Proof. This result is analogous to (1.2.9) and is an immediate consequence of (1.1.8).

1.4.5 Corollary. *Let X be any smooth space and E any vector space with a linearly generated smooth structure. Then the smooth structure of $C^\infty(X, E)$ is also linearly generated.*

We give now some standard consequences of cartesian closedness.

1.4.6 Proposition. *For any smooth spaces X, Y, Z the evaluation* ev: $C^\infty(X, Y)\sqcap X \to Y$ *and the composition* comp: $C^\infty(Y, Z)\sqcap C^\infty(X, Y) \to C^\infty(X, Z)$ *are C^∞-maps.*

Proof. Since $\mathrm{ev} = (\mathrm{id}_{C^\infty(X,\,Y)})^\wedge$, the first part is obvious. So is the second part since $\mathrm{comp}^\wedge(g, f, x) = \mathrm{ev}(g, \mathrm{ev}(f, x))$ for appropriately chosen evaluation maps. □

1.4.7 Definition. A *smooth group G* is a smooth space for which the underlying set has a given group structure such that the group multiplication $m: G\sqcap G \to G$ and the inversion $v: G \to G$ are C^∞-maps.

1.4.8 Proposition. *Let X be any smooth space. If one puts on the group* Diff(X) *of all C^∞-diffeomorphisms of X the initial smooth structure induced by the two maps i, j:* $\mathrm{Diff}(X) \to C^\infty(X, X)$, *where $i(f) := f$ and $j(f) := f^{-1}$, then* Diff(X) *is a smooth group.*

Proof. One has the following identity for the group multiplication: $i \circ m = \mathrm{comp} \circ (i \sqcap i)$. Hence, using (1.4.6), $i \circ m$ is a C^∞-map. Similarly $j \circ m = \mathrm{comp} \circ (j \sqcap j) \circ \varphi$ where $\varphi(f, g) := (g, f)$ shows that $j \circ m$ is a C^∞-map. Together this shows that m is a C^∞-map. For the inversion map v it is even simpler, since $i \circ v = j$ and $j \circ v = i$. □

1.4.9 Definition. A smooth action of a smooth group G on a smooth space X is a C^∞-map $f: G\sqcap X \to X$ such that

(i) $f(g_1 g_2, x) = f(g_1, f(g_2, x))$ for $g_1, g_2 \in G$, $x \in X$.
(ii) $f(e, x) = x$ for $x \in X$, e the neutral element in G.

Act(G, X) shall denote the smooth space formed by the C^∞-actions of G on X with the smooth structure induced by its inclusion in $C^\infty(G\sqcap X, X)$.

1.4.10 Proposition. *There is a natural isomorphism between the space* Act(G, X) *of smooth actions of G on X and the space* Hom$(G, \mathrm{Diff}(X))$ *formed by the C^∞-homomorphisms $G \to$* Diff(X), *where* Hom$(G, \mathrm{Diff}(X))$ *has the smooth structure induced by its inclusion in $C^\infty(G, \mathrm{Diff}(X))$ and* Diff(X) *the one used in* (1.4.8).