1 ADDITIONAL ISSUES OF AGGREGATION

1.1 OVERVIEW

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1 00:00:00,470 -> 00:00:03,550 [Music]
  2\ 00:00:11,480 -> 00:00:13,010\ okay
  3\ 00:00:13,010 \rightarrow 00:00:18,560 so let's continue here with aggregation
  我们继续讲聚合
  4\ 00:00:18,560 \longrightarrow 00:00:20,820 remember that we define an aggregate
  5 00:00:20.820 -> 00:00:22.230 problem a single problem
  6 00:00:22,230 -> 00:00:23,880 it's a simpler problem involving
  7\ 00:00:23,880 -> 00:00:26,490 aggregate states
  还记得么,我们定义了一个比原系统更简单的有聚合状态的聚合系统
  8\ 00:00:26,490 -> 00:00:28,590 using this disaggregation aggregation probabilities
  9\ 00:00:28,590 -> 00:00:30,689 we can construct transition
  10\ 00:00:30,689 -> 00:00:32,369 probabilities between aggregate States
  11\ 00:00:32,369 \rightarrow 00:00:36,000 and transition costs corresponding to
  12\ 00:00:36,000 -> 00:00:38,610 aggregation States and controls
   使用聚合概率我们可以构造两个聚合状态间的状态转移概率和聚合状态与控制间的转移成本
  13\ 00:00:38,610 \rightarrow 00:00:42,180 we can consider a policy duration algorithm an
  14 00:00:42,180 -> 00:00:45,059 exact policy duration algorithm or any
   15\ 00:00:45.059 -> 00:00:47.610 kind of algorithm that solves this problem exactly
  16\ 00:00:47,610 -> 00:00:50,219 it could be also by simulation
   我们可以考虑使用精确策略迭代算法或者任何算法求解这个问题的精确解,当然也可以使用仿
  17\ 00:00:50,219 -> 00:00:56,370 and however when it comes to
   18 00:00:56,370 -> 00:00:59,449 policy direction there is a certain certain deficiency
  在使用策略迭代时,有一个缺点
   19\ 00:00:59,449 \longrightarrow 00:01:04,890 it has to do with the
  20\ 00:01:04,890 -> 00:01:10,160 fact that I have to apply control u
  21\ 00:01:10,160 -> 00:01:12,869 the same control u for all the
  22\ 00:01:12,869 \rightarrow 00:01:17,000 aggregate for the entire aggregate State
   就是我必须对所有聚合状态使用同一个控制 u
  23\ 00:01:17,000 -> 00:01:20,099 so policies for this problem consists of
  24\ 00:01:20,099 -> 00:01:23,130 assignment of a common control to all
  25\ 00:01:23,130 -> 00:01:28,649 the states that correspond to X
   所以这个问题的策略需要分配给聚合状态 x 下所有的状态一个通用的控制
  26\ 00:01:28,649 \longrightarrow 00:01:30,810 it is possible to address this deficiency and
  27\ 00:01:30,810 \rightarrow 00:01:33,119 that's the first subject I want to discuss now
  这可能导致一些不足, 也是我现在要讲的第一个问题
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1.2 ALTERNATIVE POLICY ITERATION

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28\ 00:01:33,119 -> 00:01:36,420 so here's an alternative
   29\ 00:01:36,420 -> 00:01:39,959 policy Direction algorithm and it deals
   30 00:01:39,959 -> 00:01:41,429 with a faculty proceeding policy
   31\ 00:01:41,429 \rightarrow 00:01:43,259 direction method uses policy that the
   32\ 00:01:43,259 \longrightarrow 00:01:45,450 designed policies designed are controlled
   33\ 00:01:45,450 -> 00:01:48,630 which aggregate state
   这是一种策略迭代算法的替代品,用来处理策略迭代,策略被设计为控制聚合状态
   34\ 00:01:48,630 -> 00:01:51,989 an alternative is to use policy duration for a combined system
   一个替代品是用策略迭代来求解组合系统
   35\ 00:01:51,989 -> 00:01:55,920 not this aggregate system over here
   不是这个聚合系统
   36\ 00:01:55,920 -> 00:02:00.119 but rather consider a super system
   37\ 00:02:00,119 -> 00:02:03,149 a bigger system that involves a state's
   38\ 00:02:03,149 -> 00:02:09,419 both the x and also the i
   而是一个同时有 x 和 i 的更大的系统
   39\ 00:02:09,419 \rightarrow 00:02:12,340 there is a bellman equation associated with the system
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这是这个系统的 bellman 方程
40\ 00:02:12,340 -> 00:02:15,970 in wood for this system I can
41~00:02:15,970 \rightarrow 00:02:18,670 assign controls u to each I rather than to each X
在这个系统中, 我可以把控制 u 分配给每一个 i 而不是每一个 x
42\ 00:02:18,670 \longrightarrow 00:02:22,420 that's the a large system or combined system
这就是一个大系统或者叫组合系统
43\ 00:02:22,420 \longrightarrow 00:02:26,470 now bellman equations
44\ 00:02:26,470 -> 00:02:32,830 involve equations for X equations for AI
45\ 00:02:32,830 \longrightarrow 00:02:37,030 equations for J and related to equations for y
这个 bellman 方程包括 x, i, j 和 y
46\ 00:02:37,030 -> 00:02:40,750 so here are the equations here's the bellman
这就是 bellman 方程
47\ 00:02:40,750 -> 00:02:45,310 equation I'm sorry we're
48 00:02:45,310 -> 00:02:48,670 having a problem with the audio I don't
49\ 00:02:48,670 -> 00:02:51,420 know what I have done wrong
不知道话筒出什么问题了
50\ 00:03:25,470 -> 00:03:28,800 okay so the optimal costs of the
51\ 00:03:28,800 -> 00:03:32,130 aggregate states are related
52\ 00:03:32,130 \longrightarrow 00:03:34,770 using these probabilities to the optimal
53\ 00:03:34,770 -> 00:03:38,220 cost of states I
这个聚合状态的最优成本与i的最优成本与相应的概率有关
54\ 00:03:38,220 \rightarrow 00:03:41,030 the states I satisfy this bellman equation here
状态 i 满足 bellman 方程
55\ 00:03:41,030 \rightarrow 00:03:44,640 with a control chosen within the set use of I
控制在集合 U(i) 中
56\ 00:03:44,640 -> 00:03:51,510 and the we the cost of state J are
57\ 00:03:51,510 \rightarrow 00:03:53,910 related to our stash of Y according to this equation
状态j的成本与y和相应的方程有关
58\ 00:03:53,910 \longrightarrow 00:03:56,610 so this is a set of
59\ 00:03:56,610 -> 00:04:00,390 equations with unknowns both the J
60~00:04:00,390 \rightarrow 00:04:04,230 tildes and the J ones and the r stars
61 00:04:04,230 -> 00:04:07,400 or a much much larger dimensional setting
这是一个方程集合,未知数包括 J, J<sub>1</sub> 和 R* 这是一个很高维度的方程
62\ 00:04:07,400 -> 00:04:11,940 it is possible to use again
63 00:04:11,940 -> 00:04:15,480 exact policy duration and value
64\ 00:04:15,480 \rightarrow 00:04:21,959 iteration and the simulation is process
65\ 00:04:21,959 \longrightarrow 00:04:24,420 is similar involving a simulator of the
66\ 00:04:24,420 \longrightarrow 00:04:26,670 system but the computations are more complicated
可以用精确策略迭代, 值迭代和仿真求解, 但是计算量特别大
67\ 00:04:26,670 \longrightarrow 00:04:30,210 so so there is this
68\ 00:04:30,210 \rightarrow 00:04:32,130 possibility and their approximate value
69\ 00:04:32,130 -> 00:04:33,660 iteration and policy the duration
70\ 00:04:33,660 -> 00:04:35,940 methods associated with it
所以可以使用相关的近似值迭代和近似策略迭代求解
71~00:04:35,940 \longrightarrow 00:04:38,160 you'll find this in the literature and salvage it
72\ 00:04:38,160 -> 00:04:44,310 what kind of method
你可以从文章中找到他们
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1.3 RELATION OF AGGREGATION/PROJECTION

73 00:04:44,310 -> 00:04:47,160 okay the second issue on the touch upon is the relation 74 00:04:47,160 -> 00:04:49,290 between aggregation and the projection approaches 第二个话题是聚合与投影之间的关系 75 00:04:49,290 -> 00:04:52,560 the projected equation 76 00:04:52,560 -> 00:04:56,610 approach involves linear weighting of 77 00:04:56,610 -> 00:04:59,490 features combined with projection to 78 00:04:59,490 -> 00:05:03,840 solve this equation either with the T

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80\ 00:05:05,910 \rightarrow 00:05:09,590 case of single policy
投影方程包括线性加权特征投影的组合来求解这个方程,方程中算子可能是 T, 也可能是 T_{\mu}
81\ 00:05:09,590 \rightarrow 00:05:12,870 the aggregation equation looks like this again it
82\ 00:05:12,870 \rightarrow 00:05:16,710 involves linear weighting of features
聚合方程是这样的,它也包括线性加权特征
83\ 00:05:16,710 \rightarrow 00:05:18,540 now the features the the weights have a
84\ 00:05:18,540 -> 00:05:20,400 different meaning they are related to
85\ 00:05:20,400 \rightarrow 00:05:23,550 the optimal solution or the policy costs
86\ 00:05:23,550 \rightarrow 00:05:26,810 associated with the aggregate problem
这个权重就有不同的含义了, 它与聚合问题的最优解或者策略成本有关
87\ 00:05:27,380 -> 00:05:30,720 there is a reflection that capital Phi
88\ 00:05:30,720 -> 00:05:37,110 is is is is a feature it's a feature
89 00:05:37,110 \rightarrow 00:05:41,340 matrix basis a matrix of basis functions
这里有一个反射, Φ 是一个矩阵形式的基函数
90\ 00:05:41,340 -> 00:05:43,500 but the basis functions here are defined
91\ 00:05:43,500 \longrightarrow 00:05:45,090 in terms of probabilities they are not general
但是这个基函数被定义成概率的形式,并不是一般意义上的基函数
92\ 00:05:45,090 -> 00:05:50,550 and finally there is the mapping
93 00:05:50,550 -> 00:05:54,780 that multiplies T here it is Phi D the
94 00:05:54,780 -> 00:05:56,910 aggregation disaggregation probabilities
95\ 00:05:56,910 -> 00:06:00,180 here is a general weighted Euclidean projection
最后,乘以T的是聚合/分解概率\Phi,而这里是一个一般性的加权欧几里得投影
96\ 00:06:00,180 -> 00:06:06,150 now if the product of Phi and
97\ 00:06:06,150 -> 00:06:10,080\ D is a projection matrix a weighted
98\ 00:06:10.080 \rightarrow 00:06:12.330 projection matrix with respect to some
99\ 00:06:12,330 \rightarrow 00:06:15,330 way that you can Euclidean norm
这里的 ΦD 是一个带有欧几里得范数的加权投影矩阵
100\ 00{:}06{:}15{,}330 -> 00{:}06{:}18{,}330 then the aggregation approach is a special case
101\ 00:06:18,330 \rightarrow 00:06:21,690 of the projection approach
但是聚合方法只是投影方法的一种特例
102\ 00:06:21,690 -> 00:06:27,770 with just a particular instance of projection okay
投影的一种特殊形式
103\ 00:06:27,770 \rightarrow 00:06:31,800 so what kinds of aggregations are are
104\ 00:06:31,800 -> 00:06:34,470 equivalent to some form of projection
那么什么样的聚合与投影等价呢
105\ 00:06:34,470 \rightarrow 00:06:36,270 it turns out that hard aggregations a special case
硬聚合是一种特例
106\ 00:06:36,270 \longrightarrow 00:06:39,270 in hard aggregation where
107\ 00:06:39,270 -> 00:06:42,540 phi consists of zeros and ones then
108\ 00:06:42,540 -> 00:06:44,760 this matrix here can be verified to be a
109\ 00:06:44,760 -> 00:06:47,310 projection with respect to weights
110\ 00:06:47,310 -> 00:06:50,640 xi I which are proportional to the
111 00:06:50,640 \rightarrow 00:06:54,390 disaggregation probabilities
在硬聚合中 \Phi 包括 0 和 1, 这个矩阵 (\Phi D) 可以是与分解概率成比例的关于权重 \xi_i 的投影
112\ 00:06:54,390 \longrightarrow 00:06:56,280 if you assume this aggregation probabilities
113\ 00:06:56,280 \rightarrow 00:06:58,800 that are you that are positive across each aggregate state
如果你假设所有聚合状态聚合概率都是正数
114\ 00:06:58,800 -> 00:07:01,530 and you consider
115\ 00:07:01,530 \rightarrow 00:07:04,740 all of those and scale them so that they
116\ 00:07:04,740 -> 00:07:06,690 add up to one you're going to obtain
117\ 00:07:06,690 -> 00:07:09,510 weights according to which Phi D is
118\ 00:07:09,510 -> 00:07:13,800 going to be a projection
你考虑这些所有的东西并且把他们标量话,他们增加到 1, 你就可以得到权重 \Phi D 是一个投影
119\ 00:07:13,800 \longrightarrow 00:07:16,280 that's simple to verify
这可以很简单地被验证
120\ 00:07:18,300 -> 00:07:20,890 it turns out the hard aggregation is
121 00:07:20,890 -> 00:07:23,680 just about the only case for which the
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 $122\ 00:07:23,680 -> 00:07:25,900$ statement can be made if you have

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125\ 00:07:31,300 -> 00:07:33,420 and is just as good
  事实证明硬聚合是这种情况下唯一的例子,如果你有另一个聚合的方法,它会有另一个情况并
且同样好
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 $123\ 00:07:25,900 -> 00:07:27,520$ different kinds of aggregation however $124\ 00:07:27,520 -> 00:07:31,300$ there is another statement which falls

 $126\ 00:07:33.420 -> 00:07:35.950$ suppose that we consider aggregation

 $127\ 00:07:35.950 \rightarrow 00:07:39.280$ with representative features

假设我们考虑有代表性特征的聚合方法

 $128\ 00:07:39,280 \rightarrow 00:07:41,650$ this is the case where the aggregate states are

 $129\ 00:07:41,650 \rightarrow 00:07:44,620$ subsets of states of the original

 $130\ 00:07:44,620 -> 00:07:47,110$ problem but they may not exhaust the entire space

这是一种聚合状态是原问题的状态的集合但是没有覆盖整个状态空间的例子

 $131\ 00:07:47,110 -> 00:07:50,980$ remember that this was this

 $132\ 00:07:50,980 -> 00:07:54,820$ contains a special cases both the hard

 $133\ 00:07:54,820 \rightarrow 00:07:58,240$ aggregation case and the aggregation

 $134\ 00:07:58,240 \rightarrow 00:08:01,330$ with representative states will you use

135 00:08:01,330 -> 00:08:04,510 simple state aggregate States single

136 00:08:04,510 -> 00:08:07,500 States okay single States realizations

你要记住这是一个特殊的情况、代表性状态聚合会使用单独的聚合状态实现

137 00:08:07,500 -> 00:08:12,820 then for this case it turns out that phi d

 $138\ 00:08:12,820 -> 00:08:18,190$ is a projection but not with respect to a norm

这个例子证明 ΦD 是一种没有范数的投影

139 00:08:18,190 $-\!\!>$ 00:08:21,700 a projection with respect to a

140 00:08:21,700 -> 00:08:25,200 semi norm how weighted the second one

 $141\ 00:08:25,200 -> 00:08:28,740$ where the weights are proportional again

142 00:08:28,740 -> 00:08:32,890 to d however some weights may be zero

一个带有半范数权重的投影,同时权重与 d 成比例,一部分权重可能是 0

 $143\ 00:08:32,890 -> 00:08:36,010$ that's the meaning of semi-norm let me explain this

这代表 semi-norm, 我来解释一下

 $144\ 00:08:36,010 -> 00:08:40,360$ you remember the norm

 $145\ 00:08:40,360 \longrightarrow 00:08:42,490$ for the case of Euclidean weighted projection 还记得欧几里得范数加权投影么

 $146\ 00:08:42,490 \longrightarrow 00:08:47,770$ it involves the sum of the

 $147\ 00:08:47,770 \longrightarrow 00:08:53,080$ squares of the components of J weighted

 $148\ 00:08:53,080 -> 00:08:55,540$ with positive components who has

 $149\ 00:08:55,540 -> 00:08:58,420$ positive weights this is the case of a norm

它包括 J 的平方由正向量加权后累加的平方,这是一种范数

150 00:08:58,420 -> 00:09:01,480 and ξ here is the vector of this weight 这个 ξ 就是权重向量

 $151\ 00:09:01,480 \rightarrow 00:09:05,350$ however suppose that you

 $152\ 00:09:05,350 -> 00:09:08,530$ have some size to be positive and some

 $153\ 00:09:08,530 \longrightarrow 00:09:11,350$ others be zero then you don't have a norm

但是假设这个向量一部分是正数,另一部分是0,它就不是一个范数了

 $154\ 00:09:11,350 -> 00:09:16,390$ you have instead a semi norm

就是一个 semi-norm

 $155\ 00:09:16,390 -> 00:09:21,100$ a semi norm is is you corresponds to having

 $156\ 00:09:21,100 -> 00:09:24,400$ norm zero for some J's that not zero

semi-norm 表示你在计算 J 的范数的时候范数是 0 但是 J 不是 0

 $157\ 00:09:24,400 -> 00:09:27,280$ standard property of the norm

 $158\ 00:09:27,280 -> 00:09:29,920$ is that the norm of the zero vector is

 $159\ 00:09:29,920 -> 00:09:33,130$ zero and any vector whose norm is 0 must

 $160\ 00:09:33,130 \rightarrow 00:09:36,460$ be the zero vector okay

标准范数的性质是 0 向量的范数是 0, 范数是 0 的向量一定是 0 向量

 $161\ 00:09:36,460 -> 00:09:39,730\ norm of J equals zero if and only if J equals zero$

只有当 J 是 0 向量的时候 J 的范数才是 0

 $162\ 00:09:39,730 -> 00:09:41,740$ or semi norms this is not true.

但对于 semi-norm 这就不成立了

 $163\ 00:09:41,740 -> 00:09:46,000$ you may have J norm of J equals zero but

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164\ 00:09:46,000 -> 00:09:48,130\ J may be nonzero
  165\ 00:09:48,130 -> 00:09:50,170 it's just that nonzero components
  166\ 00:09:50,170 -> 00:09:54,640 correspond to zero weights
  J的范数等于 0 但是 J 可能不是 0 向量,有可能非零元素对应 0 权重
  167\ 00:09:54,640 -> 00:09:56,650 okay so this is another concept that I'm throwing at you
  这就是我要告诉你的另一个概念
  168\ 00:09:56,650 -> 00:10:00,340 it's possible to consider semi norms
  169\ 00:10:00,340 \rightarrow 00:10:02,920 and projection with respect rejected
  170\ 00:10:02,920 -> 00:10:06,150 equation with respect to semi enormous
  可以考虑 semi-norm 的投影方程
  171\ 00:10:06,150 -> 00:10:08,650 in order for this approach to make sense
  172\ 00:10:08,650 -> 00:10:11,530 it has to be the case that when you
  173\ 00:10:11,530 \rightarrow 00:10:13,480 project on the approximation subspace
  174\ 00:10:13,480 -> 00:10:15,550 with a semi norm there's a unique
  175\ 00:10:15,550 -> 00:10:19,420 projection associated with it
  为了让这个方法有意义,必须满足的条件就是你在近似子空间用 semi-norm 进行投影的时候,
存在个唯一的投影
  176\ 00:10:19,420 \rightarrow 00:10:21,850 okay if you have that property then everything I
  177\ 00:10:21,850 -> 00:10:23,380 have told you about earlier about
  178 00:10:23,380 -> 00:10:26,350 projections norm projections hold also
  179\ 00:10:26,350 -> 00:10:27,310 for semi norms
  如果有这个性质,那么我之前讲的关于范数投影对 semi-norm 同样成立
  180\ 00:10:27,310 -> 00:10:30,730 the key idea is that this pi gives you
  181\ 00:10:30,730 -> 00:10:33,430 a unique projection and then in that
  182\ 00:10:33.430 -> 00:10:35.290 case with a unique solution to this equation
  主要想法是 \Pi 给了你一个唯一的投影,这样这个方程组 (\Phi r = \Pi T(\Phi r)) 就有一个唯一解
  183\ 00:10:35,290 -> 00:10:38,350 now in order to get a unique
  184\ 00:10:38,350 -> 00:10:40,420 projection for every vector to be
  185\ 00:10:40,420 -> 00:10:43,390 approximation subspace
  为了对近似子空间的所有向量都有一个唯一的投影
  186\ 00:10:43,390 \rightarrow 00:10:46,900 what you need is this matrix here to be invertible
  你需要做的就是保证这个矩阵 (\Phi \Xi \Phi) 可逆
  187 00:10:46,900 -> 00:10:49,330 when you're calculating projections you
  188\ 00:10:49,330 -> 00:10:51,760 invert this matrix here involves
  189 00:10:51,760 -> 00:10:56,380 evolving phi prime xi the diagonal
  190 00:10:56,380 -> 00:11:01,480 matrix of weights and Phi which is this matrix here
  当你计算投影的时候,你需要求这个矩阵 (\Phi\Xi\Phi) 的逆
  191\ 00:11:01,480 -> 00:11:06,310 in the norm projection case
  192 00:11:06,310 -> 00:11:13,480 it's xi is is has all diagonal elements positive
  范数投影情况下,这个 Ξ 的所有对角元素都是正数
  193 00:11:13,480 -> 00:11:17,820 this is invertible provided
  194\ 00:11:17,820 -> 00:11:20,740 there's a full rank assumption for this
  195 00:11:20,740 -> 00:11:24,180 phi matrix
  假设矩阵 Φ 满秩,则可逆性能够保证
  196\ 00:11:24,180 \longrightarrow 00:11:27,450 however this matrix here is small dimensional
  但是这个矩阵维度很低
  197\ 00:11:27,450 -> 00:11:32,200 and and it is possible that it is
  198 00:11:32,200 -> 00:11:34,360 invertible in fact it's very common that
  199 00:11:34,360 -> 00:11:36,700 this matrix is invertible even if some
  200\ 00:11:36,700 -> 00:11:40,230 of the components of xi are zero
  但是这个矩阵可能是可逆的,实际上经常是可逆的,即使 E 的一些元素是 0
  201 00:11:40,230 -> 00:11:43,630 basically this matrix is a weighted sum
  202\ 00:11:43,630 -> 00:11:49,540 of rank one matrices
  203\ 00:11:49,540 -> 00:11:51,370 and and if sufficient number with rank one matrices
  204\ 00:11:51,370 -> 00:11:53,320 are included with positive weights in
  205\ 00:11:53,320 -> 00:11:57,360 the sum then this matrix is invertible
  基本上,这个矩阵是秩为1的矩阵的加权和,如果秩是1的矩阵数量足够多并且加权都是正
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数,那么这个矩阵就是可逆的

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206\ 00:11:59,070 -> 00:12:02,650 so if this matrix is invertible then the
   207 00:12:02,650 -> 00:12:07,360 entire theory of algorithms and and
   208\ 00:12:07,360 -> 00:12:09,660 contraction properties and so on
   209 00:12:09,660 -\!> 00:12:12,280 generalizes to semi norm projected
   210\ 00:12:12,280 -> 00:12:15,460 equations and therefore this entire
   211 00:12:15,460 -> 00:12:17,620 theory applies to aggregation problems
   212\ 00:12:17,620 -> 00:12:19,830 as well with this kind of aggregation
   如果这个矩阵可逆,那么算法和收缩性等整套理论就可以推广到 semi-norm 投影方程,同样可
以应用于所有聚合问题中
   213\ 00:12:19.830 \rightarrow 00:12:22.930 and the interesting part about it is
   214 00:12:22,930 -> 00:12:26,050 that it provides you a way to bring into
   215\ 00:12:26,050 -> 00:12:28,660 play algorithms that are well known and
   216\ 00:12:28,660 -> 00:12:30,820 trusted in the projected equation area
   217\ 00:12:30,820 -> 00:12:35,860 such as calles LSTD,LSPE,TD lambda all of
   218 00:12:35,860 -> 00:12:38,290 those who work in the context of
   219 00:12:38,290 -> 00:12:40,900 aggregation based on the fact that they
   220\ 00:12:40,900 -> 00:12:42,580 can be generalized to semi norm projections
   一个很有趣的事情就是这为投影方程提供了很多很有名的算法,比如 LSTD, LSPE 和 TD(\lambda),
所有这些算法都可以在之前的条件满足的时候对一个 semi-norm 投影方程使用
   221\ 00:12:42,580 \rightarrow 00:12:45,580 so sweeping generalization
   222\ 00:12:45,580 \rightarrow 00:12:47,890 of all these algorithms anything that
   223 00:12:47,890 -> 00:12:51,070 you can do essentially with projected
   224\ 00:12:51,070 \longrightarrow 00:12:53,970 equations you can do with aggregation
   225~00:12:53,970 \longrightarrow 00:12:59,050 equations the aggregation context the
   226\ 00:12:59.050 -> 00:13:02.250 only limitation really is the fact that
   227\ 00:13:02.250 -> 00:13:07.540 Phi and D have to be have to have roles
   228\ 00:13:07,540 -> 00:13:09,100 that are probability distributions
   229\ 00:13:09,100 -> 00:13:11,380 that's the only restriction
   概括一下就是,所有这些算法都可以用在投影方程上,你可以对他们进行状态聚合,唯一的一
个限制就是 Φ 和 D 必须表示概率分布,只有这一个限制条件
   230\ 00:13:11,380 \rightarrow 00:13:13,780 before the logically there is no there is no
   231 00:13:13,780 -> 00:13:18,190 difficulty in using projected equation
   232\ 00:13:18,190 \rightarrow 00:13:22,110 methodology to aggregation
   也就是说用投影方程来求解聚合问题一点难度都没有
   233\ 00:13:29,579 -> 00:13:32,110 this is a recent work and here's a
   234\ 00:13:32,110 \rightarrow 00:13:37,930 reference for it a paper from 2012
   这是一个最近做的工作, 2012 年的文章
   235\ 00:13:37,930 -> 00:13:42,550 it's also briefly discussed in bi9 my 2012
   236\ 00:13:42,550 \rightarrow 00:13:47,050 book but not in great detail
   这篇文章简洁地讨论了我在 2012 写的那本书,但是没有深入讨论
   237\ 00:13:47,050 -> 00:13:49,509 this paper also describes the method of freeform
   238\ 00:13:49,509 -> 00:13:51,579 sampling that I mentioned earlier in the lecture
   这篇文章同样介绍了我在之前的课程提到的 freeform sampling 方法
   239\ 00:13:51,579 \longrightarrow 00:13:56,139 and so it's a broader reference
   240\ 00:13:56,139 -> 00:14:00,629 on the material of of today's lecture
   所以这篇文章在今天的课程中被引用的很广泛
   (someone asking questions
   问题: 概率 d_{xi} 中那些元素是正数, 那些元素是 0
   回答: 硬聚合中聚合状态 x 中的状态 i 对应的元素是 1, 其他的是 0
   241\ 00:14:04,110 -> 00:14:09,149 okay I need questions on this
   有什么问题么
   242\ 00:14:20,030 \rightarrow 00:14:23,870 that is very recent probability is not
   243\ 00:14:23,870 \longrightarrow 00:14:28,970 Arabic for the data like we have all
   244\ 00:14:28,970 -> 00:14:31,360\ stayed
   245 00:14:33,369 -> 00:14:41,589 act politics or predation probability
   246 00:14:44,319 -> 00:14:48,799 probability the act is not ever only for
   247\ 00:14:48,799 \rightarrow 00:14:57,549 that yeah your question has to do with
   248\ 00:14:57,549 -> 00:15:00,049 which ones of these probabilities are
```

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249\ 00:15:00,049 -> 00:15:03,129 positive and which are zero yeah
250\ 00:15:03,129 \rightarrow 00:15:05,779 these probabilities in heart aggregation
251\ 00:15:05,779 -> 00:15:10,160 are always zero for States I outside the
252 00:15:10,160 -\!> 00:15:12,439 aggregate state so you have this group
253 00:15:12,439 -> 00:15:18,639 of state and D is 0 for X going outside
254\ 00:15:18.639 \rightarrow 00:15:22.489 and it could be positive for states
255\ 00:15:22,489 -> 00:15:25,489 inside the assumption here is that for
256\ 00:15:25,489 \rightarrow 00:15:27,230 every aggregate state all these
257 00:15:27,230 -> 00:15:31,839 probabilities are positive ok
258\ 00:15:38,010 -> 00:15:49,060 the fine matrix here no it's not
259\ 00:15:49,060 -> 00:15:53,020 identity it is it is the matrix that
260\ 00:15:53,020 -> 00:15:56,140 that involves the weights involves this
261\ 00:15:56,140 \longrightarrow 00:16:00,670 probabilities along some diagonal if all
262\ 00:16:00,670 \longrightarrow 00:16:05,560 the the probabilities are equal
263\ 00:16:05,560 -> 00:16:08,020 okay one over m let's say if m is the
264\ 00:16:08,020 -> 00:16:11,709 number of states in a good state then it
265 00:16:11,709 -> 00:16:15,430 becomes the identity matrix yes well
266 00:16:15,430 -> 00:16:17,470 even even that I'm not quite sure it may
267\ 00:16:17,470 -> 00:16:21,910 be okay if all the aggregate states have
268\ 00:16:21,910 -> 00:16:26,110 the same number of elements okay then if
269\ 00:16:26,110 \rightarrow 00:16:27,430 all the aggregate seeds have the same
270\ 00:16:27,430 -> 00:16:29,529 number of elements in all states within
271 00:16:29,529 \rightarrow 00:16:34,630 each one is equally likely then this Phi
272\ 00:16:34,630 -> 00:16:38,279\ D is going to be the identity matrix
273\ 00:16:38.279 -> 00:16:42.520 otherwise it will be a matrix that name
274\ 00:16:42,520 -> 00:16:47,680 of nonzero weights okay I mean not
275 00:16:47,680 -> 00:16:50,430 non-uniform way it's non-uniform weights
276\ 00:16:50,430 \longrightarrow 00:16:53,050 and the same thing happens here for this
277\ 00:16:53,050 -> 00:16:55,180 case except that some of the weights may
278\ 00:16:55,180 \rightarrow 00:17:02,339 be 0 it's a similar process okay
问题: 硬聚合是投影的唯一情况么
回答: 是的, 剩下的都是 semi-projection(semi-norm)
279 00:17:14,439 -> 00:17:16,069 okay you're asked an interesting
280\ 00:17:16,069 -> 00:17:19,130 question is hard aggregation the only
281\ 00:17:19,130 -> 00:17:22,849 case for which you have a projection if
282 00:17:22,849 -> 00:17:24,890 you look at the proofs if you look at
283\ 00:17:24,890 -> 00:17:28,220 the analysis you'll see that it must be
284\ 00:17:28,220 -> 00:17:32,090 the only one proving this theorem right
285\ 00:17:32,090 -> 00:17:36,429 maybe it requires some formal argument
286 00:17:36,429 -> 00:17:41,900 but if you just calculate the Phi D and
287\ 00:17:41,900 -> 00:17:43,549 you look what has to happen you know you
288\ 00:17:43,549 -> 00:17:45,799 have a projection you see that that this
289\ 00:17:45,799 -> 00:17:48,650 must be the only only case I believe
290\ 00:17:48,650 -> 00:17:50,330 that under certain assumptions is the
291\ 00:17:50,330 -> 00:17:52,820 only case which gives you a projection
292\ 00:17:52,820 -> 00:17:54,950 but there are many many other cases that
293 00:17:54,950 -> 00:17:58,360 give you a seminar projection
294 00:18:08,039 -> 00:18:10,080 yes exactly
295 00:18:10,080 \rightarrow 00:18:18,070 yeah yeah yeah if if X consists of a
296\ 00:18:18,070 -> 00:18:21,640 number of those then the requirement is
297\ 00:18:21,640 -> 00:18:26,380 that peas are all positive and the
298\ 00:18:26,380 \rightarrow 00:18:29,049 others are 0 for such they have to be in
299\ 00:18:29,049 \rightarrow 00:18:32,010 card aggregation
asking completed)
300\ 00:18:38,470 -> 00:18:41,440 okay so so there's a close connection
301\ 00:18:41,440 -> 00:18:43,059 between this methodology of the
302\ 00:18:43,059 \rightarrow 00:18:45,879 aggregation in projection and it can be
303\ 00:18:45,879 -> 00:18:48,460 exploited very nicely in many different
```

```
304\ 00:18:48,460 -> 00:18:49,149 ways
   这两种方法之间有很紧密的联系,并且可以推广到很多不同的方法
   305~00{:}18{:}49{,}149 -> 00{:}18{:}52{,}509 I mentioned error bounds for projected
   306\ 00:18:52,509 \rightarrow 00:18:55,059 equations this error bounds Terry over to aggregation
   比如误差上界投影方程, 计算聚合的误差上界
   307\ 00:18:55.059 -> 00:18:58.179\ I\ mentioned\ TD\ lambda\ L
   308~00{:}18{:}58{,}179~{-}{>}~00{:}19{:}02{,}500~\mathrm{STD} LSP for projected equations these
   309\ 00:19:02,500 -> 00:19:15,789 can also be this method can also be be
   310\ 00:19:15,789 -> 00:19:20,490 used for aggregation in the involve
   311 00:19:20,490 -> 00:19:23,980 multi-step simulation okay from
   312 00:19:23,980 -> 00:19:28,960 aggregate state two to regular state to
   313\ 00:19:28,960 \longrightarrow 00:19:31,149 aggregate state and so on over multiple steps
   我之前提到的 LSTD, LSPE 和 TD(\lambda) 投影方程都可以用来求解多阶段仿真的聚合问题, 从
聚合状态到原装胎,再到聚合状态,再到原状态
   314\ 00:19:31,149 \rightarrow 00:19:34,509 it's it's easy to generalize all
   315\ 00:19:34,509 -> 00:19:36,159 these algorithms to the aggregation
   316 00:19:36,159 -> 00:19:39,159 context and this special case is very
   317 00:19:39,159 -> 00:19:41,980 general as I mentioned earlier it sort
   318\ 00:19:41,980 -> 00:19:46,029 of includes some of the basic examples
   319\ 00:19:46,029 -> 00:19:55,570 useful examples of aggregation
   他们很容易被应用到聚合问题中,这些例子也非常一般,我之前提到的例子都是比较基本而且
有用的例子
1.4 DISTRIBUTED AGGREGATION I
   321\ 00:19:57.340 \rightarrow 00:19:59.789\ today's\ lecture
```

```
320\ 00:19:55,570 -> 00:19:57,340 okay now let me go into the last topic for
   让我们来看看今天的最后一个话题
   322\ 00:19:59,789 \longrightarrow 00:20:02,440 which has to do with parallel computation distributed
   323\ 00:20:02,440 -> 00:20:08,110 computation using aggregation
   聚合问题的并行/分布式计算
   324\ 00:20:08,110 -> 00:20:10,690 so we want to consider the composition of the state
   325\ 00:20:10,690 \longrightarrow 00:20:13,419 space and distribute solution of large
   326\ 00:20:13,419 -> 00:20:16,389 scale discounted problems using hard aggregation
   我们想要考虑的是大型折扣问题的硬聚合状态空间分解和分布式方案
   327\ 00:20:16,389 -> 00:20:23,879 we have discussed already
   328\ 00:20:23,879 \longrightarrow 00:20:27,159 distributed value iteration whereby we
   329 00:20:27,159 -> 00:20:29,100 can do value directions simultaneously
   330\ 00:20:29.100 -> 00:20:33.309 for different states in fact the
   331 00:20:33,309 \rightarrow 00:20:35,080 directions may be a synchronous and
   332\ 00:20:35,080 -> 00:20:37,720 still the method will work
   我们之前讨论过分布式值迭代,我们可以使用仿真对不同的状态进行值迭代,事实上迭代可以
是同步进行的,而且这个方法能够工作
   333\ 00:20:37,720 \longrightarrow 00:20:40,120 we're considering something similar here we
   334\ 00:20:40,120 -> 00:20:44,019 partition the big original state
   335 00:20:44,019 -> 00:20:47,740 space into subsets so how much identical
   336\ 00:20:47,740 -> 00:20:49,659 at space we cut it down into many
   337\ 00:20:49,659 \longrightarrow 00:20:50,720 pieces
   我们考虑一个相似的问题,把一个大的原始状态空间划分为几个子集合
   338\ 00:20:50,720 -> 00:20:53,210 and we have a parallel processor that
   339\ 00:20:53,210 \rightarrow 00:20:56,030 will do value iteration on part of the state space
   我们有几个并行的处理器,可以对一部分状态空间进行值迭代
   340\ 00:20:56,030 -> 00:20:58,850 okay so that would be the
   341\ 00:20:58,850 \rightarrow 00:21:02,180 way to do it and in fact with earlier
   342\ 00:21:02.180 \rightarrow 00:21:04.190 theory that we're discussing the second lecture applies
   我们在第二次课程已经讲过这个理论了
   343\ 00:21:04,190 -> 00:21:08,540 however still each
   344\ 00:21:08,540 -> 00:21:11,690 processor has to consider not only the
```

```
346\ 00:21:14,810 \rightarrow 00:21:17,270 the states in the other subsets
  在这里每一个处理器不仅需要考虑一个状态在它所属的子集合,同时还要考虑其他子集合的状
  347\ 00:21:17,270 -> 00:21:20,420 because there may be transitions from his States
  348\ 00:21:20.420 -> 00:21:23.300 into the states of other processors and
  349\ 00:21:23,300 -> 00:21:26,060 they have to be taken into account when
  350\ 00:21:26,060 -> 00:21:29,030 you write Bellman's equation or you do the value iteration
  因为状态转移有可能从当前状态到其他处理器处理的状态,因此它必须考虑到这些事情,然后
你可以写出 bellman 方程或者进行值迭代
  351\ 00:21:29,030 \longrightarrow 00:21:33,470 so the idea of
  352\ 00:21:33,470 -> 00:21:36,170 aggregation within this context is to
  353\ 00:21:36,170 -> 00:21:40,730 consider exact values for the local
  354\ 00:21:40,730 \rightarrow 00:21:44,180 states and aggregate values for the other states
  聚合的思路就是考虑当前状态子集合的值迭代和其他状态的聚合状态
  355\ 00:21:44,180 -> 00:21:47,030 and these aggregate values
  356\ 00:21:47,030 -> 00:21:49,220 will be computed by the other processors
  357\ 00:21:49,220 -> 00:21:52,580 and sent over to each processor who will
  35800:21:52,580 -\!>00:21:55,330 then do an approximate value iteration
  359\ 00:21:55,330 -> 00:21:59,780 using exact values for its own aggregate
  360\ 00:21:59,780 \rightarrow 00:22:05,750 values for the remainder of the world
  这些聚合值被其他处理器计算,使用自己的状态进行精确值迭代,使用其他处理器的聚合状态
值进行近似值迭代
  361\ 00:22:05,750 \rightarrow 00:22:07,130 so this is the distributed value iteration scheme
  这就是分布式值迭代方法
  362\ 00:22:07,130 -> 00:22:10,790 we have a processor l associated
  363\ 00:22:10,790 -> 00:22:14,360 with a subset of states as L
  我们有处理状态子集合1的处理器1
  364\ 00:22:14,360 -> 00:22:17,990 and this processor maintains detail and the exact
  365 00:22:17,990 -> 00:22:22,130 local cost J sub I for every original
  366\ 00:22:22,130 \rightarrow 00:22:26,210 system state that is local within its own subset
  这个处理器计算它所属的状态子集合中每一个原始状态的精确局部成本 J(i)
  367\ 00:22:26,210 \rightarrow 00:22:30,590 and considered aggregate
  368\ 00:22:30,590 \longrightarrow 00:22:34,850 costs for the other subsets one number for each subset
  考虑其他子集合的聚合状态的成本,每一个子集合都对应一个数
  369\ 00:22:34,850 -> 00:22:38,420 so I need to maintain in
  370\ 00:22:38,420 -> 00:22:44,420 my memory just the costs of my own state
  371\ 00:22:44,420 -> 00:22:49,810 and one number for every other processor
  所以我需要在我自己的内存中维护的只有我自己的子集合的状态的成本和其他处理器的成本
  372\ 00:22:50,710 -> 00:22:54,830 so it maintains an aggregate cost within
  373\ 00:22:54,830 -> 00:22:58,700 its own local states and it sets
  374\ 00:22:58,700 -> 00:23:00,260 this aggregate cost to the other
  375\ 00:23:00,260 -> 00:23:07,190 aggregate states or processors
  所以一个处理器需要维护的聚合成本包括他自己的状态成本和其他处理器计算的聚合状态成本
  376~00:23:07,190 -> 00:23:10,910 so how is this done processor L now the l-th
  377\ 00:23:10,910 -> 00:23:15,620 processor updates J sub I and also the
  378\ 00:23:15,620 -> 00:23:19,160 aggregate cost as follows it does a
  379\ 00:23:19,160 -> 00:23:21,940 value iteration for its local states
  380\ 00:23:21,940 \rightarrow 00:23:26,060 which involves the current local cost
  381 00:23:26,060 \rightarrow 00:23:29,080 vector and an aggregate cost vector
  382 00:23:29,080 -> 00:23:34,580 received by other processors
  所以现在的第1个处理器需要更新 J(i) 和聚合状态成本,对处理器的状态进行值迭代,包括
当前局部成本向量和其他处理器算出的聚合成本向量
  383\ 00:23:34,580 -> 00:23:40,520 so RK is the vector of aggregate cost vector of
  384\ 00:23:40,520 -> 00:23:43,370\ aggregate\ costs\ associated\ with\ the
  385\ 00:23:43,370 -> 00:23:49,130 other professors at time k
  所以 R_k 是其他处理器在时间 k 时计算的聚合成本向量
  386\ 00:23:49,130 \rightarrow 00:23:52,430 and this mapping is one of those generalized
```

 $345\ 00:21:11,690 -> 00:21:14,810$ states within its own subset but also

```
387\ 00:23:52,430 -> 00:23:55,430 mappings that I talked to heard in the
   388\ 00:23:55,430 -> 00:23:58,670 class and it involves a form of
   389\ 00:23:58,670 -> 00:24:01,580 Bellman's equation modified to
   390 00:24:01,580 -> 00:24:05,090 accommodate aggregate cost
   所以这个映射 (H_l(i,u,J,R)) 是一个一般性的映射,它包括被修正的包括聚合成本的 bellman
   391\ 00:24:05,090 -> 00:24:09,650 so this is the one stage cost this is the one
   392\ 00:24:09,650 -> 00:24:14,860 stage cost associated with state I
   这是状态 i 一个阶段的成本
   393\ 00:24:14,860 -> 00:24:20,180 this is the expected cost to go to local
   394\ 00:24:20,180 -> 00:24:26,360 states to J's within SL and get cost to
   395\ 00:24:26,360 -> 00:24:30,080 Go for going into states into other subsets
   这一项 (第二项) 是本集合的长期期望成本,这一项 (第三项) 是其他处理器计算得到的长期成
   396\ 00:24:30,080 \rightarrow 00:24:35,390 so it's an iteration that
   397\ 00:24:35,390 \rightarrow 00:24:39,080 involves a much smaller factor the
   398\ 00:24:39,080 -> 00:24:43,630 vectors of J and one number per
   399\ 00:24:43,630 -> 00:24:47,350 other processor that sort of aggregates
   40000:24:47,350 ->00:24:54,740 the the cost of aggregates the current
   401\ 00:24:54,740 -> 00:24:57,290 cost value of the other processors
   所以这个迭代只包含很少的元素,向量 J, 其他处理器的当前成本
   402\ 00:24:57,290 -> 00:24:59,660 and the interesting thing about this mapping
   403\ 00:24:59,660 -> 00:25:01,790 is that it is a contraction mapping a
   404\ 00:25:01,790 \longrightarrow 00:25:04,670 souped norm contraction and therefore
   405\ 00:25:04.670 -> 00:25:07.260 this iteration converges in
   406\ 00:25:07,260 -> 00:25:09,540 converges to a unique solution but also
   407\ 00:25:09,540 -> 00:25:11,280 converges as synchronously when
   408 00:25:11,280 -> 00:25:13,740 implemented totally synchronously
    一个很有意思的事情是这个映射是一个 sup-norm 压缩映射,所以这个迭代会收敛于一个唯一
解,使用同步算法时也会收敛
   409\ 00:25:13,740 -> 00:25:15,750 so each one of these processors computes
   410\ 00:25:15,750 -> 00:25:18,210 whenever he wishes sends out aggregate
   411\ 00:25:18,210 -> 00:25:22,250 values whenever he wishes that it seems
   412\ 00:25:22,250 -> 00:25:25,710 unpredictably values from other
   413\ 00:25:25,710 -> 00:25:28,260 processors combines them in within this
   414\ 00:25:28,260 -> 00:25:30,330 iteration and even with the
   415\ 00:25:30,330 -> 00:25:38,870 communication some health
   416\ 00:25:45,880 -> 00:25:47,680 I'm concerned that somebody may be
   417 00:25:47,680 -> 00:25:50,500 shooting at me maybe I'll just I'll just
   418\ 00:25:50,500 -> 00:25:54,580 take it off okay how about that a new
   419\ 00:25:54,580 -> 00:25:58,480 battery we don't really believe that I
   420\ 00:25:58,480 -> 00:26:00,670 would say that since I'm wearing the
   421 00:26:00,670 -> 00:26:03,640 microphone for the video I think we're
   422 00:26:03,640 -> 00:26:07,420 fine I just talk a little louder okay
   423 00:26:07,420 -> 00:26:11,410 okay can you take it off it's this one
   424 00:26:11,410 -> 00:26:14,710 right okay okay maybe a matter of battery
   425\ 00:26:14,710 -> 00:26:18,850 okay so oops that's not the
   426\ 00:26:18,850 \rightarrow 00:26:19,270\ \text{problem}
   427 00:26:19,270 -> 00:26:21,010 it may be your problem your your
   428\ 00:26:21,010 -> 00:26:28,480 microphone is is the problem okay how
   话筒突然坏了, 笑死了
   429\ 00:26:28,480 -> 00:26:36,640 about now okay okay let me review here
   430\ 00:26:36,640 -> 00:26:37,690 and maybe you can think of your questions
   我再复述一遍, 你可以想象有没有什么问题
   431 00:26:37,690 -> 00:26:42,310 a parallel computing setting
   432 00:26:42,310 -> 00:26:44,380 with a partition of the state space two
   433\ 00:26:44,380 -> 00:26:49,000 subsets each processor controls the does
   434\ 00:26:49,000 -> 00:26:52,030 value duration for the states in the subset
```

```
把状态空间费城很多子空间,每个处理器对一个子空间中的状态进行值迭代,多个处理器构成并行计算
```

- $435\ 00:26:52,030 \rightarrow 00:26:56,740$ and it does the value duration by
- $436\ 00:26:56,740 \longrightarrow 00:26:59,440$ maintaining local costs corresponding to
- $437\ 00:26:59,440 -> 00:27:02,050$ the local state and aggregate costs that
- $438\ 00:27:02.050 \rightarrow 00:27:04.930$ receives from other processors and also
- $439\ 00:27:04,930$ —> 00:27:06,940 computes aggregate cost which it sends out to other processors 处理器值迭代时需要维护自己的子集的状态和聚合状态的成本,每一个处理器计算自己的子集的成本后通知其他处理器
 - $440\ 00:27:06,940 \rightarrow 00:27:09,910$ and this is the
 - $441\ 00:27:09,910 -> 00:27:12,340$ value duration and if you look at this
 - $442\ 00:27:12,340 -> 00:27:14,320$ expression here it's not hard at all to
 - 443 00:27:14,320 -> 00:27:16,450 show that's a contraction mapping and
 - 444 00:27:16,450 -> 00:27:19,060 has a unique fixed point
- 这就是值迭代,如果你看一下这个表达式,这一点都不难发现这是一个压缩映射并且由一个不 动点
 - $445\ 00:27:19,060 \rightarrow 00:27:20,860$ now that unique fixed point of this mapping here that we
 - $446\ 00:27:20,860 -> 00:27:23,230$ compute this way is not the exact optimal cost
 - 这个映射的不动点不是精确最优成本
 - $447\ 00:27:23,230 -> 00:27:25,810$ it is something that's approximate
 - 这是一个近似解
 - $448\ 00:27:25,810 -> 00:27:28,450$ it is locally exact you
 - $449\ 00:27:28,450 -> 00:27:31,480$ might say but it uses approximate
 - $450\ 00:27:31,480 -> 00:27:34,810$ quantities from other processors
 - 它的局部解是精确的, 但是从其他处理器获得的数值是近似的
 - $451\ 00:27:34,810 \rightarrow 00:27:37,210$ so it's something that is related within some
 - $452\ 00:27:37,210 -> 00:27:39,940$ error bound of the exact optimal cost
 - $453\ 00:27:39,940 \rightarrow 00:27:43,530$ vector but it is not the optimal thing
 - 所以这个解不是最优的,是存在误差上界的
 - $454\ 00:27:43,530 \longrightarrow 00:27:48,280$ however this value Direction involves a
 - $455\ 00:27:48,280 -> 00:27:50,860$ lot less computation than if you were to
 - 456~00:27:50,860 -> 00:27:53,500 do value iteration using exact quantities
 - 这种值迭代减少了很大的计算量,如果与精确值迭代相比的话

1.5 DISTRIBUTED AGGREGATION II

- $457\ 00:27:53,500 -> 00:28:01,170$ okay so this slide shows
 - 458 00:28:01,170 -> 00:28:04,000 basically many other things I've said already
 - 这一页展示了我之前说过的基础内容
 - $459\ 00:28:04,000 -> 00:28:06,460$ we can show that this iteration
 - $460\ 00:28:06,460 -> 00:28:08,350$ involves a sup norm contraction of modules alpha
 - 我们可以看到这个迭代包括关于 α 的 sup-norm 收缩
 - $461\ 00:28:08,350 \longrightarrow 00:28:10,840$ so it converges to the
 - $462\ 00:28:10,840 \longrightarrow 00:28:13,240$ unique solution of this system of equations
 - 它会收敛到这个系统的唯一解
 - $463\ 00:28:13,240 \rightarrow 00:28:15,370$ the system of equations that
 - 464 00:28:15,370 -> 00:28:18,490 you are operating on this is not bellman
 - $465\ 00:28:18,490 -> 00:28:21,190$ equation for the original problem
 - 这个方程不是 bellman 方程
 - 466 00:28:21,190 \rightarrow 00:28:22,630 it's something else it's something approximate 这是一个近似方程
 - 467 00:28:22,630 -> 00:28:31,090 and you can show that it's
 - $468\ 00:28:31,090 -> 00:28:33,550$ actually a contraction mapping using the
 - $469\ 00:28:33,550 \rightarrow 00:28:36,630$ fact that this is a the disaggregation
 - $470\ 00:28:36,630 -> 00:28:39,670$ probabilities a former probability
 - $471\ 00:28:39,670 -> 00:28:41,760\ distribution$
 - 使用分解概率分布 dli 能够让 H 是一个压缩映射
 - $472\ 00:28:46,930 -> 00:28:48,790$ it's also possible to make a connection

```
473\ 00:28:48,790 \rightarrow 00:28:50,800 with an aggregate dynamic programming
   474\ 00:28:50,800 -> 00:28:52,930 problem the difference being that the
   475 00:28:52,930 -> 00:28:56,710 mapping H involves the exact values of
   476\ 00:28:56,710 \rightarrow 00:28:59,170\ J within the subset rather than the aggregate value
   这个方程可以叫聚合动态规划,与 bellman 方程的区别是映射 H 包括精确值 J 而不是聚合值
   477\ 00:28:59.170 \longrightarrow 00:29:04.180 I mentioned earlier a
   478 00:29:04,180 -> 00:29:06,250 synchronous implementation in an a
   479 00:29:06,250 -> 00:29:08,440 synchronous version of the algorithm The
   480\ 00:29:08,440 -> 00:29:11,170\ aggregate\ costs\ may\ be\ outdated\ to
   481\ 00:29:11,170 -> 00:29:13,090 account for communication delays between
   482\ 00:29:13,090 -> 00:29:15,730 aggregate states so the process was
   483\ 00:29:15,730 \longrightarrow 00:29:18,310 making me communicating irregularly but
   484\ 00:29:18,310 -> 00:29:20,290 no matter what the communication delays
   485\ 00:29:20,290 -> 00:29:22,330 are as long as the processes keep
   486 00:29:22,330 -> 00:29:26,200 computing this method will work
   我之前提过同步版本的算法,聚合成本由于聚合状态通信延迟而过时,处理器一直在计算,但
通信时间是不规则的,这个方法用在聚合问题上会工作
   487\ 00:29:26,200 \longrightarrow 00:29:27,640 and the proof of convergence is based on the
   488\ 00:29:27,640 -> 00:29:28,900 general theory of a synchronous
   489 00:29:28,900 -\!> 00:29:31,450 computation a single distributed
   490\ 00:29:31,450 -> 00:29:33,700 computation that we described in the
   491\ 00:29:33,700 -> 00:29:35,800 second lecture and there is a theory
   492\ 00:29:35,800 \longrightarrow 00:29:37,570 associated with that that you can find in the references
   收敛性的证明基于我们第二次课程讲的同步计算的一般性理论,你可以在引用列表中找到它
   493\ 00:29:37.570 \rightarrow 00:29:44.800 okay so we reached the
   494\ 00:29:44.800 -> 00:29:48.520 end of a lecture there is we covered in
   495\ 00:29:48,520 -> 00:29:52,350 these last two lectures a very large
   496\ 00:29:52,350 -> 00:29:57,970 domain of problems and methods
   这次课程结束了,我们在这两次课程中讲了非常多的问题和方法
   497\ 00:29:57,970 \rightarrow 00:30:00,520 the problems are challenging approximation
   498\ 00:30:00,520 -> 00:30:11,440 is is is a difficult process with the
   499\ 00:30:11,440 -> 00:30:14,650 outcome not very certain there are very
   500 00:30:14,650 -> 00:30:17,110 few guarantees in this process you try
   501\ 00:30:17,110 -> 00:30:19,120 to solve very difficult problems and
   502\ 00:30:19,120 \longrightarrow 00:30:21,520 sometimes we succeed sometimes you do not
   这个问题很有挑战性,近似是很难的,而且没有理论保证,你会尝试解决很难的问题,但是有
时候能成功有时候不能
   503\ 00:30:21,520 -> 00:30:24,700 there are some interesting methods
   504 00:30:24,700 -> 00:30:26,800 that you can choose from and you can try
   505\ 00:30:26,800 -> 00:30:30,280 none of them is guaranteed to have success
   你会尝试很多有趣的方法,但是这些方法中没有一个能够保证成功
   506\ 00:30:30,280 -> 00:30:33,820 however you can try it and if
   507\ 00:30:33,820 -> 00:30:36,040 you have some insight and the standard
   508\ 00:30:36,040 -> 00:30:37,900 theory behind them you have a better
   509 00:30:37,900 -> 00:30:42,480 probability of success
   但是你在使用那些有标准理论支撑的方法时就很可能会成功
   510\ 00:30:42,480 -> 00:30:44,920 the two signposts projected equation aggregation are
   511 00:30:44,920 -> 00:30:46,990 related I think with an important point
   我认为投影方程和聚合方程有关联是一个非常重要的点
   512\ 00:30:46,990 -> 00:30:50,020 and they are suitable for different types of problems
   他们很适合不同类型的问题
   513\ 00:30:50,020 \rightarrow 00:30:53,560 there are all sorts of
   514\ 00:30:53,560 -> 00:30:55,330 other issues that we have not covered
   还有很多话题我们没有涉及
   515\ 00:30:55,330 -> 00:30:56,890 and
   516\ 00:30:56,890 -> 00:30:59,529 I've just in the last few slides and
   517 00:30:59,529 -> 00:31:01,419 just went through some of them but both
   518 00:31:01,419 -> 00:31:03,850 projections projected equations and
```

```
519\ 00:31:03,850 \longrightarrow 00:31:09,010 aggregation involve a lot of interesting
   520\ 00:31:09,010 -> 00:31:11,169 theory and also a lot of unanswered
   521 00:31:11,169 -> 00:31:13,720 questions having to do with exploration
   522 00:31:13,720 -> 00:31:17,350 having to do with with oscillations some
   523\ 00:31:17,350 \longrightarrow 00:31:19,529 of the questions that you asked are unresolved
   我要用剩下的几页来讲一下,投影方程和聚合方程还有很多有趣的理论和没有回答的问题还没
有涉及, 比如探索, 震荡等你们问过的事情
   524\ 00:31:19,529 \rightarrow 00:31:26,769 so let me stop here and let
   525\ 00:31:26,769 -> 00:31:30,570 you ask questions have any questions
   下面你们有什么问题就可以问了
   (question and answer
   问题: 你讲的聚合动态规划方法能求解 140 页的那个方程组么
   回答: 能,这个问题不是原问题,是与原问题相关的一个问题,求到的是近似解
   526\ 00:31:41,880 \rightarrow 00:31:44,640 yes indeed you are asking whether this
   527\ 00:31:44,640 -> 00:31:48,799 method whether whether this method
   528 00:31:48,799 -> 00:31:51,809 solves this problem it will solve this
   529 00:31:51,809 -> 00:31:54,720 problem it will solve this system of
   530\ 00:31:54,720 -> 00:31:57,120 equations but that's not the exact
   531\ 00:31:57,120 -> 00:31:58,679 system of equations that we want to
   532\ 00:31:58,679 -> 00:32:01,850 solve it's only another system related
   533\ 00:32:01,850 \rightarrow 00:32:04,559 conceptually to the origin no but the
   534\ 00:32:04,559 \longrightarrow 00:32:06,000 problem is approximately an
   535~00{:}32{:}06{,}000~->~00{:}32{:}07{,}799 approximation in the solution is also an
   536\ 00:32:07,799 -> 00:32:10,530 approximation to what we want to solve
   537\ 00:32:10.530 \rightarrow 00:32:13.640 the optimal cost factor
   问题: 为什么映射 H 是一个压缩映射
   回答: 没听懂, 得去看看压缩映射的定义, 说是很简单
   538\ 00:32:21,090 -> 00:32:24,030 one day there's a website and the other
   539\ 00:32:24,030 \rightarrow 00:32:27,200 way the rest of the other states
   540\ 00:32:27,200 -> 00:32:32,300 difficult to think about why there's
   541 00:32:35,460 -> 00:32:38,619 [Music]
   542 00:32:51,760 -> 00:32:55,429 you okay so why is this mapping a
   543 00:32:55,429 -> 00:33:01,610 contraction mapping you simply verify
   544\ 00:33:01,610 -> 00:33:03,620 the definition of a soup norm
   545\ 00:33:03,620 -> 00:33:06,980 contraction in other words you take take
   546\ 00:33:06,980 -> 00:33:12,559 two vectors JR and J bar R Bar and form
   547\ 00:33:12,559 -> 00:33:14,990 the difference and then the fact that
   548\ 00:33:14,990 -> 00:33:16,789 these are probability distributions is
   549\ 00:33:16,789 -> 00:33:19,250 very critical and there is also an alpha
   550\ 00:33:19,250 -> 00:33:23,059 term here and this term is going to drop
   551\ 00:33:23.059 -> 00:33:25.820 out from B from from the from the
   552\ 00:33:25,820 -> 00:33:27,950 difference in what's left is weighted by
   553\ 00:33:27,950 -> 00:33:29,570 alpha and multiplied by probability
   554\ 00:33:29,570 \longrightarrow 00:33:33,679 distributions and and okay there are
   555\ 00:33:33,679 -> 00:33:35,630 also these distributions in here the
   556\ 00:33:35,630 -> 00:33:36,650 fact that these are probabilities
   557\ 00:33:36.650 -> 00:33:38.270 tribution and these are probability
   558\ 00:33:38,270 -> 00:33:40,460 distributions is what makes the proof go
   559\ 00:33:40,460 -> 00:33:44,110 through I think I can give a
   560~00{:}33{:}44{,}110~{-}{>}~00{:}33{:}46{,}159 mathematical proof more easily than I
   561\ 00:33:46,159 -> 00:33:48,710 can give you a conceptual proof as to
   562 00:33:48,710 -> 00:33:52,990 why this map is a contraction mapping
   回答:直接说结论了,所有的方法都是硬聚合与代表性聚合的基础上发展来的,也有可能是这
两者的综合
   563\ 00:34:01,010 -> 00:34:03,710\ \text{yeah I understand that}
   564\ 00:34:03,710 \longrightarrow 00:34:06,980 three is a special
```

 $565\ 00:34:06,980 -> 00:34:11,590$ for what's the relationship between

 $566\ 00:34:11,590 -> 00:34:15,450\ I\ think$

```
567~00:34:26,679~->~00:34:29,090 okay the question is the following I
568\ 00:34:29,090 \rightarrow 00:34:32,510 have this aggregation scheme where every
569\ 00:34:32,510 -> 00:34:34,879 aggregate state is a subset of the
570~00{:}34{:}34{,}879~{-}{>}~00{:}34{:}37{,}460 original but it is possible also that
571\ 00:34:37,460 \longrightarrow 00:34:39,800 this aggregate states do not exhaust the
572 00:34:39.800 -> 00:34:42.949 same or the entire space and I argued
573\ 00:34:42,949 -> 00:34:45,379 that with these destructions this
574\ 00:34:45,379 -> 00:34:49,369 contains special cases all the previous
575\ 00:34:49,369 -> 00:34:54,260 schemes so for case for example three I
576\ 00:34:54,260 -> 00:34:59,660 have single a single state to every
577\ 00:34:59,660 \rightarrow 00:35:03,109 representative feature state in example
578\ 00:35:03,109 -> 00:35:08,180 one which is hard aggregation then I
579\ 00:35:08,180 -> 00:35:11,869 have the sets again disjoint sets but we
580\ 00:35:11,869 -> 00:35:15,050 exhaust the entire space example two is
581\ 00:35:15,050 -> 00:35:17,390 hard aggregation is a special case of
582\ 00:35:17,390 \rightarrow 00:35:21,710 hard aggregation because I am exhausting
583\ 00:35:21,710 -> 00:35:24,500 the space of features and every state
584\ 00:35:24,500 -> 00:35:27,710 maps into a unique feature so the states
585~00{:}35{:}27{,}710 -> 00{:}35{:}29{,}750 that correspond to any subset of
586\ 00:35:29,750 -> 00:35:35,240 features is some subset over here but
587\ 00:35:35,240 -> 00:35:37,970 all of these subsets exhaust the entire
588\ 00:35:37,970 -> 00:35:39,710 space it's just that the partition he is
589\ 00:35:39,710 \longrightarrow 00:35:43,130 quite irregular it's the partitions
590\ 00:35:43,130 -> 00:35:45,200 regular in this feature space but it is
591\ 00:35:45,200 \rightarrow 00:35:48,260 irregular in the state space so this is
592\ 00:35:48,260 -> 00:35:50,930 hard aggregation in the two extreme
593 00:35:50,930 -> 00:35:53,180 cases are hard aggregation and represent
594\ 00:35:53,180 \longrightarrow 00:35:57,740 all states exhausted representative
595\ 00:35:57,740 \rightarrow 00:36:01,730 States single state eigenstate so it's
596\ 00:36:01,730 -> 00:36:03,770 it's covering these two base cases and
597\ 00:36:03,770 -> 00:36:06,430 everything in between
问题: 所有的方法都基于仿真, 那样会花费很长时间, 一般需要多久
回答: 取决于你,看你想让仿真运行多久
598~00{:}36{:}08{,}980~{-}{>}~00{:}36{:}26{,}029~{\rm yes} how long is enough yeah okay okay so
599\ 00:36:26,029 \rightarrow 00:36:27,980 the question is all of these methods
600\ 00:36:27,980 \rightarrow 00:36:31,240 require a great deal of simulation and
601 00:36:31,240 -> 00:36:33,559 potentially you don't have enough time
602\ 00:36:33,559 -> 00:36:35,809 for simulation what's enough time it
603 00:36:35,809 -> 00:36:37,369 depends on you how much patience you
604\ 00:36:37,369 \rightarrow 00:36:40,160 have if you are willing to wait for a
605\ 00:36:40,160 -> 00:36:43,730 week Vence one thing if you're ready to
606\ 00:36:43,730 -> 00:36:46,400 only for a day to another but if you are
607\ 00:36:46,400 -> 00:36:47,839 really pressed for time and you want an
608\ 00:36:47,839 \longrightarrow 00:36:49,130 answer in five seconds
609\ 00:36:49,130 -> 00:36:51,019 then probably it's not going to be
610\ 00:36:51,019 -> 00:36:54,589 enough it there are possible there are
611\ 00:36:54,589 -> 00:36:57,740 cases where enough to have enough
612\ 00:36:57,740 \rightarrow 00:36:59,630 simulation time you have to live a
613 00:36:59,630 -> 00:37:02,690 hundred years and so you should avoid
614\ 00:37:02,690 -> 00:37:04,549 methods like that
615~00:37:04,549 \rightarrow 00:37:09,170 they it's always part of the art nothing
616\ 00:37:09,170 -> 00:37:11,299 is guaranteed there are no failsafe but
617\ 00:37:11,299 -> 00:37:13,940 there's no failsafe advice here you have
618\ 00:37:13,940 -> 00:37:15,799 to look at your problem you have to look
619\ 00:37:15,799 -> 00:37:18,140 at your options you have to look at what
620\ 00:37:18,140 -> 00:37:20,630 environment you're operating in in some
621\ 00:37:20,630 \rightarrow 00:37:22,670 cases you may need an answer within five
622\ 00:37:22,670 -> 00:37:25,670 seconds if you have real-time operation
```

```
623~00:37:25,670 \longrightarrow 00:37:28,069 and you have to have a decision ready in
   624\ 00:37:28,069 -> 00:37:31,130 five seconds then you may have to do
   625\ 00:37:31,130 \longrightarrow 00:37:33,079 your simulation within five seconds and
   626 00:37:33,079 -> 00:37:35,239 then perhaps simulation may not be the
   627\ 00:37:35,239 -> 00:37:38,299 right idea maybe a simpler solution may
   628\ 00:37:38.299 \rightarrow 00:37:41.329 a simpler approach we may have to be
   629\ 00:37:41,329 -> 00:37:44,769 adopted so I can't answer your question
   630\ 00:37:44,769 -> 00:37:49,670 how much time can we afford because that
   631\ 00:37:49,670 \longrightarrow 00:37:50,839 depends on your personal circumstances
   632\ 00:37:50,839 -> 00:37:53,410 and also how you apply these methods
   633\ 00:37:53,410 \longrightarrow 00:37:56,180 these are very complicated methods I
   634\ 00:37:56,180 -> 00:37:57,529 don't want to fool you that they are
   635\ 00:37:57,529 -> 00:38:00,440 easy to apply complicated because of the
   636\ 00:38:00,440 -> 00:38:03,019 many anomalies because of oscillations
   637\ 00:38:03,019 -> 00:38:06,739 explorations what have you also because
   638\ 00:38:06,739 \longrightarrow 00:38:08,539 they require a lot of computation time
   639\ 00:38:08,539 -> 00:38:11,569 in some problems they may work but in
   640\ 00:38:11,569 -> 00:38:14,890 some other problems they may not
   641\ 00:38:16,080 -> 00:38:20,760 you have to make a judgment for yourself
   问题:一些方法会用到仿真,比如投影方程什么的,那么如何选择一个长轨迹仿真还是很多短
轨迹仿真呢
   回答:从探索和试错的角度出发,一个长轨迹效果不如多个短轨迹,特别是探索,实际上这个
东西还是得根据你对问题的认识来选择,具体问题具体分析,没有一个统一的理论指导选择
   642\ 00:38:56,250 -> 00:38:59,070 okay so the question is the following
   643\ 00:38:59.070 -> 00:39:02.980 let's say we consider some method that
   644\ 00:39:02,980 \longrightarrow 00:39:07,360 requires simulation like the simulation
   645\ 00:39:07,360 \longrightarrow 00:39:09,460 based solution of projected equations I
   646\ 00:39:09,460 -> 00:39:12,130 mentioned one possibility to use a
   647 00:39:12,130 -> 00:39:14,710 single very long simulation trajectory
   648\ 00:39:14,710 \rightarrow 00:39:17,410 and also other possibilities of using
   649 00:39:17,410 \rightarrow 00:39:19,240 shorter trajectories many shorter
   650 00:39:19,240 -> 00:39:23,950 trajectories which one to use I think
   651\ 00:39:23,950 \rightarrow 00:39:27,220 that it has to do with exploration of a
   652\ 00:39:27,220 -> 00:39:31,300 lot and also it's a matter of trial and
   653\ 00:39:31,300 -> 00:39:35,560 error I would say that if one long
   654~00:39:35,560 \rightarrow 00:39:37,990 simulation trajectory does not work well
   655\ 00:39:37,990 -> 00:39:40,390 it's unlikely that five trajectories or
   656\ 00:39:40,390 -> 00:39:42,220 four trajectories will work well you
   657\ 00:39:42,220 -> 00:39:45,360 probably need many many shorter
   658\ 00:39:45,360 -> 00:39:47,200 trajectories in order to deal with a
   659\ 00:39:47,200 -> 00:39:53,470 problem of exploration we so so the
   660\ 00:39:53,470 -> 00:39:55,060 again there is no unique answer to your
   661\ 00:39:55,060 -> 00:39:58,120 questions to your to this particular
   662\ 00:39:58,120 -> 00:40:01,450 question you need to you need to have
   663\ 00:40:01,450 -> 00:40:04,710 some insight into your problem and and
   664\ 00:40:04,710 -> 00:40:08,440 and develop a better insight on which
   665\ 00:40:08,440 \longrightarrow 00:40:16,500 method to use some other questions
   666\ 00:40:24,090 -> 00:40:28,900 okay I think we're done for today and we
   667\ 00:40:28,900 -> 00:40:31,600 have one more election Friday which has
   668 00:40:31,600 -> 00:40:35,470 to do with things that we haven't
   669\ 00:40:35,470 -> 00:40:37,330 covered so far and I'm not quite sure
   670\ 00:40:37,330 \rightarrow 00:40:39,910 what they will be
   671 00:40:39,910 -> 00:00:00,000 [Applause]
```