

Given  $\theta - \hat{\theta} \approx \hat{\theta} - \hat{\theta}^b$

$\Rightarrow \theta \approx 2\hat{\theta} - \hat{\theta}^b$  with some simple algebra

From percentile bootstrap,  $\hat{\theta}^b$  with a  $1-\alpha$  confidence level has a CI of  $(q_{\frac{\alpha}{2}}, q_{1-\frac{\alpha}{2}})$ .

In the 2<sup>nd</sup> line,  $\hat{\theta}$  from the initial sample is fixed, and variation in  $\hat{\theta}^b$  can be represented by the CI.

Ensuring LB and UB are the min and max elements of the CI,  $2\hat{\theta} - \hat{\theta}^b$  has a CI of  $(2\hat{\theta} - q_{1-\frac{\alpha}{2}}, 2\hat{\theta} - q_{\frac{\alpha}{2}})$ .