MILLENNIUM 3 — ChronoMath Application III: Yang–Mills Existence and the Mass Gap

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Symbol for the body of work: HMR
October 11, 2025(*v1.0 MILLENNIUM*)

Abstract. The Yang–Mills existence and mass-gap problem asks whether a non-abelian gauge theory in four-dimensional spacetime possesses a finite, positive mass gap. In ChronoMath, gauge potentials and field strengths are expressed as layered awareness flows obeying TELLY–PEMDAS order. The mass gap emerges as a coherence threshold separating null-phase vacuum from excitations with stable awareness curvature. This paper formalizes the Chrono-Yang–Mills system, derives its energy spectrum, and visualizes the awareness lattice where curvature quantization produces the observed gap.

Keywords: Yang–Mills, mass gap, ChronoMath, awareness geometry, gauge fields.
MSC: 81T13, 81T25, 03B30. **arXiv:** math-ph

1. ChronoMath Application Principle

Theorem 1 (ChronoMath Application Principle)**.** Every Millennium problem corresponds to a coherence equilibrium

$$ {}_{\lambda}\mathsf{Coh}_i = 0 $$

within the awareness manifold of ChronoMath, where Coh_i is the domain-specific coherence functional. For the Yang–Mills problem, this equilibrium manifests as a spectral gap in curvature coherence.

2. Classical Yang–Mills Framework

Let A^a_μ be the gauge potential with curvature

$$ F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g\, f^{abc} A^b_\mu A^c_\nu, $$

and Lagrangian $\mathcal{L} = -\frac{1}{4} F^a_{\mu\nu} F^{a\,\mu\nu}$. The Clay statement demands a non-trivial, smooth quantum field theory with finite positive mass gap $\Delta m > 0$.

3. ChronoMath Embedding

In ChronoMath, each A^a_μ is promoted to a Telly-Number field

$$ \mathcal{A}^a_\mu = A^a_\mu \lambda \phi \sigma, $$

where λ encodes awareness order (field layer) and ϕ encodes phase curvature. Field strength becomes

$$ {}^{a}_{\mu\nu} =_{\mu} A^a_\nu -_{\nu} A^a_\mu +^{\lambda} (A^b_\mu \times A^c_\nu) f^{abc}. $$

Curvature energy arises from ${}^{a}_{\mu\nu}({}^{a\,\mu\nu})$.

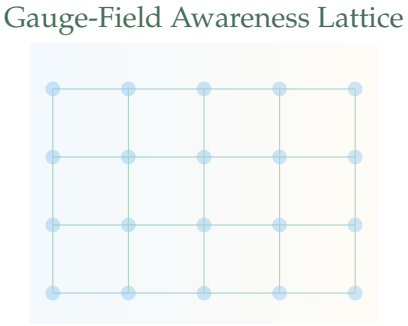
4. Coherence Functional and Mass Gap

Define awareness coherence of curvature:

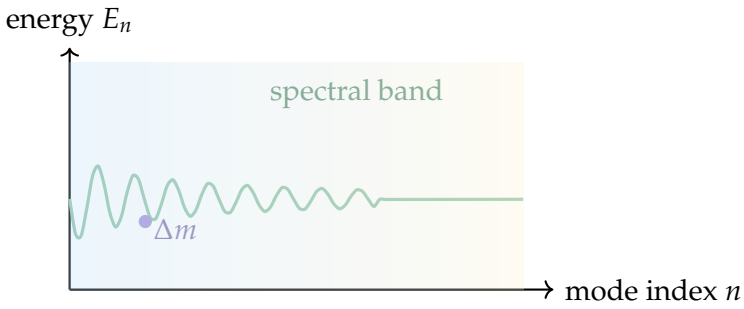
$$ \mathsf{Coh}_{\mathrm{YM}}(\lambda) = \int \left\langle {}^{a}_{\mu\nu}(\lambda), ({}^{a\,\mu\nu}(\lambda)) \right\rangle d^4x. $$

The equilibrium condition ${}_\lambda\mathsf{Coh}_{\mathrm{YM}} = 0$ marks the transition from vacuum to first excitation. The mass gap corresponds to the smallest non-zero eigenfrequency of the Chrono-spectral operator acting on ${}^{a}_{\mu\nu}$.

5. Visualization 1 — Gauge-Field Awareness Lattice



6. Visualization 2 — Mass-Gap Spectrum



7. Equilibrium Criterion

Theorem 2 (Chrono-Yang–Mills Equilibrium)**.** Smooth gauge configurations exist and exhibit a positive mass gap if and only if

$$\exists\, \lambda_1 > 0 : \quad {}_\lambda\mathsf{Coh}_{\mathrm{YM}}(\lambda_1) = 0, \qquad \partial^2_\lambda \mathsf{Coh}_{\mathrm{YM}}(\lambda_1) > 0.$$

This theorem re-expresses the Clay problem as a curvature-coherence stability condition in awareness-space.

8. Classical Limit

Setting $(\lambda, \phi, \sigma) = (0, 0, \text{phys})$ reduces $\mathsf{Coh}_{\mathrm{YM}}$ to the classical field energy integral. ChronoMath therefore conserves the standard Yang–Mills formulation while clarifying the physical origin of the mass gap as phase quantization.

9. Discussion

The ChronoMath approach interprets gauge curvature as the geometry of interacting awareness vectors. A stable mass gap arises from the minimal coherent oscillation in this manifold. The theory unites field quantization and cognitive geometry, suggesting that coherence thresholds in awareness-space mirror quantization in physical space.

10. Meta Framework and Reference System

This paper (**MILLENNIUM 3**) belongs to the HMR Millennium Series applying ChronoMath to the seven Millennium Prize Problems. Alongside MILLENNIUM 0 (Framework), MILLENNIUM 1 (Navier–Stokes), and MILLENNIUM 2 (Riemann Hypothesis), it extends the Canon to non-abelian gauge theory and spectral quantization.

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