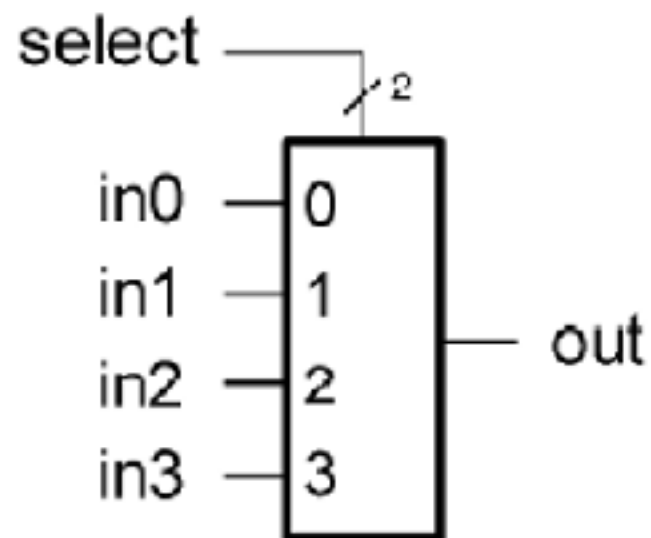
# 2-1 MUX in Verilog

---

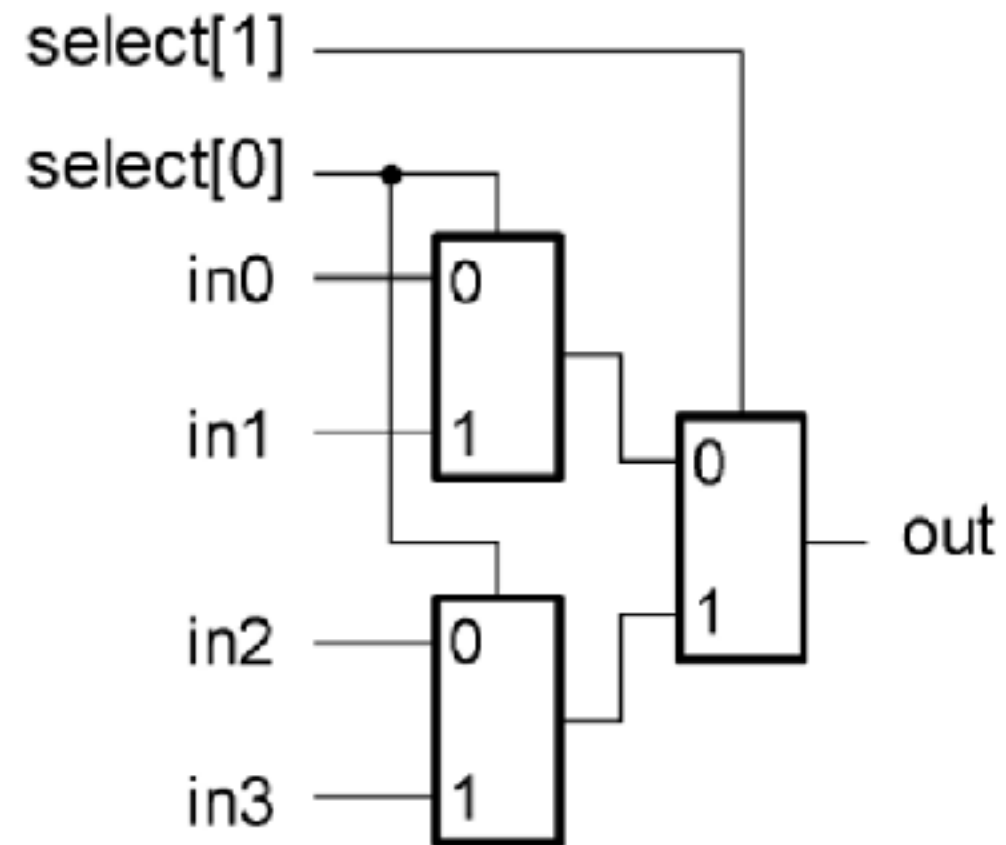
- ❖ `module mux2 (in0,in1,select,out);`
- ❖ `input in0,in1,select;`
- ❖ `output out;`
- ❖ `reg out;`
- ❖ `always @ (in0 or in1 or select) // sensitivity list`
- ❖ `if (select) out=in1;`
- ❖ `else out=in0;`
- ❖ `endmodule // mux2`

# Larger MUX Design

- ❖ 4-to-1 mux out of 2-to-1 muxes



a) 4-input mux symbol



b) 4-input mux implemented with 2-input muxes

---

# Verilog Code

---

## ❖ 4-to-1 mux out of 2-to-1 muxes

```
module mux4(in0, in1, in2, in3, select, out);  
    input [1:0] select;  
    input in0, in1, in2, in3;  
    output out;  
    wire w0, w1;  
  
    mux2  
        m0 (.select(select[0]), .in0(in0), .in1(in1), .out(w0)),  
        m1 (.select(select[0]), .in0(in2), .in1(in3), .out(w1)),  
        m2 (.select(select[1]), .in0(w0), .in1(w1), .out(out));  
  
endmodule // mux4
```

---

# Testbench for 4-to-1 MUX

---

```
module testmux4;
  reg [5:0] count = 6'b000000;
  reg a,b,c,d;
  reg [1:0] s;
  reg expected;
  wire f;

  mux4 myMux(.select(s), .in0(a), .in1(b), .in2(c), .in3(d), .out(f));
  initial
  begin
    repeat(64)
      begin
        a = count[0];
        b = count[1];
        c = count[2];
        d = count[3];
        s = count[5:4];
      end
  end
endmodule
```

---

# Testbench for 4-to-1 MUX (2)

---

```
case (s)
    2'b00: expected = a;
    2'b01: expected = b;
    2'b10: expected = c;
    2'b11: expected = d;
endcase
#5
$strobe("select=%b in0=%b in1=%b in2=%b in3= %b out = %b,
        expected = %b time=%d", s, a, b, c, d, f, expected, $time);
#5 count = count + 1'b1;
end
$stop;
end
endmodule //testbench
```

---

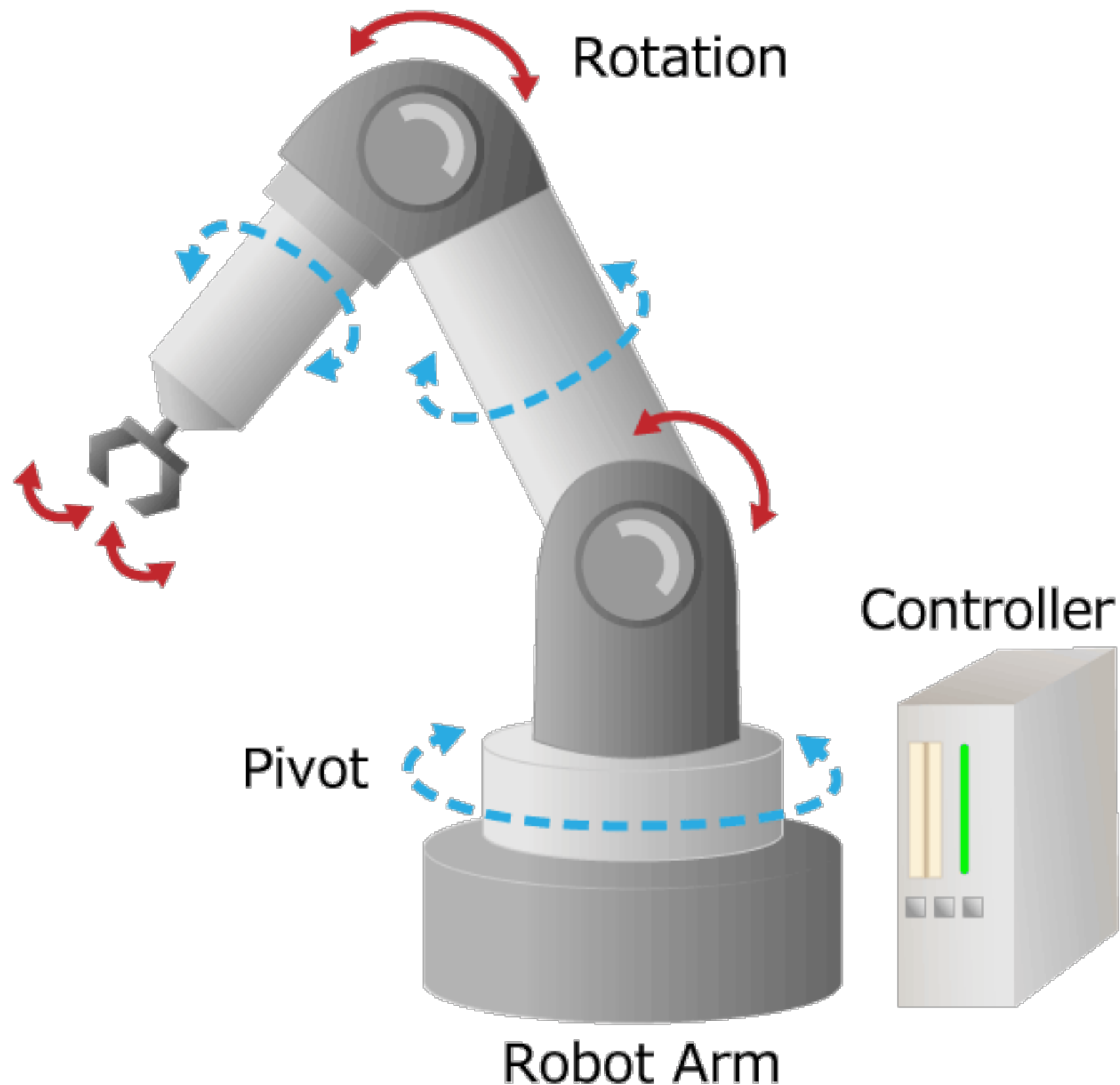
# case in Verilog

---

- ❖ Checks if the given expression matches one of the other expressions in the list and branches accordingly
- ❖ **Mostly used in building MUXes (see why later)**

```
case (<expression>)
  case_item1 : <single statement>
  case_item2,
  case_item3 : <single statement>
  case_item4 : begin
                  <multiple statements>
                end
  default    : <statement>
endcase
```

# Real-World Digital Applications





# Real-World Digital Applications

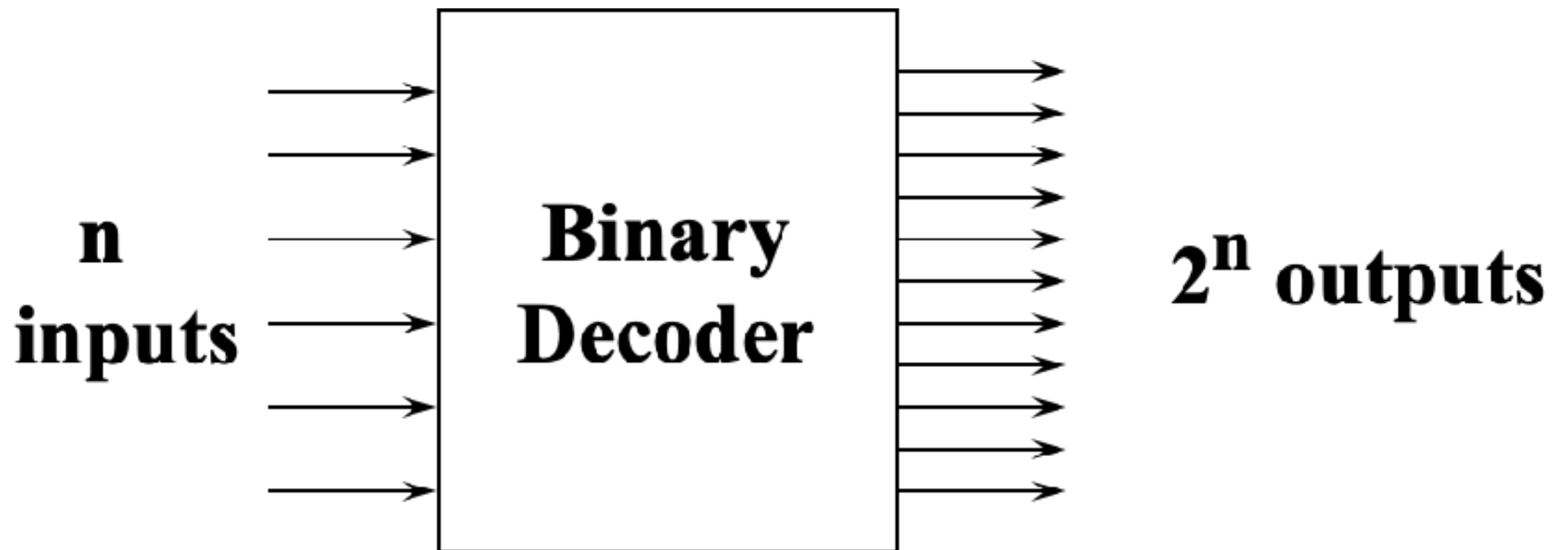


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# Decoder

---

- ❖ Logic with  $n$  input lines and  $2^n$  output lines
- ❖ Only one output is a 1 for any given input
- ❖



---

# Decoder Example

---

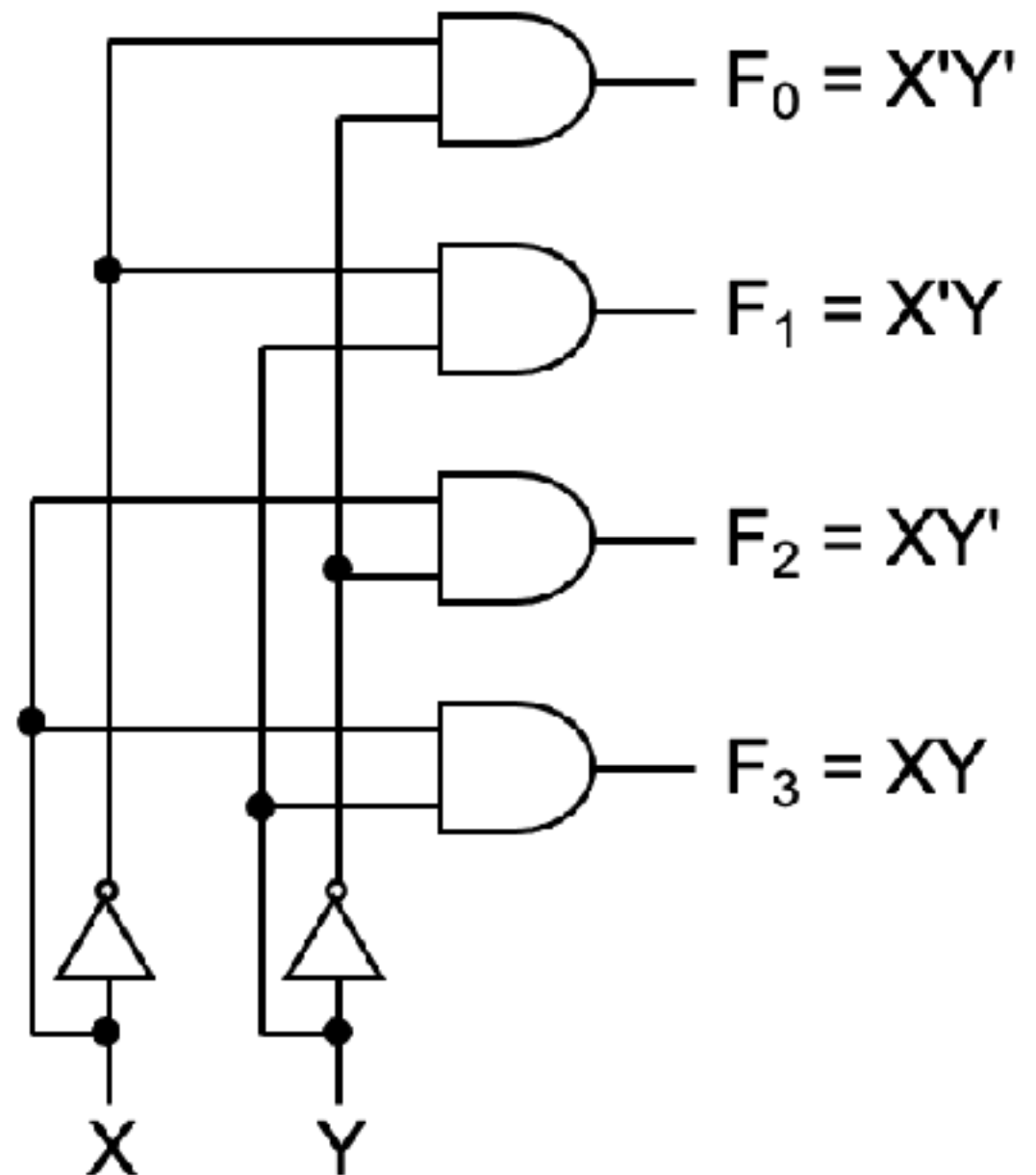
❖ 2-to-4 decoder

<b>X</b>	<b>Y</b>	<b>F<sub>0</sub></b>	<b>F<sub>1</sub></b>	<b>F<sub>2</sub></b>	<b>F<sub>3</sub></b>
0	0	1	0	0	0
0	1	0	1	0	0
1	0	0	0	1	0
1	1	0	0	0	1

# Decoder Example

## ❖ 2-to-4 decoder

<b>X</b>	<b>Y</b>	<b>F<sub>0</sub></b>	<b>F<sub>1</sub></b>	<b>F<sub>2</sub></b>	<b>F<sub>3</sub></b>
0	0	1	0	0	0
0	1	0	1	0	0
1	0	0	0	1	0
1	1	0	0	0	1

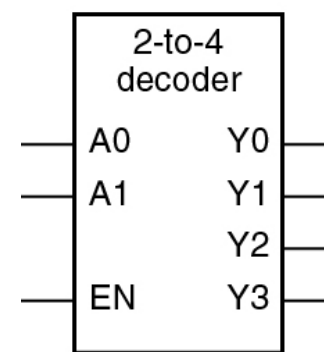


# Binary Decoder with ENable

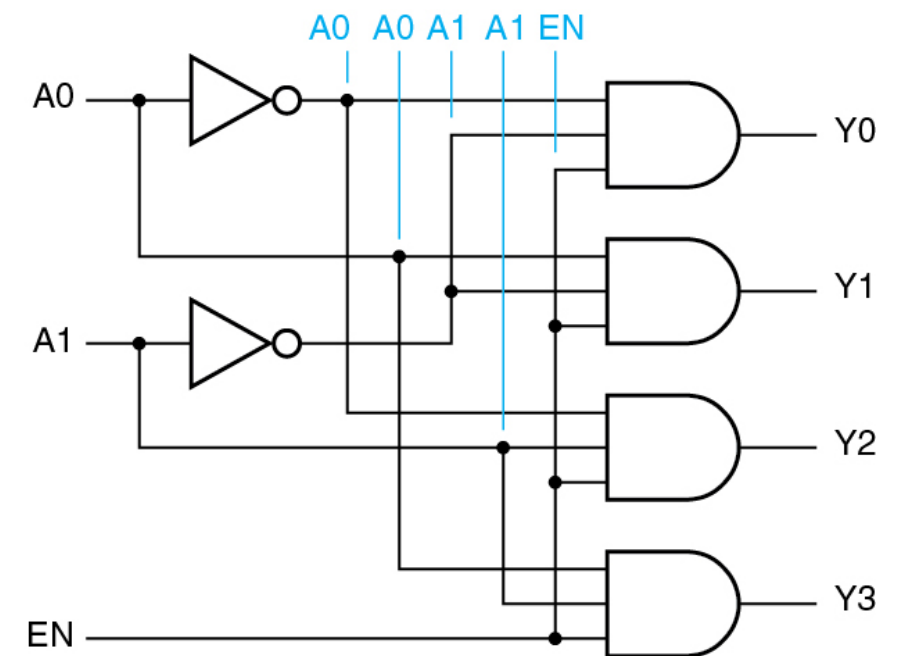
<i>Inputs</i>			<i>Outputs</i>			
EN	A1	A0	Y3	Y2	Y1	Y0
0	x	x	0	0	0	0
1	0	0	0	0	0	1
1	0	1	0	0	1	0
1	1	0	0	1	0	0
1	1	1	1	0	0	0

# Binary Decoder with Enable

Inputs			Outputs			
EN	A1	A0	Y3	Y2	Y1	Y0
0	x	x	0	0	0	0
1	0	0	0	0	0	1
1	0	1	0	0	1	0
1	1	0	0	1	0	0
1	1	1	1	0	0	0



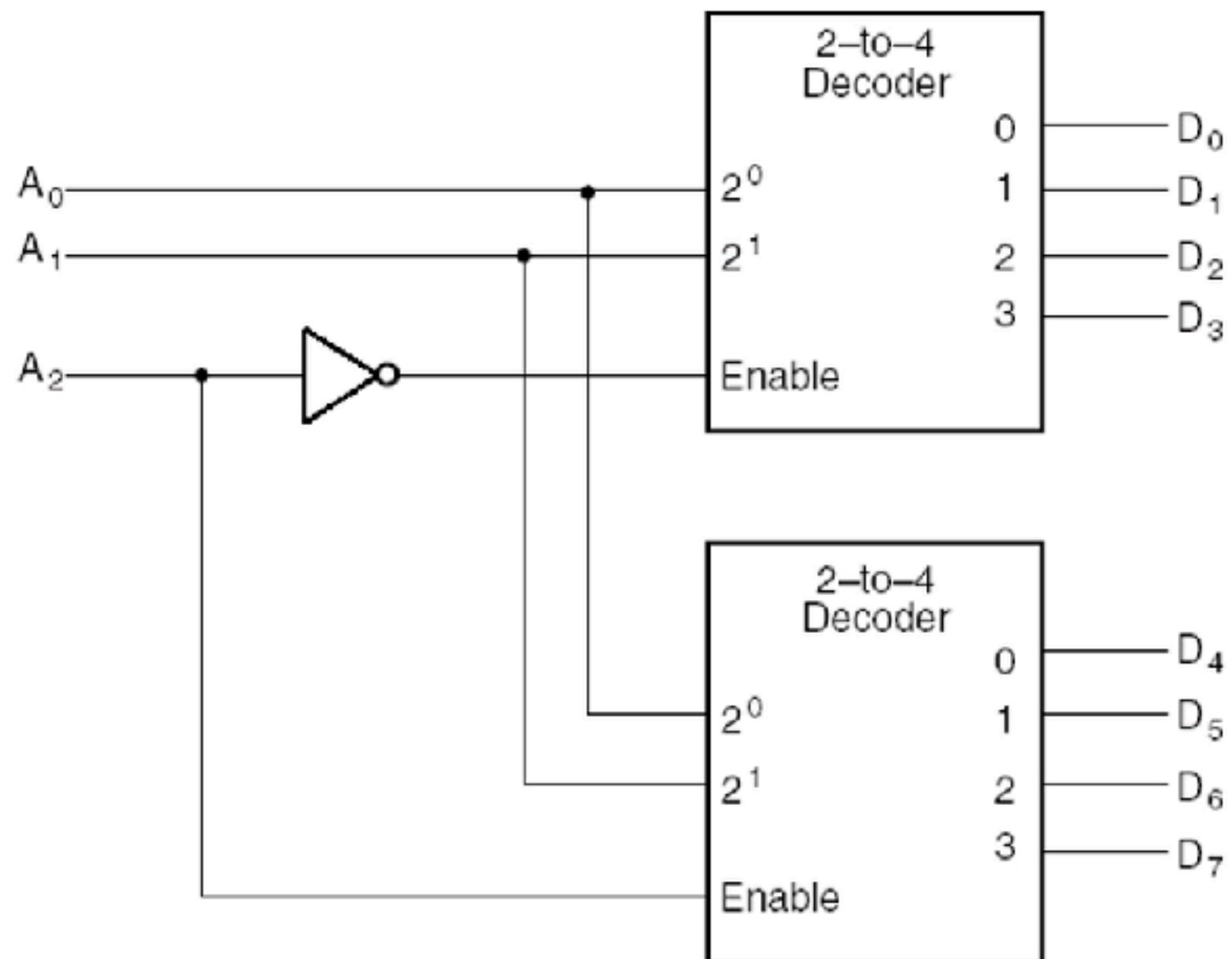
(a)



(b)

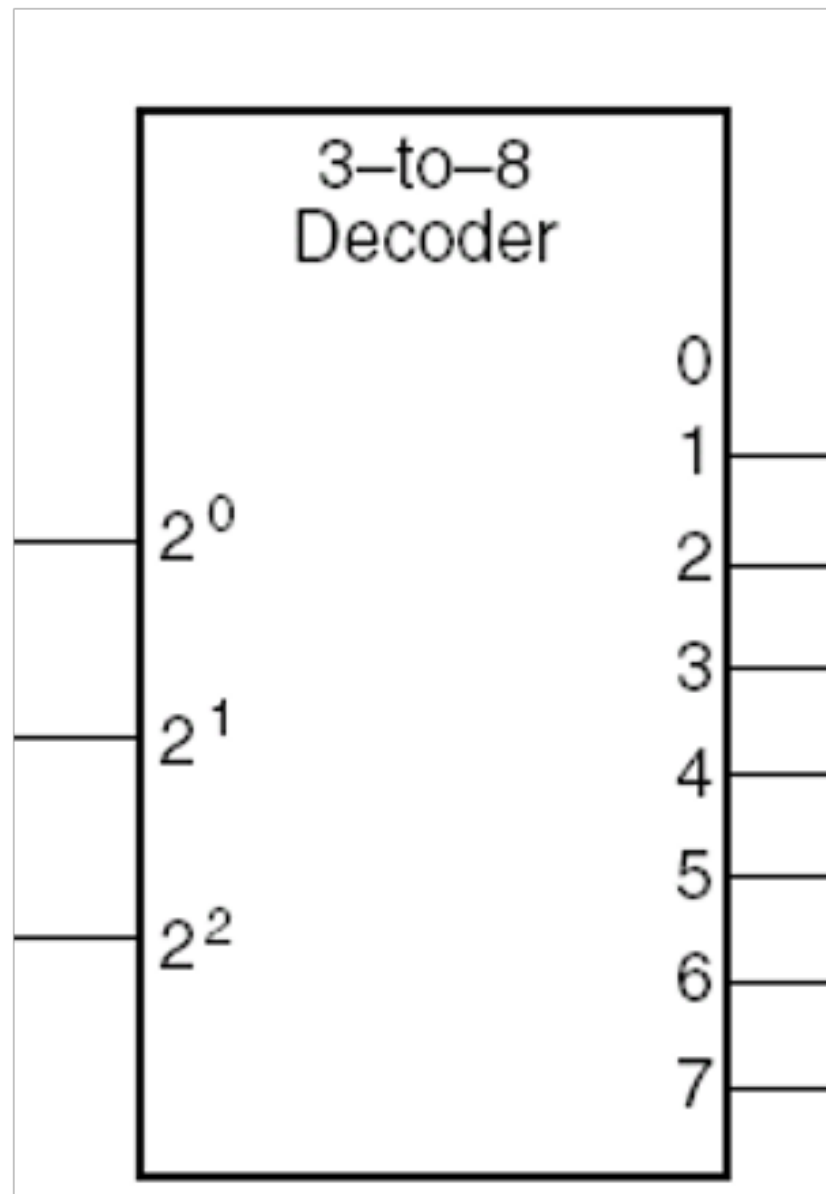
# Building Larger Decoder

- ❖ Build larger decoders out of two or more smaller decoders



# Decoder Example

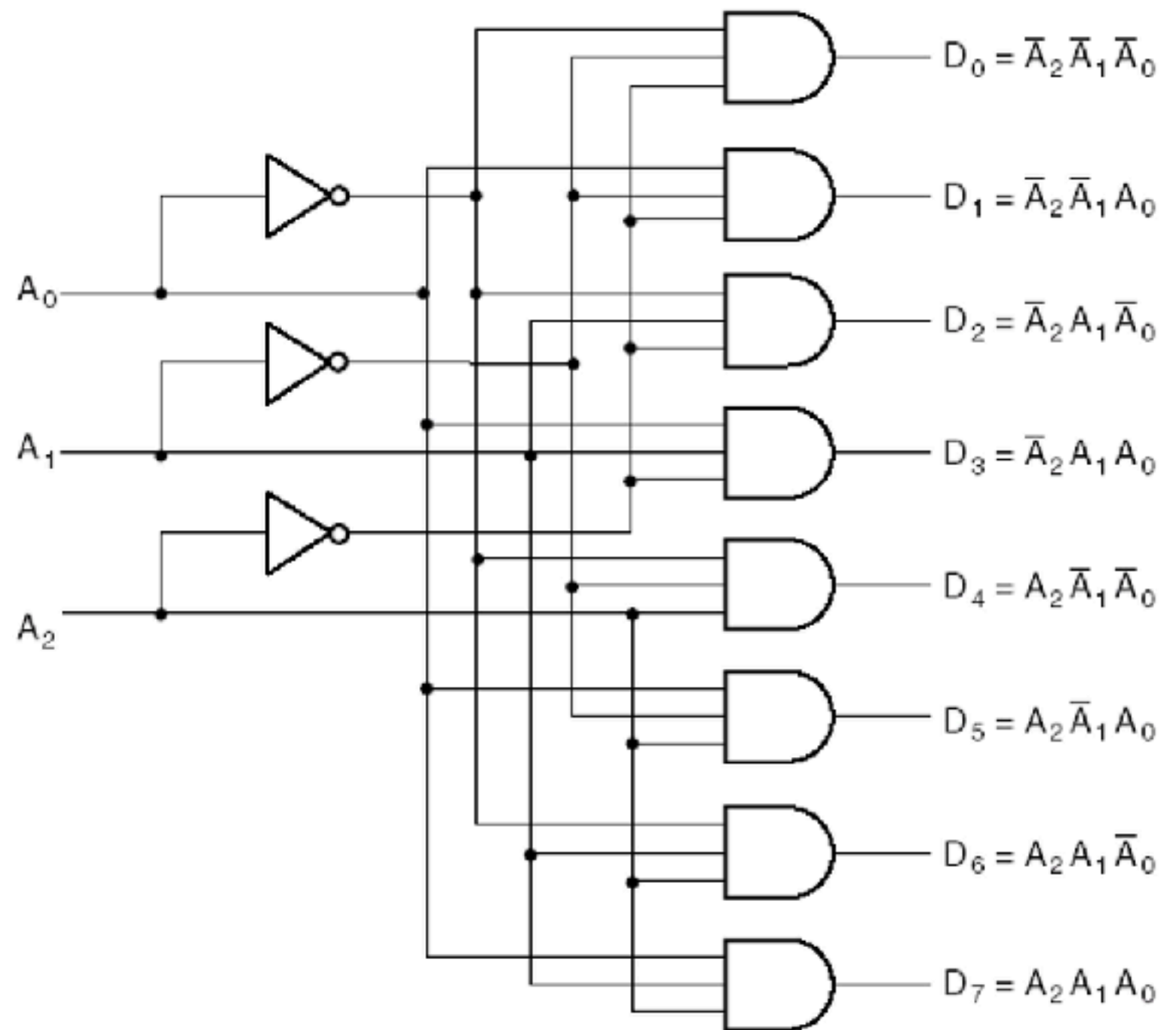
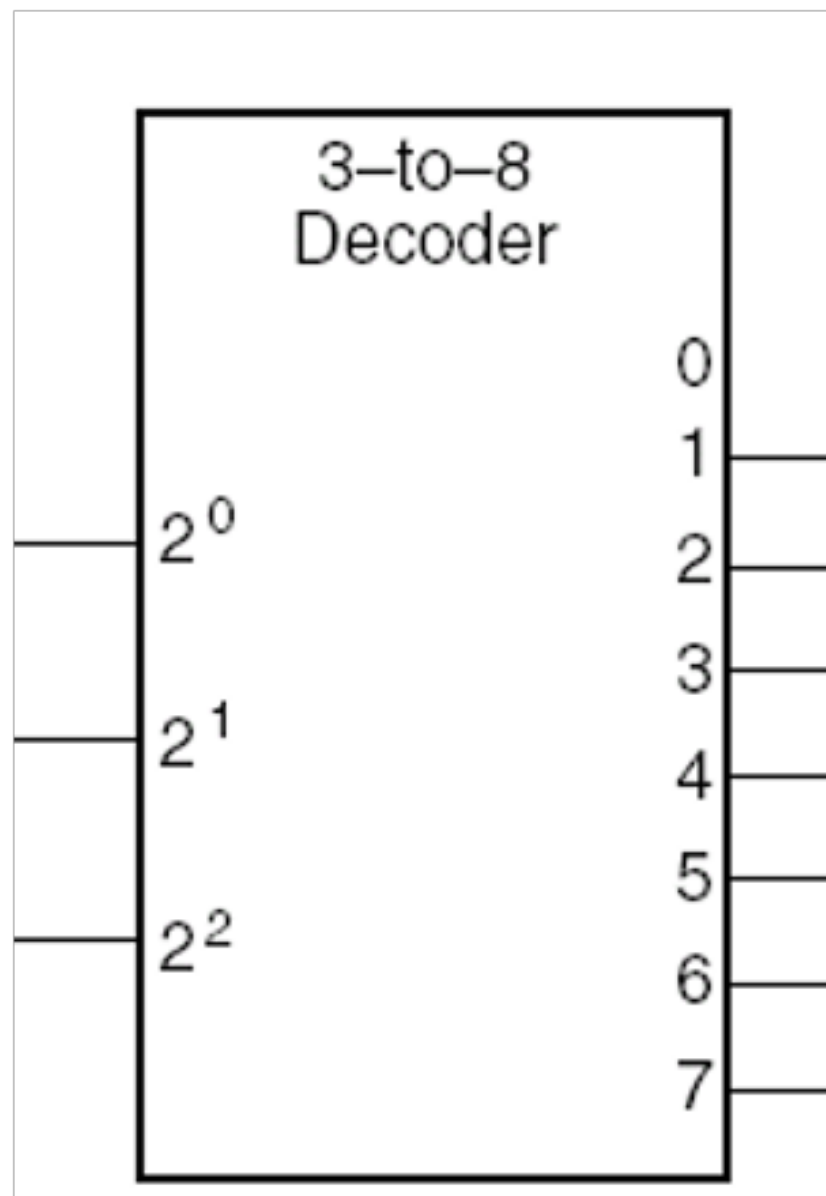
## ❖ 3-to-8 decoder





# Decoder Example

## ❖ 3-to-8 decoder



---

# 5-To-32 Decoder

---

❖ How?

---

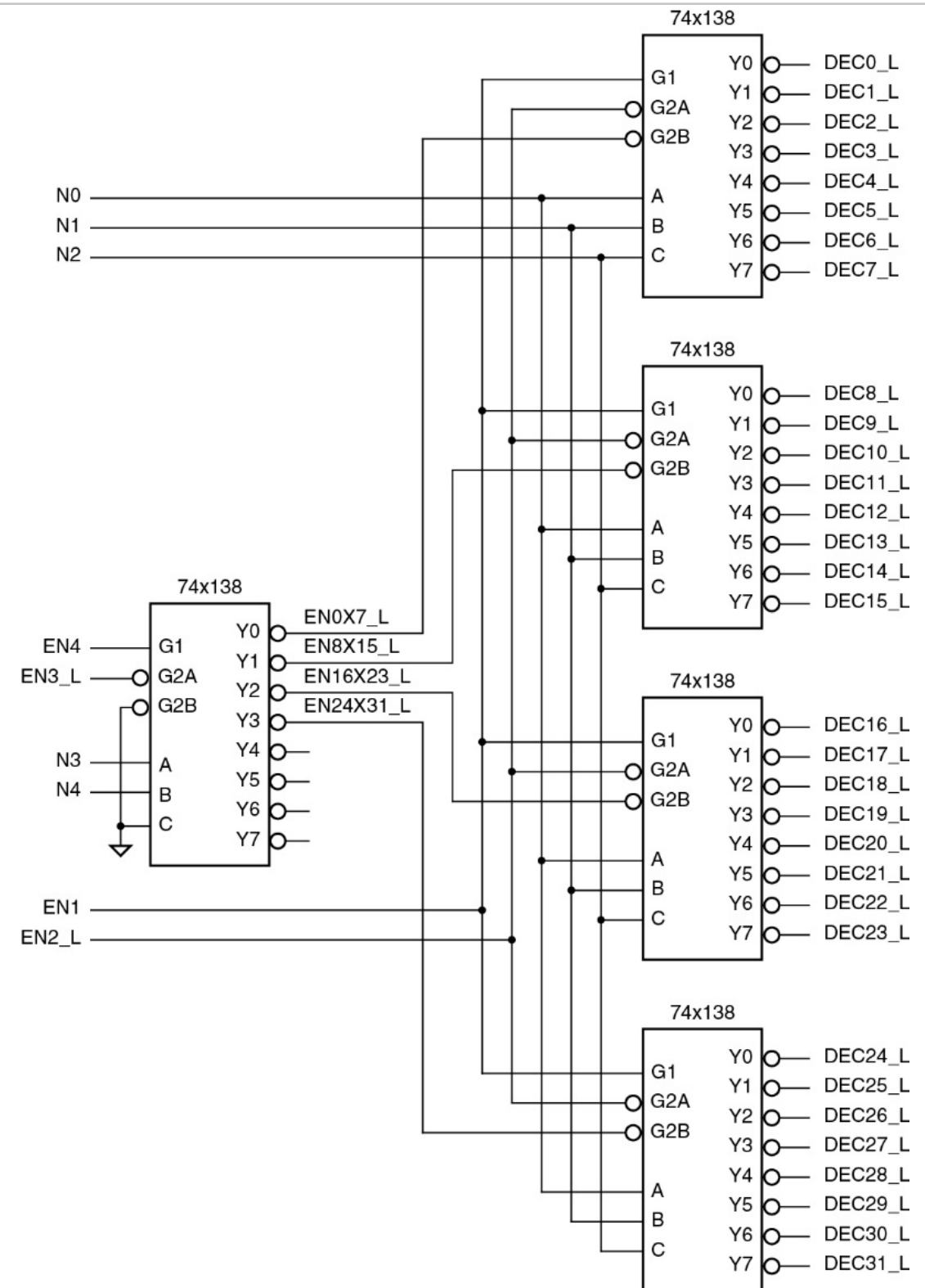
# 5-To-32 Decoder

---

- ❖ How?
- ❖ Cascading 3-To-8 decoders

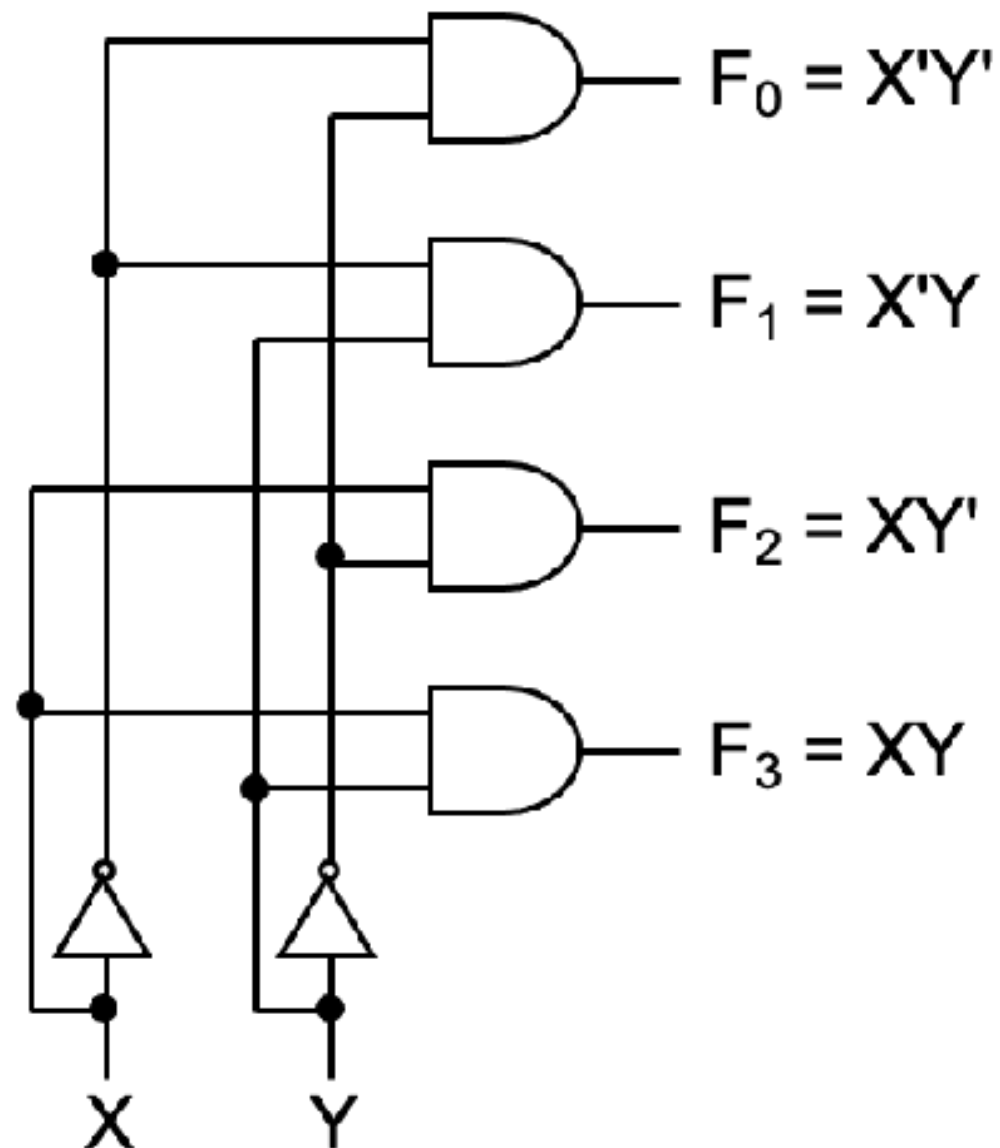
# 5-To-32 Decoder

- ❖ How?
- ❖ Cascading 3-To-8 decoders



# 2-to-4 Decoder in Verilog

❖ Think about how?



❖ Structural-style Verilog Module

---

# 2-to-4 Decoder in Verilog

---

❖ Think about how?

```
module Vr2to4dec(I0, I1, EN, Y0, Y1, Y2, Y3);  
    input I0, I1, EN;  
    output Y0, Y1, Y2, Y3;  
    wire NOTI0, NOTI1;  
  
    not U1 (NOTI0, I0);  
    not U2 (NOTI1, I1);  
    and U3 (Y0, NOTI0, NOTI1, EN);  
    and U4 (Y1, I0, NOTI1, EN);  
    and U5 (Y2, NOTI0, I1, EN);  
    and U6 (Y3, I0, I1, EN);  
  
endmodule
```

❖ Structural-style Verilog Module

# 2-to-4 Decoder in Verilog

```
module dec2to4 (Y, W, En);  
    input [1:0] W;  
    input En;  
    output [3:0] Y;  
    reg [3:0] Y;  
  
    always@ (W or En)  
        case ({En, W}) // concatenation of En and W  
            3'b100: Y = 4'b0001;  
            3'b101: Y = 4'b0010;  
            3'b110: Y = 4'b0100;  
            3'b111: Y = 4'b1000;  
            default: Y = 4'b0000;  
        endcase  
    endmodule
```

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# A Decoder with ENable in Verilog

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❖ Think about how?

❖ Data flow style Verilog module



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# A Decoder with ENable in Verilog

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❖ Think about how?

```
module dec24_dat (  
  output [3:0] y,  
  input [1:0] a,  
  input en);  
  
  assign y = en ? (4'b0001 << a) : 0;  
  
endmodule
```

❖ Data flow style Verilog module