Homework 1

ISyE 6420
Spring 2021

1. Circuit. A circuit consisting of seven independent elements $A_1, \ldots, A_7$ is connected as in Figure 1.

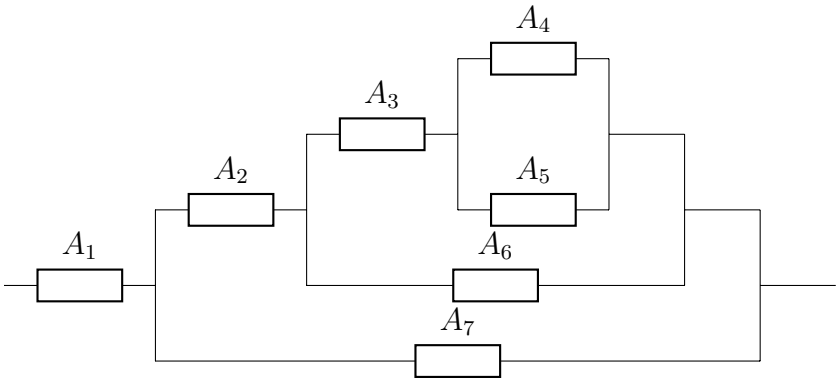

Figure 1: Circuit S with seven independent elements

The elements are operational during time interval T with probabilities

	A_1	A_2	A_3	A_4	A_5	A_6	A_7
Probability of working (p)	0.4	0.8	0.6	0.5	0.9	0.4	0.3

Find the probability that the circuit is operational during time interval T.

2. Classifier. In a machine learning classification procedure the items are classified as 1 or 0. Based on a training sample of size 120 in which there are 65 1's and 55 0's, the classifier predicts 70 1's and 50 0's. Out of 70 items predicted by the classifier as 1, 52 are correctly classified.

From the population of items where the proportion of 0-labels is 99% (and 1-labels 1%), an item is selected at random. What is the probability that the item is of label 1, if the classifier says it was.

Hint. Think about the following interpretation. If **1** is a specific disease present, **0** no disease present, and the classifier is a medical test for the disease, then you are asked to find a positive predictive value of a test for a subject coming from population where the prevalence of the disease is 1%.

3. System. A system operates under normal conditions with probability 0.9, and under stress with probability 0.1. Under normal conditions the system will fail with probability 0.05, and under stress with probability 0.4.

(a) What is the probability that the system will fail if no information about working conditions is available.

(b) If the system failed, what are updated probabilities of the two working conditions?