My University Name

My College Name
My Department Name

My Course Name

Thesis Title

John Doe
Student Number: 1234567

Supervisor:
Dr. Richard Smith

Year of Submission: 2016

Contents

Chapter 1

Sequences

1.1 Sequences and convergence

A *sequence* is an infinite list of numbers. Examples of sequences are

$$\begin{array}{ll} 1, 2, 3, 4, 5, 6, \ldots & 1, -1, 1, -1, 1, -1, \ldots \\ 4, 2, 1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \ldots & 2, 3, 5, 7, 11, 13, \ldots \\ 3, 10, 5, 16, 8, 4, 2, \ldots & 3, 3.1, 3.14, 3.141, 3.1415, 3.14159, \ldots \end{array}$$

The dots indicate that the sequence does not stop after 6 elements, but continues indefinitely. It is the finiteness of paper that forces us to stop at some point. In this regard the mathematical usage of the term 'sequence' differs from the colloqial one: in mathematics a finite, ordered list of numbers is a *tuple*. We can speak of a 3-tuple $(7, 2, 5)$ or an n-tuple $(1, 2, \ldots, n)$. A sequence however is an infinite list of numbers. Formally we have the following definition.

Definition 1.1. A *sequence* is a map[1] $x : \mathbb{N} \to \mathbb{R}$. We will write $x_n := x(n)$ for the *nth element* of the sequence and denote the whole sequence by $\{x_n\}_{n=1}^{\infty}$ or $\{x_n\}$.

How do we capture infinitely many elements on a finite sheet of paper? We do this by providing the *general rule* that allows us to calculate any element of the sequence. For example, the first four sequences shown above are defined by the following rules:

$$\begin{array}{ll} x_n = n & x_n = (-1)^n \\ x_n = 2^{2-n} & x_n \text{ is the } n\text{-th prime number}. \end{array}$$

[1]For the purpose of these notes, $\mathbb{N} = \{1, 2, 3, 4, \ldots\}$.

Bibliography

[1] S. Abbott. *Understanding analysis*. Second. Undergraduate Texts in Mathematics. Springer, New York, 2015, pp. xii+312.

[2] R. E. Bradley and C. E. Sandifer. *Cauchy's Cours d'analyse*. Sources and Studies in the History of Mathematics and Physical Sciences. An annotated translation. Springer, New York, 2009, pp. xx+411.

[3] D. M. Burton. *The History of Mathematics*. Second. An introduction. W. C. Brown Publishers, Dubuque, IA, 1991, pp. xii+678.

[4] C. H. Edwards Jr. *The Historical Development of the Calculus*. Springer-Verlag, New York-Heidelberg, 1979, pp. xii+351.

[5] N. Huggett. *Zeno's paradoxes — The Stanford Encyclopedia of Philosophy*. Ed. by E. N. Zalta. Winter 2010 Edition. URL: `http://plato.stanford.edu/archives/win2010/entries/paradox-zeno/`.

[6] S. Kifowit. *More proofs of divergence of the harmonic series*. Online addendum to the article "The harmonic series diverges again and again". 2006. URL: `http://stevekifowit.com/pubs/harm2.pdf`.

[7] S. Kifowit and T. A. Stamps. *Serious about the harmonic series II*. Handout to presentation at 31st Annual Conference of the Illinois Mathematics Association of Community Colleges. 2006. URL: `http://stevekifowit.com/pubs/al1.pdf`.

[8] S. Kifowit and T. A. Stamps. "The harmonic series diverges again and again". In: *The AMATYC Review* 27 (2006), pp. 31–43.

[9] J. J. O'Connor and E. F. Robertson. *Augustin-Louis Cauchy — MacTutor History of Mathematics archive*. Accessed 11/11/2016. URL: `http://www-history.mcs.st-andrews.ac.uk/Biographies/Cauchy.html`.

[10] A. Stubhaug. *Niels Henrik Abel and his times*. Called too soon by flames afar, Translated from the second Norwegian (1996) edition by Richard H. Daly. Springer-Verlag, Berlin, 2000, pp. x+580.