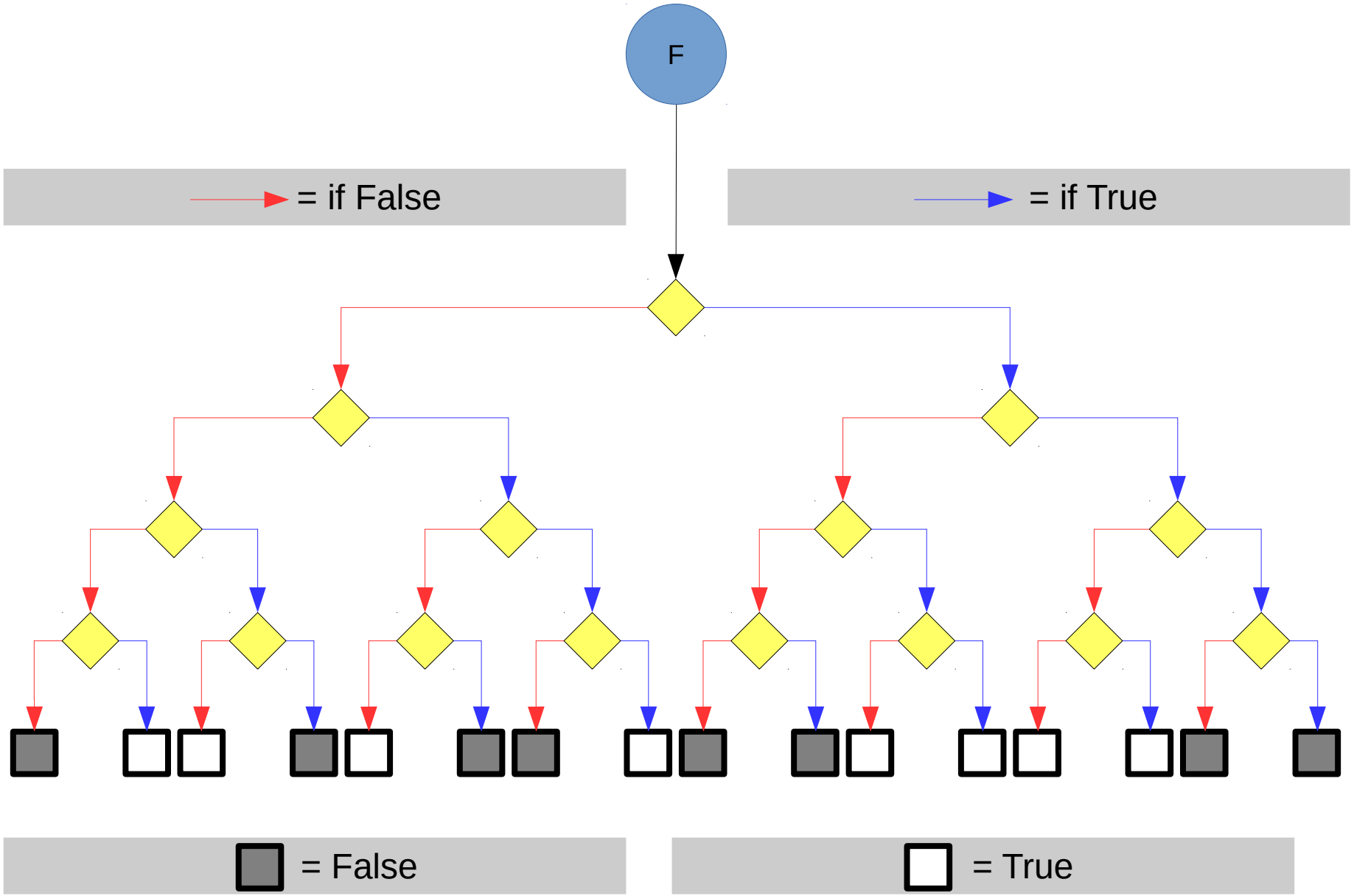
A Generalized Reduction of Ordered Binary Decision Diagram (GroBdd)

Joan Thibault

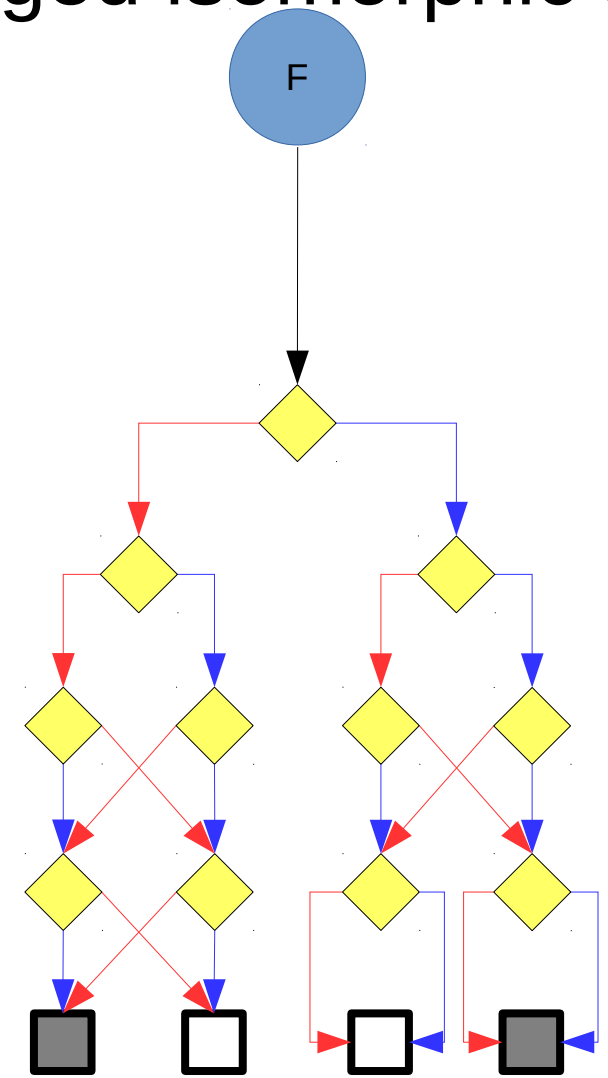
Reduced Ordered Binary Decision Diagram (ROBDD)

- Applications
  - Computer Aided Design (e.g. equivalence checking)
  - Knowledge Representation (e.g. Artificial Intelligence)
  - Combinatorial Problems (e.g. N-Queens problem)
- What are required operation ?
  - Compact representation
  - Operations (e.g. composing, concatening, evaluation)
  - Operators (e.g. AND, XOR, ITE, NOT)
  - Reductions (e.g. quantification, partial evaluation, SAT)

Shanon's Binary Decision Tree



Shannon's Decision Diagram
(We merged isomorphic sub-trees)



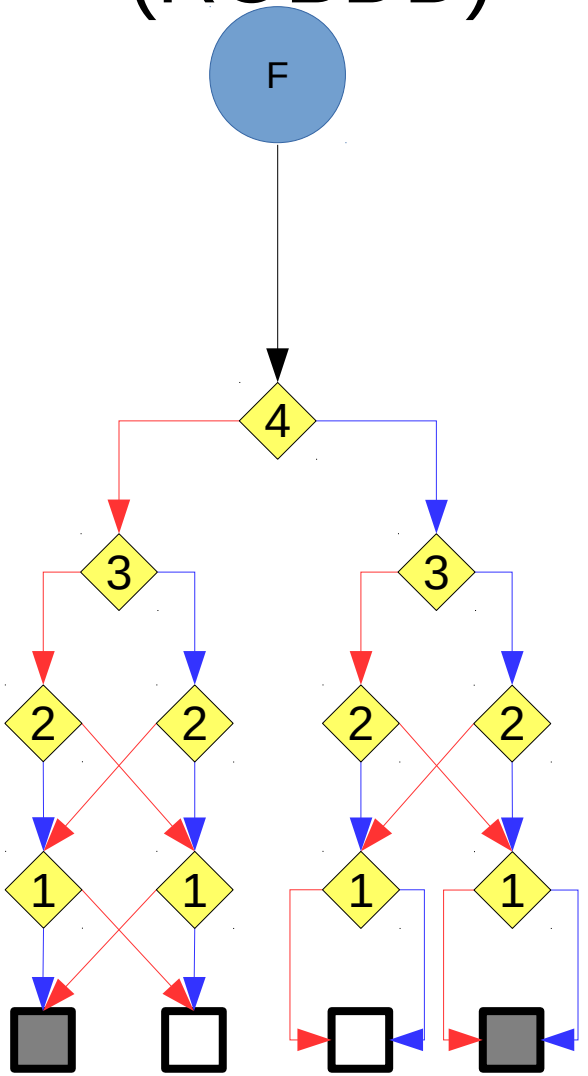
= if False
= if True

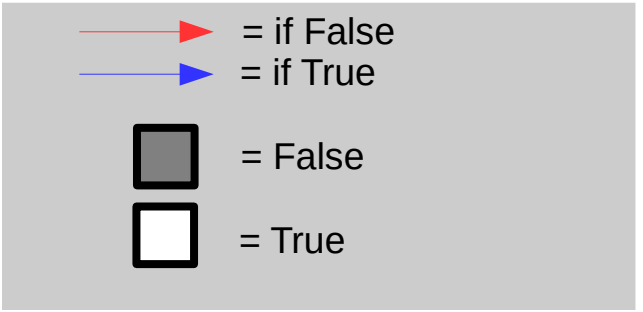
= False

= True

For aesthetic reasons, we do not merge all terminals

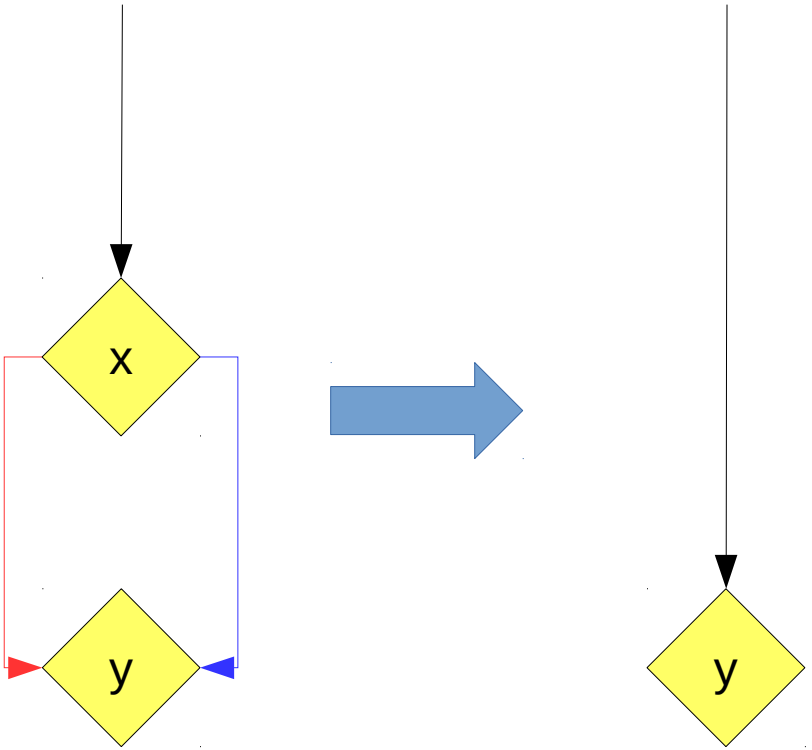
Reduced Ordered Binary Decision Diagram (ROBDD)

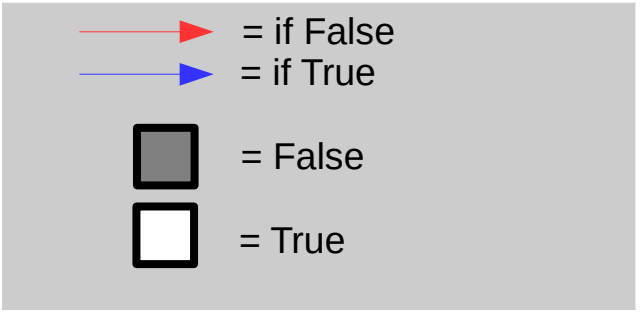




For aesthetic reasons, we do not merge all terminals

ROBDD : reduction rule





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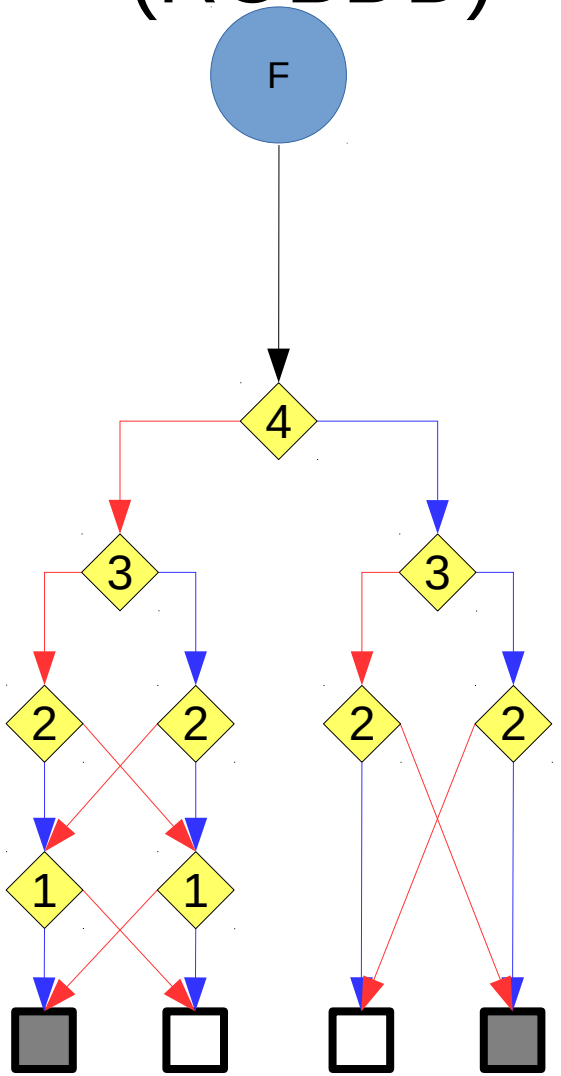
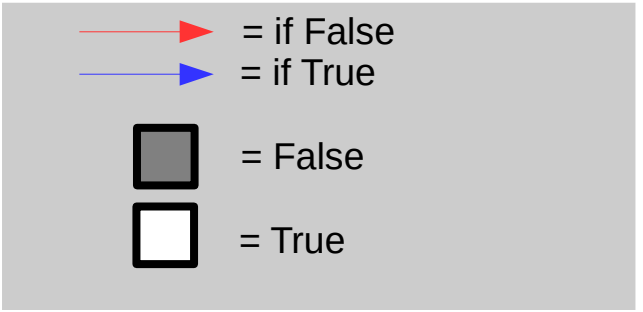
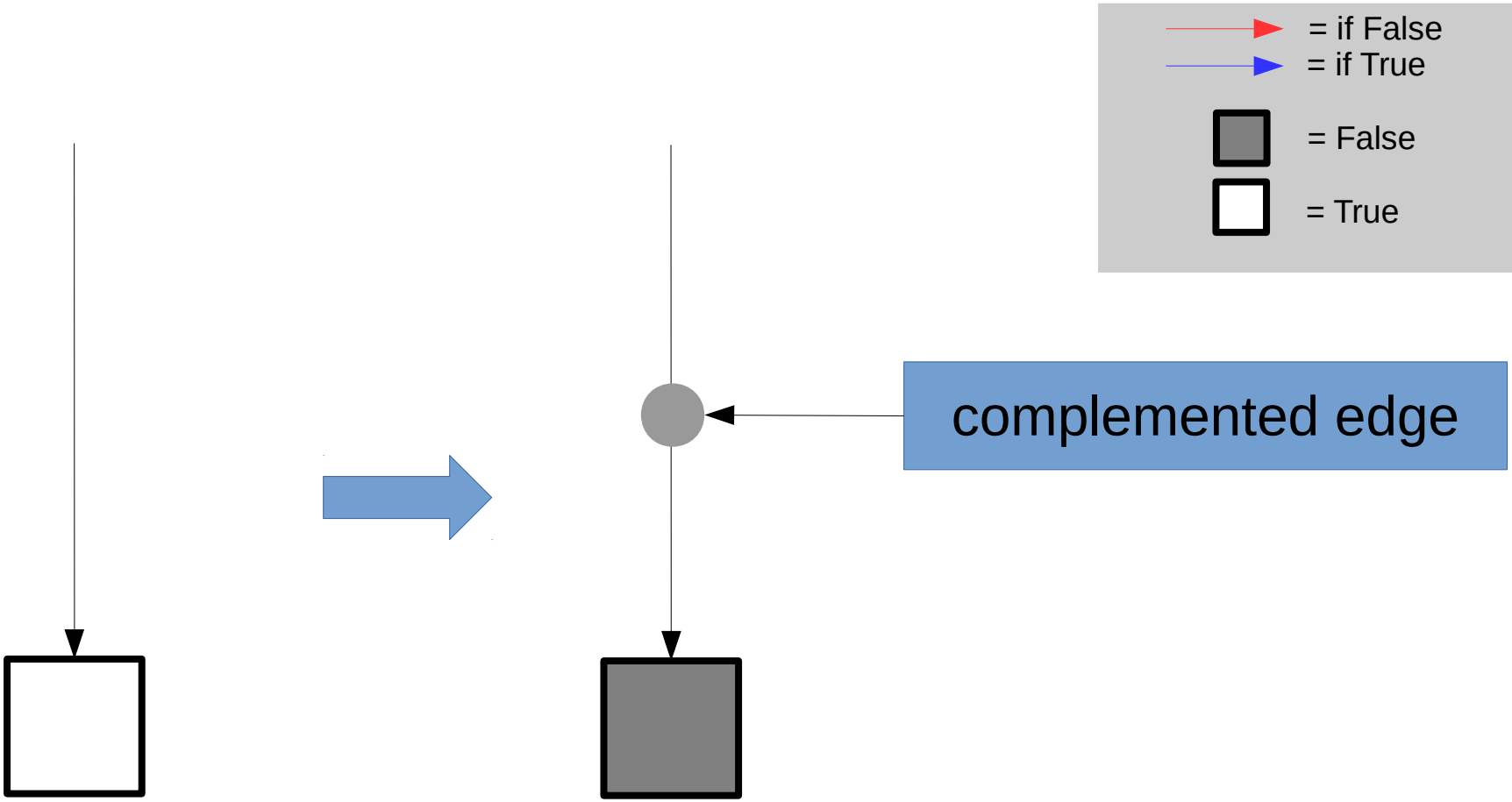
Reduced Ordered Binary Decision Diagram (ROBDD)



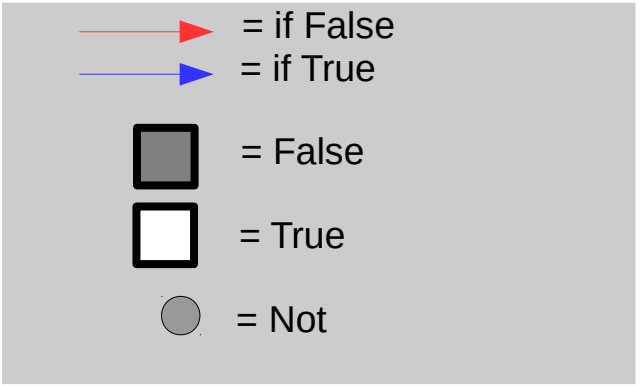


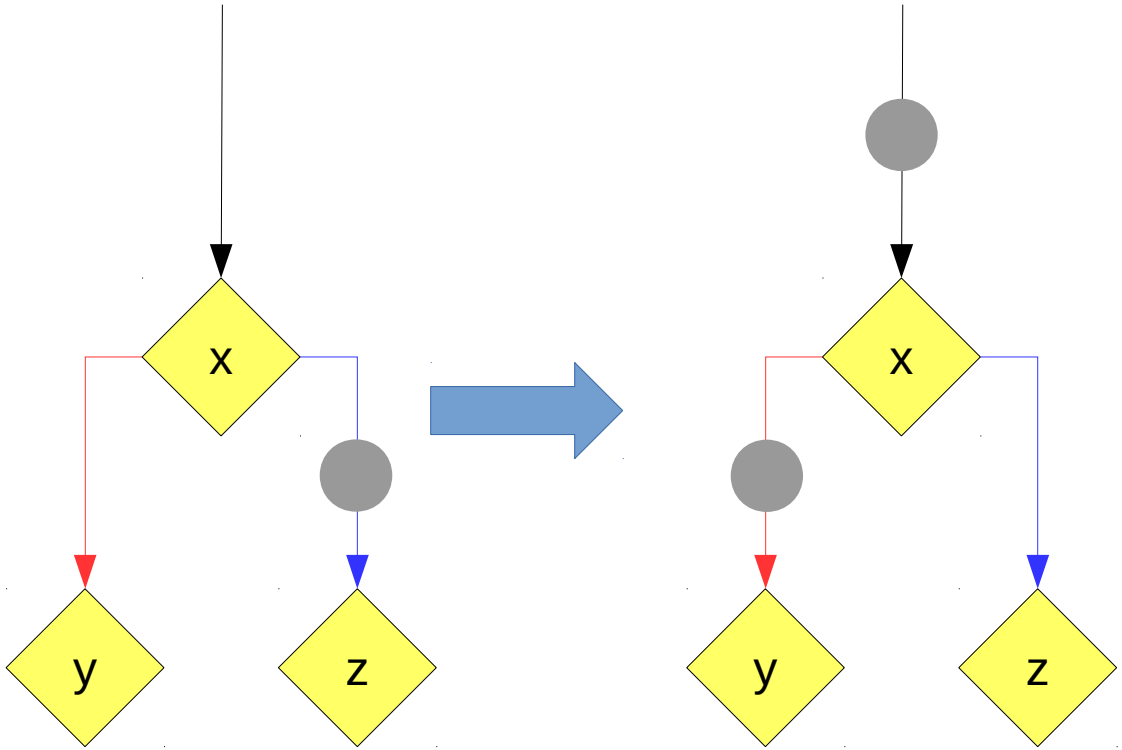
“output negation” : reduction rule (N1)



For aesthetic reasons, we do not merge all terminals

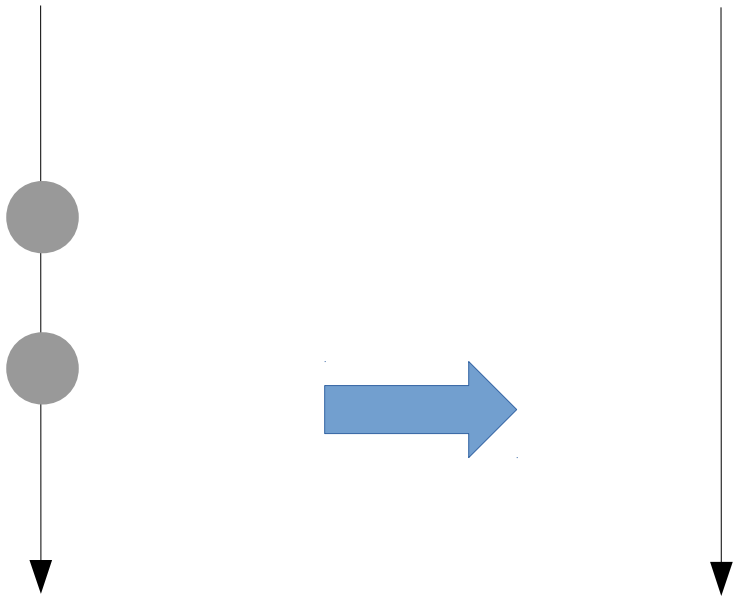
“output negation” : reduction rule (N2)

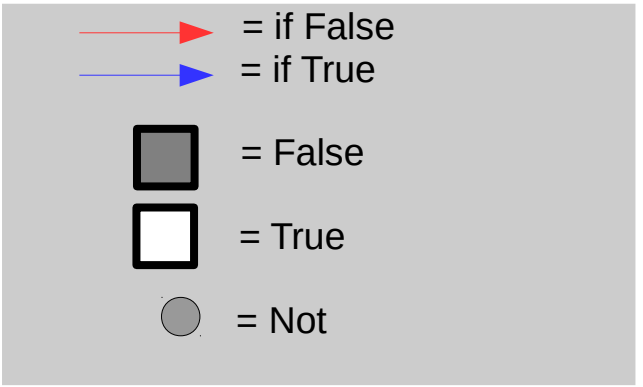




For aesthetic reasons, we do not merge all terminals

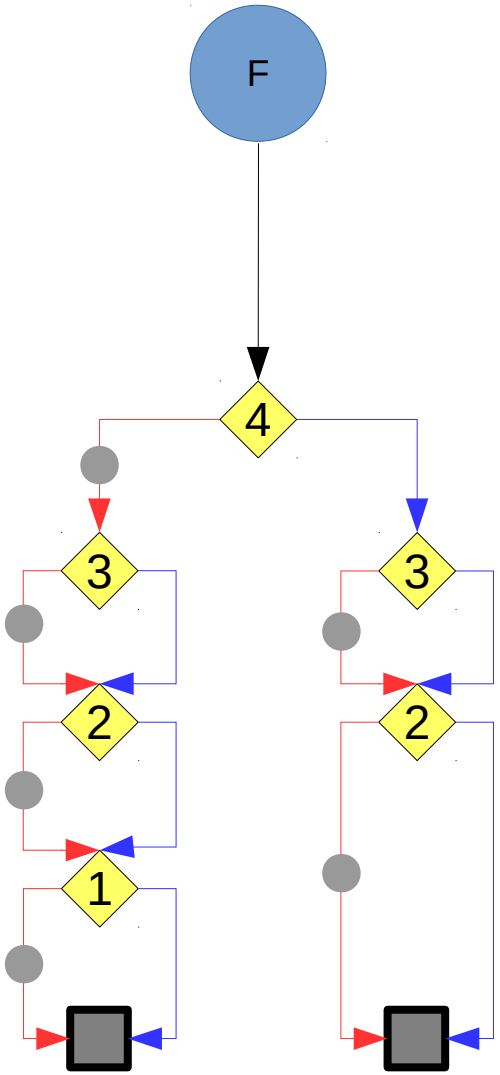
“output negation” : reduction rule (N3)

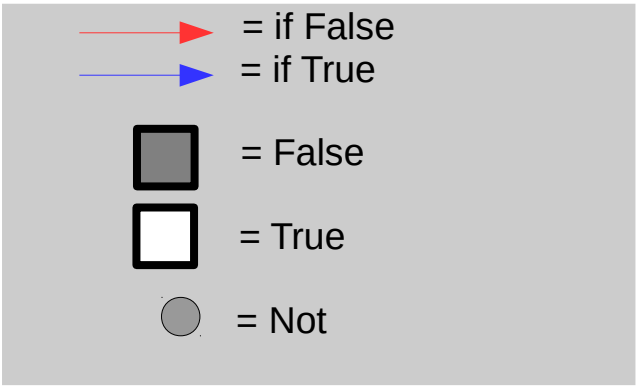




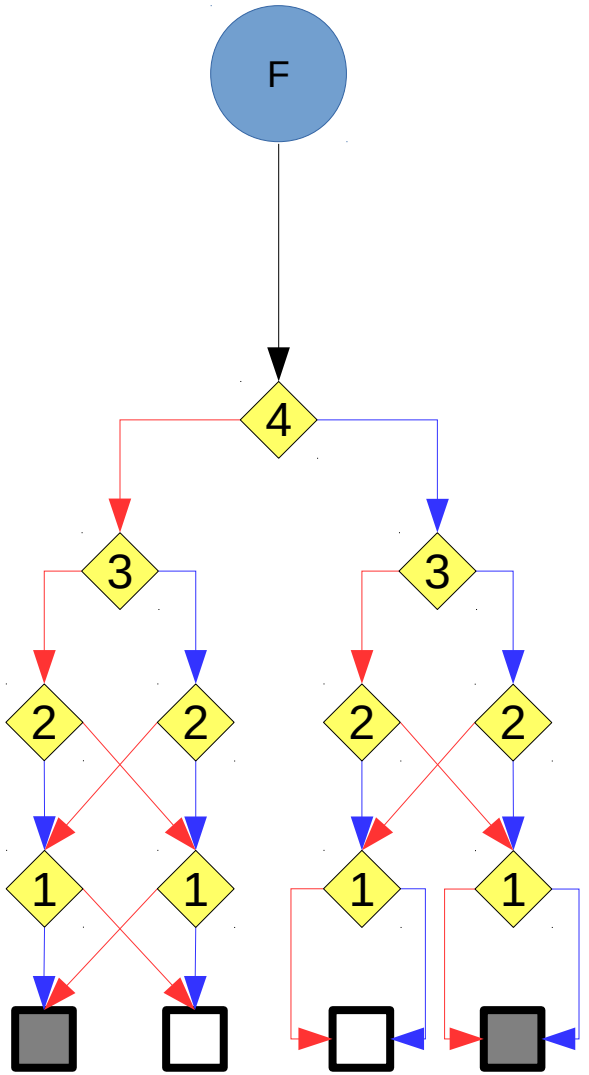
For aesthetic reasons, we do not merge all terminals

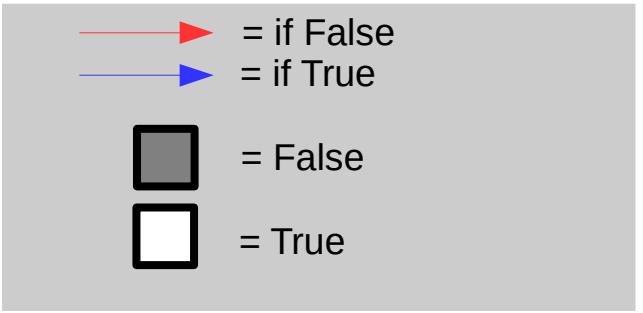
ROBDD + “output negation”





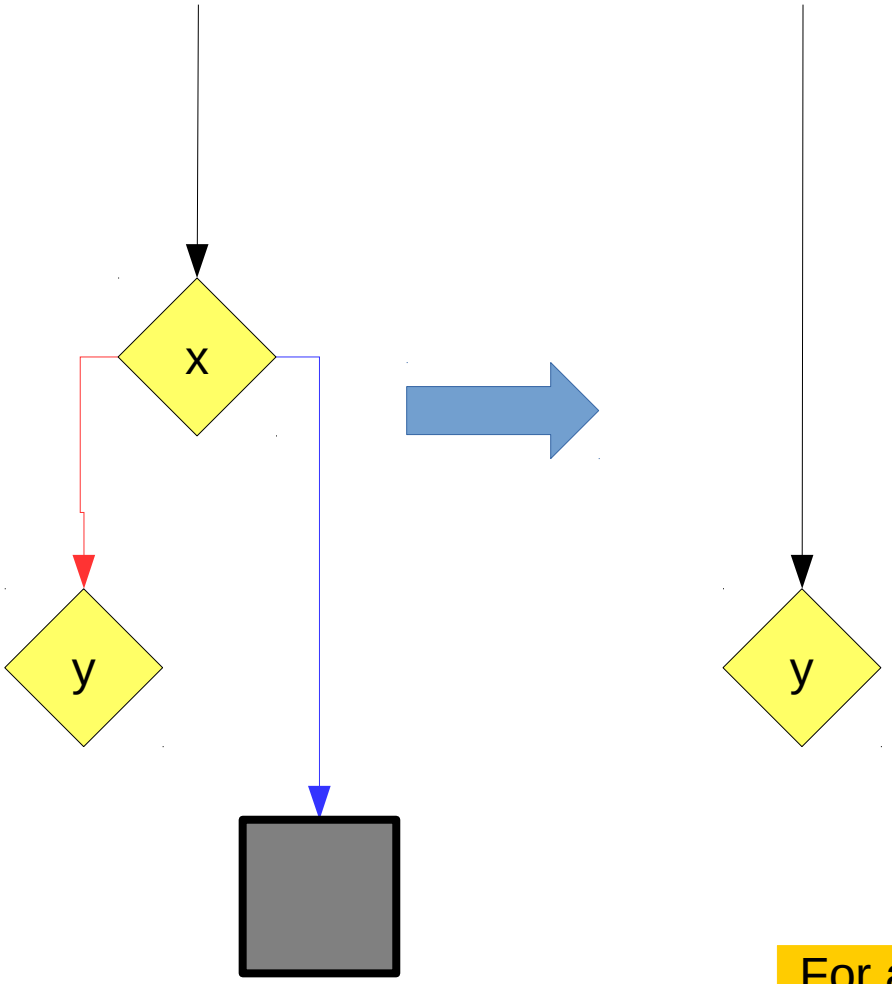
Zero supressed Binary Decision Diagram (ZBDD)

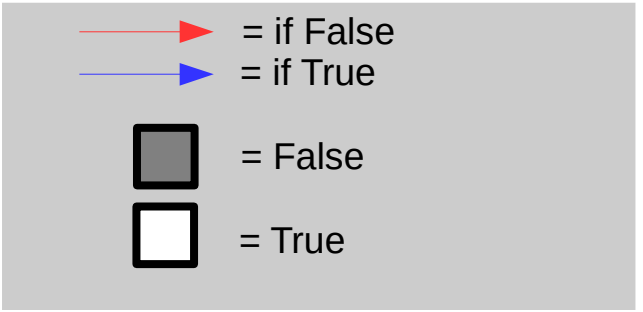




For aesthetic reasons, we do not merge all terminals

ZBDD : reduction rule





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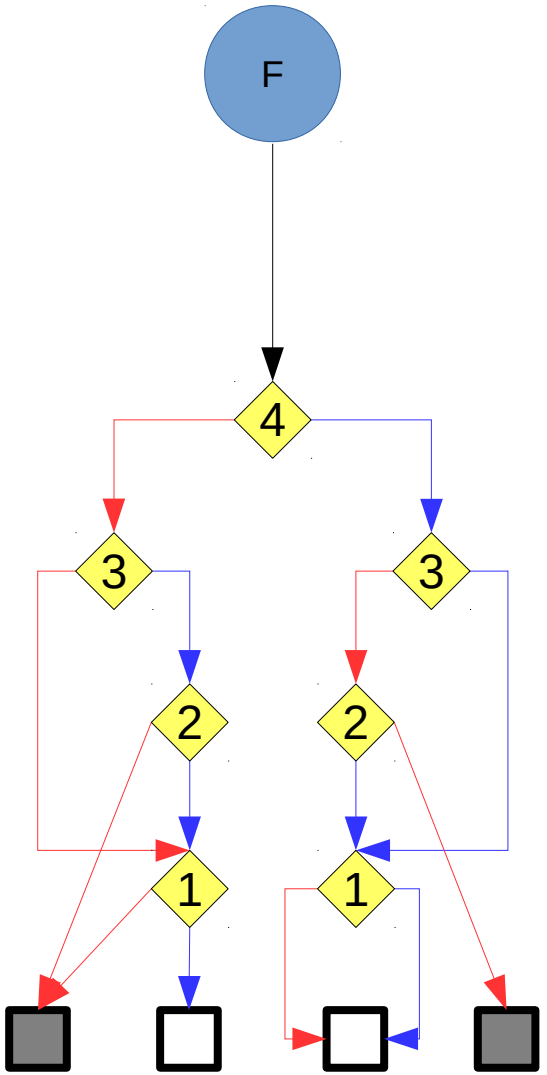
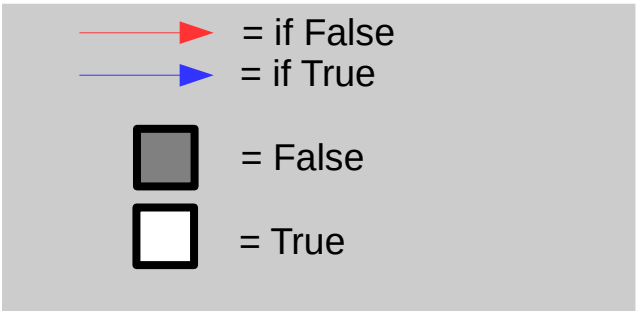
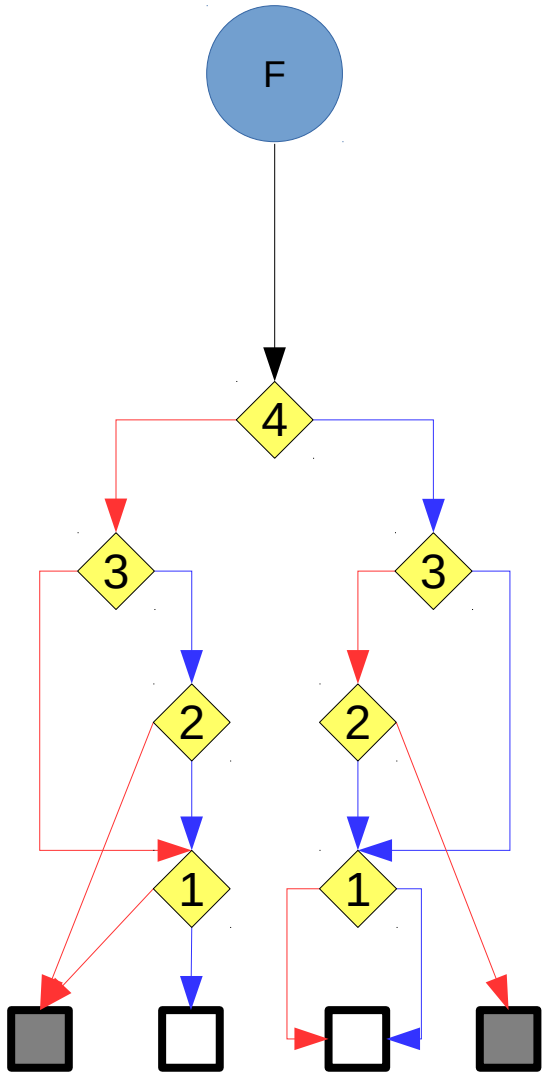
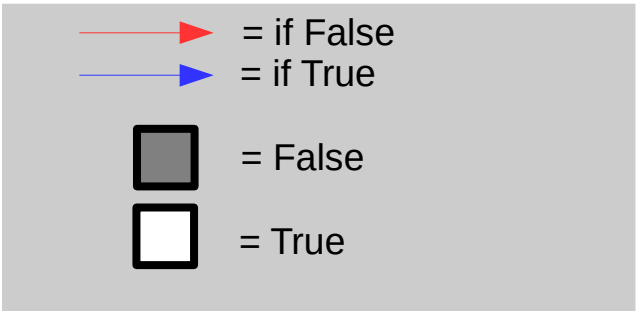




For aesthetic reasons, we do not merge all terminals

ZBDD + “output negation”





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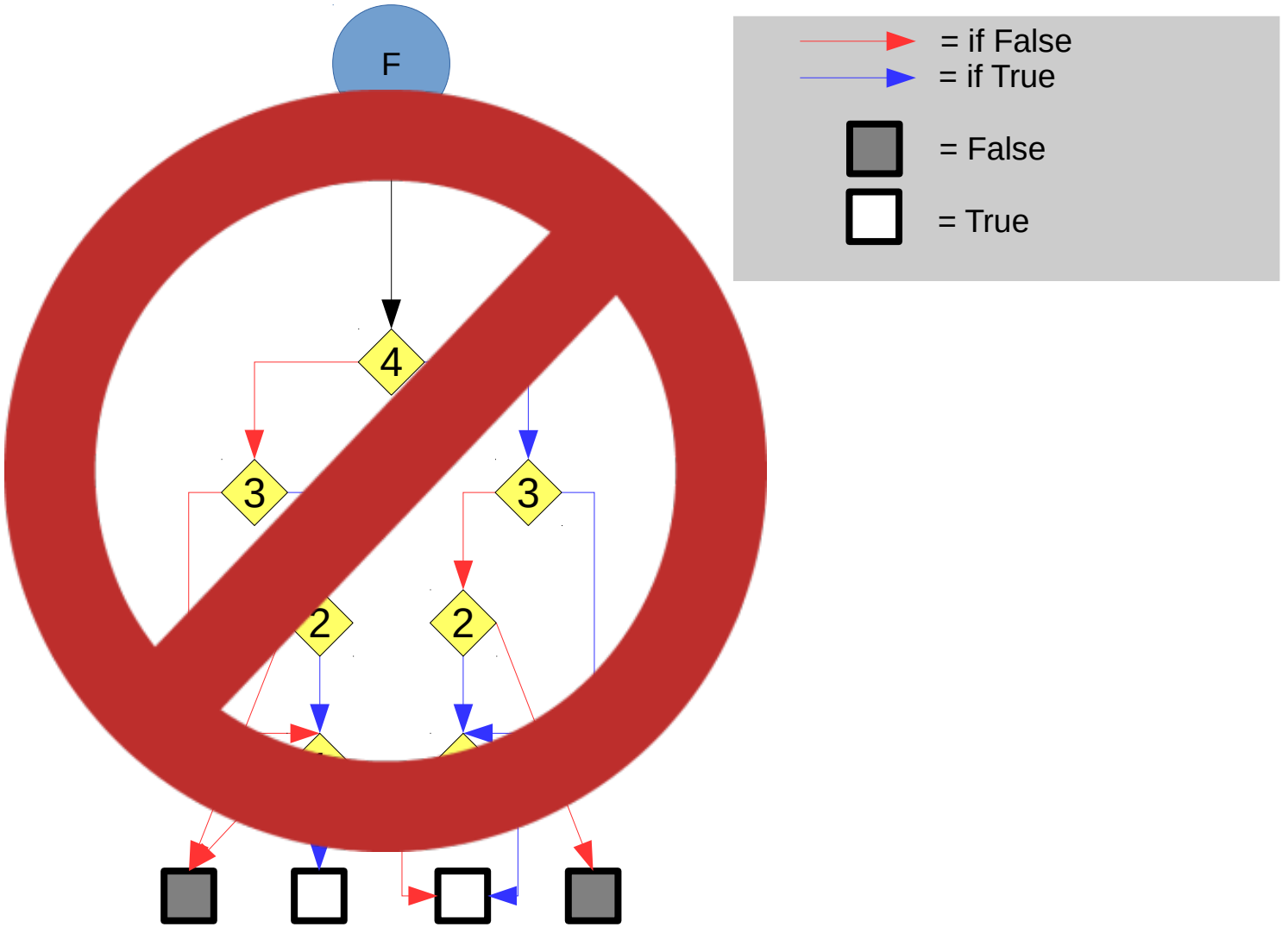

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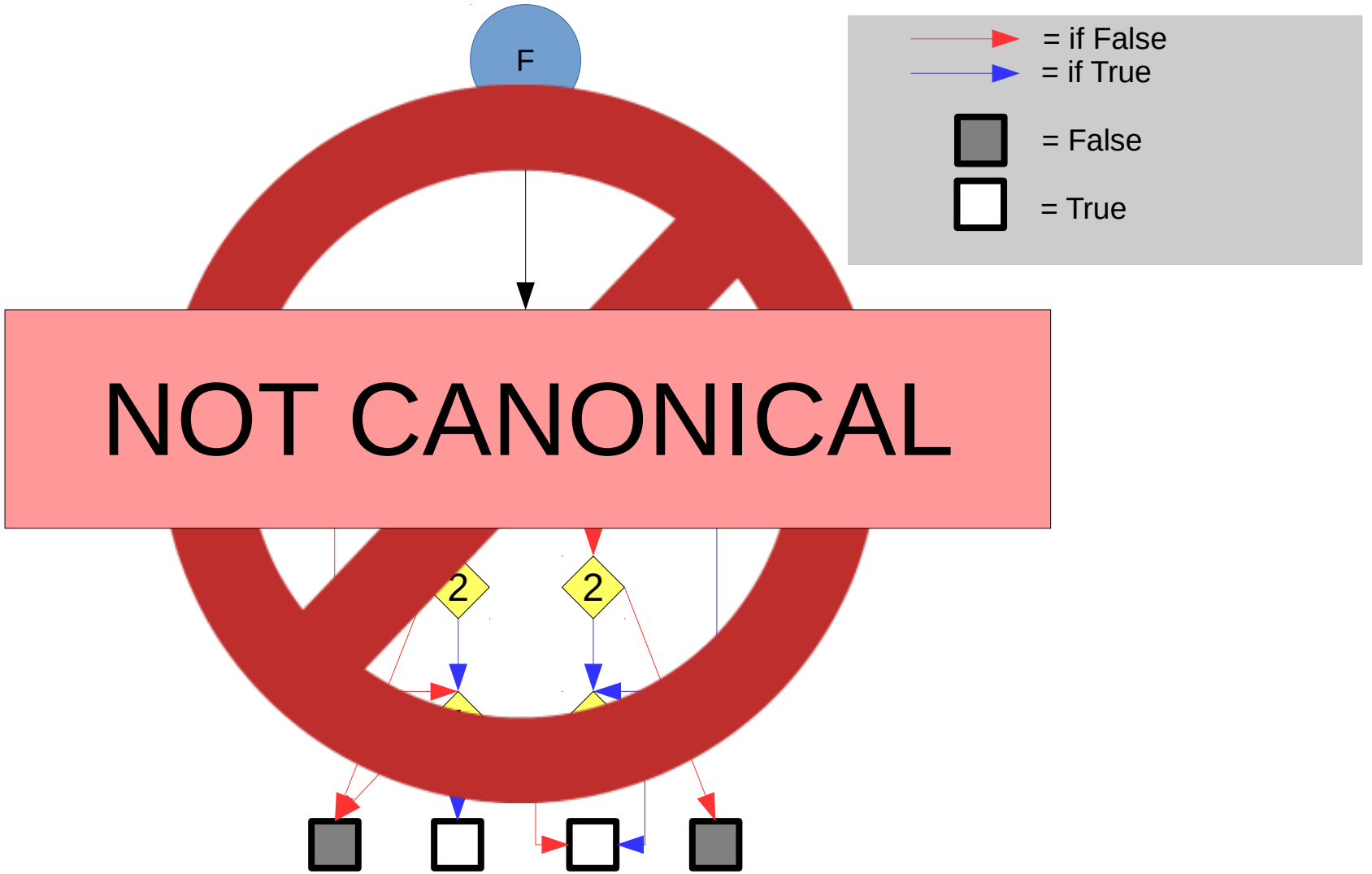
ZBDD + “output negation”



For aesthetic reasons, we do not merge all terminals

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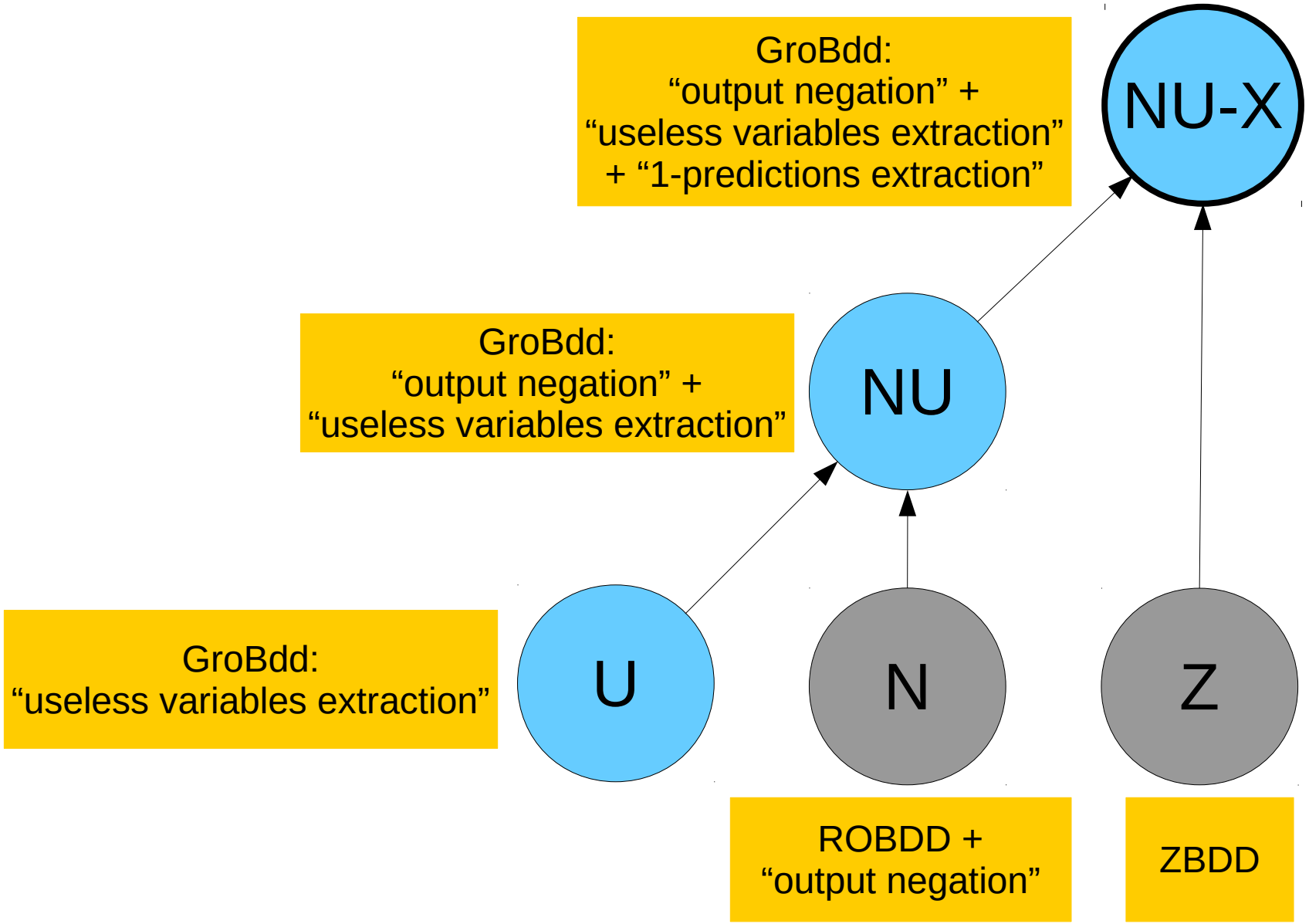
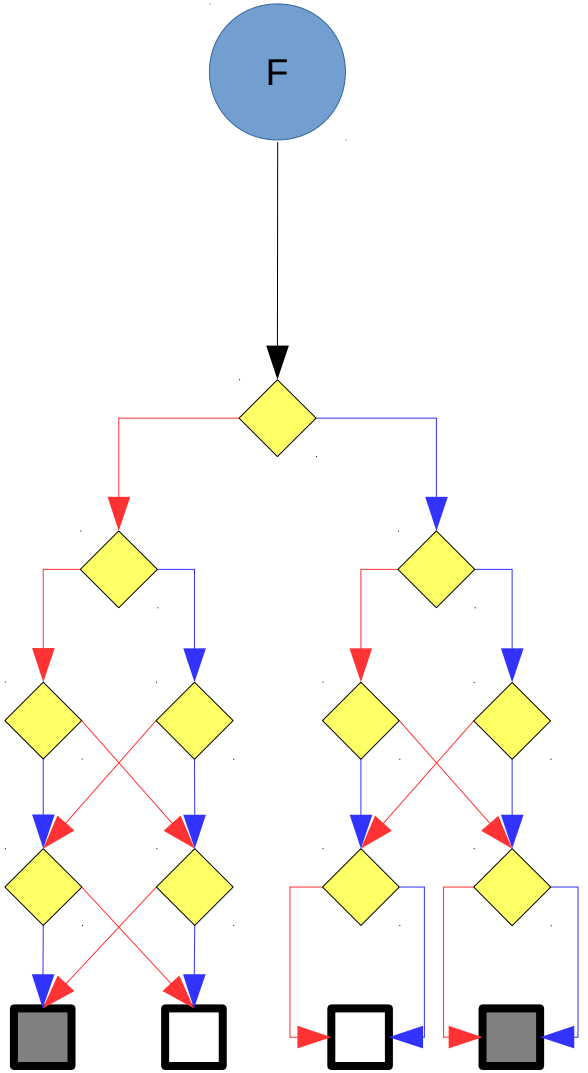
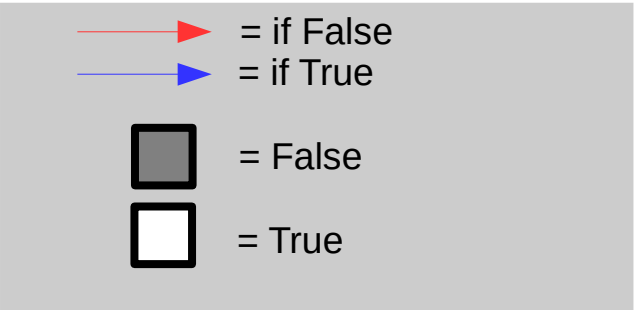


Shannon’s Decision Diagram





For aesthetic reasons, we do not merge all terminals

Section 1 : model U

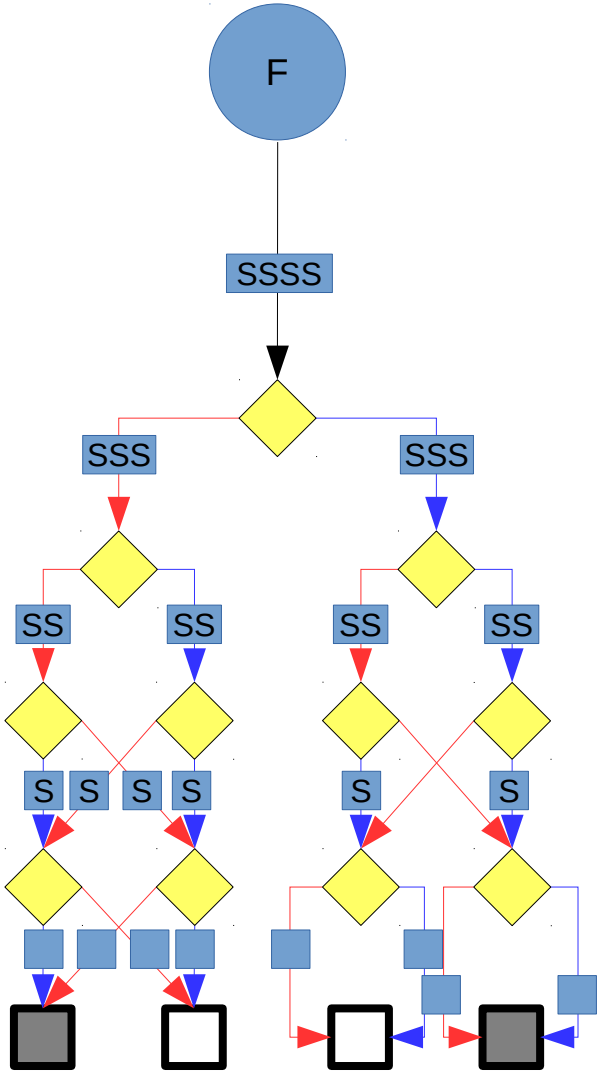
Functions of arity 4

Functions of arity 3

Functions of arity 2

Functions of arity 1

Functions of arity 0



= if False
= if True

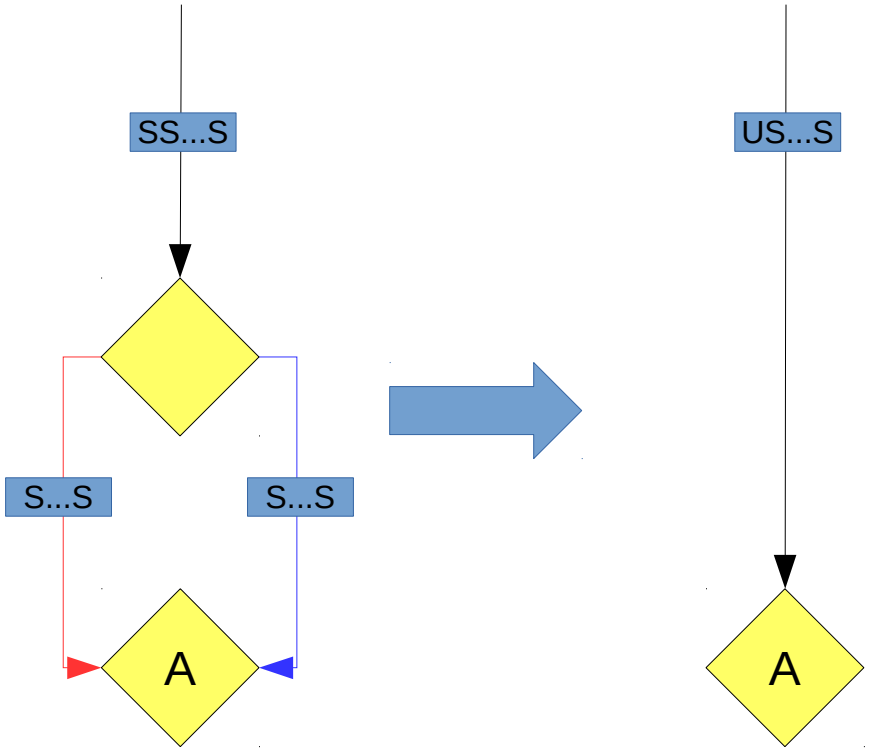
= False

= True

S = significant

For aesthetic reasons, we do not merge all terminals

“Useless variables extraction” : reduction rule (U1)



= if False
= if True

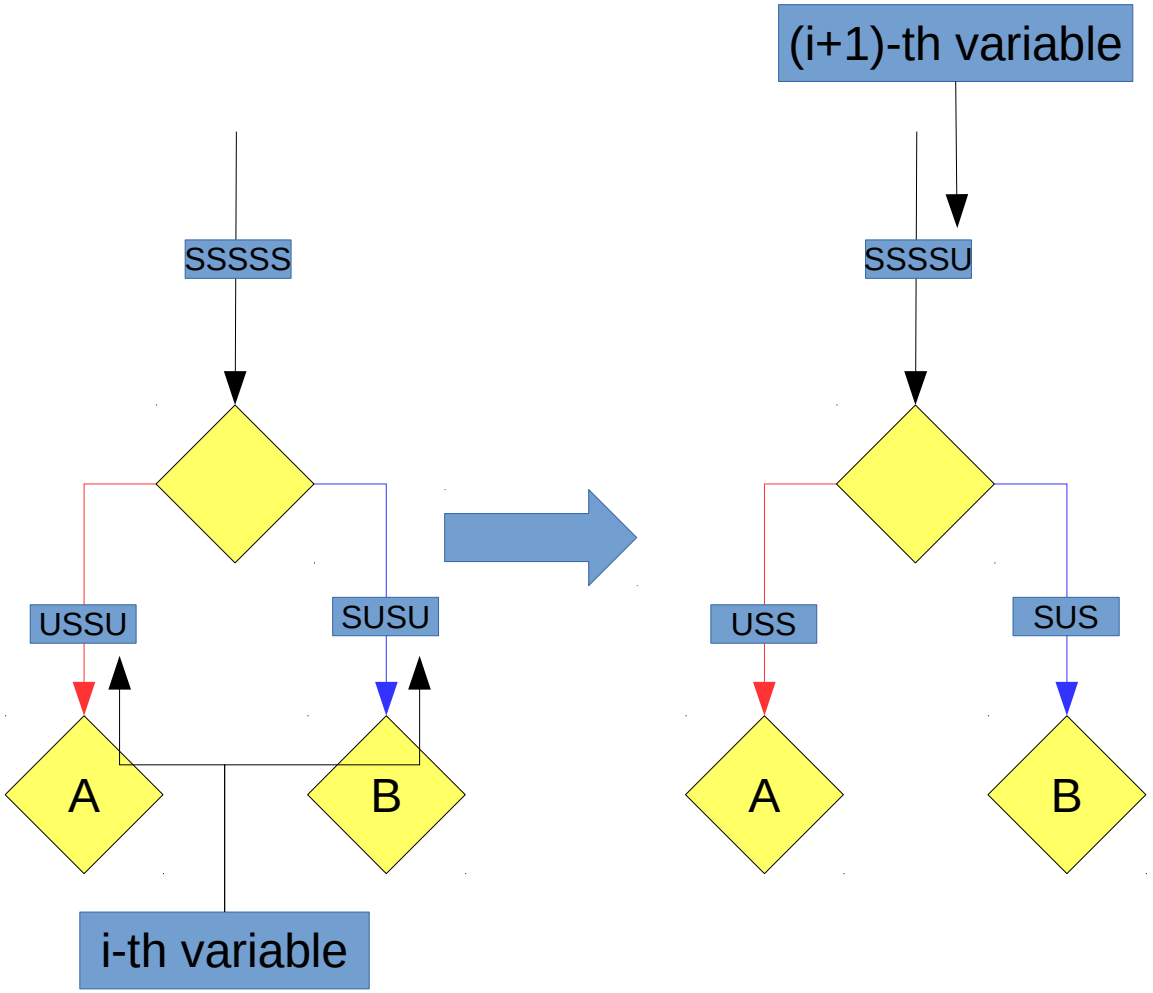
= False

= True

S = significant
U = useless

For aesthetic reasons, we do not merge all terminals

“Useless variables extraction” : reduction rule (U2)



= if False
= if True

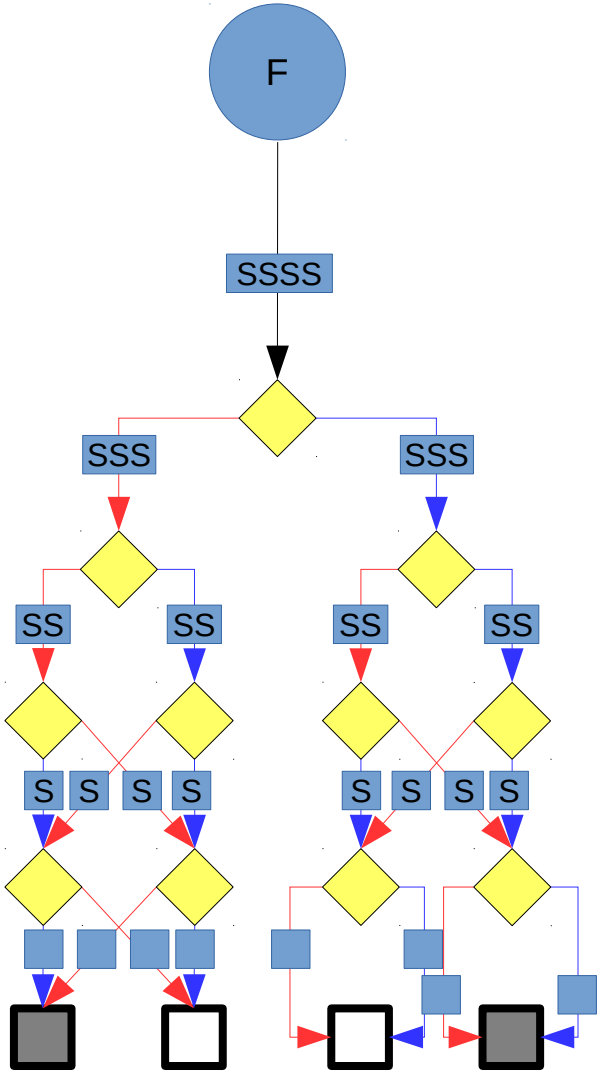
= False

= True

S = significant
U = useless

For aesthetic reasons, we do not merge all terminals

Section 1 : model U



= if False
= if True

= False

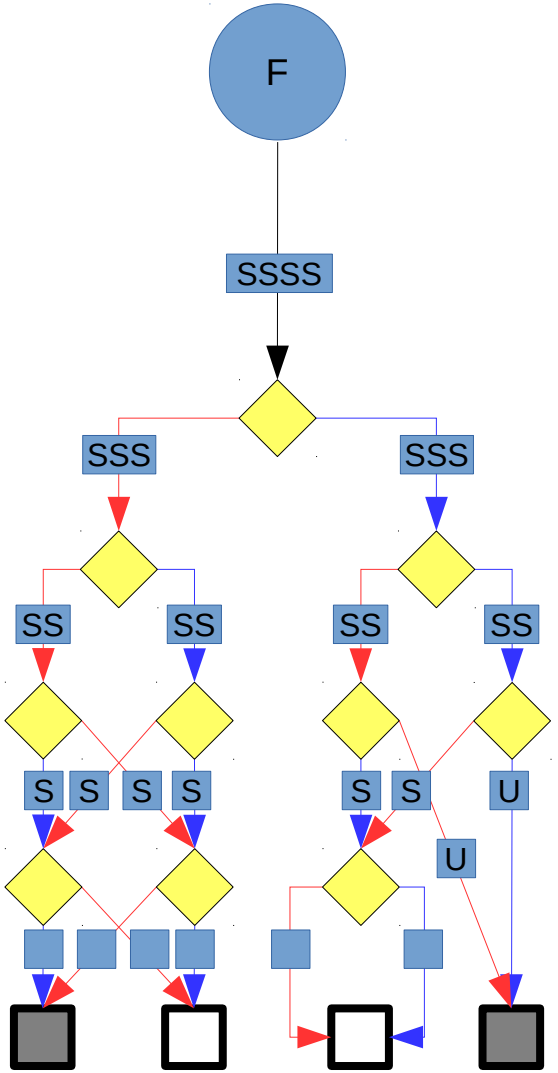
= True

S = significant
U = useless

Apply U1

For aesthetic reasons, we do not merge all terminals

Section 1 : model U



= if False
= if True

= False

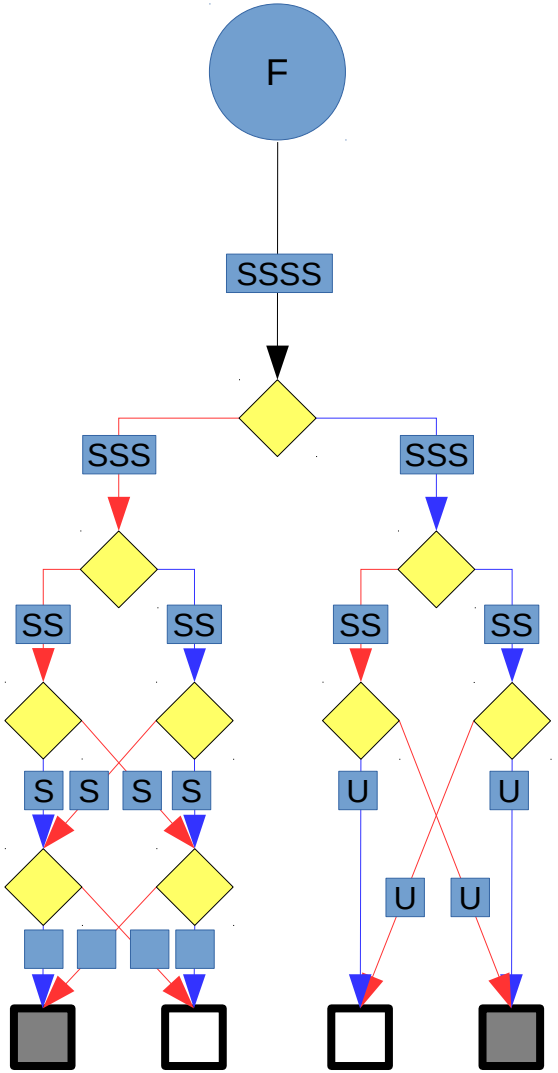
= True

S = significant
U = useless

Apply U1

For aesthetic reasons, we do not merge all terminals

Section 1 : model U



= if False
= if True

= False

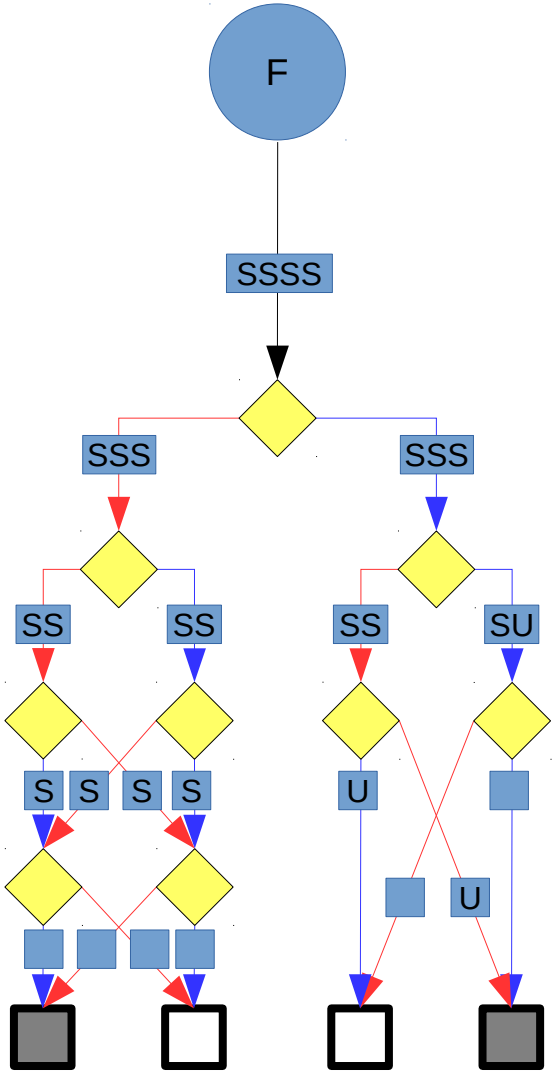
= True

S = significant
U = useless

Apply U2

For aesthetic reasons, we do not merge all terminals

Section 1 : model U



= if False
= if True

= False

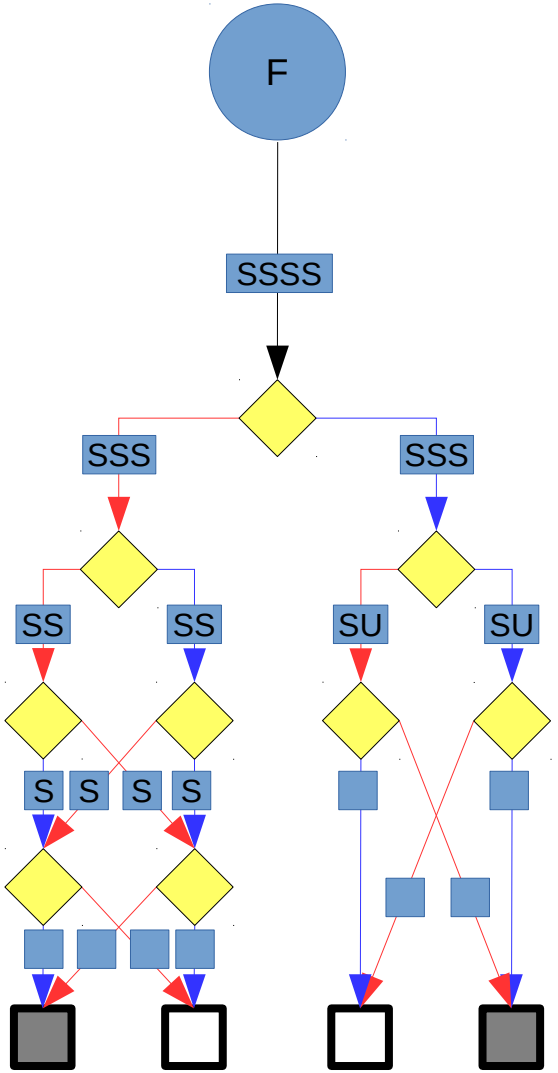
= True

S = significant
U = useless

Apply U2

For aesthetic reasons, we do not merge all terminals

Section 1 : model U



= if False
= if True

= False

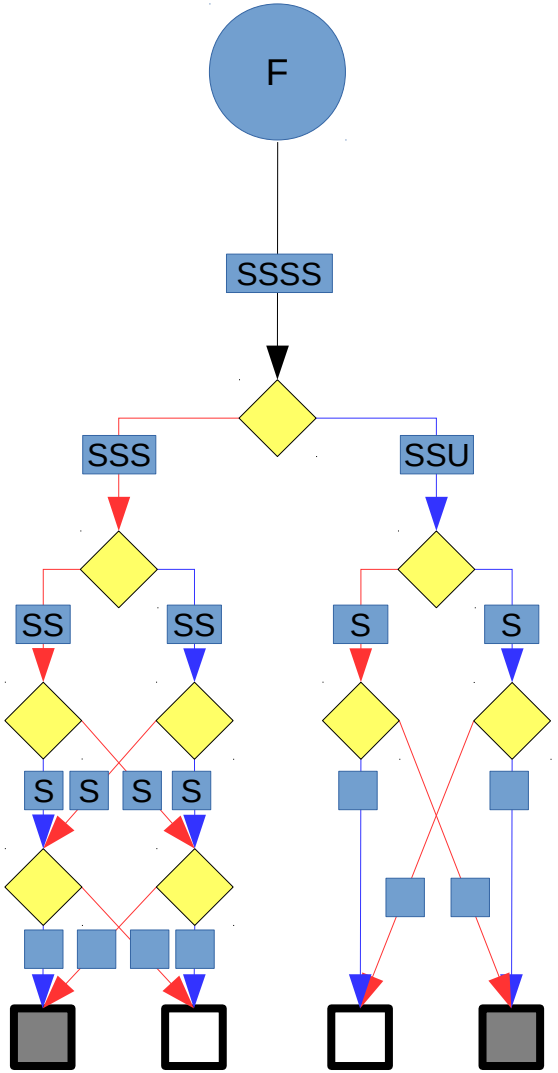
= True

S = significant
U = useless

Apply U2

For aesthetic reasons, we do not merge all terminals

Section 1 : model U



= if False
= if True

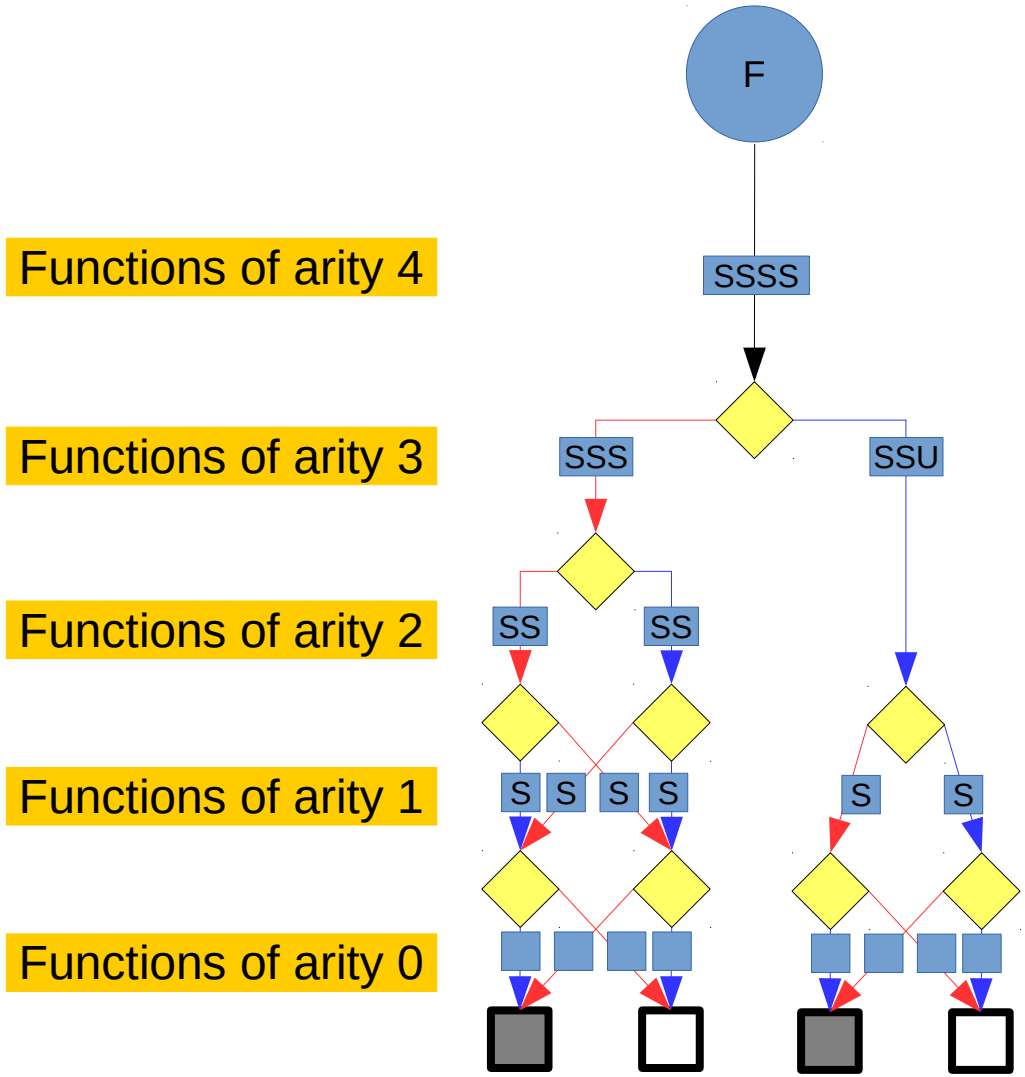
= False

= True

S = significant
U = useless

For aesthetic reasons, we do not merge all terminals

Section 1 : model U



= if False
= if True

= False

= True

S = significant
U = useless

Merge

For aesthetic reasons, we do not merge all terminals

Section 1 : model U

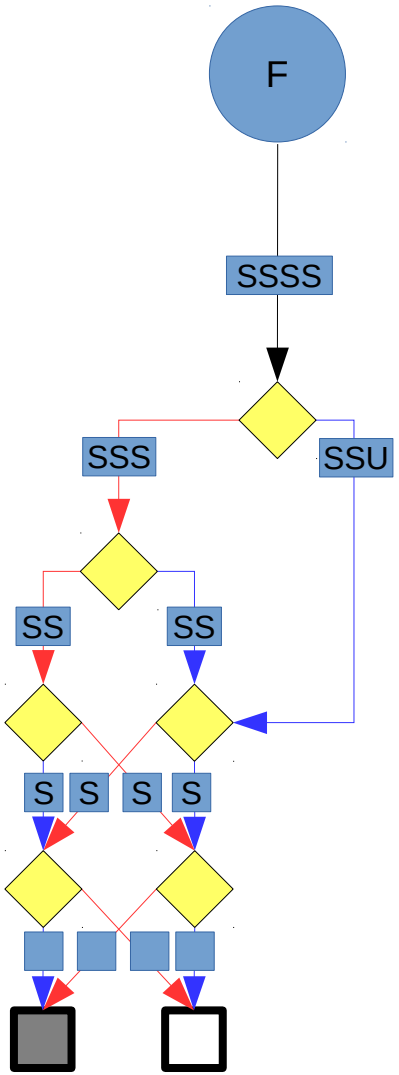
Functions of arity 4

Functions of arity 3

Functions of arity 2

Functions of arity 1

Functions of arity 0



= if False
= if True

= False

= True

S = significant
U = useless

For aesthetic reasons, we do not merge all terminals

Section 1 : model U

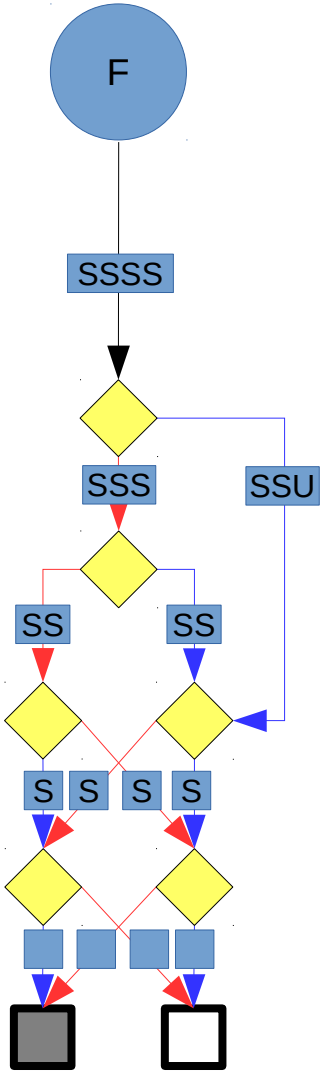
Functions of arity 4

Functions of arity 3

Functions of arity 2

Functions of arity 1

Functions of arity 0



= if False
= if True

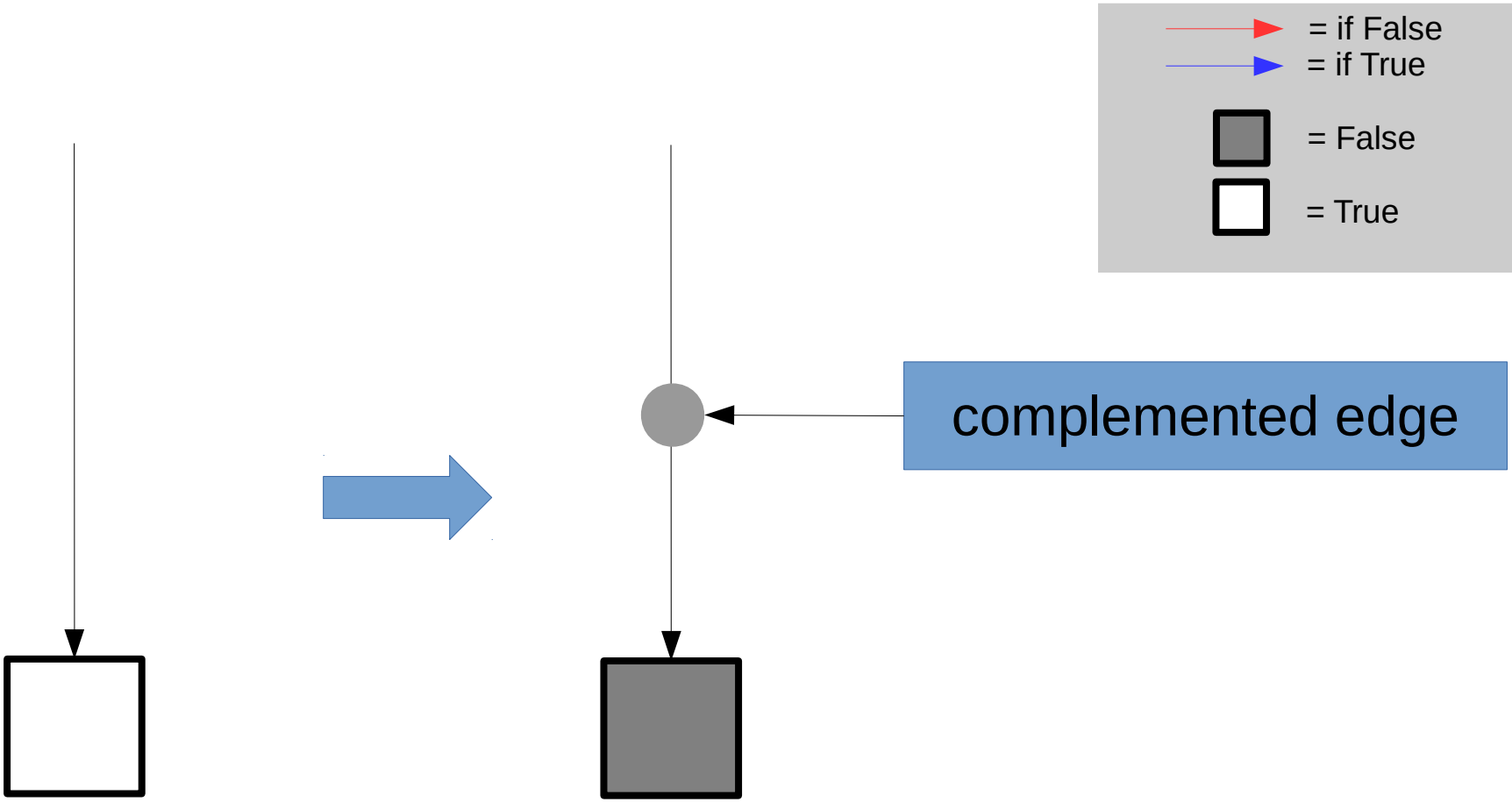
= False

= True

S = significant
U = useless

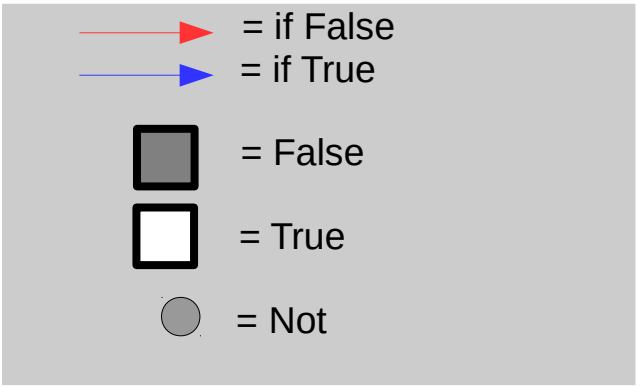
For aesthetic reasons, we do not merge all terminals

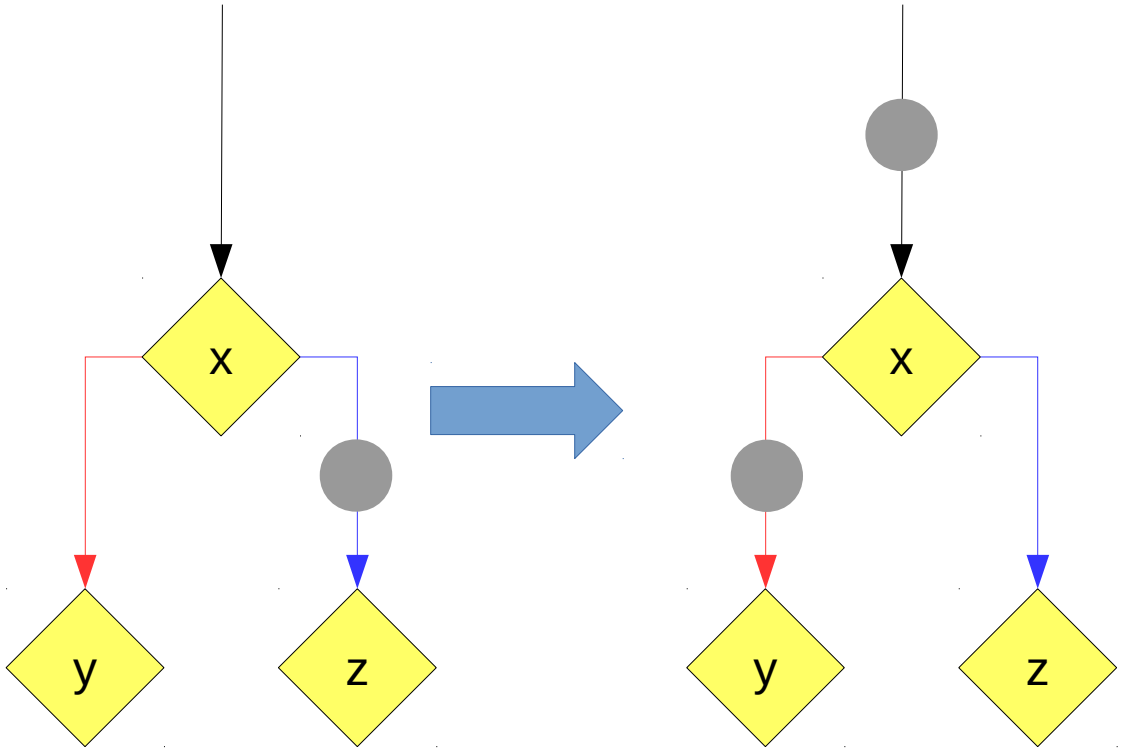
“output negation” : reduction rule (N1)



For aesthetic reasons, we do not merge all terminals

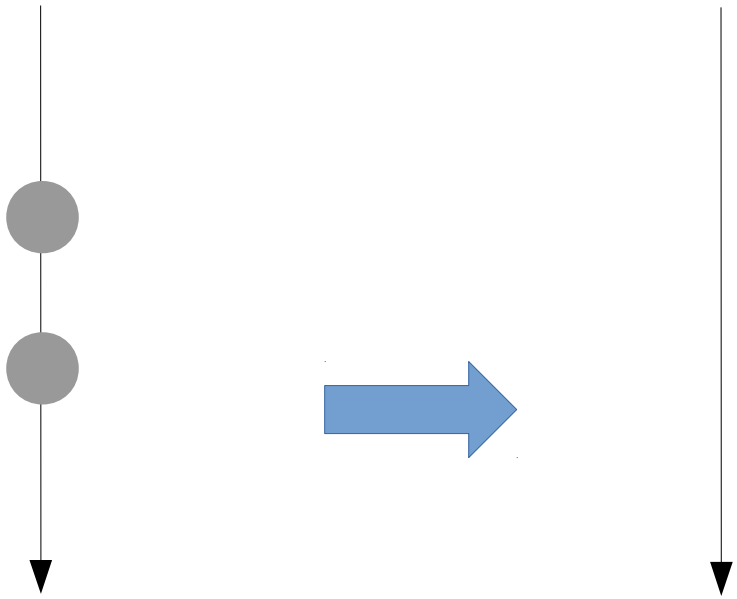
“output negation” : reduction rule (N2)

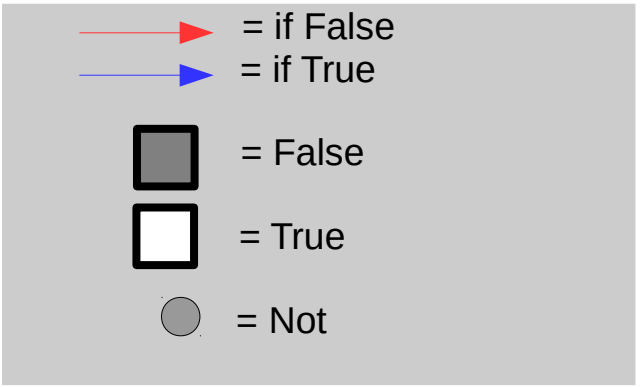




For aesthetic reasons, we do not merge all terminals

“output negation” : reduction rule (N3)





For aesthetic reasons, we do not merge all terminals

Section 2 : model NU

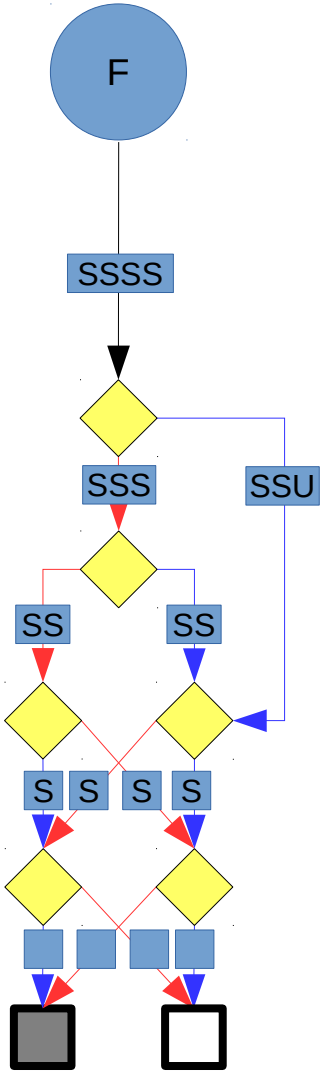
Functions of arity 4

Functions of arity 3

Functions of arity 2

Functions of arity 1

Functions of arity 0



= if False
= if True
= False
= True
= Not

S = significant
U = useless

Apply N1

For aesthetic reasons, we do not merge all terminals

Section 2 : model NU

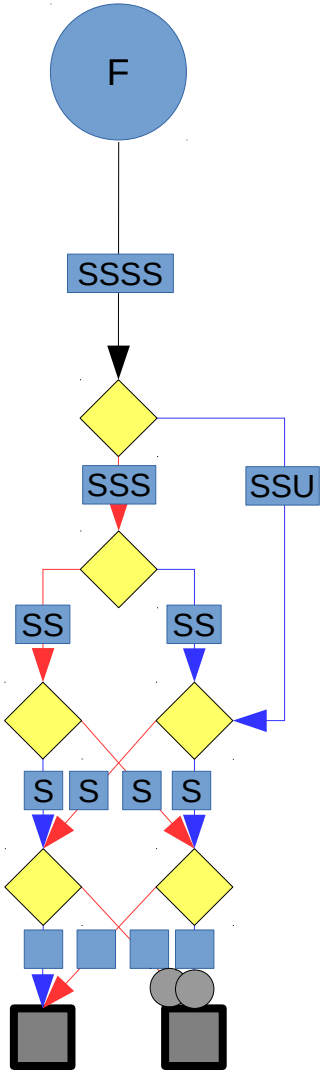
Functions of arity 4

Functions of arity 3

Functions of arity 2

Functions of arity 1

Functions of arity 0



= if False
= if True
= False
= True
= Not

S = significant
U = useless

Apply N2

For aesthetic reasons, we do not merge all terminals

Section 2 : model NU

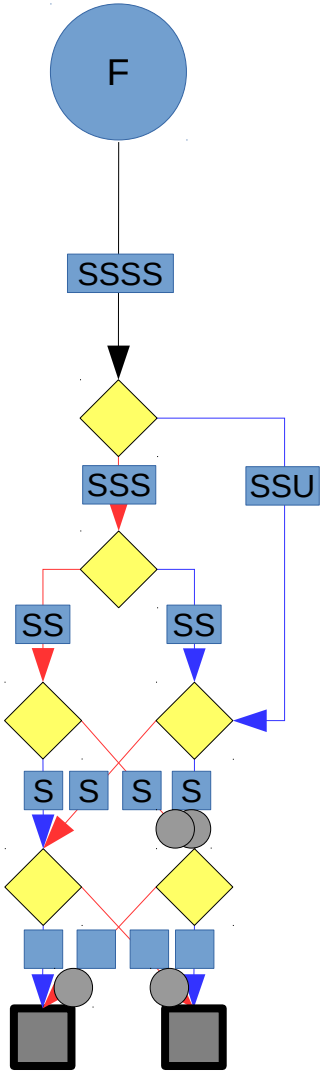
Functions of arity 4

Functions of arity 3

Functions of arity 2

Functions of arity 1

Functions of arity 0



= if False
= if True
= False
= True
= Not

S = significant
U = useless

Apply N2

For aesthetic reasons, we do not merge all terminals

Section 2 : model NU

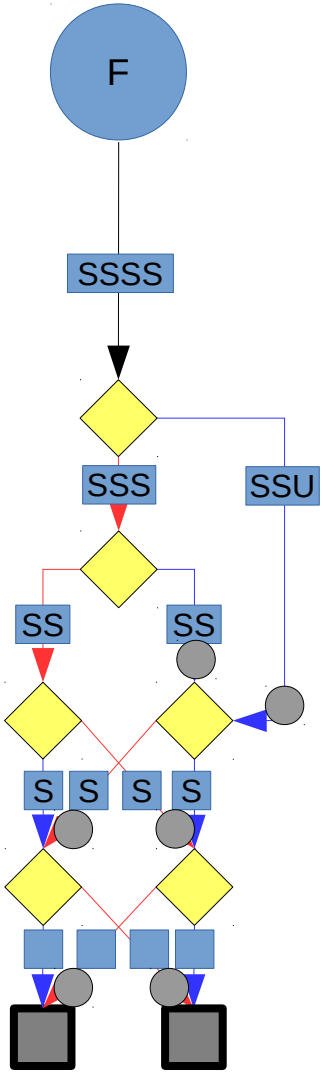
Functions of arity 4

Functions of arity 3

Functions of arity 2

Functions of arity 1

Functions of arity 0



= if False
= if True
= False
= True
= Not

S = significant
U = useless

Apply N2

For aesthetic reasons, we do not merge all terminals

Section 2 : model NU

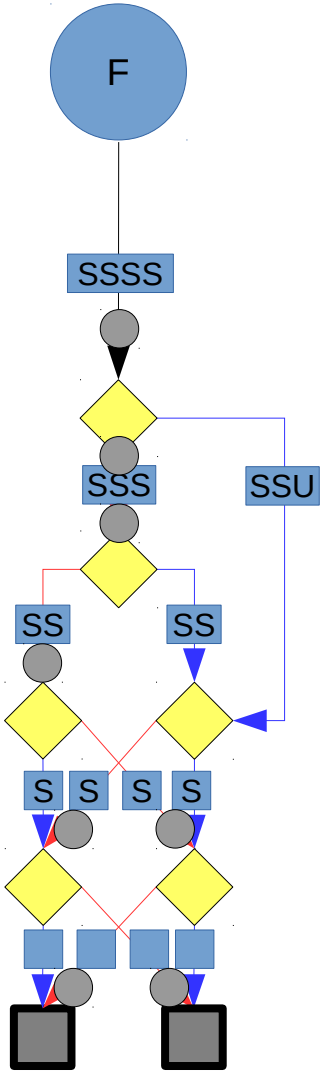
Functions of arity 4

Functions of arity 3

Functions of arity 2

Functions of arity 1

Functions of arity 0



= if False
= if True
= False
= True
= Not

S = significant
U = useless

Apply N3

For aesthetic reasons, we do not merge all terminals

Section 2 : model NU

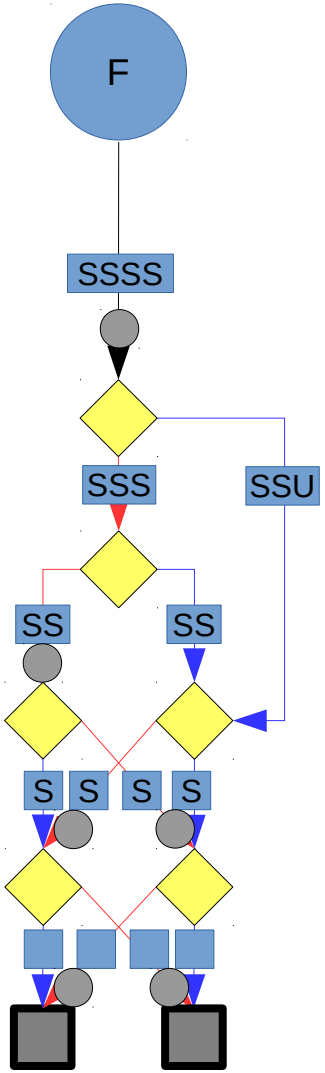
Functions of arity 4

Functions of arity 3

Functions of arity 2

Functions of arity 1

Functions of arity 0



= if False
= if True

= False

= True

= Not

S = significant
U = useless

Merge

For aesthetic reasons, we do not merge all terminals

Section 2 : model NU

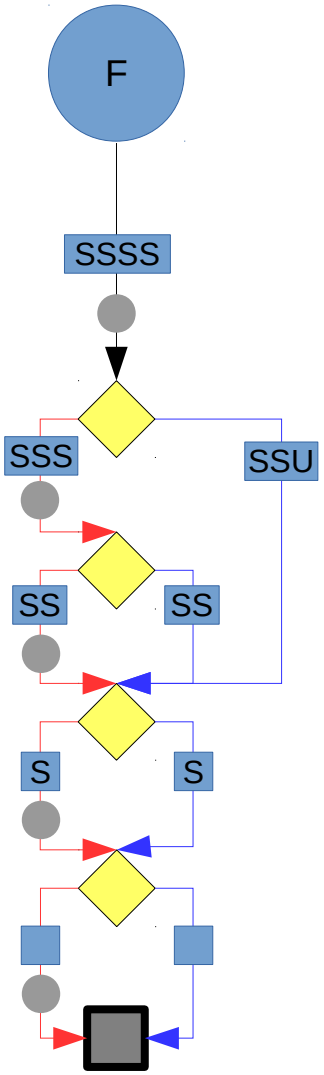
Functions of arity 4

Functions of arity 3

Functions of arity 2

Functions of arity 1

Functions of arity 0



= if False
= if True

= False

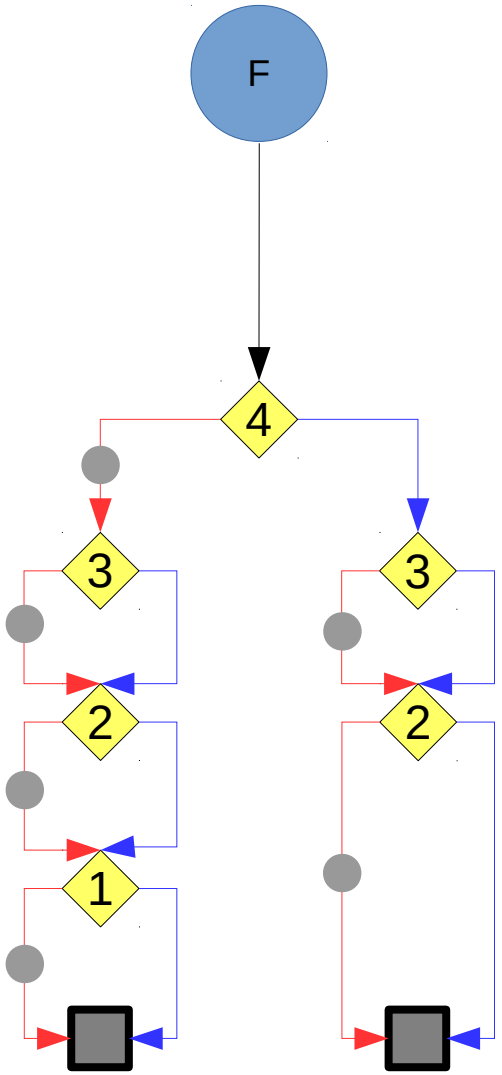
= True

= Not

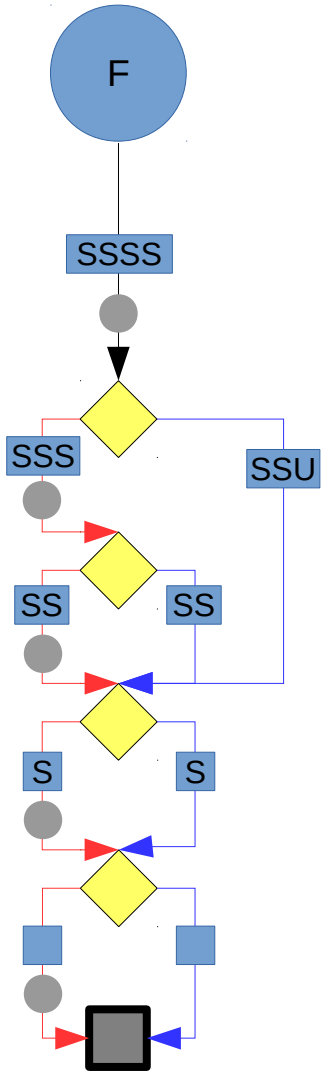
S = significant
U = useless

For aesthetic reasons, we do not merge all terminals

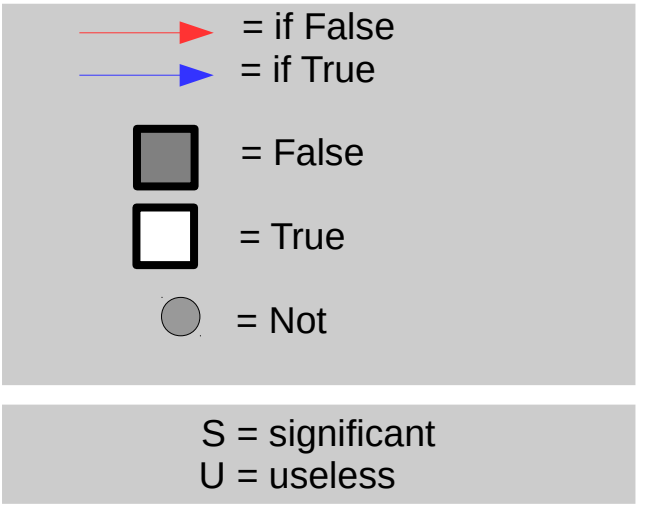
N vs NU



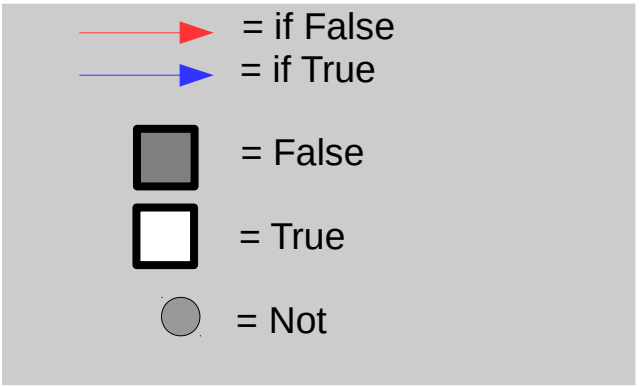
N =
ROBDD + “output negation”



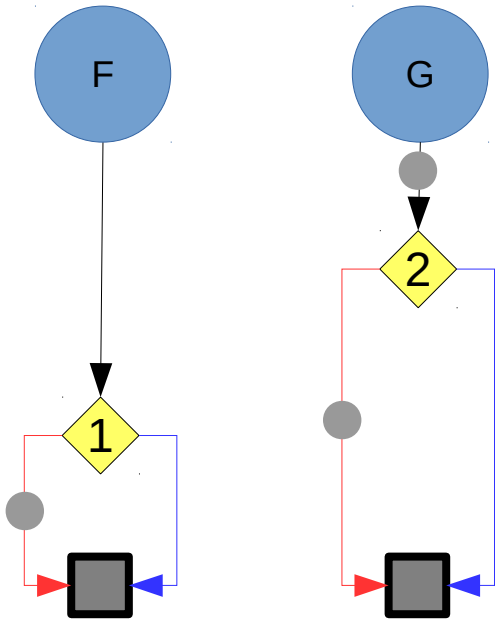
Model NU



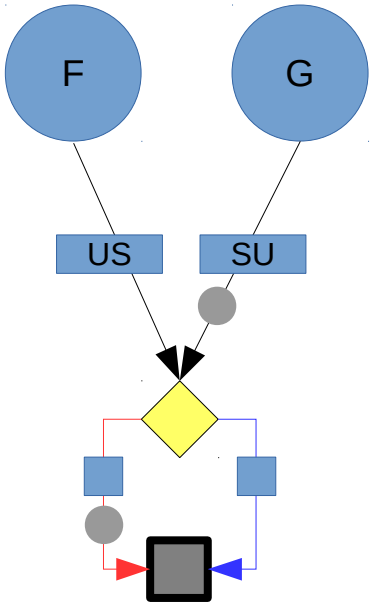
N vs NU: Example 2



S = significant
U = useless

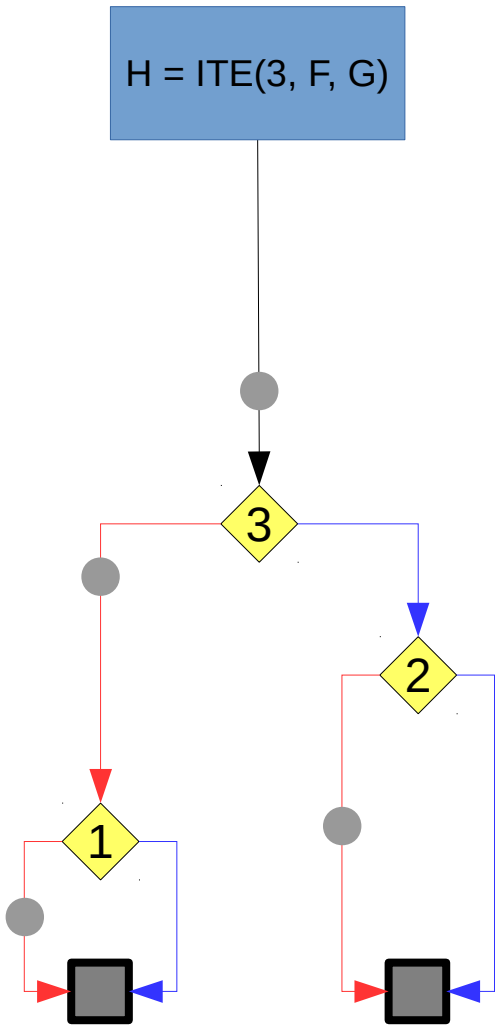


N =
ROBDD + “output negation”

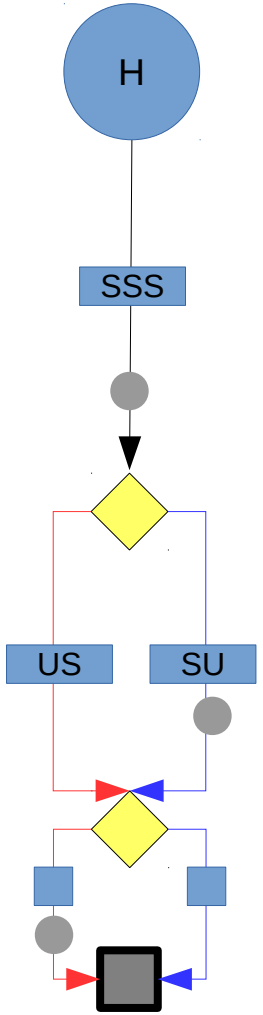


Model NU

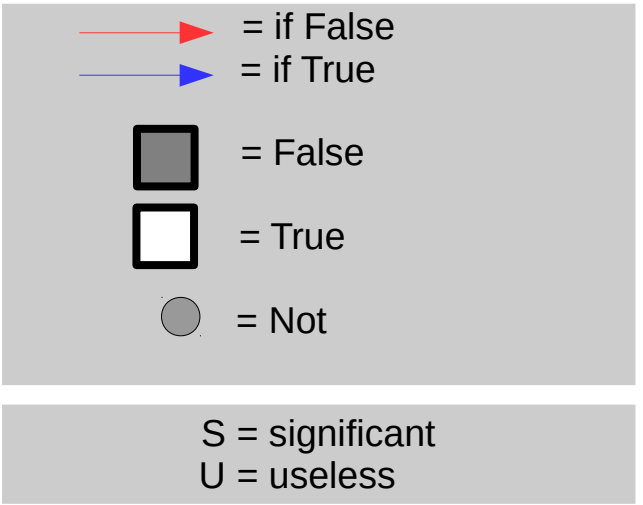
N vs NU: Example 3



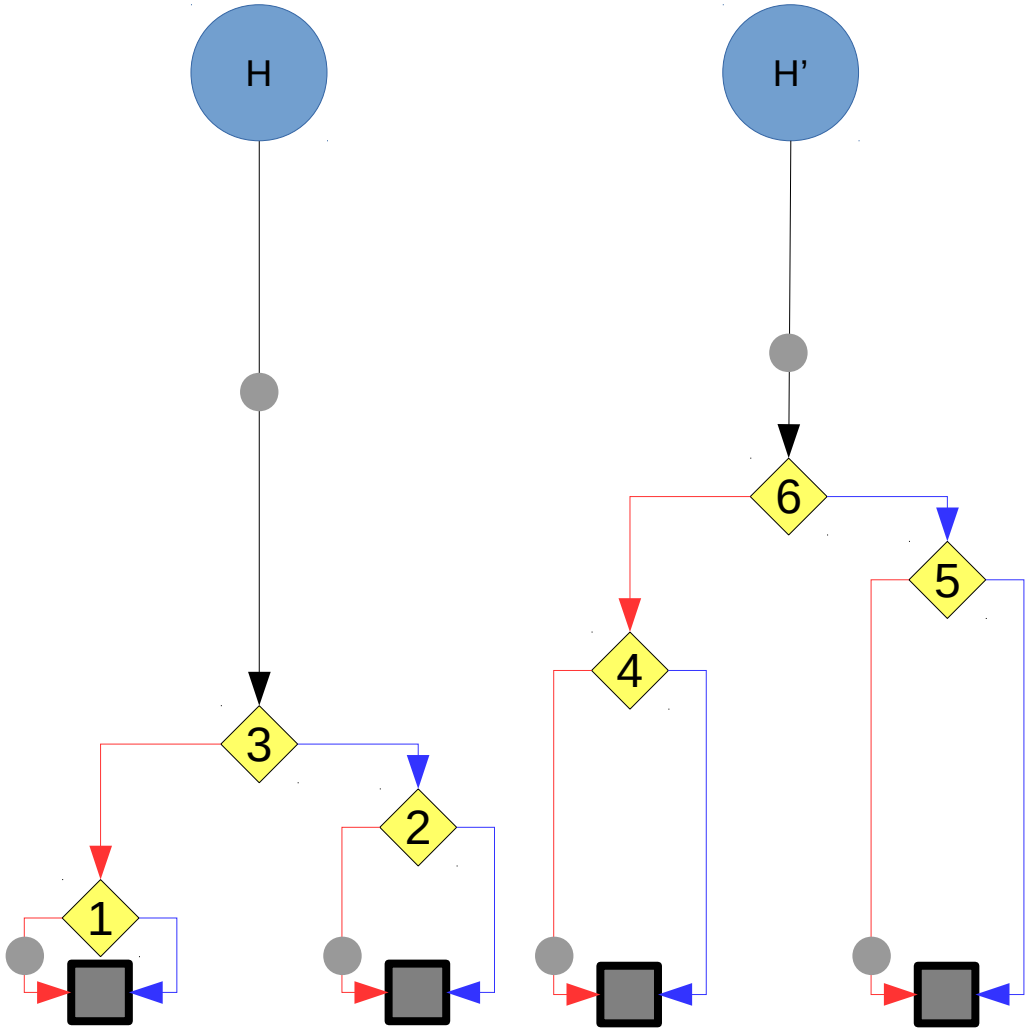
N =
ROBDD + “output negation”



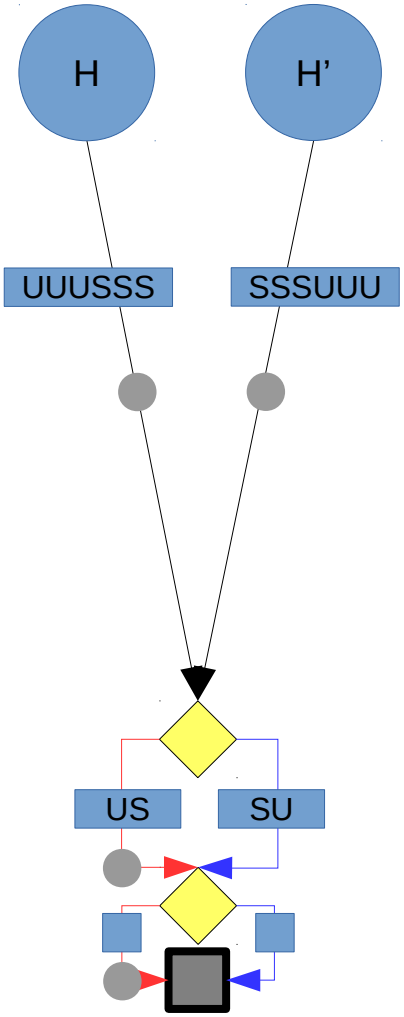
Model NU



N vs NU: Example 4

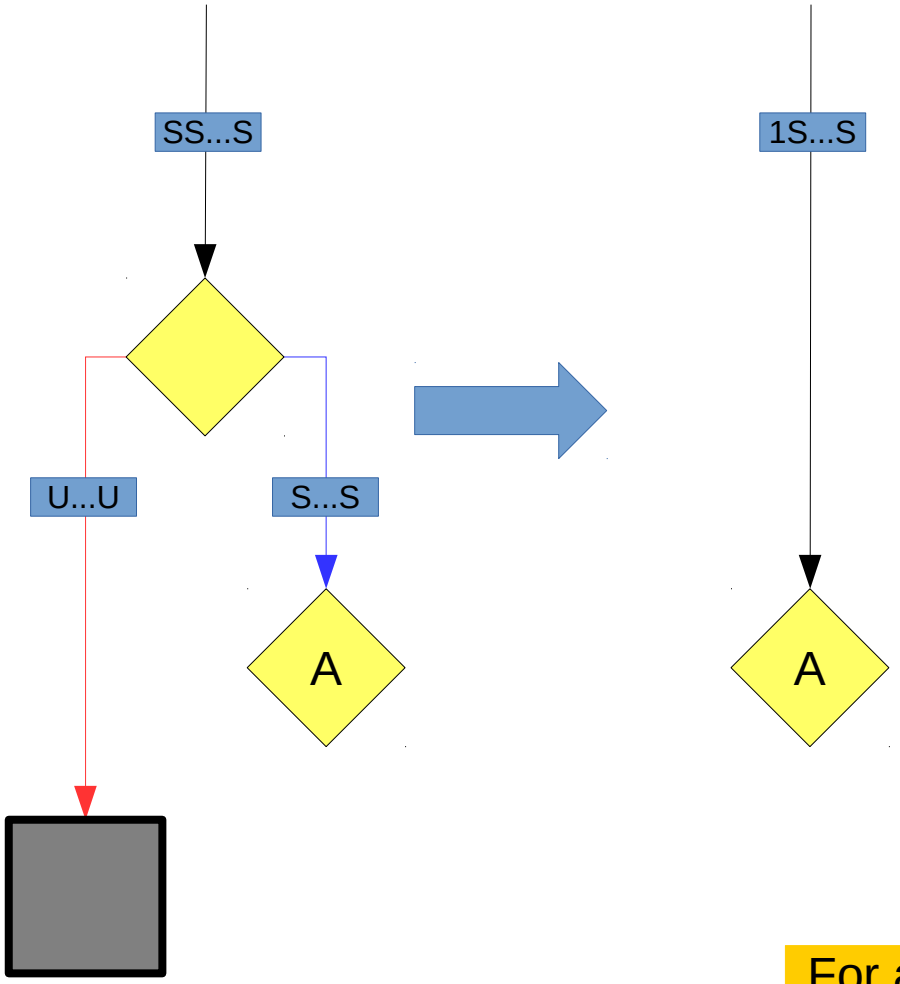


N =
ROBDD + “output negation”



Model NU

“1-prediction” : reduction rule (X1-’1’)

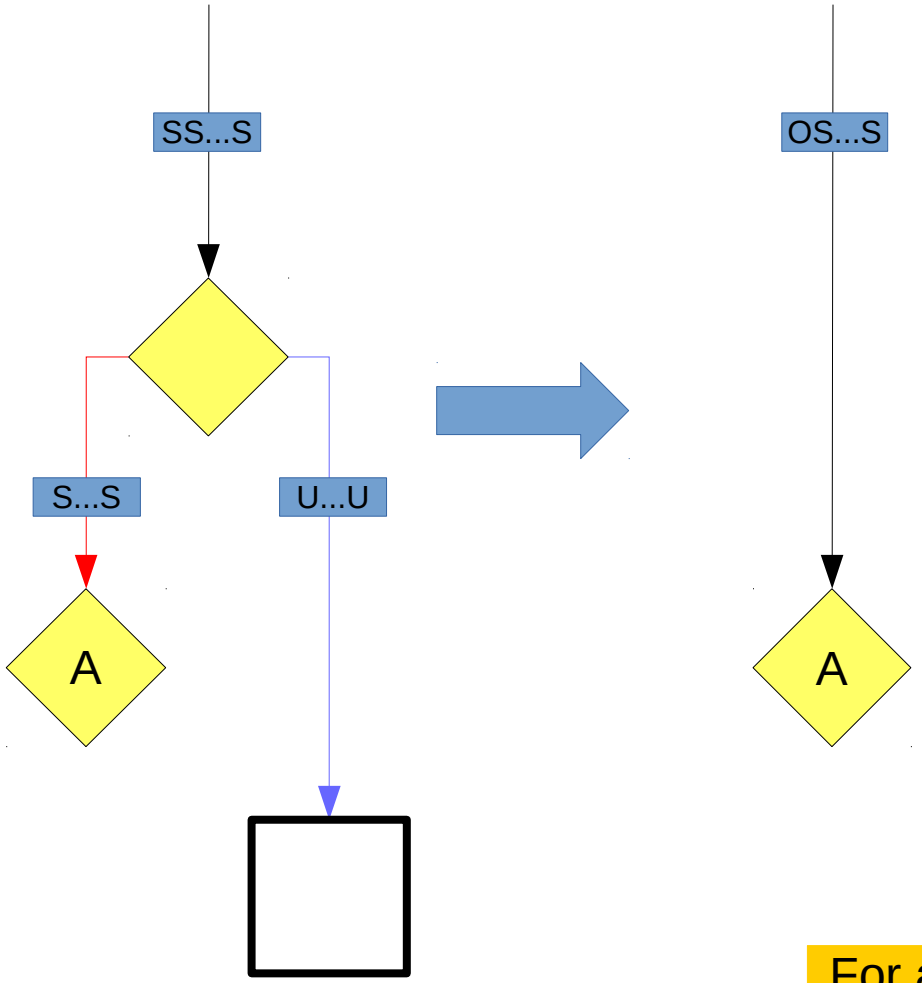


= if False
= if True
= False
= True
= Not

S = significant
U = useless
1 = If 0 then 0
0 = If 1 then 0
I = If 0 then 1
O = If 1 then 1

For aesthetic reasons, we do not merge all terminals

“1-prediction” : reduction rule (X1-’O’)

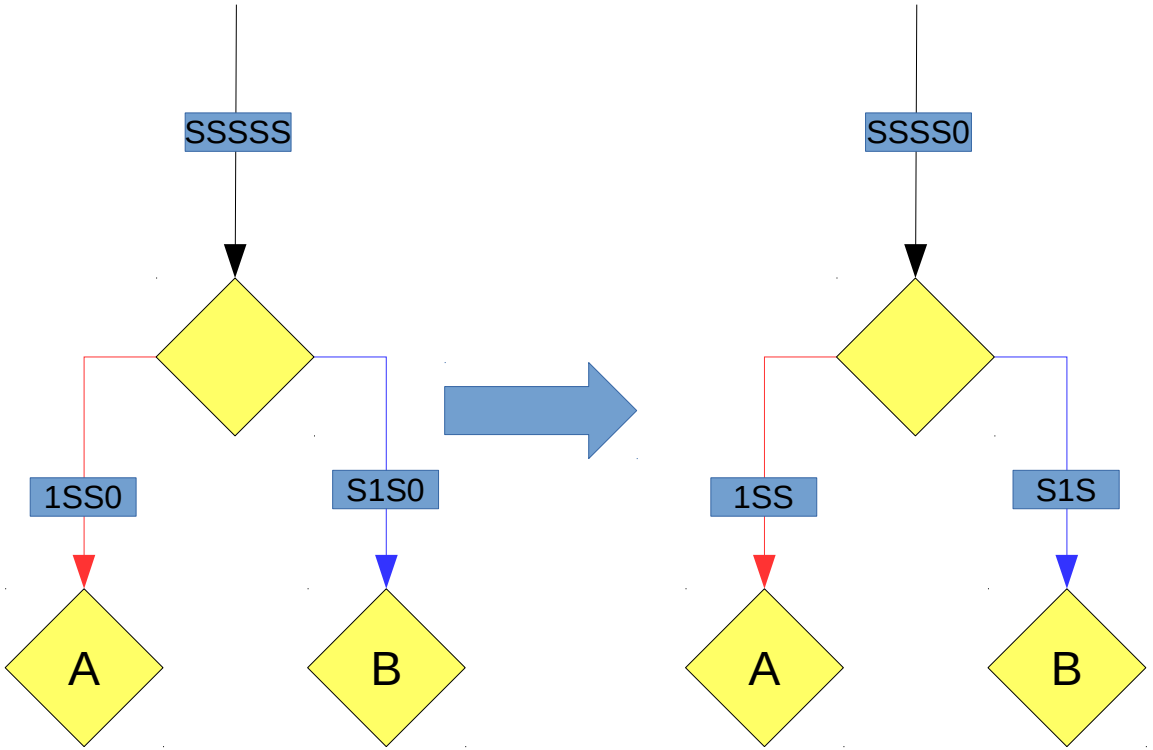


= if False
= if True
= False
= True
= Not

S = significant
U = useless
1 = If 0 then 0
0 = If 1 then 0
I = If 0 then 1
O = If 1 then 1

For aesthetic reasons, we do not merge all terminals

“1-prediction” : reduction rule (X2-’0’)



= if False
= if True
= False
= True
= Not

S = significant
U = useless
1 = If 0 then 0
0 = If 1 then 0
I = If 0 then 1
O = If 1 then 1

For aesthetic reasons, we do not merge all terminals

Section 2 : model NU

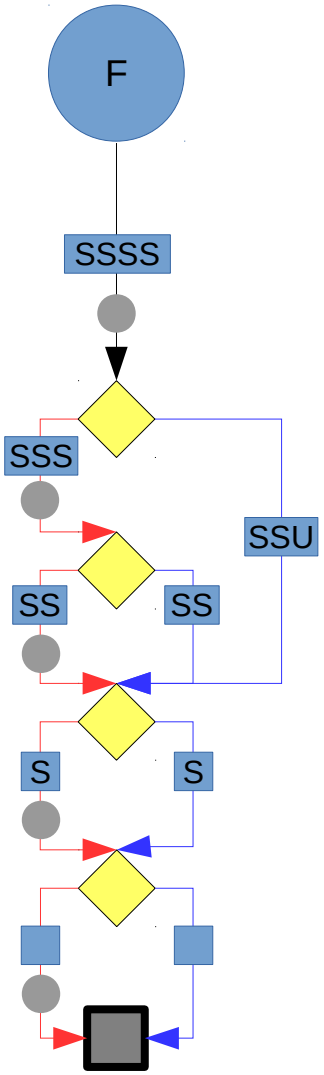
Functions of arity 4

Functions of arity 3

Functions of arity 2

Functions of arity 1

Functions of arity 0



= if False
= if True
= False
= True
= Not

S = significant
U = useless
1 = If 0 then 0
0 = If 1 then 0
I = If 0 then 1
O = If 1 then 1

Apply X1-'I'

For aesthetic reasons, we do not merge all terminals

Section 3 : model NU-X

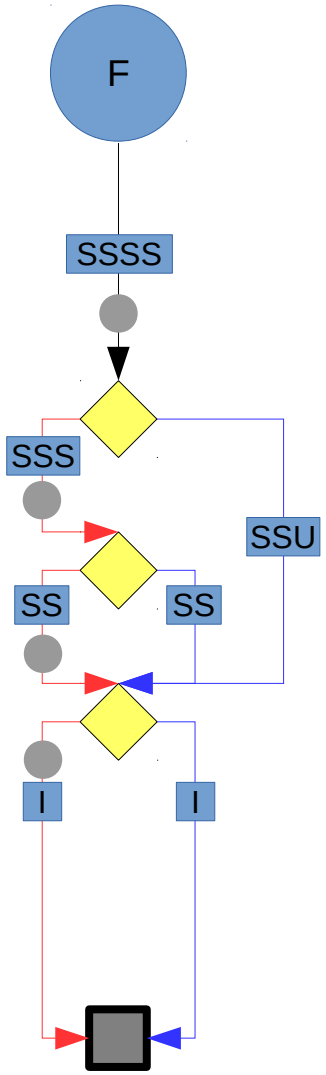
Functions of arity 4

Functions of arity 3

Functions of arity 2

Functions of arity 1

Functions of arity 0

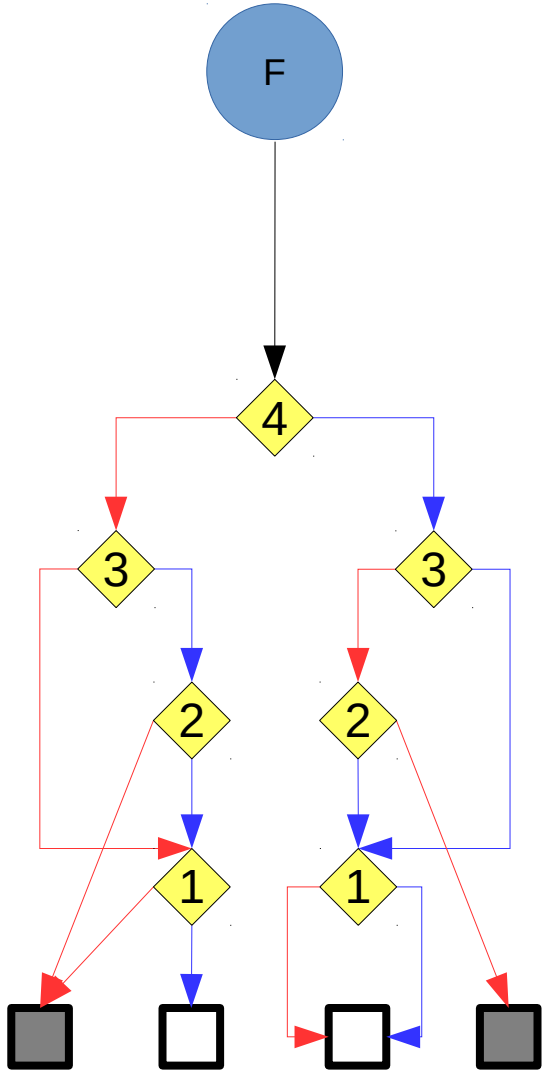


= if False
= if True
= False
= True
= Not

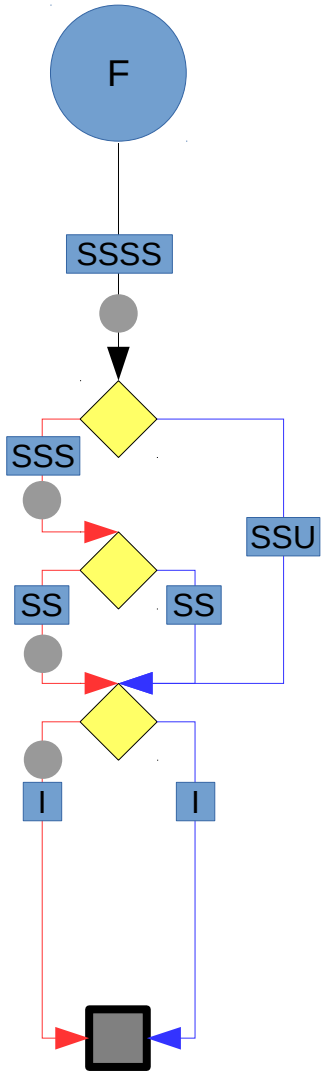
S = significant
U = useless
1 = If 0 then 0
0 = If 1 then 0
I = If 0 then 1
O = If 1 then 1

For aesthetic reasons, we do not merge all terminals

Section 3 : model NU-X



Z =
ZBDD

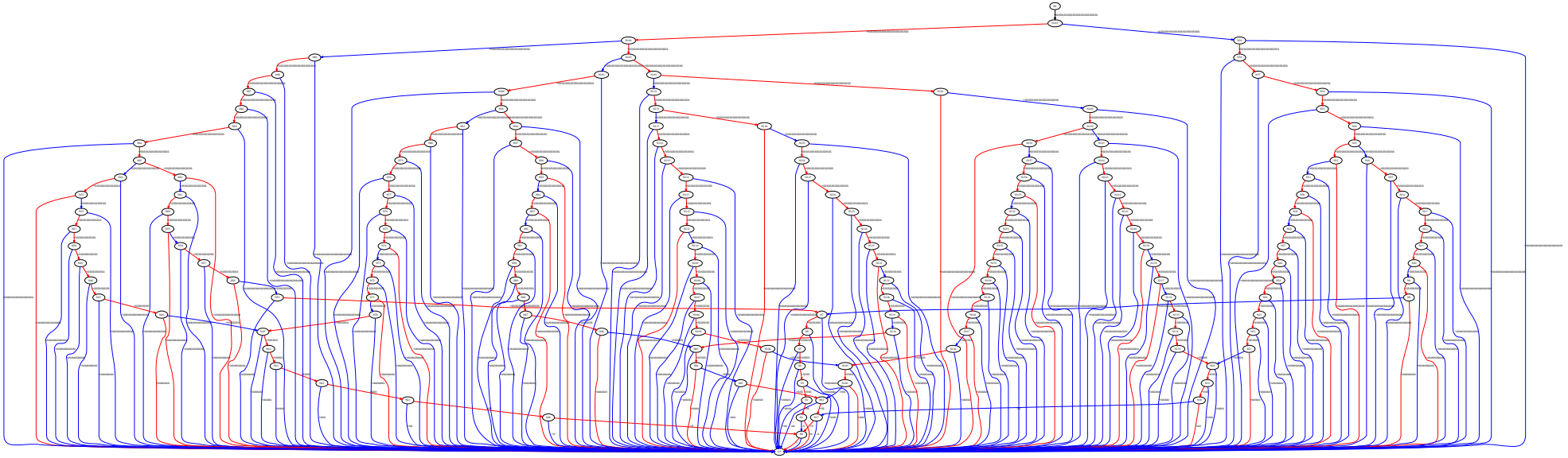


Model NU-X

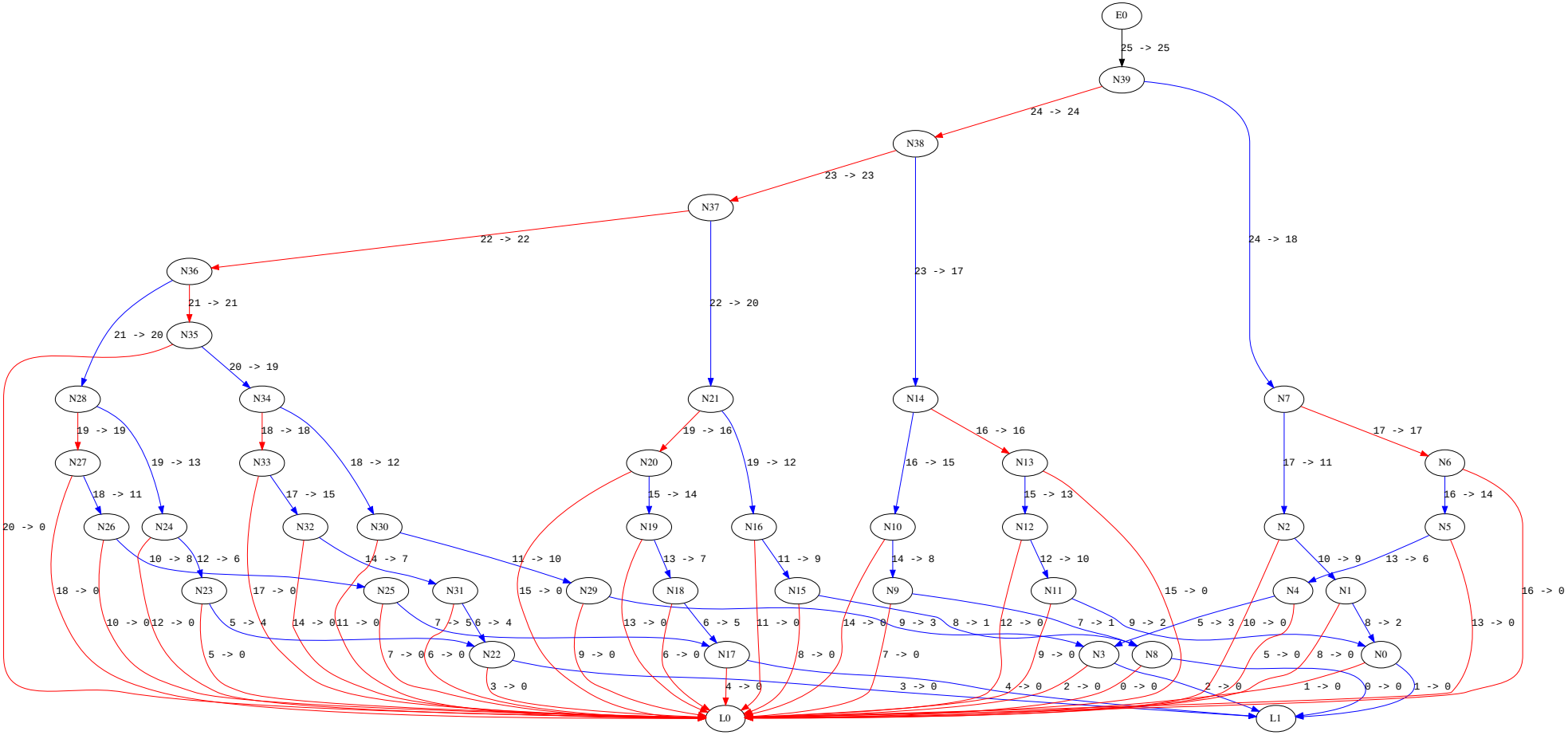
= if False
= if True
= False
= True
= Not

S = significant
U = useless
1 = If 0 then 0
0 = If 1 then 0
I = If 0 then 1
O = If 1 then 1

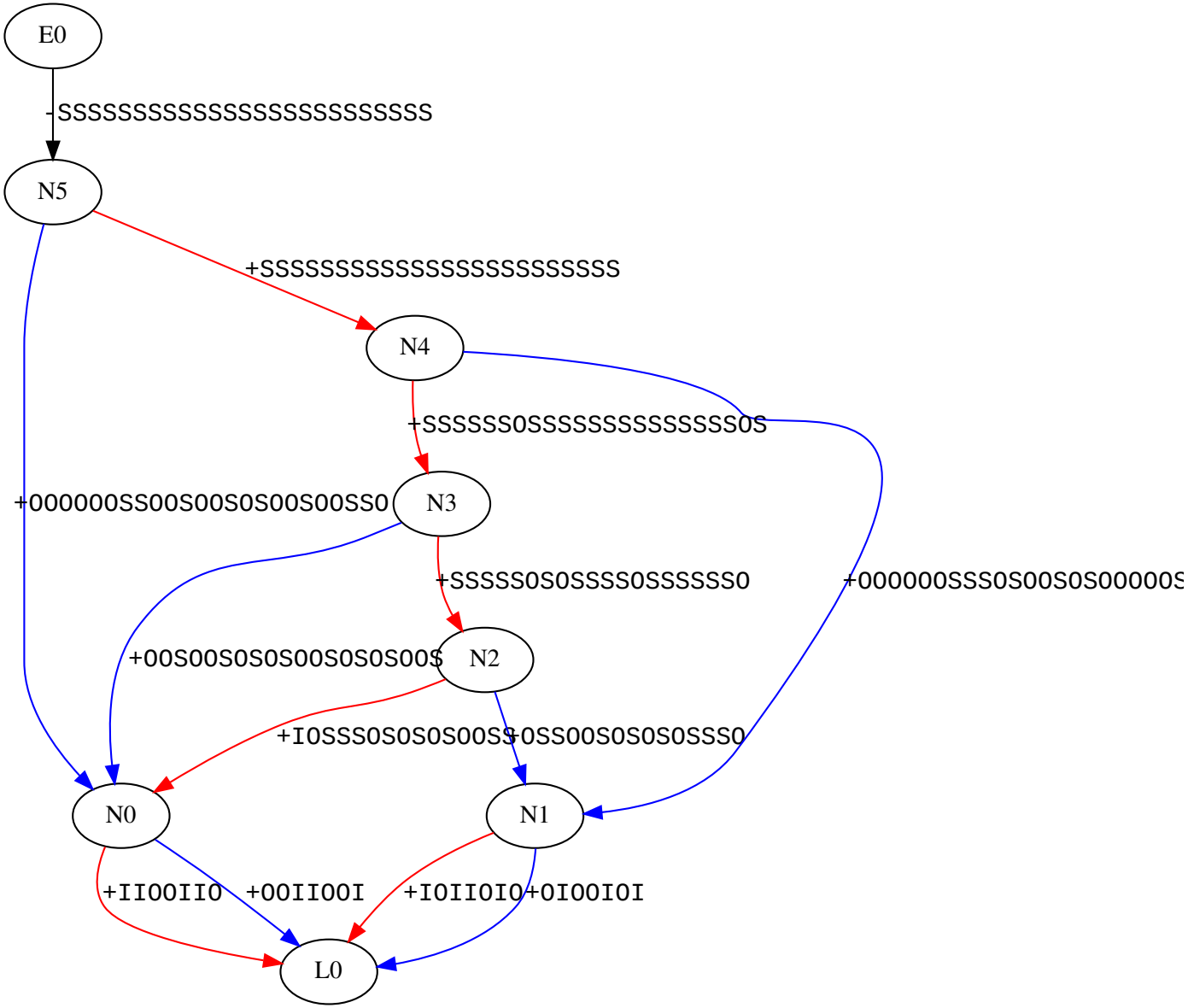
5-Queens: N or NU



5-Queens: Z


E0
25 -> 25
N39
24 -> 24
N38
23 -> 23
N37
22 -> 22
24 -> 18
23 -> 17
22 -> 20
N36
21 -> 21
21 -> 20
N35
20 -> 19
N28
N34
N21
N14
N7
19 -> 19
18 -> 18
19 -> 16
16 -> 16
17 -> 17
N27
N33
N20
N13
N6
19 -> 13
18 -> 12
16 -> 15
17 -> 11
18 -> 11
17 -> 15
15 -> 14
15 -> 13
16 -> 14
20 -> 0
N26
N24
N32
N30
N19
N16
N10
N12
N2
N5
10 -> 8
12 -> 6
14 -> 7
11 -> 10
13 -> 7
11 -> 9
14 -> 8
12 -> 10
10 -> 9
13 -> 6
18 -> 0
N23
17 -> 0
N25
N31
15 -> 0
N29
N18
N15
N9
N11
15 -> 0
N4
N1
16 -> 0
10 -> 0
12 -> 0
5 -> 4
14 -> 0
11 -> 0
7 -> 5
6 -> 4
13 -> 0
6 -> 5
11 -> 0
14 -> 0
9 -> 3
8 -> 1
12 -> 0
7 -> 1
9 -> 2
5 -> 3
10 -> 0
8 -> 2
13 -> 0
5 -> 0
N22
7 -> 0
6 -> 0
9 -> 0
6 -> 0
N17
8 -> 0
7 -> 0
9 -> 0
N3
N8
5 -> 0
8 -> 0
N0
3 -> 0
4 -> 0
3 -> 0
4 -> 0
2 -> 0
0 -> 0
2 -> 0
1 -> 0
0 -> 0
1 -> 0
L0
L1

5-Queens: NU-X


E0
-SSSSSSSSSSSSSSSSSSSSSSSSS
N5
+SSSSSSSSSSSSSSSSSSSSSSSS
N4
+SSSSSSSSSSSSSSSSSSSSSSSS0S
N3
+000000SS00S00S0S00S00SS0
+SSSSS0S0SSSS0SSSSSS0
+000000SSS0S00S0S00000S
N2
+00S00S0S0S00S0S0S00S
+I0SSS0S0S0S00SS+0SS00S0S0S0SSS0
N0
N1
+II00II0
+00II00I
+I0II0I0
+0I00I0I
L0

Results

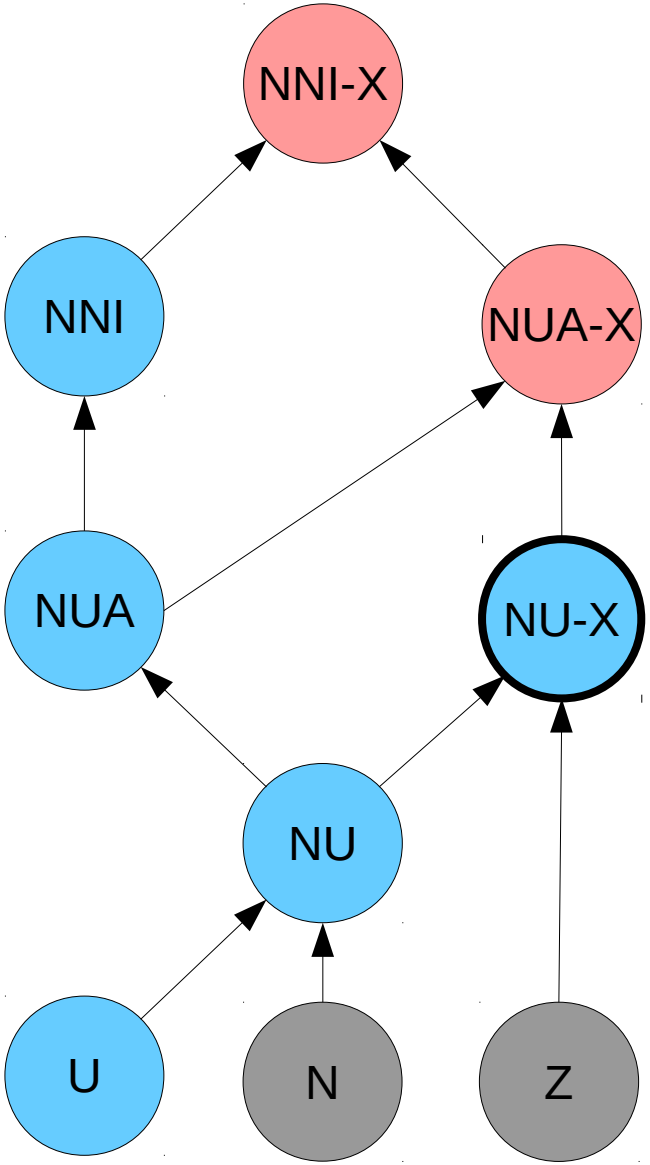
Average reduction of the {number of nodes / **estimated** memory cost} on four benchmarks

	Circuits (dense functions)				CNF formulas (sparse functions)			
	lgsynth91		iscas99		uf20-91		uf50-218	
variants	#node	mem	#node	mem	#node	mem	#node	mem
Z	+233%	+233%	+162%	+162%	-41%	-41%	-42%	-42%
NU	-26%	-39%	-25%	-20%	-3%	+7%	-3%	+22%
NU-X	-64%	-67%	-55%	-46%	-96%	-95%	-97%	-96%

Can we go further ?

“input/output negation”

“output negation” + “useless variables extraction” + “anti-variables extraction”



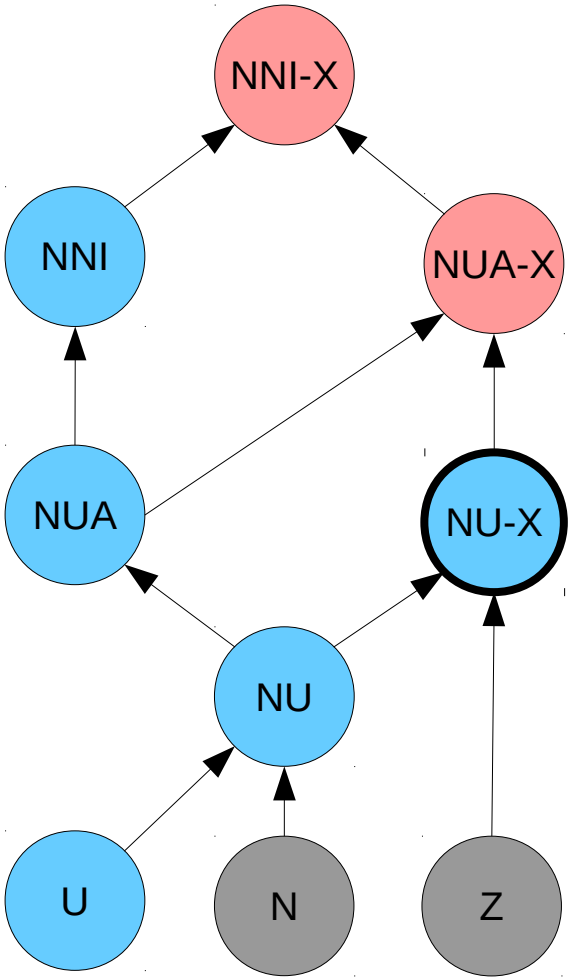
“input/output negation” + “1-prediction extraction”

“output negation” + “useless variables extraction” + “anti-variables extraction” + “1-prediction extraction”

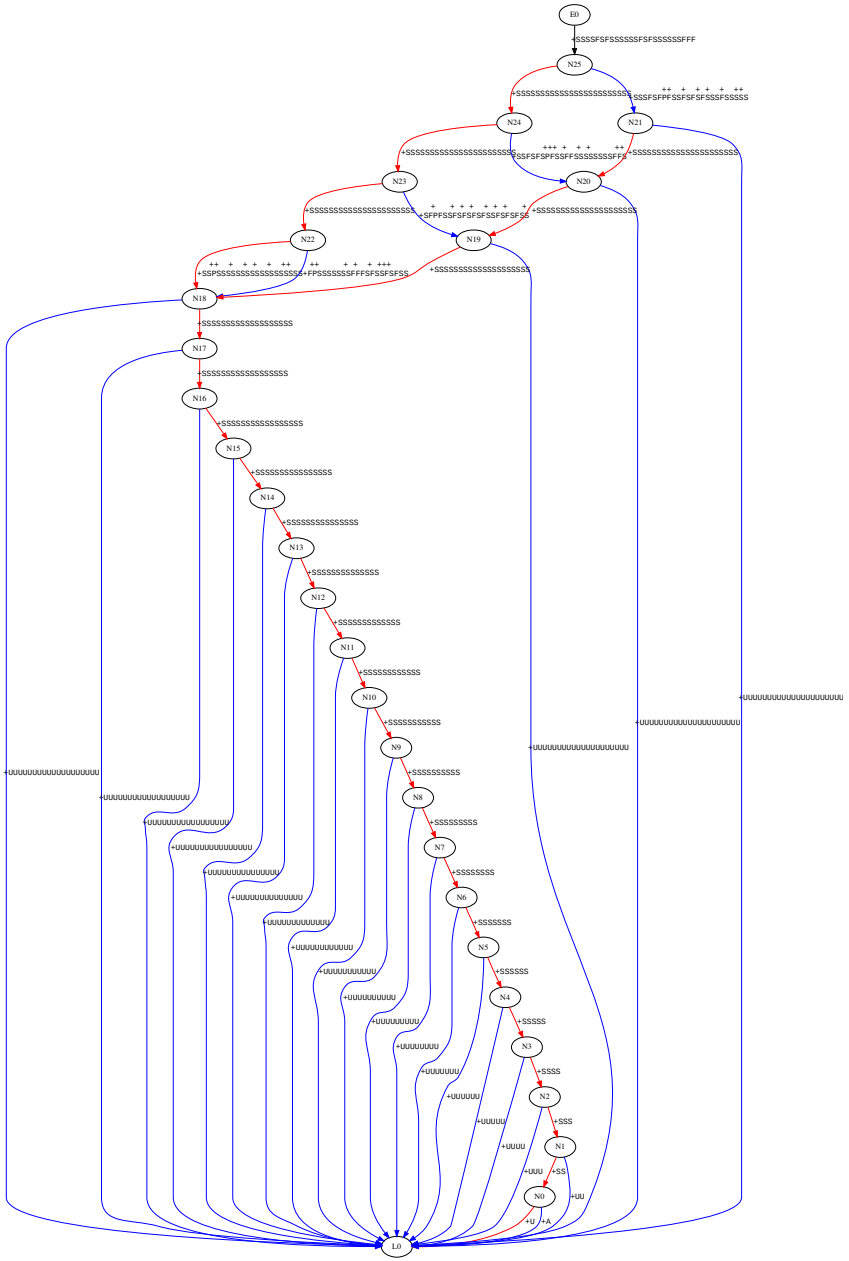
“output negation” + “useless variables extraction” + “1-prediction extraction”

Conclusion

- Software implemented in OCaml:
  - https://github.com/JoanThibault/DAGaml/tree/grobdd-dev
  - ~ 12 000 lines of OCaml
- Fewer nodes & Less memory
- Future Work
  - Quantify the dependency between variables' order and #node
  - Solve & Implement NUA-X and NNI-X versions
- TO DO
  - Quantification Operators
  - Variable Reordering
  - Parallelism & hardware acceleration
- Other Applications
  - Apply similar strategies to compress other DAG
    - DAG / Graph isomorphism



5-Queens: NNI


E0
+SSSSFSFSSSSSSFSFSSSSSSFFF
N25
+SSSSSSSSSSSSSSSSSSSSSSSSS
++ + + + ++
+SSSSFSFPFSSFSFSFSSSFSSSSS
N24
N21
+SSSSSSSSSSSSSSSSSSSSSSSS
+++ + + + ++
+SSSFSFSPFSSFFSSSSSSSFFF
+SSSSSSSSSSSSSSSSSSSSSSSS
N23
N20
+SSSSSSSSSSSSSSSSSSSSSSS
+ + + + + + + + +
+SFPFSSFSFSFSFSFSFSFSS
+SSSSSSSSSSSSSSSSSSSSS
N22
N19
++ + + + + ++ ++ + + +++
+SSPSSSSSSSSSSSSSSSSS+FPSSSSSSSFFFSFSSFSFSS
+SSSSSSSSSSSSSSSSSSS
N18
+SSSSSSSSSSSSSSSSSS
N17
+SSSSSSSSSSSSSSSSS
N16
+SSSSSSSSSSSSSSSS
N15
+SSSSSSSSSSSSSSS
N14
+SSSSSSSSSSSSSS
N13
+SSSSSSSSSSSSS
N12
+SSSSSSSSSSSS
N11
+SSSSSSSSSSS
N10
+SSSSSSSSSS
N9
+SSSSSSSSS
N8
+SSSSSSSS
N7
+SSSSSSS
N6
+SSSSSS
N5
+SSSSS
N4
+SSSS
N3
+SSS
N2
+SS
N1
+S
N0
+U
+A
+UU
+UUU
+UUUU
+UUUUU
+UUUUUU
+UUUUUUU
+UUUUUUUU
+UUUUUUUUU
+UUUUUUUUUU
+UUUUUUUUUUU
+UUUUUUUUUUUU
+UUUUUUUUUUUUU
+UUUUUUUUUUUUUU
+UUUUUUUUUUUUUUU
+UUUUUUUUUUUUUUUU
+UUUUUUUUUUUUUUUUU
+UUUUUUUUUUUUUUUUUU
+UUUUUUUUUUUUUUUUUUU
+UUUUUUUUUUUUUUUUUUUU
L0

Results

Average reduction of the {number of nodes / **estimated** memory cost} on four benchmarks

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NU	-26%	-39%	-25%	-20%	-3%	+7%	-3%	+22%
NNI	-60%	-63%	-56%	-49%	-30%	-10%	-39%	+5%
NU-X	-64%	-67%	-55%	-46%	-96%	-95%	-97%	-96%