

Chapter 3. Uncalibrated stereo

Prof. Slimane LARABI, USTHB

Chapitre 3. Uncalibrated stereo

3.1 Overview

3.2 Problem of uncalibrated stereo

3.3 Epipolar Geometry

3.4 Estimating Fundamental Matrix

3.5 Finding Correspondances

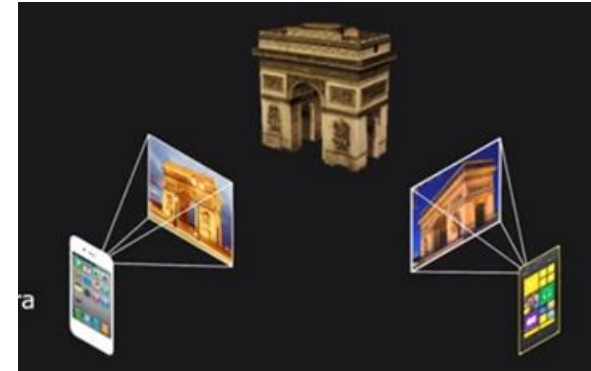
3.6 Computing Depth

Chapitre 3. Uncalibrated stereo

3.1 Overview

If you visit a monument and you take a picture and sometime later your friend visits the same monument and take another picture.

We have no idea where these pictures we taken from.



If we know the internal parameters of the two cameras, then from these two views we can compute the translation and rotation of one camera with respect to another camera.

And once that's done, we can then compute a 3D model of the monument.

Chapitre 3. Uncalibrated stereo

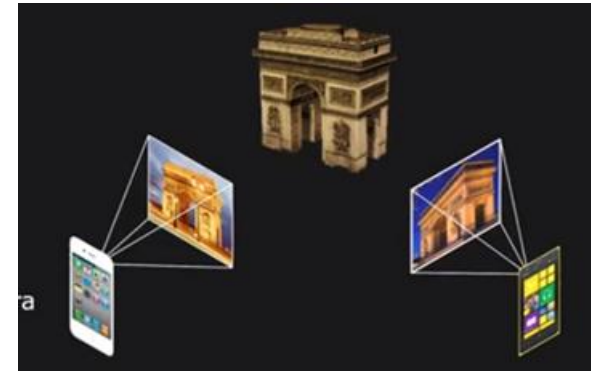
3.1 Overview

In this chapter we present a method to estimate 3D structure of a static scene from two arbitrary views. We will study the following topics:

- The problem of uncalibrated stereo
- The epipolar geometry
- Estimating Fundamental matrix
- Finding dense correspondences
- Computing depth

Note that:

- intrinsic parameters of cameras are known.
- extrinsic parameters (relative position, orientation of cameras) are unknown.

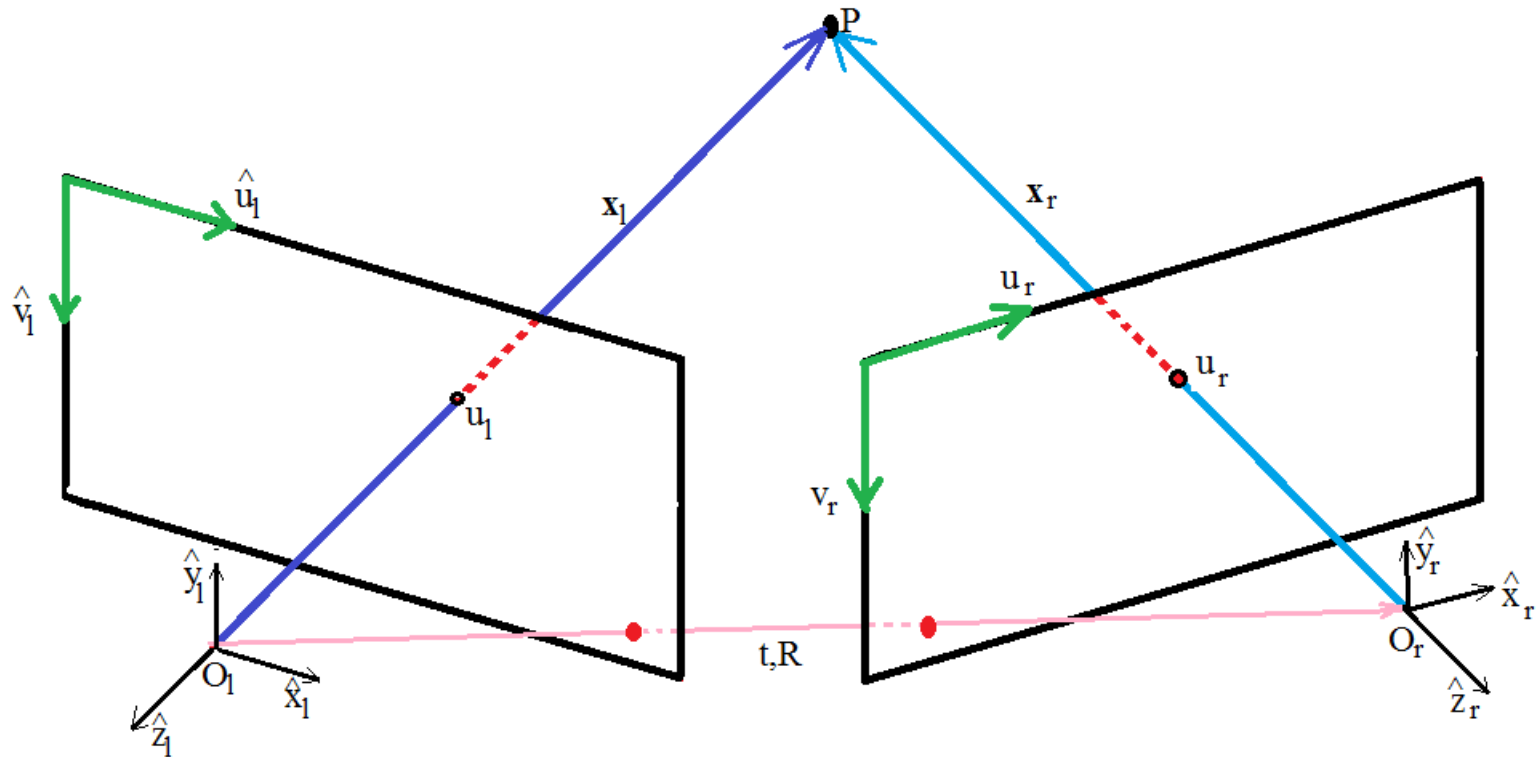


Chapitre 3. Uncalibrated stereo

3.2 Problem of uncalibrated stereo

The method is based on the following steps:

- The camera matrix K of each camera is available.
- We need to find reliable corresponding points on the two images



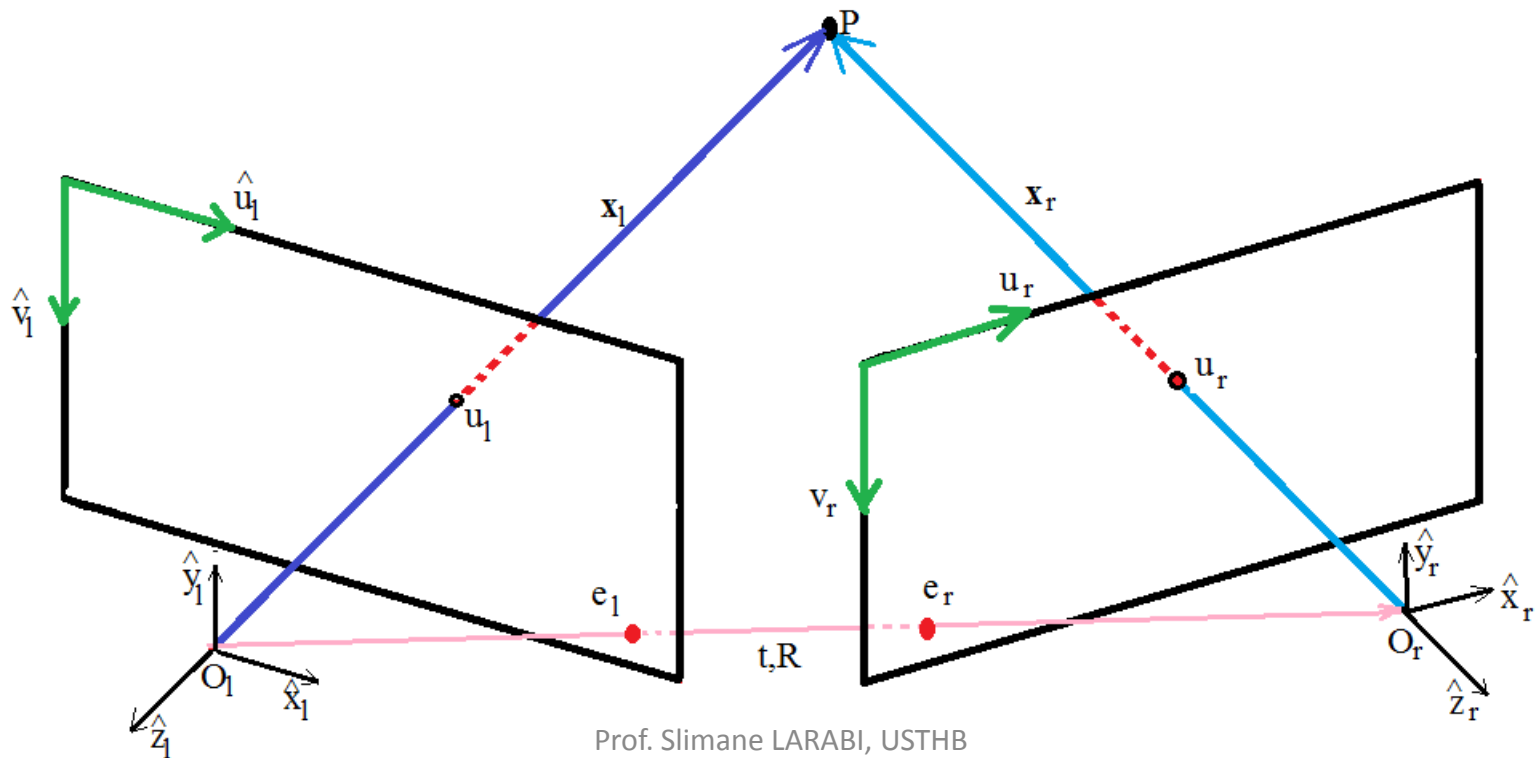
Chapitre 3. Uncalibrated stereo

3.3 Epipolar Geometry

The epipoles

Is defined as the image point of pinhole of one camera as viewed by the other camera.

In the figure e_l and e_r are the two epipoles, they are unique for a given stereo pair.



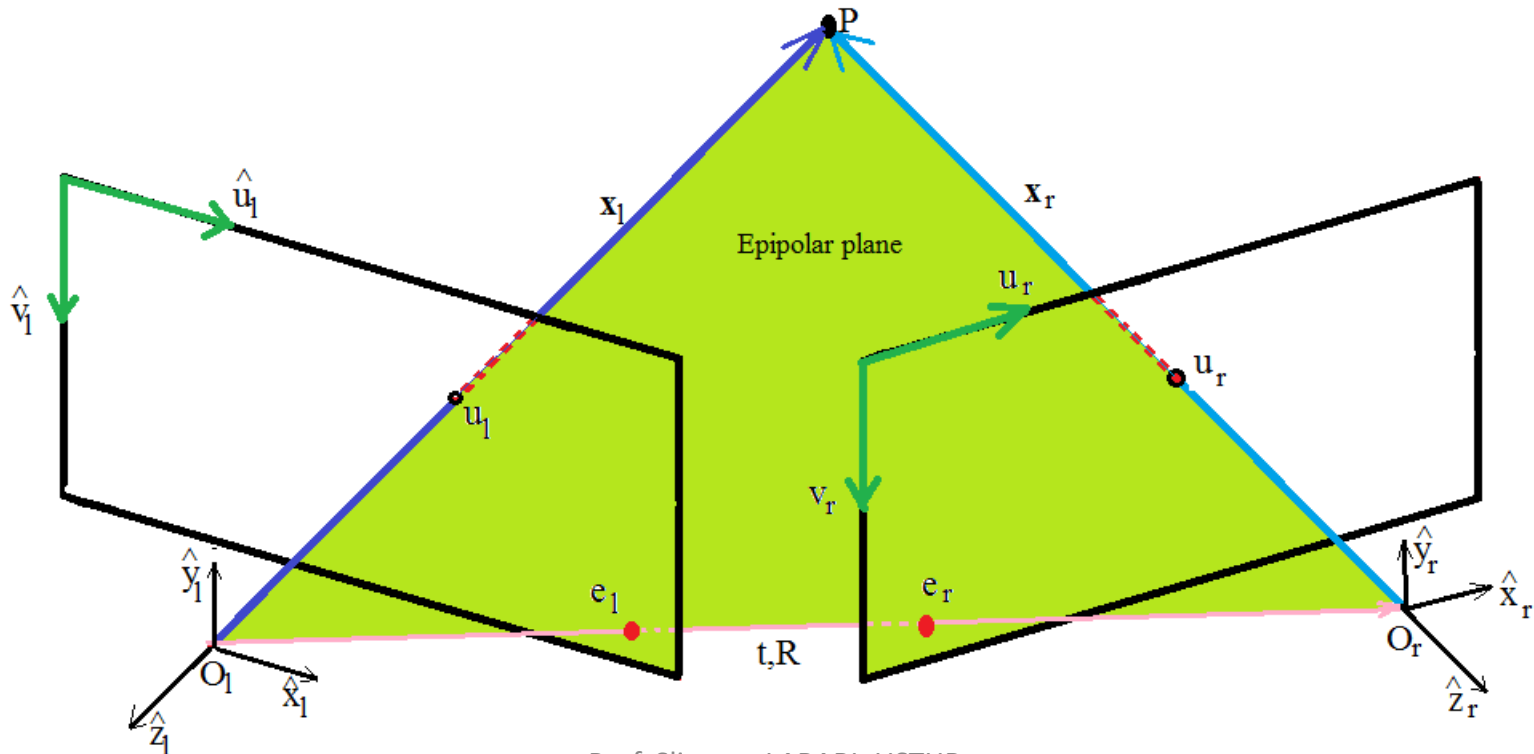
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3.3 Epipolar Geometry

Epipolar plane

Is associated to a scene point P : is formed by camera origins O_l and O_r , epipoles e_l and e_r and scene point P .

Every scene point P lies on a unique epipolar plane.



Chapitre 3. Uncalibrated stereo

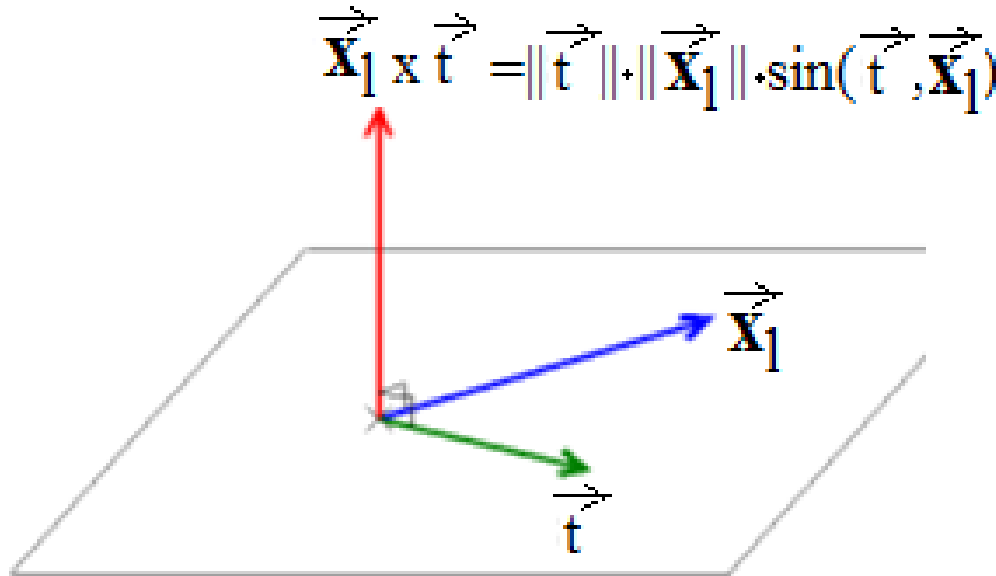
3.3 Epipolar Geometry

Epipolar constraint

Let n be the vector normal to the epipolar plane.

$$n = t \times x_l$$

$$x_l \cdot n = x_l \cdot (t \times x_l) = 0 \quad : \text{epipolar constraint}$$



Chapitre 3. Uncalibrated stereo

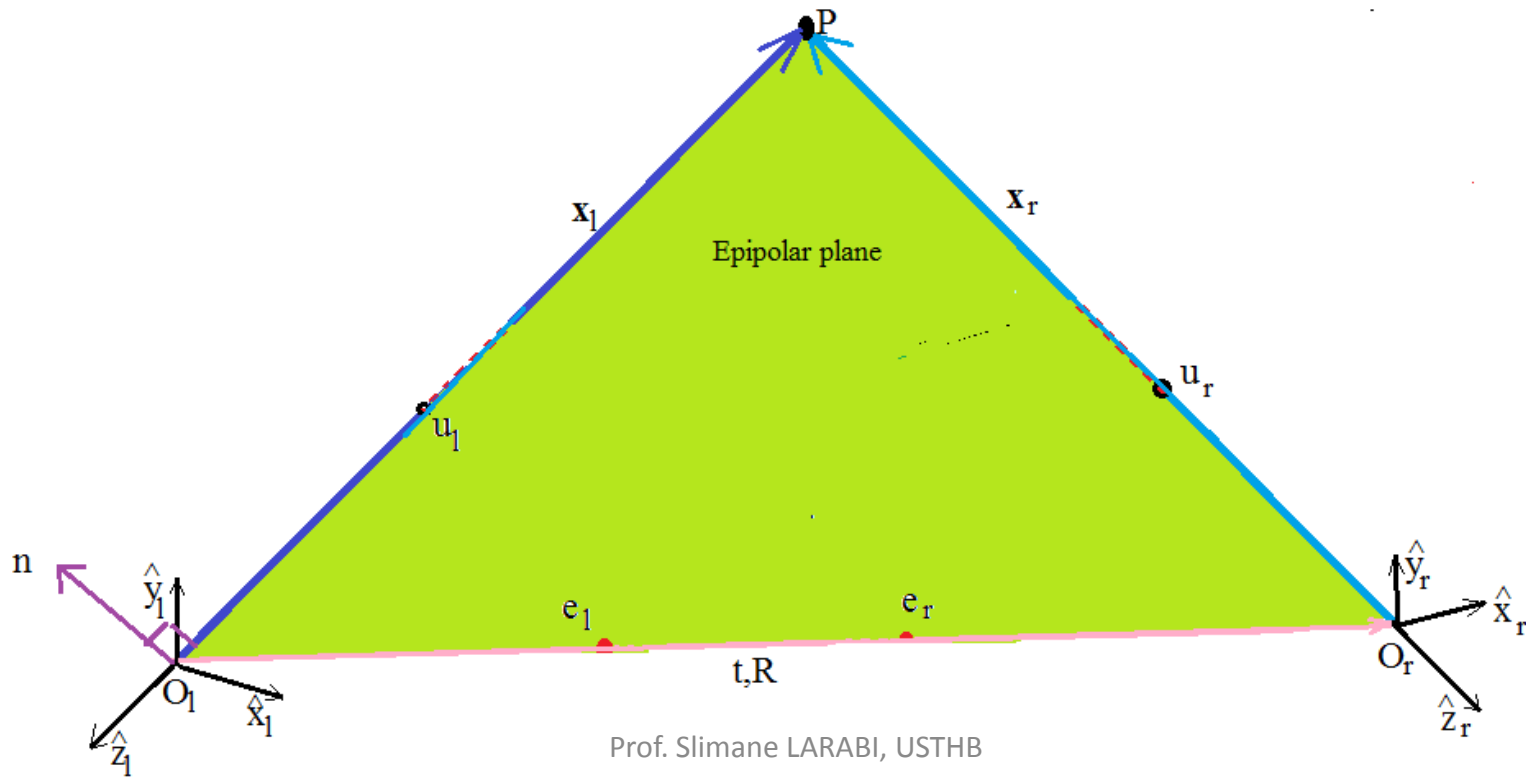
3.3 Epipolar Geometry

Epipolar constraint

Let n be the vector normal to the epipolar plane.

$$n = t \times x_l$$

$x_l \cdot n = x_l \cdot (t \times x_l) = 0$ (dot product) : *epipolar constraint*



Chapitre 3. Uncalibrated stereo

3.3 Epipolar Geometry

Epipolar constraint

Writing the epipolar constraint in matrix form:

$$\begin{bmatrix} t_x \\ t_y \\ t_z \end{bmatrix} \times \begin{bmatrix} x_l \\ y_l \\ z_l \end{bmatrix} = \begin{bmatrix} t_y z_l - t_z y_l \\ t_z x_l - t_x z_l \\ t_x y_l - t_y x_l \end{bmatrix}$$

$$x_l \cdot (t \times x_l) = 0$$

$$\begin{bmatrix} x_l & y_l & z_l \end{bmatrix} \begin{bmatrix} t_y z_l - t_z y_l \\ t_z x_l - t_x z_l \\ t_x y_l - t_y x_l \end{bmatrix} = 0 \quad \begin{bmatrix} x_l & y_l & z_l \end{bmatrix} \begin{bmatrix} 0 & -t_z & t_y \\ t_z & 0 & -t_x \\ -t_y & t_x & 0 \end{bmatrix} \begin{bmatrix} x_l \\ y_l \\ z_l \end{bmatrix} = 0 \quad \begin{bmatrix} x_l & y_l & z_l \end{bmatrix} T_x \begin{bmatrix} x_l \\ y_l \\ z_l \end{bmatrix} = 0$$

We note $t_{3 \times 1}$: $\begin{bmatrix} t_x & t_y & t_z \end{bmatrix}$ is the position of the right camera in the left camera frame.

Chapitre 3. Uncalibrated stereo

3.3 Epipolar Geometry

Epipolar constraint

$$[x_l \ y_l \ z_l] \begin{bmatrix} 0 & -t_z & t_y \\ t_z & 0 & -t_x \\ -t_y & t_x & 0 \end{bmatrix} \begin{bmatrix} x_l \\ y_l \\ z_l \end{bmatrix} = 0 \quad [x_l \ y_l \ z_l] T_x \begin{bmatrix} x_l \\ y_l \\ z_l \end{bmatrix} = 0$$

We note $t_{3 \times 1}$: $[t_x \ t_y \ t_z]$ is the position of the right camera in the left camera frame.

We note $R_{3 \times 3}$: the orientation of the left camera in the right camera's frame

$$x_l = R x_r + t \quad \begin{bmatrix} x_l \\ y_l \\ z_l \end{bmatrix} = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix} \begin{bmatrix} x_r \\ y_r \\ z_r \end{bmatrix} + \begin{bmatrix} t_x \\ t_y \\ t_z \end{bmatrix}$$

Chapitre 3. Uncalibrated stereo

3.3 Epipolar Geometry

Epipolar constraint

$$[x_l \ y_l \ z_l] \begin{bmatrix} 0 & -t_z & t_y \\ t_z & 0 & -t_x \\ -t_y & t_x & 0 \end{bmatrix} \begin{bmatrix} x_l \\ y_l \\ z_l \end{bmatrix} = 0 \quad \begin{bmatrix} x_l \\ y_l \\ z_l \end{bmatrix} = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix} \begin{bmatrix} x_r \\ y_r \\ z_r \end{bmatrix} + \begin{bmatrix} t_x \\ t_y \\ t_z \end{bmatrix}$$

$$[x_l \ y_l \ z_l] \begin{bmatrix} 0 & -t_z & t_y \\ t_z & 0 & -t_x \\ -t_y & t_x & 0 \end{bmatrix} \left(\begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix} \begin{bmatrix} x_r \\ y_r \\ z_r \end{bmatrix} + \begin{bmatrix} t_x \\ t_y \\ t_z \end{bmatrix} \right) = 0$$

$$[x_l \ y_l \ z_l] \begin{bmatrix} 0 & -t_z & t_y \\ t_z & 0 & -t_x \\ -t_y & t_x & 0 \end{bmatrix} \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix} \begin{bmatrix} x_r \\ y_r \\ z_r \end{bmatrix} + [x_l \ y_l \ z_l] \begin{bmatrix} 0 & -t_z & t_y \\ t_z & 0 & -t_x \\ -t_y & t_x & 0 \end{bmatrix} \begin{bmatrix} t_x \\ t_y \\ t_z \end{bmatrix} = 0$$

=0

Chapitre 3. Uncalibrated stereo

3.3 Epipolar Geometry

Epipolar constraint

$$[x_l \ y_l \ z_l] \begin{bmatrix} 0 & -t_z & t_y \\ t_z & 0 & -t_x \\ -t_y & t_x & 0 \end{bmatrix} \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix} \begin{bmatrix} x_r \\ y_r \\ z_r \end{bmatrix} = 0$$

$$[x_l \ y_l \ z_l] T_x R \begin{bmatrix} x_r \\ y_r \\ z_r \end{bmatrix} = 0 \qquad [x_l \ y_l \ z_l] \begin{bmatrix} \mathbf{e}_{11} & \mathbf{e}_{12} & \mathbf{e}_{13} \\ \mathbf{e}_{21} & \mathbf{e}_{22} & \mathbf{e}_{23} \\ \mathbf{e}_{31} & \mathbf{e}_{32} & \mathbf{e}_{33} \end{bmatrix} \begin{bmatrix} x_r \\ y_r \\ z_r \end{bmatrix} = 0$$

$E = T_x R$ is the **essential matrix**

This is the equation relating the 3D coordinates of P with respect to right and left coordinates frames

Longuet-Higgins 1981

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3.3 Epipolar Geometry

Epipolar constraint

Essential matrix E: Decomposition

It is possible to decouple R and T_x from E *using SVD*

How to find E?

$$[x_l \ y_l \ z_l] \begin{bmatrix} e_{11} & e_{12} & e_{13} \\ e_{21} & e_{22} & e_{23} \\ e_{31} & e_{32} & e_{33} \end{bmatrix} \begin{bmatrix} x_r \\ y_r \\ z_r \end{bmatrix} = 0$$

$$x_l^T E x_r = 0$$

x_l and x_r are unknown, but we know corresponding points in image coordinates

Chapitre 3. Uncalibrated stereo

3.3 Epipolar Geometry

Epipolar constraint

How to find E?

The perspective projection of each camera:

$$u = f_x \frac{x_c}{z_c} + O_x \quad v = f_y \frac{y_c}{z_c} + O_y$$

$$z_l u_l = f_x^l x_l + z_l O_x^l \quad z_l v_l = f_y^l y_l + z_l O_y^l$$

$$z_l \begin{bmatrix} u_l \\ v_l \\ 1 \end{bmatrix} = \begin{bmatrix} z_l u_l \\ z_l v_l \\ z_l \end{bmatrix} = \begin{bmatrix} f_x^l x_l + z_l O_x^l \\ f_y^l y_l + z_l O_y^l \\ z_l \end{bmatrix} = \underbrace{\begin{bmatrix} f_x^l & 0 & O_x^l \\ 0 & f_y^l & O_y^l \\ 0 & 0 & 1 \end{bmatrix}}_{\text{Known: Calibration matrix}} \begin{bmatrix} x_l \\ y_l \\ z_l \end{bmatrix}$$

Known: Calibration matrix

Chapitre 3. Uncalibrated stereo

3.3 Epipolar Geometry

Epipolar constraint

How to find E?

$$z_l \begin{bmatrix} u_l \\ v_l \\ 1 \end{bmatrix} = \underbrace{\begin{bmatrix} f_x^l & 0 & O_x^l \\ 0 & f_y^l & O_y^l \\ 0 & 0 & 1 \end{bmatrix}}_{K_l} \begin{bmatrix} x_l \\ y_l \\ z_l \end{bmatrix}$$

K_l : Calibration matrix
of left camera

$$z_r \begin{bmatrix} u_r \\ v_r \\ 1 \end{bmatrix} = \underbrace{\begin{bmatrix} f_x^r & 0 & O_x^r \\ 0 & f_y^r & O_y^r \\ 0 & 0 & 1 \end{bmatrix}}_{K_r} \begin{bmatrix} x_r \\ y_r \\ z_r \end{bmatrix}$$

K_r : Calibration matrix
of right camera

We obtain:

$$x_l^T = [u_l \ v_l \ 1] z_l K_l^{-1 \ T}$$

$$x_r = K_r^{-1} z_r \begin{bmatrix} u_l \\ v_r \\ 1 \end{bmatrix}$$

Chapitre 3. Uncalibrated stereo

3.3 Epipolar Geometry

Epipolar constraint

How to find E?

Epipolar constraint:

$$[x_l \ y_l \ z_l] \begin{bmatrix} \mathbf{e}_{11} & \mathbf{e}_{12} & \mathbf{e}_{13} \\ \mathbf{e}_{21} & \mathbf{e}_{22} & \mathbf{e}_{23} \\ \mathbf{e}_{31} & \mathbf{e}_{32} & \mathbf{e}_{33} \end{bmatrix} \begin{bmatrix} x_r \\ y_r \\ z_r \end{bmatrix} = 0$$

Rewriting in terms of image coordinates:

$$x_l^T = [u_l \ v_l \ 1] z_l K_l^{-1 T}$$

$$x_r = K_r^{-1} z_r \begin{bmatrix} u_r \\ v_r \\ 1 \end{bmatrix}$$

$$[u_l \ v_l \ 1] z_l K_l^{-1 T} \begin{bmatrix} \mathbf{e}_{11} & \mathbf{e}_{12} & \mathbf{e}_{13} \\ \mathbf{e}_{21} & \mathbf{e}_{22} & \mathbf{e}_{23} \\ \mathbf{e}_{31} & \mathbf{e}_{32} & \mathbf{e}_{33} \end{bmatrix} K_r^{-1} z_r \begin{bmatrix} u_r \\ v_r \\ 1 \end{bmatrix} = 0$$

As z_l and z_r are non zero:

Chapitre 3. Uncalibrated stereo

3.3 Epipolar Geometry

Epipolar constraint

How to find E?

Faugeras, Luong 1992,

$$[u_l \ v_l \ 1] K_l^{-1 T} \begin{bmatrix} \mathbf{e}_{11} & \mathbf{e}_{12} & \mathbf{e}_{13} \\ \mathbf{e}_{21} & \mathbf{e}_{22} & \mathbf{e}_{23} \\ \mathbf{e}_{31} & \mathbf{e}_{32} & \mathbf{e}_{33} \end{bmatrix} K_r^{-1} \begin{bmatrix} u_r \\ v_r \\ 1 \end{bmatrix} = 0$$

$$[u_l \ v_l \ 1] \begin{bmatrix} \mathbf{f}_{11} & \mathbf{f}_{12} & \mathbf{f}_{13} \\ \mathbf{f}_{21} & \mathbf{f}_{22} & \mathbf{f}_{23} \\ \mathbf{f}_{31} & \mathbf{f}_{32} & \mathbf{f}_{33} \end{bmatrix} \begin{bmatrix} u_r \\ v_r \\ 1 \end{bmatrix} = x_l^T F x_r = 0$$

F= Fundamental matrix

$$F = K_l^{-1 T} E K_r^{-1}$$

$$K_l^T F K_r = K_l^T K_l^{-1 T} E K_r^{-1} K_r$$

E: Essential matrix, $E = K_l^T F K_r$

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3.4 Estimating Fundamental Matrix

Initial correspondence

Find a set of corresponding features (at least 8) in left and right images (using SIFT or hand-picked)



$$(u_1^l, v_1^l), (u_2^l, v_2^l), \dots (u_m^l, v_m^l)$$



$$(u_1^r, v_1^r), (u_2^r, v_2^r), \dots (u_m^r, v_m^r)$$

Chapitre 3. Uncalibrated stereo

3.4 Estimating Fundamental Matrix

For each stereo correspondence (i), write out the epipolar constraint:

$$\begin{bmatrix} u_l^i & v_l^i & 1 \end{bmatrix} \begin{bmatrix} \mathbf{f}_{11} & \mathbf{f}_{12} & \mathbf{f}_{13} \\ \mathbf{f}_{21} & \mathbf{f}_{22} & \mathbf{f}_{23} \\ \mathbf{f}_{31} & \mathbf{f}_{32} & \mathbf{f}_{33} \end{bmatrix} \begin{bmatrix} u_r^i \\ v_r^i \\ 1 \end{bmatrix} = 0$$

We obtain for all stereo correspondences the linear system:

Chapitre 3. Uncalibrated stereo

3.4 Estimating Fundamental Matrix

$$\begin{bmatrix} u_l^1 u_r^1 & u_l^1 v_r^1 & u_l^1 & v_l^1 u_r^1 & v_l^1 v_r^1 & v_l^1 & u_r^1 & v_r^1 & 1 \\ - & - & - & - & - & - & - & - & - \\ u_l^i u_r^i & u_l^i v_r^i & u_l^i & v_l^i u_r^i & v_l^i v_r^i & v_l^i & u_r^i & v_r^i & 1 \\ - & - & - & - & - & - & - & - & - \\ u_l^m u_r^m & u_l^m v_r^m & u_l^m & v_l^m u_r^m & v_l^m v_r^m & v_l^m & u_r^m & v_r^m & 1 \end{bmatrix} \begin{bmatrix} f_{11} \\ f_{12} \\ f_{13} \\ f_{21} \\ f_{22} \\ f_{23} \\ f_{31} \\ f_{32} \\ f_{33} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \cdot \\ \cdot \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$Af = 0$$

Chapitre 3. Uncalibrated stereo

3.4 Estimating Fundamental Matrix

The fundamental matrix F and kF describe the same epipolar geometry. F is then defined only up to scale.

$$\begin{bmatrix} u_l^i & v_l^i & 1 \end{bmatrix} \begin{bmatrix} kf_{11} & kf_{12} & kf_{13} \\ kf_{21} & kf_{22} & kf_{23} \\ kf_{31} & kf_{32} & kf_{33} \end{bmatrix} \begin{bmatrix} u_r^i \\ v_r^i \\ 1 \end{bmatrix} = 0$$

We set the matrix F to some arbitrary scale: $\|f\|^2 = 1$

We want Af close to zero as possible and $\|f\|^2 = 1$.

$$\min_f \|Af\|^2 \text{ such that } \|f\|^2 = 1$$

This is the same problem like solving Projection matrix during camera calibration, or Homography matrix for image stitching.

Chapitre 3. Uncalibrated stereo

3.4 Estimating Fundamental Matrix

The next step is the computation of the **Essential** matrix E :

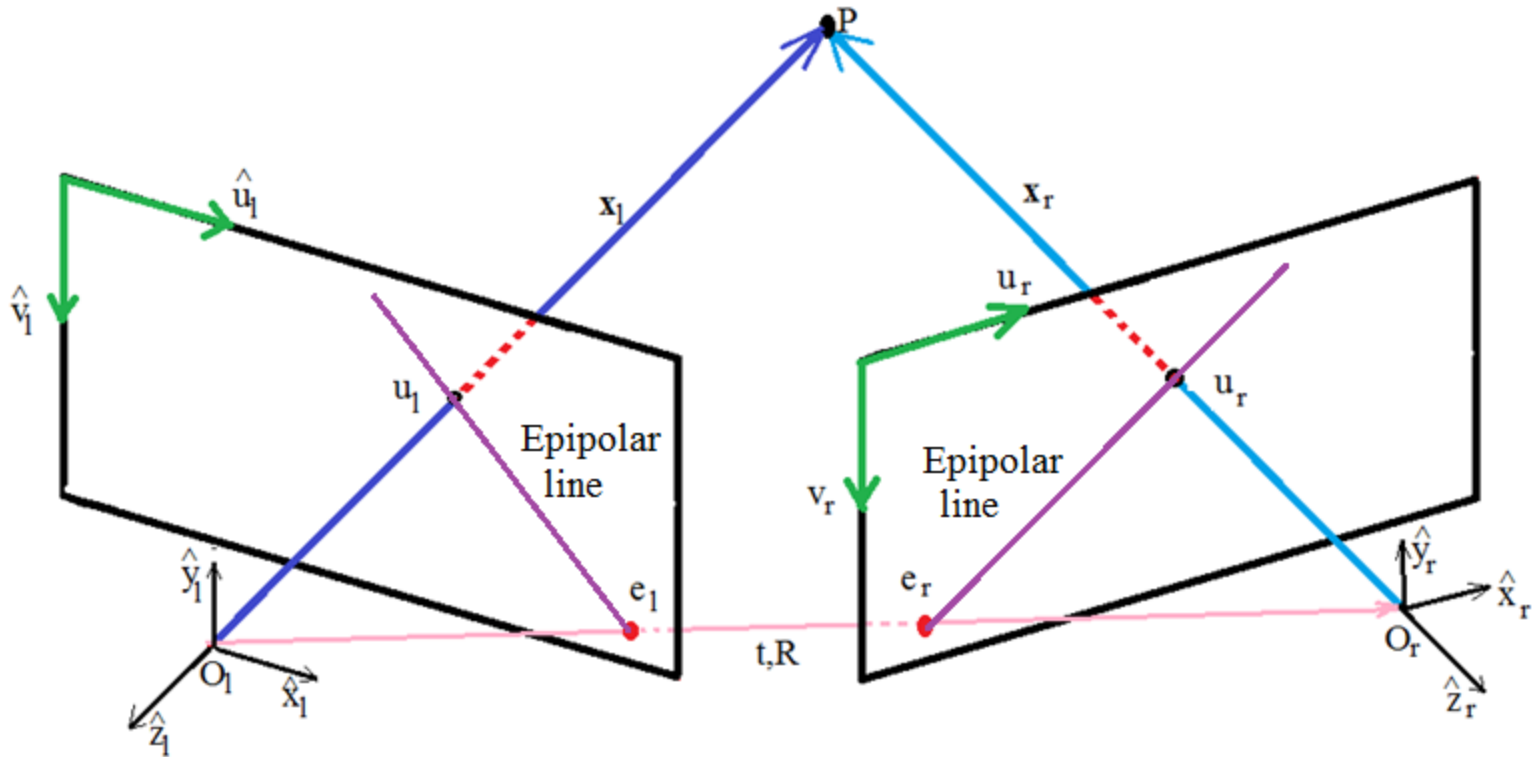
$$E = K_l^T F K_r$$

And extract the matrix R and vector t from E *using SVD (Singular Value Decomposition)*

$$E = R \times t$$

Chapitre 3. Uncalibrated stereo

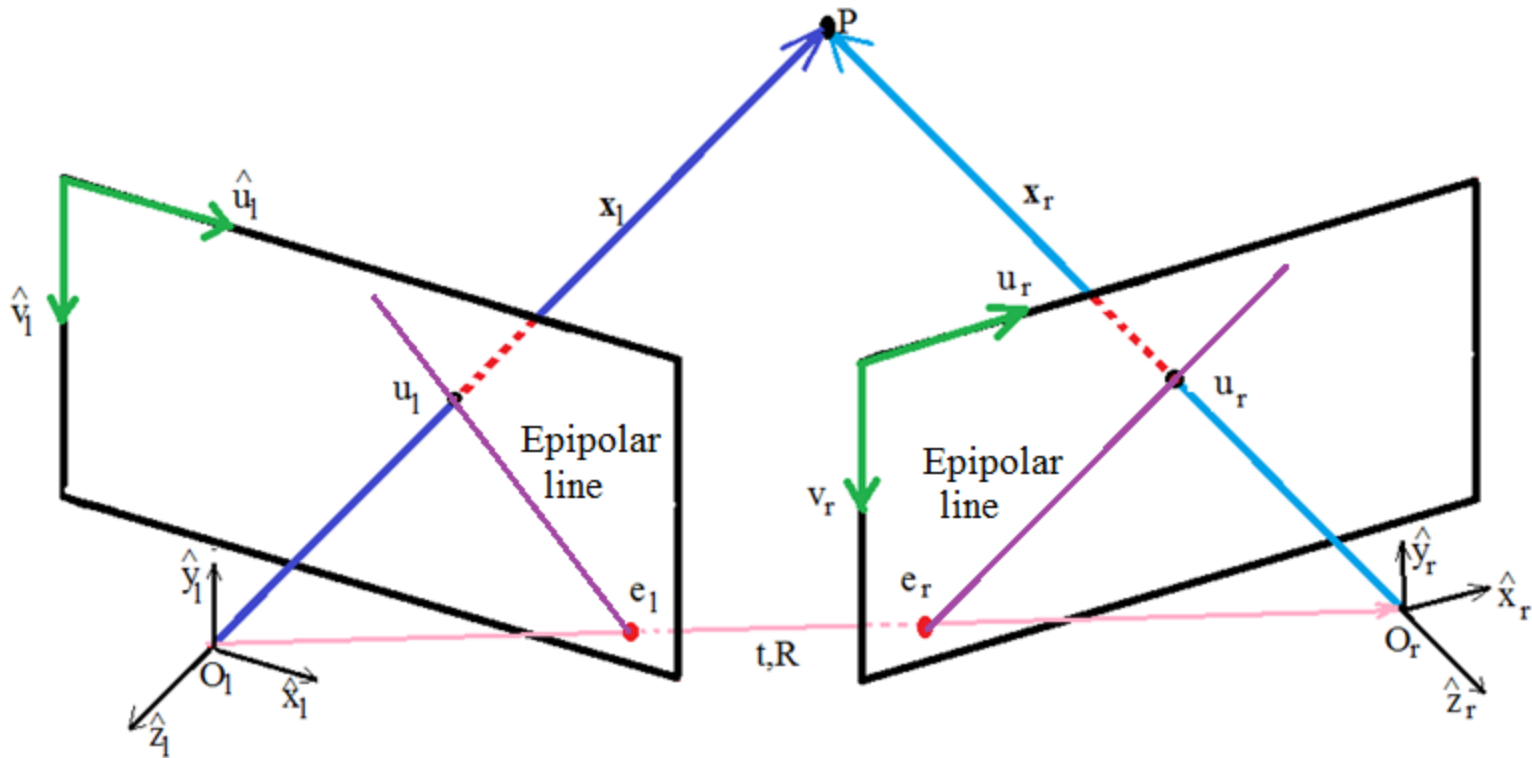
3.5 Finding Correspondences



Epipolar line is the intersection of image plane and epipolar plane.
At each scene point, there are two corresponding epipolar line,
one each on the two image planes.

Chapitre 3. Uncalibrated stereo

3.5 Finding Correspondences



Given one point on the left image, the corresponding point on the right image must lie on the epipolar line.

Finding correspondences is then reduced to 1D search.

Chapitre 3. Uncalibrated stereo

3.5 Finding Correspondances

Finding Epipolar lines:

Given the Fundamental matrix and point on left image ()

Find the epipolar line in the right image.

$$\begin{bmatrix} u_l & v_l & 1 \end{bmatrix} \begin{bmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{bmatrix} \begin{bmatrix} u_r \\ v_r \\ 1 \end{bmatrix} = 0$$

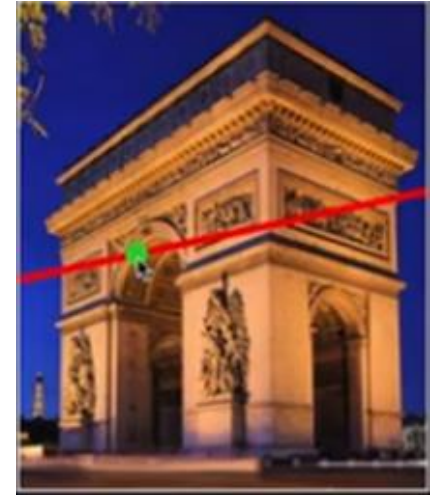
Expanding this equation:

$$u_r[u_l f_{11} + v_l f_{12} + f_{13}] + v_r[u_l f_{21} + v_l f_{22} + f_{23}] + [u_l f_{31} + v_l f_{32} + f_{33}] = 0$$

$$a_l u_r + b_l v_r + c_l = 0$$

Chapitre 3. Uncalibrated stereo

3.5 Finding Correspondences



Chapitre 3. Uncalibrated stereo

3.6 Computing Depth

Given the intrinsic parameters, the projections of scene point are:

$$\begin{bmatrix} u_l \\ v_l \\ 1 \end{bmatrix} = \begin{bmatrix} f_x^l & 0 & O_x^l & 0 \\ 0 & f_y^l & O_y^l & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_l \\ y_l \\ z_l \\ 1 \end{bmatrix} \quad \begin{bmatrix} u_r \\ v_r \\ 1 \end{bmatrix} = \begin{bmatrix} f_x^r & 0 & O_x^r & 0 \\ 0 & f_y^r & O_y^r & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_r \\ y_r \\ z_r \\ 1 \end{bmatrix}$$

Given also the relative position and orientation between the two cameras:

$$\begin{bmatrix} x_l \\ y_l \\ z_l \end{bmatrix} = \begin{bmatrix} r_{11} & r_{12} & r_{13} & t_x \\ r_{21} & r_{22} & r_{23} & t_y \\ r_{31} & r_{32} & r_{33} & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_r \\ y_r \\ z_r \\ 1 \end{bmatrix}$$

Chapitre 3. Uncalibrated stereo

3.6 Computing Depth

We obtain then for the left camera:

$$\begin{bmatrix} u_l \\ v_l \\ 1 \end{bmatrix} = \begin{bmatrix} f_x^l & 0 & O_x^l & 0 \\ 0 & f_y^l & O_y^l & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} r_{11} & r_{12} & r_{13} & t_x \\ r_{21} & r_{22} & r_{23} & t_y \\ r_{31} & r_{32} & r_{33} & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_r \\ y_r \\ z_r \\ 1 \end{bmatrix} \quad \tilde{u}_l = P_l \tilde{\mathbf{x}}_r$$

For the right camera:

$$\begin{bmatrix} u_r \\ v_r \\ 1 \end{bmatrix} = \begin{bmatrix} f_x^r & 0 & O_x^r & 0 \\ 0 & f_y^r & O_y^r & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_r \\ y_r \\ z_r \\ 1 \end{bmatrix} \quad \tilde{u}_r = M_{intr} \tilde{\mathbf{x}}_r$$

Chapitre 3. Uncalibrated stereo

3.6 Computing Depth

We obtain then for the left camera:

$$\begin{bmatrix} u_l \\ v_l \\ 1 \end{bmatrix} = \begin{bmatrix} p_{11} & p_{12} & p_{13} & p_{14} \\ p_{21} & p_{22} & p_{23} & p_{24} \\ p_{31} & p_{32} & p_{33} & p_{34} \\ p_{41} & p_{42} & p_{43} & p_{44} \end{bmatrix} \begin{bmatrix} x_r \\ y_r \\ z_r \\ 1 \end{bmatrix} \quad \tilde{u}_l = P_l \tilde{\mathbf{x}}_r$$

For the right camera:

$$\begin{bmatrix} u_r \\ v_r \\ 1 \end{bmatrix} = \begin{bmatrix} m_{11} & m_{12} & m_{13} & m_{14} \\ m_{21} & m_{22} & m_{23} & m_{24} \\ m_{31} & m_{32} & m_{33} & m_{34} \\ m_{41} & m_{42} & m_{43} & m_{44} \end{bmatrix} \begin{bmatrix} x_r \\ y_r \\ z_r \\ 1 \end{bmatrix} \quad \tilde{u}_r = M_{intr} \tilde{\mathbf{x}}_r$$

Chapitre 3. Uncalibrated stereo

3.6 Computing Depth

We rearrange the terms:

$$\begin{bmatrix} u_r m_{31} - m_{11} & u_r m_{32} - m_{12} & u_r m_{33} - m_{13} \\ v_r m_{31} - m_{21} & v_r m_{32} - m_{22} & v_r m_{33} - m_{23} \\ u_l p_{31} - p_{11} & u_l p_{32} - p_{12} & u_l p_{33} - p_{13} \\ v_l p_{31} - p_{21} & v_l p_{32} - p_{22} & v_l p_{33} - p_{23} \end{bmatrix} \begin{bmatrix} x_r \\ y_r \\ z_r \end{bmatrix} = \begin{bmatrix} m_{34} - m_{14} \\ m_{34} - m_{24} \\ p_{34} - p_{14} \\ p_{34} - p_{24} \end{bmatrix}$$

A
Known

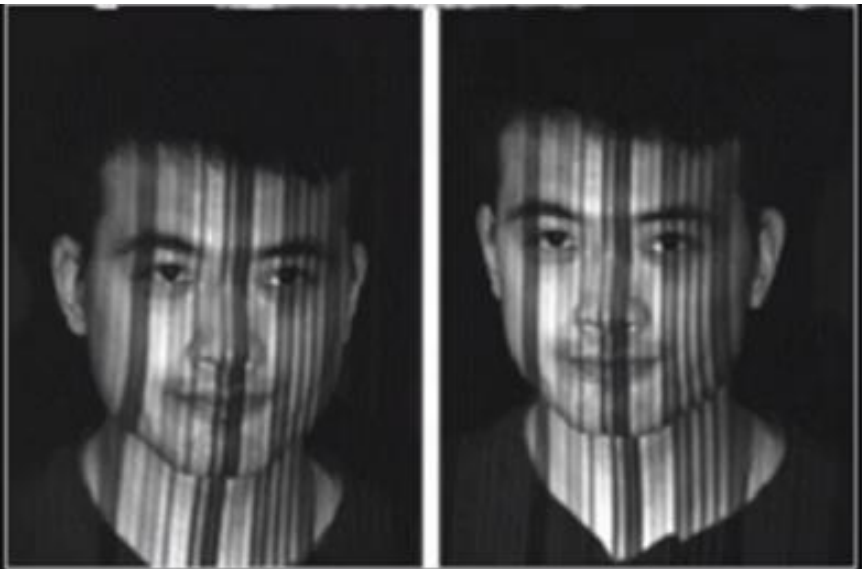
x_r = b
Unknown Known

$$\begin{aligned} Ax_r &= b \\ A^T Ax_r &= A^T b \\ x_r &= (A^T A)^{-1} A^T b \end{aligned}$$

Chapitre 3. Uncalibrated stereo

3.6 Computing Depth

Example of computing depth.



Chapitre 3. Uncalibrated stereo

3.6 Computing Depth

Example of computing depth.

