2 Manifolds with corners

2.1 Definition: A quadrant $Q \subseteq \mathbb{R}^n$ is a subset of the form $Q = \{x \in \mathbb{R}^n: l_1(x) \geq 0, \ldots, l_k(x) \geq 0\}$ where $\{l_1, \ldots, l_k\}$ is a linearly independent subset of $(\mathbb{R}^n)^*$. Here $0 \leq k \leq n$ and k is called the index of Q.

If $x \in Q$ and exactly j of the l_i's satisfy $l_i(x) = 0$, then x is called a corner of index j. The index of a corner depends only on x and Q and not on the special system $\{l_1, \ldots, l_k\}$ describing Q.

2.2 Let $U \subseteq Q$ be an open subset of a quadrant Q. A function $f: U \to \mathbb{R}^p$ is called C^r $(0 \leq r \leq \infty)$ if all partial derivatives of f of order $\leq r$ exist and are continuous on U. By the Whitney extension theorem (cf. H. WHITNEY (1936), J.C. TOUGERON (1972)) this is the case iff f can be extended to a C^r function $\tilde{f}: \tilde{U} \to \mathbb{R}^p$, where $\tilde{U} \subseteq \mathbb{R}^n$ is open and $U = Q \cap \tilde{U}$.

2.3 The border ∂Q of a quadrant Q is $\{x \in \mathbb{R}^n: l_1(x) = 0$ or $l_2(x) = 0$ or ... or $l_k(x) = 0\}$; it is the disjoint union of finitely many (plane) submanifolds of $\mathbb{R}^n$, the faces, edges, corners etc. of Q. ∂Q is "stratified" by this set of submanifolds.

Let $U \subseteq Q$, $U' \subseteq Q'$ be open subsets of quadrants in $\mathbb{R}^n$. A mapping $f: U \to U'$ is a diffeomorphism iff f is bijective and locally of maximal rank. It follows that f maps corners of index j in U to corners of index j in U'. So: $x \in U \subseteq Q$ is of index j iff $f(x) \in U' \subseteq Q'$ is of index j.