

EECE 2322: Fundamentals of Digital Design and Computer Organization

Lecture 4_3: ALU

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Mid-term Exam1

- ❖ When

- ❖ Thursday, Feb. 17, 2022, 1:35—2:40PM

- ❖ Where

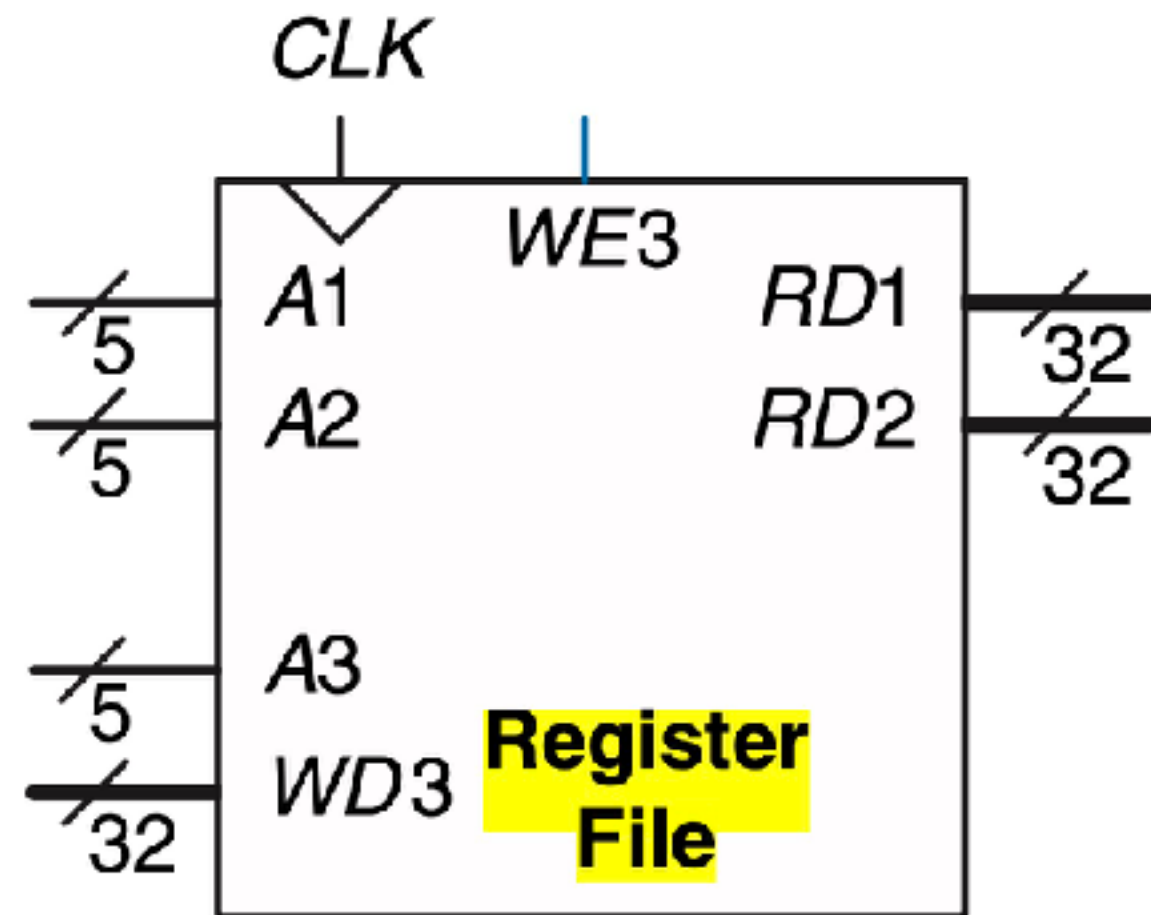
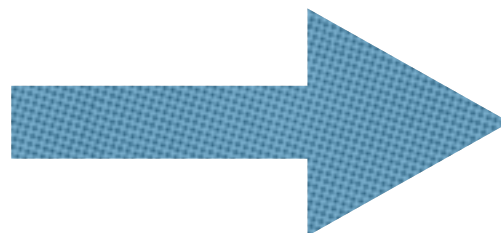
- ❖ Online, as a regular assignment in Canvas
 - ❖ So, no lecture that day!

- ❖ What

- ❖ Slides, Week1—Week4
 - ❖ HWs

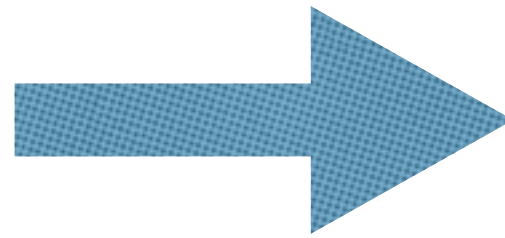
Register File

- ❖ Register file or Regfile: a number of registers are used to store variables
 - ❖ Could be a number of memory cells, e.g., SRAM array
- ❖ *32 × 32 register file with two read ports and one write port*

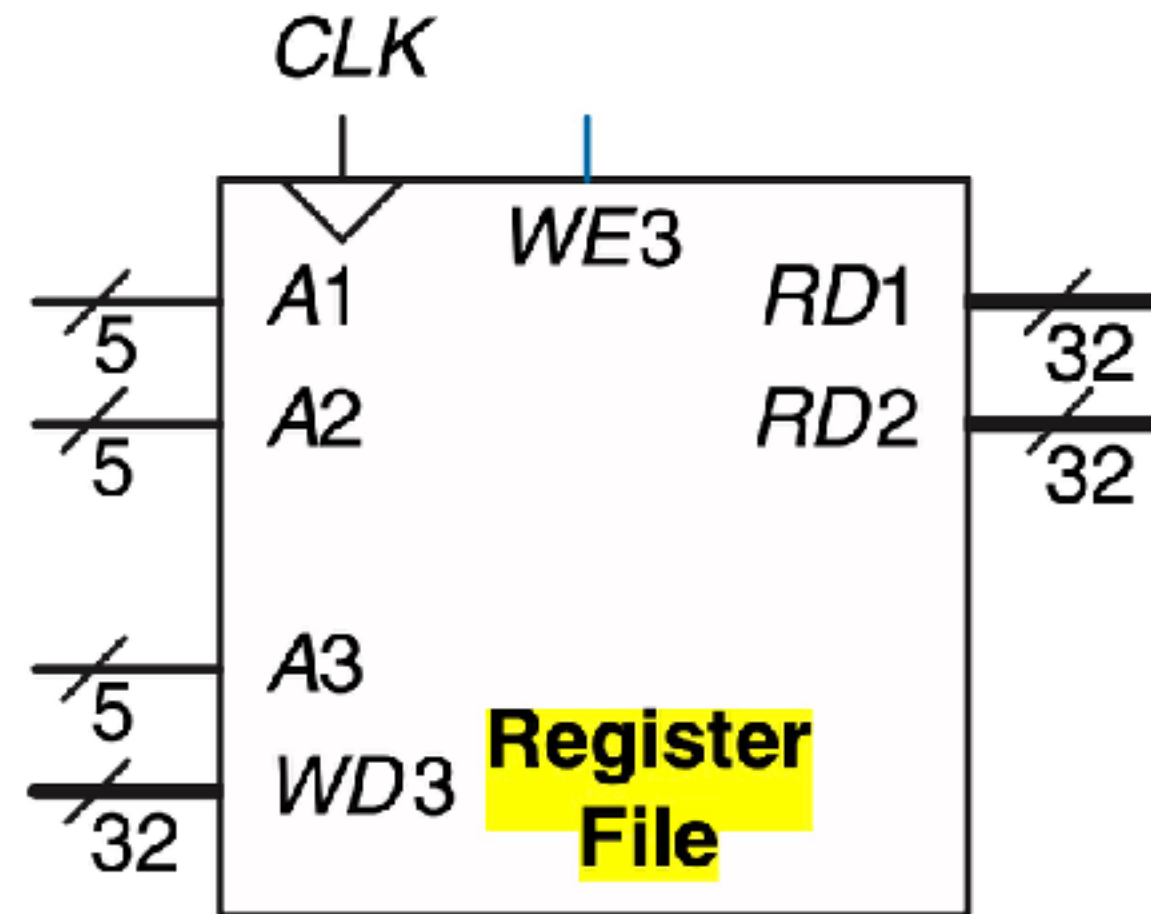


Register File

- ❖ *32 × 32 register file with two read ports and one write port*

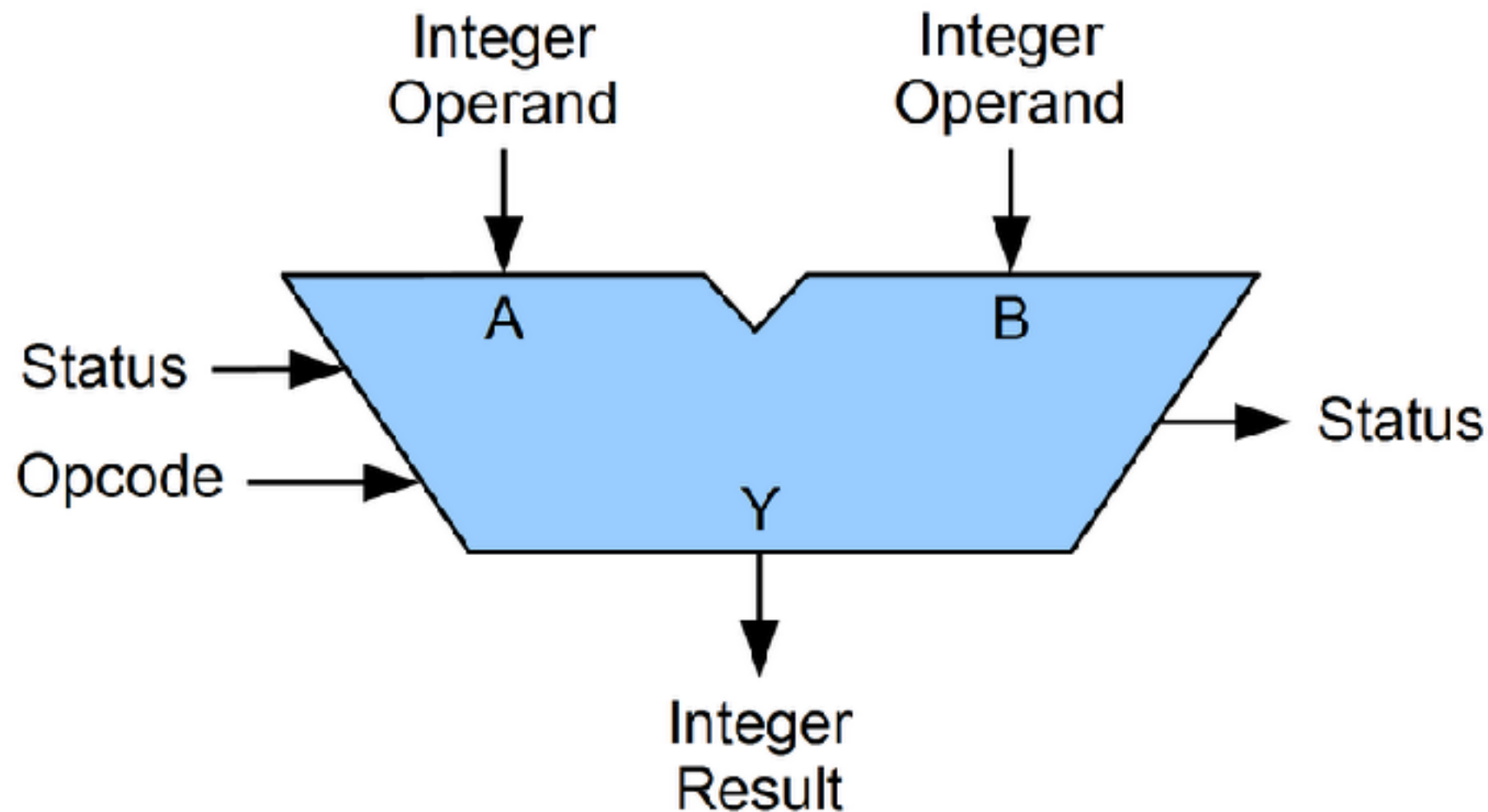


- ❖ Two read ports (A1 / RD1 and A2 / RD2)
- ❖ One write port (A3 / WD3).
- ❖ 5-bit addresses (A1, A2, and A3) can each access all 32 registers.
- ❖ *Two registers can be read and one register written simultaneously.*



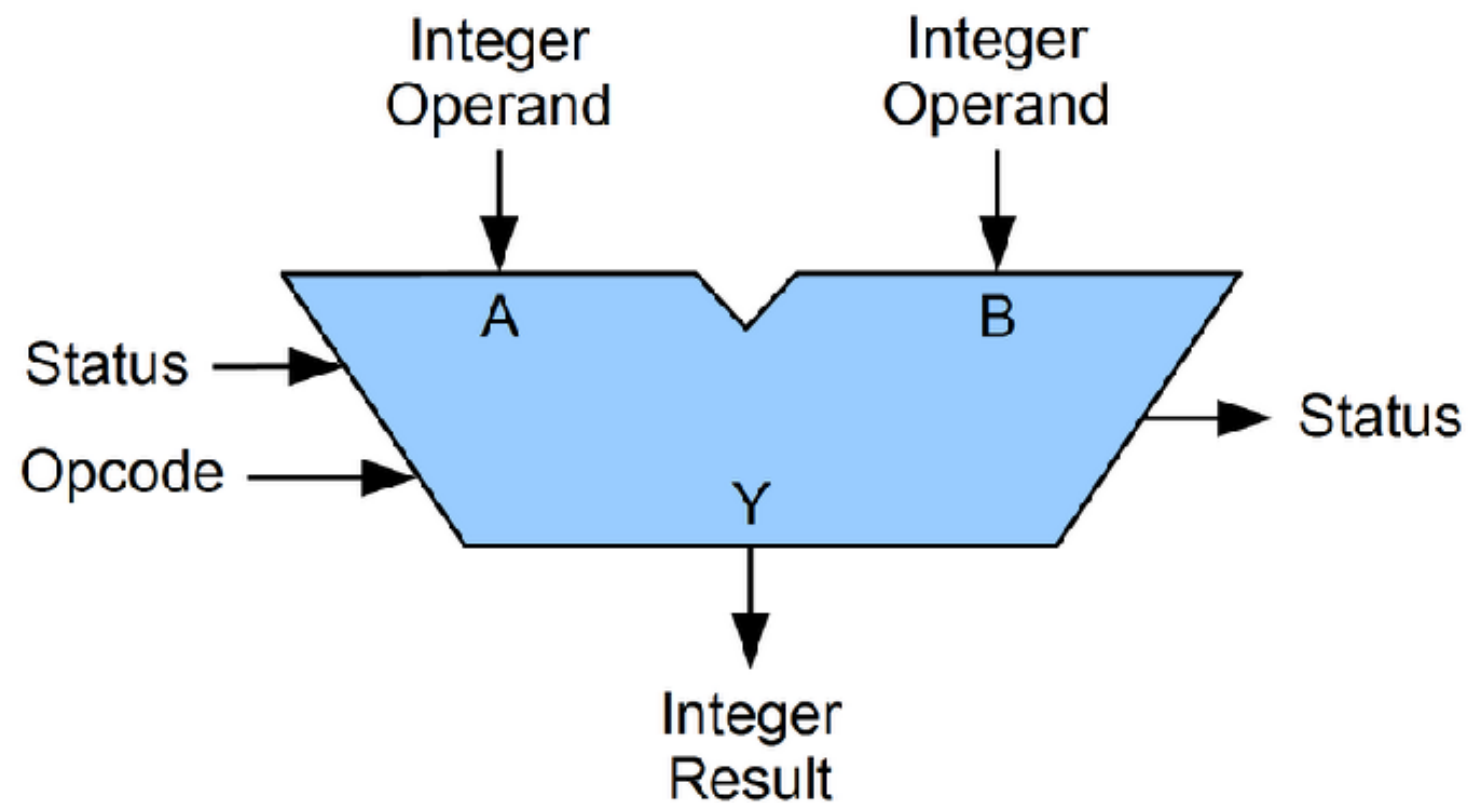
ALU: Arithmetic Logic Unit

- ❖ Digital circuit that performs arithmetic and bitwise operations



ALU: Arithmetic Logic Unit

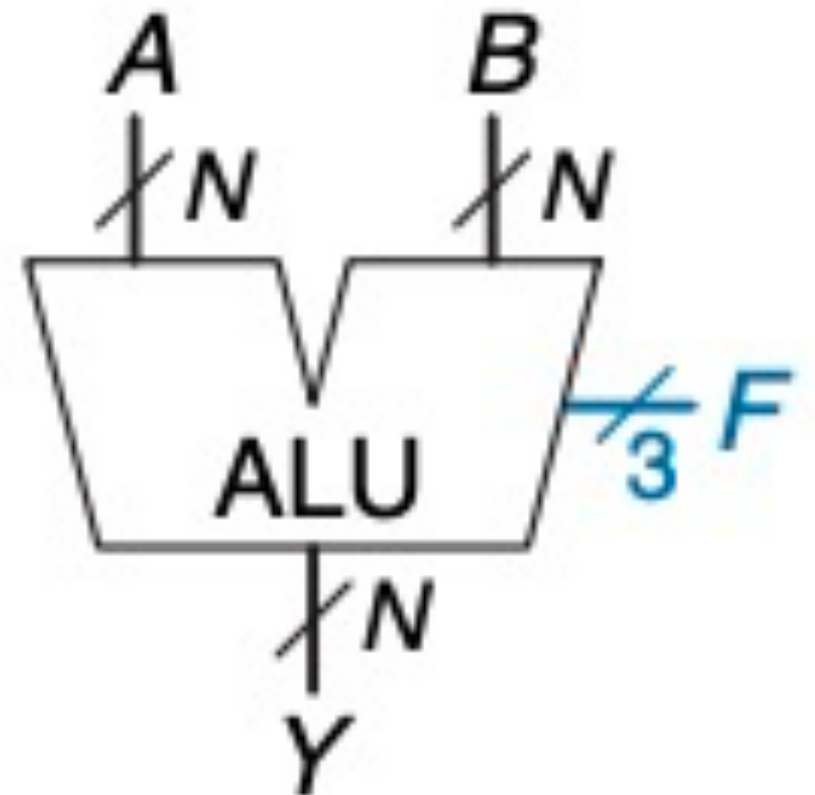
- ❖ Digital circuit that performs arithmetic and bitwise operations
 - ❖ Data, Opcode, Status
- ❖ Arithmetic
 - ❖ Add, Add with carry, Subtract, Subtract with borrow ...
- ❖ Bite-wise logic
 - ❖ AND, OR, XOR ...



ALU Schematic

- ❖ Abstraction of ALU, three components
 - ❖ Two inputs (A, B)
 - ❖ One output (Y)
 - ❖ Control signal F (3-bit)

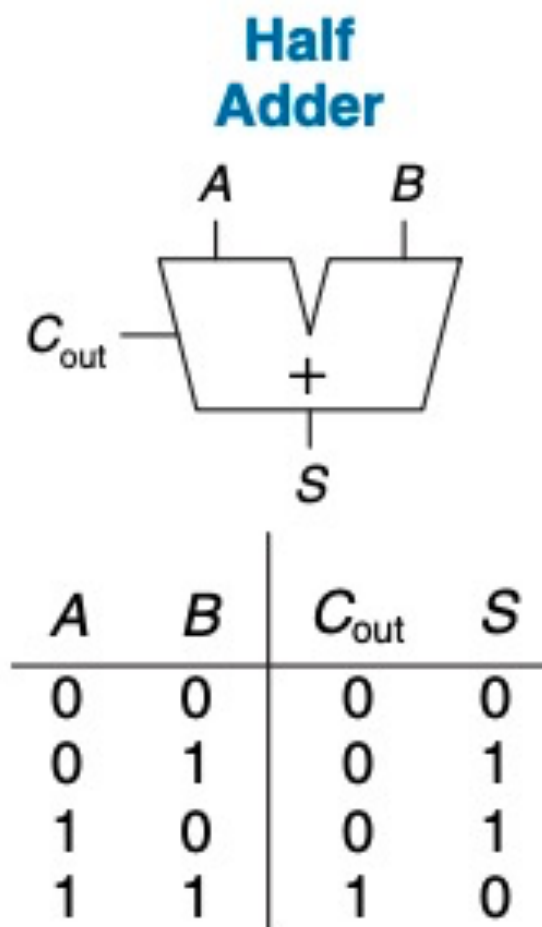
$F_{2:0}$	Function
000	A AND B
001	A OR B
010	A + B
011	not used
100	A AND \bar{B}
101	A OR \bar{B}
110	A - B
111	SLT



SLT: Set less than (later)

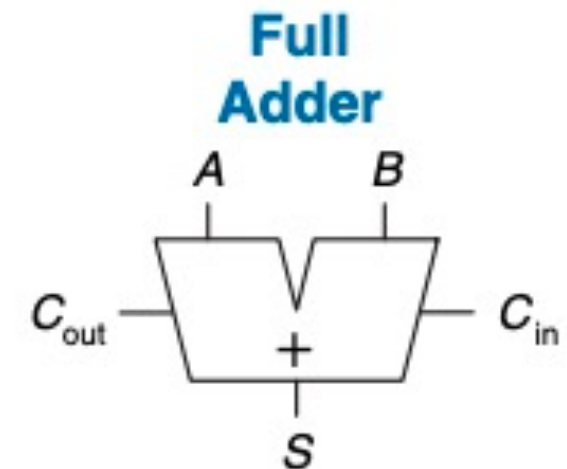
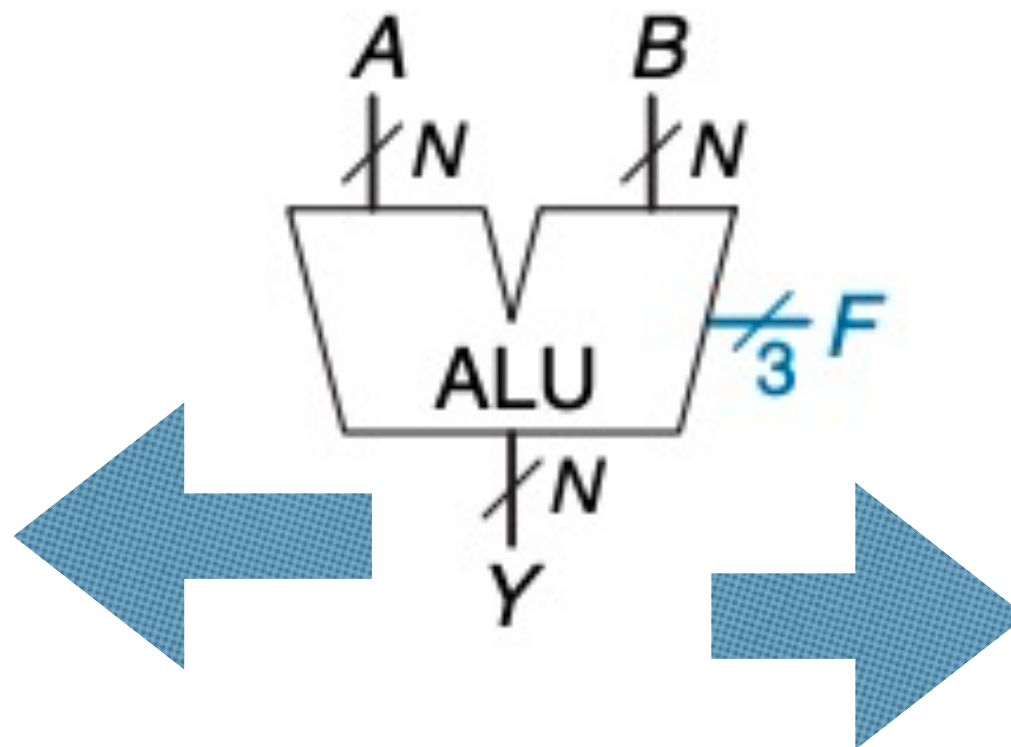
ALU Operations: Addition

- ❖ Adder, already introduced
- ❖ Replace the ALU with +



$$S = A \oplus B$$

$$C_{out} = AB$$



C _{in}	A	B	C _{out}	S
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	1

$$S = A \oplus B \oplus C_{in}$$

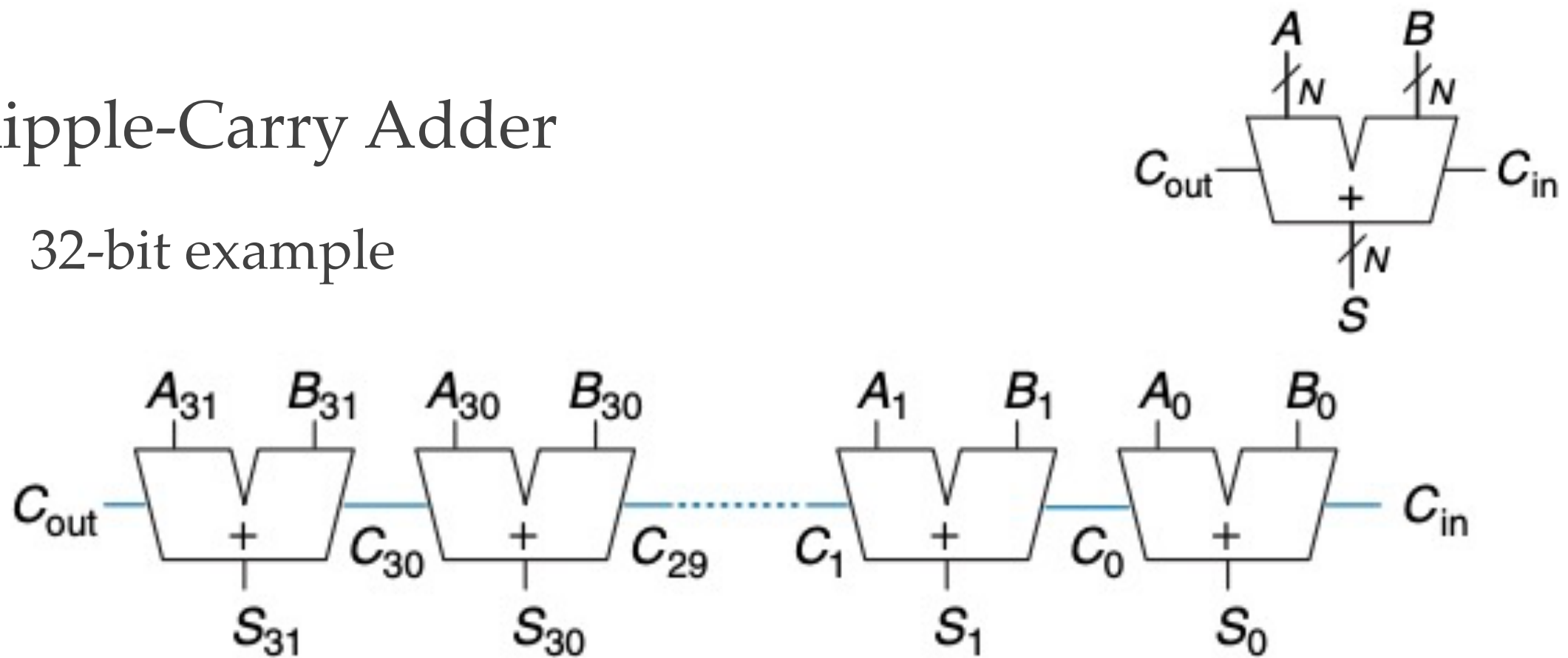
$$C_{out} = AB + AC_{in} + BC_{in}$$

N-bit Adder

- ❖ Carry Propagate Adder (CPA)
 - ❖ Full adder symbol, A, B, and S are busses rather than single bits

- ❖ Ripple-Carry Adder

- ❖ 32-bit example



- ❖ *Performance issue?*

Ripple-Carry Adder

❖ RCA

Ripple-Carry Adder

- ❖ RCA

- ❖ The Cout of LSB is cascaded/used as the Cin of its LSB+1

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- ❖ Pros

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- ❖ The Cout of LSB is cascaded/used as the Cin of its LSB+1

- ❖ Pros

- ❖ Straightforward, easy topology

Ripple-Carry Adder

- ❖ RCA

- ❖ The Cout of LSB is cascaded/used as the Cin of its LSB+1

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- ❖ Straightforward, easy topology

- ❖ Cons

Ripple-Carry Adder

- ❖ RCA

- ❖ The Cout of LSB is cascaded / used as the Cin of its LSB+1

- ❖ Pros

- ❖ Straightforward, easy topology

- ❖ Cons

- ❖ Long-latency

Ripple-Carry Adder

- ❖ RCA

- ❖ The Cout of LSB is cascaded / used as the Cin of its LSB+1

- ❖ Pros

- ❖ Straightforward, easy topology

- ❖ Cons

- ❖ Long-latency

- ❖ MSB will have to wait for the results from LSB!

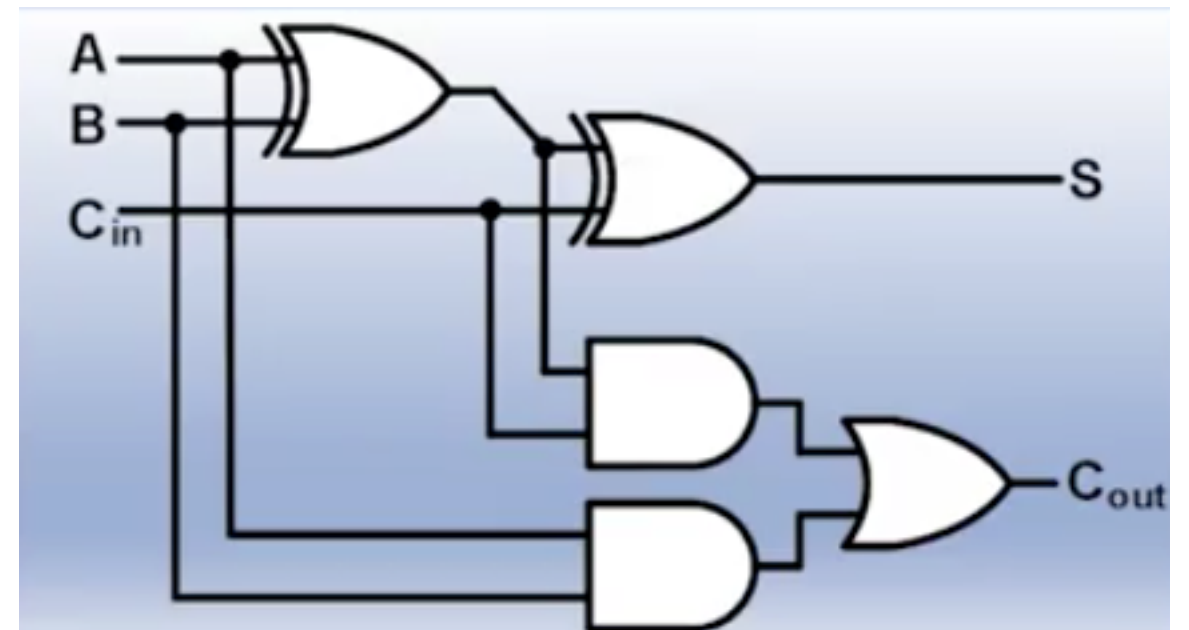
Delay Analysis

Delay Analysis

- ❖ Basic component

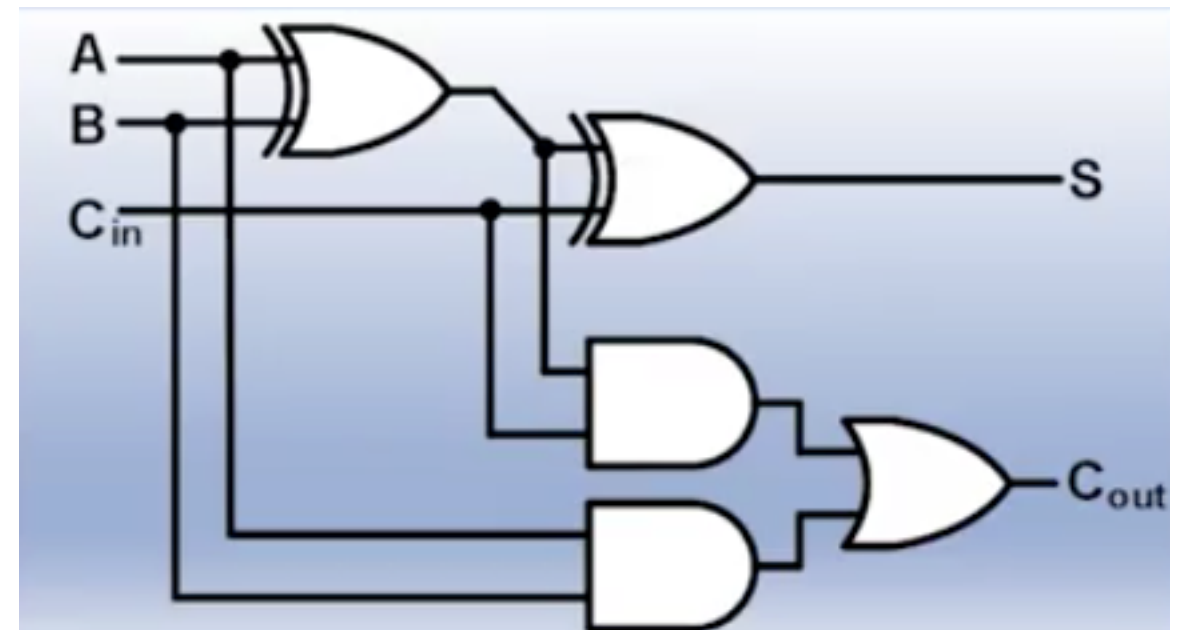
Delay Analysis

❖ Basic component



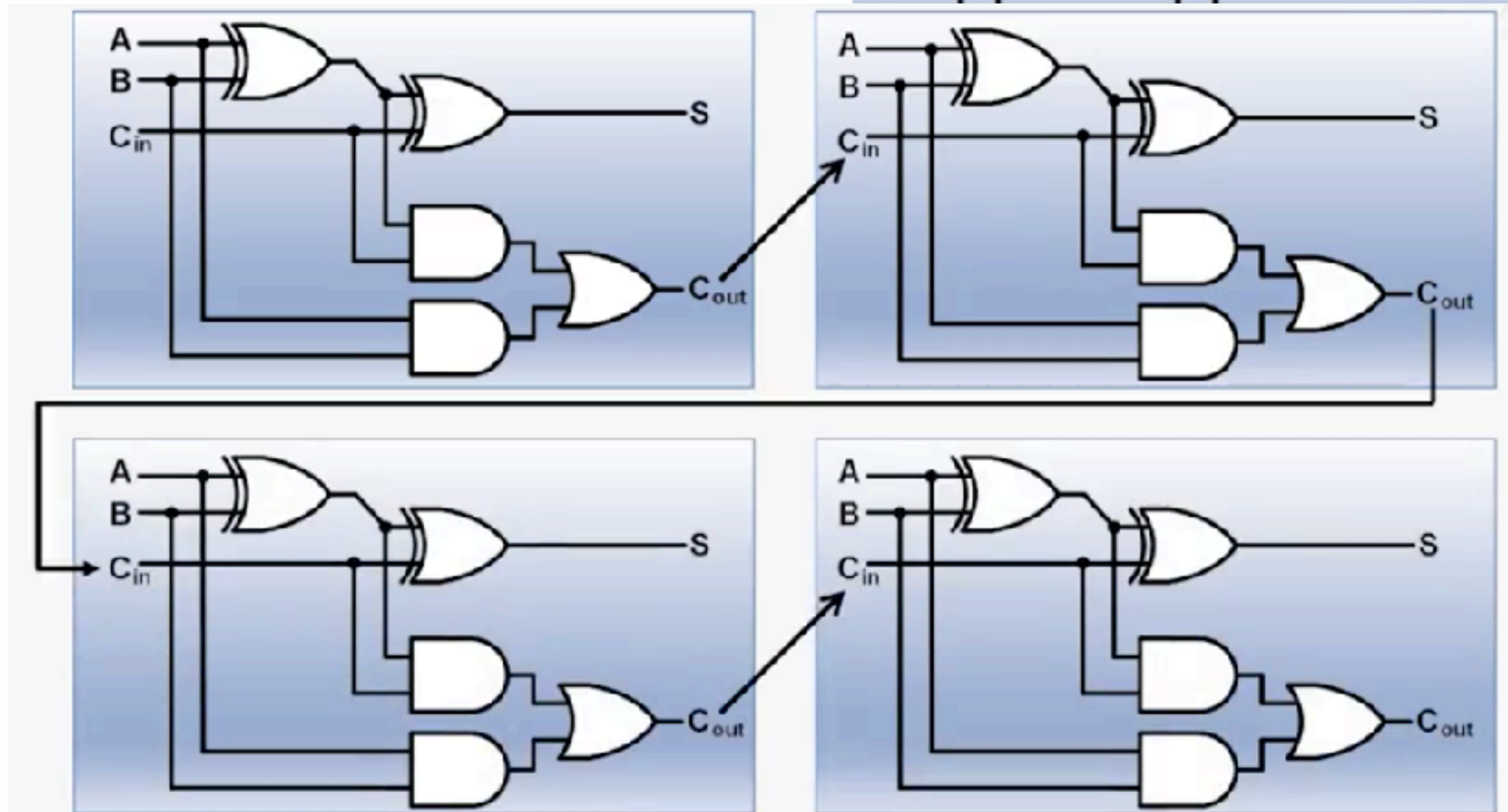
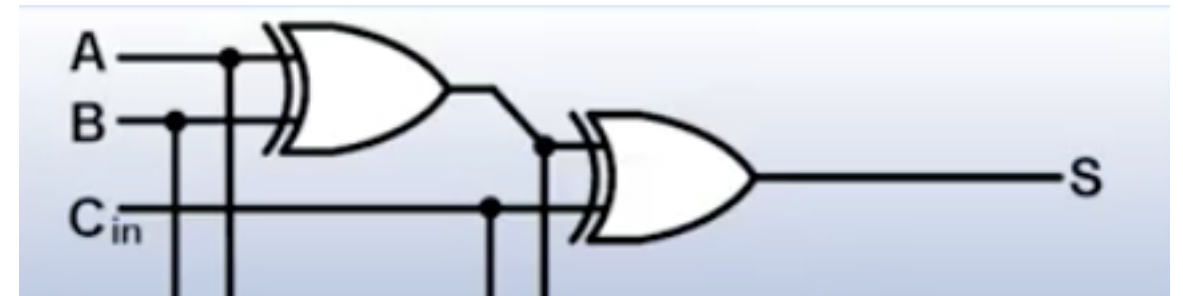
Delay Analysis

- ❖ Basic component
- ❖ Cascaded



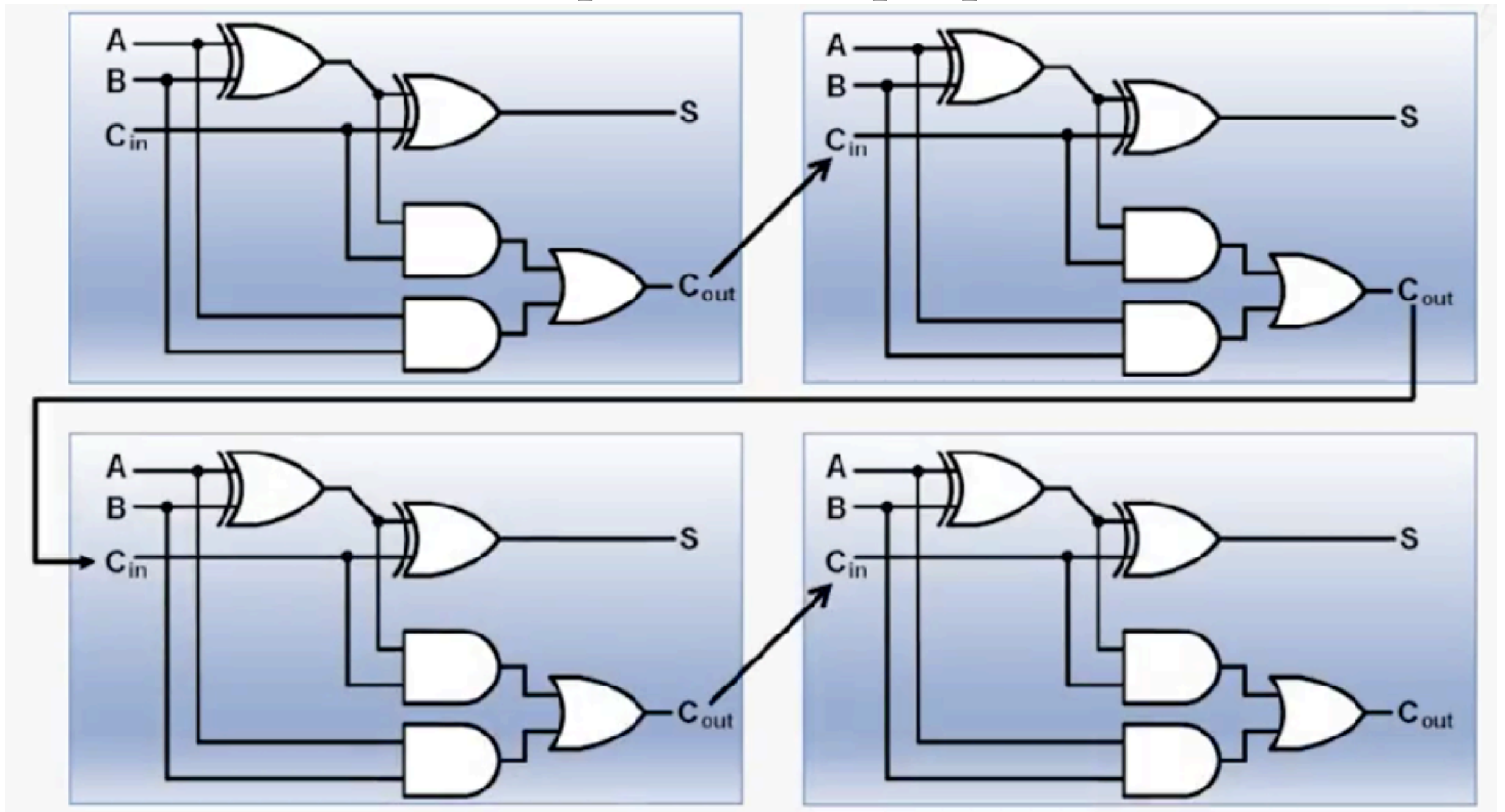
Delay Analysis

- ❖ Basic component
- ❖ Cascaded



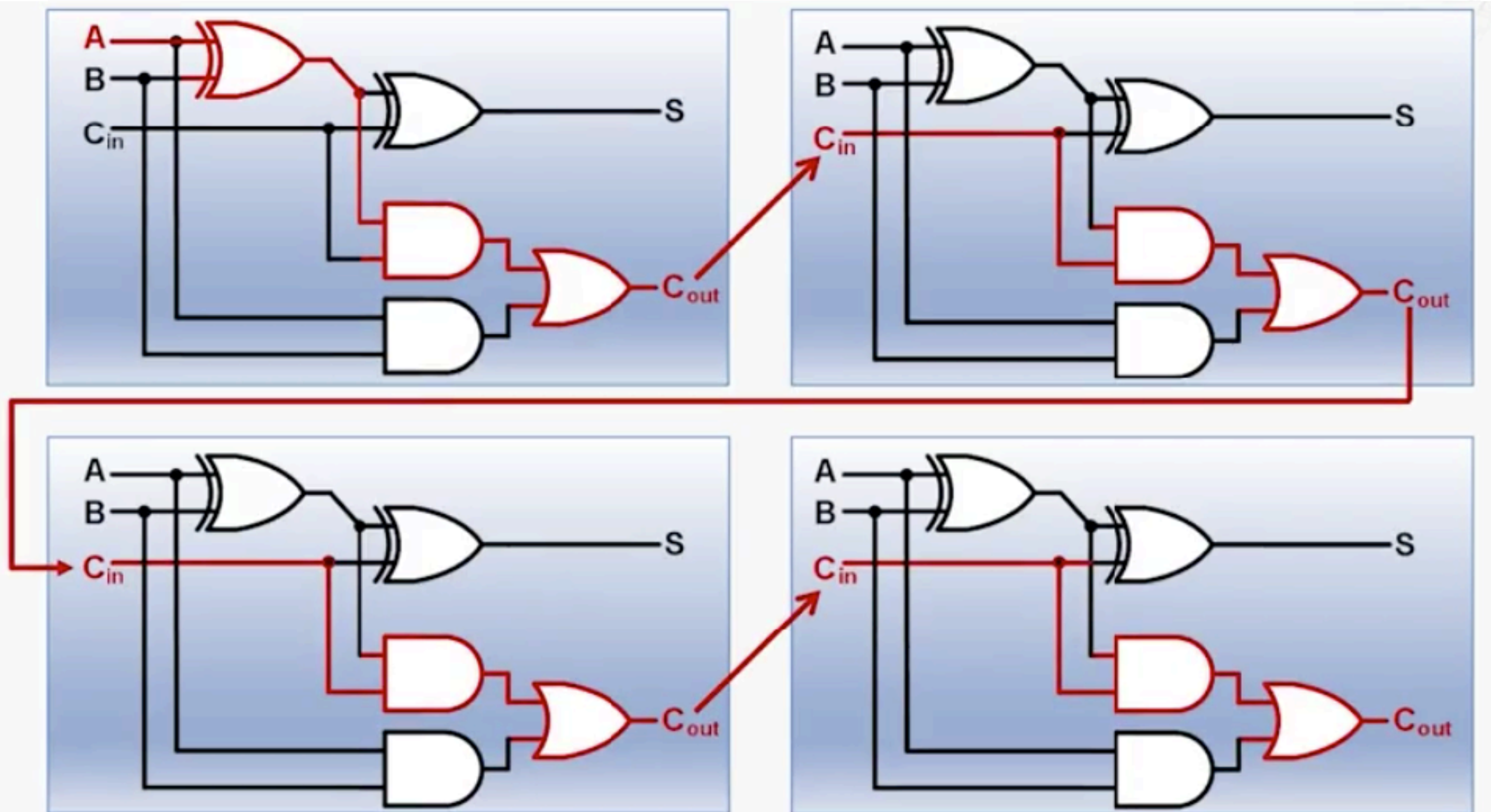
Critical Path Analysis

- ❖ Where is the critical path of the proposed schematic?



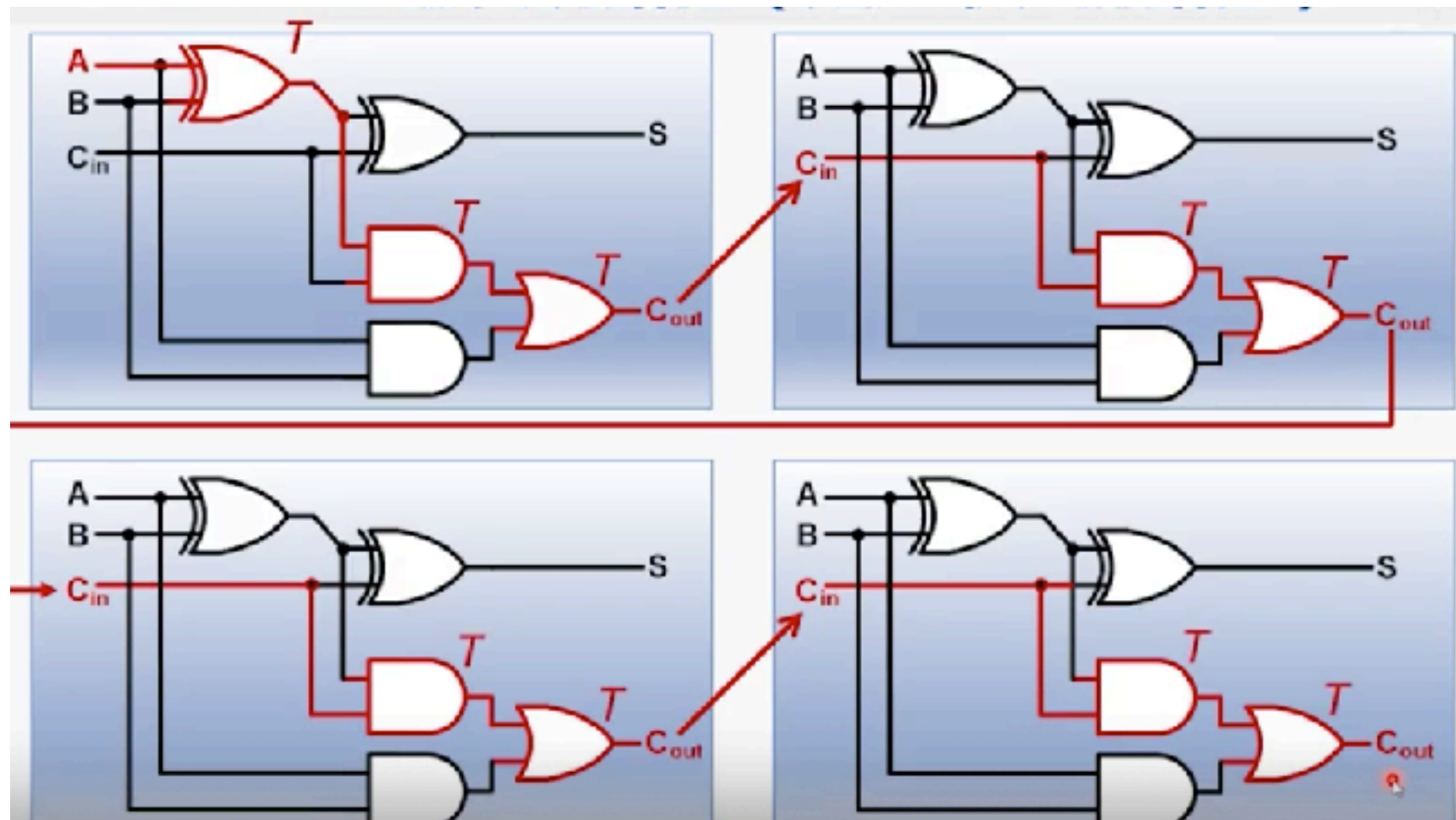
Critical Path Analysis

- ❖ Where is the critical path of the proposed schematic?



Delay Formulation

- ❖ Each gate has a delay of T (ideal)
- ❖ How much delay in total?
 - ❖ *hand calculation*

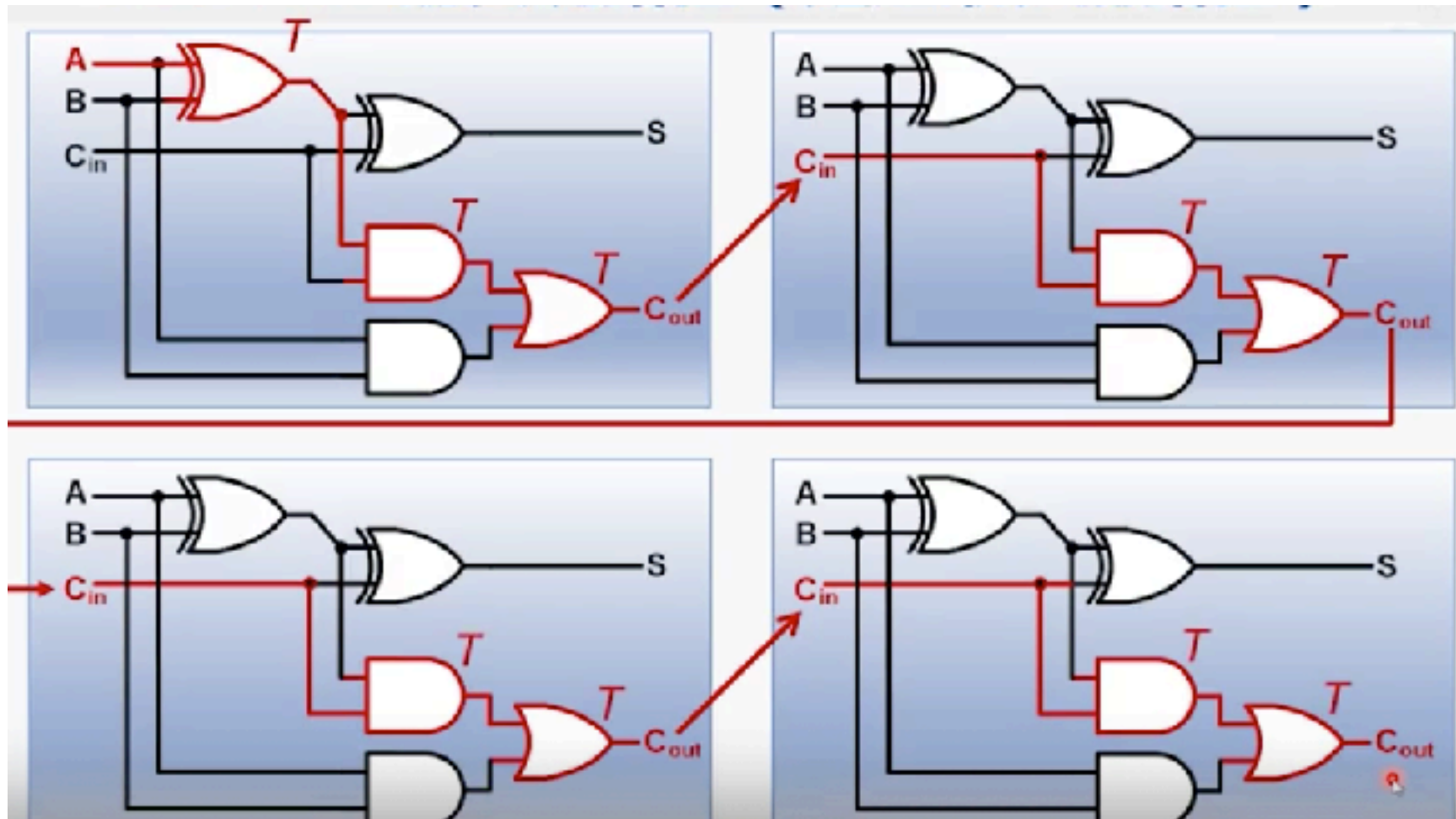


Delay Formulation

- ❖ Each gate has a delay of T (ideal)
- ❖ How much delay in total?

- ❖ *hand calculation*

- ❖ $2T*N+T$

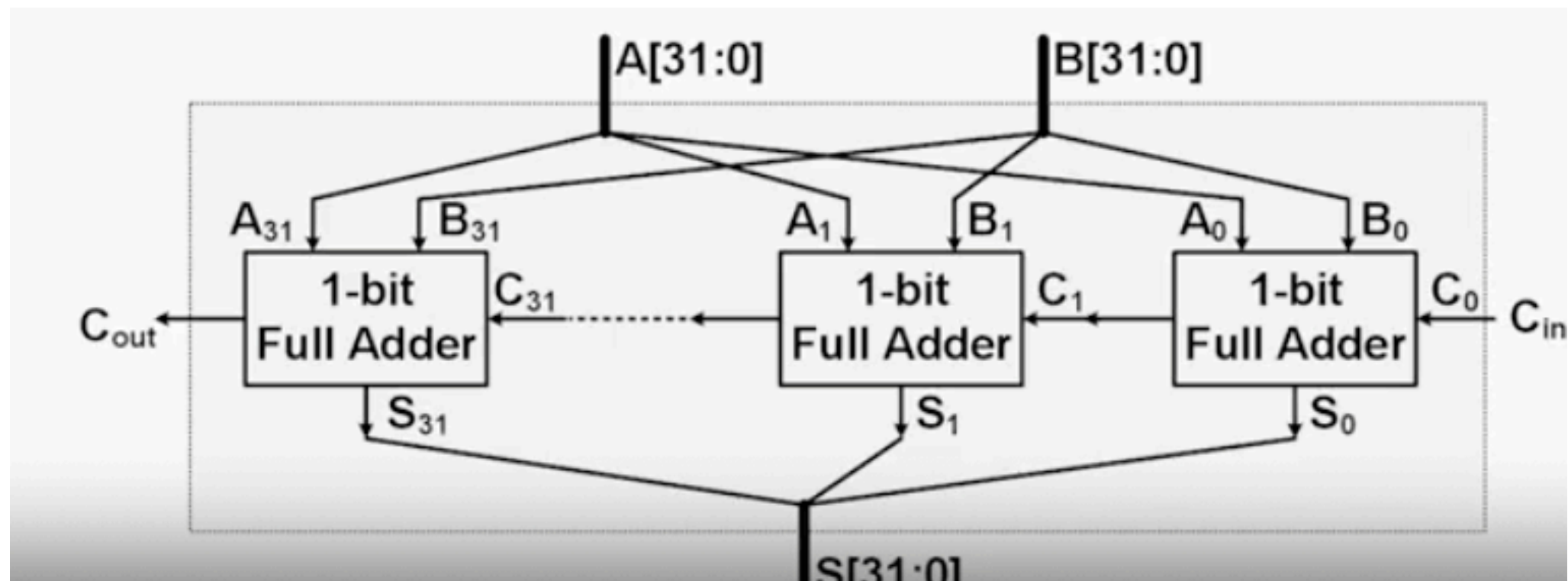


Delay Formulation

- ❖ How much delay for a 32-bit full adder?
- ❖ $(T+T)*N+T$ where $N=32 = 65T$

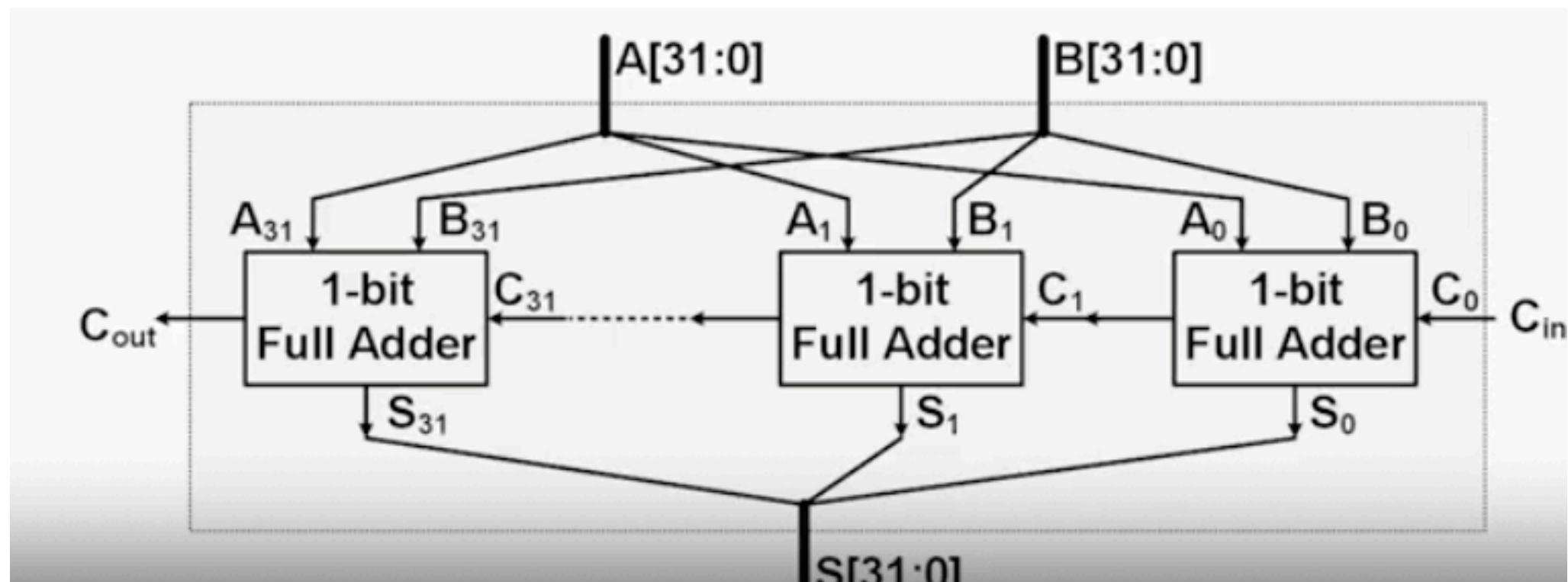
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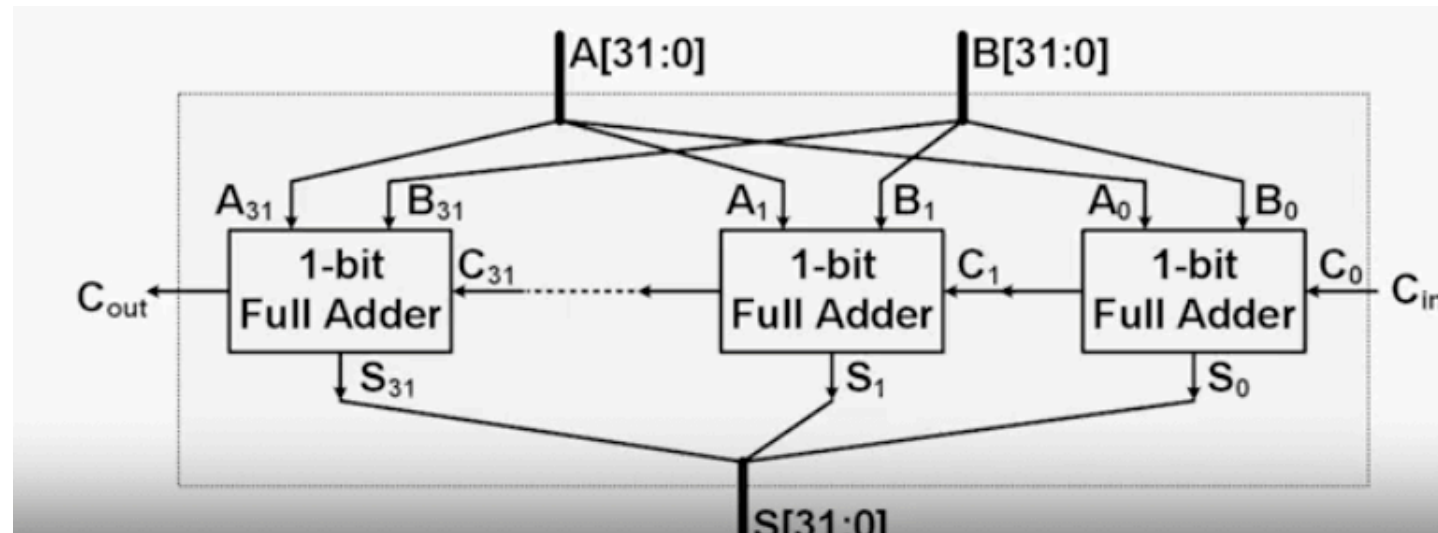
Delay Formulation

- ❖ How much delay for a 32-bit full adder?
- ❖ $(T+T)*N+T$ where $N=32 = 65T$
- ❖ Good? Bad? What this number means?



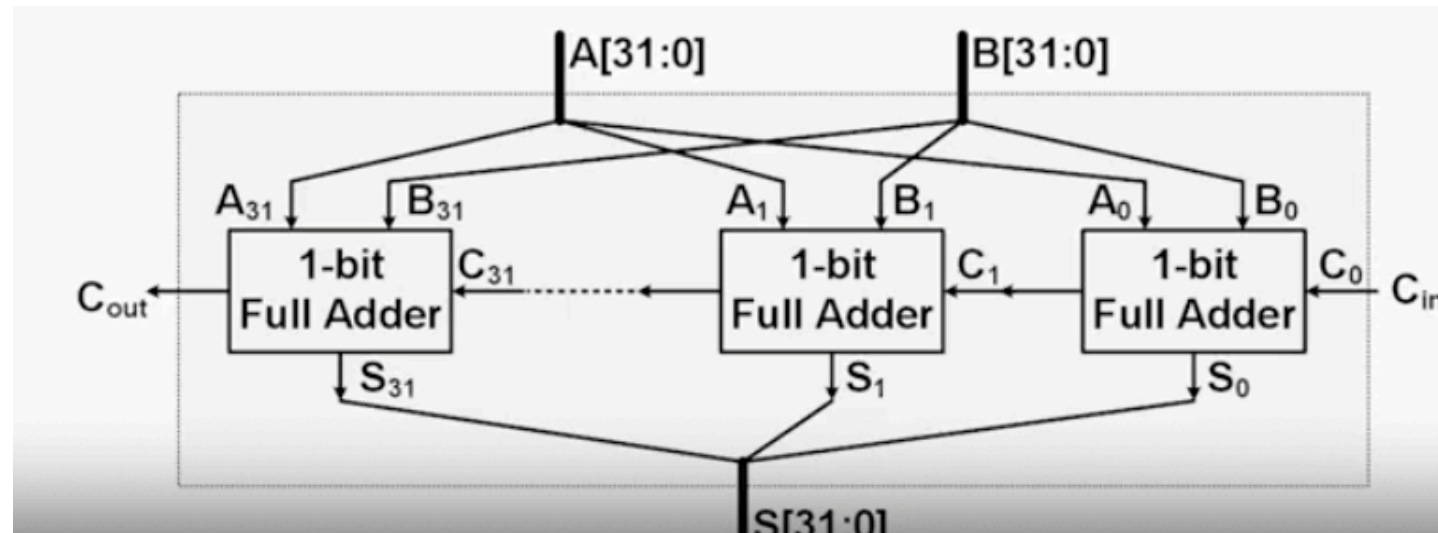
A Real-World Example

- ❖ 65T
- ❖ A7 SoC of iPhone 5s
 - ❖ 28nm CMOS
 - ❖ 1.3GHz (~0.66ns)



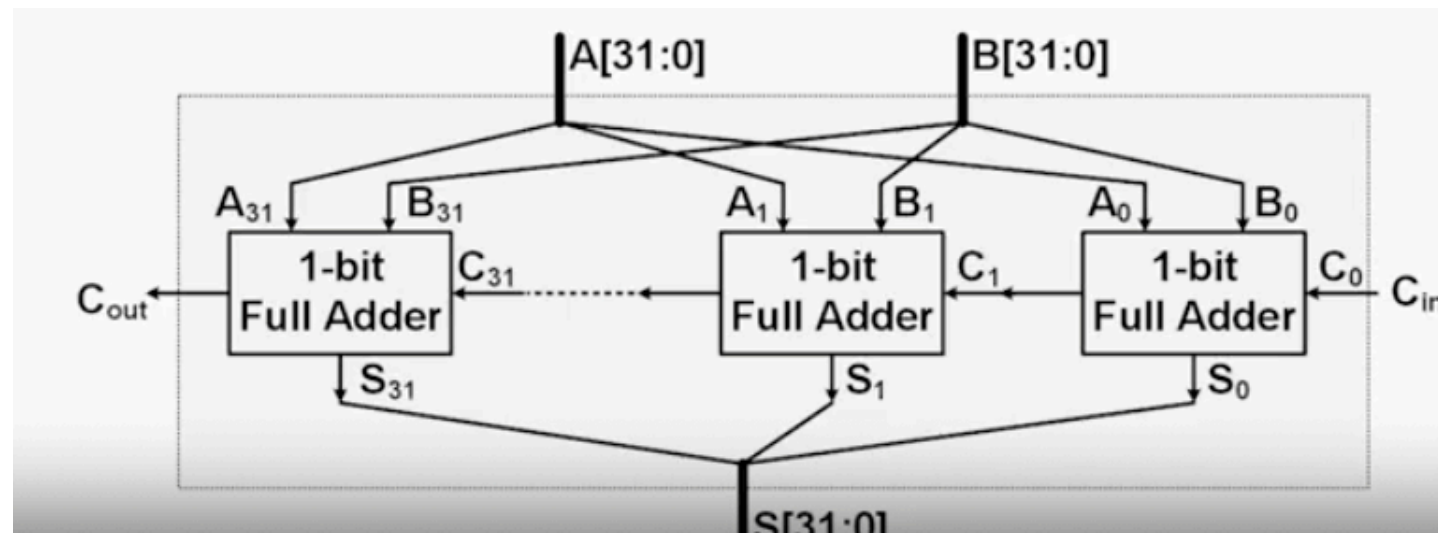
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 - ❖ 28nm CMOS
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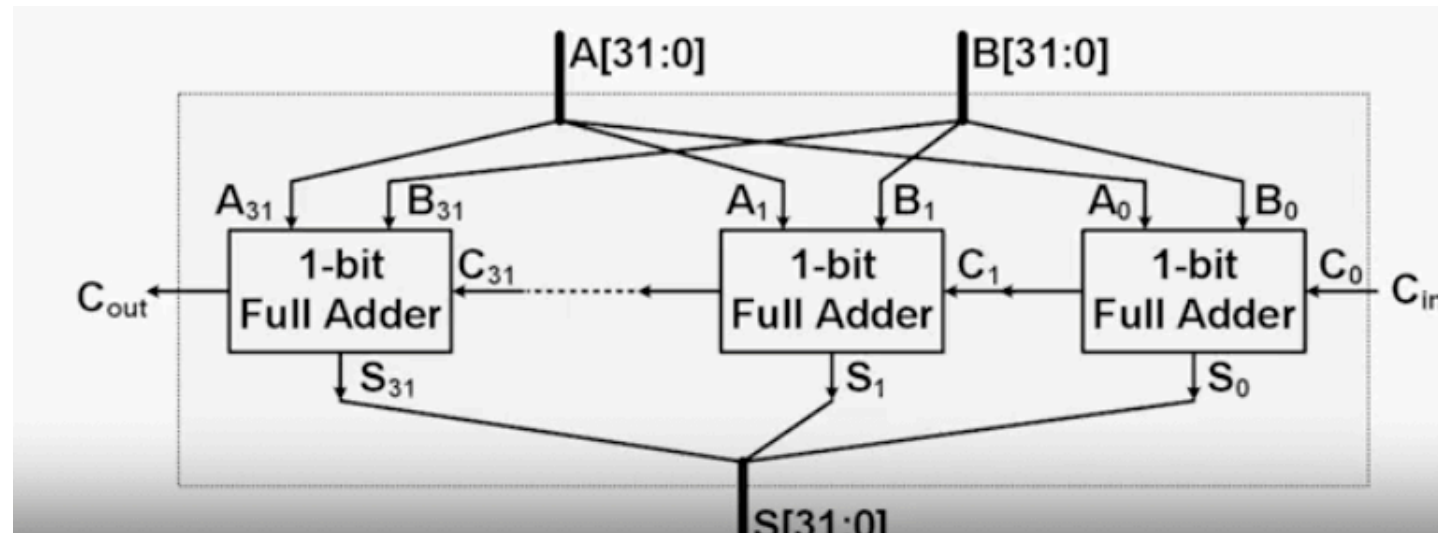
A Real-World Example

- ❖ 65T
- ❖ A7 SoC of iPhone 5s
 - ❖ 28nm CMOS
 - ❖ 1.3GHz (~0.66ns)
- ❖ Satisfying??
 - ❖ $T=0.02\text{ns}$ for 28nm
 - ❖ No! Not even considering setup/hold!



A Real-World Example

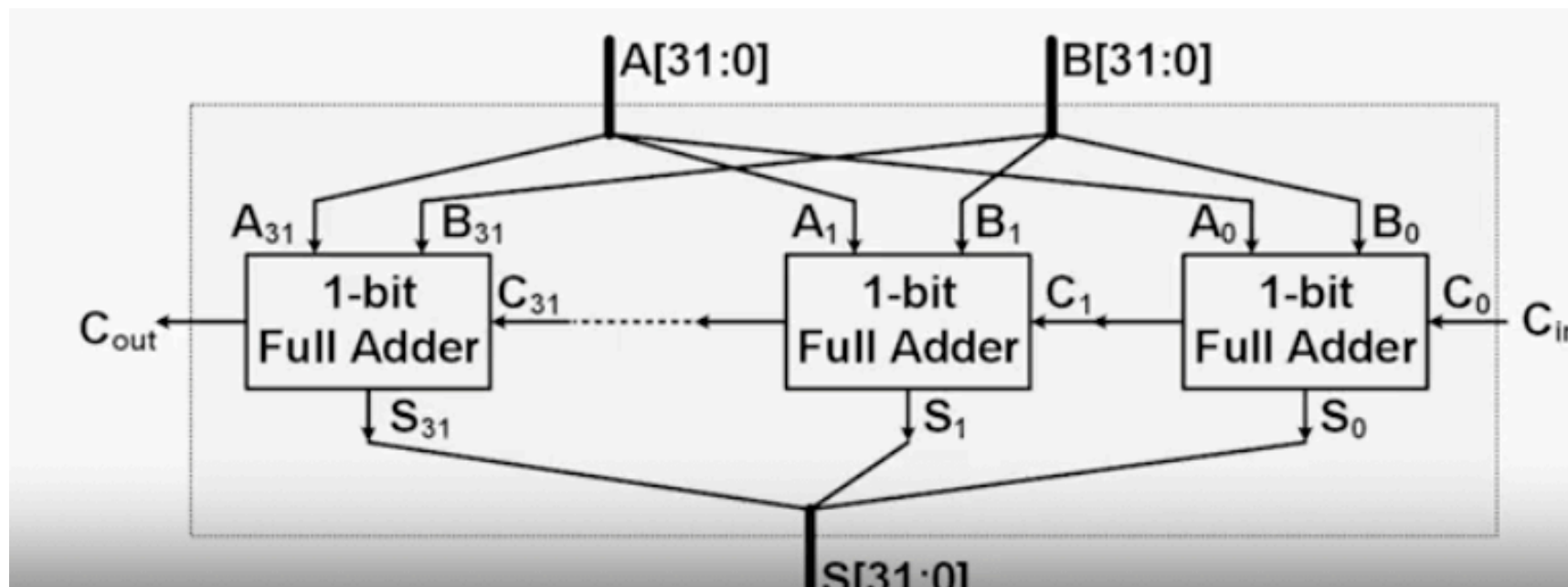
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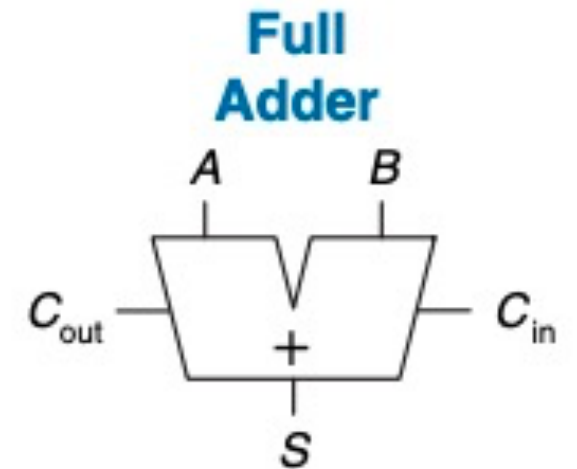
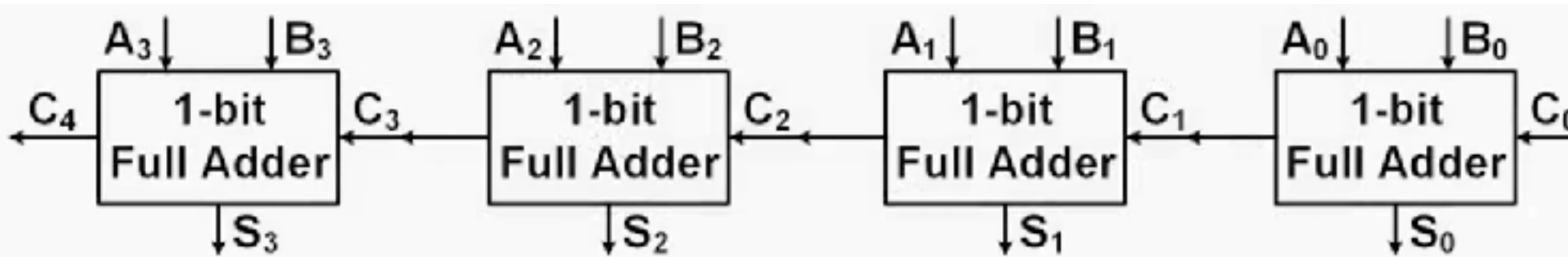
4-bit RCA	0.18ns	5.56GHz
32-bit RCA	1.3ns	769MHz

Need to Optimize the Adder

- ❖ Key issue
 - ❖ MSBs has to wait for Cout from LSBs
- ❖ How to optimize?
 - ❖ What if we can predict the Cout?



Analysis of Cout

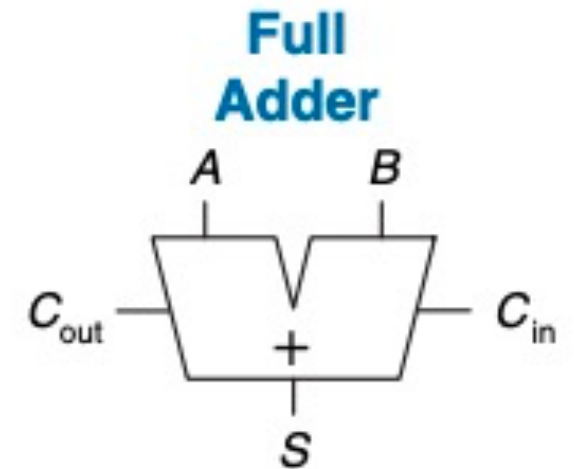
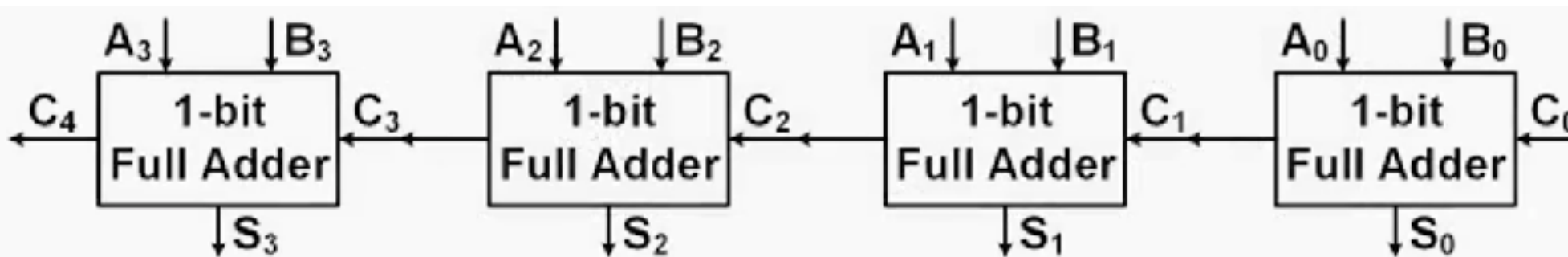


C_{in}	A	B	C_{out}	S
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	1

$$S = A \oplus B \oplus C_{in}$$

$$C_{out} = AB + AC_{in} + BC_{in}$$

Analysis of Cout



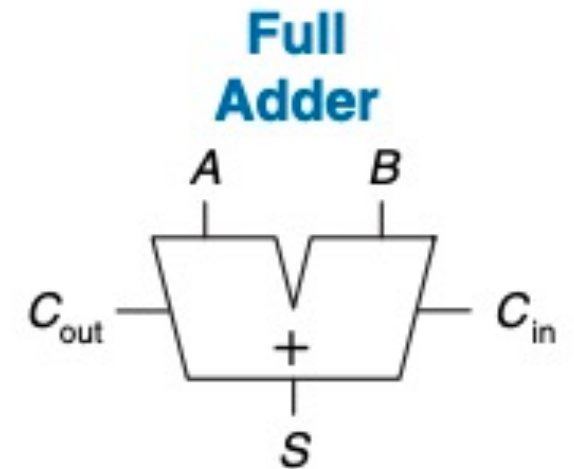
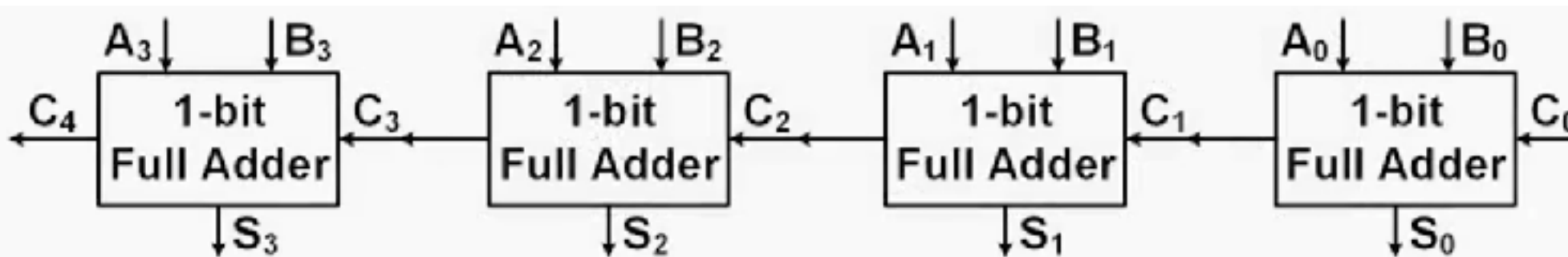
$$C_{i+1} =$$

C_{in}	A	B	C_{out}	S
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
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Analysis of Cout



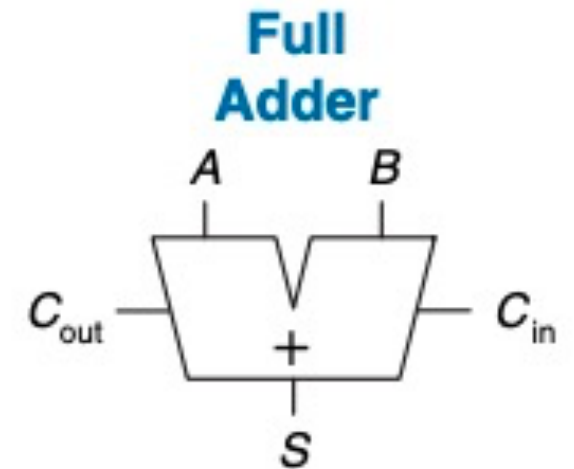
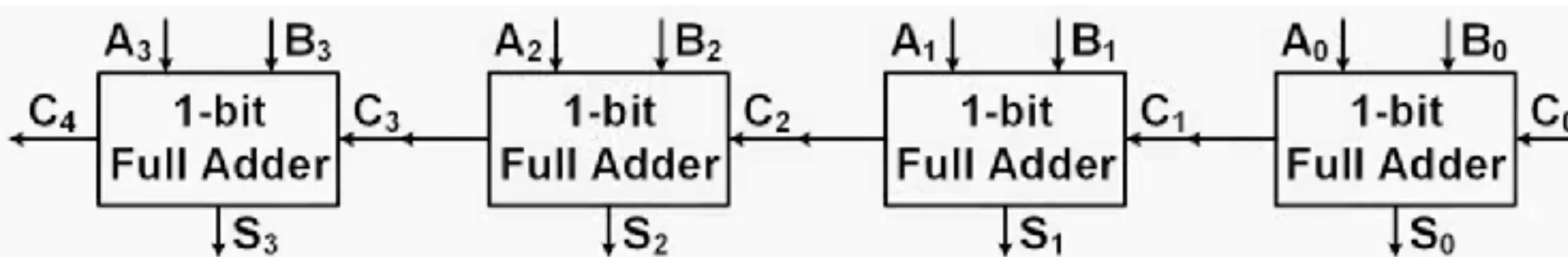
$$C_{i+1} = (A_i \cdot B_i) + (A_i \cdot C_i) + (B_i \cdot C_i)$$

C_{in}	A	B	C_{out}	S
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	1

$$S = A \oplus B \oplus C_{in}$$

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Analysis of Cout



$$C_{i+1} = (A_i \cdot B_i) + (A_i \cdot C_i) + (B_i \cdot C_i)$$

$$= (A_i \cdot B_i) + (A_i + B_i) \cdot C_i$$

C_{in}	A	B	C_{out}	S
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1	1	0	1	0
1	1	1	1	1

$$S = A \oplus B \oplus C_{in}$$

$$C_{out} = AB + AC_{in} + BC_{in}$$

Carry-Lookahead Adder

- ❖ Proposed for addressing the low-performance of ripple-carry adder

$$\begin{aligned}C_{i+1} &= (A_i \cdot B_i) + (A_i \cdot C_i) + (B_i \cdot C_i) \\ &= (A_i \cdot B_i) + (A_i + B_i) \cdot C_i\end{aligned}$$

- ❖ How?

- ❖ The i-th adder will **generate** a carry-out, if both A and B are 1, *independent of carry_in!*
- ❖ The i-th adder will **propagate** a carry-out, if either A or B is 1, *and there is a of carry_in!*

Analysis of Cout

❖ Define two new parameters:

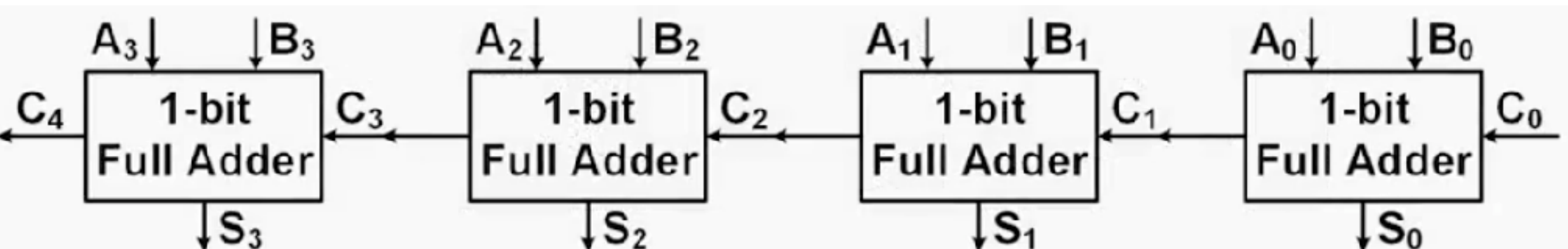
❖ Generate: $G_i = A_i \text{ AND } B_i$

❖ Propagate: $P_i = A_i + B_i$

$$C_{i+1} = G_i + P_i \cdot C_i$$

$$C_{i+1} = (A_i \cdot B_i) + (A_i \cdot C_i) + (B_i \cdot C_i)$$

$$= (A_i \cdot B_i) + (A_i + B_i) \cdot C_i$$



Analysis of Cout

❖ Define two new parameters:

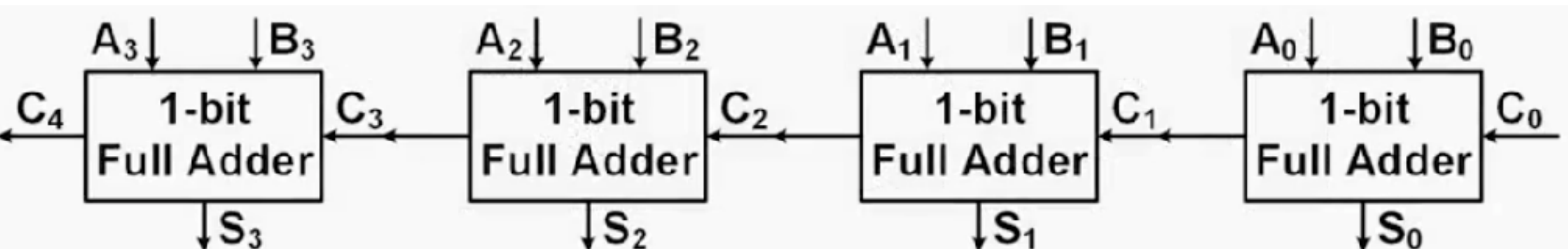
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$$= (A_i \cdot B_i) + (A_i + B_i) \cdot C_i$$



Know the Values in Advance

$$C_{i+1} = G_i + P_i \cdot C_i$$

Know the Values in Advance

$$C_1 = G_0 + P_0 \cdot C_0$$

$$C_{i+1} = G_i + P_i \cdot C_i$$

Know the Values in Advance

$$C_1 = G_0 + P_0 \cdot C_0$$

$$C_2 = G_1 + P_1 \cdot C_1$$

$$C_{i+1} = G_i + P_i \cdot C_i$$

Know the Values in Advance

$$C_1 = G_0 + P_0 \cdot C_0$$

$$C_2 = G_1 + P_1 \cdot C_1$$

$$= G_1 + P_1 \cdot (G_0 + P_0 \cdot C_0)$$

$$C_{i+1} = G_i + P_i \cdot C_i$$

Know the Values in Advance

$$C_1 = G_0 + P_0 \cdot C_0$$

$$C_2 = G_1 + P_1 \cdot C_1$$

$$= G_1 + P_1 \cdot (G_0 + P_0 \cdot C_0)$$

$$= G_1 + P_1 \cdot G_0 + P_1 \cdot P_0 \cdot C_0$$

$$C_{i+1} = G_i + P_i \cdot C_i$$

Know the Values in Advance

$$C_1 = G_0 + P_0 \cdot C_0$$

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$$= G_1 + P_1 \cdot (G_0 + P_0 \cdot C_0)$$

$$= G_1 + P_1 \cdot G_0 + P_1 \cdot P_0 \cdot C_0$$

$$C_3 = G_2 + P_2 \cdot C_2$$

$$= G_2 + P_2 \cdot (G_1 + P_1 \cdot G_0 + P_1 \cdot P_0 \cdot C_0)$$

$$= G_2 + P_2 \cdot G_1 + P_2 \cdot P_1 \cdot G_0 + P_2 \cdot P_1 \cdot P_0 \cdot C_0$$

$$C_{i+1} = G_i + P_i \cdot C_i$$

Know the Values in Advance

$$C_1 = G_0 + P_0 \cdot C_0$$

$$C_2 = G_1 + P_1 \cdot C_1$$

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$$= G_1 + P_1 \cdot G_0 + P_1 \cdot P_0 \cdot C_0$$

$$C_3 = G_2 + P_2 \cdot C_2$$

$$= G_2 + P_2 \cdot (G_1 + P_1 \cdot G_0 + P_1 \cdot P_0 \cdot C_0)$$

$$= G_2 + P_2 \cdot G_1 + P_2 \cdot P_1 \cdot G_0 + P_2 \cdot P_1 \cdot P_0 \cdot C_0$$

$$C_4 = G_3 + P_3 \cdot C_3$$

$$= G_3 + P_3 \cdot (G_2 + P_2 \cdot G_1 + P_2 \cdot P_1 \cdot G_0 + P_2 \cdot P_1 \cdot P_0 \cdot C_0)$$

$$= G_3 + P_3 \cdot G_2 + P_3 \cdot P_2 \cdot G_1 + P_3 \cdot P_2 \cdot P_1 \cdot G_0 + P_3 \cdot P_2 \cdot P_1 \cdot P_0 \cdot C_0$$

$$C_{i+1} = G_i + P_i \cdot C_i$$

Detailed Analysis of C4

$$\begin{aligned}\mathbf{C}_4 &= G_3 + P_3 \cdot C_3 \\ &= G_3 + P_3 \cdot (G_2 + P_2 \cdot G_1 + P_2 \cdot P_1 \cdot G_0 + P_2 \cdot P_1 \cdot P_0 \cdot C_0) \\ &= \mathbf{G_3 + P_3 \cdot G_2 + P_3 \cdot P_2 \cdot G_1 + P_3 \cdot P_2 \cdot P_1 \cdot G_0 + P_3 \cdot P_2 \cdot P_1 \cdot P_0 \cdot C_0}\end{aligned}$$

Detailed Analysis of C4

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$$\begin{aligned}\mathbf{C_4} &= \\ &G_3 \\ &+ \\ &P_3 \cdot G_2 \\ &+ \\ &P_3 \cdot P_2 \cdot G_1 \\ &+ \\ &P_3 \cdot P_2 \cdot P_1 \cdot G_0 \\ &+ \\ &P_3 \cdot P_2 \cdot P_1 \cdot P_0 \cdot C_0\end{aligned}$$

Detailed Analysis of C4

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$\mathbf{C_4=}$

$\mathbf{G_3}$

$\mathbf{+}$

$\mathbf{P_3 \cdot G_2}$

$\mathbf{+}$

$\mathbf{P_3 \cdot P_2 \cdot G_1}$

$\mathbf{+}$

$\mathbf{P_3 \cdot P_2 \cdot P_1 \cdot G_0}$

$\mathbf{+}$

$\mathbf{P_3 \cdot P_2 \cdot P_1 \cdot P_0 \cdot C_0}$

Detailed Analysis of C4

$$\begin{aligned}\mathbf{C}_4 &= G_3 + P_3 \cdot C_3 \\ &= G_3 + P_3 \cdot (G_2 + P_2 \cdot G_1 + P_2 \cdot P_1 \cdot G_0 + P_2 \cdot P_1 \cdot P_0 \cdot C_0) \\ &= \mathbf{G_3 + P_3 \cdot G_2 + P_3 \cdot P_2 \cdot G_1 + P_3 \cdot P_2 \cdot P_1 \cdot G_0 + P_3 \cdot P_2 \cdot P_1 \cdot P_0 \cdot C_0}\end{aligned}$$

$$\begin{aligned}C_4 &= \\ &G_3 \\ &+ \\ &P_3 \cdot G_2 \\ &+ \\ &P_3 \cdot P_2 \cdot G_1 \\ &+ \\ &P_3 \cdot P_2 \cdot P_1 \cdot G_0 \\ &+ \\ &P_3 \cdot P_2 \cdot P_1 \cdot P_0 \cdot C_0\end{aligned}$$

Detailed Analysis of C4

$$\begin{aligned} \mathbf{C}_4 &= G_3 + P_3 \cdot C_3 \\ &= G_3 + P_3 \cdot (G_2 + P_2 \cdot G_1 + P_2 \cdot P_1 \cdot G_0 + P_2 \cdot P_1 \cdot P_0 \cdot C_0) \\ &= \mathbf{G_3 + P_3 \cdot G_2 + P_3 \cdot P_2 \cdot G_1 + P_3 \cdot P_2 \cdot P_1 \cdot G_0 + P_3 \cdot P_2 \cdot P_1 \cdot P_0 \cdot C_0} \end{aligned}$$

$C_4 =$

- G_3
- $+$
- $P_3 \cdot G_2$
- $+$
- $P_3 \cdot P_2 \cdot G_1$
- $+$
- $P_3 \cdot P_2 \cdot P_1 \cdot G_0$
- $+$
- $P_3 \cdot P_2 \cdot P_1 \cdot P_0 \cdot C_0$

Xu

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- G_3
- $+$
- $P_3 \cdot G_2$
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- $+$
- $P_3 \cdot P_2 \cdot P_1 \cdot G_0$
- $+$
- $P_3 \cdot P_2 \cdot P_1 \cdot P_0 \cdot C_0$

Xu

Detailed Analysis of C4

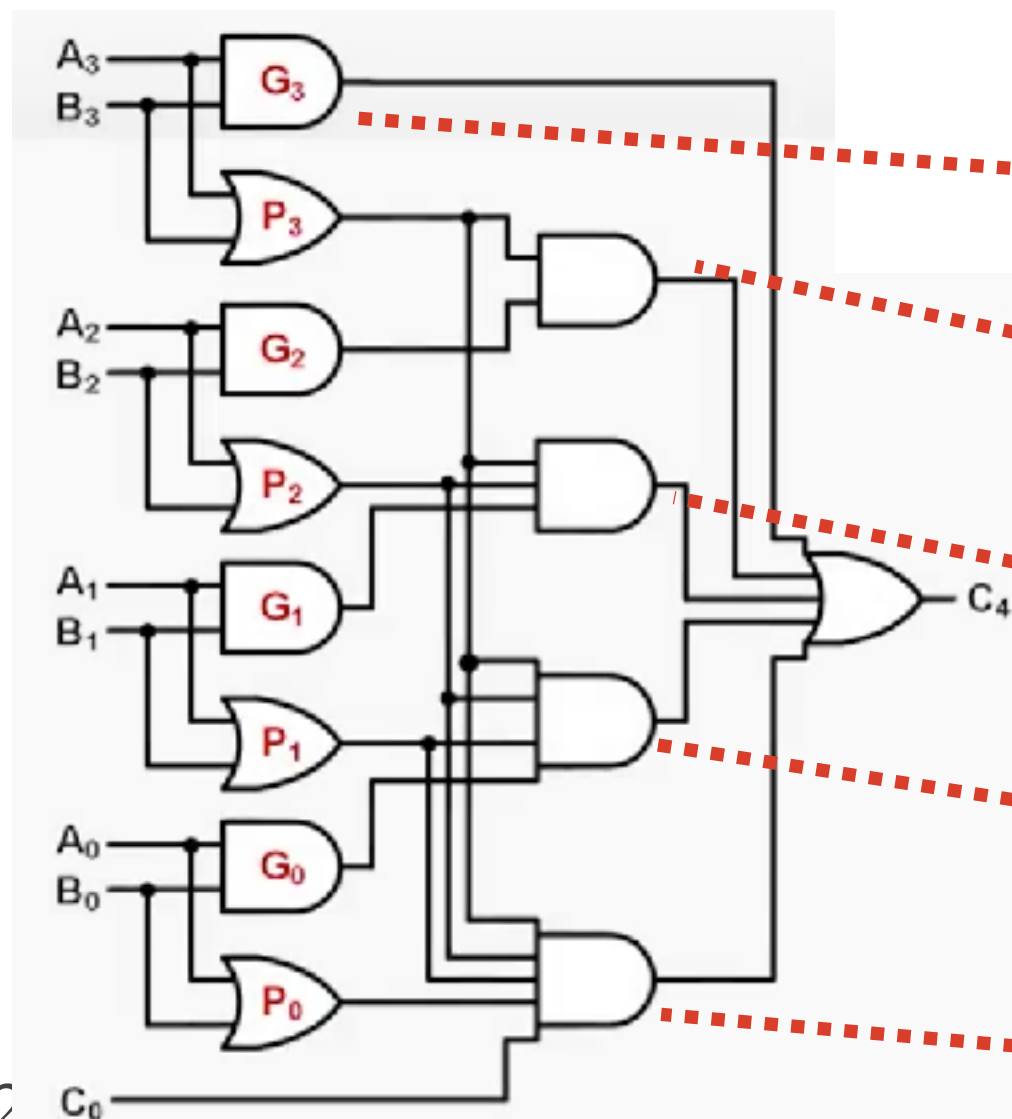
$$\begin{aligned}\mathbf{C}_4 &= G_3 + P_3 \cdot C_3 \\ &= G_3 + P_3 \cdot (G_2 + P_2 \cdot G_1 + P_2 \cdot P_1 \cdot G_0 + P_2 \cdot P_1 \cdot P_0 \cdot C_0) \\ &= \mathbf{G_3 + P_3 \cdot G_2 + P_3 \cdot P_2 \cdot G_1 + P_3 \cdot P_2 \cdot P_1 \cdot G_0 + P_3 \cdot P_2 \cdot P_1 \cdot P_0 \cdot C_0}\end{aligned}$$

The diagram illustrates the expansion of the expression C_4 into its constituent terms. Red dashed lines connect the terms in the expanded equation to the corresponding terms in the original equation. The expanded equation is shown as follows:

$$\begin{aligned}C_4 &= \\ &G_3 \\ &+ \\ &P_3 \cdot G_2 \\ &+ \\ &P_3 \cdot P_2 \cdot G_1 \\ &+ \\ &P_3 \cdot P_2 \cdot P_1 \cdot G_0 \\ &+ \\ &P_3 \cdot P_2 \cdot P_1 \cdot P_0 \cdot C_0\end{aligned}$$

Detailed Analysis of C4

$$\begin{aligned}
 \mathbf{C_4} &= G_3 + P_3 \cdot C_3 \\
 &= G_3 + P_3 \cdot (G_2 + P_2 \cdot G_1 + P_2 \cdot P_1 \cdot G_0 + P_2 \cdot P_1 \cdot P_0 \cdot C_0) \\
 &= \mathbf{G_3 + P_3 \cdot G_2 + P_3 \cdot P_2 \cdot G_1 + P_3 \cdot P_2 \cdot P_1 \cdot G_0 + P_3 \cdot P_2 \cdot P_1 \cdot P_0 \cdot C_0}
 \end{aligned}$$

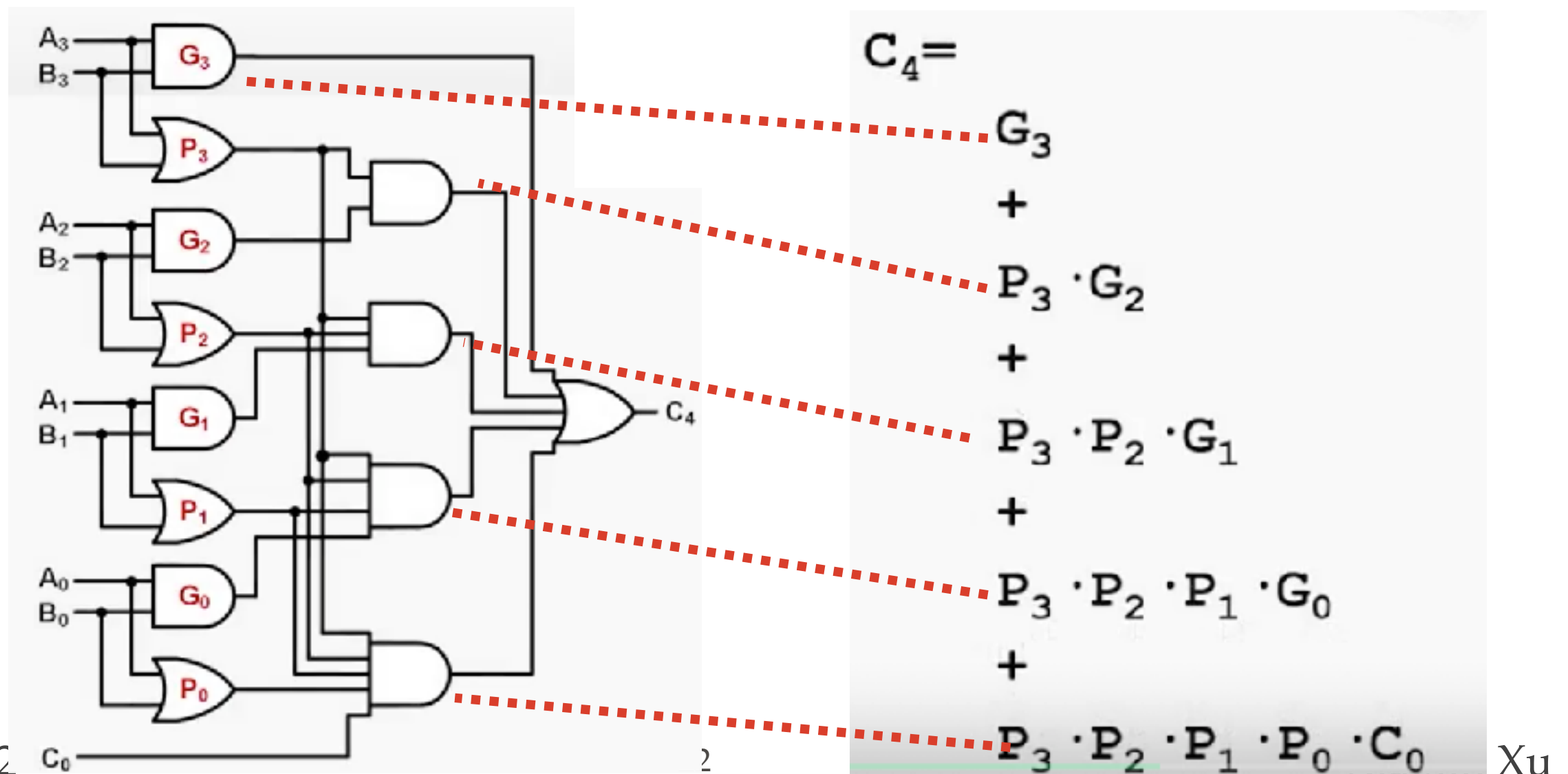


$$\begin{aligned}
 \mathbf{C_4} &= \\
 &+ G_3 \\
 &+ P_3 \cdot G_2 \\
 &+ P_3 \cdot P_2 \cdot G_1 \\
 &+ P_3 \cdot P_2 \cdot P_1 \cdot G_0 \\
 &+ P_3 \cdot P_2 \cdot P_1 \cdot P_0 \cdot C_0
 \end{aligned}$$

Detailed Analysis of C4

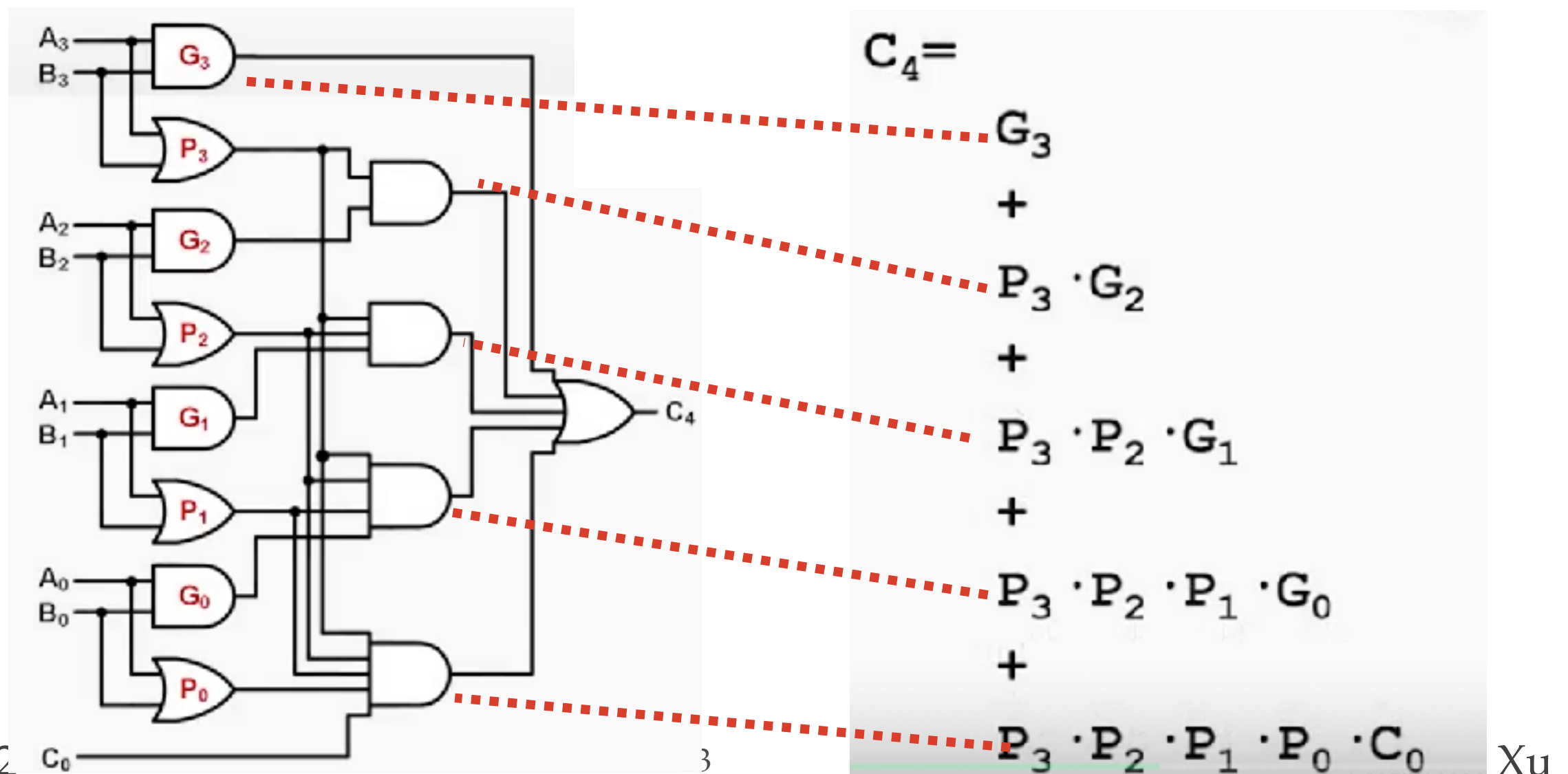
❖ Advantages?

- ❖ The latency of C_{i+1} is constant! Not dependent on N anymore



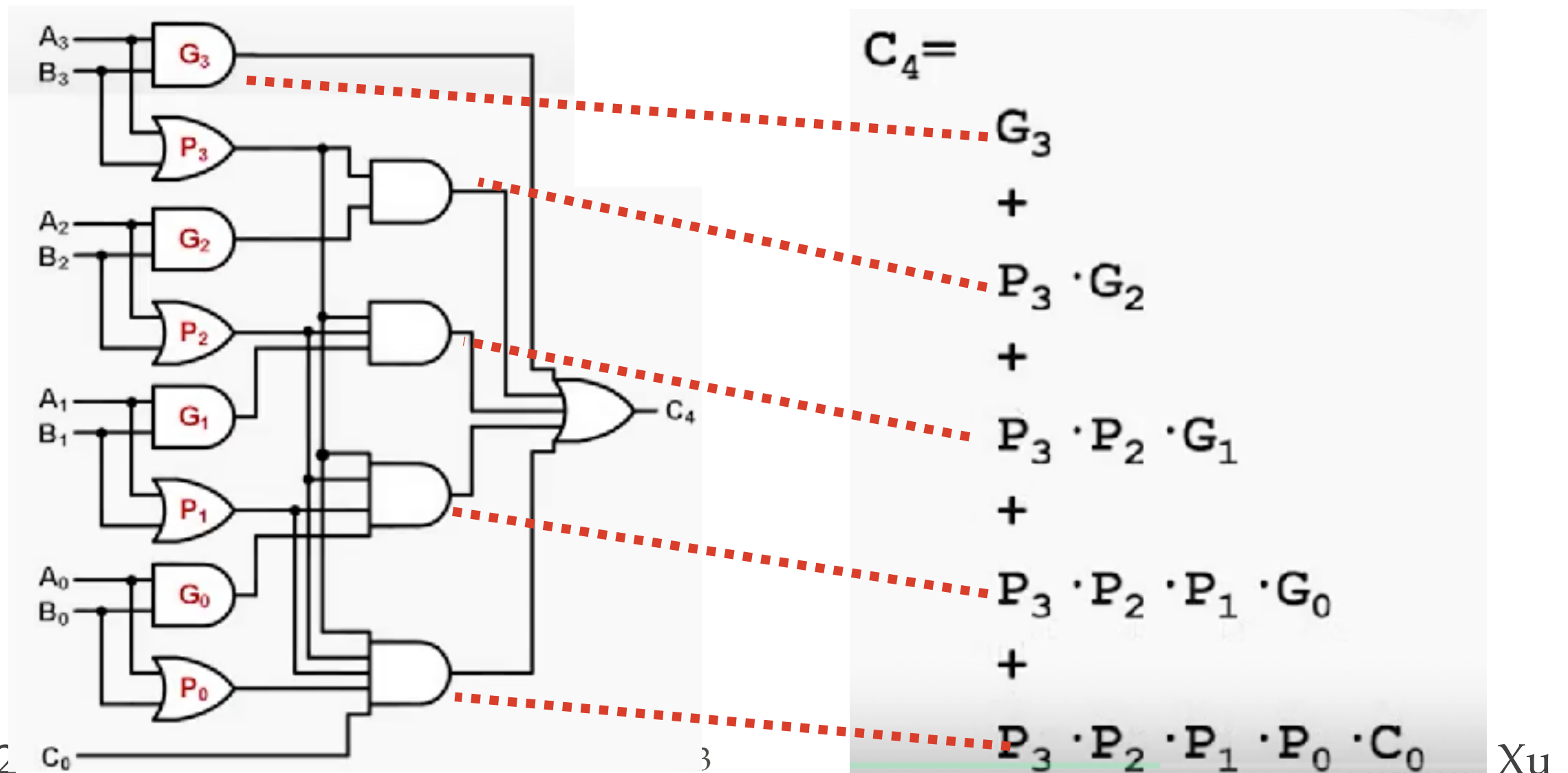
Detailed Analysis of C4

❖ Any disadvantages?



Detailed Analysis of C4

- ❖ Any disadvantages?
 - ❖ With the adder becomes wider, the circuit will be very complicated!



Further Optimization: Block of Adders

- ❖ Applying the concepts of “generate” and “propagate” to multiple-bit blocks / adders
 - ❖ A block *generates* a carry_out independent of the carry_in
 - ❖ A block *propagates* a carry_out if there is a carry_in
- ❖ Two new variables:
 - ❖ $G_{i:j}$ and $P_{i:j}$

$$G_{3:0} = G_3 + P_3(G_2 + P_2(G_1 + P_1G_0))$$

$$P_{3:0} = P_3P_2P_1P_0$$

Further Optimization: Block of Adders

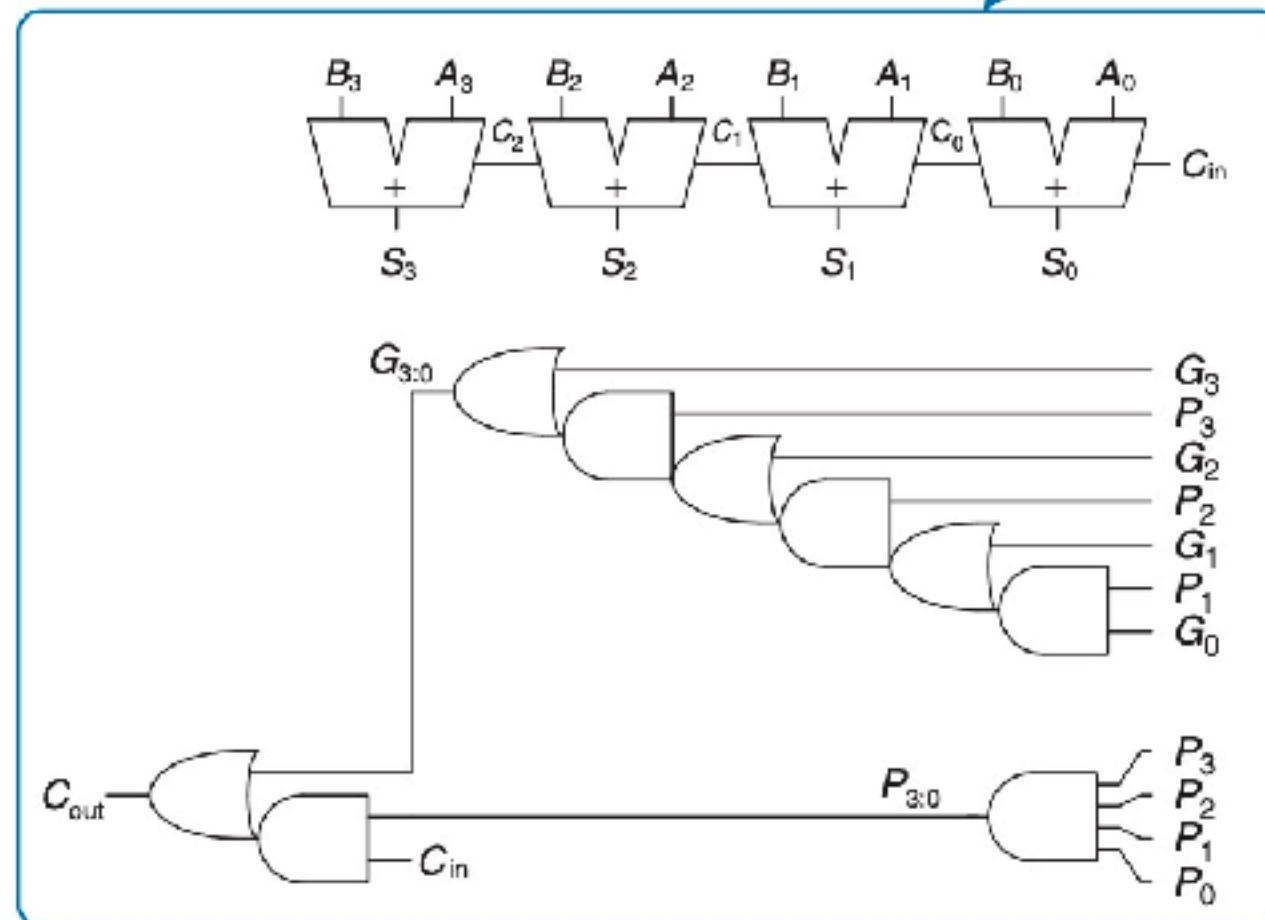
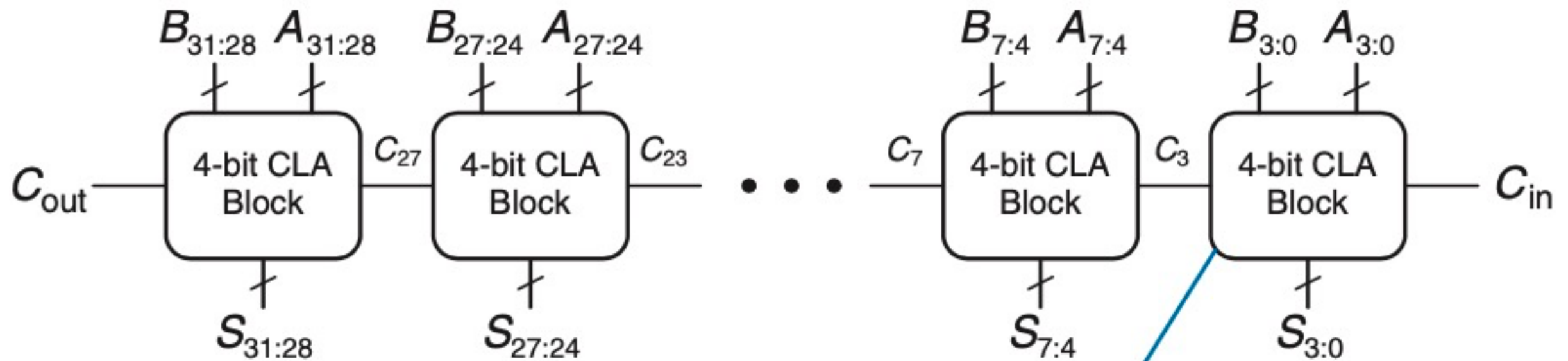
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$$G_{3:0} = G_3 + P_3(G_2 + P_2(G_1 + P_1G_0))$$

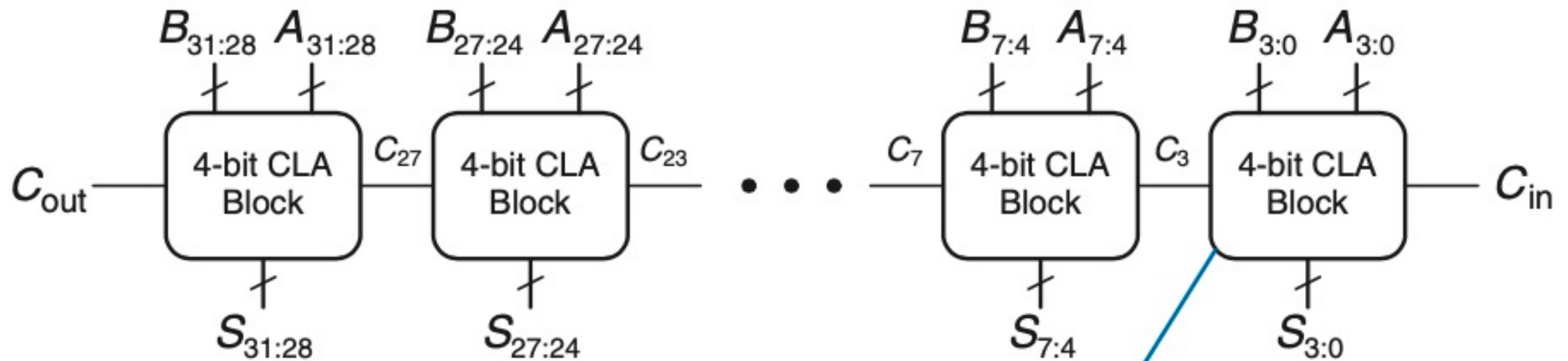
$$P_{3:0} = P_3P_2P_1P_0$$

$$C_i = G_{i:j} + P_{i:j}C_j$$

32-bit CLA Example

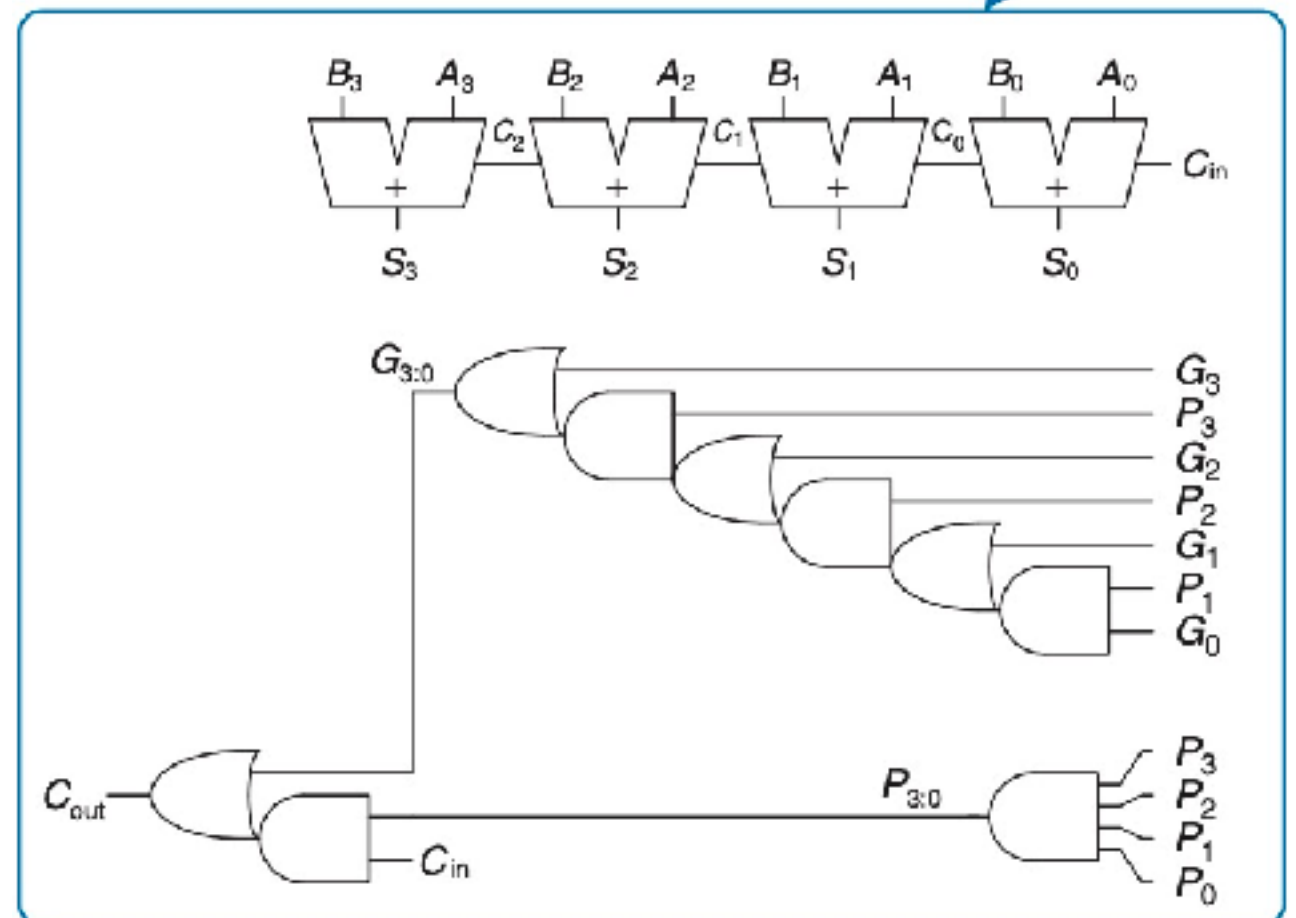


32-bit CLA Example

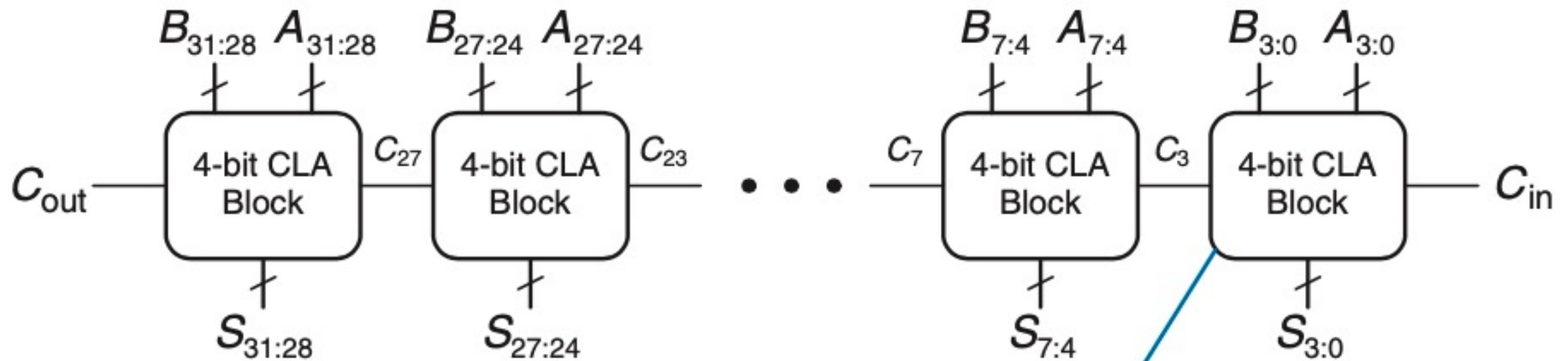


❖ Timing analysis

- ❖ t_{pg} : Propagation delay of a single AND/OR gate for generate/propagate
- ❖ t_{pg_block} : Propagation delay of a single AND/OR gate for generate/propagate *of a block*
- ❖ t_{AND_OR} : Propagation delay from C_{in} to C_{out}



32-bit CLA Example

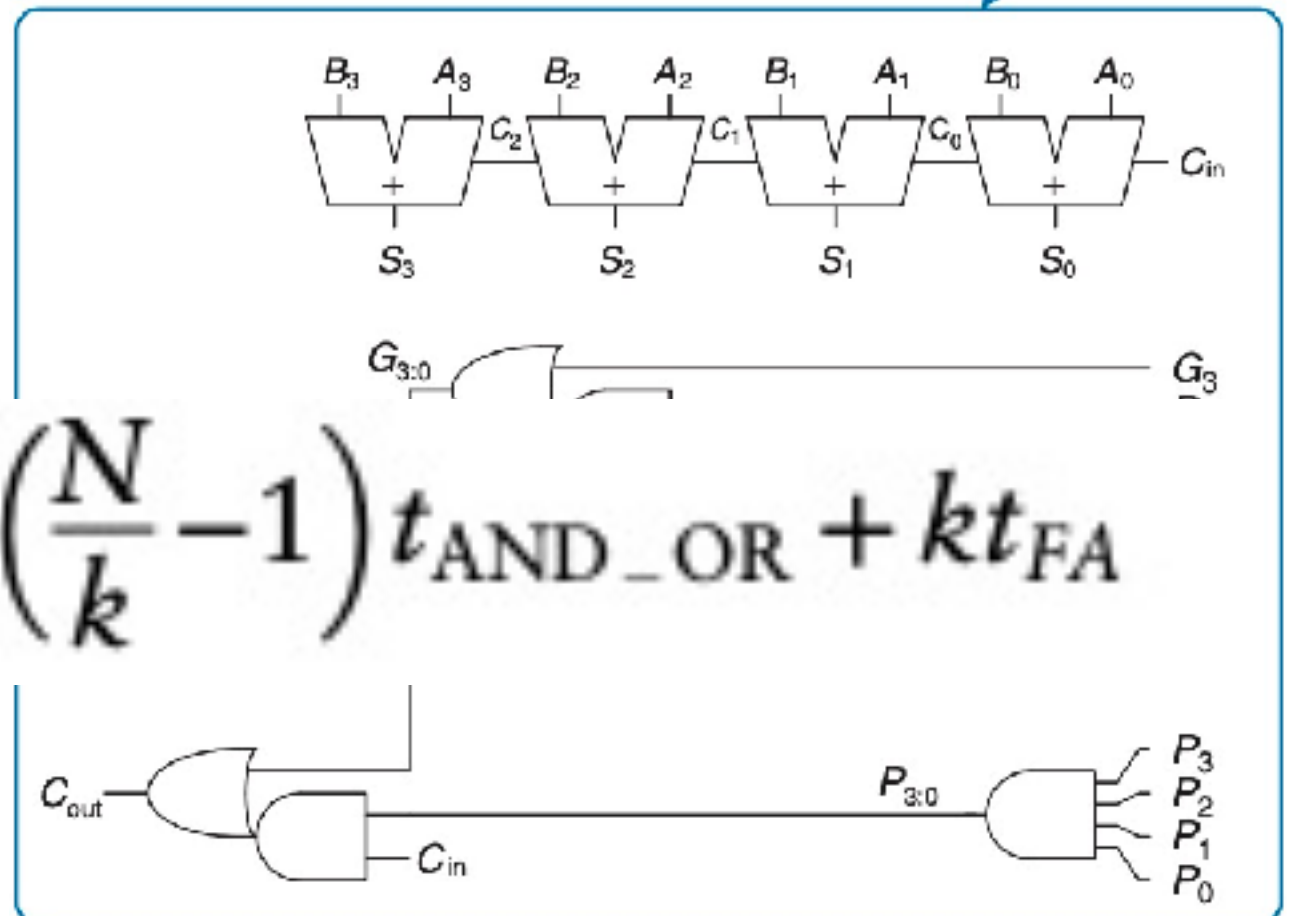


❖ Timing analysis

- ❖ t_{pg} : Propagation delay of a single AND/OR gate for generate/propagate

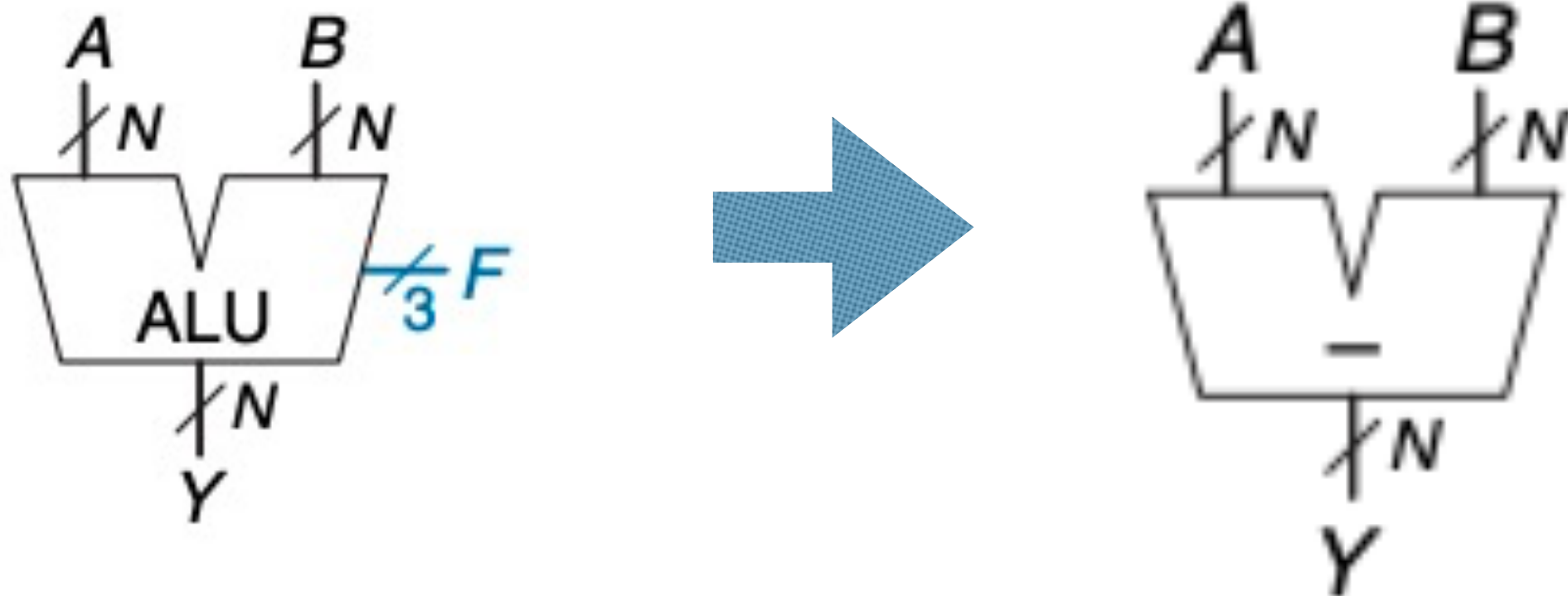
$$t_{CLA} = t_{pg} + t_{pg_block} + \left(\frac{N}{k} - 1\right) t_{AND_OR} + k t_{FA}$$

- ❖ t_{AND_OR} : Propagation delay from C_{in} to C_{out}



ALU Operations: Subtraction

- ❖ Replace the ALU with — (easy!)
- ❖ How to implement? Do we need a new circuit? (easy?)



- ❖ Recall the two's complement number representation

Two's Complement Number

- ❖ $17_{10} = 11111111_2 - 00010001_2 + 1$
 - ❖ $= 11101110_2 + 1$
 - ❖ $= 11101111_2$
 - ❖ $= -17_{10}$
 - ❖ The MSB has a negative weight $-2^{(n-1)}$
- ❖ Hardware mapping?

$$Y = A - B$$

Two's Complement Number

- ❖ $17_{10} = 11111111_2 - 00010001_2 + 1$
 - ❖ $= 11101110_2 + 1$
 - ❖ $= 11101111_2$
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- ❖ Hardware mapping?

$$Y = A - B = A + \overline{B} + 1$$

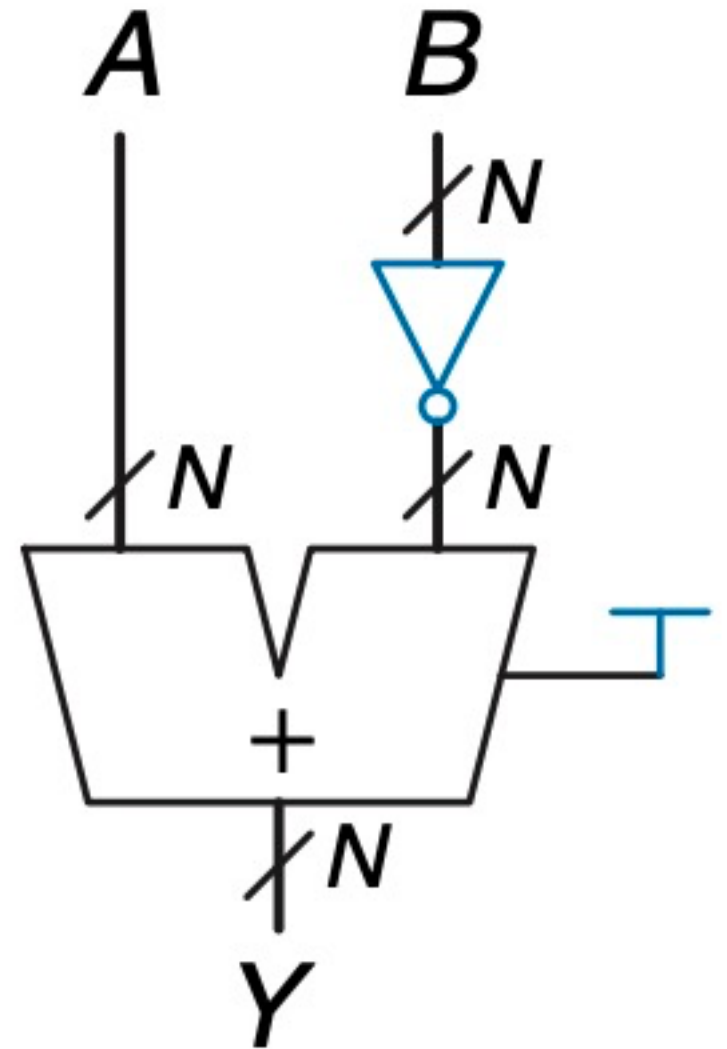
Two's Complement Number

- ❖ Hardware mapping?
 - ❖ Step1: inverting all bit in B
 - ❖ Step2: add “1” to $\sim B$
 - ❖ Step3: adding two variables

$$Y = A - B = A + \overline{B} + 1$$

Two's Complement Number

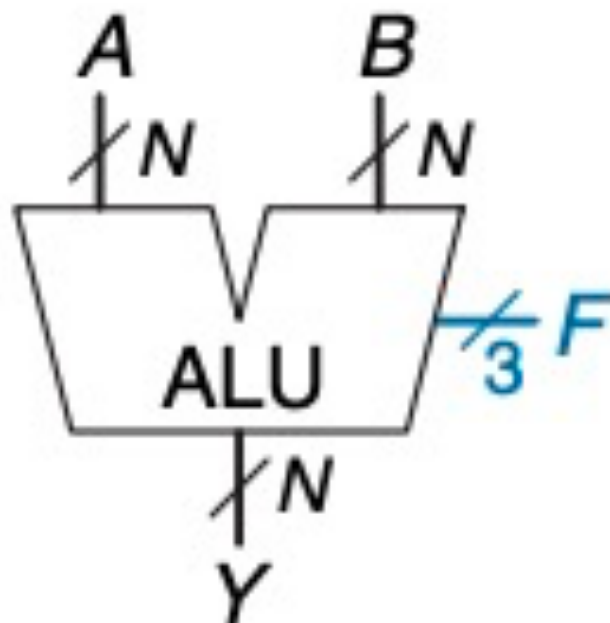
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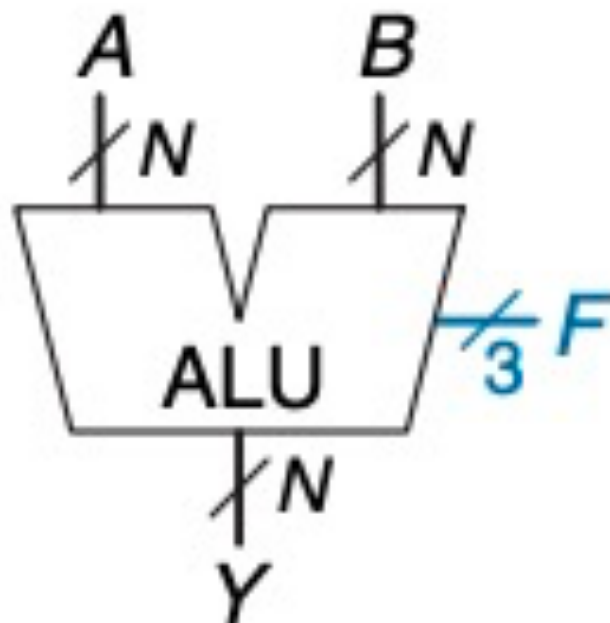
ALU Operations: Comparator

- ❖ Determines whether two binary numbers are equal or if one is greater or less than the other
- ❖ How to implement?



ALU Operations: Comparator

- ❖ Determines whether two binary numbers are equal or if one is greater or less than the other
- ❖ How to implement?
 - ❖ Equal

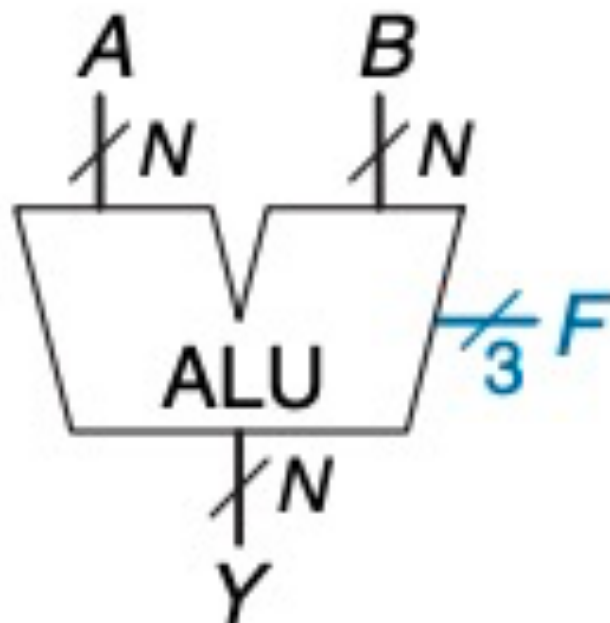
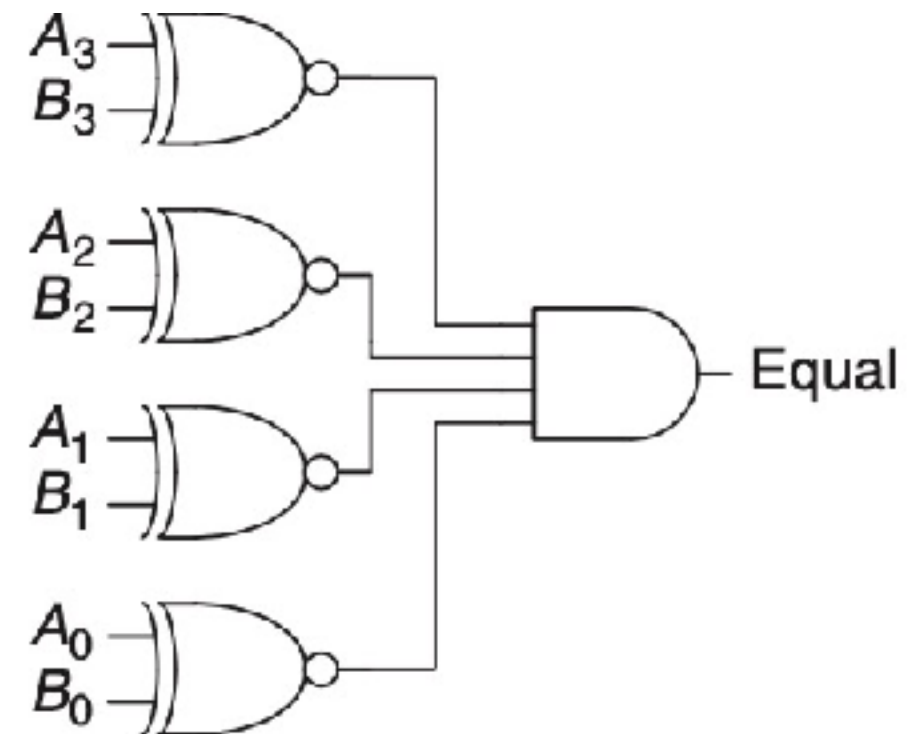
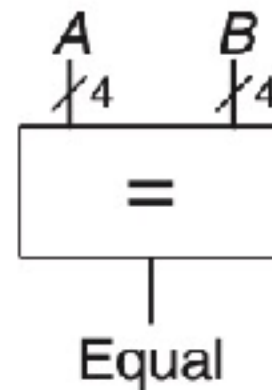


ALU Operations: Comparator

- ❖ Determines whether two binary numbers are equal or if one is greater or less than the other

- ❖ How to implement?

- ❖ Equal

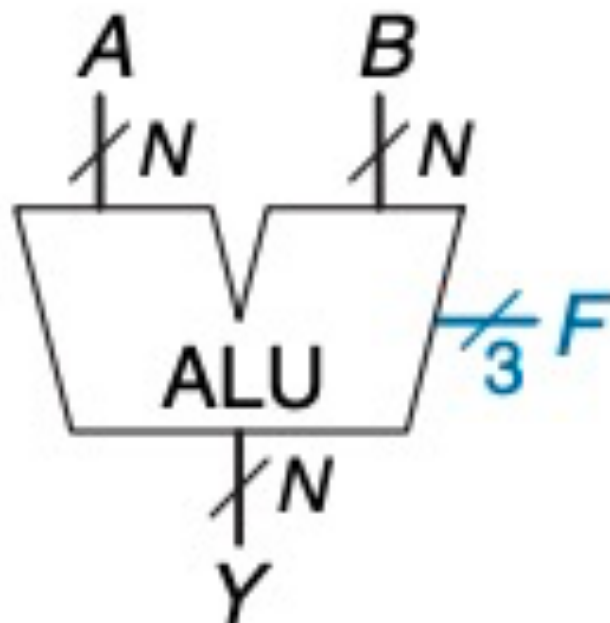
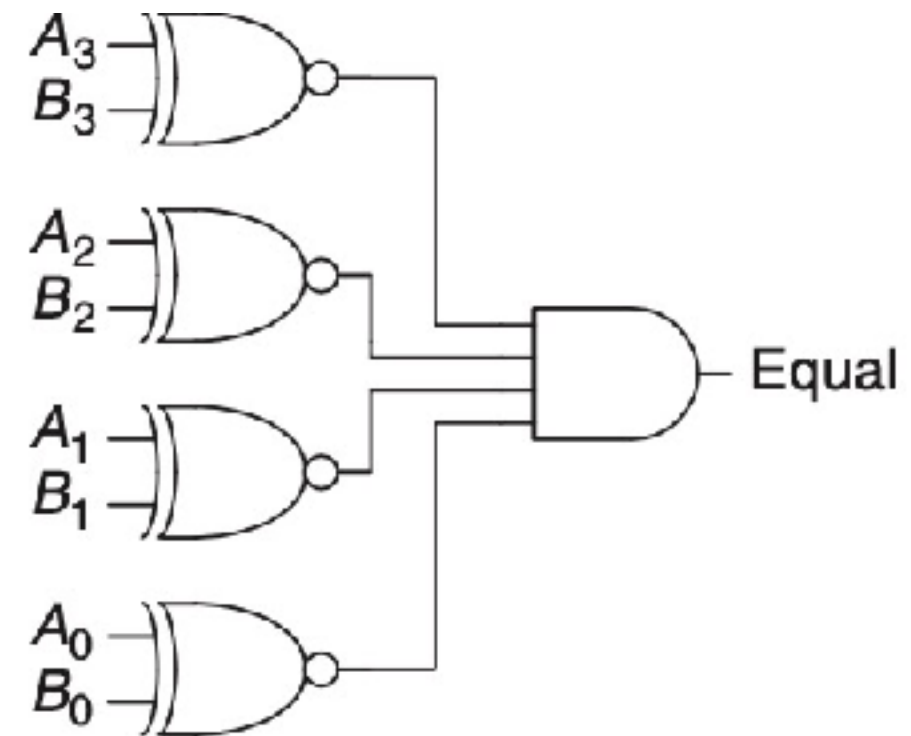
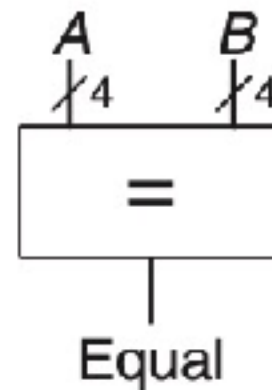


ALU Operations: Comparator

- ❖ Determines whether two binary numbers are equal or if one is greater or less than the other

- ❖ How to implement?

- ❖ Equal
- ❖ Unequal

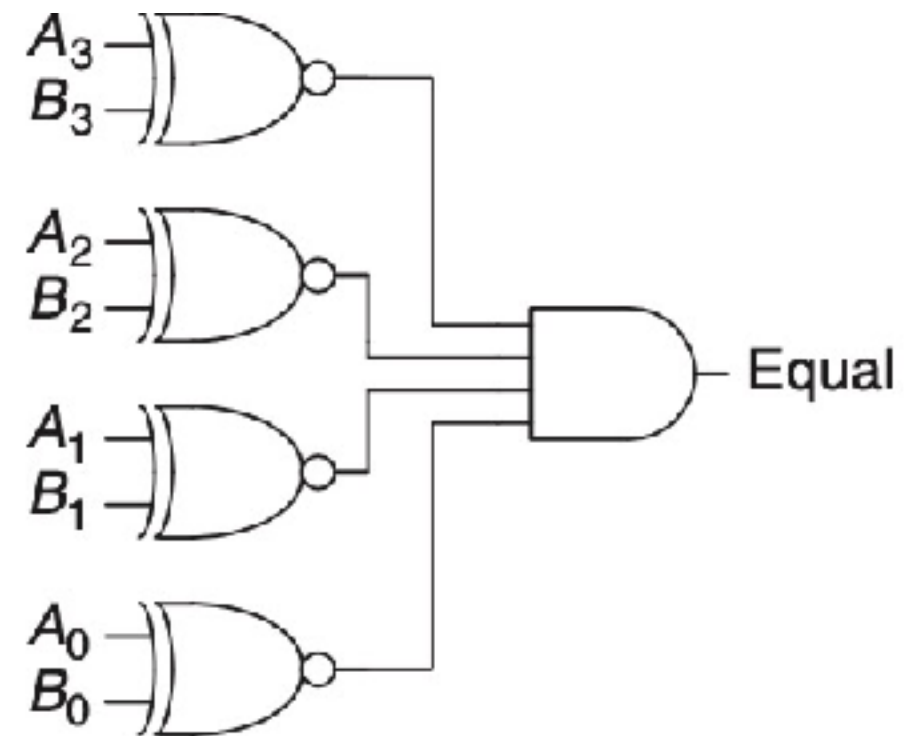
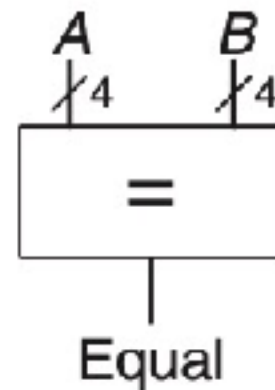
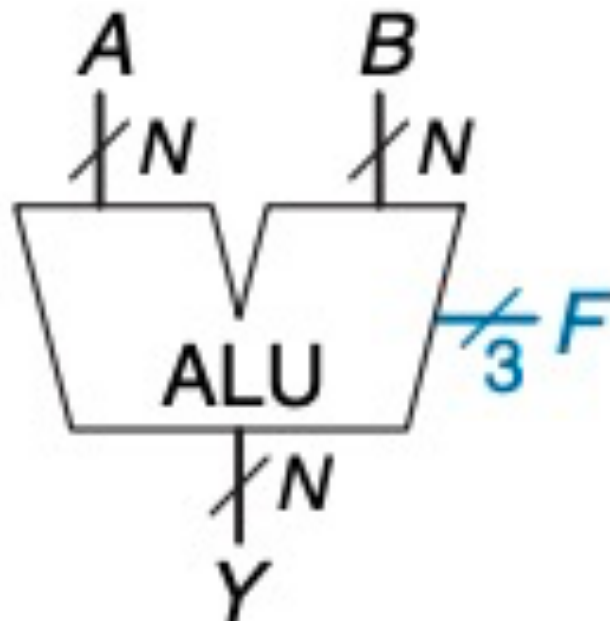
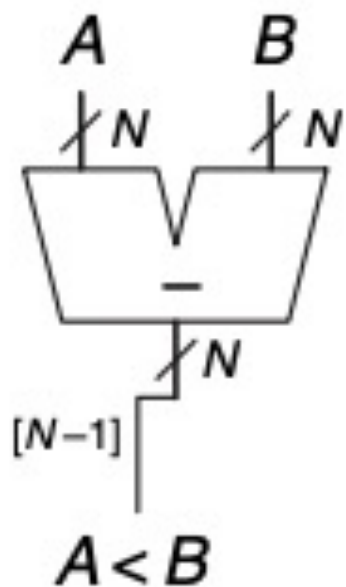


ALU Operations: Comparator

- ❖ Determines whether two binary numbers are equal or if one is greater or less than the other

- ❖ How to implement?

- ❖ Equal
- ❖ Unequal



Recall the ALU Schematic

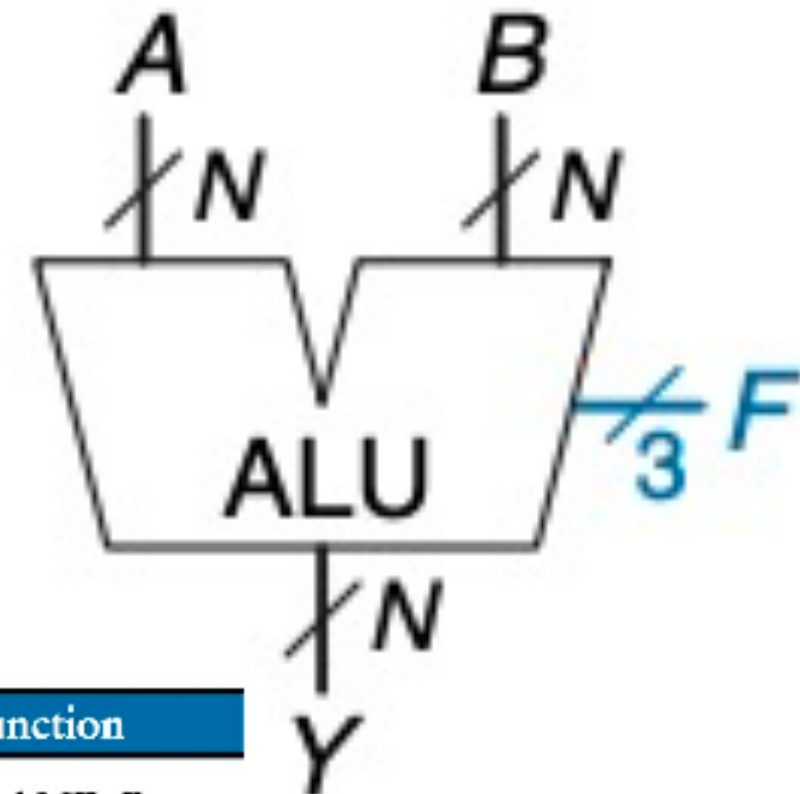
- ❖ Abstraction of ALU, three components

- ❖ Two inputs (A, B)

- ❖ One output (Y)

- ❖ Control signal F (3-bit)

- ❖ **Most operations done!**



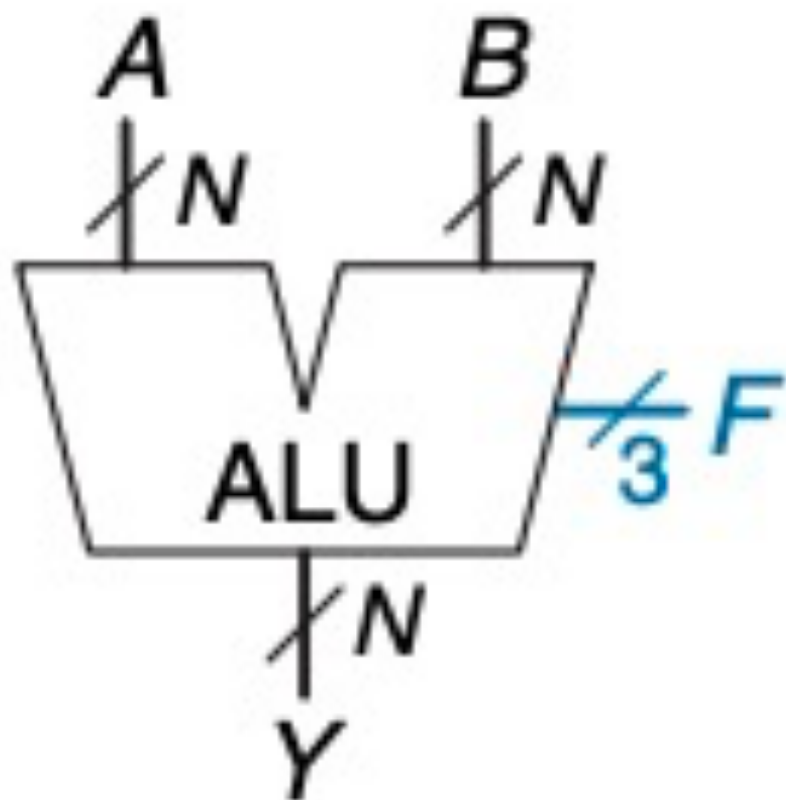
$F_{2:0}$	Function
000	A AND B
001	A OR B
010	A + B
011	not used
100	A AND \bar{B}
101	A OR \bar{B}
110	A - B
111	SLT

SLT: Set if Less Than

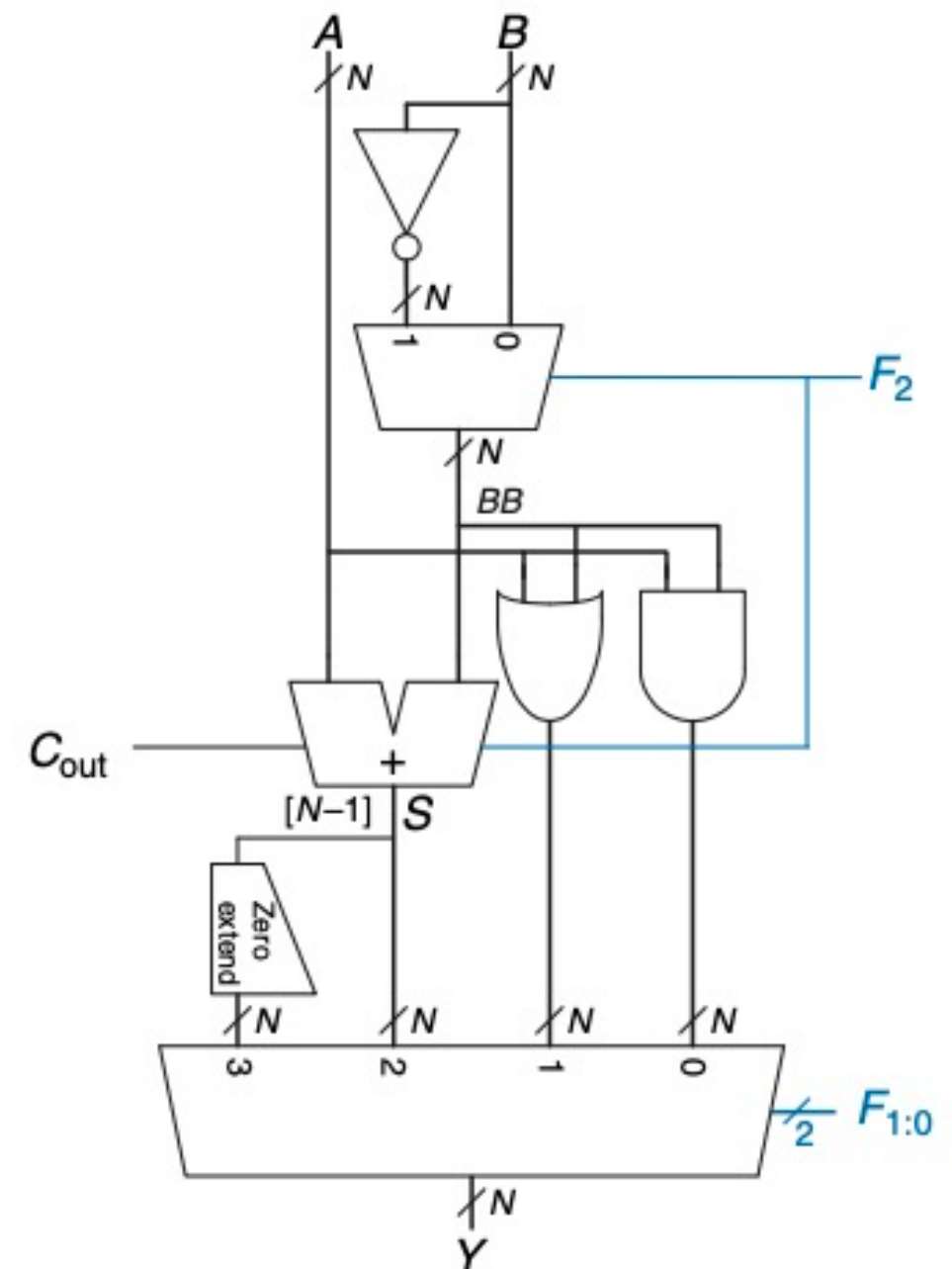
- ❖ Y is set to 1 if A is less than B
 - ❖ $A < B$, $Y = 1$
 - ❖ Otherwise, $Y = 0$
- ❖ Simply computing $Y = A - B$
 - ❖ If negative, then $Y = 1$
 - ❖ Otherwise, $Y = 0$

An ALU Design

- ❖ Abstraction of ALU, three components
- ❖ All operations done!



$F_{2:0}$	Function
000	A AND B
001	A OR B
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100	A AND \bar{B}
101	A OR \bar{B}
110	A - B
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Other Operations: Shifter

- ❖ Move bits toward left or right
 - ❖ **Logical shifter** — shifts the number to the left (LSL) or right (LSR) and fills empty spots with 0s
 - ❖ $11001 \text{ LSR } 2 = \underline{\hspace{2cm}}$; $11001 \text{ LSL } 2 = \underline{\hspace{2cm}}$;
 - ❖ **Arithmetic shifter**
 - ❖ Same as a logical shifter
 - ❖ Right shifts fills the most significant bits with a copy of the old most significant bit (msb)
 - ❖ $11001 \text{ ASR } 2 = \underline{\hspace{2cm}}$; $11001 \text{ ASL } 2 = \underline{\hspace{2cm}}$;

Other Operations: Shifter

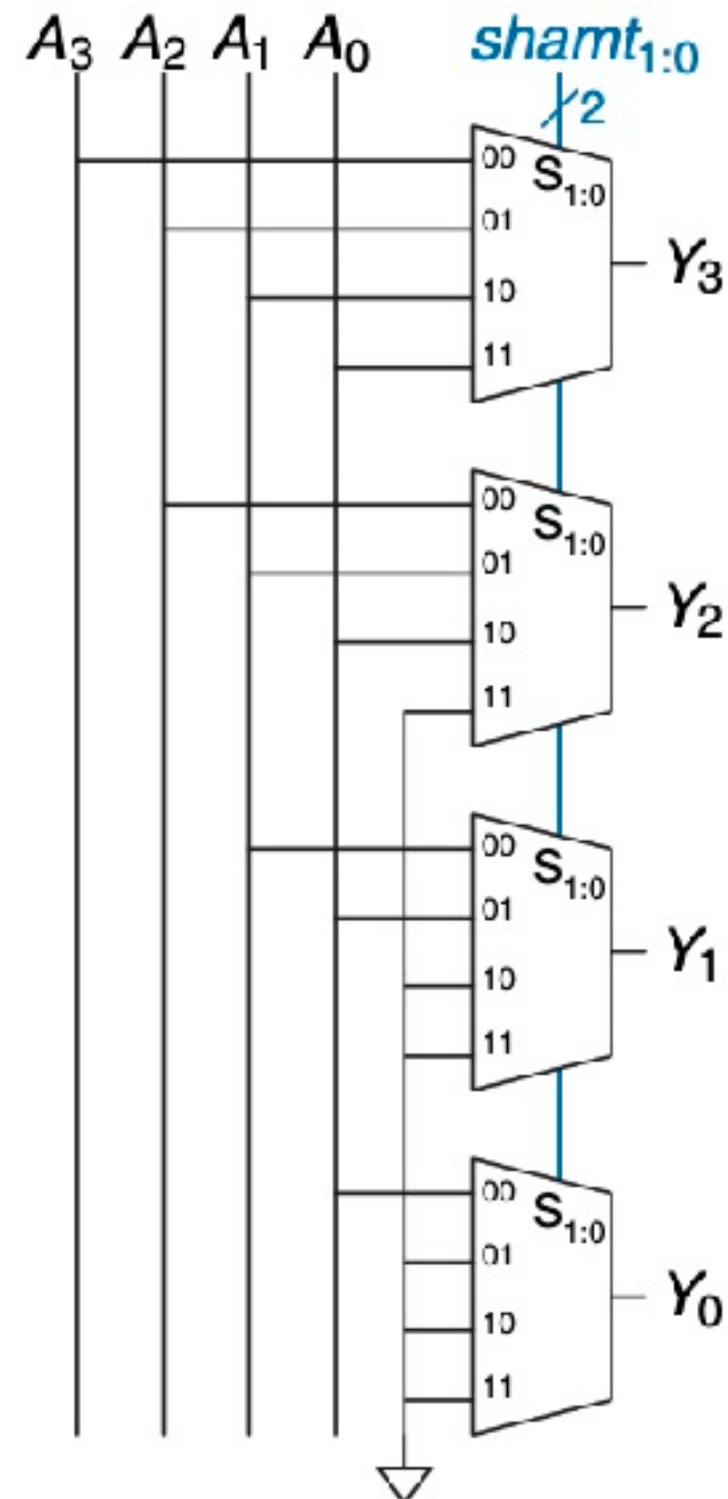
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- ❖ Arithmetic shifter

- ❖ Same as a logical shifter
 - ❖ Right shifts fills the most significant bits with a copy of the old most significant bit (msb)
 - ❖ 11001 ASR 2 = _____; 11001 ASL 2 = _____;



Other Operations: Rotator

- ❖ Rotator—rotates number in circle such that empty spots are filled with bits shifted off the other end.
- ❖ $11001 \text{ ROR } 2 = \underline{\hspace{2cm}}$; $11001 \text{ ROL } 2 = \underline{\hspace{2cm}}$;