Caijun Qin
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1. For smoothing splines, the effective degrees of freedom df_λ moves in opposite direction to λ such that as $\lambda \to \infty$, $df_\lambda \to 2$. However, computing the coefficients for a smoothing spline with $\lambda = 0$ maximizes df_λ to the greatest possible value with the spline only using RSS for OLS to determine coefficients. Since p covariates exist for linear regression, the df without any constraints or penalties maximize to p.

2.

(a) Since $\lambda = \infty$, $\hat{f}$ essentially focuses on minimizing the term, $\int [f(x)]^2 dx$. This leads to a flat spline with estimated values falling on the zero (0) line.

(b) Since $\lambda = \infty$, $\hat{f}$ essentially focuses on minimizing the term, $\int [f'(x)]^2 dx$. This leads to a spline that tries to minimize the absolute value of slope at each x-value, while also minimizing the RSS (the first term in arg min argument). The optimal spline would look like a flat line (minimizing absolute slope to zero (0)) falling on the average of true y-values (minimizing RSS).

(c) Since $\lambda = \infty$, $\hat{f}$ essentially focuses on minimizing the term, $\int [f''(x)]^2 dx$. This leads to a flat spline that attempts to have low curvature, while minimizing RSS. The spline would be very smooth with no rapid changes of slope direction. Since a straight line has the a 2nd derivative of zero (smoothest possible), the spline would be effectually linear regression. The predicted y-values of f would be computed the same as linear regression, $f = (X^T X)^{-1} X^T Y$.

(d) Since $\lambda = 0$, $\hat{f}$ has no penalty term effectually regardless of m. The spline would go through each training data point exactly and be overfitted to the training data. This may look like a very squiggly curved line (think Lagrangian interpolating polynomial with many points).

HW5

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Install packages

```
r = getOption("repos")
r["CRAN"] = "http://cran.us.r-project.org"
options(repos = r)

packages <- c('gam', 'randomForest', 'tree', 'Metrics')
install.packages(packages)
```

```
## Installing packages into 'D:/Users/qcaij/OneDrive - University of Florida/DESKTOP-R7MUAPV/DATA/Users/qcaij/Doc
uments/R/win-library/4.1'
## (as 'lib' is unspecified)
```

```
## package 'gam' successfully unpacked and MD5 sums checked
## package 'randomForest' successfully unpacked and MD5 sums checked
## package 'tree' successfully unpacked and MD5 sums checked
## package 'Metrics' successfully unpacked and MD5 sums checked
##
## The downloaded binary packages are in
##  C:\Users\qcaij\AppData\Local\Temp\RtmpOcg2Lz\downloaded_packages
```

```
lapply(packages, library, character.only = TRUE)
```

```
## Loading required package: splines
```

```
## Loading required package: foreach
```

```
## Loaded gam 1.20
```

```
## randomForest 4.6-14
```

```
## Type rfNews() to see new features/changes/bug fixes.
```

```
## [[1]]
##  [1] "gam"       "foreach"   "splines"   "stats"     "graphics"  "grDevices"
##  [7] "utils"     "datasets"  "methods"   "base"
##
## [[2]]
##  [1] "randomForest" "gam"          "foreach"      "splines"      "stats"
##  [6] "graphics"     "grDevices"    "utils"        "datasets"     "methods"
## [11] "base"
##
## [[3]]
##  [1] "tree"         "randomForest" "gam"          "foreach"      "splines"
##  [6] "stats"        "graphics"     "grDevices"    "utils"        "datasets"
## [11] "methods"      "base"
##
## [[4]]
##  [1] "Metrics"      "tree"         "randomForest" "gam"          "foreach"
##  [6] "splines"      "stats"        "graphics"     "grDevices"    "utils"
## [11] "datasets"     "methods"      "base"
```

Load data

```
data.train <- read.csv(file = './Data/Problem3train.csv', header = TRUE)
data.test <- read.csv(file = './Data/Problem3train.csv', header = TRUE)
```

Question 3a

```
tree.obj <- tree(
  formula = y ~ .,
  data = data.train,
  split = 'deviance'
)

tree.obj
```

```
## node), split, n, deviance, yval
##       * denotes terminal node
##
##   1) root 153 912.600 -0.38520
##     2) x2 < 2.05946 147 593.200 -0.11440
##       4) x2 < -1.85081 5  90.290  5.71300 *
##       5) x2 > -1.85081 142 327.200 -0.31960
##        10) x2 < 1.33595 121 246.000 -0.06526
##          20) x3 < 0.122611 65 115.200 -0.60140
##            40) x2 < -1.43456 6   2.400  1.24900 *
##            41) x2 > -1.43456 59  90.160 -0.78970
##              82) x5 < -0.399483 21  28.780 -1.39300
##               164) x6 < -0.922876 8   4.027 -0.27140 *
##               165) x6 > -0.922876 13   8.491 -2.08300 *
##              83) x5 > -0.399483 38  49.500 -0.45610
##               166) x8 < 0.14313 23  16.590  0.07244 *
##               167) x8 > 0.14313 15  16.630 -1.26700 *
##          21) x3 > 0.122611 56  90.380  0.55710
##            42) x4 < 1.29418 49  74.770  0.40380 *
##            43) x4 > 1.29418 7   6.398  1.63000 *
##        11) x2 > 1.33595 21  28.280 -1.78500 *
##     3) x2 > 2.05946 6  44.370 -7.02100 *
```

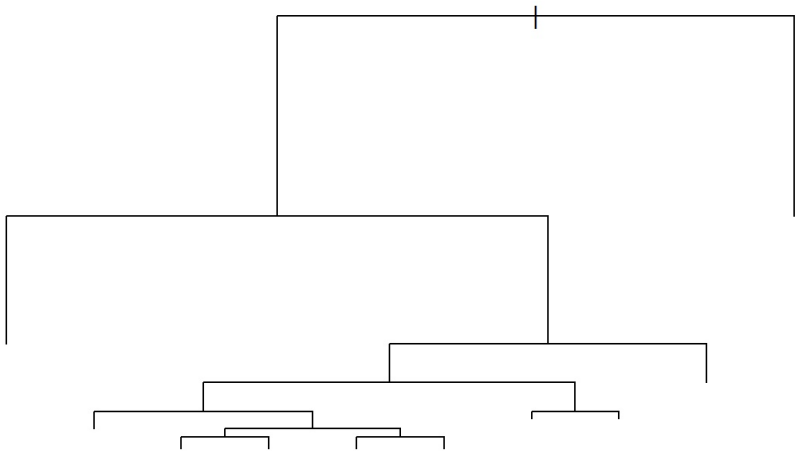
```
tree.cv <- tree::cv.tree(object = tree.obj, FUN = prune.tree)
tree.cv$dev
```

```
## [1] 611.4880 611.7997 561.0164 539.8354 590.4065 597.5310 833.0016 934.5437
```

```
tree.cv$size # size of 5 corresponds to smallest deviance
```

```
## [1] 10  9  6  5  4  3  2  1
```

```
tree.pruned <- prune.tree(tree = tree.obj, k = 5)
plot(tree.pruned)
```



```
tree.pred <- predict(object = tree.pruned, newdata = data.test)
tree.mse <- Metrics::mse(actual = data.test$y, predicted = tree.pred)
tree.mse
```

```
## [1] 1.910123
```

The prediction error (MSE) for decision tree on test data set is 1.910123.

Question 3b

```
rf.obj <- randomForest::randomForest(
  formula = y ~ .,
  data = data.train,
  mtry = ncol(x = data.train) - 1,
  ntree = 1000
)

rf.pred <- predict(object = rf.obj, newdata = data.test)
rf.mse <- Metrics::mse(actual = data.test$y, predicted = rf.pred)
rf.mse
```

```
## [1] 0.4875422
```

```
rf.tree.diff <- tree.mse - rf.mse
rf.tree.diff
```

```
## [1] 1.422581
```

The prediction error (MSE) for bagging (random forest) on test data set is 0.4404094, outperforming a single decision tree by 1.469714.

Question 3c

```
gam.obj <- gam::gam(
  formula = y ~ s(x1, 4) + s(x2, 4) + s(x3, 4) + s(x4, 4) + s(x5, 4) + s(x6, 4) + s(x7, 4) + s(x8, 4) + s(x9, 4)
+ s(x10, 4),
  data = data.train
)

gam.pred <- predict(object = gam.obj, newdata = data.test)
gam.mse <- Metrics::mse(actual = data.test$y, predicted = gam.pred)
gam.mse
```

```
## [1] 0.9799886
```

The prediction error (MSE) for GAM on test data set is 0.9799886.