
About the Lab Re-enroll

- ❖ Both my lab sections are full
- ❖ If you have requested to re-enroll my lab sections, I will approve it as well. But very possibly it will be denied.
- ❖ If this applied to your case, you should now resort to other sections, we have enough (4 in total)

Mid-term Review

- ❖ Data conversion
- ❖ FPGA design
- ❖ Combinational circuit
- ❖ Sequential Circuit
- ❖ Encoder / Decoder
- ❖ Verilog

Mid-term Review

- ❖ Data conversion
 - ❖ Binary to decimal
 - ❖ Signed and unsigned
 - ❖ One's and Two's complement
 - ❖ Hexadecimal to and from Octal

Mid-term Review

❖ FPGA design

❖

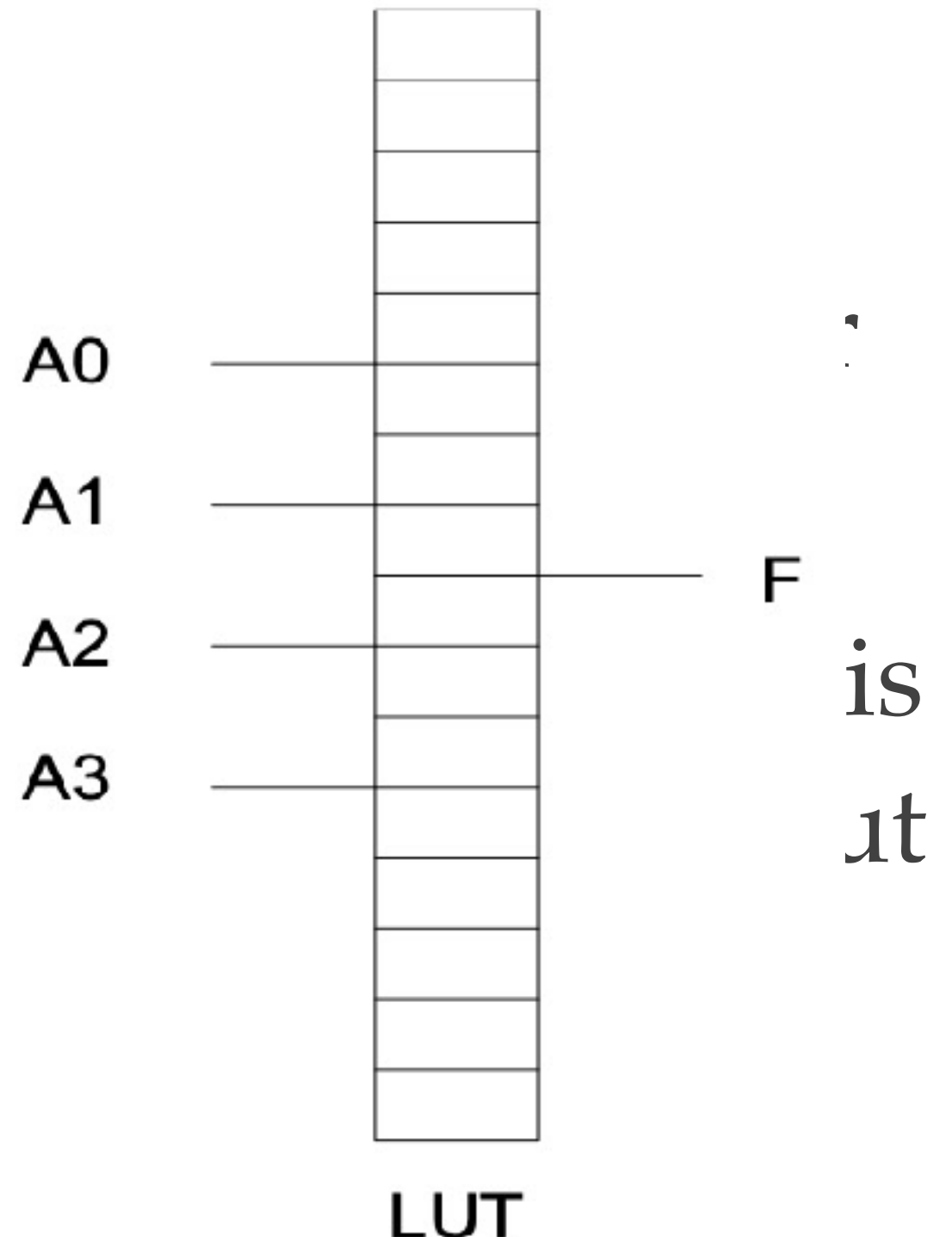
LUT: Look-Up Table

- ❖ Look-up tables are how your logic actually gets implemented. Users can program what the output should be for every single possible input
- ❖ A LUT consists of a block of RAM that is indexed by the LUT's inputs. The output of the LUT is whatever value is in the indexed location in its RAM cell

LUT: Look-Up Table

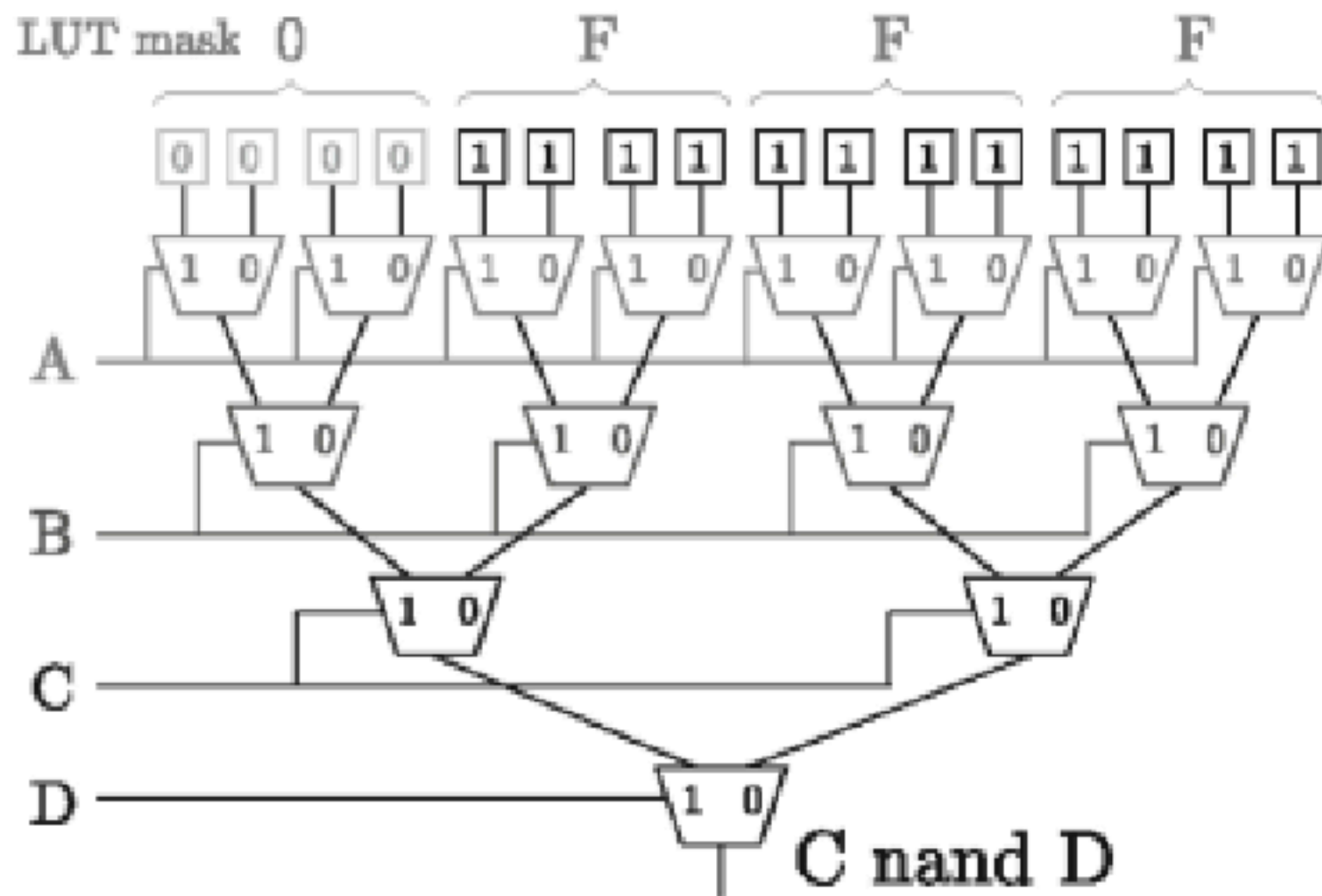
- ❖ Look-up tables are how a function that a program actually gets implemented by looking up what the output is for every single possible input

- ❖ A LUT consists of a block of memory indexed by the LUT's address. The output of the LUT is whatever is stored at the indexed location in its memory



FPGA Design Example

- ❖ Lets reverse the function from LUT



FPGA Design Example

- ❖ A Boolean Function of four input variables A, B, C and D using a 4-input LUT.
- ❖ Here, let the output become high only when any of the two input variables are one.
- ❖ What is the hardware realization?

FPGA Design Example

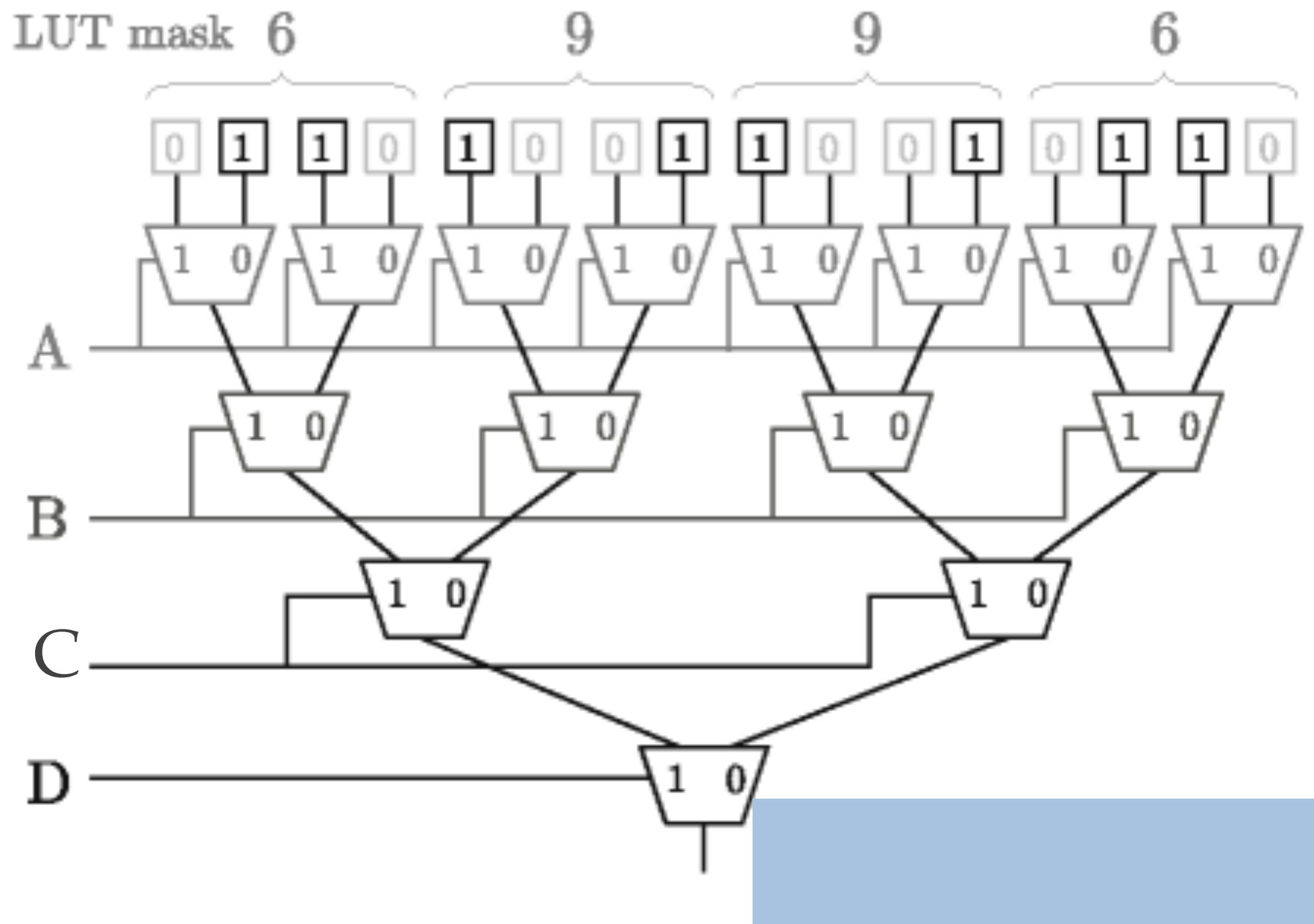
- ❖ A Boolean Function of four inputs A, B, C and D using a 4-input LUT.
- ❖ Here, let the output become high if exactly two input variables are one.
- ❖ What is the hardware realization of the function?

Truth Table

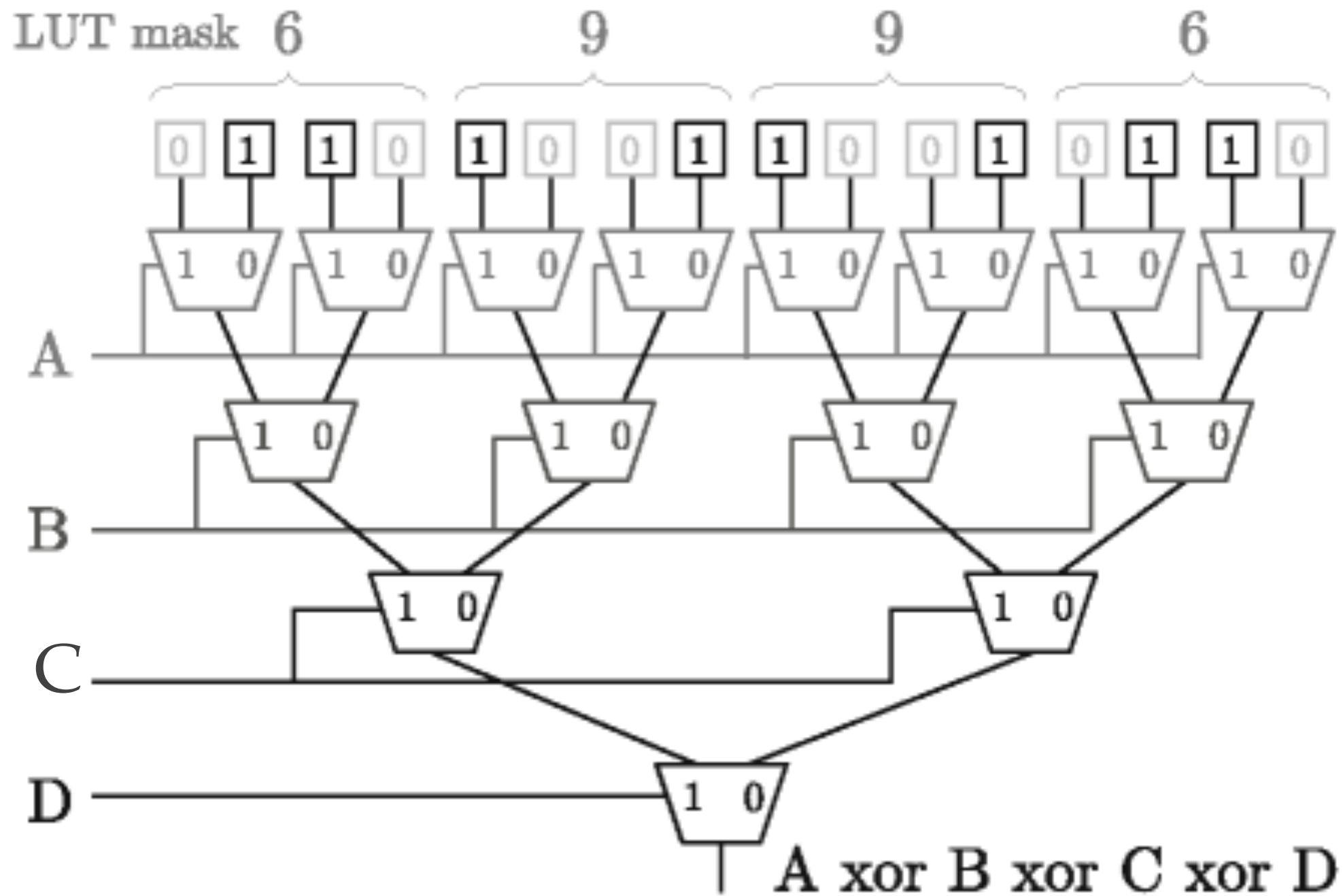
Inputs				Output
A	B	C	D	Y
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	1
0	1	0	0	0
0	1	0	1	1
0	1	1	0	1
0	1	1	1	0
1	0	0	0	0
1	0	0	1	1
1	0	1	0	1
1	0	1	1	0
1	1	0	0	1
1	1	0	1	0
1	1	1	0	0
1	1	1	1	0

C and
of the

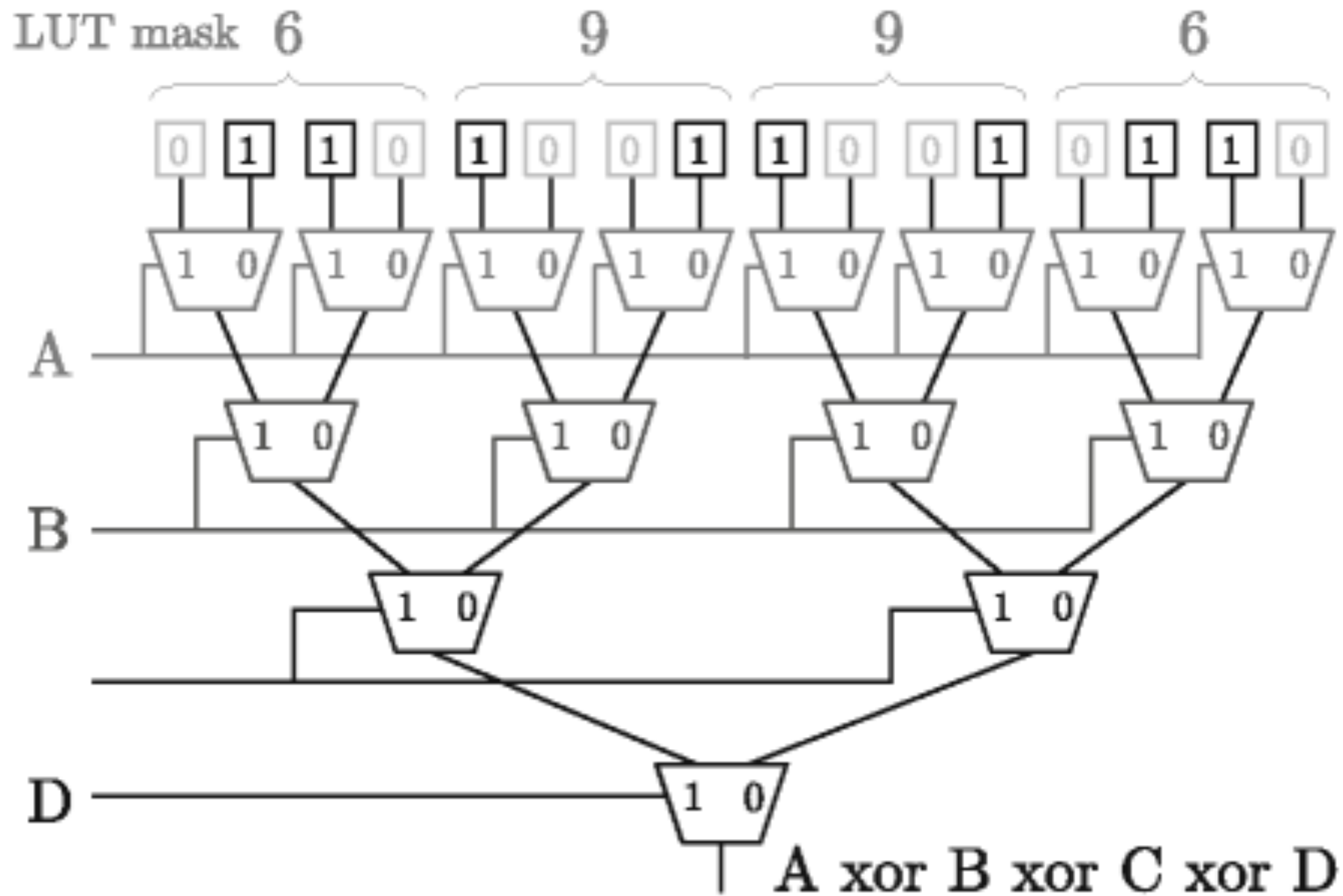
Practice



Practice



Practice

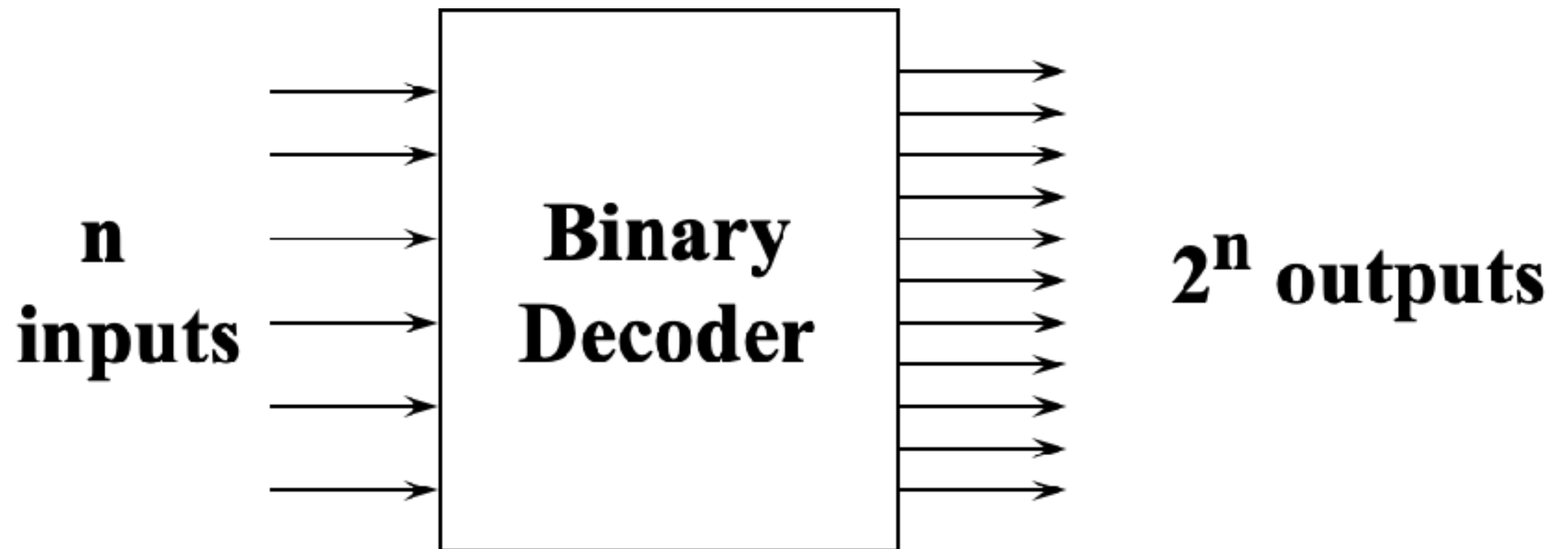


Mid-term Review

- ❖ Combinational circuit
- ❖ Logic gates
- ❖ Encoder
- ❖ Decoder

Decoder

- ❖ Logic with n input lines and 2^n output lines
- ❖ Only one output is a 1 for any given input
- ❖



Decoder Example

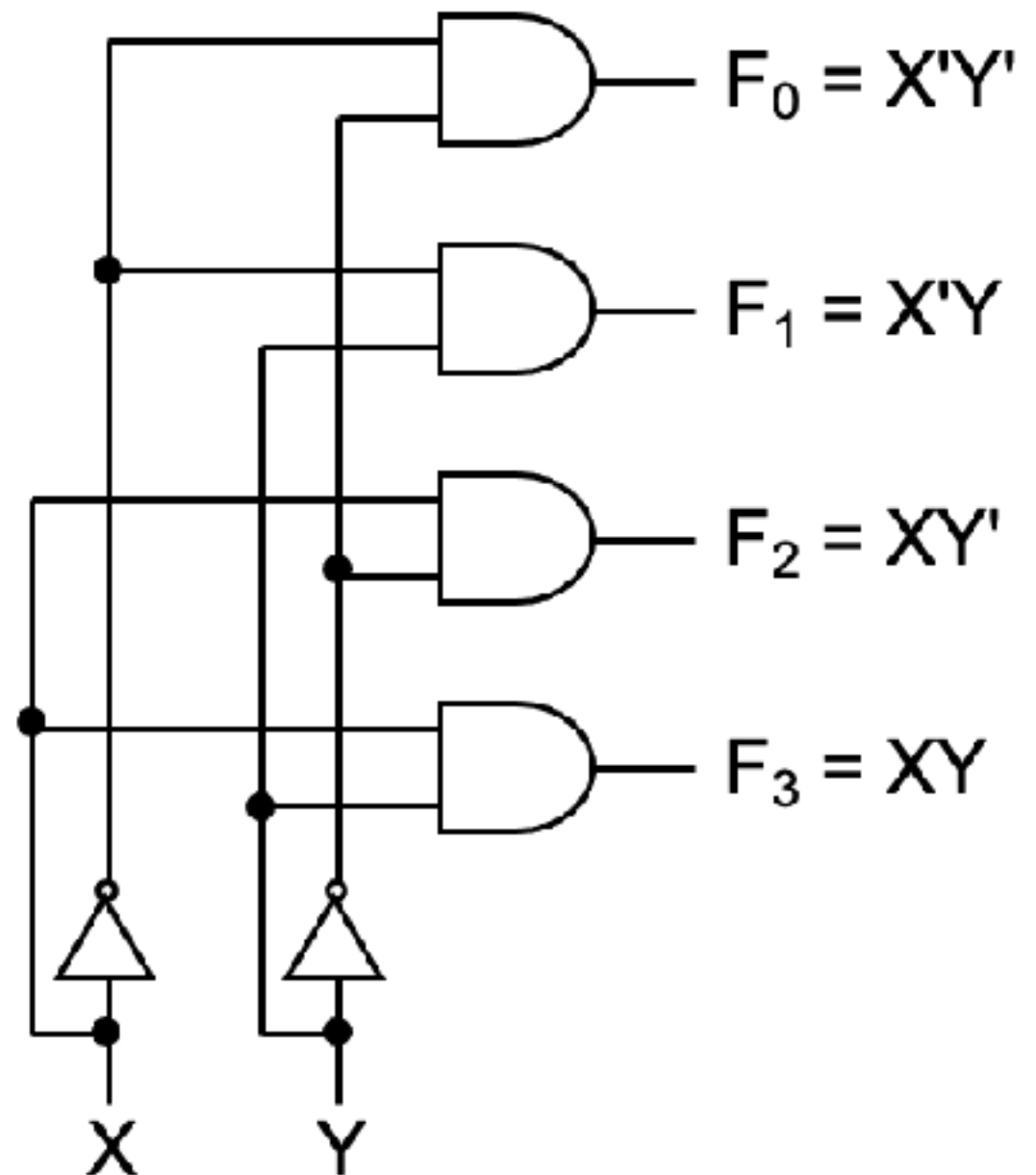
❖ 2-to-4 decoder

X	Y	F₀	F₁	F₂	F₃
0	0	1	0	0	0
0	1	0	1	0	0
1	0	0	0	1	0
1	1	0	0	0	1

Decoder Example

❖ 2-to-4 decoder

X	Y	F₀	F₁	F₂	F₃
0	0	1	0	0	0
0	1	0	1	0	0
1	0	0	0	1	0
1	1	0	0	0	1

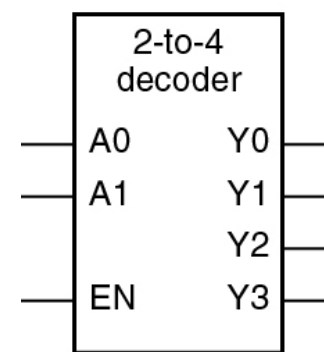


Binary Decoder with ENable

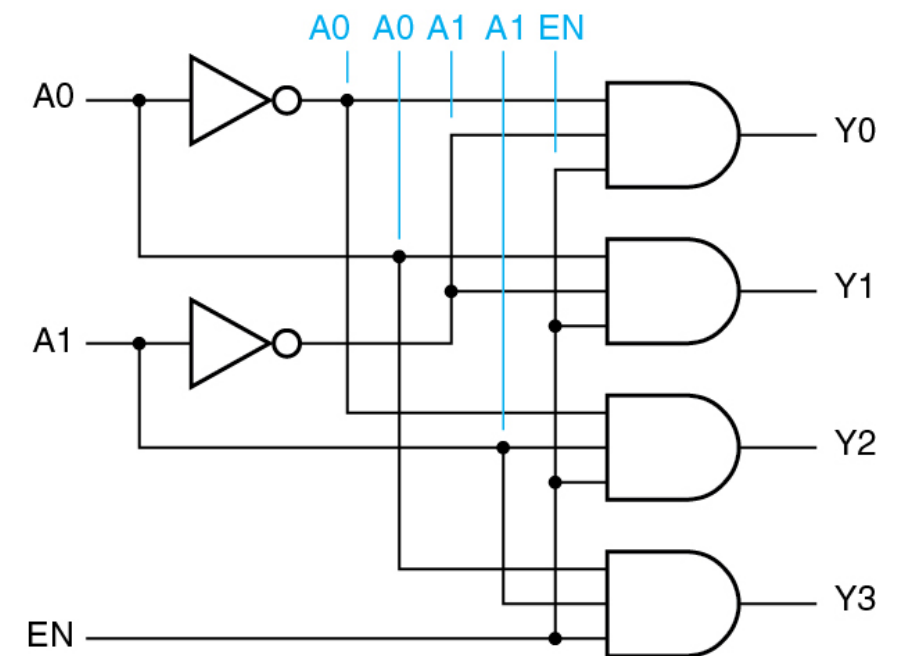
<i>Inputs</i>			<i>Outputs</i>			
EN	A1	A0	Y3	Y2	Y1	Y0
0	x	x	0	0	0	0
1	0	0	0	0	0	1
1	0	1	0	0	1	0
1	1	0	0	1	0	0
1	1	1	1	0	0	0

Binary Decoder with Enable

Inputs			Outputs			
EN	A1	A0	Y3	Y2	Y1	Y0
0	x	x	0	0	0	0
1	0	0	0	0	0	1
1	0	1	0	0	1	0
1	1	0	0	1	0	0
1	1	1	1	0	0	0



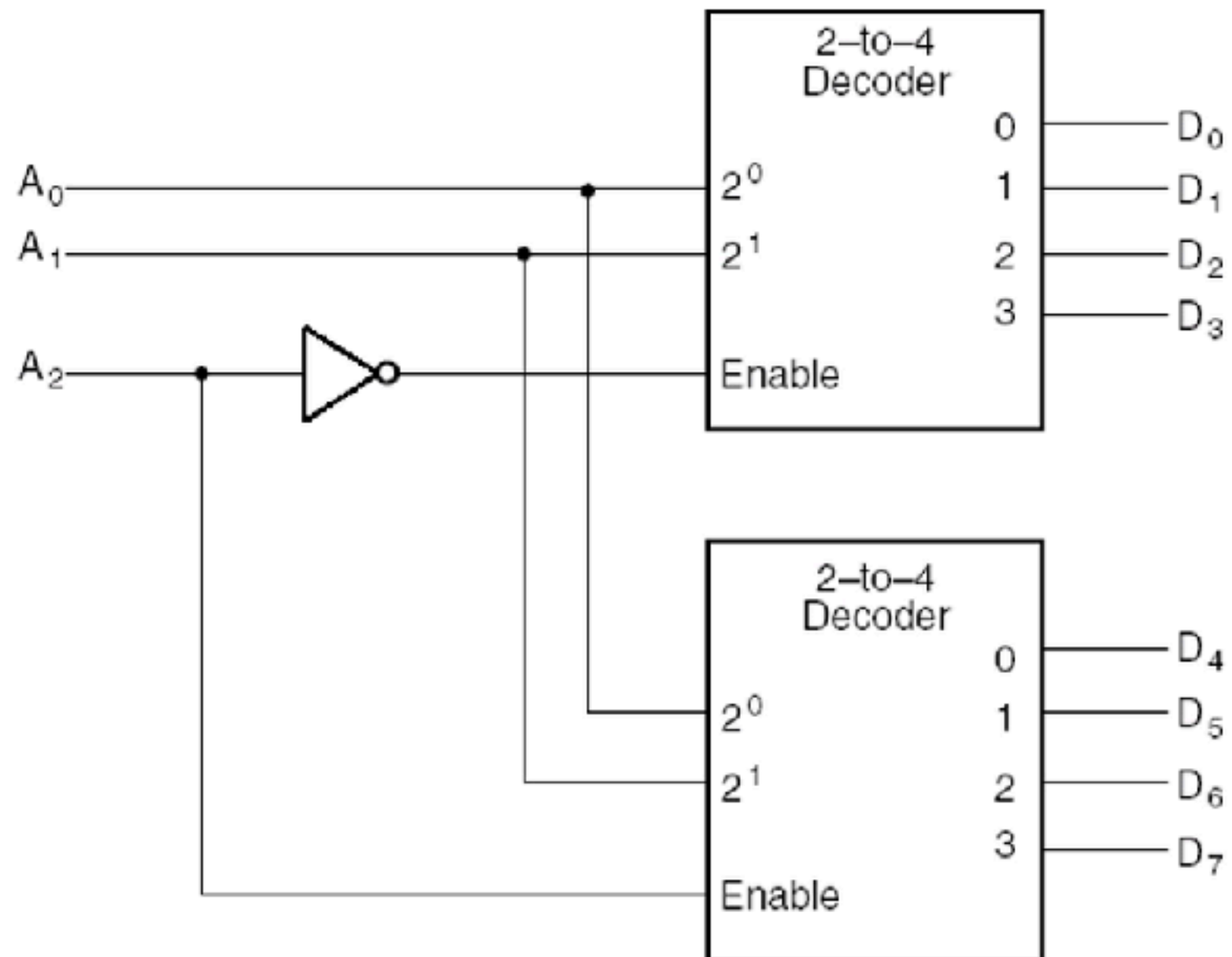
(a)



(b)

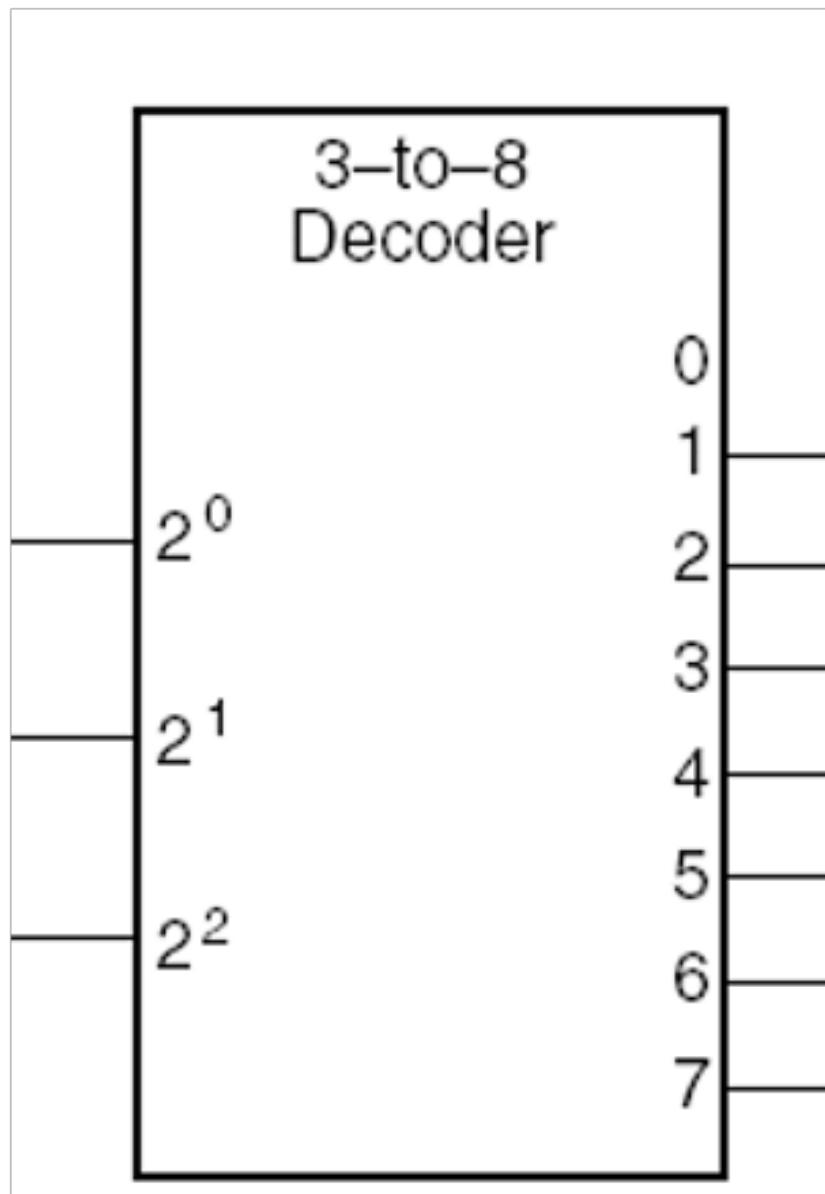
Building Larger Decoder

- ❖ Build larger decoders out of two or more smaller decoders



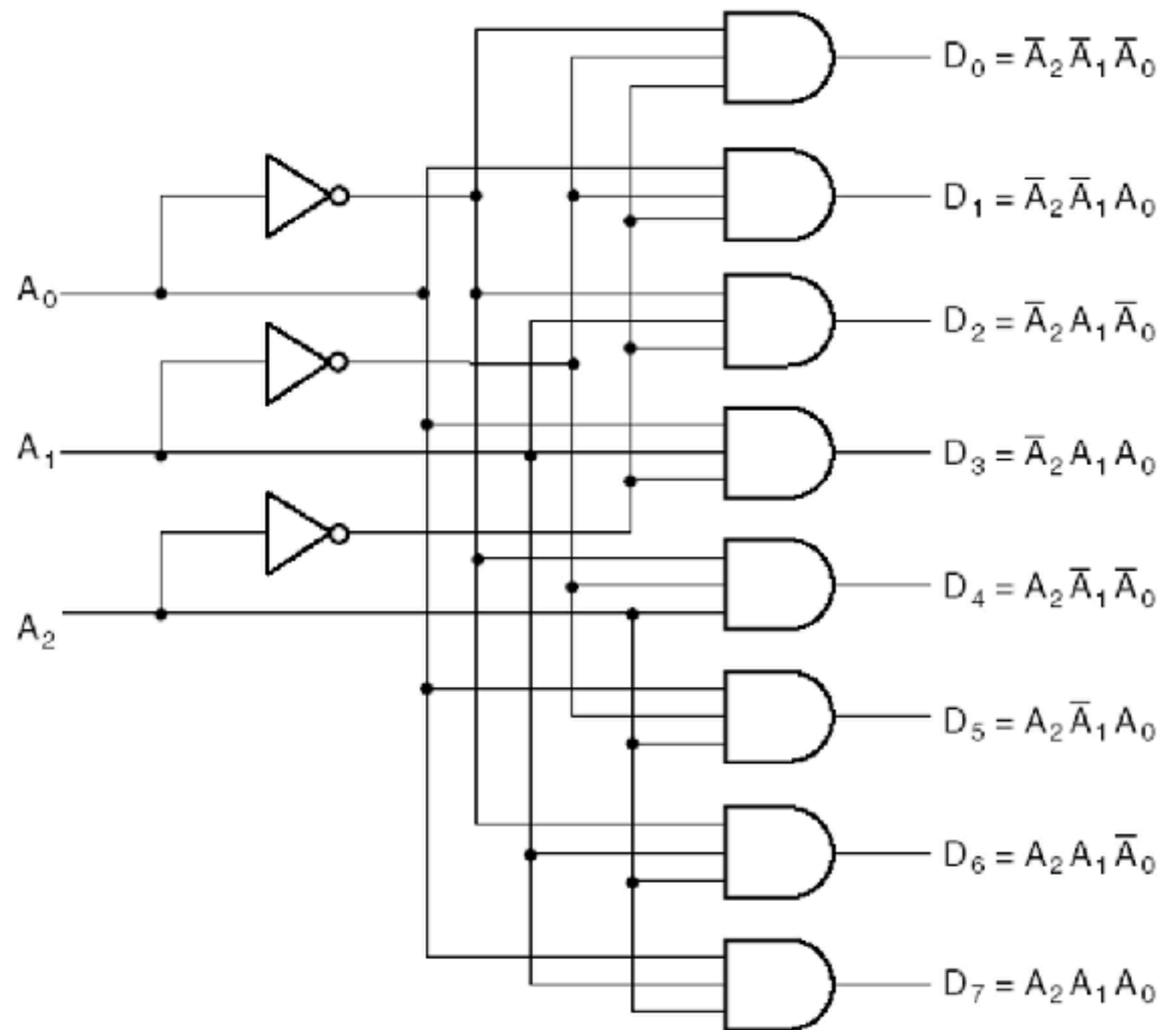
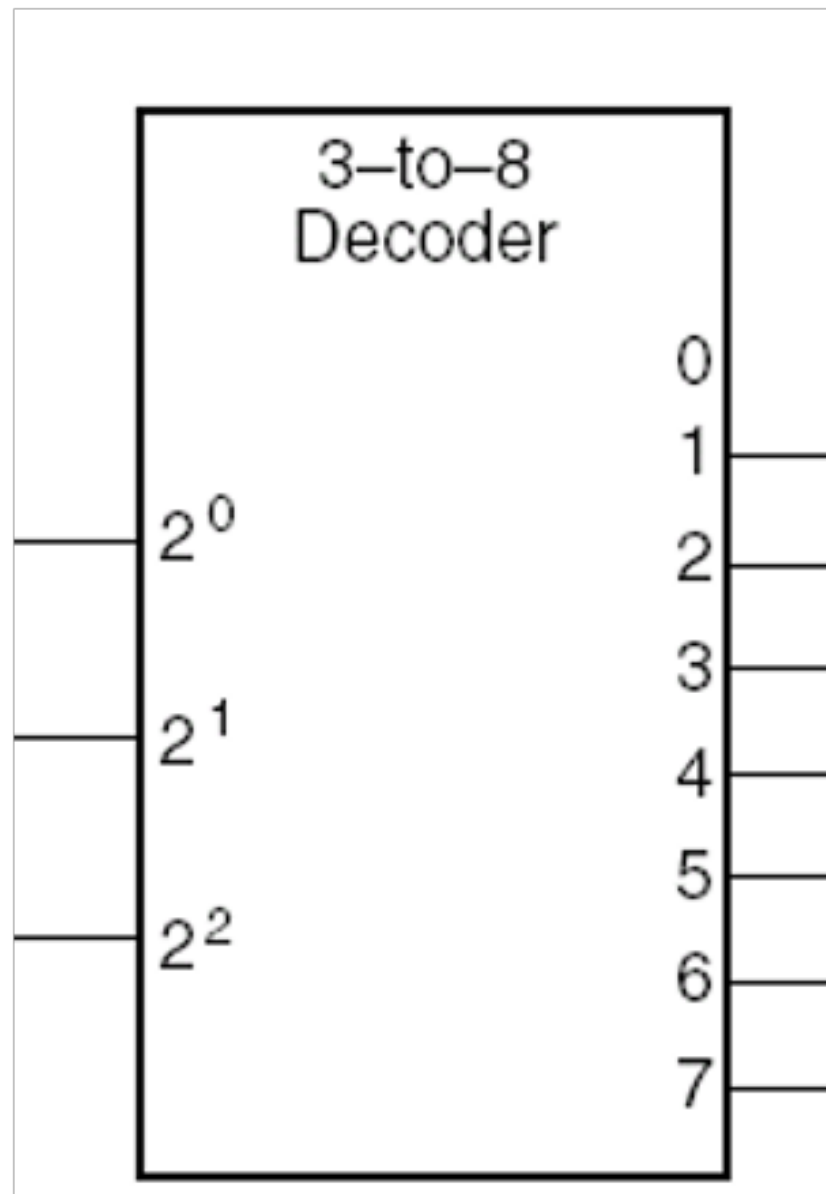
Decoder Example

❖ 3-to-8 decoder



Decoder Example

❖ 3-to-8 decoder



5-To-32 Decoder

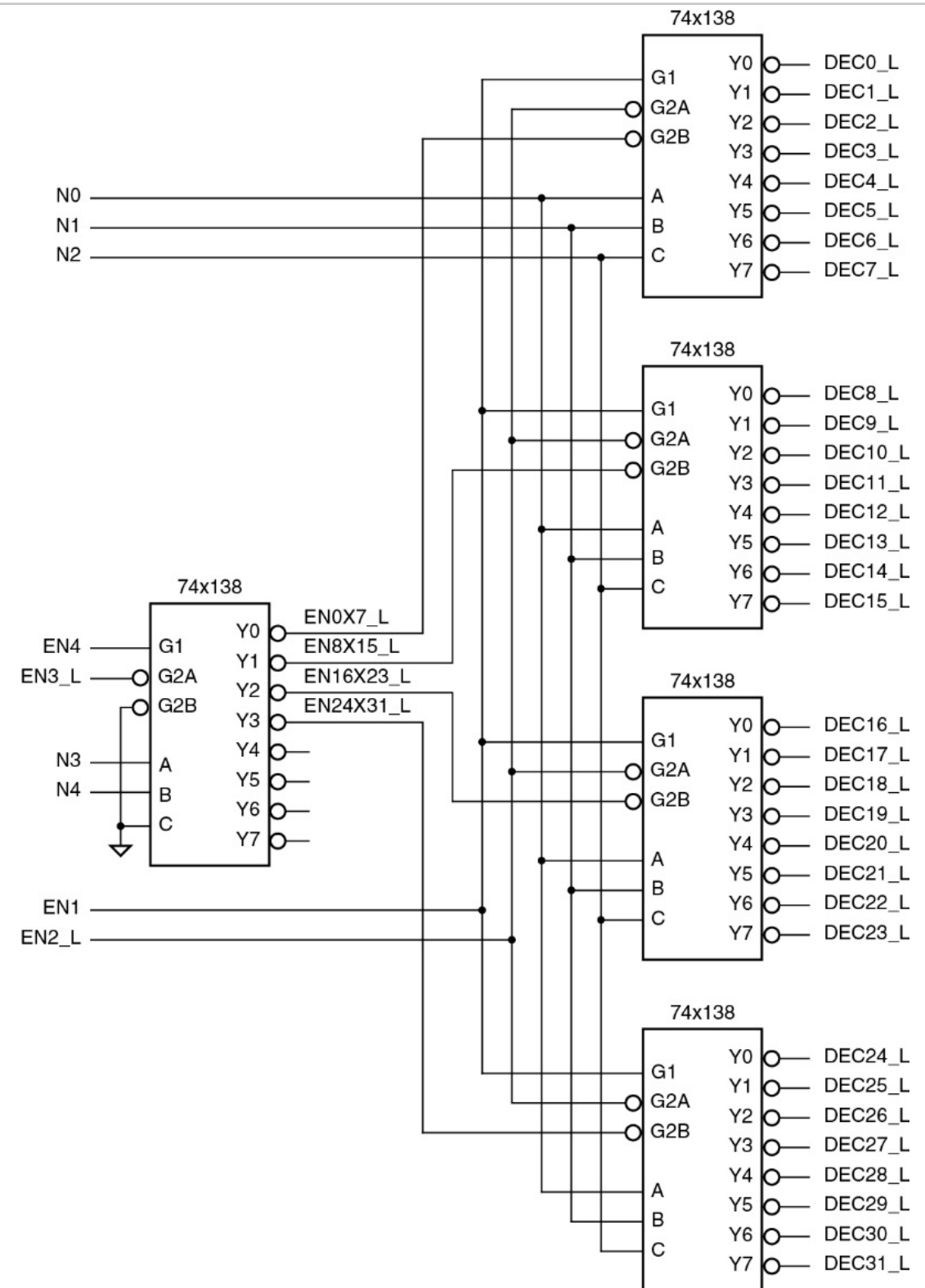
❖ How?

5-To-32 Decoder

- ❖ How?
- ❖ Cascading 3-To-8 decoders

5-To-32 Decoder

- ❖ How?
- ❖ Cascading 3-To-8 decoders



Encoder Example

TABLE 3-5
Truth Table for Octal-to-Binary Encoder

Inputs								Outputs		
D_7	D_6	D_5	D_4	D_3	D_2	D_1	D_0	A_2	A_1	A_0
0	0	0	0	0	0	0	1	0	0	0
0	0	0	0	0	0	1	0	0	0	1
0	0	0	0	0	1	0	0	0	1	0
0	0	0	0	1	0	0	0	0	1	1
0	0	0	1	0	0	0	0	1	0	0
0	0	1	0	0	0	0	0	1	0	1
0	1	0	0	0	0	0	0	1	1	0
1	0	0	0	0	0	0	0	1	1	1

4-to-2 Binary Encoder in Verilog

```
module encoder (Y, W);  
    input [3:0] W;  
    output [1:0] Y;  
    reg [1:0] Y;  
  
    always@ (W)  
        case (W)  
            4'b1000: Y = 2'b11;  
            4'b0100: Y = 2'b10;  
            4'b0010: Y = 2'b01;  
            4'b0001: Y = 2'b00;  
            default: Y = 2'bxx; //don't care case  
        endcase  
    endmodule
```

Priority Encoder

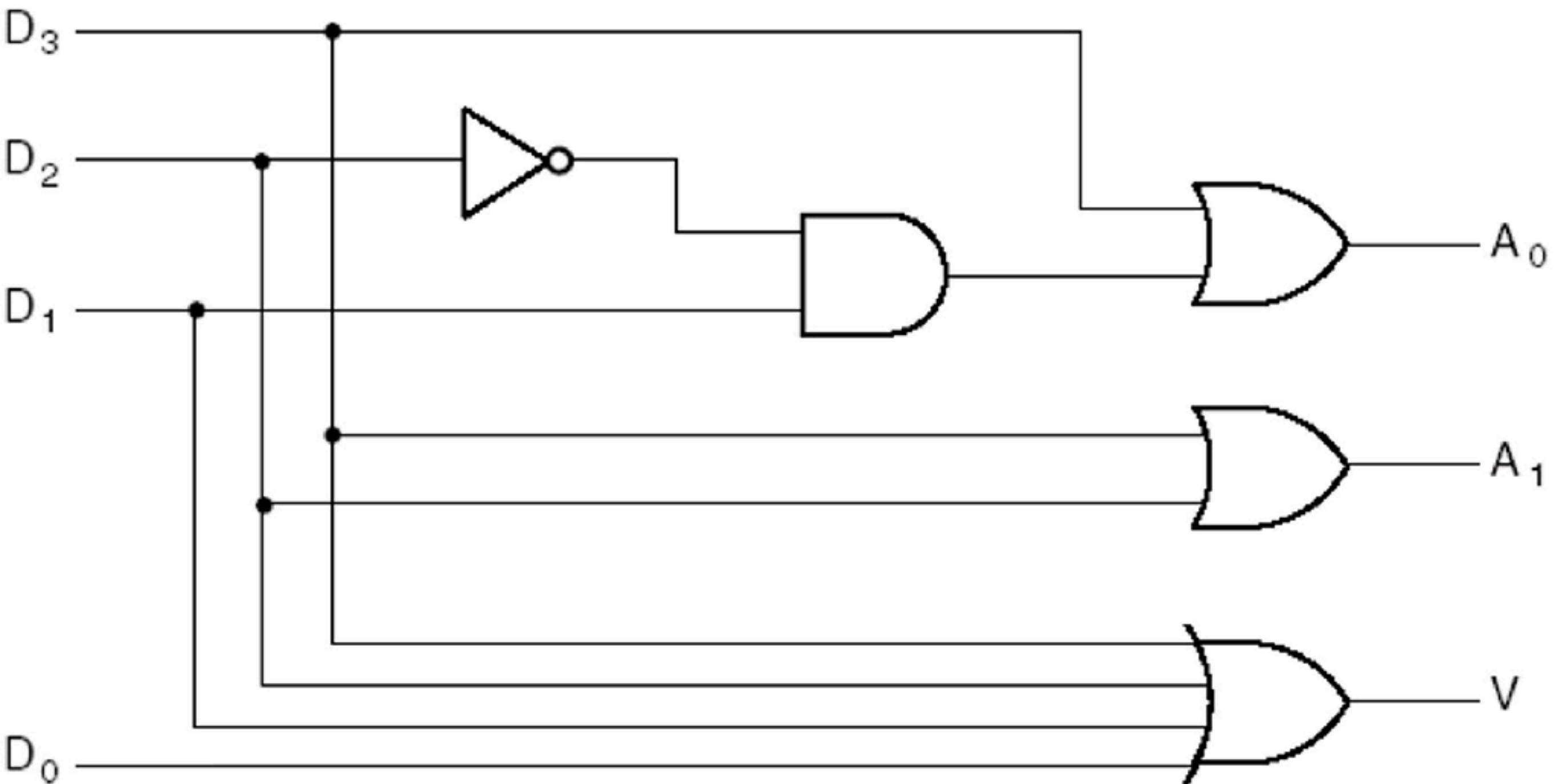
- ❖ If two or more inputs are equal to 1 at the same time, the input with highest priority takes precedence.
- ❖ Truth Table of 4-Input Priority Encoder
- ❖

Priority Encoder

- ❖ If two or more inputs are equal to 1 at the same time, the input with highest priority takes precedence.
- ❖ Truth Table of 4-Input Priority Encoder

Inputs				Outputs		
D_3	D_2	D_1	D_0	A_1	A_0	V
0	0	0	0	X	X	0
0	0	0	1	0	0	1
0	0	1	X	0	1	1
0	1	X	X	1	0	1
1	X	X	X	1	1	1

Priority Encoder: Circuit Implementation



4-to-2 Priority Encoder in Verilog

```
module priority (Y, V, W);
    input [3:0] W;
    output [1:0] Y;
    output V;
    reg [1:0] Y;
    reg V;
    always@ (W) begin
        V = 1; // assume valid output
        casex (W)
            4'b1xxx: Y = 2'b11;
            4'b01xx: Y = 2'b10;
            4'b001x: Y = 2'b01;
            4'b0001: Y = 2'b00;
            default: begin
                V = 0;
                Y = 2'bxx; //don't care case
            end
        endcase
    end
endmodule
```

Mid-term Review

- ❖ Sequential Circuit
- ❖ Latch
- ❖ Flip-flop

Difference Between Combinational and Sequential Circuits

- ❖ Combinational Circuit

- ❖ Time independent circuits
- ❖ Do not depend on previous inputs to generate any output



Figure: Combinational Circuits

Difference Between Combinational and Sequential Circuits

❖ Sequential Circuit

- ❖ Dependent on
 - ❖ Clock cycles
 - ❖ Present inputs
 - ❖ Past inputs
- ❖ to generate any

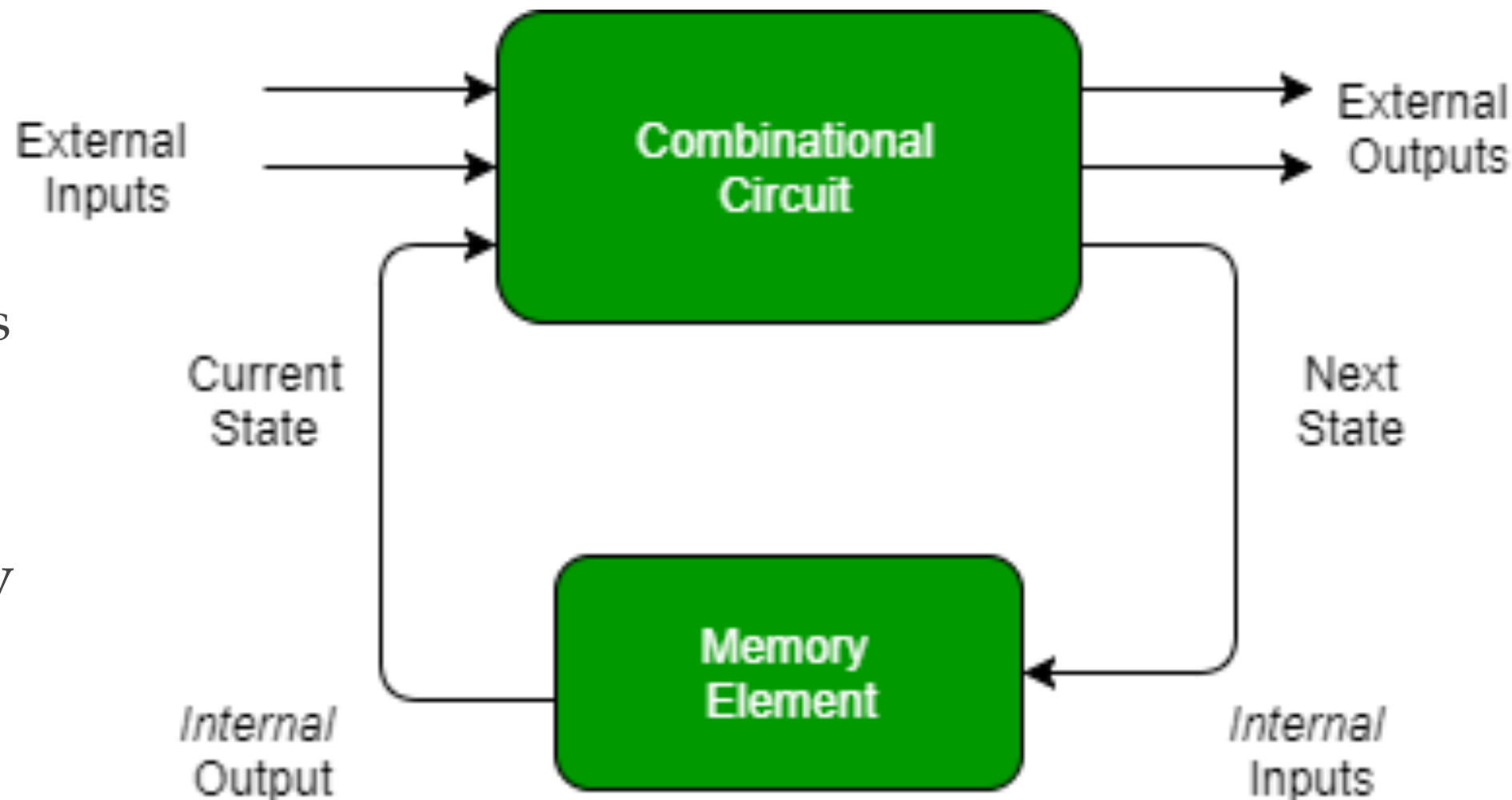


Figure: Sequential Circuit

Sequential Logic (2): SR Latch

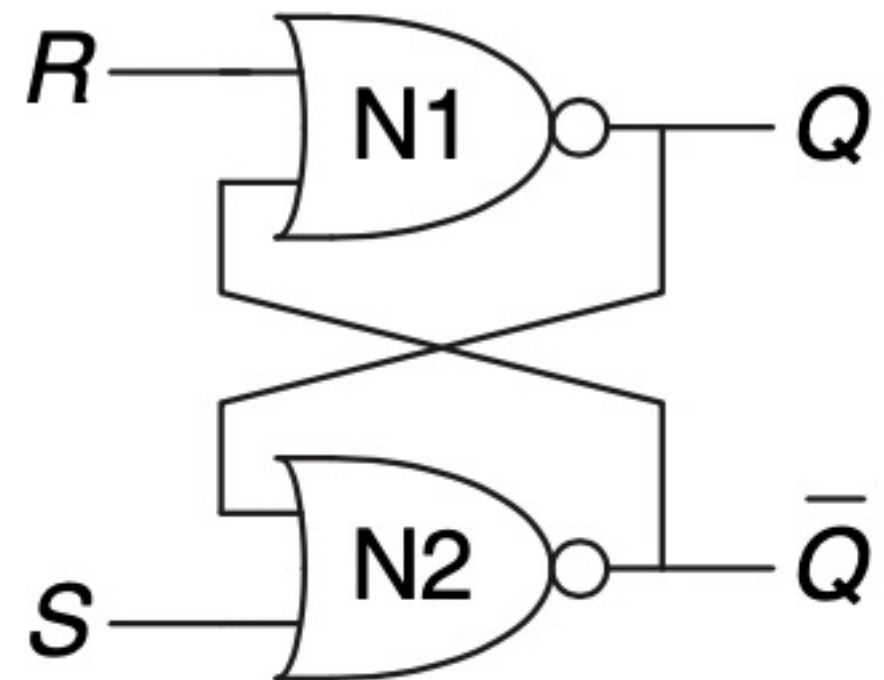
- ❖ SR latch

- ❖ S: Set

- ❖ R: Reset

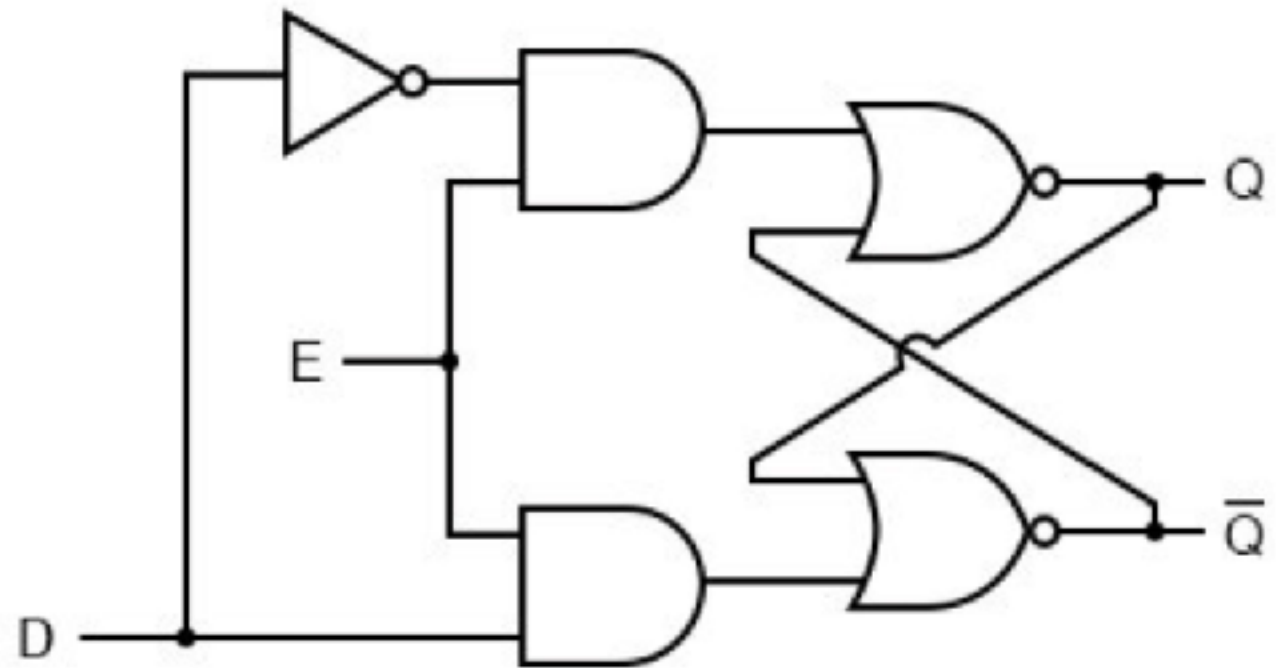
- ❖ Truth table of SR latch

R	S	Q	$\sim Q$
1	0		
0	1		
1	1		
0	0		



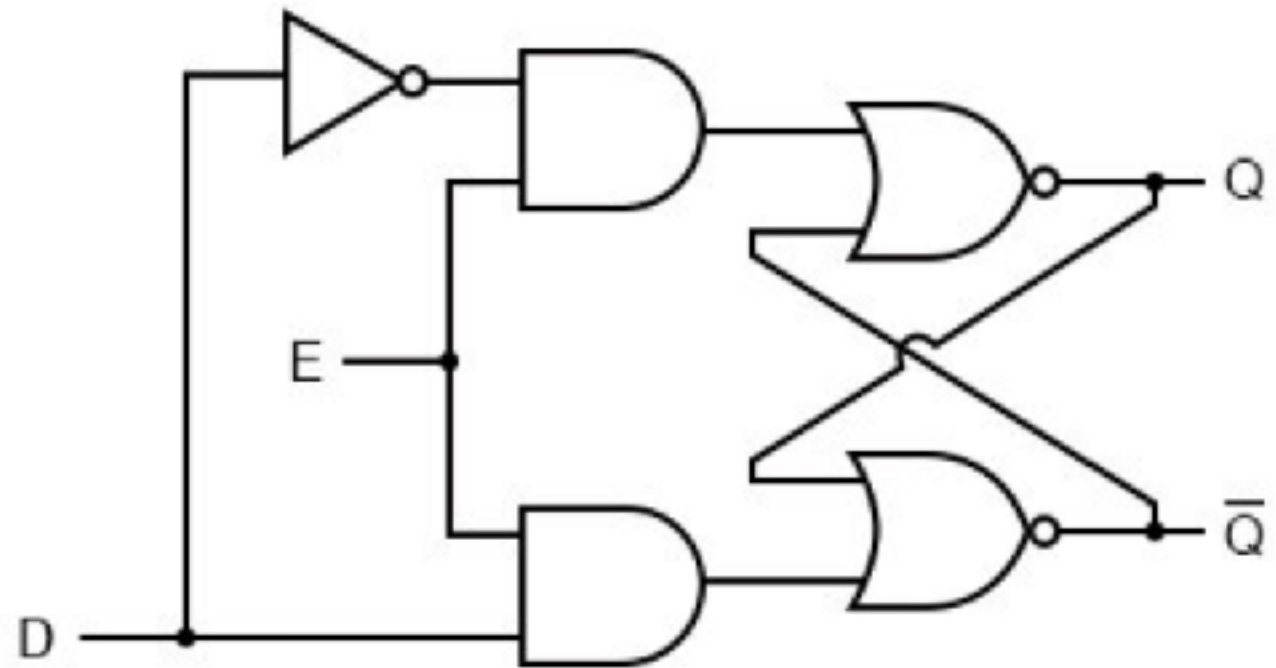
Sequential Logic (2): D Latch

- ❖ *The current design CANNOT latch the D value*
- ❖ How to solve this issue?
 - ❖ Adding an *Enable*



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- ❖ Truth table



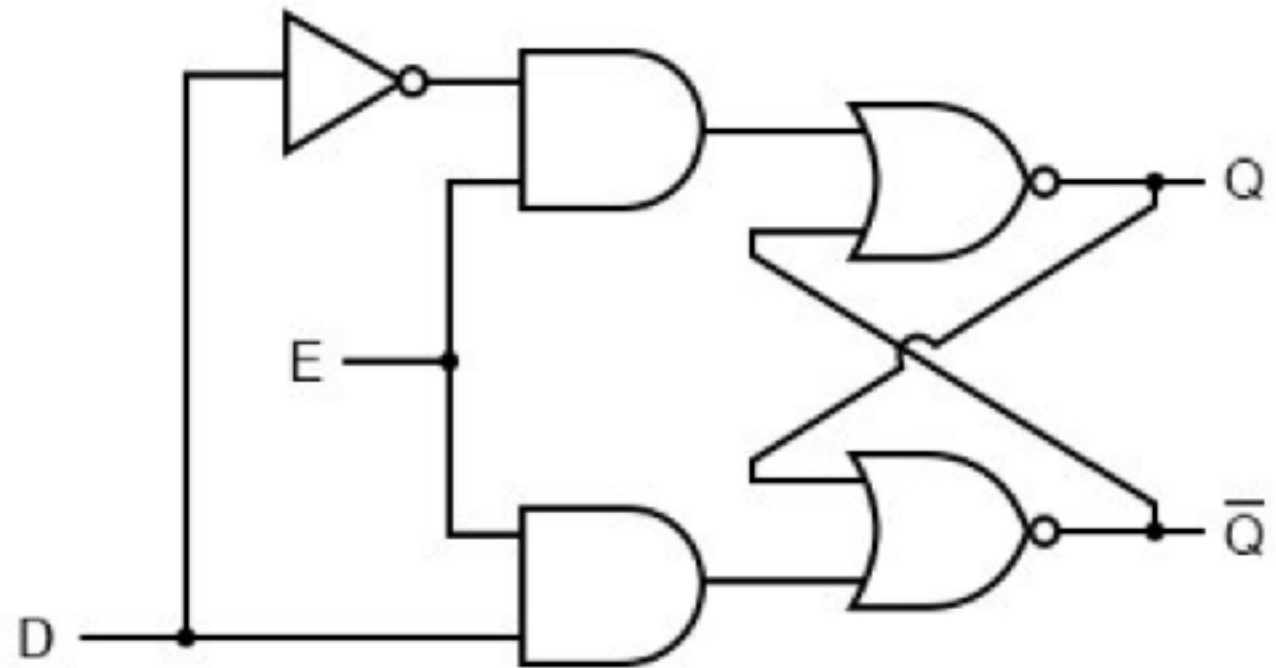
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- ❖ *The current design CANNOT latch the D value*
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- ❖ Adding an *Enable*

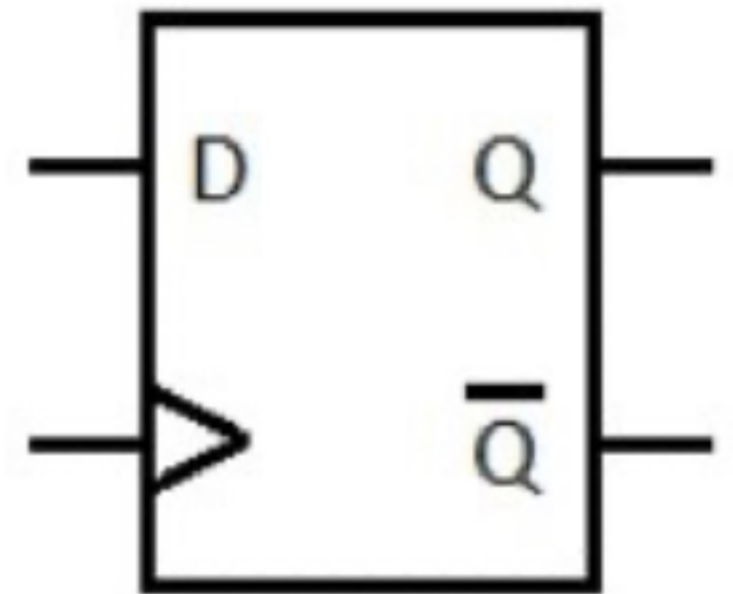
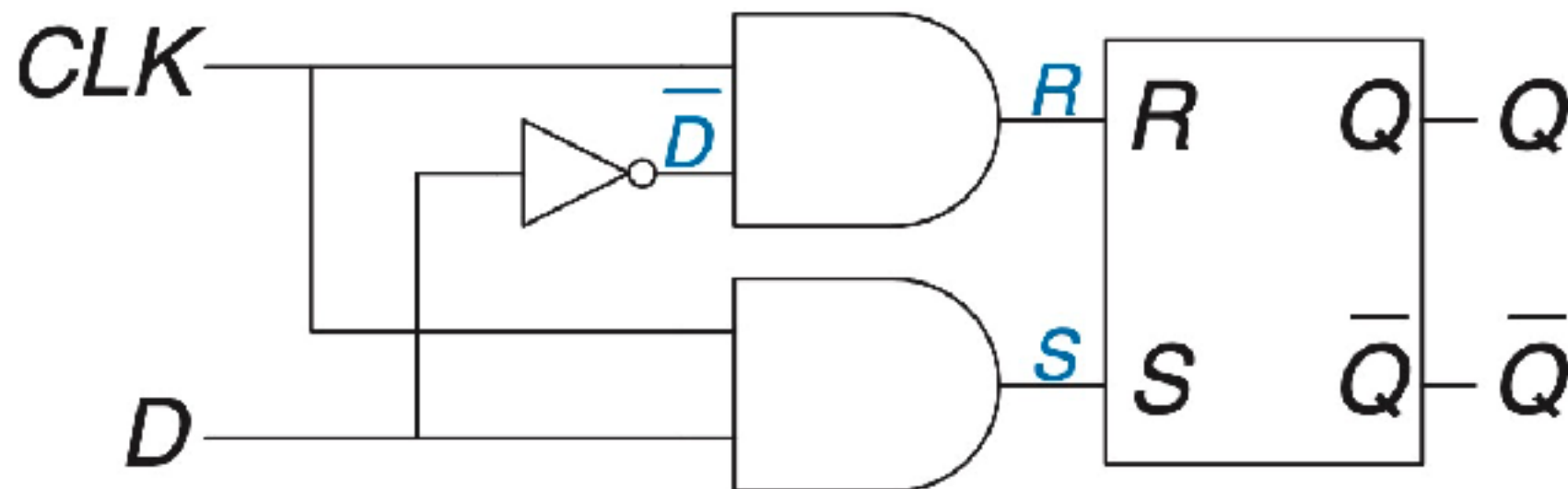
- ❖ Truth table

E	D	Q	$\sim Q$
1	0		
0	1		
0	1		
0	0		



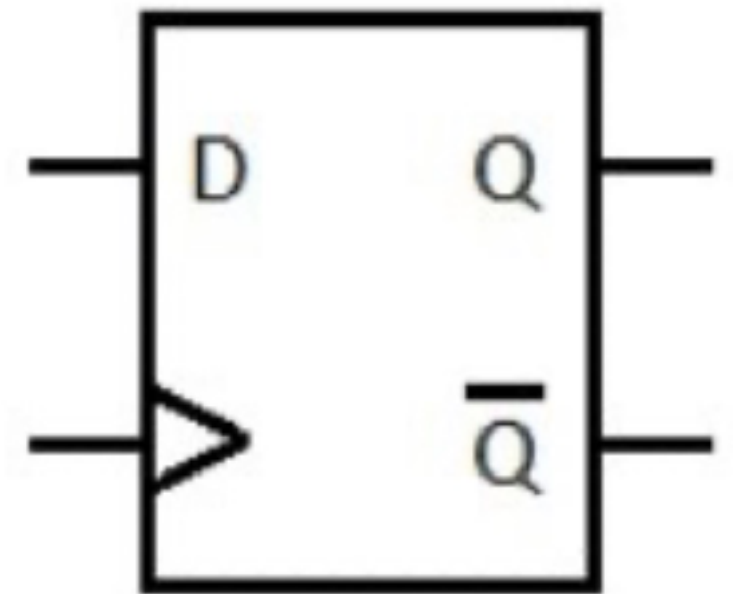
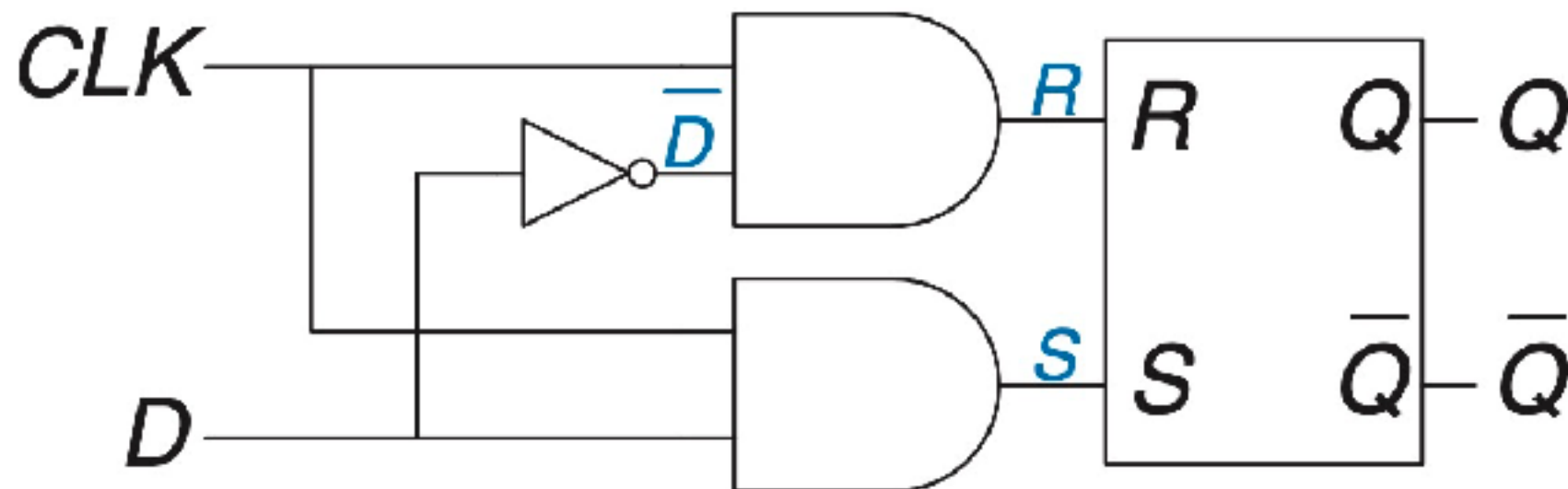
Sequential Logic (2): D Latch

- ❖ Replacing Enable with **CLK** (clock)



Sequential Logic (2): D Latch

- ❖ Replacing Enable with **CLK** (clock)



CLK	D	\overline{D}	S	R	Q	\overline{Q}
0	X	\overline{X}	0	0	Q_{prev}	\overline{Q}_{prev}
1	0	1	0	1	0	1
1	1	0	1	0	1	0

Sequential Logic (3): D Flip-flop

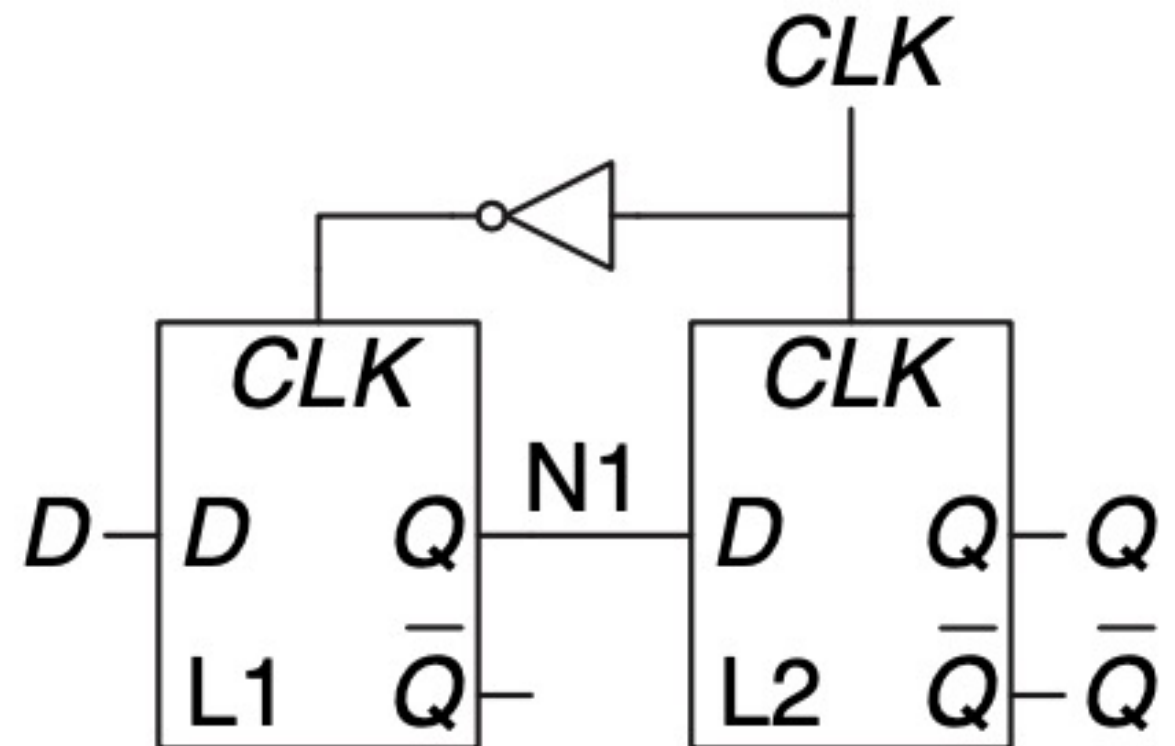
❖ Sequential circuit is edge-sensitive!

❖ Truth table

❖ CLK = 0, first transparent

❖ CLK = 1, second transparent

CLK	D	N1	Q	$\sim Q$
0				
1				
0				
1				



A D flip-flop copies D to Q on the rising edge of the clock, and remembers its state at all other times

Timing Diagram / Behavior of Latch and Flip-flop

Ripple-Carry Adder

❖ RCA

Ripple-Carry Adder

- ❖ RCA

- ❖ The Cout of LSB is cascaded/used as the Cin of its LSB+1

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- ❖ Pros

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- ❖ Straightforward, easy topology

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- ❖ Cons

Ripple-Carry Adder

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- ❖ The Cout of LSB is cascaded / used as the Cin of its LSB+1

- ❖ Pros

- ❖ Straightforward, easy topology

- ❖ Cons

- ❖ Long-latency

Ripple-Carry Adder

- ❖ RCA

- ❖ The Cout of LSB is cascaded / used as the Cin of its LSB+1

- ❖ Pros

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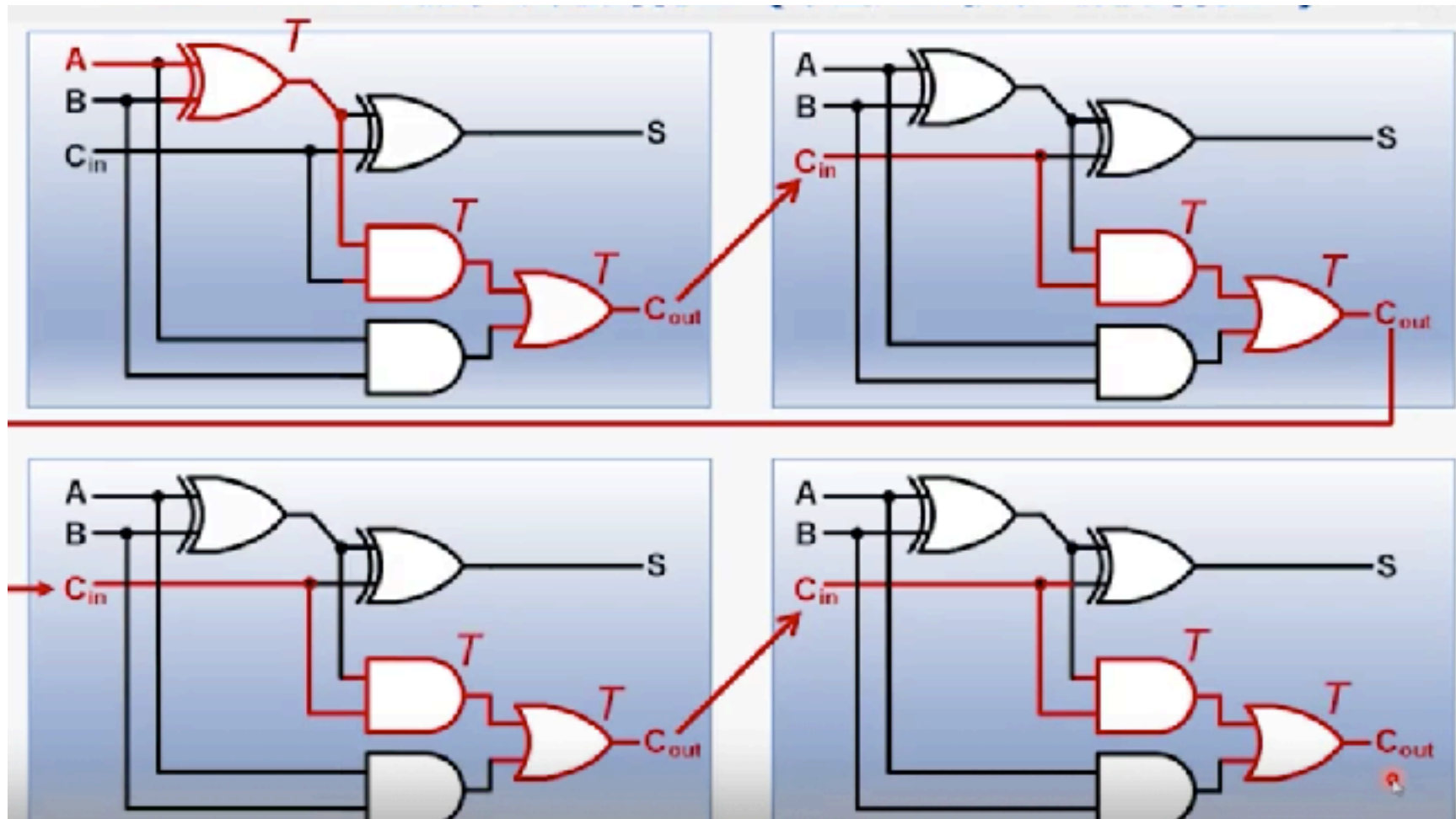
- ❖ Cons

- ❖ Long-latency

- ❖ MSB will have to wait for the results from LSB!

Delay Formulation

- ❖ Each gate has a delay of T (ideal)
- ❖ How much delay in total?
 - ❖ *hand calculation*

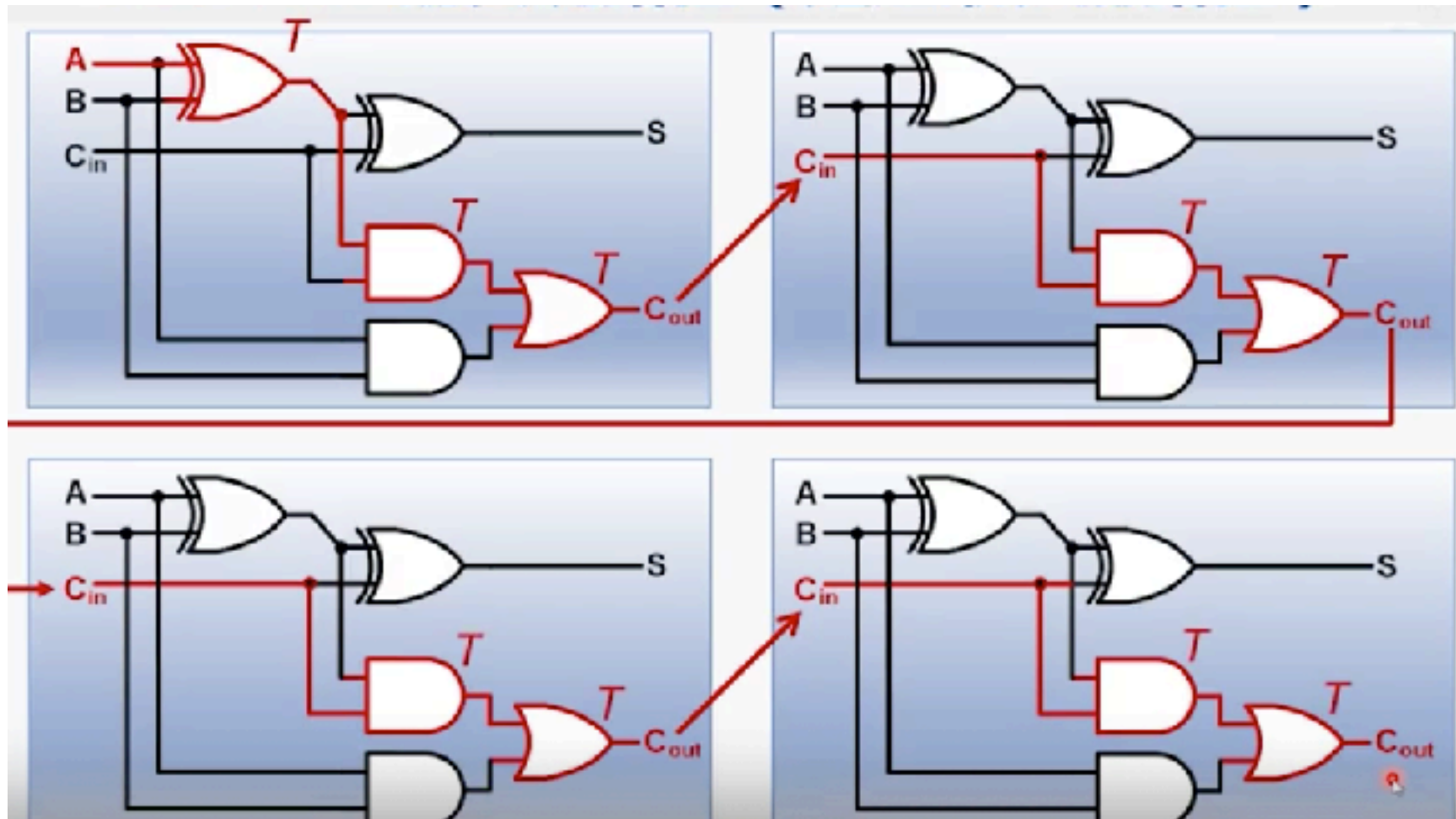


Delay Formulation

- ❖ Each gate has a delay of T (ideal)
- ❖ How much delay in total?

- ❖ *hand calculation*

- ❖ $2T*N+T$

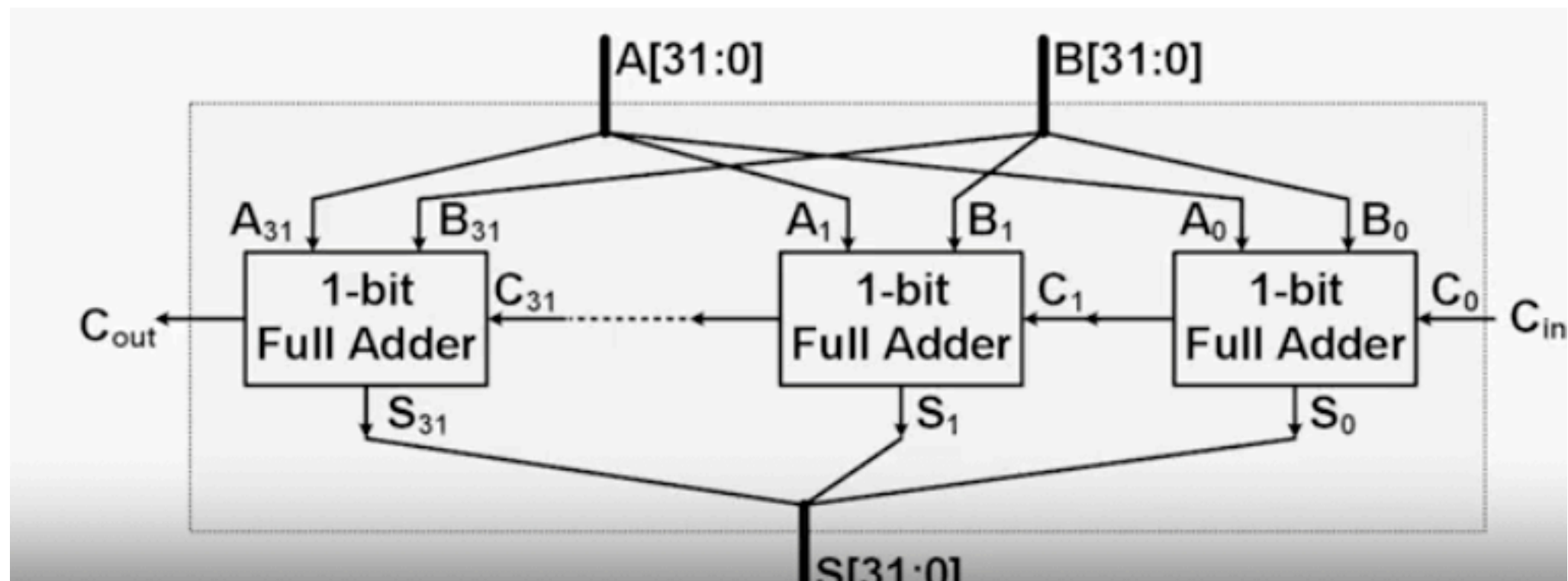


Delay Formulation

- ❖ How much delay for a 32-bit full adder?
- ❖ $(T+T)*N+T$ where $N=32 = 65T$

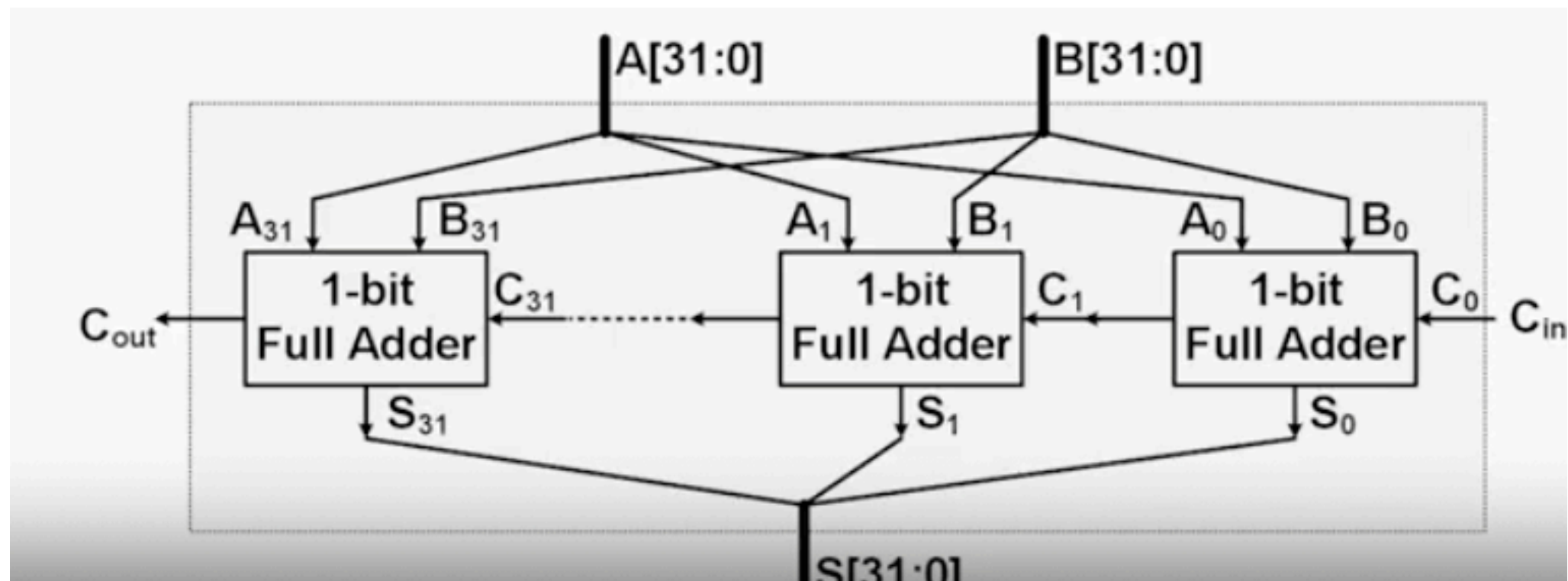
Delay Formulation

- ❖ How much delay for a 32-bit full adder?
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Delay Formulation

- ❖ How much delay for a 32-bit full adder?
- ❖ $(T+T)*N+T$ where $N=32 = 65T$
- ❖ Good? Bad? What this number means?



Carry-Lookahead Adder

- ❖ Proposed for addressing the low-performance of ripple-carry adder

$$\begin{aligned}C_{i+1} &= (A_i \cdot B_i) + (A_i \cdot C_i) + (B_i \cdot C_i) \\ &= (A_i \cdot B_i) + (A_i + B_i) \cdot C_i\end{aligned}$$

- ❖ How?

- ❖ The i -th adder will **generate** a carry-out, if both A and B are 1, *independent of carry_in!*
- ❖ The i -th adder will **propagate** a carry-out, if either A or B is 1, *and there is a of carry_in!*

Analysis of Cout

❖ Define two new parameters:

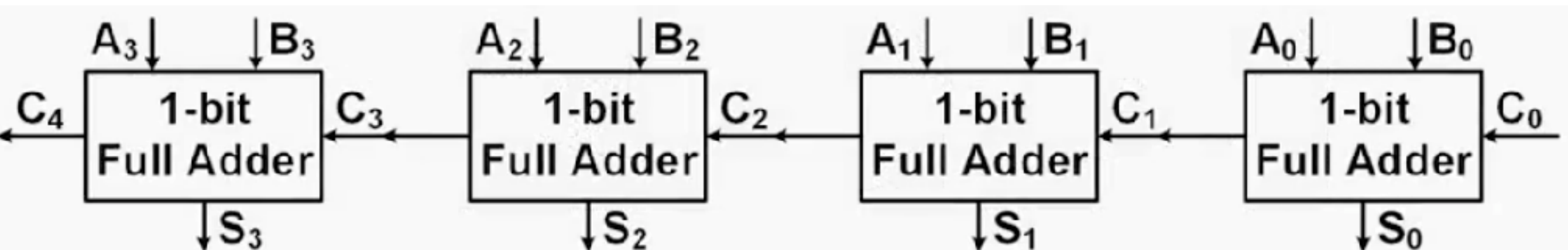
❖ Generate: $G_i = A_i \text{ AND } B_i$

❖ Propagate: $P_i = A_i + B_i$

$$C_{i+1} = G_i + P_i \cdot C_i$$

$$C_{i+1} = (A_i \cdot B_i) + (A_i \cdot C_i) + (B_i \cdot C_i)$$

$$= (A_i \cdot B_i) + (A_i + B_i) \cdot C_i$$



Analysis of Cout

❖ Define two new parameters:

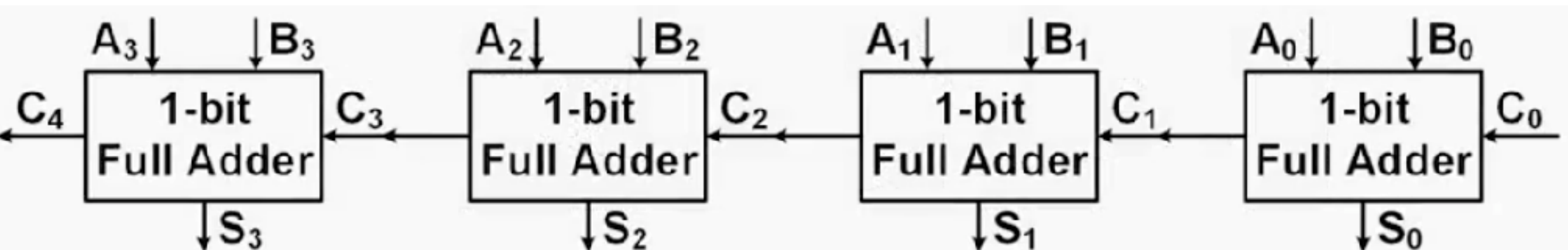
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$$C_{i+1} = (A_i \cdot B_i) + (A_i \cdot C_i) + (B_i \cdot C_i)$$

$$= (A_i \cdot B_i) + (A_i + B_i) \cdot C_i$$



Know the Values in Advance

$$C_{i+1} = G_i + P_i \cdot C_i$$

Know the Values in Advance

$$C_1 = G_0 + P_0 \cdot C_0$$

$$C_{i+1} = G_i + P_i \cdot C_i$$

Know the Values in Advance

$$C_1 = G_0 + P_0 \cdot C_0$$

$$C_2 = G_1 + P_1 \cdot C_1$$

$$C_{i+1} = G_i + P_i \cdot C_i$$

Know the Values in Advance

$$C_1 = G_0 + P_0 \cdot C_0$$

$$C_2 = G_1 + P_1 \cdot C_1$$

$$= G_1 + P_1 \cdot (G_0 + P_0 \cdot C_0)$$

$$C_{i+1} = G_i + P_i \cdot C_i$$

Know the Values in Advance

$$C_1 = G_0 + P_0 \cdot C_0$$

$$C_2 = G_1 + P_1 \cdot C_1$$

$$= G_1 + P_1 \cdot (G_0 + P_0 \cdot C_0)$$

$$= G_1 + P_1 \cdot G_0 + P_1 \cdot P_0 \cdot C_0$$

$$C_{i+1} = G_i + P_i \cdot C_i$$

Know the Values in Advance

$$C_1 = G_0 + P_0 \cdot C_0$$

$$C_2 = G_1 + P_1 \cdot C_1$$

$$= G_1 + P_1 \cdot (G_0 + P_0 \cdot C_0)$$

$$= G_1 + P_1 \cdot G_0 + P_1 \cdot P_0 \cdot C_0$$

$$C_3 = G_2 + P_2 \cdot C_2$$

$$= G_2 + P_2 \cdot (G_1 + P_1 \cdot G_0 + P_1 \cdot P_0 \cdot C_0)$$

$$= G_2 + P_2 \cdot G_1 + P_2 \cdot P_1 \cdot G_0 + P_2 \cdot P_1 \cdot P_0 \cdot C_0$$

$$C_{i+1} = G_i + P_i \cdot C_i$$

Know the Values in Advance

$$C_1 = G_0 + P_0 \cdot C_0$$

$$C_2 = G_1 + P_1 \cdot C_1$$

$$= G_1 + P_1 \cdot (G_0 + P_0 \cdot C_0)$$

$$= G_1 + P_1 \cdot G_0 + P_1 \cdot P_0 \cdot C_0$$

$$C_3 = G_2 + P_2 \cdot C_2$$

$$= G_2 + P_2 \cdot (G_1 + P_1 \cdot G_0 + P_1 \cdot P_0 \cdot C_0)$$

$$= G_2 + P_2 \cdot G_1 + P_2 \cdot P_1 \cdot G_0 + P_2 \cdot P_1 \cdot P_0 \cdot C_0$$

$$C_4 = G_3 + P_3 \cdot C_3$$

$$= G_3 + P_3 \cdot (G_2 + P_2 \cdot G_1 + P_2 \cdot P_1 \cdot G_0 + P_2 \cdot P_1 \cdot P_0 \cdot C_0)$$

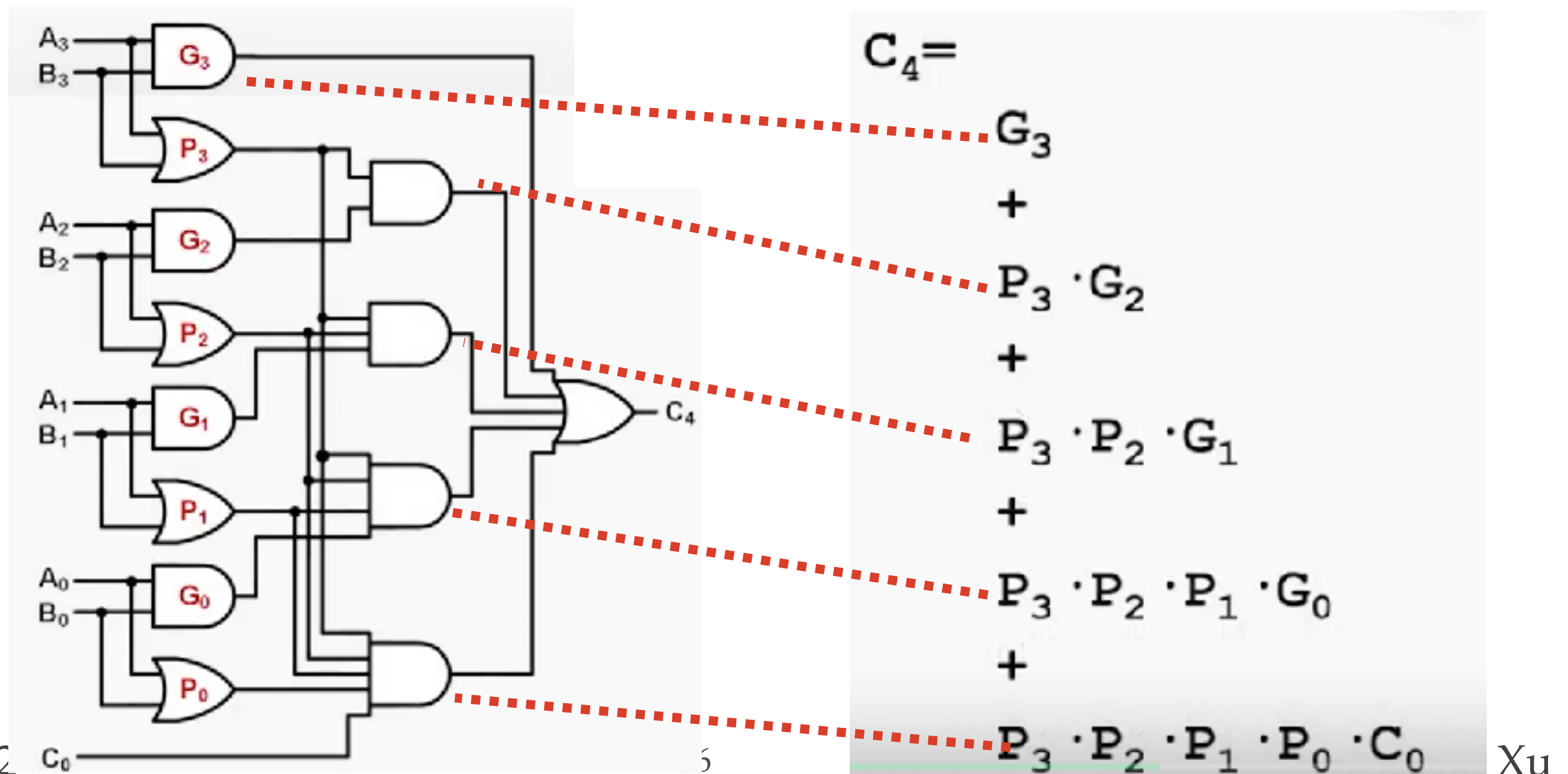
$$= G_3 + P_3 \cdot G_2 + P_3 \cdot P_2 \cdot G_1 + P_3 \cdot P_2 \cdot P_1 \cdot G_0 + P_3 \cdot P_2 \cdot P_1 \cdot P_0 \cdot C_0$$

$$C_{i+1} = G_i + P_i \cdot C_i$$

Detailed Analysis of C4

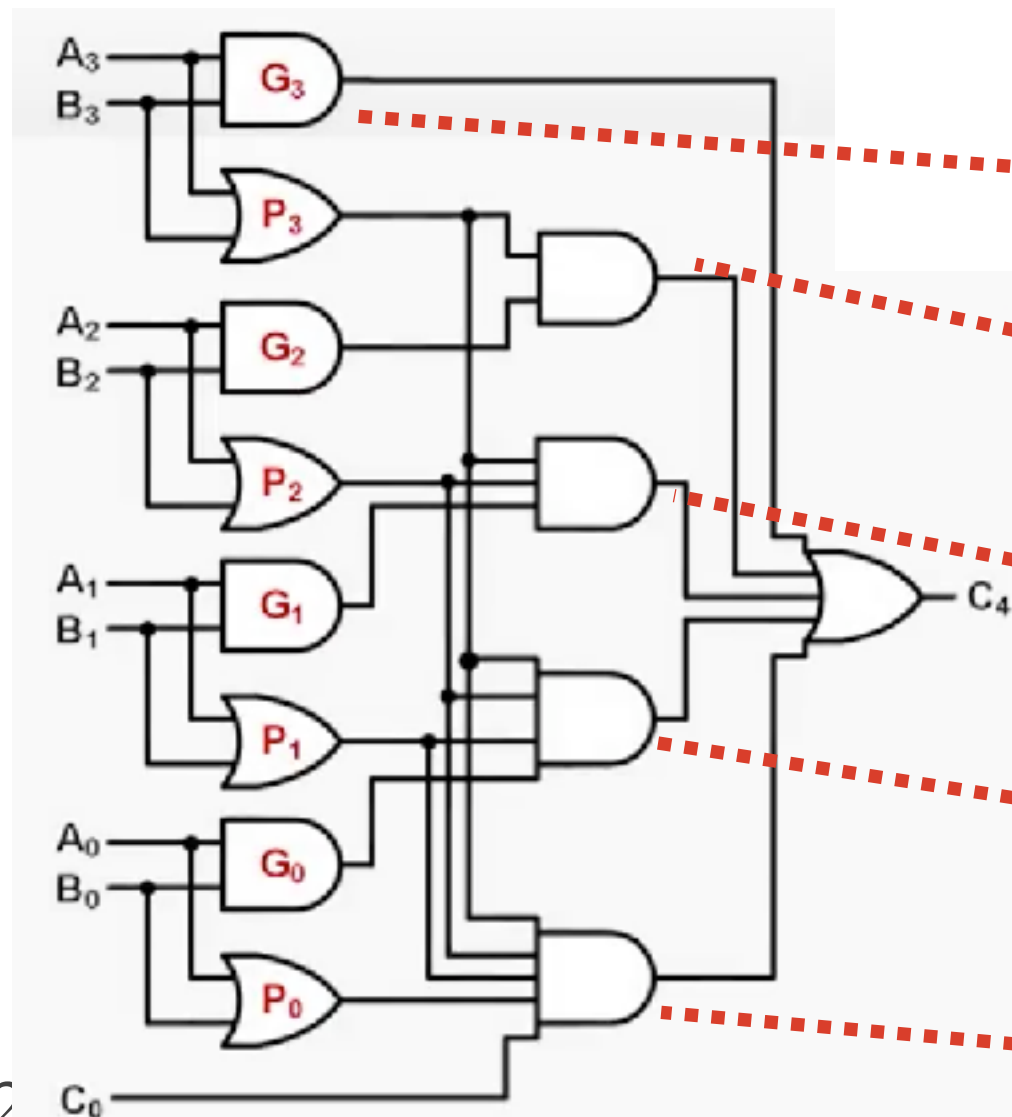
❖ Advantages?

- ❖ The latency of C_{i+1} is constant! Not dependent on N anymore



Detailed Analysis of C4

❖ Any disadvantages?

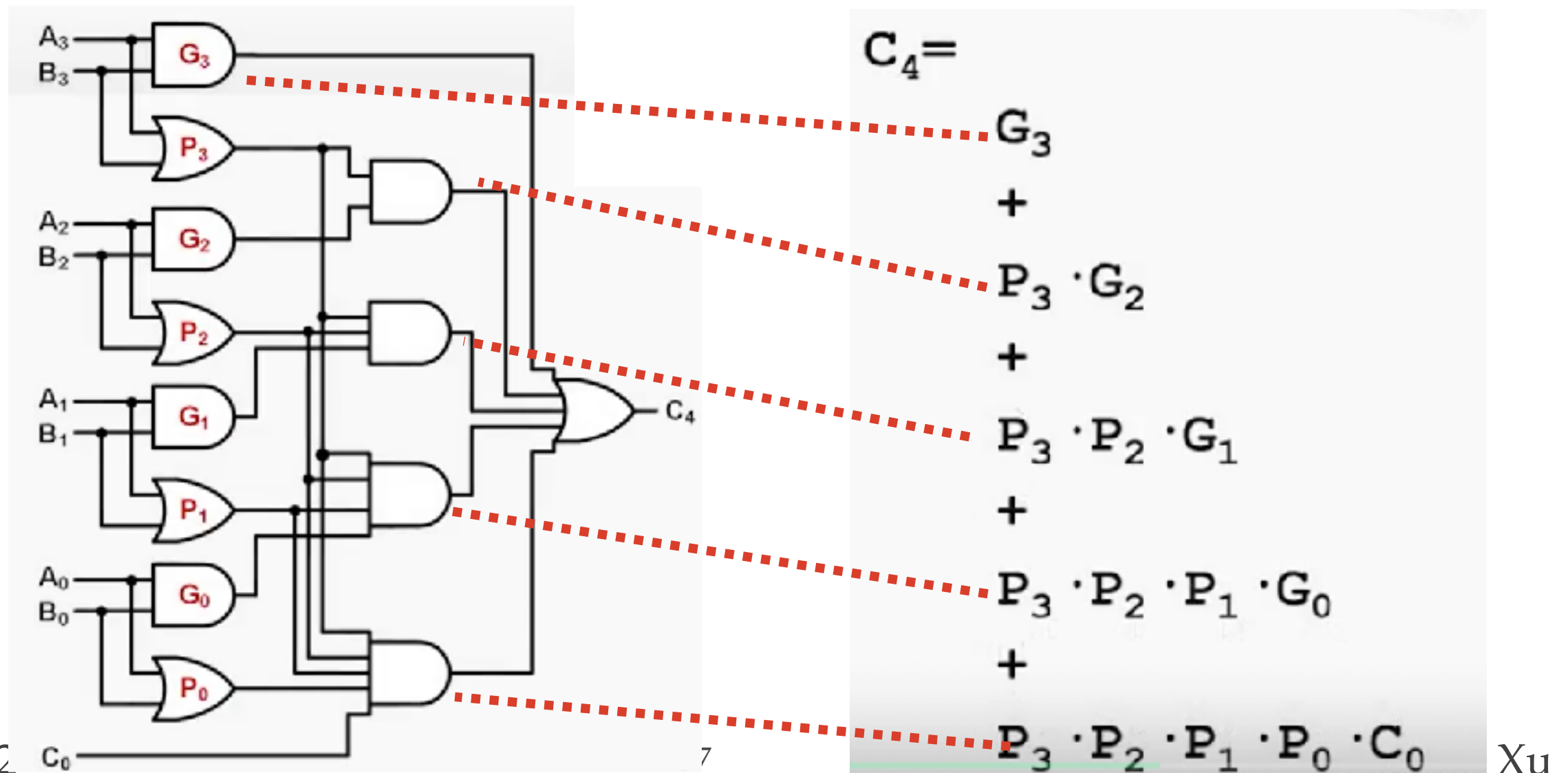


$C_4 =$

$$\begin{aligned} C_4 = & G_3 \\ & + P_3 \cdot G_2 \\ & + P_3 \cdot P_2 \cdot G_1 \\ & + P_3 \cdot P_2 \cdot P_1 \cdot G_0 \\ & + P_3 \cdot P_2 \cdot P_1 \cdot P_0 \cdot C_0 \end{aligned}$$

Detailed Analysis of C4

- ❖ Any disadvantages?
 - ❖ With the adder becomes wider, the circuit will be very complicated!



Further Optimization: Block of Adders

- ❖ Applying the concepts of “generate” and “propagate” to multiple-bit blocks / adders
 - ❖ A block *generates* a carry_out independent of the carry_in
 - ❖ A block *propagates* a carry_out if there is a carry_in
- ❖ Two new variables:
 - ❖ $G_{i:j}$ and $P_{i:j}$

$$G_{3:0} = G_3 + P_3(G_2 + P_2(G_1 + P_1G_0))$$

$$P_{3:0} = P_3P_2P_1P_0$$

Further Optimization: Block of Adders

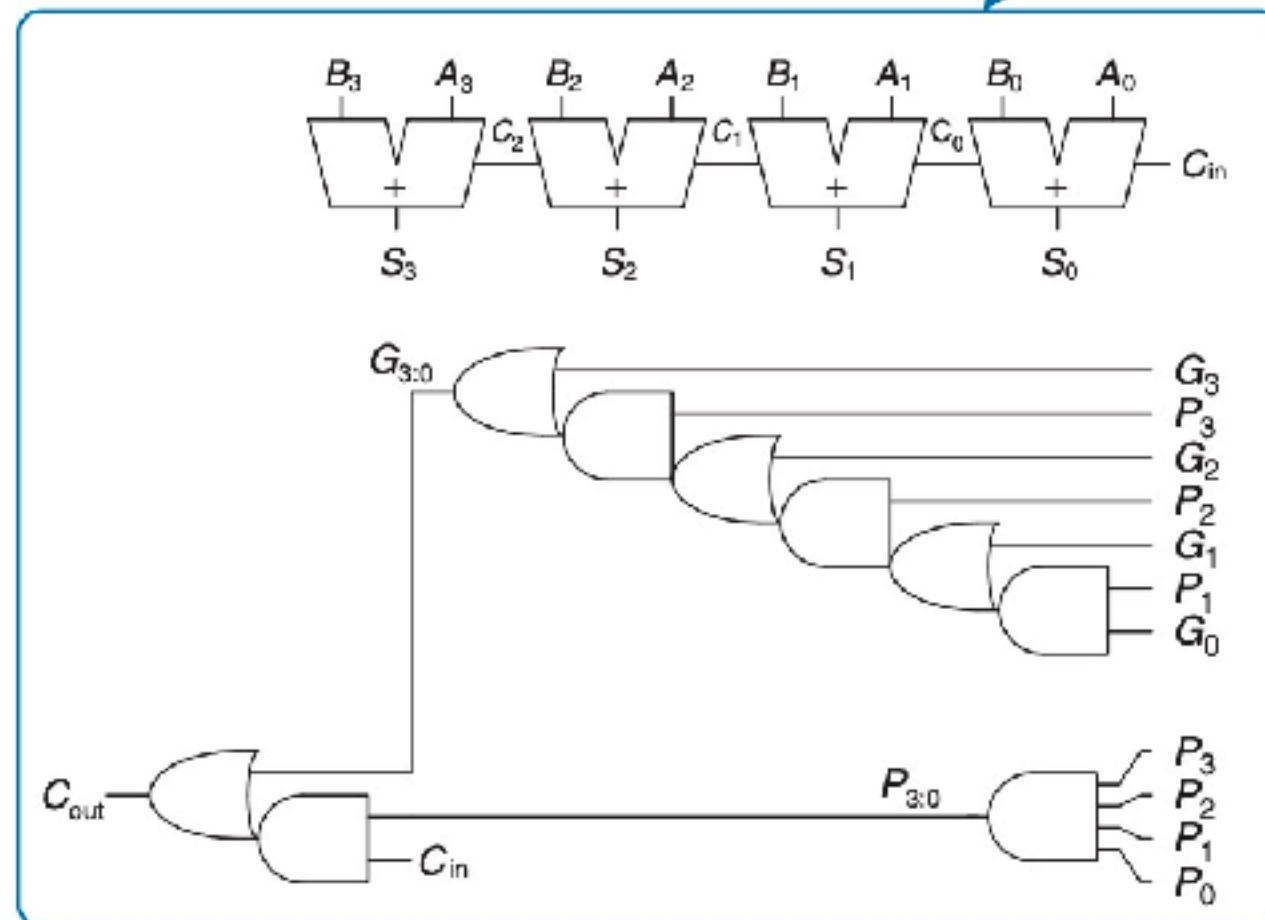
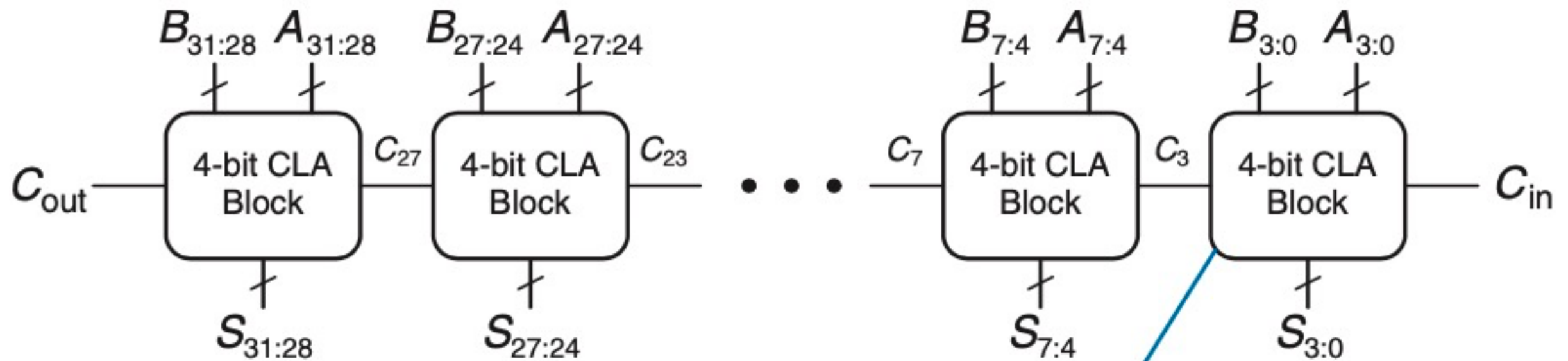
- ❖ Applying the concepts of “generate” and “propagate” to multiple-bit blocks / adders
 - ❖ A block *generates* a carry_out independent of the carry_in
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$$G_{3:0} = G_3 + P_3(G_2 + P_2(G_1 + P_1G_0))$$

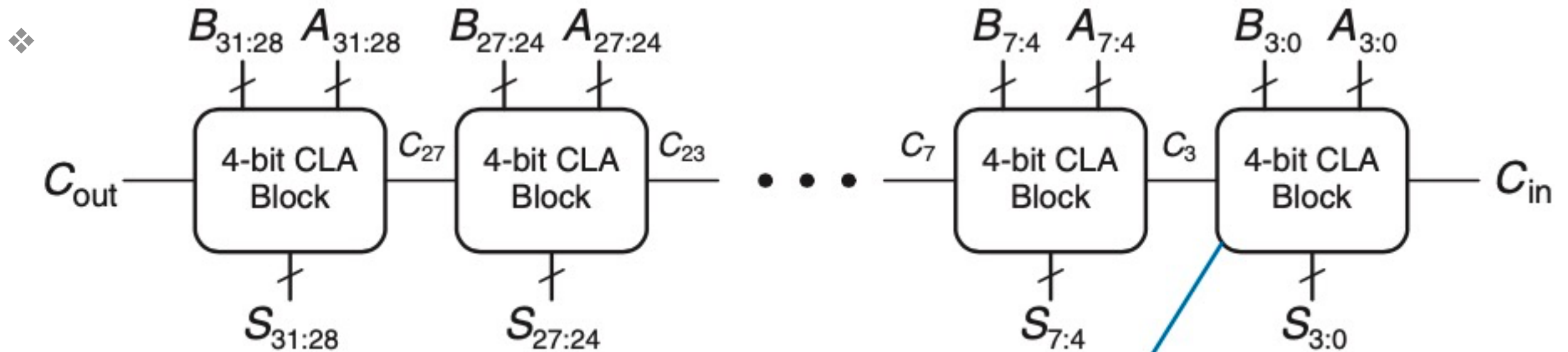
$$P_{3:0} = P_3P_2P_1P_0$$

$$C_i = G_{i:j} + P_{i:j}C_j$$

32-bit CLA Example

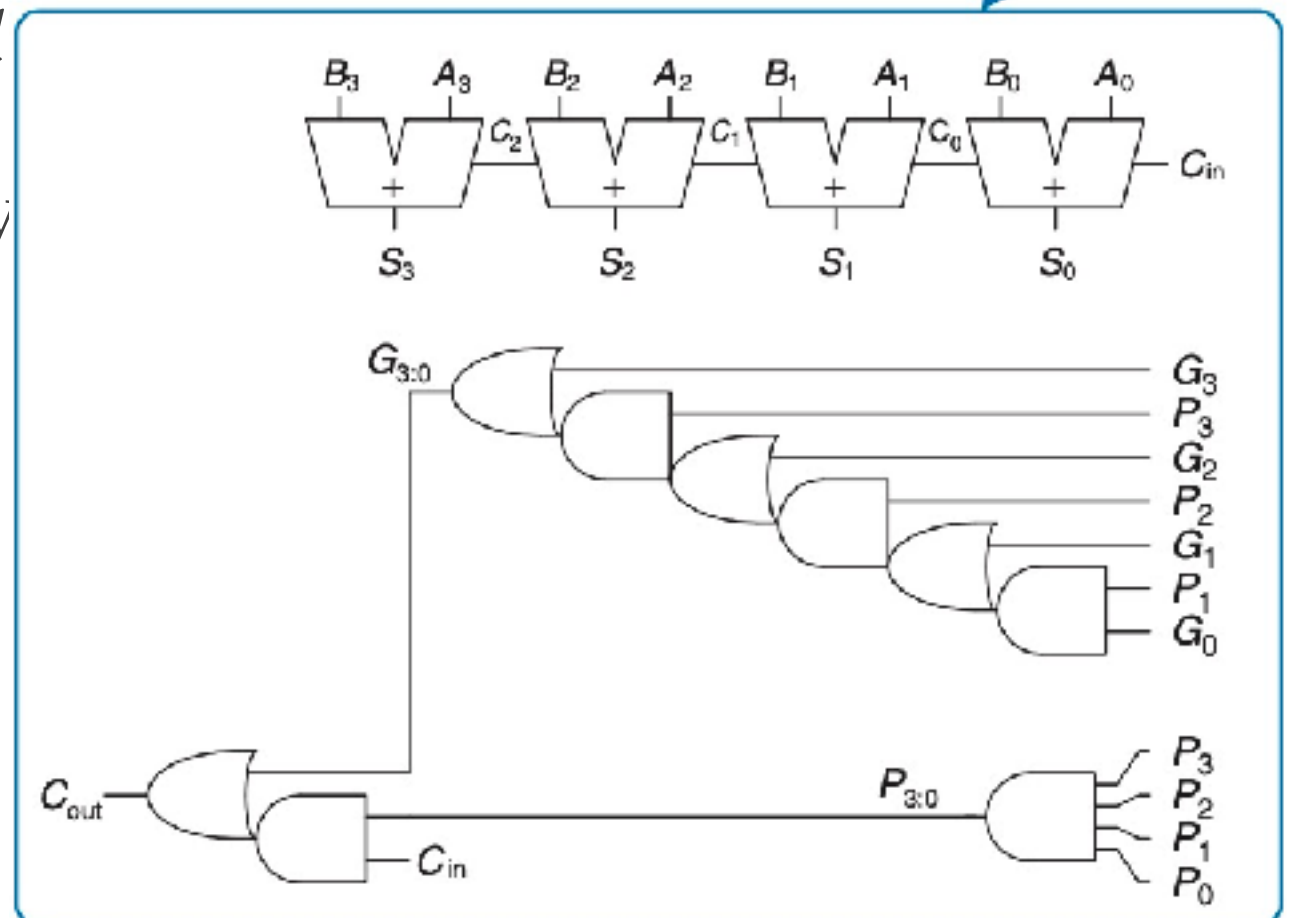


32-bit CLA Example

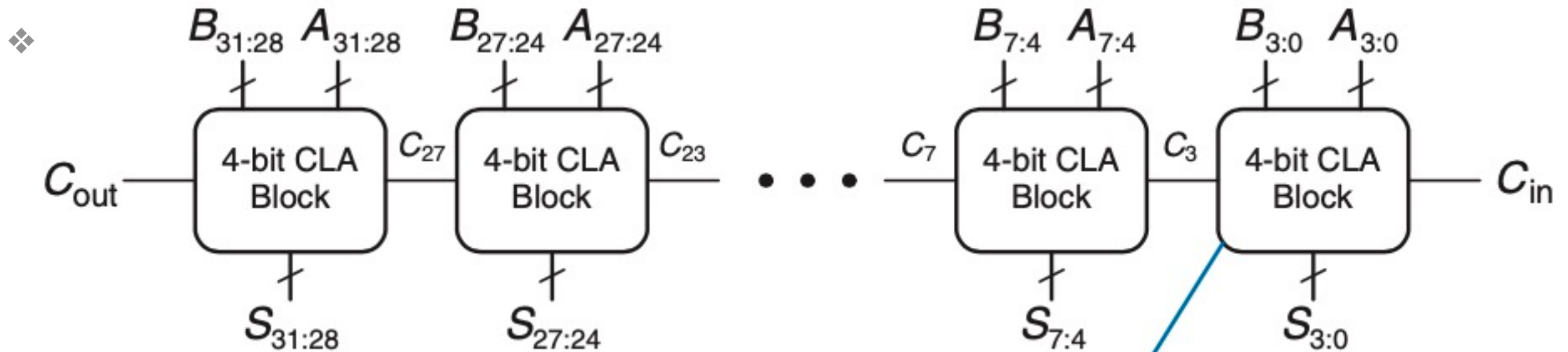


generate/propagate of a bit

❖ t_{AND_OR} : Propagation delay

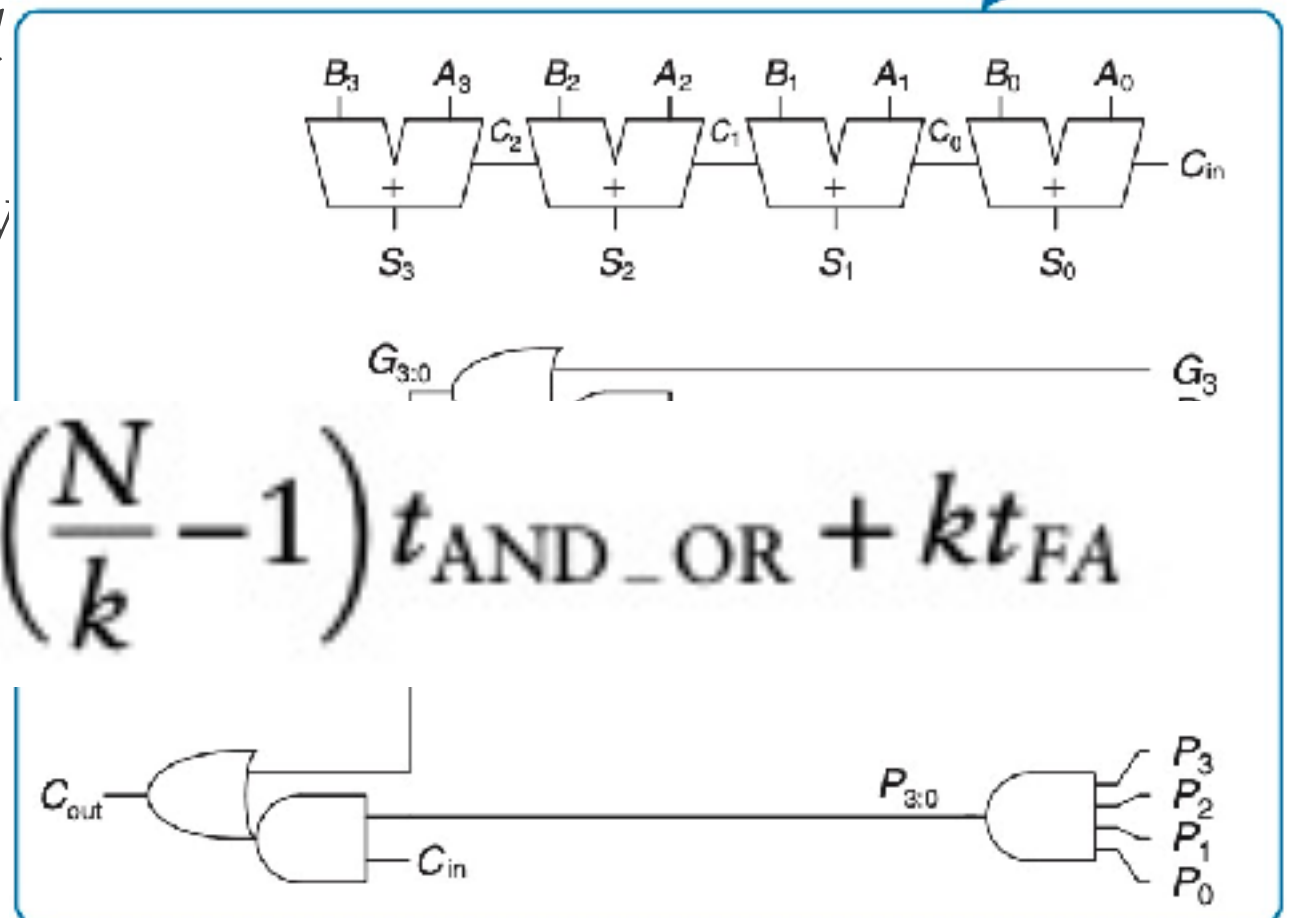


32-bit CLA Example



generate/propagate of a bit

❖ t_{AND_OR} : Propagation delay



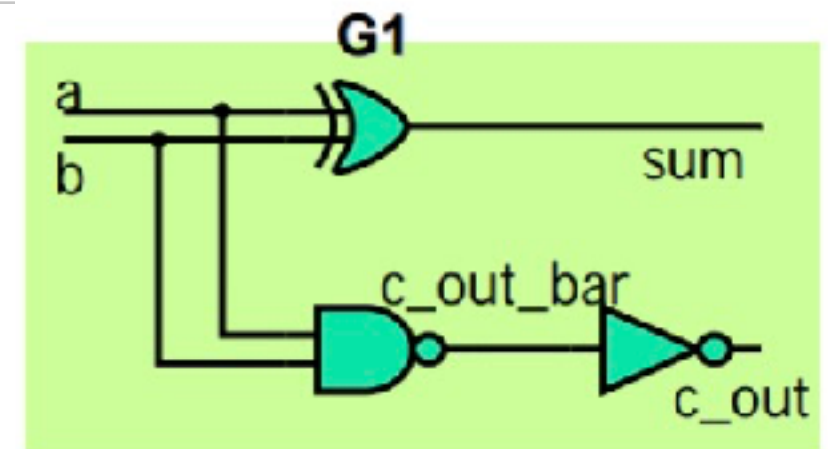
$$t_{CLA} = t_{pg} + t_{pg_block} + \left(\frac{N}{k} - 1\right) t_{AND_OR} + k t_{FA}$$

Mid-term Review

- ❖ Verilog
 - ❖ Write Verilog program for a given circuitry
 - ❖ Blocking and Non-blocking

Write the Code By Yourself

- ❖ A Multiplexer Using Basic Gates
 - ❖ Basic components?



Module name

Module ports

```
module Add_half ( input a, input b, output sum, output c_out );  
wire c_out_bar;  
  
xor (sum, a, b);  
// xor G1(sum, a, b); ← the label G1 is optional for logic gate primitives  
nand (c_out_bar, a, b);  
not (c_out, c_out_bar);  
  
endmodule  
//Verilog keywords in blue
```

Declaration of internal signals

Instantiation of primitive gates

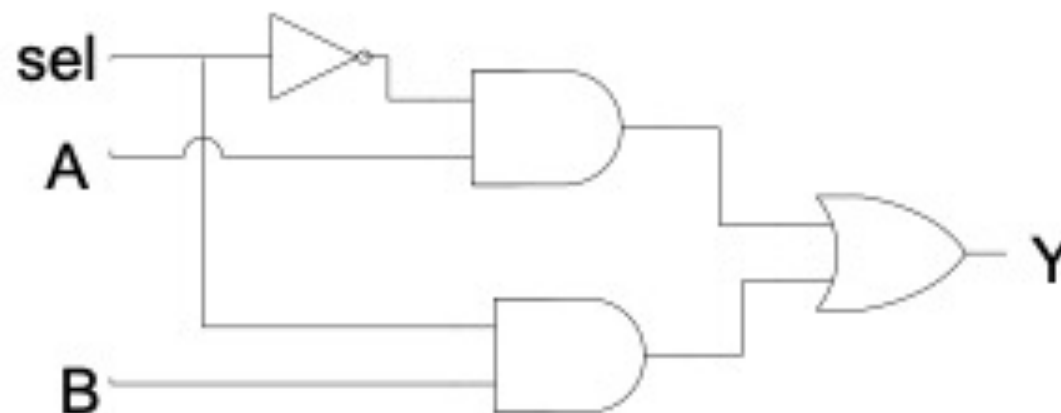
2-1 MUX

2-to-1 Multiplexer Behavior

when sel = 0 choose A to send to output Y
sel = 1 choose B

$$Y = \overline{\text{Sel}} \bullet A + \text{Sel} \bullet B$$

sel	Y
0	A
1	B



Design Code of 2-1 MUX

❖ Structural Model: 2-1 mux

```
❖ module mux2 (in0, in1, select, out);  
    input in0,in1,select;  
    output out;  
    wire s0,w0,w1;  
  
    not (s0, select);  
    and (w0, s0, in0),  
        (w1, select, in1);  
    or  (out, w0, w1);  
  
endmodule // mux2
```

Blocking and Non-blocking

❖ initial

❖ begin

❖ $B \leq A;$

❖ $C \leq B;$

❖ end

❖ Results?

❖ initial

❖ begin

❖ $B = A;$

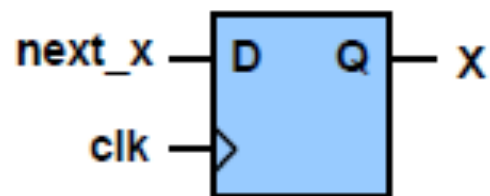
❖ $C = B;$

❖ end

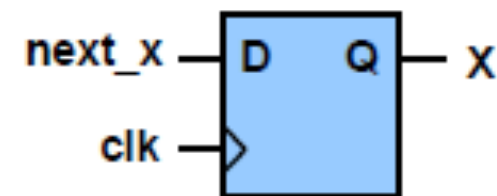
❖ Result?

Blocking and Nonblocking Statements

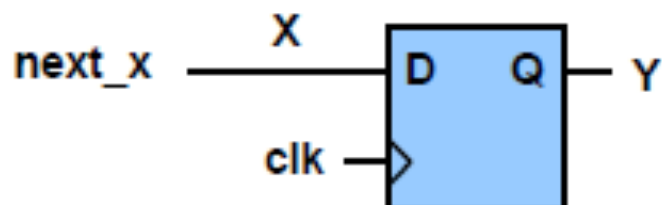
```
always @( posedge clk )  
begin  
    x = next_x;  
end
```



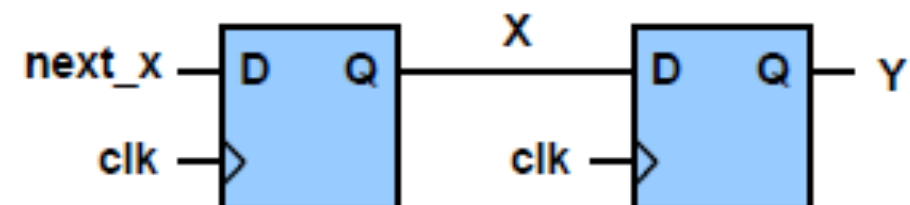
```
always @( posedge clk )  
begin  
    x <= next_x;  
end
```



```
always @( posedge clk )  
begin  
    x = next_x;  
    y = x;  
end
```

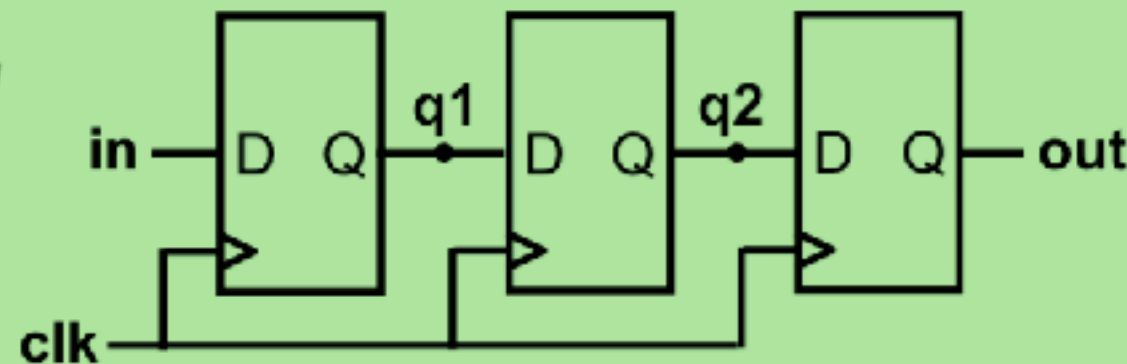


```
always @( posedge clk )  
begin  
    x <= next_x;  
    y <= x;  
end
```



Blocking and Nonblocking Statements

Flip-Flop Based Digital Delay Line



- Will nonblocking and blocking assignments both produce the desired result?

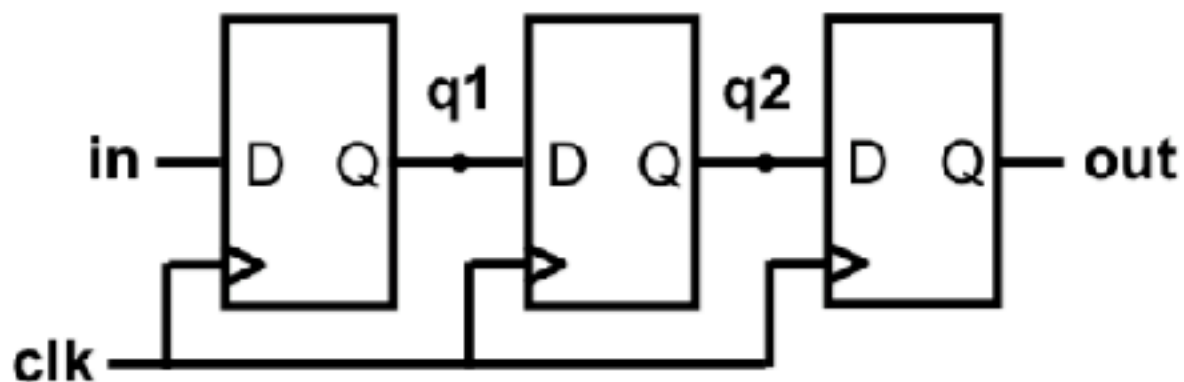
```
module nonblocking(in, clk, out);  
  input in, clk;  
  output out;  
  reg q1, q2, out;  
  always @ (posedge clk)  
  begin  
    q1 <= in;  
    q2 <= q1;  
    out <= q2;  
  end  
endmodule
```

```
module blocking(in, clk, out);  
  input in, clk;  
  output out;  
  reg q1, q2, out;  
  always @ (posedge clk)  
  begin  
    q1 = in;  
    q2 = q1;  
    out = q2;  
  end  
endmodule
```


Blocking and Nonblocking Statements

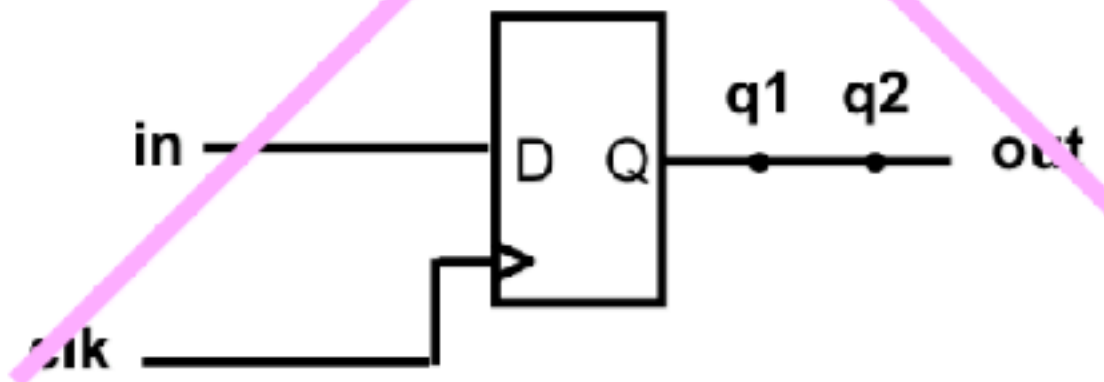
```
always @ (posedge clk)
begin
    q1 <= in;
    q2 <= q1;
    out <= q2;
end
```

“At each rising clock edge, $q1$, $q2$, and out **simultaneously receive the old values** of in , $q1$, and $q2$.”



```
always @ (posedge clk)
begin
    q1 = in;
    q2 = q1;
    out = q2;
end
```

“At each rising clock edge, $q1 = in$.
After that, $q2 = q1 = in$; **After that**,
 $out = q2 = q1 = in$; **Finally** $out = in$.”



Blocking and Non-blocking

❖ initial

❖ begin

❖ $B \leq A;$

❖ $C \leq B;$

❖ end

❖ Results?

❖ initial

❖ begin

❖ $B = A;$

❖ $C = B;$

❖ end

❖ Result?