12. *i*) $\tau = 1 + \frac{1}{\tau} = 1 + \frac{1}{1+\frac{1}{\tau}} = 1 + \frac{1}{1+\frac{1}{1+\frac{1}{\tau}}} = \cdots,$

and the "pieces" of the limiting "continued fraction" $1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cdots}}}$ are

$1,\ 1+\frac{1}{1} = 2,\ 1 + \frac{1}{1+\frac{1}{1}} = \frac{3}{2},\ 1 + \frac{1}{1+\frac{1}{1+\frac{1}{1}}} = \frac{5}{3}, \cdots$

with the general one being $\frac{u_{n+1}}{u_n}$;

ii) it is entirely reasonable to write

$$\tau = 1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1+\cdots}}} .$$

13. For $m \geq 1$, $n \geq 1$:

i) $u_{m+n} = u_{m-1}u_{n-1} + u_m u_n$;

ii) u_{n-1} divides u_{nm-1} ;

iii) $(u_{n-1}, u_{m-1}) = u_{(n,m)-1}$.

14. Using matrix multiplication one has

$$\begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}^n = \begin{pmatrix} u_n & u_{n-1} \\ u_{n-1} & u_{n-2} \end{pmatrix} \text{ for } n \geq 2 .$$