1 LECTURE OUTLINE

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1\ 00:00:30,310 \longrightarrow 00:00:35,300 okay let's get started let me welcome
   2\ 00:00:35,300 -> 00:00:38,690 you to this last lecture last time the
   3\ 00:00:38,690 \rightarrow 00:00:41,480 six lectures when approximate dynamic
   4\ 00:00:41,480 \longrightarrow 00:00:45,860 programming I have a feeling that my the
   5\ 00:00:45,860 -> 00:00:49,250 sound is not very good let me just make
   6 00:00:49,250 \rightarrow 00:00:56,239 an adjustment I'll put it over here and
   7.00:00:56,239 \rightarrow 00:00:57,670 see what happens
   8\ 00:00:57,670 -> 00:01:01,990 okay is that are you guys doing okay
   9\ 00:01:01,990 -> 00:01:03,790\ okay
   10\ 00:01:03,790 -> 00:01:06,170 okay I think this a little better at
   11\ 00:01:06,170 -> 00:01:08,710 least it sounds my ears a little better
   开场白,没啥用,就不翻译了
   12\ 00:01:08,710 -> 00:01:11,330 so this is the last lecture lecture six
   13\ 00:01:11,330 \rightarrow 00:01:14,450 out of in an approximate dynamic programming
   这是近似动态规划的第六次课, 也是最后一次课
   14\ 00:01:14,450 -> 00:01:18,440 and we have discussed quite
   15\ 00:01:18,440 \to 00:01:22,970 a few topics so far and mostly having to
   16\ 00:01:22,970 \longrightarrow 00:01:26,330 do with policy duration cost function approximations
   我们讨论了几个问题, 主要包括策略迭代和成本函数近似
   17 00:01:26,330 -> 00:01:29,479 today we're going to look
   18\ 00:01:29,479 -> 00:01:32,380 at Q factors and model free
   19\ 00:01:32,380 \rightarrow 00:01:34,310 implementations of approximate dynamic
   20\ 00:01:34,310 -> 00:01:37,369 programming algorithms
   今天我要讲 Q 函数和近似动态规划算法的无模型求解
   21\ 00:01:37,369 \rightarrow 00:01:40,280 so we are going to review Q factors in the corresponding
bellman equations
   我要对 bellman 方程相关的 Q 函数进行一下综述
   22\ 00:01:40,280 -> 00:01:43,910 then talk about value
   23 00:01:43,910 -> 00:01:47,390 and policy duration for Q factors
   然后讨论 Q 函数的值迭代和策略迭代
   24~00{:}01{:}47{,}390~->~00{:}01{:}50{,}899 and then discuss Q learning it's an
   25~00:01:50,899 \longrightarrow 00:01:53,450 important algorithm that can be viewed
   26\ 00:01:53,450 \longrightarrow 00:01:56,619 as a sampled version of value iteration
   然后是 Q 学习, 一种采样视角的值迭代
   27\ 00{:}01{:}56{,}619 -> 00{:}02{:}00{,}679 you can do Q learning for Q factors
   28\ 00:02:00,679 \longrightarrow 00:02:02,420 but there is no corresponding algorithms
   29\ 00:02:02,420 \rightarrow 00:02:04,340 are great for costs that's an interesting fact
   你可以使用 Q 值运行 Q 学习, 但是这俩不是一个东西, 这个事情很有意思
   30\ 00:02:04,340 \longrightarrow 00:02:07,039 then we are going to
   31 00:02:07,039 -> 00:02:09,949 talk about Q learning in combination
   32\ 00:02:09,949 -> 00:02:12,620 with cost function approximation then
   33\ 00:02:12,620 -> 00:02:15,110 say a few things about adaptive dynamic
   34\ 00:02:15,110 -> 00:02:16,459 programming a few things about
   35\ 00:02:16,459 -> 00:02:19,340 approximation policy space
   然后我们要讲一些与成本函数近似结合的 Q 学习,然后是自适应动态规划中近似策略空间的
内容
   36\ 00:02:19,340 -> 00:02:19,700 and then we are
   37\ 00:02:19,700 \longrightarrow 00:02:23,450 to have a more general discussion to close the course
   然后我们会讲一些比较通用的内容来结束这个课程
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2 REVIEW

2.1 DISCOUNTED MDP

38 00:02:23,450 -> 00:02:30,200 so let's do a little review 我要简单回顾一下

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39\ 00:02:30,200 -> 00:02:36,099 we continue to talk about a
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- $40\ 00:02:36,099 \rightarrow 00:02:40,519$ finite state system involving states 1
- $41\ 00:02:40,519 -> 00:02:43,459$ up to n a finite set of controls for
- $42\ 00:02:43,459 -> 00:02:46,549$ each state I the system is defined by
- $43\ 00:02:46,549 \rightarrow 00:02:48,680$ transition probabilities that depend on control

我要谈论的是一个状态从 1 到 n 的有限状态系统,每一个阶段的控制都是一个有限集合,系统的转移概率依赖于控制

- $44\ 00:02:48,680 -> 00:02:51,860$ and we consider policies which
- $45\ 00:02:51,860 -> 00:02:53,930$ are sequences of functions from state to
- $46\ 00:02:53,930 \rightarrow 00:02:59,120\ control\ nu\ 0\ mu\ 1\ and\ so\ on$
- 考虑的策略是一个状态的函数序列,从 μ_0 , μ_1 等等
- $47\ 00:02:59,120 \longrightarrow 00:03:03,349$ and given a policy starting at a state I we can
- $48\ 00:03:03,349 \rightarrow 00:03:05,840$ consider the sequence of generated costs
- $49\ 00:03:05.840 \rightarrow 00:03:09.560$ over time discounted by a factor of by a discount factor alpha
- 对于一个给定的起始状态 i 和策略, 我可以考虑根据时间生成一个序列成本, 这些成本被一个折扣因子 α 衰减
 - $50\ 00:03:09,560 -> 00:03:13,579$ and take the limit
 - $51\ 00:03:13,579 \rightarrow 00:03:17,090$ of the series as n goes to infinity this
 - $52\ 00:03:17,090 \rightarrow 00:03:19,069$ defines a number for every policy in every state
 - 对于每一个策略和每一个状态, 求 n 趋于无穷的时候的数值
 - $53\ 00:03:19,069 -> 00:03:21,769$ so J pi is a cost function
 - J_{π} 是一个成本函数
 - $54\ 00:03:21,769 -> 00:03:23,209$ we want to find the minimal cost
 - $55\ 00:03:23,209 \rightarrow 00:03:25,489$ function in a policy pi that attains the minimum
 - 我想要找到一个策略 π 下的最小成本函数
 - $56\ 00:03:25.489 -> 00:03:27.650$ we have seen the importance
 - $57\ 00:03:27,650 \rightarrow 00:03:30,109$ of stationary policies policies where
 - $58\ 00:03:30,109 \rightarrow 00:03:32,420$ all the mutates are the same equal to some mu
- 我们需要知道平稳策略的重要性,平稳策略就是每一个阶段都用同一个策略 μ ,也就是时间无关的策略
 - $59\ 00:03:32,420 -> 00:03:37,910$ and optimal optimal policies
 - $60\ 00:03:37,910 -> 00:03:39,560$ can be find among the stationary
 - $61\ 00:03:39,560 -> 00:03:41,480$ policies in those of the stationary
 - $62\ 00:03:41,480 \longrightarrow 00:03:43,430$ policies and in the various algorithms
 - 63 00:03:43,430 -> 00:03:46,930 such as policy duration and so on
 - 用各种各样的算法, 比如策略迭代什么的就可以在平稳策略中找到最优策略
 - $64\ 00:03:46,930 \longrightarrow 00:03:49,519$ let me remind you also of our standard
 - 65 00:03:49,519 $-\!>$ 00:03:52,639 notation which we use occasionally to
 - $66\ 00:03:52,639 \rightarrow 00:03:55,370$ denote the dynamic programming mapping
 - 67 00:03:55,370 -> 00:03:56,810 and the corresponding mapping
 - 68 00:03:56,810 -> 00:03:59,930 corresponding policies Maps j Park
 - $69\ 00:03:59,930 -> 00:04:02,120$ functions or n-dimensional vectors in
 - $70\ 00:04:02,120 -> 00:04:04,040$ this case into other n-dimensional
 - $71\ 00:04:04,040 -> 00:04:09,560\ vectors\ TJ$ or T mu use of J

我要提醒你我们用过的标准符号, 表示动态规划映射, 包括映射, 相应的策略, 函数 J 和 n 维向量, 在这个例子中, 另一个 n 维向量 TJ 或者 $T_{\mu}J$

2.2 BELLMAN EQUATIONS FOR Q-FACTORS

72 00:04:09,560 -> 00:04:13,639 and now let me remind you about Q factors we

- $73\ 00:04:13,639 -> 00:04:16,160$ discussed those learning the second
- 74 00:04:16,160 -> 00:04:20,450 lecture so first the second lecture
- 我要提醒一下你关于 Q 函数, 我们在第二个课程的时候讨论过了
- $75\ 00:04:20,450 -> 00:04:25,479$ the given J star the optimal cost function
- 给定了最后成本函数 J^*
- 76 00:04:25,479 \rightarrow 00:04:30,860 the optimal Q factor at the state I can
- $77\ 00:04:30,860 \longrightarrow 00:04:32,180\ control\ you$
- $78\ 00:04:32,180 -> 00:04:35,660$ is given by this expression

```
状态 i 处控制 u 的最优 Q 函数被如下表达式定义
   79\ 00:04:35,660 \rightarrow 00:04:38,919 it's the expected one stage cost starting from I
   80\ 00:04:38,919 \rightarrow 00:04:44,600 using you plus the optimal cost to go
   81\ 00:04:44,600 -> 00:04:48,410 starting from J so it's cost associated
   82\ 00:04:48,410 -> 00:04:51,560 will start me state I but using you
   83 00:04:51,560 -> 00:04:57,070 first and then optimal after that okay
   他是一阶段成本加上后续最优长期成本 J 的期望, 所以成本在当前状态与控制给定后最小化后
续成本获得的
   84\ 00:04:57,400 -> 00:05:00,560 so this is a bigger function than J star
   这是一个比 J* 更大的函数
   85\ 00:05:00,560 -> 00:05:03,590\ J\ star\ is\ defined\ for\ every\ step\ you
   86\ 00:05:03,590 \longrightarrow 00:05:05,180 started to find for every state and control
   J^* 是定义在每一个阶段的,需要做的就是找每一个阶段的最优控制
   87\ 00:05:05,180 -> 00:05:10,669 okay now we have balanced
   88\ 00:05:10,669 -> 00:05:15,020 equation J star satisfying equal to this expression here
   我们有 bellman 方程, J^* 也需要满足这个方程 (这一页第一个方程)
   89\ 00:05:15,020 -> 00:05:18,080 therefore if we minimize
   90\ 00:05:18,080 -> 00:05:21,169 on both sides of this expression we see
   91 00:05:21,169 -> 00:05:23,930 that J starts of I is the minimum of
   92\ 00:05:23,930 -> 00:05:26,570 the Q factors so if I have the optimal
   93 00:05:26,570 -> 00:05:28,880 few factors I can obtain the optimal
   94\ 00:05:28,880 -> 00:05:33,160 cost by minimizations over you
   因此最小化这个表达式的等号两边, 我们看到 J^*(i) 是状态 i 下所有 u 中最小的 Q(i,u)
   95\ 00:05:33,160 \rightarrow 00:05:36,169 moreover if I substitute this expression here in
   96 00:05:36,169 -> 00:05:39,110 place of j* I have an equation that
   97\ 00:05:39,110 -> 00:05:42,380 does not involve a star at all
   如果我用这个表达式 (J^*\left(i\right) = \min_{u \in U(i)} Q^*\left(i,u\right)) 代替上面的表达式中的 J^*,就可以得到一个完
全不含有 J* 的表达式
   98\ 00:05:42,380 \rightarrow 00:05:46,130 it's an equation for Q factors
   这是一个 Q 值的方程
   99 00:05:46,130 -> 00:05:51,789 this equation holds for every I and you
   这个方程对所有的 i 和 u 都成立
   100\ 00:05:51,789 -> 00:05:54,889 so it is a number of nonlinear equations with equal number
of unknowns
   所以这是一个方程数与未知数数相等的非线性方程组
   101\ 00:05:54,889 -> 00:05:57,740 but a bigger equation
   102\ 00:05:57,740 -> 00:05:59,479 that Bellman's equation has more
   103\ 00:05:59,479 \longrightarrow 00:06:05,330 variables in more equations
   但是 bellman 方程组规模更大,变量数量更多
   104\ 00:06:05,330 \rightarrow 00:06:07,490 okay now we can use this equation also as a bellman equation
   我们把这个方程也叫做 bellman 方程
   105\ 00:06:07,490 -> 00:06:10,430 we can view Q star has been the
   106\ 00:06:10,430 -> 00:06:13,159 fixed point of a certain mapping
   我们用 Q* 表示这个映射的不动点
   107\ 00:06:13,159 -> 00:06:16,190 okay and the mapping is the one here on the right-hand side
   这个映射在这个公式(本页第二个公式)的等号右边的内容
   108\ 00:06:16,190 -> 00:06:20,810 so it is really a
   109\ 00:06:20,810 -> 00:06:24,310 bellman equation such this one here and
   110\ 00:06:24,310 \longrightarrow 00:06:27,050 inherits the same properties as other bellman equations
   这是一个 bellman 方程, 与其他 bellman 方程具备的性质相同
   111\ 00:06:27,050 -> 00:06:30,289 in fact you can
   112 00:06:30,289 -> 00:06:34,370 consider i u as a state of a bigger
   113\ 00:06:34,370 -> 00:06:39,889 system and we move from I you to other J
   114\ 00:06:39,889 -> 00:06:41,490 u Prime
   115\ 00:06:41.490 -> 00:06:43.650 and the minimum over here denotes the
   116\ 00:06:43,650 -> 00:06:46,380 minimization over control
   事实上你可以把 i, u 当成一个更大的系统的状态, 然后从状态 (i, u) 转移到另一个状态 (i, u'),
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然后找到合适的 u 最小化新状态的 Q 值

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117\ 00:06:46,380 -> 00:06:50,220 so there is an underlying dynamic programming problem
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- $118\ 00:06:50,220 -> 00:06:53,069$ on a bigger space that involves the Q
- $119\ 00:06:53,069 \longrightarrow 00:06:55,860$ factors in involves of states the pairs I nu
- 这是一个底层的在更大空间上的动态规划问题,状态空间是(i, u)
- $120\ 00:06:55,860 -> 00:06:59,160$ and the bellman equation for that
- $121\ 00:06:59,160 \longrightarrow 00:07:01,349$ problem is this one
- 这个是这个问题的 bellman 方程
- $122\ 00:07:01,349 \longrightarrow 00:07:03,479$ so everything we have said about bellman equations for J
- $123\ 00:07:03,479 -> 00:07:08,550\ star\ or\ J\ mu\ applies\ to\ Q\ factors\ as\ well$
- J^* 和 J_μ 也作用在这个 Q 值上了
- $124\ 00:07:08,550 -> 00:07:11,550$ so there's a unique solution to
- $125\ 00:07:11,550 -> 00:07:13,590$ this equation this map here is a
- 126 00:07:13,590 -> 00:07:15,690 contraction mapping a lot of cool stuff
- $127\ 00:07:15,690 \rightarrow 00:07:18,139$ that we discussed earlier apply
- 这是这个映射 (中间的红公式) 的唯一解, 它是一个压缩映射, 我们之前讨论过很多关于压缩映射的东西
 - $128\ 00:07:18,139 \rightarrow 00:07:21,330$ and there is also similar mapping describing
 - 129 00:07:21,330 -> 00:07:24,690 the Q factors of a policy and let me get into those
 - 这是一个相似的映射, Q 值的策略, 我们来讲一下

2.3 BELLMAN EQ FOR Q-FACTORS OF A POLICY

- $130\ 00:07:24,690 -> 00:07:29,220$ given a stationary policy
 - 给定一个平稳策略
 - $131\ 00:07:29,220 -> 00:07:31,830$ we can consider some larger system that
 - $132\ 00:07:31,830 -> 00:07:35,490$ moves from IU to our use of J and then
 - $133\ 00:07:35,490 \longrightarrow 00:07:38,150$ continues according to this mechanism
 - 可以考虑在一个大系统中,根据这个转移机制 (概率) 从状态 (i,u) 转移到状态 $(u,\mu(j))$
 - $134\ 00:07:38,150 -> 00:07:42,680$ and the Q factors of a policy satisfy
 - $135\ 00:07:42,680 \rightarrow 00:07:49,380$ this equation
 - 这个策略的 Q 值满足这个等式 (从上往下第一个公式)
 - $136\ 00:07:49,380 \rightarrow 00:07:52,199$ equivalently q mu is a fixed point of this mapping
 - Q_{μ} 是这个映射 (第二个公式) 的不动点
 - $137\ 00:07:52,199 -> 00:07:54,960$ this is a linear equation now
 - 这是一个线性方程(第一个方程)
 - $138\ 00:07:54,960 -> 00:07:56,849$ has all equations that have to do with single policies are
 - $139\ 00:07:56,849 \rightarrow 00:07:59,340$ a linear equation involving these
 - $140\ 00:07:59,340 -> 00:08:02,370$ unknowns defined for every policy in
 - 141 00:08:02,370 -> 00:08:04,680 having an unknown for every I and u
 - 所有的方程都是对于同一个策略来说的,这个线性方程 (第一个) 的未知数包括策略和 (i, u)
 - $142\ 00:08:04,680 -> 00:08:08,490$ and we can use this equation we can
 - $143\ 00:08:08,490 -> 00:08:13,710$ solve this equation we can evaluate the
 - 144 00:08:13,710 -> 00:08:16,680 Q factors of a policy
 - 我们可以评价这个策略的 Q 值
 - $145\ 00{:}08{:}16{,}680 -> 00{:}08{:}20{,}060$ and that's useful in policy direction
 - 这可以用在策略迭代中

(someone asking

- 146 00:08:23,070 -> 00:08:35,820 I didn't understand yes it's a number of
- $147\ 00:08:35,820 -> 00:08:55,440$ states that's true yes yes yes this in
- $148\ 00:08:55,440 -> 00:08:58,560$ this is only one of the several possible
- $149\ 00:08:58,560 -> 00:09:01,020$ transitions from I you okay
- $150\ 00{:}09{:}01{,}020 -> 00{:}09{:}04{,}920$ it's a several address there are n
- 151 00:09:04,920 \rightarrow 00:09:17,610 possibilities yes okay P IJ you are the
- $152\ 00:09:17,610 \rightarrow 00:09:20,970$ transition probabilities starting at I
- $153\ 00:09:20,970 -> 00:09:24,450$ going to J over you will be using those
- 154 00:09:24,450 -> 00:09:29,360 for six lectures so so okay that's fine

asking completed)

155 00:09:34,430 \rightarrow 00:09:39,210 okay so we have Q factor optimal Q

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156\ 00:09:39,210 -> 00:09:41,840\ factors\ Q\ factors\ of\ a\ policy
   我们求得了某个策略的 Q 值
   157 00:09:41,840 -> 00:09:44,190 and generally you value direction in policy
   158\ 00:09:44,190 \longrightarrow 00:09:47,070 direction can be carried out in terms of Q factors
   一般你可以用 Q 函数进行值迭代和策略迭代
   159\ 00:09:47.070 -> 00:09:54,600 now if you do value duration
   160\ 00:09:54,600 -> 00:09:56,970 for Q factors in all policy direction
   161 00:09:56,970 -> 00:09:58,980 for Q factors you're not getting
   162\ 00:09:58,980 -> 00:10:01,040 anything different than for course
   163 00:10:01,040 -> 00:10:03,240 mathematically they're equivalent
   如果你对 Q 值进行值迭代或者对 Q 值进行策略迭代,他们没什么区别,从数学上说他们是等
价的
   164 00:10:03,240 -> 00:10:05,340 you just operate on other things you're
   165 00:10:05,340 -> 00:10:07,710 you're operating on you're using a
   166\ 00:10:07,710 \rightarrow 00:10:10,250 transform versions of the same algorithm
   你只是从不同的角度来看待这个算法
   167\ 00:10:10,250 -> 00:10:16,170 so for exact so for exact policy
   168 00:10:16,170 -> 00:10:18,960 evaluation policy direction and value
   169\ 00:10:18,960 -> 00:10:21,920 iteration there is no difference between
   170\ 00:10:21,920 -> 00:10:26,340 using two factors or policies except for a few things
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所以对于精确策略迭代和值迭代来说,用Q值或者策略除了一些东西几乎没什么不同的

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2.4 WHAT IS GOOD AND BAD ABOUT Q-FACTORS
171\ 00:10:26,340 \longrightarrow 00:10:28,320 so let that leads us to a
   172\ 00:10:28,320 -> 00:10:28,900 question
   173\ 00:10:28,900 -> 00:10:31,390 why do we want to use Q factors what's
   174\ 00:10:31,390 -> 00:10:35,440\ good\ and\ what's\ bad\ about\ them
   下面我们来讨论一个问题,为什么要用Q值,他有什么好处和坏处呢
   175\ 00:10:35,440 \longrightarrow 00:10:38,170 first of all all the exact theory and algorithms
   176 00:10:38,170 -> 00:10:41,080 for costs applies to Q factors
   首先, 所有的精确理论和算法都用 Q 值计算成本
   177\ 00:10:41,080 \rightarrow 00:10:42,760 bellman equations all the things that we were
   178\ 00:10:42,760 -> 00:10:45,160 discussing the last in the preceding
   179 00:10:45,160 -> 00:10:46,600 lectures contraction mappings
   180\ 00:10:46,600 -> 00:10:47,920 contraction properties optimality
   181 00:10:47,920 -> 00:10:50,320 conditions convergence and value of
   182\ 00:10:50,320 \rightarrow 00:10:52,150 valuing policy direction all of these things apply
   bellman 中的所有事情,我们之前讨论过的内容,压缩映射,收缩性,最优条件,收敛性,值
迭代和策略迭代,都在用Q值
   183\ 00:10:52,150 -> 00:10:56,020 all the approximate theory
   184\ 00:10:56,020 \rightarrow 00:10:58,480 that we discussed also applies projected
   185 00:10:58,480 -> 00:11:01,110 equation sampling exploration issues
   186\ 00:11:01,110 \longrightarrow 00:11:05,260 oscillation aggregation and all of these
   187 00:11:05,260 -> 00:11:08,230 phenomena manifest themselves in Q
   188\ 00:11:08,230 -> 00:11:11,850 factor land just as well as in cost land
   我们讨论过的所有的近似理论也都在用,投影方程、采样、探索、震荡、聚合、所有的理论都
在用 Q 值计算成本
   189 00:11:11,850 -> 00:11:15,010 now here's one big thing about Q factors
   现在关于 Q 值有一个很重要的事情
   190\ 00:11:15,010 -> 00:11:18,850 it allows if you can find them a
   191\ 00:11:18,850 \longrightarrow 00:11:22,260 model free controller implementation
   如果你能够找到一个模型无关的控制器
   192\ 00:11:22,260 -> 00:11:25,420 once we calculate these Q factors for all I and u
   如果我能计算所有 (i,u) 的 Q 值
   193\ 00:11:25,420 -> 00:11:28,210 we can calculate an
   194\ 00:11:28,210 -> 00:11:31,630 optimal policy not by minimizing on the
   195\ 00:11:31,630 -> 00:11:33,570 right hand side of the bellman equation
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196\ 00:11:33,570 -> 00:11:36,760 but rather by just looking at this list
   197\ 00:11:36,760 -> 00:11:38,860 of Q factors and selecting the one
   198\ 00:11:38,860 -> 00:11:40,540 that's numerically smaller
   那么就不用通过最小化 bellman 方程来计算最优策略了,可以直接在 Q 值列表中找一个值最
小的控制就行了
   199 00:11:40,540 -> 00:11:43,720 okay so implementation once you have
   200\ 00:11:43,720 -> 00:11:47,230 these Q stars is very simple
   如果你能找到 Q*, 实施起来就非常简单
   201\ 00:11:47,230 -> 00:11:49,510 and moreover I do not need to know the
   202\ 00:11:49,510 \rightarrow 00:11:51,779 transition probabilities of the system
   更多地,我不需要知道系统的状态转移概率
   203\ 00:11:51,779 -> 00:11:55,060 ordinarily you need to minimize the
   204\ 00:11:55,060 \longrightarrow 00:11:58,240 expected value of the current stage cost
   205 00:11:58,240 -> 00:12:01,720 plus the expected optimal costs ago that
   206 00:12:01,720 -> 00:12:04,110 involves calculating an expectation and
   207\ 00:12:04,110 \rightarrow 00:12:06,339 involves knowing the transition probabilities
   按理说你需要最小化当前成本和最优长期成本的和的期望,同时还得知道状态转移概率
   208\ 00:12:06,339 -> 00:12:08,500 here you don't need to
   209\ 00:12:08,500 -> 00:12:10,360 know the transition probabilities you
   210\ 00:12:10,360 -> 00:12:12,580\ don't\ need to\ know\ the\ model
   这里你不需要知道转移概率,也不需要知道模型
   211 00:12:12,580 -> 00:12:16,089 if you can find these q stars okay
   如果你能找到 Q^*
   212 00:12:16,089 -> 00:12:20,620 that's the big incentive for using q factors
   这是大家用 Q 值得一个很大的原因
   213 00:12:20,620 -> 00:12:23,200 similarly if you calculate a parametric
   214\ 00:12:23,200 -> 00:12:25,630 approximation of the q factors involved
   215\ 00:12:25,630 \rightarrow 00:12:28,240 in some approximation architecture with
   216 00:12:28,240 -> 00:12:31,420 a parameter vector r you can obtain a
   217 00:12:31,420 -> 00:12:34,870 suboptimal policy by minimizing
   218\ 00:12:34,870 \longrightarrow 00:12:37,510 again these Q factors without the need for a model
   相似地,如果你要计算一个 Q 值使用参数向量 r 和相应近似结构得参数近似问题, 你可以不知
道模型直接最小化 Q 来得到次优策略
   219\ 00:12:37,510 -> 00:12:42,640 what are the bad things
   220 00:12:42,640 -> 00:12:44,050 about Q factors
   那么 Q 值得缺点是什么呢
   221 00:12:44,050 -> 00:12:47,320 well greater dimension more storage and
   222\ 00:12:47,320 \rightarrow 00:12:50,170 some complications in calculating these or that
   更高的维度, 更大的存储空间和更大的计算量
   223\ 00:12:50,170 \rightarrow 00:12:55,390 that's a rough cut about what
   224\ 00:12:55,390 -> 00:12:59,050 what we can expect from two factors in
   225\ 00:12:59,050 -> 00:13:00,880 particular if you want to calculate
   226\ 00:13:00,880 -> 00:13:04,240 exactly factors then the difficult is
   227\ 00:13:04,240 \longrightarrow 00:13:05,980 increased for large-scale problems and
   228 00:13:05,980 -> 00:13:08,230 you have to think about aggregation or
   229\ 00:13:08,230 \rightarrow 00:13:10,839 other approximation procedures in order
   230\ 00:13:10,839 -> 00:13:19,690 to calculate Q factors
   这是一个策略的划分,如果你想要计算精确的 Q 值,大规模会增加求解的难度,这时候你就不
得不考虑聚合或者其他近似方法来求解 Q 值
```

3 Q-LEARNING

231 00:13:19,690 -> 00:13:21,190 okay now here's one thing about Q factors that we 232 00:13:21,190 -> 00:13:22,630 don't have for cause it's an algorithm called Q learning 下面要讲的关于 Q 值得算法叫做 Q 学习 233 00:13:22,630 -> 00:13:28,330 in this is in addition 234 00:13:28,330 -> 00:13:30,070 to everything else we discuss this is new 我们接下来要讲的是一个新东西

```
235\ 00:13:30,070 \longrightarrow 00:13:33,880 and Q learning is a sampled form of value iteration
   这是值迭代的采样形式
   236\ 00:13:33,880 -> 00:13:36,460 it is a stochastic
   237\ 00:13:36,460 -> 00:13:39,420 iterative algorithm
   也是一个随机迭代算法
   238\ 00:13:39,420 -> 00:13:42,850 that generates iterated values of these Q factors by
   239\ 00:13:42,850 \longrightarrow 00:13:47,310 using samples from the system
   这个算法使用从系统中获取的样本对 Q 值进行迭代
   240\ 00:13:47,310 \rightarrow 00:13:50,230 and it resembles value generation but it's not
   241\ 00:13:50,230 -> 00:13:52,600 value duration it's a sampled form of value iteration
   它很像值迭代,但是又不是值迭代,而是一种采样形式的值迭代
   242\ 00:13:52,600 -> 00:13:56,020 okay so here's the
   243\ 00:13:56,020 -> 00:13:58,270 description of Q learning
   这是 Q 学习的介绍
   244\ 00:13:58,270 \longrightarrow 00:13:59,740 in the classical form there are many variations
   245\ 00:13:59,740 \longrightarrow 00:14:01,600 but this is the standard Q learning algorithm
   Q 学习有很多不同的形式, 这是标准 Q 学习
   246\ 00:14:01,600 -> 00:14:05,040 remember we want to calculate
   247\ 00:14:05,040 -> 00:14:09,970 Q of all I and you so we select a
   248 00:14:09,970 -> 00:14:12,810 sequence of pairs of states and controls
   249\ 00:14:12,810 -> 00:14:15,880 we have a system sample we get one state
   250\ 00:14:15,880 -> 00:14:17,440 control pair here another one another
   251\ 00:14:17,440 -> 00:14:20,170 one we have a whole large number of them
   我们想要计算所有 (i,u) 对应的 Q 值,我们先选择一个序列的状态动作对,从系统中采样可以
获得非常多的状态动作对
   252\ 00:14:20,170 -> 00:14:23,050 and we can use any mechanism for doing
   253\ 00:14:23,050 -> 00:14:25,660 that probabilistic mechanism or
   254\ 00:14:25,660 -> 00:14:28,870 deterministic mechanism as long as all
   255\ 00:14:28,870 -> 00:14:32,980 the state control pairs are represented
   256\ 00:14:32,980 -> 00:14:36,850 and are sampled infinitely often was
   257~00{:}14{:}36{,}850~{-}{>}~00{:}14{:}38{,}800 this an idealized algorithm practice we
   258 00:14:38,800 -> 00:14:40,720 would not sound every state control pair
   259\ 00:14:40,720 \rightarrow 00:14:42,900 infinitely often but that's the
   260\ 00:14:42,900 -> 00:14:44,380 theoretical
   261\ 00:14:44,380 \rightarrow 00:14:49,260 version of the algorithm
   我们可以用任何机制进行采样、概率机制或者确定性机制来获得无数个状态动作对,实际上我
们不能获得无数个状态动作对,但是这是算法从理论上的要求
   262\ 00:14:49,260 -> 00:14:53,170 so a iteration case we had some two
   263 00:14:53,170 -> 00:14:56,560 factors QK a whole big vector of Q factors
   迭代是我们获得了巨大的 Q 值向量中的一部分
   264\ 00:14:56,560 -> 00:15:00,370 and we update we said we have a
   265\ 00:15:00,370 -> 00:15:04,360 sample we have this ik uk and we
   266\ 00:15:04,360 -> 00:15:06,790 sample the next state according to this
   267\ 00:15:06,790 \longrightarrow 00:15:10,560 transition probabilities so we have I K
   268\ 00:15:10,560 -> 00:15:16,060\ \mathrm{UK} and JK and we have we an and we
   269\ 00:15:16,060 -> 00:15:19,180 update the Q factor of that particular
   270\ 00:15:19,180 -> 00:15:22,660 pair according to this formula and we
   271 00:15:22,660 -> 00:15:25,380 live only all the other Q factors unchanged
   我们使用这个 i_k, u_k 和 j_k 根据这个表达式来更新这些 Q 值,被更新的只有这部分 Q 值,其
他的没有变
   272\ 00:15:25,380 -> 00:15:31,900 okay so here i am i select ik
   273 00:15:31,900 -> 00:15:35,650 UK and i pick the Q factor of only
   274\ 00:15:35,650 -> 00:15:40,750 that pair and i sample the next state to
   275\ 00:15:40,750 -> 00:15:43,810 obtain JK and i calculate this
   276\ 00:15:43,810 -> 00:15:47,170 expression which is a sample of the
   277\ 00:15:47,170 -> 00:15:50,140 expected value that appears in bellman
   278\ 00:15:50,140 -> 00:15:51,880 equation okay
   279 00:15:51,880 \rightarrow 00:15:55,090 sample because it's not some over p IJ s
```

```
280 00:15:55,090 -> 00:16:01,120 but in rather just j k okay and we move
```

 $281\ 00:16:01,120 -> 00:16:05,080$ the Q factor of only that pair in the

 $282\ 00:16:05,080 -> 00:16:08,290$ direction of this sample but we also

283 00:16:08,290 -> 00:16:11,740 interpolate with a current value gamma K

284 00:16:11,740 $-\!>$ 00:16:14,500 is a step size typically a small step

285 00:16:14,500 $-\!>$ 00:16:17,740 size in fact it is necessary but the

286 00:16:17,740 -> 00:16:20,800 step size goes to 0 lightly at a rate of

287 00:16:20,800 -> 00:16:24,850 1 over K proportional to a constant over K

我有 i_k 和 u_k ,选择这些状态和控制对应的 Q 值,然后对下一阶段状态进行采样得到 j_k ,然后根据这个表达式计算样本的期望值,这里没有概率 p_{ij} ,只有 j_k ,现在我使用样本在当前的 γ_k 下更新 Q 值,这个步长必须非常小才行,一般与 $\frac{1}{k}$ 成正比

 $288\ 00:16:24,850 \longrightarrow 00:16:31,690$ so we calculate the sample which is like value direction

我们用样本来进行这个迭代时很像值迭代

 $289\ 00:16:31,690 -> 00:16:36,460$ okay but it's only

 $290\ 00:16:36,460 -> 00:16:39,840$ a sample and then we make this update

291 00:16:39,840 \rightarrow 00:16:43,180 leave all the others and change then I

 $292\ 00:16:43,180 -> 00:16:47,110$ pick out the next sample i ka plus 1 u k

 $293\ 00:16:47,110 \rightarrow 00:16:51,370$ plus 1 i do again a change for that you

 $294\ 00:16:51,370 -> 00:16:55,149$ factor and keep going like the

只有一个样本时用这个样本更新相应的 Q 值,其他 Q 值不变,然后选择新的状态和控制进行 迭代,迭代一直这么进行下去

 $295\ 00:16:55,149 -> 00:16:57,589$ all the few factors are going to be

 $296~00:16:57,589 \longrightarrow 00:17:00,560$ updated infinitely often but only one at a time

所有的 Q 值都会被更新无数次, 但是每次之更新一个

 $297\ 00:17:00.560 \rightarrow 00:17:08.770$ okay so that's the idea

298 00:17:08,770 -> 00:17:14,329 value direction would move in that

 $299\ 00:17:14,329 \rightarrow 00:17:17,150$ direction fully with step size equal to

300 00:17:17,150 -> 00:17:21,289 one Q learning moves in the direction of

 $301\ 00:17:21,289 -> 00:17:27,400$ a sample of this by a step size gamma K

这个想法是这样的,值迭代会沿着步长是 1 的方向迭代,而 Q 学习会根据样本以 $gamma_k$ 的步长进行迭代

3.1 NOTES AND QUESTIONS ABOUT Q-LEARNING

 $302\ 00:17:37,240 \rightarrow 00:17:39,850$ so let's say let's before we go into

 $303\ 00:17:39,850 -> 00:17:42,400$ questions as to why this works and so on

 $304\ 00:17:42,400 \rightarrow 00:17:44,010$ let's just look at it and try to

 $305\ 00:17:44,010 -> 00:17:46,450$ understand it a little bit better

我们讨论为什么它能工作之前先来看看这个好好理解一下

 $306\ 00:17:46,450 -> 00:17:49,240$ here is the algorithm I've just restated the algorithm here 我要讲的算法在这里

 $307\ 00:17:49,240 \longrightarrow 00:17:57,400$ the first thing is that

 $308\ 00:17:57,400 -> 00:18:00,250$ to implement the algorithm I don't need

 $309\ 00:18:00,250 -> 00:18:02,830$ to have a detailed model of the system

首先实现这个算法的时候我不需要知道这个系统的细节

 $310\ 00:18:02,830 -> 00:18:05,350\ I\ don't\ need to\ have the\ pIJ\ probabilities$

我不需要知道概率 p_{ij}

 $311\ 00:18:05,350 \rightarrow 00:18:08,950$ all I need is to have a simulator or

 $312\ 00:18:08,950 -> 00:18:12,880$ some mechanism so that given AI K and UK

313 00:18:12,880 -> 00:18:16,030 I can generate the next state

我需要的是一个给定 i_k 和 u_k 能够生成新状态的模拟器

 $314\ 00:18:16,030 -> 00:18:18,580$ so all I need is a box that samples next States

 $315~00{:}18{:}18{,}580~{-}{>}~00{:}18{:}21{,}610$ and also gives me values of one state cost

所以我需要的是一个盒子, 能够给我新状态和一阶段成本

 $316\ 00:18:21,610 \longrightarrow 00:18:30,610$ there is no need for a model

不需要知道模型的信息

 $317\ 00:18:30,610 -> 00:18:32,950$ the other thing is that it operates at only

318 00:18:32,950 -> 00:18:35,020 one state control pair at a time and

```
319\ 00:18:35,020 -> 00:18:38,140 this is convenient if you want to
320\ 00:18:38,140 \rightarrow 00:18:40,600 generate the state control pairs by simulation
另一个事情就是每次通过仿真给定一个状态动作对很方便
321\ 00:18:40,600 -> 00:18:43,960 so you have one to state
322\ 00:18:43,960 -> 00:18:45,790 control pair that moves you to a next
323\ 00:18:45,790 -> 00:18:47,950 state and then you use that next state
324\ 00:18:47,950 -> 00:18:51,970 as as part of the next state control pair and so on
所以你有一个状态控制对,转移到下一个状态,然后继续转移到新状态,这样产生状态动作对
325\ 00:18:51,970 \longrightarrow 00:18:55,030 so even though the
326\ 00:18:55,030 -> 00:18:58,060 mechanism for generating this this state
327\ 00:18:58,060 -> 00:19:00,460 control pair arbitrary often times
328\ 00:19:00,460 -> 00:19:02,380 you use us the simulator of the system
329\ 00:19:02,380 -> 00:19:05,830 and in that context it's convenient that
330\ 00:19:05,830 -> 00:19:08,620 you operate at only one state control pair at a time
331\ 00:19:08,620 -> 00:19:11,110 because that's consistent with simulation
这种每次通过系统仿真产生一个状态动作对的方法很方便
332\ 00:19:11,110 -> 00:19:16,060 however with this type
333\ 00:19:16,060 -> 00:19:18,640 of operation the algorithm becomes a synchronous
这种类型的操作让算法变成一个同步算法
334\ 00:19:18,640 -> 00:19:21,280 remember we talked about
335\ 00:19:21,280 \rightarrow 00:19:24,910 asynchronous algorithms backing in second
336\ 00:19:24,910 -> 00:19:27,520 lecture where we operate somewhat
337\ 00:19:27,520 \longrightarrow 00:19:30,640 chaotically among the components of the
338\ 00:19:30,640 -> 00:19:33,550 vector that we update
记得我们在第二次课程讲的异步算法么,迭代更新时成本向量更新得很混乱(不是一起更新的)
339\ 00:19:33,550 -> 00:19:36,310 so and one possibility is to update components one at a time
一种可能是每次更新一个元素
340\ 00:19:36,310 -> 00:19:39,070 well that's what we do here we
341\ 00:19:39,070 -> 00:19:42,580 operate on Q factors of only one
342\ 00:19:42,580 -> 00:19:46,179 component at operate and only one factor at a time
这就是这个地方做的, 更新时每次只更新一个 Q 值
343\ 00:19:46,179 \longrightarrow 00:19:52,090 now there are no
344\ 00:19:52,090 -> 00:19:53,980 approximations in this out good we aim
345\ 00:19:53,980 -> 00:19:56,230 to find the exactly optimal Q factors
我们的目的是找到精确的最优 Q 值
346\ 00:19:56,230 -> 00:19:59,259 okay we don't cook multiple suboptimal
347\ 00:19:59,259 -> 00:20:02,320 here we want to we aiming straight for
348\ 00:20:02,320 \longrightarrow 00:20:05,769 the optimal Q factors
我们不想计算次优解, 我们的目标就是得到最优 Q 向量
349\ 00:20:05,769 -> 00:20:09,100 and then the question is why does this work why does
350\ 00:20:09,100 \rightarrow 00:20:12,490 this algorithm converge to Q star the optimal Q factors
问题是为什么这个算法会收敛到最优 Q 值
351\ 00:20:12,490 -> 00:20:16,389 and also why can I
352\ 00:20:16,389 -> 00:20:19,179 not use a sample version of value
353\ 00:20:19,179 \longrightarrow 00:20:23,700 iteration to calculate optimal costs
和为什么我不用采样版本的值迭代计算最优成本
354\ 00:20:23,700 \rightarrow 00:20:27,850 well the reason for this is actually Mathematica
355\ 00:20:27,850 -> 00:20:31,720 it's just a fine point in the mathematics
原因是基于数学的一些比较好的理论
356\ 00:20:31,720 -> 00:20:35,220 generally speaking for
357\ 00:20:35,220 \longrightarrow 00:20:38,460 stochastic iterative algorithms to work
一般来说,随即迭代算法工作时
358\ 00:20:38,460 -> 00:20:41,950 what you need to have is an expected
359\ 00:20:41,950 -> 00:20:43,779 value an iteration that involves an
360\ 00:20:43,779 -> 00:20:46,679 expected value that you can sample
你想要的是通过采样计算成本的期望值
361\ 00:20:46,679 -> 00:20:50,919 however in bellman's equation you have a
362\ 00:20:50,919 -> 00:20:52,570 mapping that does not involve an
```

```
363\ 00:20:52,570 -> 00:20:55,119 expected value only but rather the
   364\ 00:20:55,119 -> 00:20:58,389 minimum of an expected value if I were
   365\ 00:20:58,389 -> 00:21:01,749 to sample this expected value and
   36600:21:01,749 ->00:21:03,429 consider an algorithm that invoke
   367\ 00:21:03,429 -> 00:21:06,220 minimization over example this would just not work
   bellman 方程中你有一个不包括期望只有最小化操作的映射,如果我对这个期望进行采样然后
对样本进行最小化,算法就不会工作
   368\ 00:21:06,220 -> 00:21:10,059 the sample of minimum of
   369\ 00:21:10,059 -> 00:21:13,419 minimum works but minimum sample the
   370 00:21:13,419 -> 00:21:16,029 mathematics don't work
   最小化的采样工作,但是最小化采样不工作
   371\ 00:21:16,029 -> 00:21:18,340 because when you take expected values of a minimum of the
   372\ 00:21:18,340 -> 00:21:19,990 sample you don't get the same as the
   373\ 00:21:19,990 \rightarrow 00:21:22,990 minimum of the expected value
   因为你计算样本的最小化期望值时,它的值与最小化期望值是不一样的
   374\ 00:21:22,990 \rightarrow 00:21:25,269 so it's a fine mathematical point but it makes a
   375 00:21:25,269 -> 00:21:27,789 big difference because I would like to
   376\ 00:21:27,789 -> 00:21:30,490 work with I would like to have a sampled
   377\ 00:21:30,490 -> 00:21:34,179 value iteration of involved in cost but
   378\ 00:21:34,179 -> 00:21:36,759\ I\ can't\ do\ it\ because the\ minimum\ and
```

这是一个数学上很好的概念,但是会导致这两种算法很大的差别,因为我想要进行采样值迭代

的时候由于最小化和期望值的原因而不可行 380 00:21:38,590 -> 00:21:44,559 okay we're not going to go back

381 00:21:44,559 -> 00:21:47,470 to this point but instead let's consider

 $379\ 00:21:36,759 -> 00:21:38,590$ the expectation are in the wrong relation

 $382\ 00:21:47,470 -> 00:21:49,210$ why do we get convergence

我们不会再一次讲这个地方了, 我们来考虑一下为什么会收敛吧

383 00:21:49,210 -> 00:21:54,990 I think questions at this point 有什么问题么

3.2 CONVERGENCE ASPECTS OF Q-LEARNING

 $384\ 00:22:07,360 -> 00:22:14,500$ okay you can show convergence to the

 $385\ 00:22:14,500 \rightarrow 00:22:16,660$ true exact Q factors and the fairly mild assumptions

在这里你可以看到精确 Q 值收敛和比较温和的假设

 $386\ 00:22:16,660 -> 00:22:20,220$ and the line of proof

 $387\ 00:22:20,220 -> 00:22:23,650$ involves two types of arguments two

 $388\ 00:22:23,650 -> 00:22:26,050$ types of theories why is the theory of

389 00:22:26,050 -> 00:22:28,780 stochastic approximation iterations and

 $390\ 00:22:28,780 -> 00:22:32,230$ the other is the theory of a synchronous algorithms

这一行证明包括两种类型(包括了两种理论)的讨论,一种是随机近似迭代,另一种是同步算法 理论

 $391\ 00:22:32,230 \rightarrow 00:22:35,560$ mathematically the key fact

 $392\ 00:22:35,560 -> 00:22:38,380$ is that the q-learning mapping that

393 00:22:38,380 $-\!\!>$ 00:22:38,860 map's

 $394\ 00:22:38,860 -> 00:22:43,090\ Q$ into FQ is a contraction with respect

 $395~00:22:43,090 \longrightarrow 00:22:44,740$ to the soup norm okay the maximum norm

从数学上讲,关键的地方在 Q 学习映射上,也就是把 Q 映射到 FQ,这是一个带有 sup-norm 的压缩映射

 $396\ 00:22:44,740 \longrightarrow 00:22:47,620$ contraction is

397~00:22:47,620 -> 00:22:49,630 important in order to be able to use the stochastic approximation

为了使用随机近似方法,压缩性是很重要的

 $398\ 00:22:49,630 -> 00:22:52,510 \text{ sup norm is}$

399 00:22:52,510 \rightarrow 00:22:55,390 important because of the a synchronous operation sup norm 重要是因为同步操作

 $400\ 00:22:55,390 \rightarrow 00:22:57,430$ you may remember

 $401\ 00:22:57,430 -> 00:22:59,320$ on my discussion about a synchronous

```
402\ 00:22:59,320 -> 00:23:01,510 algorithms a synchronous values are
403 00:23:01,510 -> 00:23:04,150 valid provided there's some kind of
404 00:23:04,150 -> 00:23:06,430 contraction light property with respect
405\ 00:23:06,430 -> 00:23:08,410 to the soup norm if you remember this
406\ 00:23:08,410 -> 00:23:11,740 box conditions and so on
你应该还记得我们讨论过的同步算法, sup norm 为同步值提供了收缩性, 还记得那个方框么
407\ 00:23:11,740 \longrightarrow 00:23:15,190 that's what in effect here when you look into in depth into
如果你深入地看他的证明,就可以知道它的影响是什么了
408\ 00:23:15,190 -> 00:23:18,880 soup norm works well with
409 00:23:18,880 -> 00:23:21,070 a synchronous contraction works well
410\ 00:23:21,070 -> 00:23:25,960 with stochastic approximation okay
sup norm 在同步算法中工作的很好,收缩性在随机近似方法中过的得很好
411 00:23:25,960 -> 00:23:27,940 now let's understand the connection with
412 00:23:27,940 -> 00:23:34,300 stochastic approximation
我们来看看它与随机近似方法的联系
413\ 00:23:34,300 \rightarrow 00:23:36,700 let's step take one step back from Q learning and look
414\ 00:23:36,700 -> 00:23:39,130 at a generic fixed point problem
415\ 00:23:39,130 -> 00:23:42,850 involving an expectation
我们回到 Q 学习看一下带期望的通用的不动点问题
416\ 00:23:42,850 \rightarrow 00:23:46,360 we want to find a fixed point of this expected value of
417\ 00:23:46,360 -> 00:23:50,070\ f mapping where W is a random variable
我们想要找到这个以w 为随机变量的映射f 的期望值的不动点
418\ 00:23:50,070 -> 00:23:54,550 okay so in our case of course this
419\ 00:23:54,550 -> 00:23:56.890 involves a bellman equation so one but
420\ 00:23:56,890 \rightarrow 00:23:59,410 let's in what in greater generality
在这个例子中这个表达式是一个一般性的 bellman 方程,
421\ 00:23:59,410 -> 00:24:01,330 and let's assume that this mapping is a
422\ 00:24:01,330 \rightarrow 00:24:03,250 contraction mapping with respect to some norm
我们假设这是一个与某些 nrom 相关的压缩映射
423\ 00:24:03,250 -> 00:24:06,820 so that this equation has a unique
424\ 00:24:06,820 -> 00:24:09,400 solution and also this iteration
425\ 00:24:09,400 -> 00:24:13,590 converges to the unique fixed point okay
所以这个方程有一个不动点, 迭代过程会收敛与不动点
426 00:24:13,590 -> 00:24:17,710 now how would you work this how do you
427 00:24:17,710 -> 00:24:19,120 modify this algorithm
428\ 00:24:19,120 -> 00:24:21,760 so that instead of iterating with full
429\ 00:24:21,760 -> 00:24:24,520 expected values you just use samples sampled approximations
那么我们该如何修改这个算法让他使用采样近似代替迭代时的完全期望
```

3.3 STOCH. APPROX. CONVERGENCE IDEAS

```
430\ 00:24:24.520 \longrightarrow 00:24:30.700 well we could
  431\ 00:24:30,700 -> 00:24:35,050 generate a sequence of damages and use
  432\ 00:24:35,050 \rightarrow 00:24:38,680 the sequence to approximate the expected
  433\ 00:24:38,680 \rightarrow 00:24:43,180 value appears in this direction
   我们生成了一个序列的样本,然后在迭代中用这个序列去近似期望值
  434\ 00:24:43,180 -> 00:24:48,130 so the duration K to approximate this
  435\ 00:24:48,130 -> 00:24:49,830 expression here this expected value
  436 00:24:49,830 -> 00:24:54,070 let's use a Monte Carlo average using
  437\ 00:24:54,070 -> 00:24:56,850 the first k samples okay
   第 k 次迭代时用前 k 个样本进行蒙特卡洛平均近似期望值
  438\ 00:24:56,850 -> 00:24:59,410 that's certainly an algorithm that makes sense
  这是一个很有用的算法
  439\ 00:24:59,410 -> 00:25:03,310 and as K increases this Monte
  440\ 00:25:03,310 -> 00:25:06,100 Carlo approximation is going to be more
  441\ 00:25:06,100 -> 00:25:08,140 and more accurate
   随着 k 的值增加, 蒙特卡洛近似越来越精确
```

```
442\ 00:25:08,140 \longrightarrow 00:25:10,870 so asymptotically you're getting this algorithm in this
  443\ 00:25:10,870 -> 00:25:13,090 it's very simple to show that this kind
  444\ 00:25:13,090 \rightarrow 00:25:14,200 of algorithm works
  从渐进角度上讲,这就是这个算法,可以很简单地说明它能够工作
  445 00:25:14,200 -> 00:25:16,840 it's like iterating with a contraction
  446 00:25:16,840 -> 00:25:20,230 fixed point iteration that converges
  在迭代过程中收缩, 并且收敛到不动点
  447\ 00:25:20,230 -> 00:25:22,060 but there is a problem with this and the
  448\ 00:25:22,060 -> 00:25:24,360 problem is the following
  但是这种方法有一些问题, 如下
  449\ 00:25:24,360 -> 00:25:29,170 here at every Direction K I have to go
  450\ 00:25:29,170 -> 00:25:31,960 back and calculate this expression for
  451\ 00:25:31,960 -> 00:25:34,600 all the previous samples
  每一次迭代 k, 我都必须回去对所有之前的样本按照这个表达式进行一次计算
  452\ 00:25:34,600 -> 00:25:37,230 so there's a lot of computation involved here
  需要很大的计算量
  453\ 00:25:37,230 -> 00:25:41,530 an increasing amount of averaging as the time progresses
  随着迭代进行, 计算量越来越大
  454\ 00:25:41,530 -> 00:25:44,890 so we need to compute
  455\ 00:25:44,890 -> 00:25:46,690 all these values here for all the previous WT
  所以我们需要对所有 w_t 进行计算
  456\ 00:25:46,690 \rightarrow 00:25:50,110 so a more convenient
  457\ 00:25:50,110 -> 00:25:57,520 iteration is to replace this with the
  458\ 00:25:57,520 -> 00:25:59,380 expression that you have already
  459\ 00:25:59.380 -> 00:26:02.380 computed at the duration T
  现在有一个更方便地方法,就是你用已经算过的数据来代替之前的数据
  460\ 00:26:02,380 \rightarrow 00:26:04,900 that's a major simplification because you don't
  461\ 00:26:04,900 -> 00:26:06,790 have to recompute all of these things
  462 00:26:06,790 -> 00:26:10,690 again from time 1 to K but rather you
  463\ 00:26:10,690 -> 00:26:14,520 can use the previous values that you had
  这种方法非常简单因为你不需要重新计算所有值,只用之前算过的值就可以了
  464\ 00:26:15,960 -> 00:26:18,790 so this is similar but requires much
  465 00:26:18,790 -> 00:26:20,650 less computation because it leaves only
  466\ 00:26:20,650 -> 00:26:25,180 one value of F per sample
  这两种方法很相似,但是新的那个需要的计算量更少因为它只留下了一个每个样本的函数值
  467\ 00:26:25,180 -> 00:26:27,010 now it turns out that this algorithm is kind of slow
  468 00:26:27,010 -> 00:26:28,960 because it involves average
  469\ 00:26:28,960 -> 00:26:34,929 and and then then the XS do not change
  470\ 00:26:34,929 -> 00:26:38,919 very much over time
  这种算法被证实比较慢因为它包括求平均,就导致了 x 的值随时间变化得非常慢
  471\ 00:26:38,919 \rightarrow 00:26:41,679 so X of T at previous Direction is fairly close to
  472\ 00:26:41,679 -> 00:26:44,590 the current XK and this thing is close
  473\ 00:26:44,590 \longrightarrow 00:26:46,779 enough to this so that this replacement is valid
  所以之前的迭代中 x_t 与当前的 x_t 非常相近,而且函数 f 的值也非常相近,所以这个替换是有
效的
  474\ 00:26:46,779 \rightarrow 00:26:50,230 but that's what the successive
  475\ 00:26:50,230 -> 00:26:52,960 approximation algorithm does
  这就是近似算法成功的地方
  476\ 00:26:52,960 -> 00:26:56,649 it replaces the naive Monte Carlo approximation with
  477\ 00:26:56,649 \longrightarrow 00:26:58,720 a more convenient computationally approximation
  用一个更方便地近似计算方法来代替原始的蒙特卡洛
  478\ 00:26:58,720 \longrightarrow 00:27:02,769 now if you take this
  479\ 00:27:02,769 -> 00:27:07,389 expression and you can rewrite it in a
  480~00:27:07,389 -> 00:27:09,669 recursive form like this with gamma K equals 1 over K
  现在你可以把这个表达式重新写成这样的, \gamma_k 等于 \frac{1}{k}
  481 00:27:09,669 -> 00:27:13,960 so that you only need
  482\ 00:27:13,960 \longrightarrow 00:27:16,690 one sample represented at each direction
  这样每次迭代你只需要一个样本就可以了
```

```
483\ 00{:}27{:}16{,}690 -> 00{:}27{:}19{,}419 and now you can see that this iteration
  484\ 00:27:19,419 \rightarrow 00:27:22,179 is sort of the same thing as the Q learning direction
  现在你可以看到这个迭代方法与 Q 学习迭代方法是一样的
  485\ 00:27:22,179 \longrightarrow 00:27:25,450 it involves moving at
   486\ 00:27:25,450 \rightarrow 00:27:27,450 the edge in the direction of a sample
   487\ 00:27:27,450 \rightarrow 00:27:29,980 doing interpolation between the current
  488\ 00:27:29,980 -> 00:27:36,809 point a and the and the other sample
   在当前样本和其他样本间做插值让 x 从当前值变化到新的值
  489\ 00:27:37,559 -> 00:27:40,600 so Q learning is stochastic
  490\ 00:27:40,600 \rightarrow 00:27:43,600 approximation if it were done for all
  491\ 00:27:43,600 -> 00:27:46,570\ Q factors simultaneously
   如果对所有的 Q 值都进行仿真, 那么 Q 学习就是一种随机近似方法
  492\ 00:27:46,570 \rightarrow 00:27:50,049 it is stochastic approximation with just this F mapping
   493\ 00:27:50,049 \rightarrow 00:27:54,399 being this mapping here
  这是一种随机近似, 迭代公式中的 F 映射 (倒数第二个公式) 也就是这个映射 (最后一个公式)
  494\ 00:27:54,399 \rightarrow 00:27:57,909 and using a single sample in here instead of the expected
  用一个样本代替这里面(最后一个公式)的期望值
  495\ 00:27:57,909 -> 00:28:02,440 so this provides the
   496\ 00:28:02,440 -> 00:28:04,419 connection between Q learning and stochastic approximation
   所以这提供了 Q 学习和随机近似方法的联系
  497\ 00:28:04,419 \rightarrow 00:28:08,919 to learning now
  498\ 00:28:08,919 -> 00:28:11,980 has a little bit more into it because
  499\ 00:28:11,980 -> 00:28:14,889 the durations are done 1q factor at that
  500\ 00:28:14.889 \rightarrow 00:28:17.620 time but then again this will involve
  501\ 00:28:17,620 -> 00:28:19,600 the theory of a synchronous computation
  502\ 00:28:19,600 -> 00:28:22,600 and that's also what will come into the proof of convergence
  现在将这个还有点早,因为每次迭代更新一个Q值,涉及同步计算,这部分也会出现在Q学
习的收敛性证明里
  503\ 00:28:22,600 \rightarrow 00:28:25,840 so this is not a
  504\ 00:28:25,840 -> 00:28:28,659 proof of convergence by any means
   从任何角度上讲这都不是收敛性的证明
  505\ 00:28:28,659 -> 00:28:31,179 this step has to be justified and also there
  506\ 00:28:31,179 -> 00:28:32,350 are other things that we have omitted
  507\ 00:28:32,350 \rightarrow 00:28:34,480 but gives you an idea about the
  508~00:28:34,480 \longrightarrow 00:28:36,600 mathematical connections with other
  509\ 00:28:36,600 -> 00:28:45,650 class of methodologies okay
  这些东西都被省略了,我主要是想给你一个想法,关于它与其他方法有什么数学上的联系的
3.4 Q-LEARNING COMBINED WITH OPTIMISTIC PI
510\ 00:28:49,669 -> 00:28:52,279 let me mention a variation of Q learning
```

```
511 00:28:52,279 -> 00:28:55,119 that's computationally more efficient
我来介绍一下各种计算效率更高的 Q 学习
512\ 00:28:55,119 -> 00:28:57,999 this is the Q learning direction right
这是 Q 学习迭代公式
513\ 00:28:57,999 -> 00:29:03,859 we pick a pair i k UK and we calculate
514\ 00:29:03,859 -> 00:29:05,749 this expression and we move in the
515 00:29:05,749 -> 00:29:08,450 direction of that expression
我们选择了 (i_k,u_k) 然后在迭代中计算这个表达式
516\ 00:29:08,450 -> 00:29:11,479 now this expression involves minimization over all controls
这个表达式包括对这个表达式进行最小化,找到合适的 u
517\ 00:29:11,479 \longrightarrow 00:29:14,419 and if you have a lot of
518\ 00:29:14,419 \rightarrow 00:29:16,759 controls this may involve a substantial amount of computation
如果 u 非常多, 计算量就会非常大
519\ 00:29:16,759 -> 00:29:21,200 it would be more
520\ 00:29:21,200 -> 00:29:24,379 efficient to remember the good controls
521\ 00:29:24,379 \rightarrow 00:29:27,469 instead of minimizing over all the
```

```
522 00:29:27,469 -> 00:29:29,629 controls minimize or over the good
  523 00:29:29,629 -> 00:29:32,239 controls or perhaps just one control
  把所有好的决策记住,不再在所有控制中选择最小化成本的控制而是在好的控制中选择
  524\ 00:29:32,239 \rightarrow 00:29:35,239 this leads into the idea of maintaining
  525 00:29:35,239 -> 00:29:38,719 a policy a sort of current policy which
  526\ 00:29:38,719 -> 00:29:41,059 you use in here instead of do using a minimum
  这就产生了一种想法、维护一个策略、在最小化时使用当前策略选择控制
  527\ 00:29:41,059 -> 00:29:43,879 and if you try to connect it
  528\ 00:29:43,879 -> 00:29:46,849 back to two policy direction and
  529 00:29:46,849 -> 00:29:50,690 optimistic policy direction
   如果你想要把它与策略迭代和乐观策略迭代联系起来
  530\ 00:29:50,690 -> 00:29:53,359 you may consider a sample version of an
  531\ 00:29:53,359 \longrightarrow 00:29:55,690 optimistic policy Direction algorithm
   你可以考虑采样版本的乐观策略迭代
  532\ 00:29:55,690 \longrightarrow 00:30:00,259 which evaluates the current policy we
  533 00:30:00,259 -> 00:30:02,269 evaluate the Q factors of the current
  534\ 00:30:02,269 \rightarrow 00:30:04,999 policy by using a finite number of
  535\ 00:30:04.999 -> 00:30:07.639 values directions which would not be
  536\ 00:30:07,639 -> 00:30:10,369 important in minimization here
   使用有限次数的值迭代对策略进行评价,是不是最小化就不重要了
  537\ 00:30:10,369 \rightarrow 00:30:12,709 and then improves the policy occasionally after
  538\ 00:30:12,709 -> 00:30:16,329 new K steps by means of this iteration
   然后使用这个公式对策略进行改进
  539\ 00:30:16,329 -> 00:30:18,799 so this is a more efficient version of this
  这是更有效率的算法
  540\ 00:30:18,799 -> 00:30:21,109 but it turns out not to work
   但是事实证明它不工作
  541\ 00:30:21,109 -> 00:30:23,299 there's an all the work
  542 00:30:23,299 -> 00:30:26,749 dating back to 1993 which says that if
  543\ 00:30:26,749 -> 00:30:28,879 you implement Q learning are
  544\ 00:30:28,879 \rightarrow 00:30:33,379 synchronously with with maintaining both
  545\ 00:30:33,379 -> 00:30:35,929 a policy and Q factors then you may
  546\ 00:30:35,929 -> 00:30:37,089 get oscillation
  追溯到 1993 年的工作就说明了在使用同步算法与策略和 Q 值的方式实现 Q 学习的时候会产
生震荡
  547\ 00:30:37,089 \rightarrow 00:30:39,589 however it turns out that this can be
   548\ 00:30:39,589 -> 00:30:42,440 rectified by there's a simple
  549 00:30:42,440 -> 00:30:44,570 modification of this algorithm which is
  550 00:30:44,570 -> 00:30:46,940 does not involve much overhead that
  551\ 00:30:46,940 -> 00:30:49,489 overcomes this convergence problem
   但是事实证明这个问题可以通过修改很少的东西校正这个算法
  552\ 00:30:49,489 -> 00:30:52,519 and this is recent research and I refer you
  553\ 00:30:52,519 -> 00:30:56,179 to a series of papers and I'll leave it
  554\ 00:30:56,179 -> 00:30:58,419 at that
  这些工作可以看这些论文, 我把他们给你们了
  555\ 00:31:02,239 -> 00:31:05,519 okay so we have Q learning for exactly
  556\ 00:31:05,519 \rightarrow 00:31:08,460 factors we have optimistic versions of Q
  557 00:31:08,460 -> 00:31:09,989 learning combinations with policy
  558 00:31:09,989 -> 00:31:12,809 iteration all of those for exact Q factors
  我们讲了精确的 Q 学习,带有策略迭代的乐观版本的 Q 学习,都是用来计算精确值的
```

3.5 Q-FACTOR APPROXIMATIONS

559 00:31:12,809 -> 00:31:20,700 however we mentioned that for 560 00:31:20,700 -> 00:31:23,070 large-scale problems the exact form of Q 561 00:31:23,070 -> 00:31:26,669 learning is not is not it cannot be 562 00:31:26,669 -> 00:31:28,379 applied because of the storage and dimension problem 我提到过对于大规模问题精确 Q 学习由于存储空间和维度问题没法用

```
563\ 00:31:28,379 \longrightarrow 00:31:30,690 so let's try to
   564 00:31:30,690 -> 00:31:33,330 consider basis function approximations
   所以我们考虑基函数近似
   565\ 00:31:33,330 \rightarrow 00:31:40,759 so we introduce a Q factor approximation
   566\ 00:31:40,759 -> 00:31:44,669 which is linear and involved this basis
   567 00:31:44.669 -> 00:31:46.639 functions phi is a feature vector
   568\ 00:31:46,639 -> 00:31:50,729 associated with I u and R is a of weights vector
   我们介绍的近似是一种线性近似,基函数 \phi 是关于 i 和 u 的特征向量, r 是权重向量
   569\ 00:31:50,729 -> 00:31:56,359 and we can try to train r
   570 00:31:56,359 -> 00:31:59,279 by for example approximate policy
   571\ 00:31:59,279 -> 00:32:02,039 direction pick a policy calculate the
   572\ 00:32:02,039 -> 00:32:04,109 corresponding r policy improve and so
   573\ 00:32:04,109 -> 00:32:06,389 on and you can use several language for
   574\ 00:32:06,389 -> 00:32:09,450 policy evaluation like LSTD LSPE and
   575\ 00:32:09,450 -> 00:32:14,940 others that we have discussed
   我们尝试样本近似策略迭代训练 r,选择一个策略计算相关的 r 然后进行策略改进,你可以用
我们之前讲过的 LSTD,LSPE 什么的方法进行策略评价
   576\ 00:32:14,940 \rightarrow 00:32:19,129 very often in practice people use optimistic
   577~00{:}32{:}19{,}129 -> 00{:}32{:}21{,}749 versions of the policy duration method
   实际操作种人们经常用乐观策略迭代
   578 00:32:21,749 -> 00:32:28,589 whereby they have a policy they collect
   579\ 00:32:28,589 -> 00:32:31,229 a few samples maybe one maybe ten maybe
   580\ 00:32:31,229 -> 00:32:32,999 a hundred but not a very large number
   581\ 00:32:32,999 -> 00:32:36,479 and then do an approximate evaluation
   582\ 00:32:36.479 -> 00:32:38.999 based on the current weight the current
   583\ 00:32:38,999 -> 00:32:42,239 policy then do a policy improvement or a
   584 00:32:42,239 -> 00:32:45,149 step towards policy improvement collect
   585\ 00:32:45,149 \longrightarrow 00:32:48,629 more samples and so on
   现在有一个策略,收集一些样本,可能一个,可能十个可能一百个,总之不是很多,然后基于
当前的权重r和策略进行近似策略评价,再进行策略改进,然后继续收集样本评价和改进
   586\ 00:32:48,629 -> 00:32:51,179 so policy evaluation with a few samples policy
   587~00:32:51,179 -> 00:32:53,279 improvement by adjusting the weight
   588 00:32:53,279 -> 00:32:56,489 vector again more policy evaluation and
   589\ 00:32:56,489 -> 00:32:59,519 with samples and so on
   根据已有的样本进行策略评价,通过调整权重向量改进策略,然后继续进行策略评价,这么一
直进行下去
   590\ 00:32:59,519 \rightarrow 00:33:01,529 the most extreme type of algorithm of this type uses just
   591\ 00:33:01,529 -> 00:33:04,169 a single sample a single sample in
   592 00:33:04,169 -> 00:33:07,850 between policy updates
   这个算法的极端情况是只用一个样本进行策略的更新
   593\ 00:33:07,850 \rightarrow 00:33:11,790 okay so here this algorithm generates a
   594\ 00:33:11,790 \rightarrow 00:33:14,490 trajectory of state control pairs as follows
   这个算法生成状态控制对的轨迹是这样的
   595\ 00:33:14,490 -> 00:33:18,660 given the current weight vector
   596\ 00:33:18,660 \longrightarrow 00:33:22,470 that represents an estimate of the okay
   597\ 00:33:22,470 -> 00:33:24,470 together with phi gives you an
   598\ 00:33:24,470 \rightarrow 00:33:27,299 approximate cost function of a current policy
   给定当前权重向量和当前策略的近似成本函数
   599 00:33:27,299 -> 00:33:31,650 you simulate the transition using
   600\ 00:33:31,650 \longrightarrow 00:33:35,640 this this the transition probabilities
   601\ 00:33:35,640 -> 00:33:39,030 of phi of the system
   使用这个系统的 \phi,也就是转移概率仿真生成状态状态转移样本
   602\ 00:33:39,030 -> 00:33:43,740 then generate a control by minimizing the Q factors at the
new point
   然后在新的点通过最小化 Q 值产生控制
   603\ 00:33:43,740 \rightarrow 00:33:48,990 and using that control you
   604 00:33:48,990 -> 00:33:51,960 update the corresponding policy by
```

 $605\ 00:33:51,960 \longrightarrow 00:33:56,120$ changing the the weight vector R and

```
606\ 00:33:56,120 -> 00:33:59,880 this is this is done in the direction of
   607\ 00:33:59,880 -> 00:34:04,410 a TD lambda type of iteration correction
   608\ 00:34:04,410 -> 00:34:07,110 or lsbe direction some kind of iterative
   609\ 00:34:07,110 -> 00:34:10,050 algorithm that makes a correction after
   610\ 00:34:10,050 \rightarrow 00:34:14,219 a single sample
   用这个控制通过改变权重向量 r 的值更新相关的策略, 更新方式通过 LSPE 或者 TD 相关的算
法给出
   611\ 00:34:14,219 \rightarrow 00:34:19,859 so I have a state control pair I use that to generate a
   612 00:34:19,859 -> 00:34:25,500 new new state then reevaluate the
   613\ 00:34:25,500 -> 00:34:29,850 control at that state by minimizing over
   614\ 00:34:29,850 -> 00:34:33,239 the Q factors then move again and so on
   我有一个状态控制对,用这个产生一个新的状态,通过最小化 Q 值生成一个新的状态控制对,
然后估值,改进,估值,一直这么进行下去
   615\ 00:34:33,239 \rightarrow 00:34:37,199 a very chaotic type of algorithm with
   616\ 00:34:37,199 \rightarrow 00:34:39,418 some connection to theory some
   617\ 00:34:39,418 -> 00:34:43,109 connection to validity but very unclear properties
   这种算法很混乱,有的与理论相关,有的与有效性相关,总之没有很明确的性质
   618 00:34:43,109 -> 00:34:46,379 very complex behavior and
   619\ 00:34:46,379 \rightarrow 00:34:48,929 clear validity oscillations all over the place
   620\ 00:34:48,929 \longrightarrow 00:34:55,230 but people have used it a lot
   有效性与震荡什么的很复杂,但是人们用的很多
   621\ 00:34:55,230 -> 00:34:59,490 they claim claim success with it
   他们说这个算法很成功
   622\ 00:34:59,490 -> 00:35:01,050 you have to judge for yourself what kind of success
   623\ 00:35:01.050 -> 00:35:06.450 this is or you know what it means
   你需要自己判断这个成功到底是指什么
   624\ 00:35:06,450 -> 00:35:09,359 there is however solid basis for a version of
   625\ 00:35:09,359 \rightarrow 00:35:13,619 this algorithm for one for one type of
   626 00:35:13,619 -> 00:35:16,790 problem problems of optimal stopping
   在最优停止问题上,这个算法还是很有效的
   627\ 00:35:16,790 \longrightarrow 00:35:19,030 optimal stopping problems we have talked
   628 00:35:19,030 -> 00:35:20,980 very much optimal stopping problems
   629\ 00:35:20,980 \rightarrow 00:35:22,570 involved just two controls for each
   630\ 00:35:22,570 -> 00:35:25,930 stage either stop or continue okay
   我们经常会讨论最优停止问题,每一个阶段都有两个控制,继续或者停止
   631\ 00:35:25,930 \rightarrow 00:35:29,320 an example is when you're trying to
   632 00:35:29,320 -> 00:35:36,250 optimize optimize the time that you
   633\ 00:35:36,250 -> 00:35:38,140 exercise an option like a financial
   634\ 00:35:38,140 -> 00:35:40,660 option what you exercise the option you
   635\ 00:35:40,660 -> 00:35:42,490 sell the option or you keep it for one more step
   636\ 00:35:42,490 -> 00:35:44,850 you either stop or continue
   637\ 00:35:44,850 \rightarrow 00:35:46,060\ okay
   一个例子就是你试图优化一个金融操作,卖出去或者留着,这就是继续或者停止的决定
   638\ 00:35:46,060 -> 00:35:48,070 turns out that Q learning is used very
   639\ 00:35:48,070 -> 00:35:49,890 widely in practice in financial
   640\ 00:35:49,890 \rightarrow 00:35:53,500 engineering financial programs that that
   641\ 00:35:53,500 -> 00:35:58,060 value options and and the optimal
   642\ 00:35:58,060 \rightarrow 00:36:00,960 stopping formulation is the one that
   643\ 00:36:00,960 -> 00:36:04,480 comes in there and there are and in this
   644\ 00:36:04,480 \longrightarrow 00:36:06,730 particular algorithm is valid for this
   645\ 00:36:06,730 -> 00:36:12,210 for this for this problem
   事实证明 Q 学习在金融问题中用的非常多,最优停止问题就是这个,上面的方法都可以很有效
```

地用在这个问题上

3.6 BELLMAN EQUATION ERROR APPROACH

```
646\ 00:36:17,370 \rightarrow 00:36:21,100 okay now let's talk about another type
   647\ 00:36:21,100 -> 00:36:23,710 of algorithm that involves Q factor
   648\ 00:36:23,710 \longrightarrow 00:36:25,740 approximation
   我们来讨论另一个近似 Q 值得方法
   649\ 00:36:25,740 -> 00:36:31,830 instead of using projected equations and
   650\ 00:36:31,830 \longrightarrow 00:36:34,180 the algorithmic we have discussed this
   651\ 00:36:34,180 -> 00:36:35,680 another approach that we mentioned but
   652\ 00:36:35.680 \rightarrow 00:36:37.750 it did not discuss very much the so
   653\ 00:36:37,750 \rightarrow 00:36:40,530 called bellman equation error approach
   能够代替投影方程但是我们讨论的比较少得算法就是 bellman 误差方法
   654\ 00:36:40,530 \rightarrow 00:36:43,360 okay so we'll describe this approach for Q factors
   我们要介绍一下这个方法关于 Q 值的版本
   655\ 00:36:43,360 -> 00:36:47,950 given a policy new we
   656\ 00:36:47,950 -> 00:36:51,150 approximate the Q factors of that policy
   657\ 00:36:51,150 -> 00:36:53,880 using a linear approximation
   658\ 00:36:53,880 -> 00:36:57,100 architecture that is obtained by
   659\ 00:36:57,100 -> 00:36:59,980 minimizing the error in satisfying
   660\ 00:36:59,980 -> 00:37:05,430\ Bellman's equation okay
   给定一个策略 \mu,我们用线性结构对这个策略的 Q 值今昔,也就是满足这个最小化 bellman 误
差的表达式
   661 00:37:05,430 \rightarrow 00:37:13,120 so this is Phi R Phi R is the approximation to Q mu
   \Phi r 是 Q_{\mu} 的近似
   662\ 00:37:13,120 \longrightarrow 00:37:17,380 and F nm is the mapping associated with with
   663\ 00:37:17,380 -> 00:37:20,770 the the policy mu involves expected
   664\ 00:37:20,770 \longrightarrow 00:37:24,280 value and so on
   F_{\mu} 是关于策略 \mu 的映射,这个映射包括期望值
   665\ 00:37:24,280 -> 00:37:28,890 so this is a least squares problem
   这是一个最小二乘问题
   666\ 00:37:28,890 \longrightarrow 00:37:32,470 and the norm here is Euclidean work with respect to some
   667\ 00:37:32,470 -> 00:37:35,950 distribution xi
   这个范数是关于分布 ξ 的欧几里得范数
   668\ 00:37:35,950 -> 00:37:38,200 this approach is related to somewhere to projected
   669\ 00:37:38,200 \longrightarrow 00:37:43,540 equation approach
   这个方法与投影方程有关
   670\ 00:37:43,540 \longrightarrow 00:37:45,700 and and you can and it works as follows given a policy you
   671\ 00:37:45,700 -> 00:37:48,910 solve this problem to find our new then
   672\ 00:37:48,910 \longrightarrow 00:37:52,600 you improve the policy
   这个算法是这样工作的,给定一个策略,逆求解这个最小二乘问题找到 r,然后对策略进行改进
   673\ 00:37:52,600 -> 00:37:53,980 and that allows you to solve this problem for the new
   674\ 00:37:53,980 \longrightarrow 00:37:59,410 policy and so on
   然后用这个最小二乘问题计算新的策略
   675\ 00:37:59,410 \rightarrow 00:38:02,680 now let's look at this for deterministic problems okay
   我们来看看它是怎么作用在确定性问题上的
   676\ 00:38:02,680 -> 00:38:04,840 this approach has found more application
   677\ 00:38:04,840 \longrightarrow 00:38:06,160 for deterministic problems rather than
   678\ 00:38:06,160 -> 00:38:07,600 for stochastic problems and there are reasons for that
   这个方法被发现相对于随机问题,更适用于确定性问题,原因如下
   679\ 00:38:07,600 -> 00:38:09,430 let's look at the
   680\ 00:38:09,430 \longrightarrow 00:38:11,140 special case of deterministic problem
   我们来看这个确定性问题的特殊例子
   681\ 00:38:11,140 \longrightarrow 00:38:13,960 now in the deterministic problem given a
   682\ 00:38:13,960 -> 00:38:16,960 state in the control pair the next state
   683\ 00:38:16,960 -> 00:38:19,900 is completely determined okay
   对于一个确定性问题的状态控制对,下一个状态是完全确定的
   684\ 00:38:19,900 \longrightarrow 00:38:21,460 there are no transition probabilities there is a
   685 00:38:21,460 -> 00:38:25,450 unique state which you go
```

没有转移概率,你将要得到的是一个唯一的状态

```
686\ 00:38:25,450 -> 00:38:27,440 so here's how this approach would be implemented
  这就是这个方法是如何被使用的
  687\ 00:38:27,440 -> 00:38:31,500 given a policy will generate a large set
  688\ 00:38:31,500 -> 00:38:34,790 of sample pairs i K and UK
  689\ 00:38:34,790 \longrightarrow 00:38:38,120 corresponding deterministic costs
  给定一个策略,产生很多状态控制对样本和相应的成本
  690\ 00:38:38,120 \longrightarrow 00:38:40,350 and corresponding transitions which are also
  691\ 00:38:40,350 -> 00:38:46,290 deterministic to the state determines
  692\ 00:38:46,290 -> 00:38:50,070 the state jk jk is the next state in
  693\ 00:38:50,070 -> 00:38:53,540 news of JK is determined from the policy
  状态转移会根据策略确定地到达状态 jk
  694\ 00:38:53,540 -> 00:38:55,980 now you can use a simulator of the
  695\ 00:38:55,980 \rightarrow 00:38:59,310 deterministic system to generate such sample pairs
   现在你可以使用这个确定性系统仿真器来生成这样的样本
  696\ 00:38:59,310 -> 00:39:03,210 give an i and u the
  697\ 00:39:03,210 -> 00:39:05,940 next u tell the simulator to generate JK for mu
   给定一个 i 和 \mu, 可以生成控制 \mu(j_k)
  698\ 00:39:05,940 \rightarrow 00:39:08,610 and you have a policy that
  699\ 00:39:08,610 -> 00:39:12,720 gives you the control at the next state
  700\ 00:39:12,720 \longrightarrow 00:39:16,290 which you need for me you need to plug in here
   然后这个策略就可以给你下一个状态的控制
   701\ 00:39:16,290 \rightarrow 00:39:21,660 so given all these samples you
  702\ 00:39:21,660 -> 00:39:24,030 put them together into a least squares
  703\ 00:39:24,030 \longrightarrow 00:39:29,010 problem that emulates this
  给定了这些样本,把这些样本放到最小二乘问题中并计算
  704\ 00:39:29.010 -> 00:39:33.030 so it's the sum of the errors in satisfying bellman
  705\ 00:39:33,030 -> 00:39:34,350 equation right
   这个是 bellman 误差的和
   706\ 00:39:34,350 -> 00:39:37,650 it's a deterministic bellman equation
  这是一个确定性 bellman 方程
  707\ 00:39:37,650 -> 00:39:40,020 so this is the error corresponding this
  708\ 00:39:40,020 -> 00:39:43,140 error here but instead of taking over
  709\ 00:39:43,140 -> 00:39:45,450 all states you take it only over those samples
  这是相关的 bellman 误差,与原来的对所有状态计算不同,这里对所有的样本进行计算
  710\ 00:39:45,450 -> 00:39:48,900 if this summation includes all
  711\ 00:39:48,900 -> 00:39:50,310 states these two would be a dime equivalent
   如果这个表达式是对所有状态进行的,那么这两个表达式就是等价的
   712\ 00:39:50,310 -> 00:39:54,450 so instead of sampling all
  713\ 00:39:54,450 \rightarrow 00:39:56,370 the states I sample some representative
  714\ 00:39:56,370 -> 00:39:59,340 states i form the error to satisfy
  715\ 00:39:59.340 \rightarrow 00:40:00.420 balanced equation
  716\ 00:40:00,420 -> 00:40:04,590 minimize that to obtain the weight
  717\ 00:40:04,590 -> 00:40:08,180 vector that corresponds to the policy
   用采集有代表性的样本取代对所有状态做计算,我这样处理误差并且最小化来得到相关策略的
权重向量
  718 00:40:08,180 \rightarrow 00:40:10,920 it's a linear least squares problem it
  719\ 00:40:10,920 -> 00:40:13,590 can be solved the solution of this can
  720\ 00:40:13,590 \rightarrow 00:40:17,130 be done in many different ways
  这是一个线性最小二乘问题, 可以被很多方法求解
  721\ 00:40:17,130 -> 00:40:20,190 you need to have enough samples in order for the
  722\ 00:40:20,190 -> 00:40:22,980 matrix involved here to be invertible to
  723\ 00:40:22,980 -> 00:40:25,140 have a unique solution but this can be
   724\ 00:40:25,140 \longrightarrow 00:40:26,190 easily arranged
  725\ 00:40:26,190 -> 00:40:31,910 it's not not a problem okay
   你需要有足够的样本来保证矩阵可逆,然后得到一个唯一解,这很容易做到
  726\ 00:40:31,910 -> 00:40:35,520 let me mention also that for stochastic
  727\ 00:40:35,520 \rightarrow 00:40:37,500 problems is a similar approach but it's more complicated
   随机问题和这个相似但是更复杂
```

```
728\ 00:40:37,500 -> 00:40:39,750 it involves taking Q
729 00:40:39,750 -> 00:40:42,090 samples per iteration it's it's more and
730 00:40:42,090 -> 00:40:46,440 it's it's it's more complicated and and
731 00:40:46,440 -> 00:40:48,330 there are reasons for that
732\ 00:40:48,330 -> 00:40:53,580 refer you to the text
 每一个阶段都要计算 Q 值,这就是复杂的原因
733\ 00:40:53,580 -> 00:40:57,090 however the approach is model free and for the
734\ 00:40:57,090 -> 00:40:58,650 deterministic problems it is very
735\ 00:40:58,650 \longrightarrow 00:41:01,260 attacked attractive for adaptive control
736\ 00:41:01,260 -> 00:41:05,070 of systems with unknown dynamics
 确定性问题的方法不需要知道模型,同样非常适合于不知道动态性的自适应系统
737\ 00:41:05,070 -> 00:41:06,920 for example you may have a linear system
738\ 00:41:06,920 -> 00:41:10,430 like a process or some kind of a robot
 比如你有一个线性系统,比如一个过程或者某种类型的机器人
 739\ 00:41:10,430 \longrightarrow 00:41:13,260 that does things but because of the
740 00:41:13,260 -> 00:41:16,290 circumstances which operates you don't
741\ 00:41:16,290 -> 00:41:18,150 know the dynamics of the system may be
742\ 00:41:18,150 -> 00:41:20,370 linear but you don't know them the
743\ 00:41:20,370 \longrightarrow 00:41:22,350 coefficients the coefficients that
744\ 00:41:22,350 \rightarrow 00:41:25,110 multiply the state coefficient that
745\ 00:41:25,110 \rightarrow 00:41:29,760 multiplies the control and so on
 由于系统的参数你不知道系统的动态性你也不知道,状态和控制的系数都不知道,只知道这是
个线性系统
746 00:41:29,760 -> 00:41:33,360 this approach provides an avenue for dealing
747\ 00:41:33.360 -> 00:41:35.970 with such problems without knowing the
748 00:41:35,970 -> 00:41:38,970 dynamics you go directly towards finding
749\ 00:41:38,970 \longrightarrow 00:41:41,550 an optimal policy what more Q factors optimal policy
这个方法提供了一个不知道系统细节的情况下直接计算最优策略的途径
```

4 ADAPTIVE CONTROL BASED ON ADP

```
750\ 00:41:41,550 \longrightarrow 00:41:47,010 so that's the next topic
  751\ 00:41:47,010 -> 00:41:48,690\ I want to talk about but we'll take a
  752\ 00:41:48,690 -> 00:41:52,680 break this relates to something called
  753\ 00:41:52,680 \rightarrow 00:41:55,500 adaptive dynamic programming which is
  754\ 00:41:55,500 -> 00:41:57,420 really adaptive control based on
  755\ 00:41:57,420 \rightarrow 00:42:00,090 approximate dynamic programming adaptive
  756 00:42:00,090 -> 00:42:02,670 dynamic programming a DP has the same
  757 00:42:02,670 -> 00:42:03,930 acronym as approximate dynamic
  758 00:42:03,930 -> 00:42:08,790 programming which is a DP also but
  759 00:42:08,790 -> 00:42:10,890 adaptive dynamic programming is really a
  760\ 00:42:10,890 -> 00:42:14,070 subset of approximate dynamic program so
  761\ 00:42:14,070 -> 00:42:15,900 we'll talk about that after we take a
  762\ 00:42:15,900 -> 00:42:20,330 break any questions
  这是我要讲的下一个话题,但是我要先休息一会,然后讲自适应动态规划。一种基于近似动态
规划的自适应控制。自适应动态规划与近似动态规划相同,或者说自适应动态规划是近似动态规
划的一个子集。休息之后再讲吧,谁有什么问题么
   (questions
  问题:连续状态空间是否有精确 Q 学习算法
  回答:没有
  763 00:42:31,240 -> 00:42:34,320 [Music]
  764\ 00:42:35,020 -> 00:42:38,440 okay but that's a good question
  765\ 00:42:38,440 -> 00:42:41,260 we have been talking so far about
  766 00:42:41,260 -> 00:42:44,780 discrete space systems finite state
  767 00:42:44,780 -> 00:42:48,320 systems however this approach also has
  768 00:42:48,320 \rightarrow 00:42:50,570 an analogue for continuous space systems
  769 00:42:50,570 -> 00:42:52,700 and in fact it is continuous space
  770\ 00:42:52,700 -> 00:42:55,520 systems that involve for which adaptive
```

```
771 00:42:55,520 \rightarrow 00:42:58,820 control is most applicable basically you
   772\ 00:42:58,820 \rightarrow 00:43:00,920 do the same thing for continuous space
   773\ 00:43:00,920 -> 00:43:16,850 systems you just sample States okay we
   774 00:43:16,850 -> 00:43:19,880 have to distinguish between okay your
   775\ 00:43:19,880 -> 00:43:21,740 question is the following suppose I have
   776\ 00:43:21.740 -> 00:43:23.690 a continuous space problem to begin with
   777 00:43:23,690 -> 00:43:26,240 does there exist an exact Q learning
   778\ 00:43:26,240 -> 00:43:28,160 algorithm that operates on the
   779\ 00:43:28,160 -> 00:43:30,710 continuous space of states the answer is
   780 00:43:30,710 -> 00:43:34,880 no we don't I don't play the literature
   781\ 00:43:34,880 -> 00:43:38,270 has there's no non version of the
   782\ 00:43:38,270 -> 00:43:39,800 algorithm probably it's much more
   783 00:43:39,800 -> 00:43:43,040 complicated perhaps you can you can
   784 00:43:43,040 -> 00:43:44,960 figure out how it might be but there's
   785\ 00:43:44,960 -> 00:43:47,570 no analysis of such an algorithm now
   786\ 00:43:47,570 \longrightarrow 00:43:49,790\ \text{that's one thing Q learning per}
   787 00:43:49,790 -> 00:43:51,950 continuous space system in exact form
   788\ 00:43:51,950 -> 00:43:54,440 that we don't know how it how it
   789\ 00:43:54,440 -> 00:43:57,190 operates but it is possible to apply
   790 00:43:57,190 -\!> 00:44:00,770 your learning for a continuous space
   791\ 00:44:00,770 -> 00:44:03,140 system with cost function approximation
   792\ 00:44:03,140 -> 00:44:06,470 and the R vector then the vector of
   793 00:44:06,470 -> 00:44:08,360 weights the basis functions is only a
   794 00:44:08,360 -> 00:44:11,300 finite number and for that theory so
   795\ 00:44:11,300 \rightarrow 00:44:14,500 there has been a lot of work
   问题:如何保证这个算法的性能
   回答: 没法保证, 收敛和收敛率 (convergence rate) 都没法保证, 最后一段在说成本函数 g
有解析表达的时候会不会有帮助,没听清,但是再说解析表达会有帮助的
   796\ 00:44:21,630 -> 00:44:24,519 okay so the question is how fast
   797\ 00:44:24,519 -> 00:44:26,079 what's the put what are the performance
   798\ 00:44:26,079 \longrightarrow 00:44:34,950 guarantees for good for disabilities III
   799 00:44:34,950 -> 00:44:36,190 okay
   800 00:44:36,190 -> 00:44:37,539 the Q learning algorithm involves
   801\ 00:44:37,539 -> 00:44:41,740 several complicating factors one is the
   802\ 00:44:41,740 \rightarrow 00:44:43,630 resemblance to stochastic approximation
   803\ 00:44:43,630 \rightarrow 00:44:46,569 in the use of samples and the other is
   804\ 00:44:46,569 \rightarrow 00:44:50,380 one Q factor at a time if we were to do
   805\ 00:44:50,380 \rightarrow 00:44:52,869 all two factors simultaneously and
   806\ 00:44:52,869 -> 00:44:54,579 obtain basically a stochastic
   807\ 00:44:54,579 -> 00:44:58,200 approximation algorithm then there are
   808\ 00:44:58,200 -> 00:45:00,519 convergence rate results about
   809 00:45:00,519 \rightarrow 00:45:02,529 stochastic approximation algorithms
   810 00:45:02,529 -> 00:45:04,630 however the Q learning algorithm more
   811\ 00:45:04,630 -> 00:45:07,089 complicated because it involves a
   812 00:45:07,089 -> 00:45:14,230 chaotic kind of chaotic trajectories
   813\ 00:45:14,230 -> 00:45:16,390 over state control pairs and you update
   814 00:45:16,390 -> 00:45:18,640 only one at a time and I don't believe
   815\ 00:45:18,640 \rightarrow 00:45:21,910 that there are interesting convergence
   816\ 00:45:21,910 \rightarrow 00:45:25,829 rate results for this type of algorithm
   817 00:45:25,829 -> 00:45:29,079 if you want a qualitative answer the
   818\ 00:45:29,079 -> 00:45:31,930 algorithm is very slow very slow very
   819\ 00:45:31,930 \rightarrow 00:45:36,190 often and that's typical of stochastic
   820\ 00:45:36,190 -> 00:45:38,829 approximation algorithms so if you are
   821 00:45:38,829 -> 00:45:43,359 interested in high accuracy then that's
   822 00:45:43,359 -> 00:45:44,890 probably something that you cannot yet
   823\ 00:45:44,890 -> 00:45:46,809 but sometimes you get convergence
   824 00:45:46,809 -> 00:45:49,240 reasonably fast particularly if you've
   825 00:45:49,240 -> 00:45:51,339 got no good starting points if you have
```

```
826 00:45:51,339 -> 00:45:54,430 if you have if you do the sampling in an 827 00:45:54,430 -> 00:46:01,119 intelligent way if you've your first 828 00:46:01,119 -> 00:46:03,190 experiences with Q learning are likely 829 00:46:03,190 -> 00:46:07,470 to be frustrating frustratingly slow 830 00:46:07,870 -> 00:46:24,110 form of it it if G were continued okay 831 00:46:24,110 -> 00:46:27,080 so so what kind of G function here would 832 00:46:27,080 -> 00:47:16,790 help perhaps yeah would be also if you 833 00:47:16,790 -> 00:47:19,190 use cost function approximation if you 834 00:47:19,190 -> 00:47:21,020 know the form of G you may be able to 835 00:47:21,020 -> 00:47:23,360 exploit it in order to pick your basis 836 00:47:23,360 -> 00:47:25,850 functions appropriately and that problem 837 00:47:25,850 -> 00:47:28,540 is going to be helpful 838 00:47:30,670 -> 00:47:34,360 any other questions 839 00:47:36,400 -> 00:47:38,319 okay so let's take a break for ten 840 00:47:38,319 -> 00:00:00,000 minutes come back
```