Computational Data Analysis

Machine Learning

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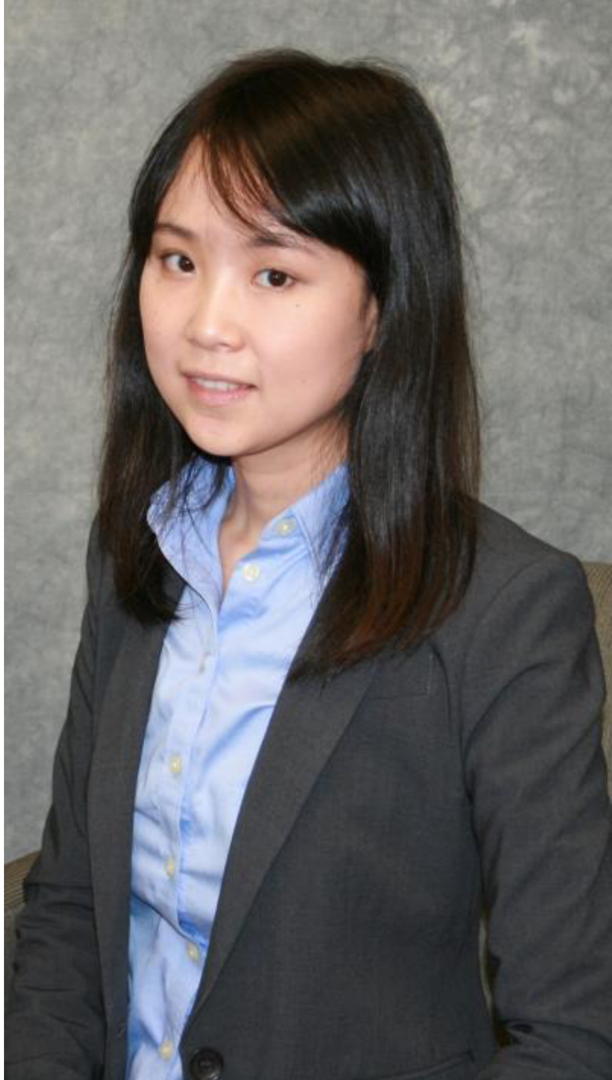


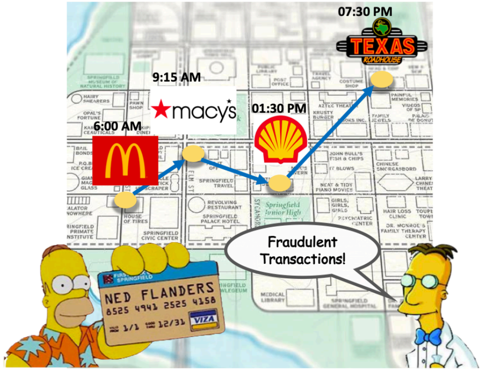
Anomaly detection

Motivation

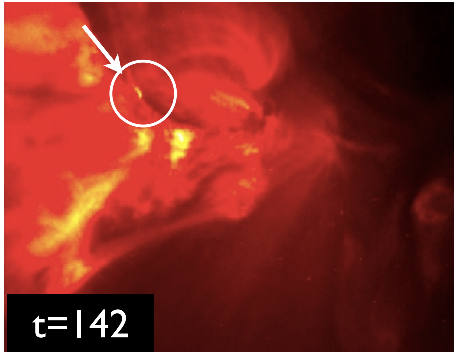
Power network monitoring

Northeast blackout, 2003

Credit card fraud detection

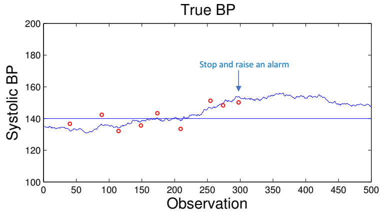


Solar flare detection

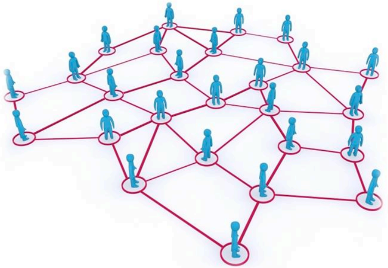


Blood pressure monitoring

Omron Project Zero 2.0.

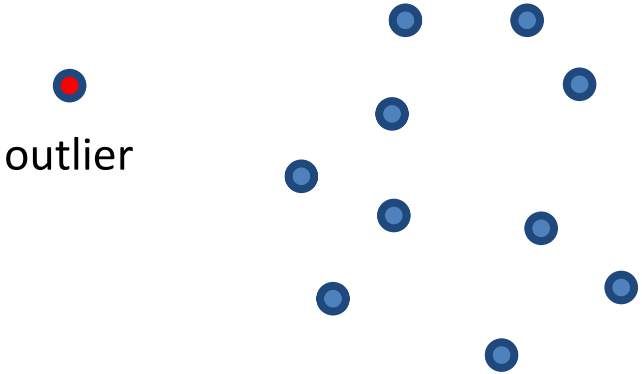


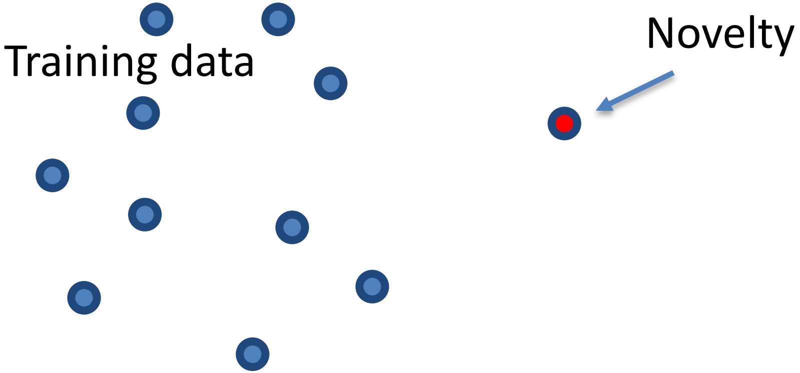
Event detection from social networks



Anomaly detection

- Outlier detection: identify outliers in training data, which are defined as observations that are "far" from the others in a chosen metric. --- As pre-processing step to clean data.
- Novelty detection: Detecting whether a **new** observation is an outlier. Related to change point detection (e.g., detecting change-points in time series).



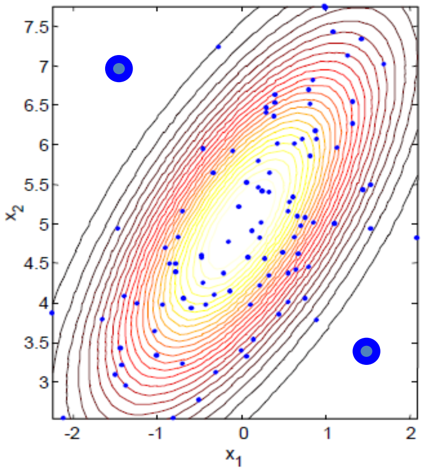


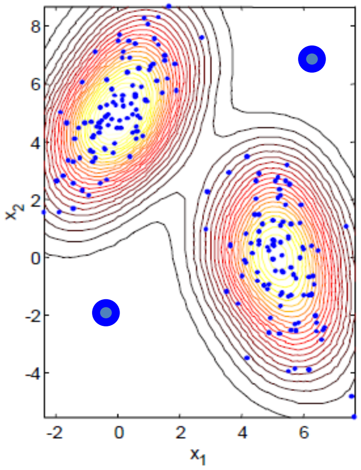
Anomaly detection

- Statistical methods: likelihood based
- Geometric methods: one-class SVM
- Some theory

Outlier detection: density-based

- Declare data points in low density region as outliers
  - First fit density to normal data $\widehat{f_0}(x)$, and then set a threshold ϵ (e.g. using kernel density estimation (KDE)
  - Data points in region with density small than ϵ, i.e. $\widehat{f_0}(x) \leq \epsilon$





Covariance based detector

- Multivariate Gaussian

$$f_0(x) = \frac{1}{(2\pi)^{n/2}|\Sigma|^{1/2}} e^{-\frac{1}{2}(x-\mu)^T\Sigma^{-1}(x-\mu)}$$

- Find parameter estimation from data

$$\hat{\mu} = \frac{1}{m}\sum_{i=1}^{m} x^i, \hat{\Sigma} = \frac{1}{m}\sum_{i=1}^{m}(x^i - \hat{\mu})(x^i - \hat{\mu})^T$$

- For a new data, having $\widehat{f_0}(x_{new}) \leq \epsilon$ is equivalent to having

$$(x_{new} - \hat{\mu})^T \hat{\Sigma}^{-1}(x_{new} - \hat{\mu}) \geq b$$

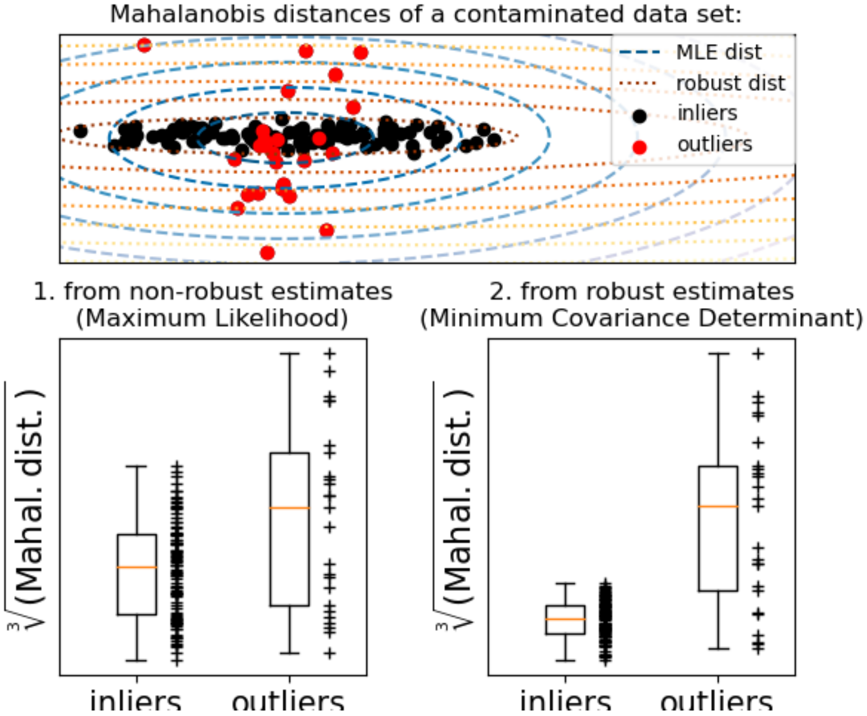
Mahalanobis distance

for some threshold b

Robust covariance estimation

- Mahalanobis distance can be affected by outliers
- Minimum Covariance Determinant estimator (MCD):

find a subset of observations whose empirical covariance has the smallest determinant $|\hat{\Sigma}|$



Anomaly detection

- Statistical methods: likelihood based
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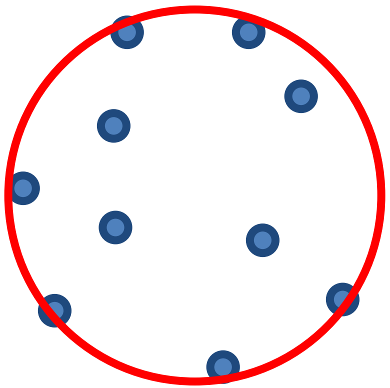
Geometric method for anomaly detector

Rather than first fit density and then find threshold, directly find the boundary



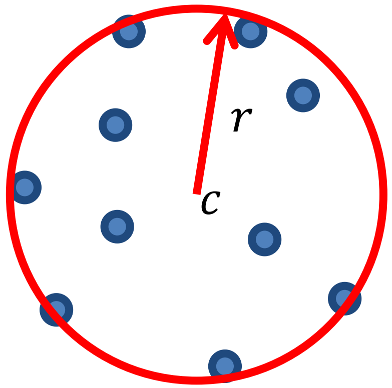
SVM based one-class classification (OCC)

Find the smallest ball such that it include all training data points.

Parameterized the ball with a center c and radius r

Optimization problem

$$\min_{r,c} \quad r^2$$
$$s.t. ||x^i - c||^2 \leq r^2,$$
$$r \geq 0$$
$$\forall i = 1, \ldots, m$$



Estimating the support of a high-dimensional distribution Schölkopf, Bernhard, et al. Neural computation 13.7 (2001): 1443-1471.

Solving one-class SVM

Lagrangian function

$$L(r, c, \alpha) \;=\; r^2 + \sum_{i=1}^{m} \alpha_i \left(\left(x^i - c\right)^\top \left(x^i - c\right) - r^2 \right), \alpha_i \geq 0$$

Set partial derivative of L to 0

$$\frac{\partial L}{\partial r} = 2r - 2\sum_{i=1}^{m} \alpha_i \, r = 0 \Rightarrow \sum_{i=1}^{m} \alpha_i \;= 1$$

$$\frac{\partial L}{\partial c} = \sum_{i=1}^{m} \alpha_i(-2x^i + 2c) = 0 \Rightarrow c = \sum_{i=1}^{m} \alpha_i x^i$$

Expressing Lagrangian function L in α_i

$$\max_{\alpha} g(\alpha) := L(r^*, c^*, \alpha) = \sum_{i=1}^{m} \alpha_i {x^i}^\top x^i - \sum_{i=1}^{m} \sum_{j=1}^{m} \alpha_i \alpha_j {x^i}^\top x^j$$

$$s.t. \ \sum_{i=1}^{m} \alpha_i = 1, \alpha_i \geq 0$$

A quadratic programming problem $g(\alpha) = b^\top \alpha - \alpha^\top A \alpha$

Can be solved efficiently

Optimal radius can be

found based on c^*

$A_{ij} = {x^i}^\top x^j$ $\quad b_i = {x^i}^\top x^i$ $\quad \alpha$

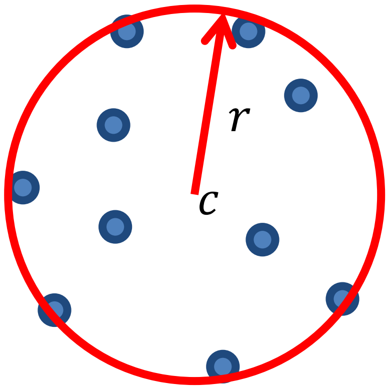
Sparsity in α_i

Complementary slackness (for all i):

$$\alpha_i\left(\left(x^i - c\right)^\top \left(x^i - c\right) - r^2\right) = 0, \quad \alpha_i \geq 0$$

Interesting observation:

- Many data points will be inside the circle
  - $\left(x^i - c\right)^\top \left(x^i - c\right) - r^2 < 0$
  - $\alpha_i = 0$
- Very few data point will be exactly on the boundary
  - $\left(x^i - c\right)^\top \left(x^i - c\right) - r^2 = 0$
  - α_i can be nonzero



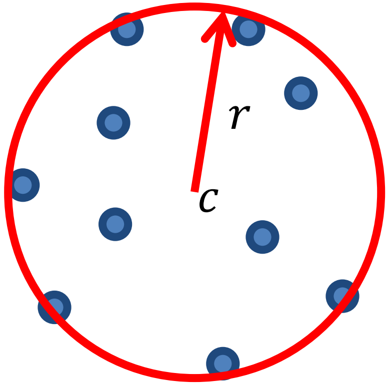
Solution in α is very sparse!

Support vectors

- Each α_i is associated with a one data point

- Data points with nonzero α_i are called support vectors. They determine the center and radius

$$c^* = \sum_{i=1}^{m} \alpha_i x^i$$
$$r = \min_{i:\alpha_i > 0} ||x^i - c^*||$$



- Data points with zero α_i is not affect c and r

Kernalize the algorithm

Dual problem

$$\max_{a} g(a) = b^\top \alpha - \alpha^\top A \alpha$$

$$s.t. \sum_{i=1}^{m} \alpha_i = 1, \alpha_i \geq 0$$

$A_{ij} = {x^i}^\top x^j$

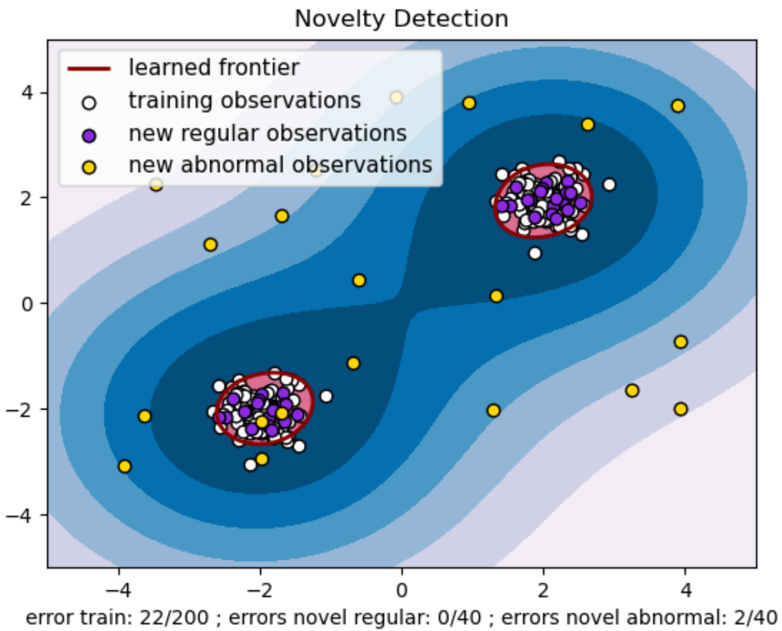
$b_i = {x^i}^\top x^i$

α

$k(x^i, x^j)$

$k(x^i, x^i)$

Demo: Novelty detection

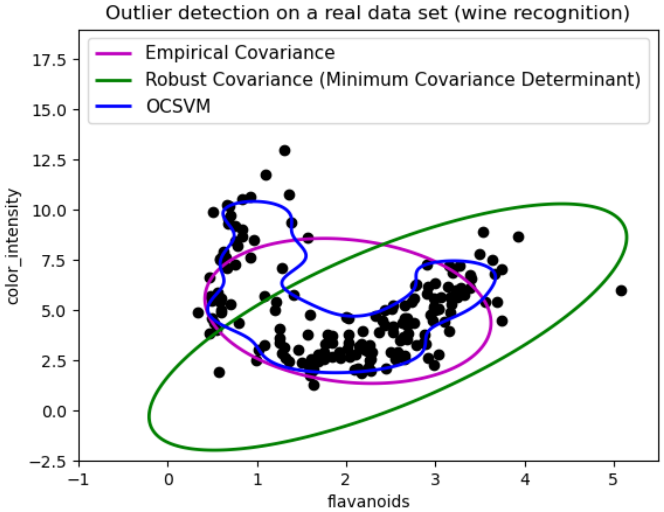


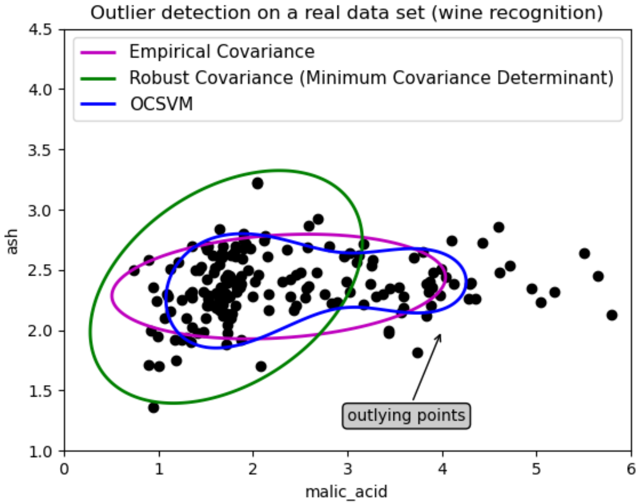
https://scikit-learn.org/stable/auto_examples/svm/plot_oneclass.html#sphx-glr-auto-examples-svm-plot-oneclass-py

Demo: Wine data

- The wine data set was introduced by Forina et al. (1986).
- It originally included the results of 27 chemical measurements on 178 wines made in the same region in Italy but derived from three different cultivars: Barolo, Grignolino and Barbera.

https://scikit-learn.org/stable/auto_examples/applications/plot_outlier_detection_wine.html#sphx-glr-auto-examples-applications-plot-outlier-detection-wine-py





Anomaly detection

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Hypothesis test

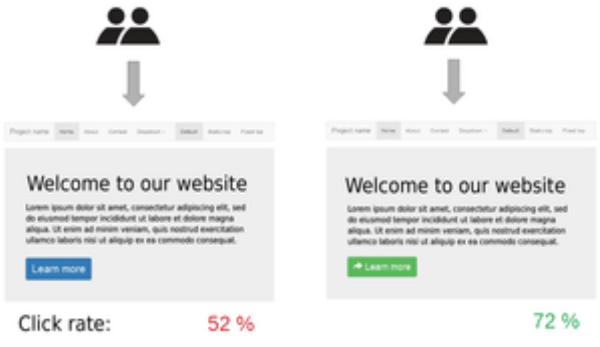
- Common hypothesis test

$$H_0: x^i \sim f_0, \qquad i = 1, \dots, m$$
$$H_1: x^i \sim f_1, \qquad i = 1, \dots, m$$

- Anomaly detection

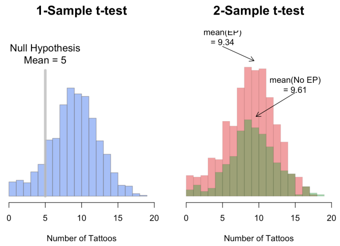
$$H_0: x^i \sim f_0, \qquad i = 1, \dots, m$$
$$H_1: x^i \nsim f_0, \qquad i = 1, \dots, m$$

- A/B testing



- Two-sample test

$$H_0: x^i \sim f_0, z^i \sim f_0, \qquad i = 1, \dots, m$$
$$H_0: x^i \sim f_0, z^i \sim f_1. \qquad i = 1, \dots, m$$

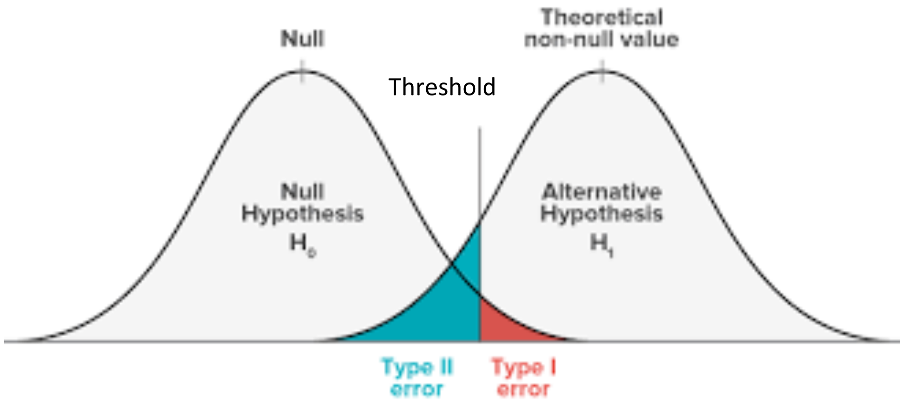


Performance metrics

Decision making: Choose H_0 or H_1

Performance metrics

- Type-I error = $P_{H_0}\{\text{reject } H\_0\}$
- Type-II error = $P_{H_1}\{\text{accept } H\_0\}$



Decision	H_0 Is True	H_0 Is False
Fail to reject H_0	no error	type II error
Reject H_0	type I error	no error

Likelihood ratio test

- The likelihood ratio test is optimal when f_0 and f_1 are known exactly (Neyman-Pearson lemma)

$$\text{Decide } H_1 \text{ when } \frac{f_1(x)}{f_0(x)} > b$$

- In practice, we can estimate the distributions
- Set the threshold b to control false alarms (Type-I error)

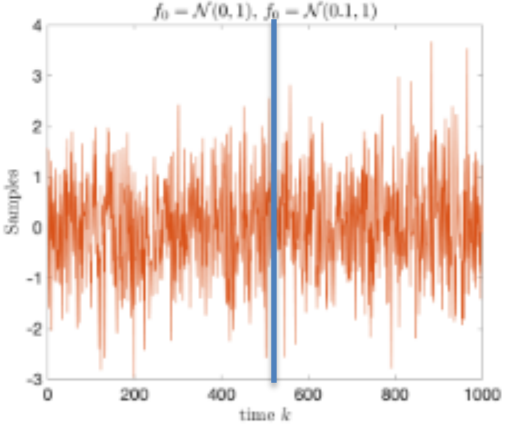
Novelty detection for sequential data

Assume the distribution before the change: $f_0(x)$, after the change: $f_1(x)$

CUSUM procedure (cumulative sum over log-likelihood ratios)

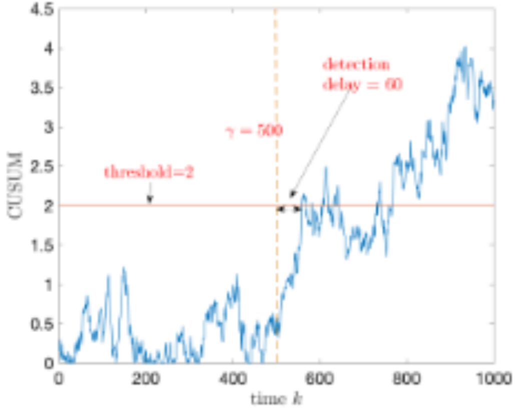
$$W_0 = 0, \qquad W_t = \max\left(W_{t-1} + \log\frac{f_1(X_t)}{f_0(X_t)}, 0\right)$$

Sequential data



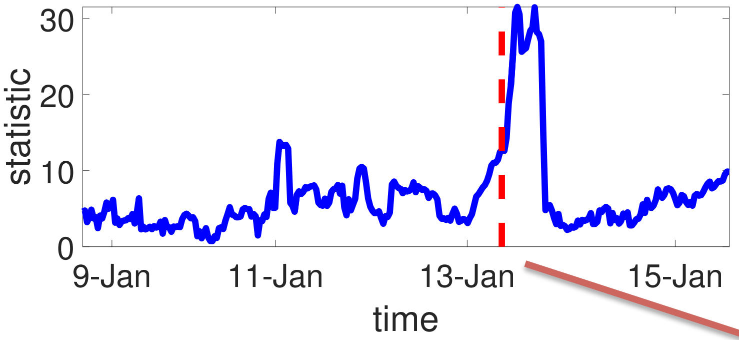
Change-point

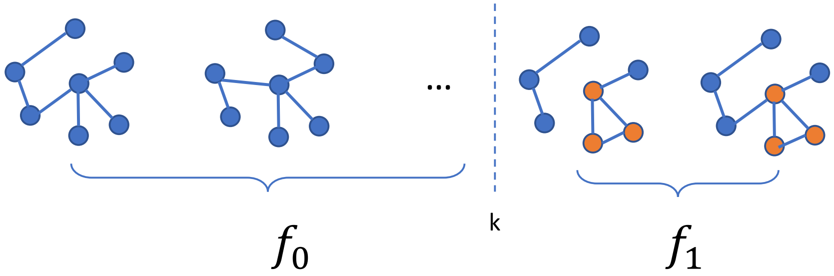
CUSUM statistics

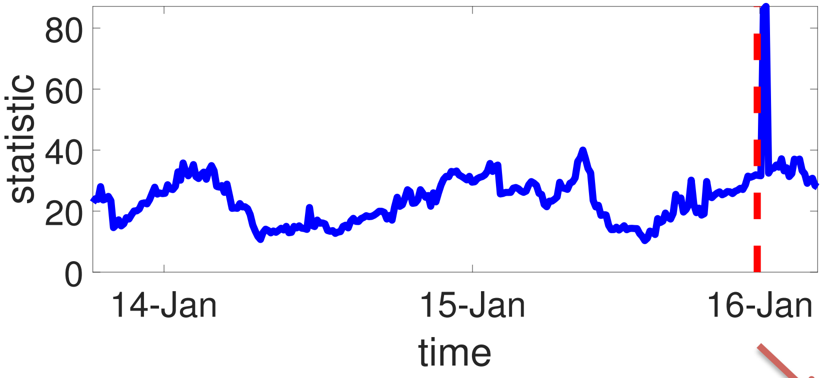


Example: Social network change detection

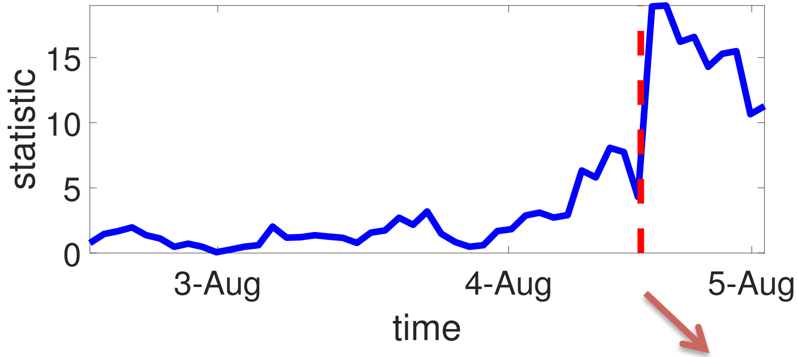
Detecting change in random graph, using twitter data







Israel announces ceasefire in Gaza War in 2009v



Beijing Olympic opening in 2008

Other anomaly detection algorithms

- Local outlier factor (LOF): distance metric based
- Neural network based: e.g., anomalous sequence detection using RNN

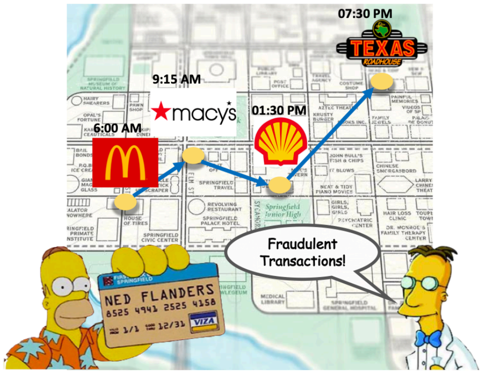
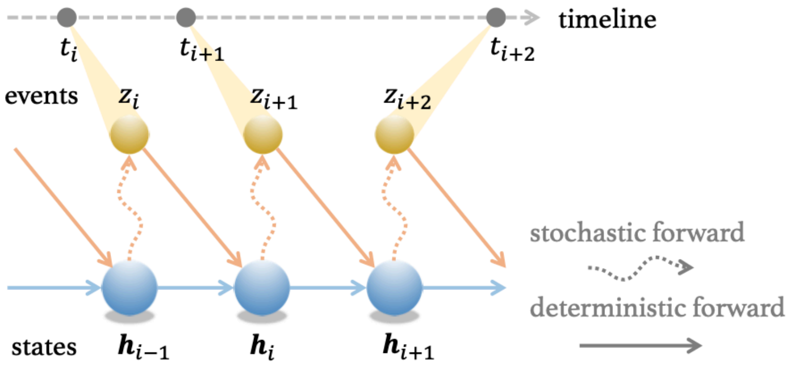


Fig. 1: An example of a sequence of anomalous events that are dependent: one leads to another.

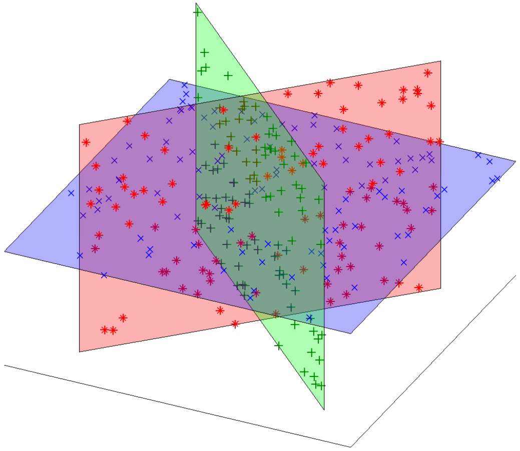


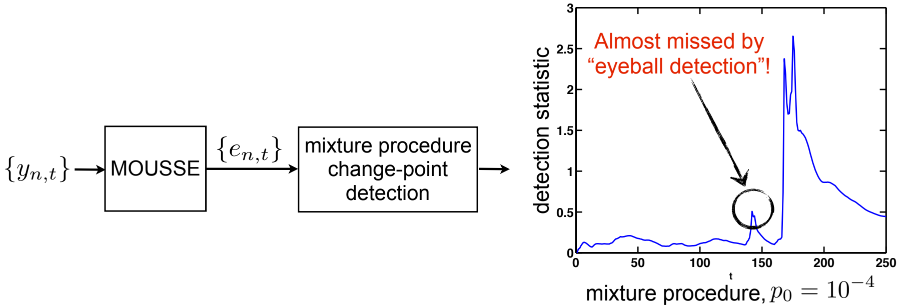
RNN-based anomalous seq. detection

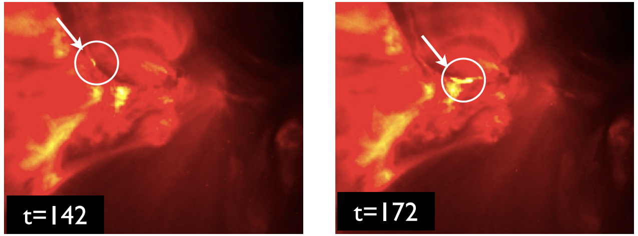
Other anomaly detection algorithms (cont.)

- Subspace based anomaly detection

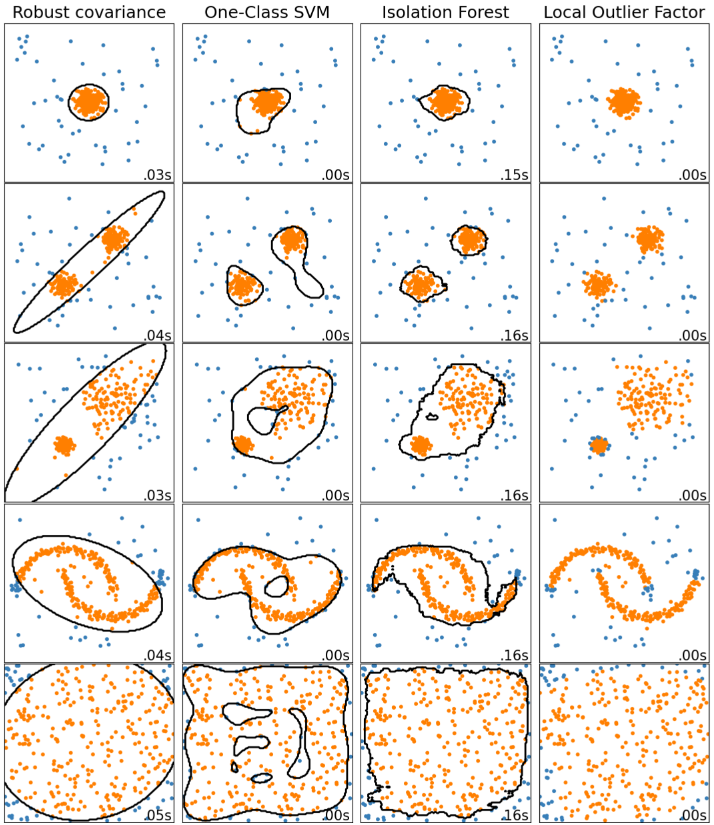
Represent normal data using subspace







Comparison of algorithms



https://scikit-learn.org/stable/auto_examples/miscellaneous/plot_anomaly_comparison.html#sphx-glr-auto-examples-miscellaneous-plot-anomaly-comparison-py


SUMMARY