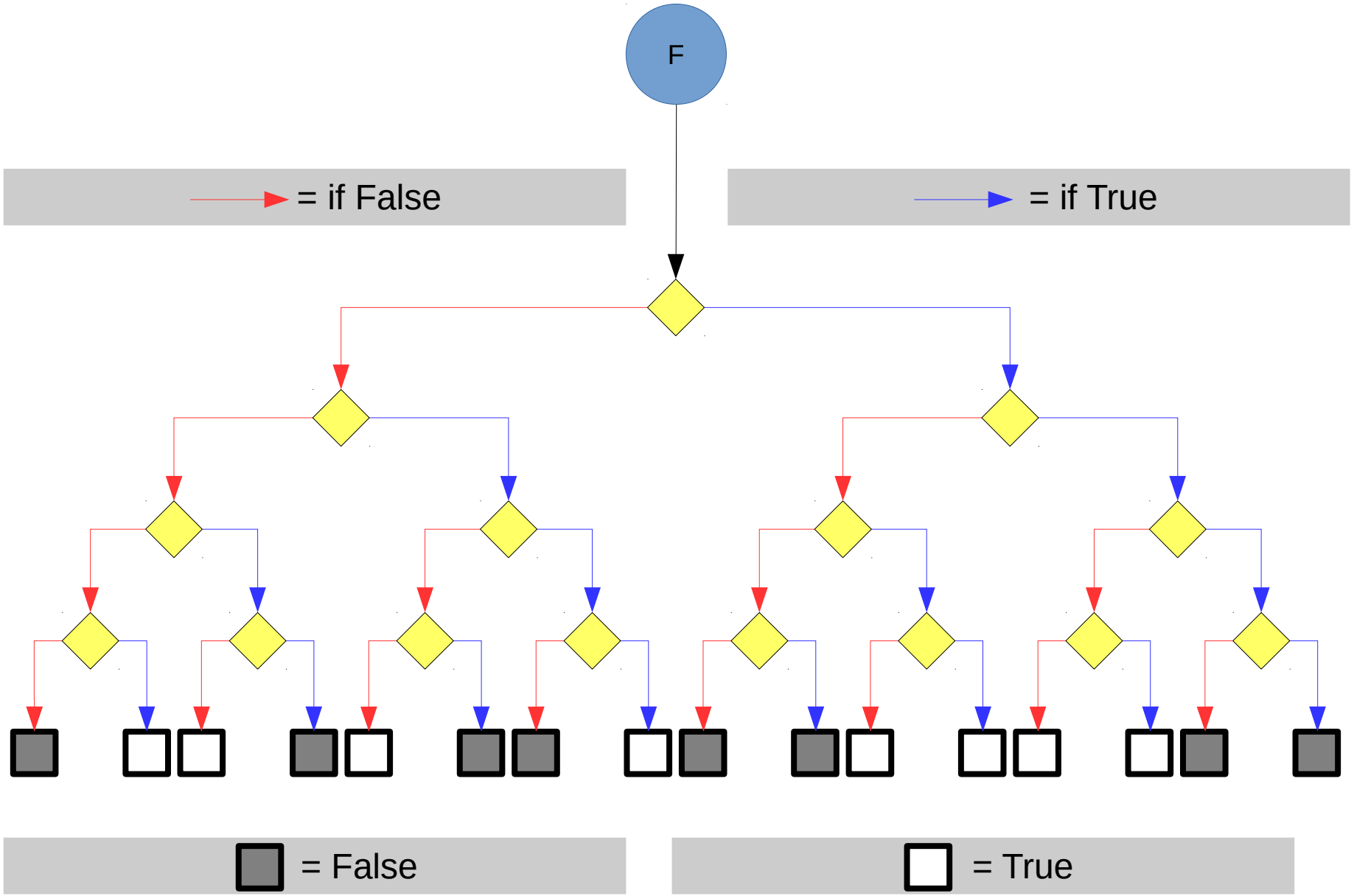
A Generalized Reduction of Ordered Binary Decision Diagram (GroBdd)

Joan Thibault

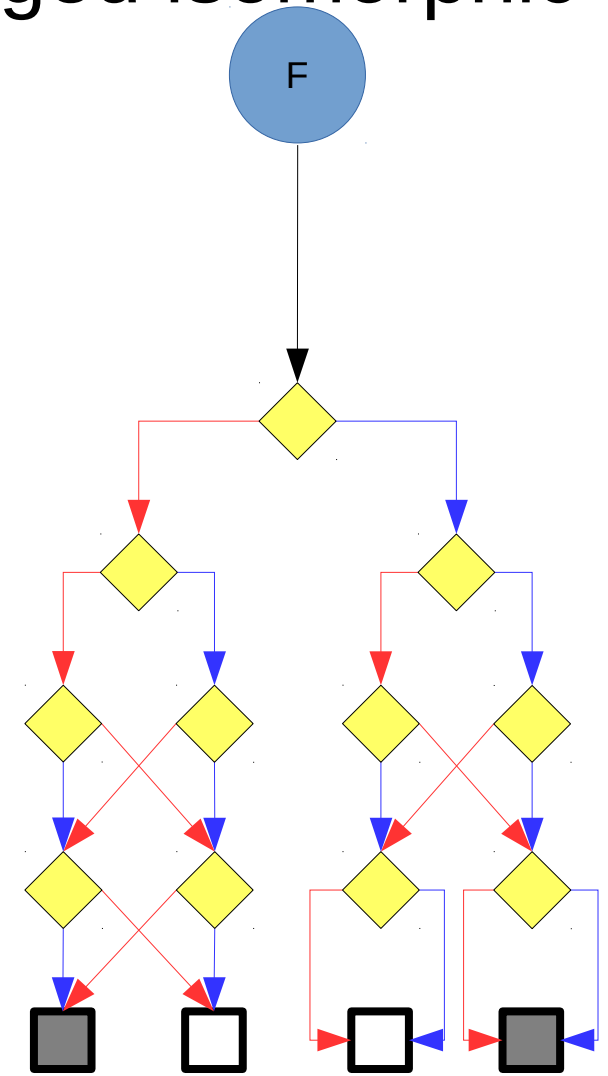
Reduced Ordered Binary Decision Diagram (ROBDD)

- Applications
  - Computer Aided Design (e.g. equivalence checking)
  - Knowledge Representation (e.g. Artificial Intelligence)
  - Combinatorial Problems (e.g. N-Queens problem)
- What are required operation ?
  - Compact representation
  - Operations (e.g. composing, concatening, evaluation)
  - Operators (e.g. AND, XOR, ITE, NOT)
  - Reductions (e.g. quantification, partial evaluation, SAT)

Shannon's Binary Decision Tree



Shannon's Decision Diagram
(We merged isomorphic sub-trees)



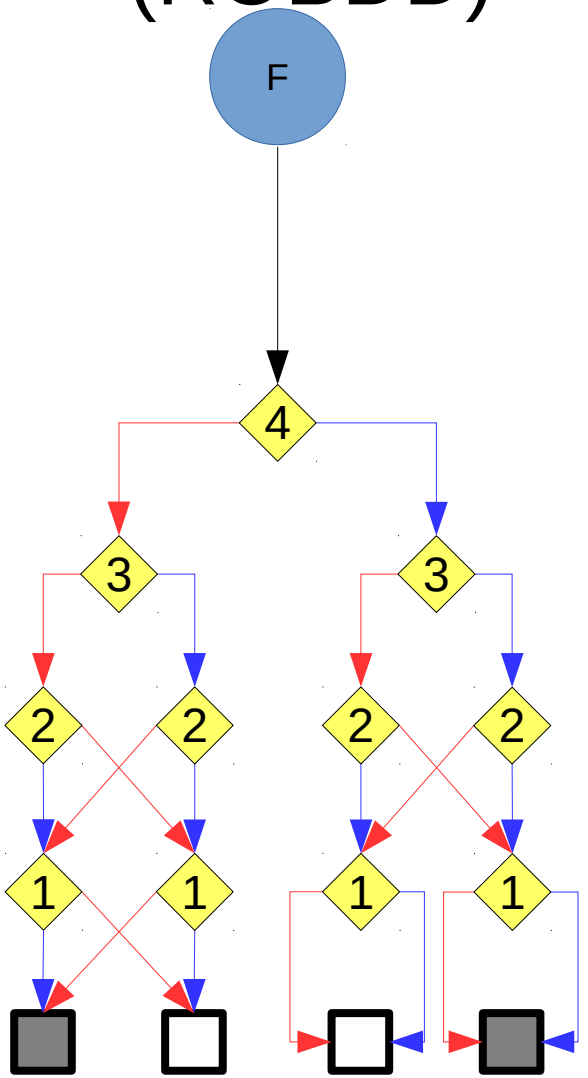
= if False
= if True

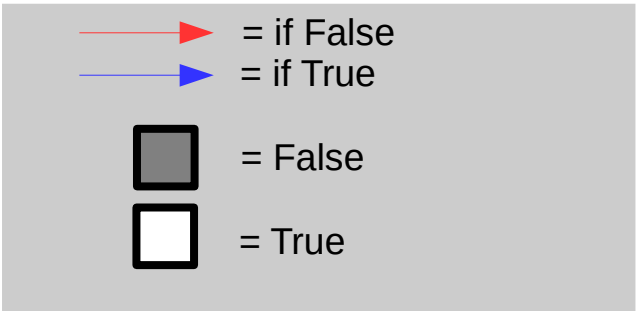
= False

= True

For aesthetic reasons, we do not merge all terminals

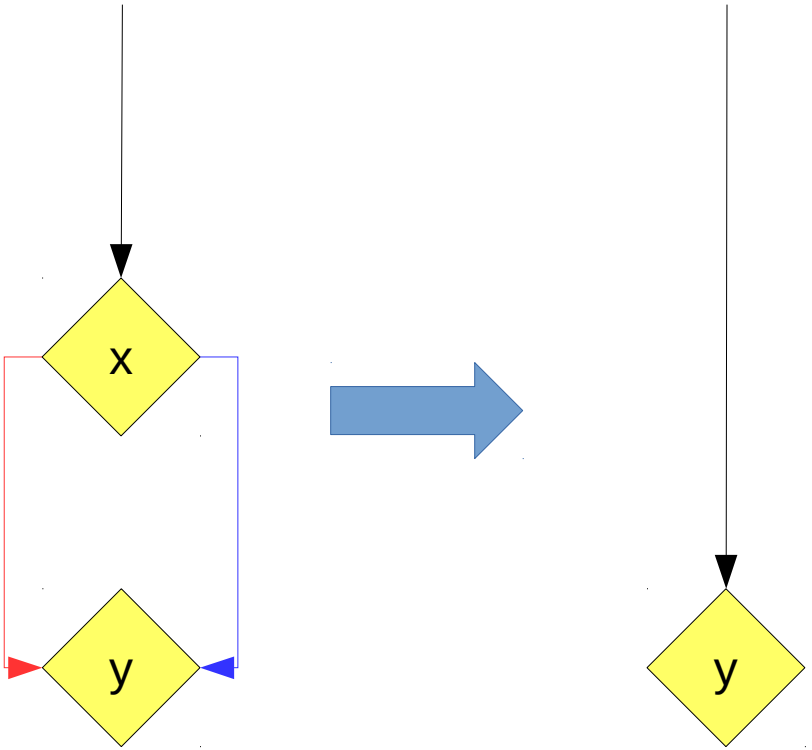
Reduced Ordered Binary Decision Diagram (ROBDD)





For aesthetic reasons, we do not merge all terminals

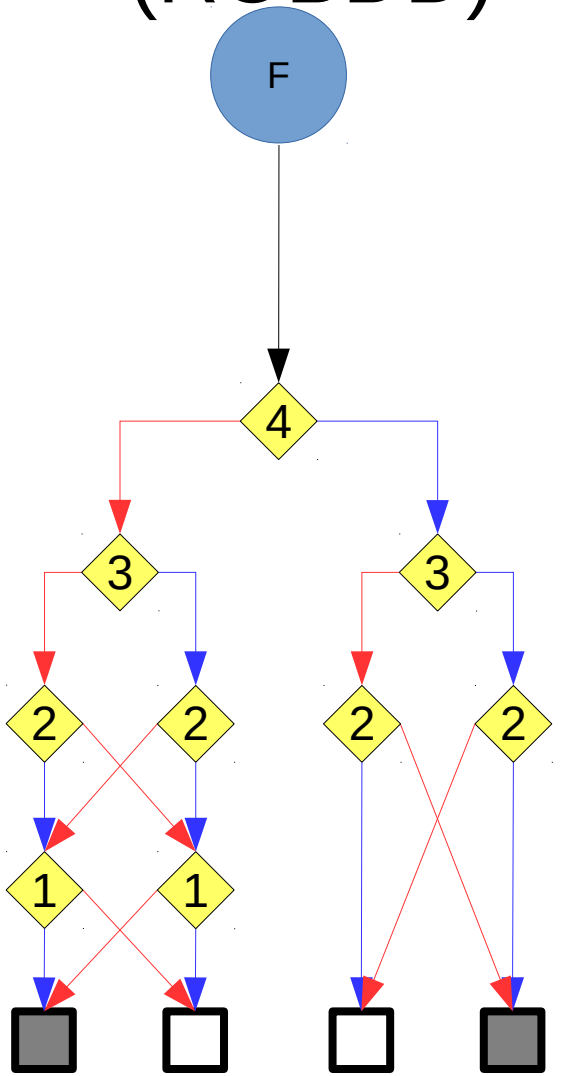
ROBDD : reduction rule

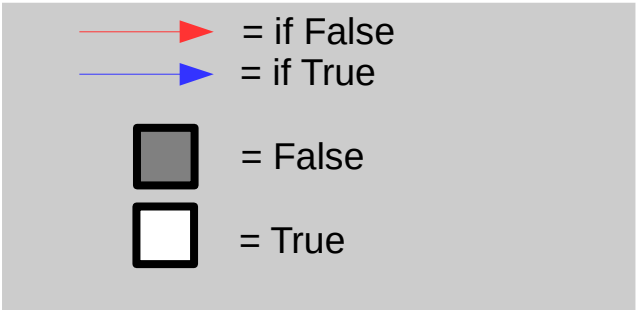


= if False
= if True
= False
= True

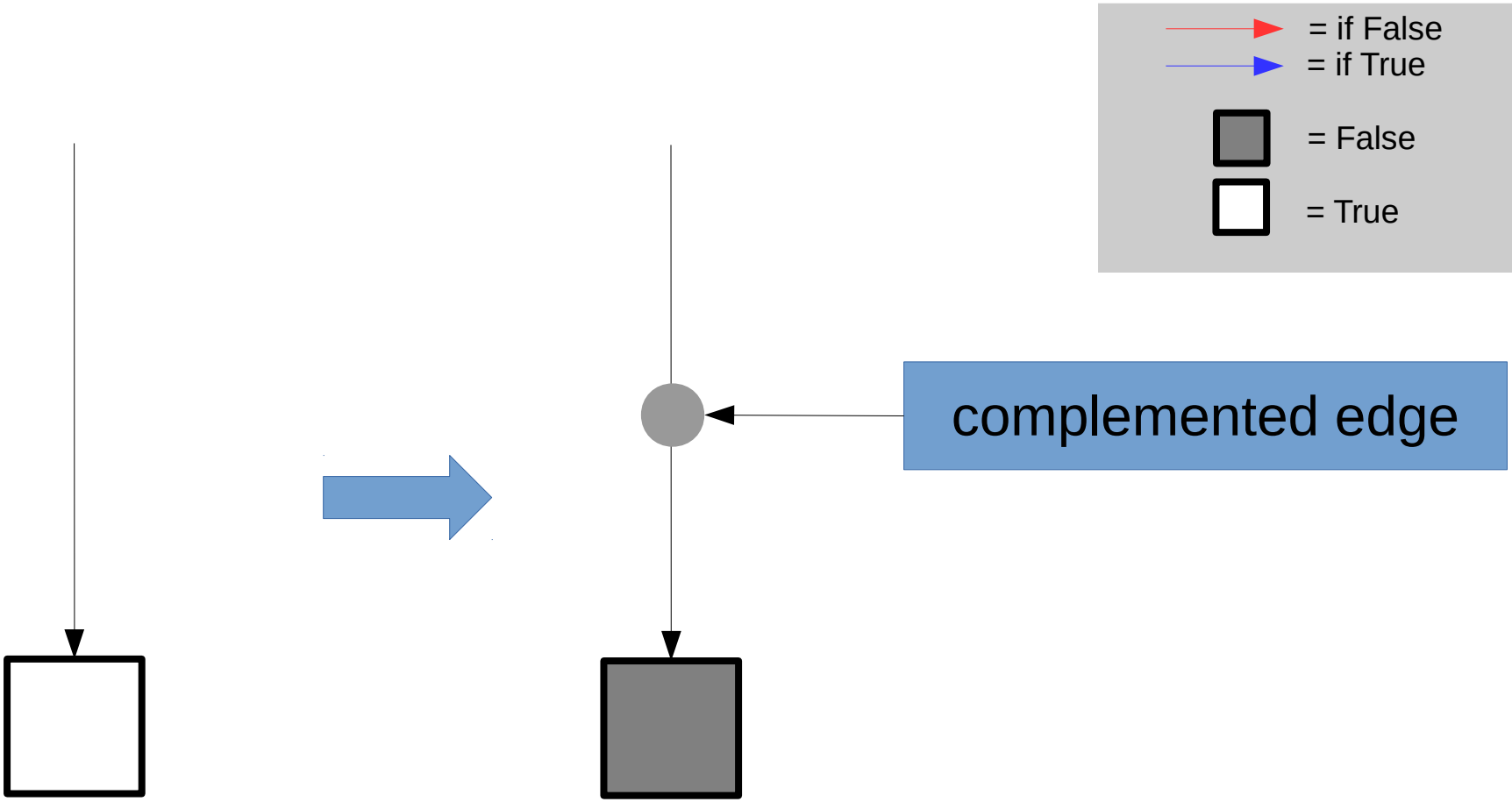
For aesthetic reasons, we do not merge all terminals

Reduced Ordered Binary Decision Diagram (ROBDD)



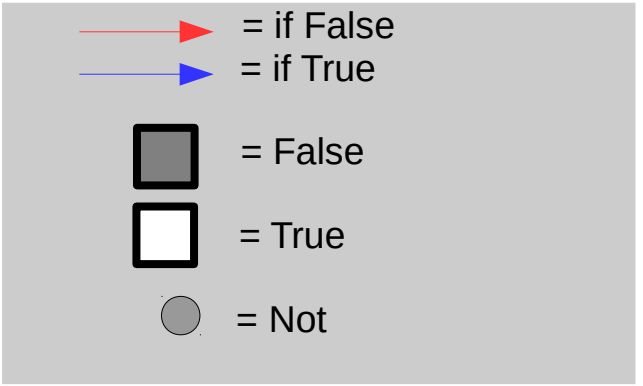


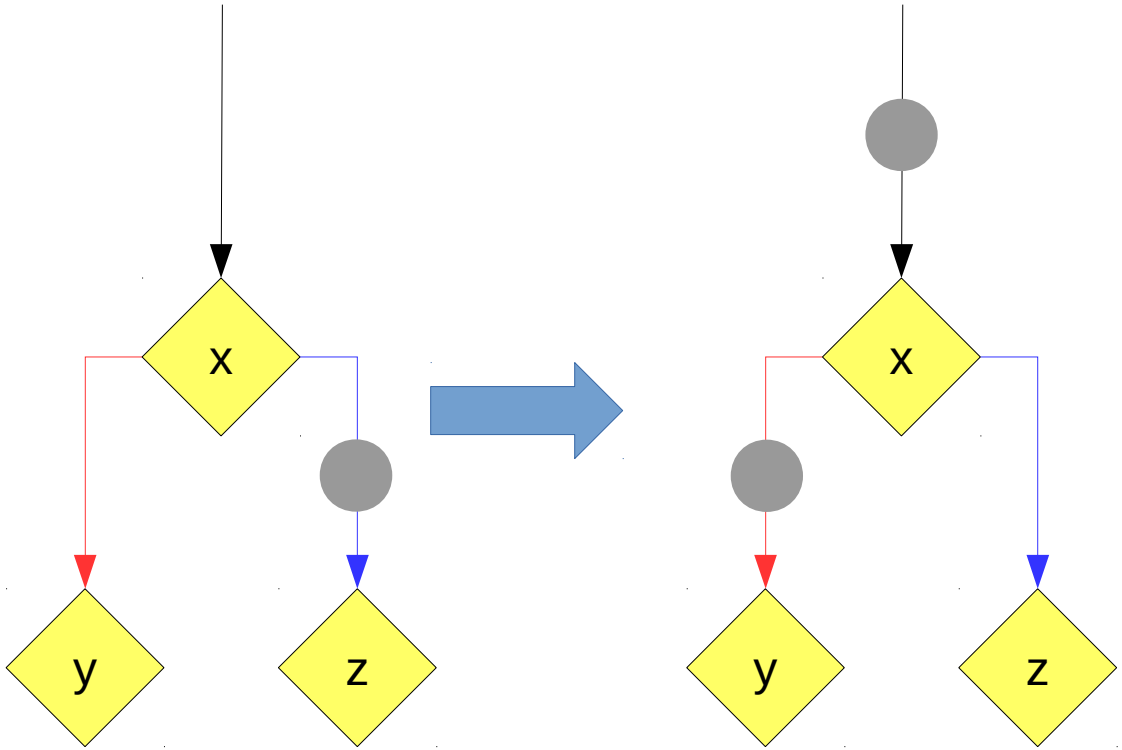
“output negation” : reduction rule (N1)



For aesthetic reasons, we do not merge all terminals

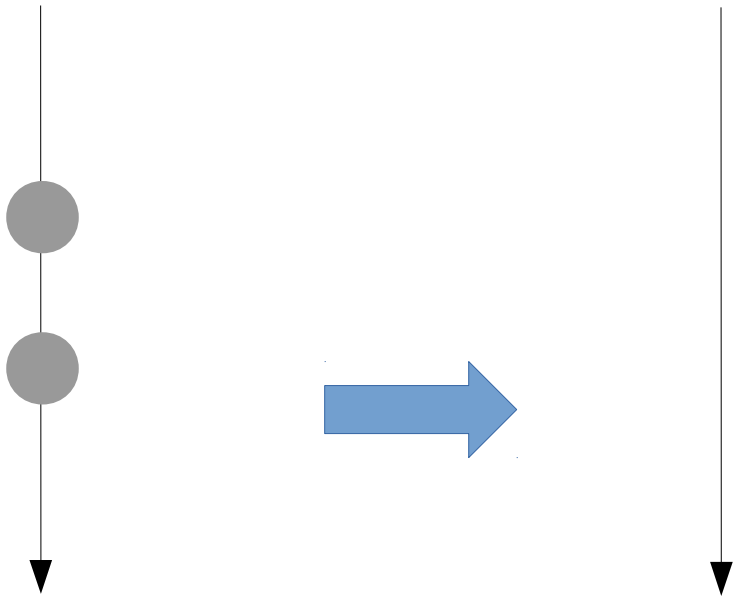
“output negation” : reduction rule (N2)

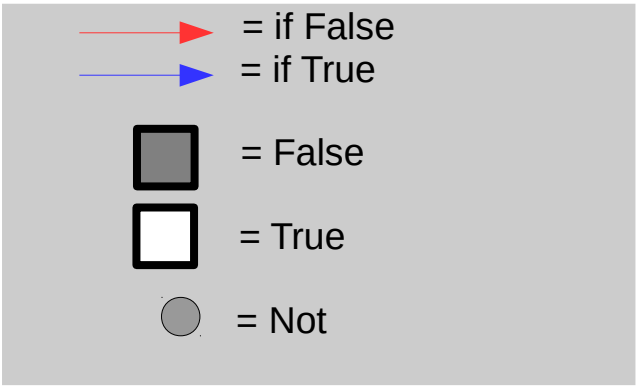




For aesthetic reasons, we do not merge all terminals

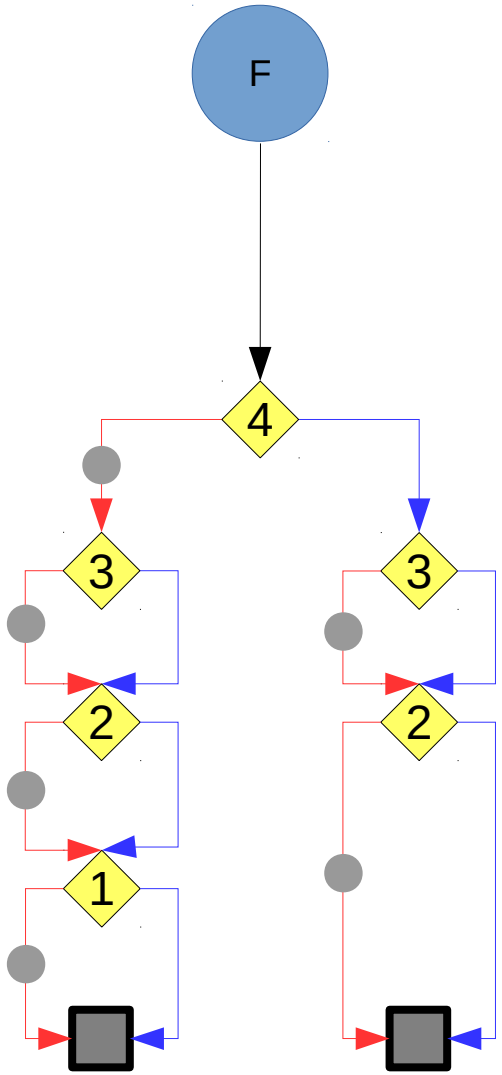
“output negation” : reduction rule (N3)

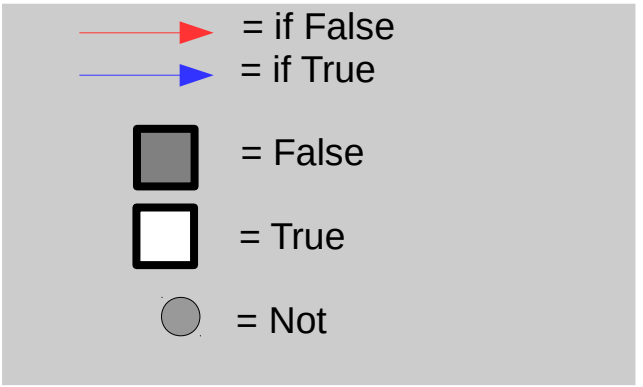




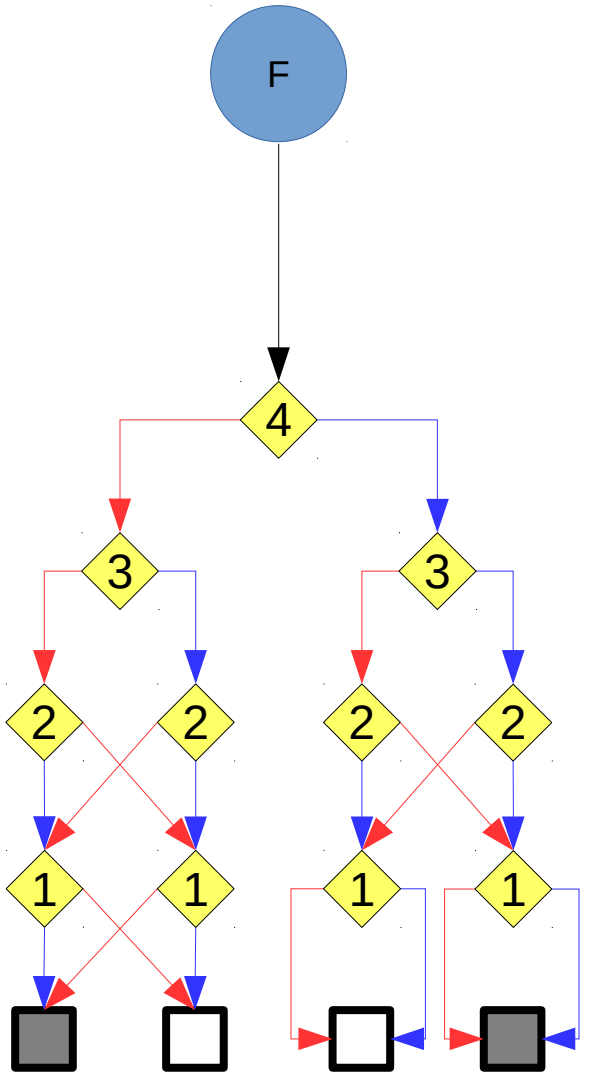
For aesthetic reasons, we do not merge all terminals

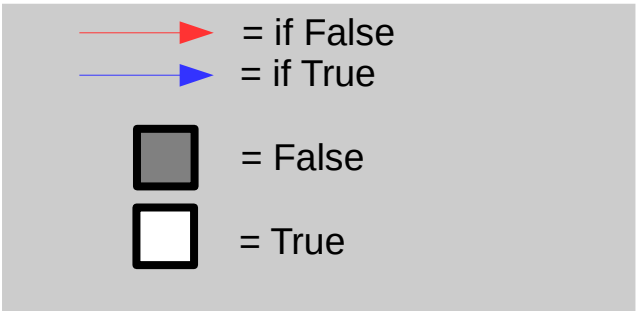
ROBDD + “output negation”





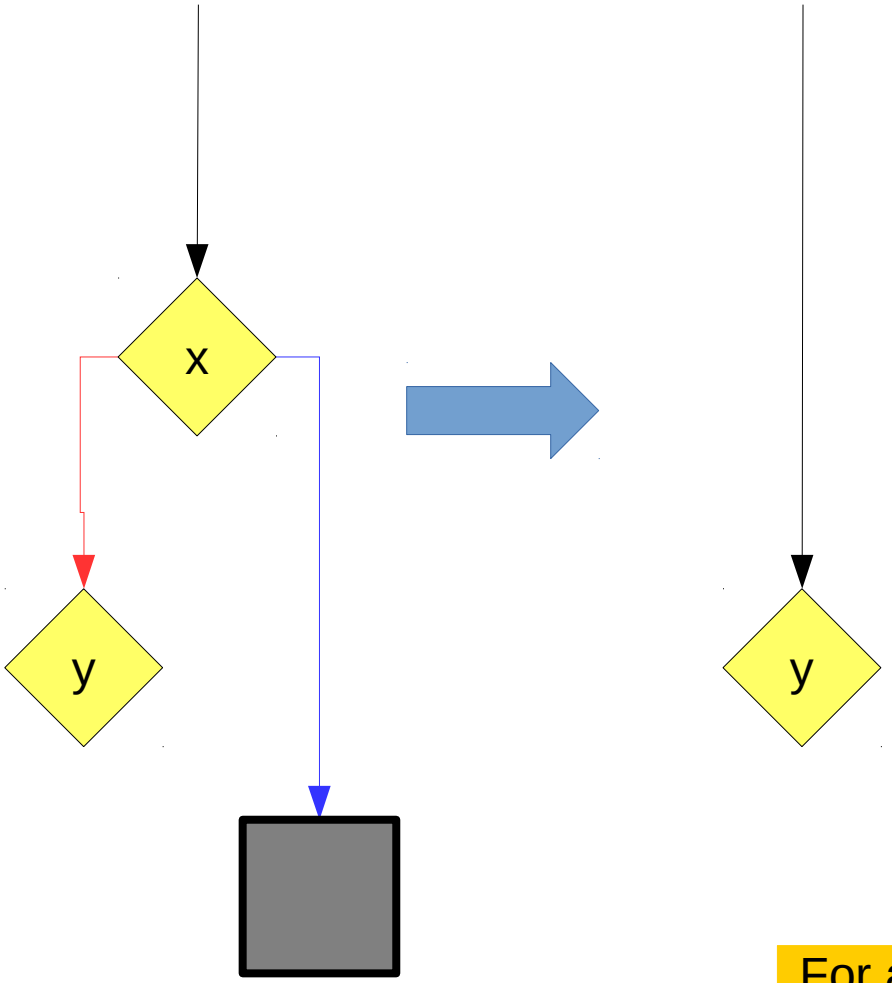
Zero supressed Binary Decision Diagram (ZBDD)

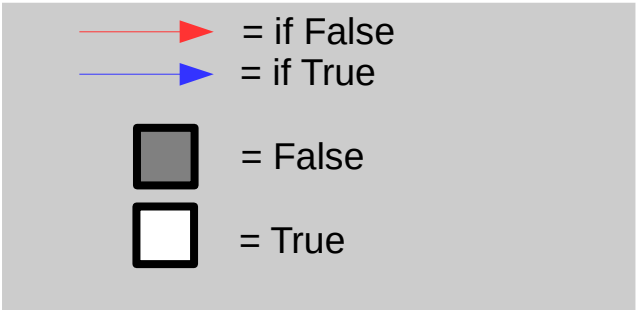




For aesthetic reasons, we do not merge all terminals

ZBDD : reduction rule





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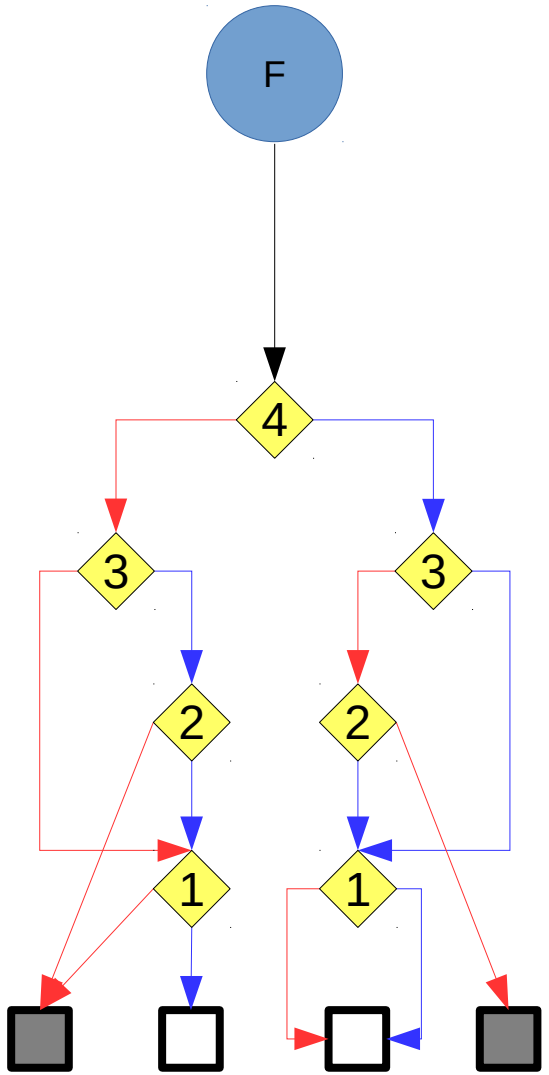
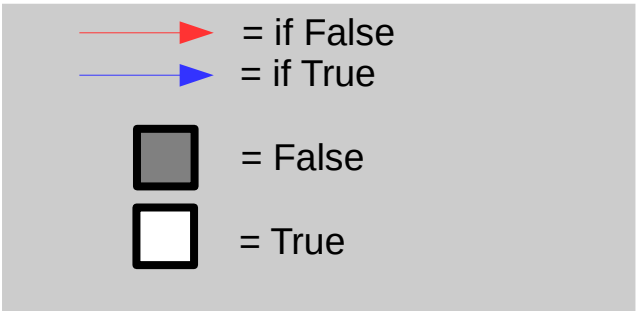
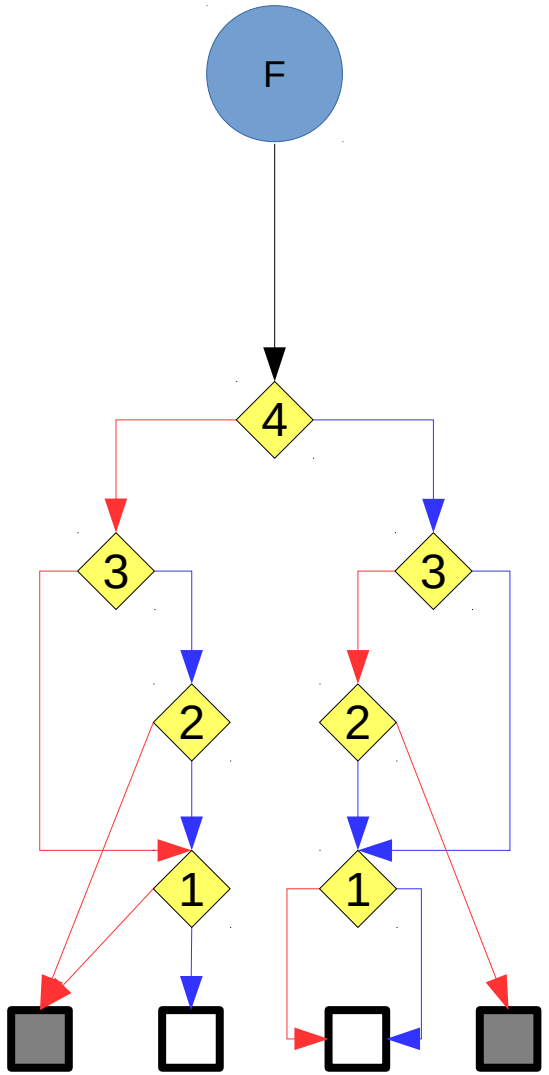
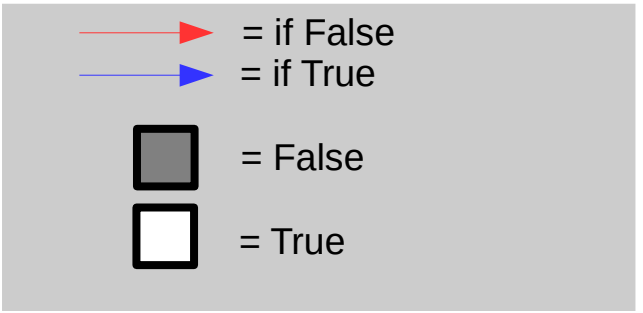




For aesthetic reasons, we do not merge all terminals

ZBDD + “output negation”





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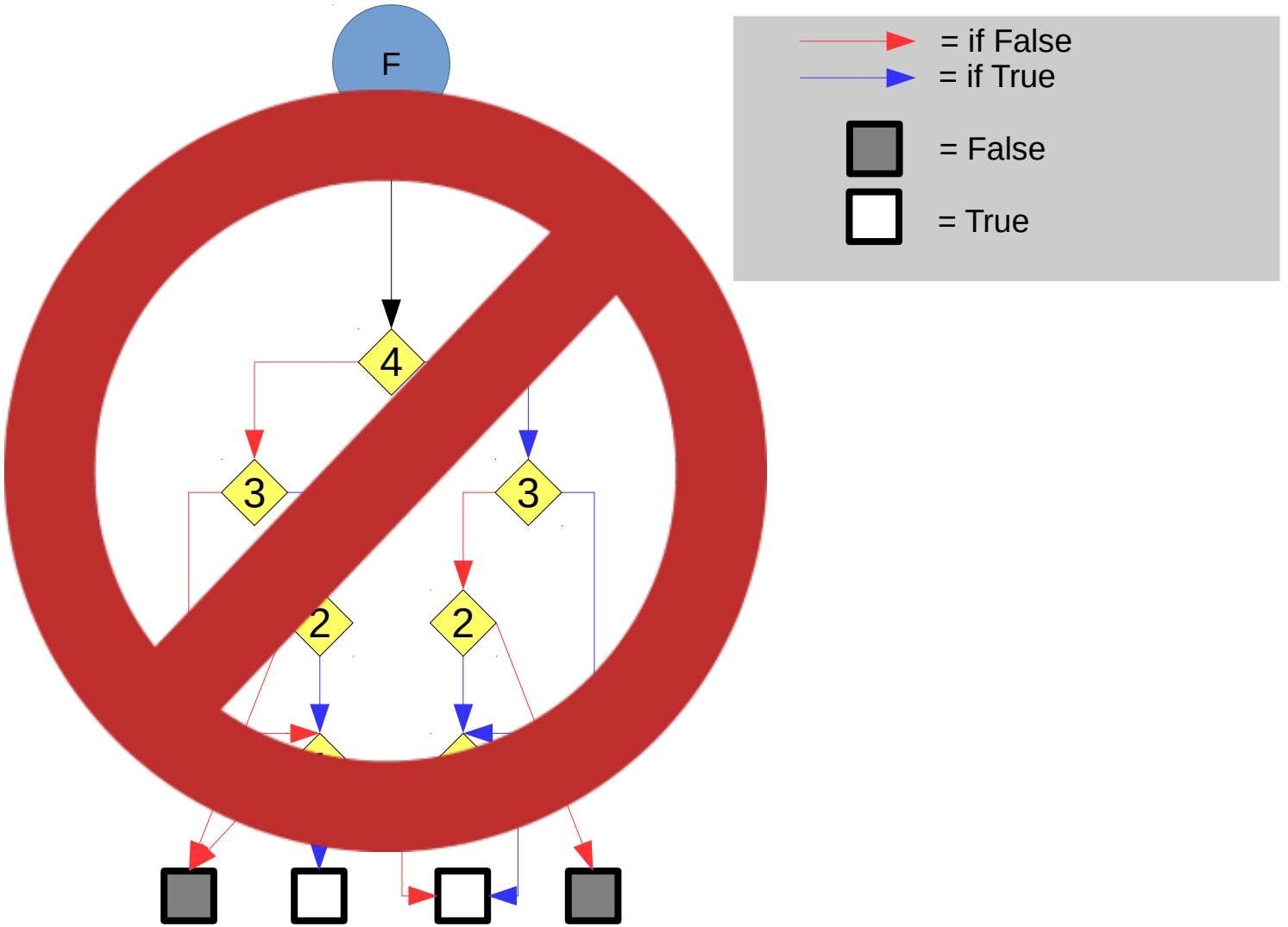
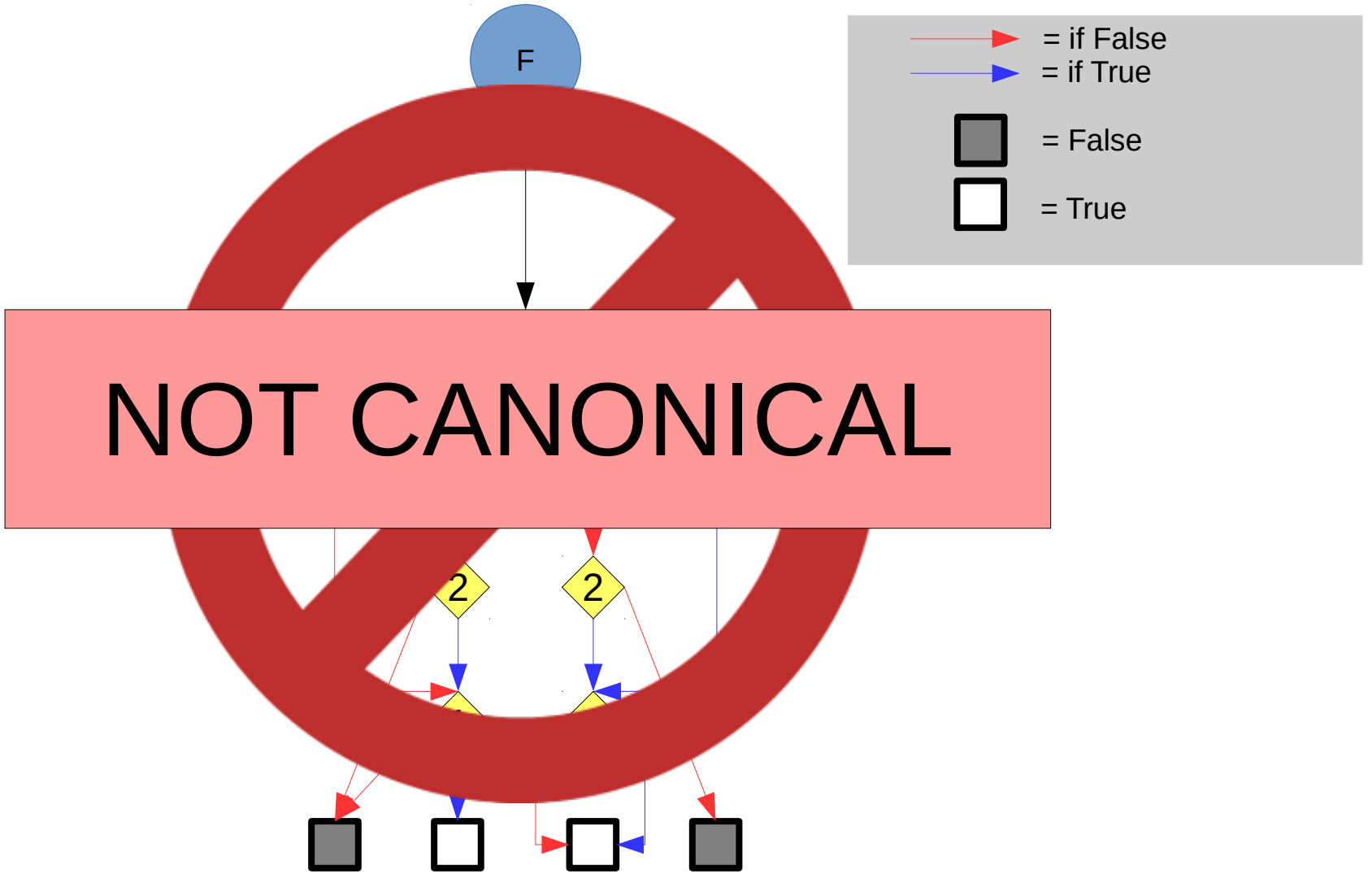
ZBDD + “output negation”



For aesthetic reasons, we do not merge all terminals

ZBDD + “output negation”



For aesthetic reasons, we do not merge all terminals

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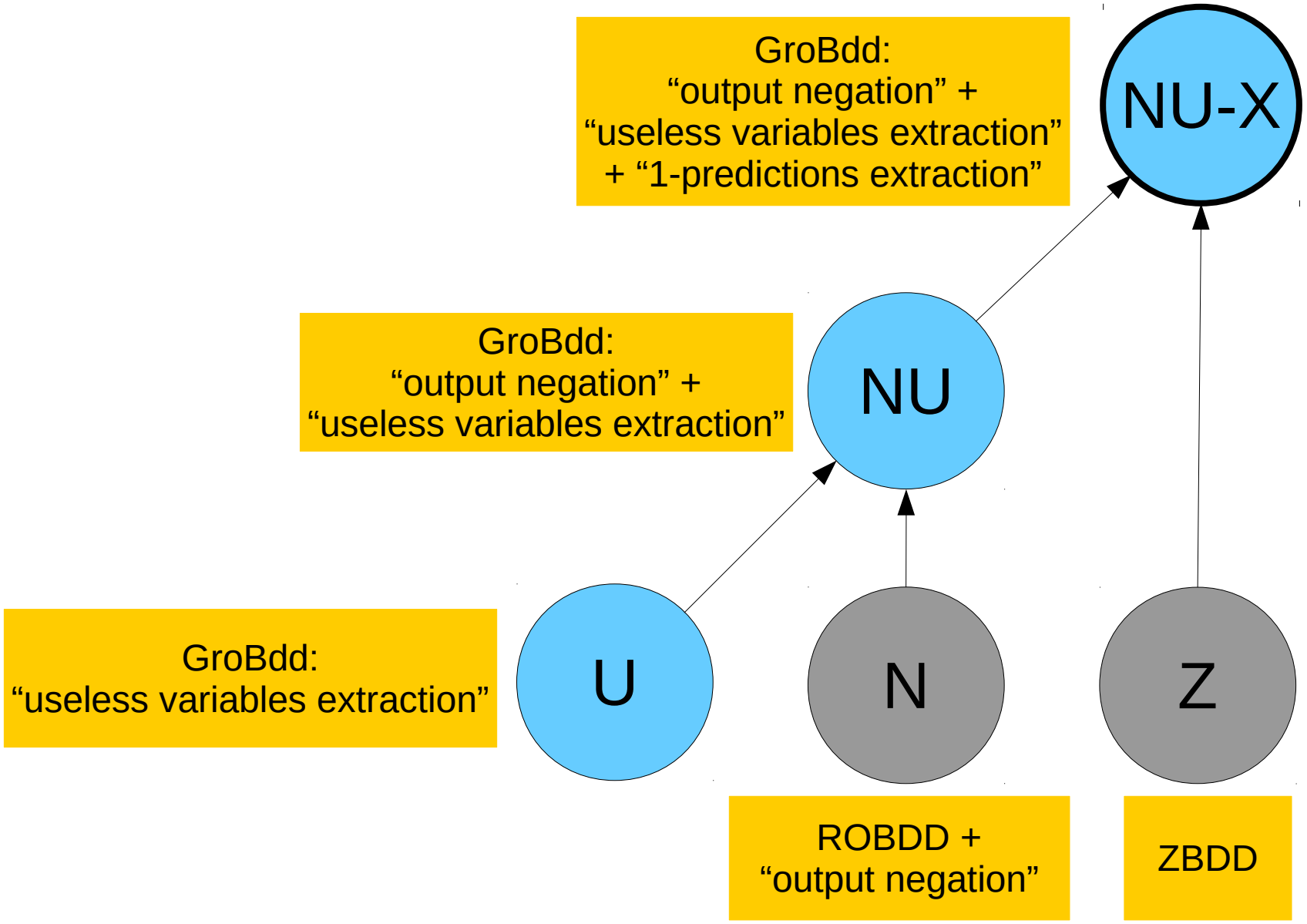
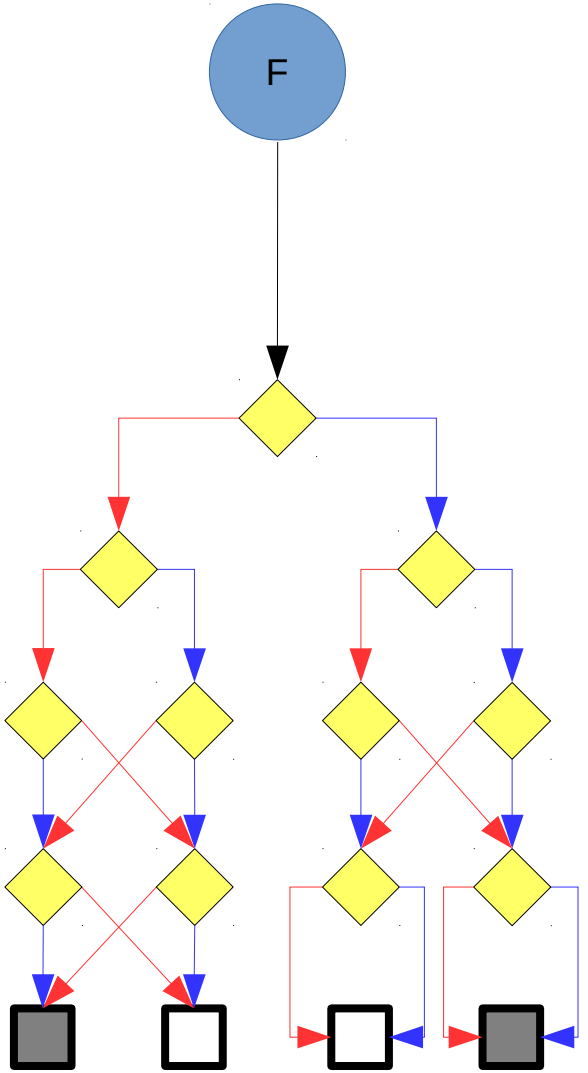
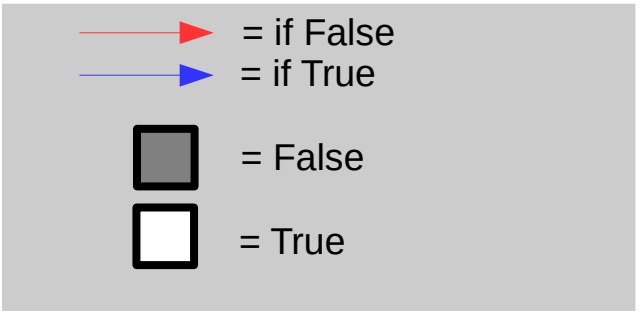


Shannon’s Decision Diagram





For aesthetic reasons, we do not merge all terminals

Section 1 : model U

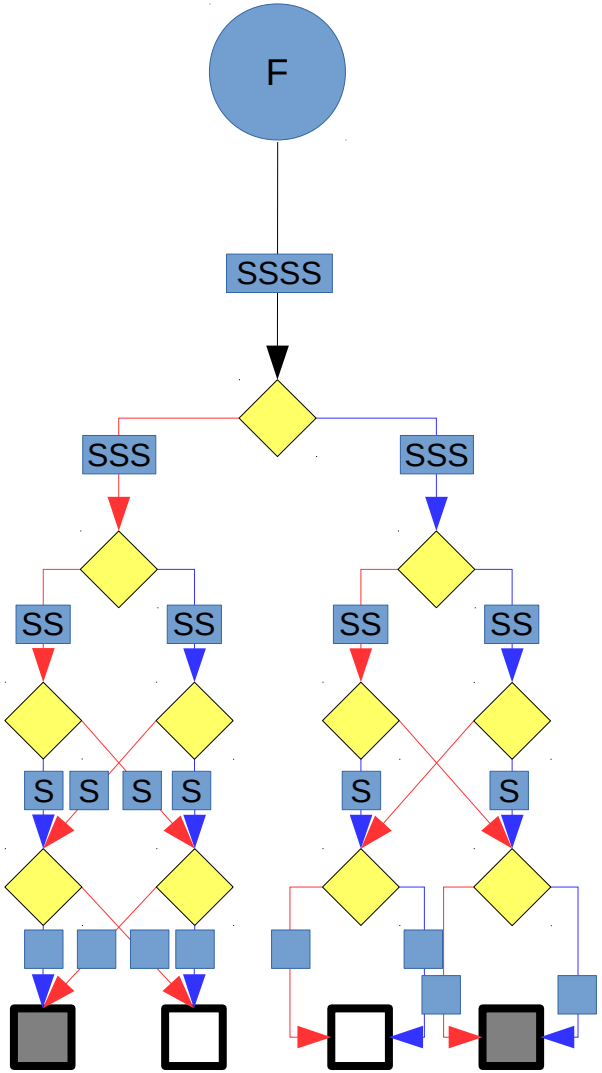
Functions of arity 4

Functions of arity 3

Functions of arity 2

Functions of arity 1

Functions of arity 0



= if False
= if True

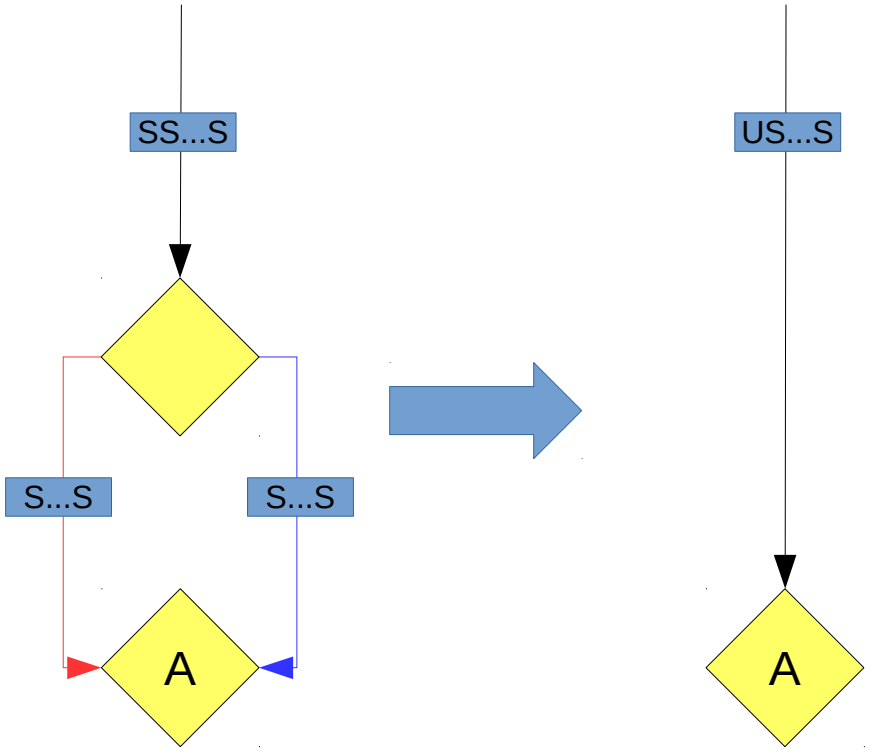
= False

= True

S = significant

For aesthetic reasons, we do not merge all terminals

“Useless variables extraction” : reduction rule (U1)



= if False
= if True

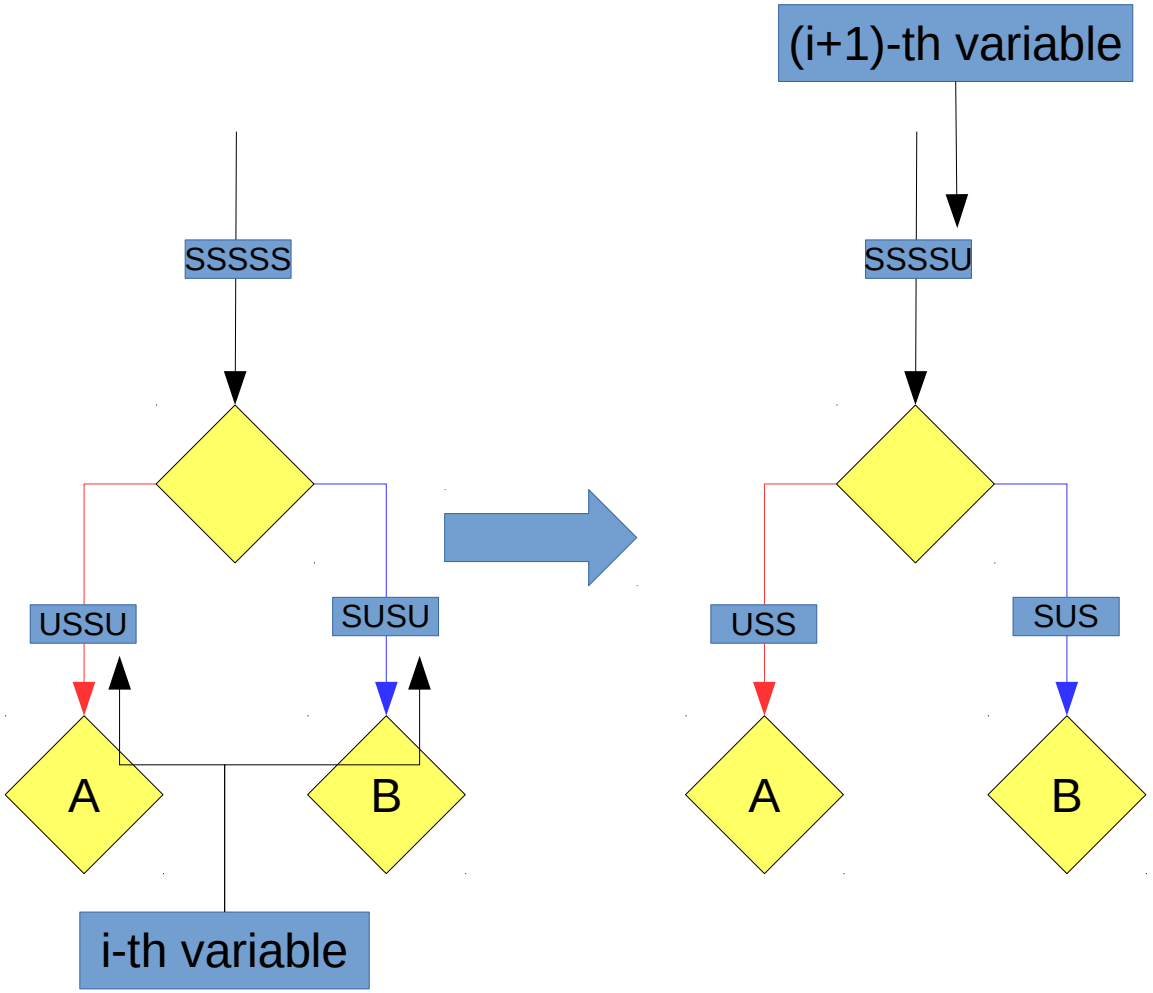
= False

= True

S = significant
U = useless

For aesthetic reasons, we do not merge all terminals

“Useless variables extraction” : reduction rule (U2)



= if False
= if True

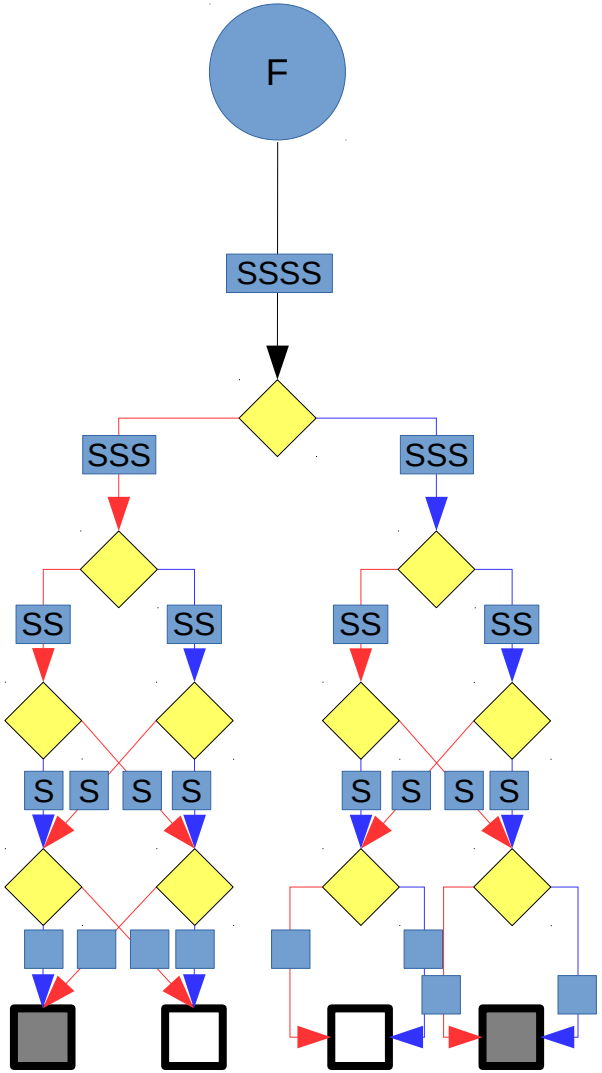
= False

= True

S = significant
U = useless

For aesthetic reasons, we do not merge all terminals

Section 1 : model U



= if False
= if True

= False

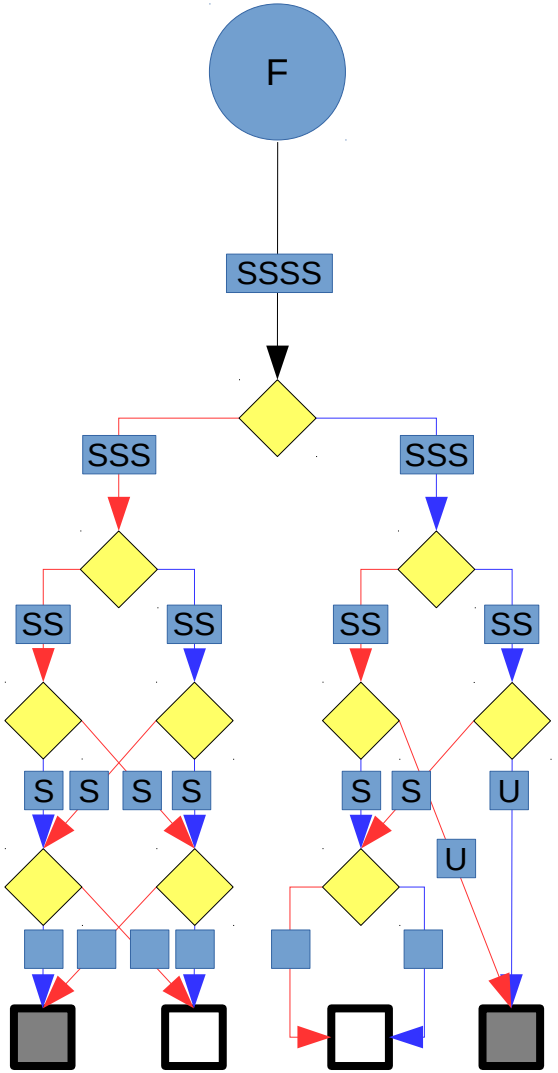
= True

S = significant
U = useless

Apply U1

For aesthetic reasons, we do not merge all terminals

Section 1 : model U



= if False
= if True

= False

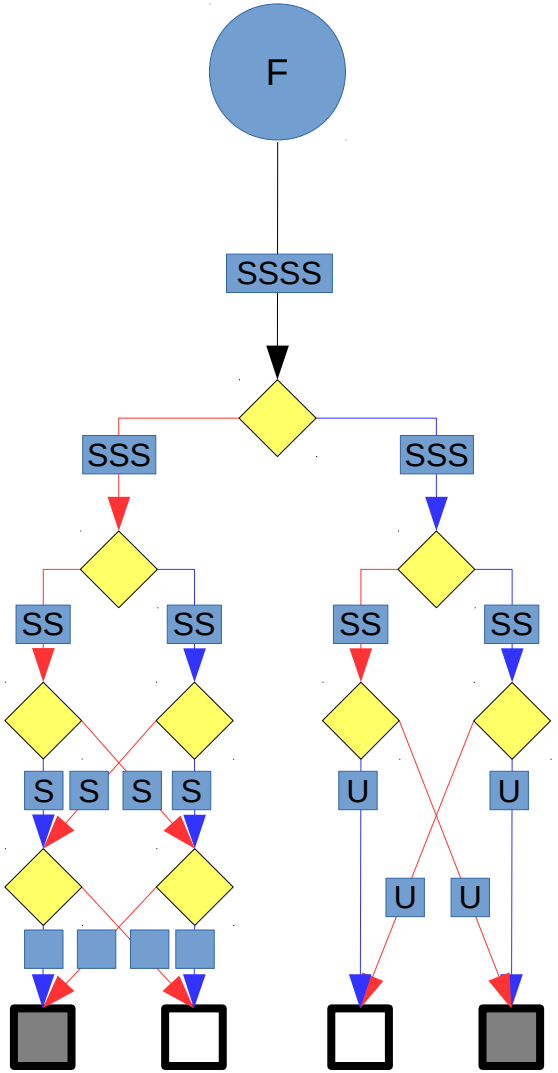
= True

S = significant
U = useless

Apply U1

For aesthetic reasons, we do not merge all terminals

Section 1 : model U



= if False
= if True

= False

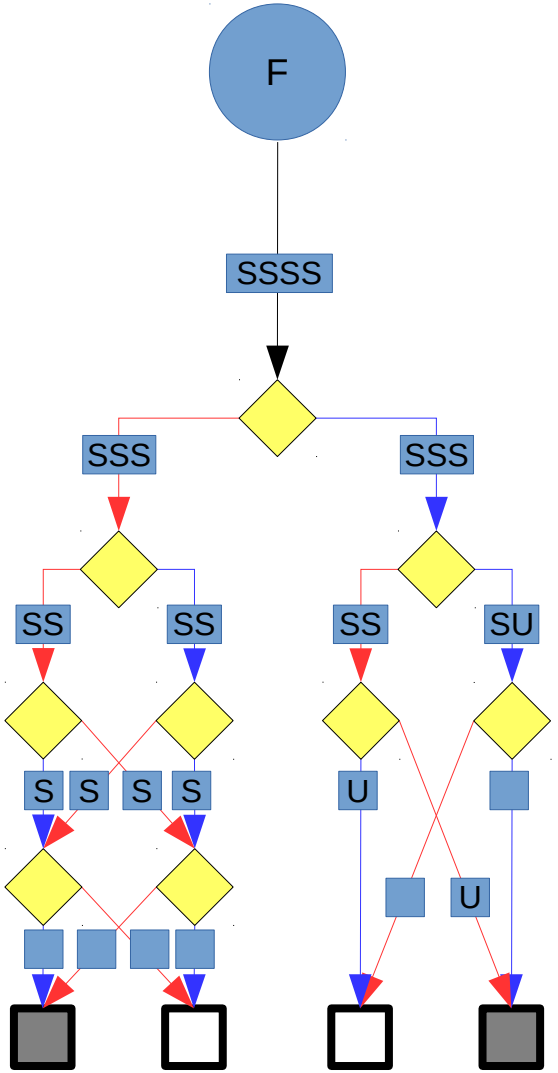
= True

S = significant
U = useless

Apply U2

For aesthetic reasons, we do not merge all terminals

Section 1 : model U



= if False
= if True

= False

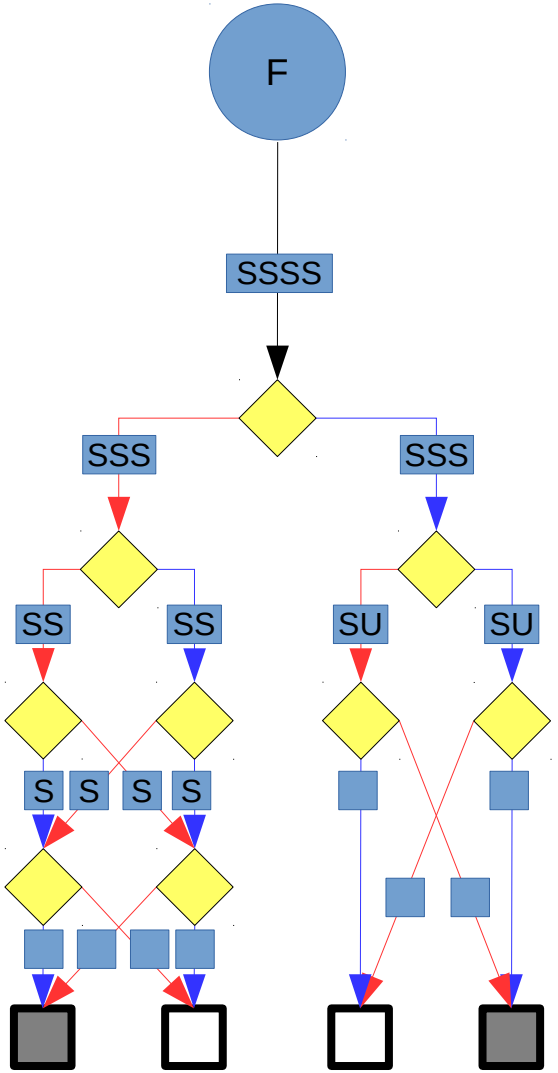
= True

S = significant
U = useless

Apply U2

For aesthetic reasons, we do not merge all terminals

Section 1 : model U



= if False
= if True

= False

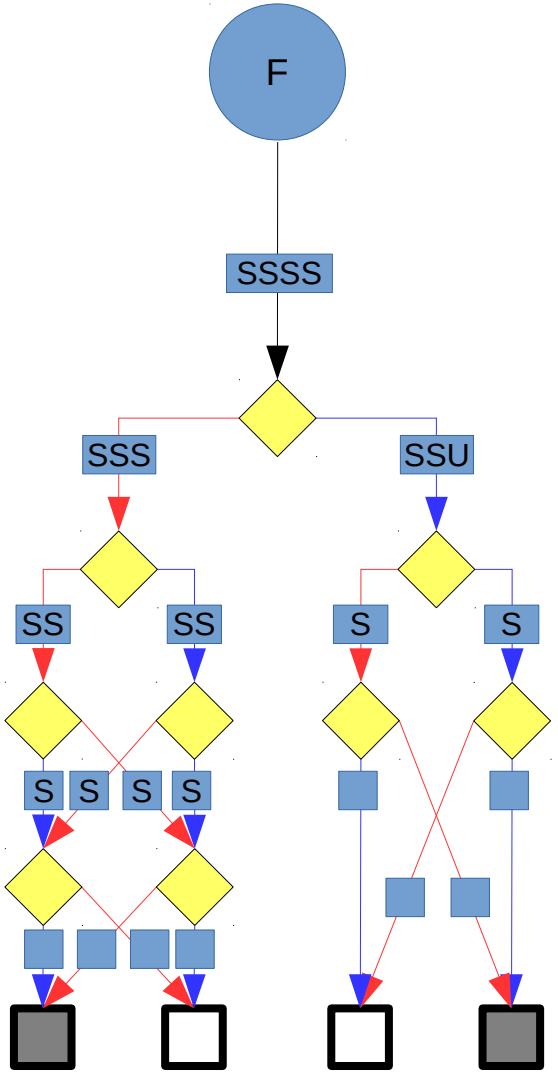
= True

S = significant
U = useless

Apply U2

For aesthetic reasons, we do not merge all terminals

Section 1 : model U



= if False
= if True

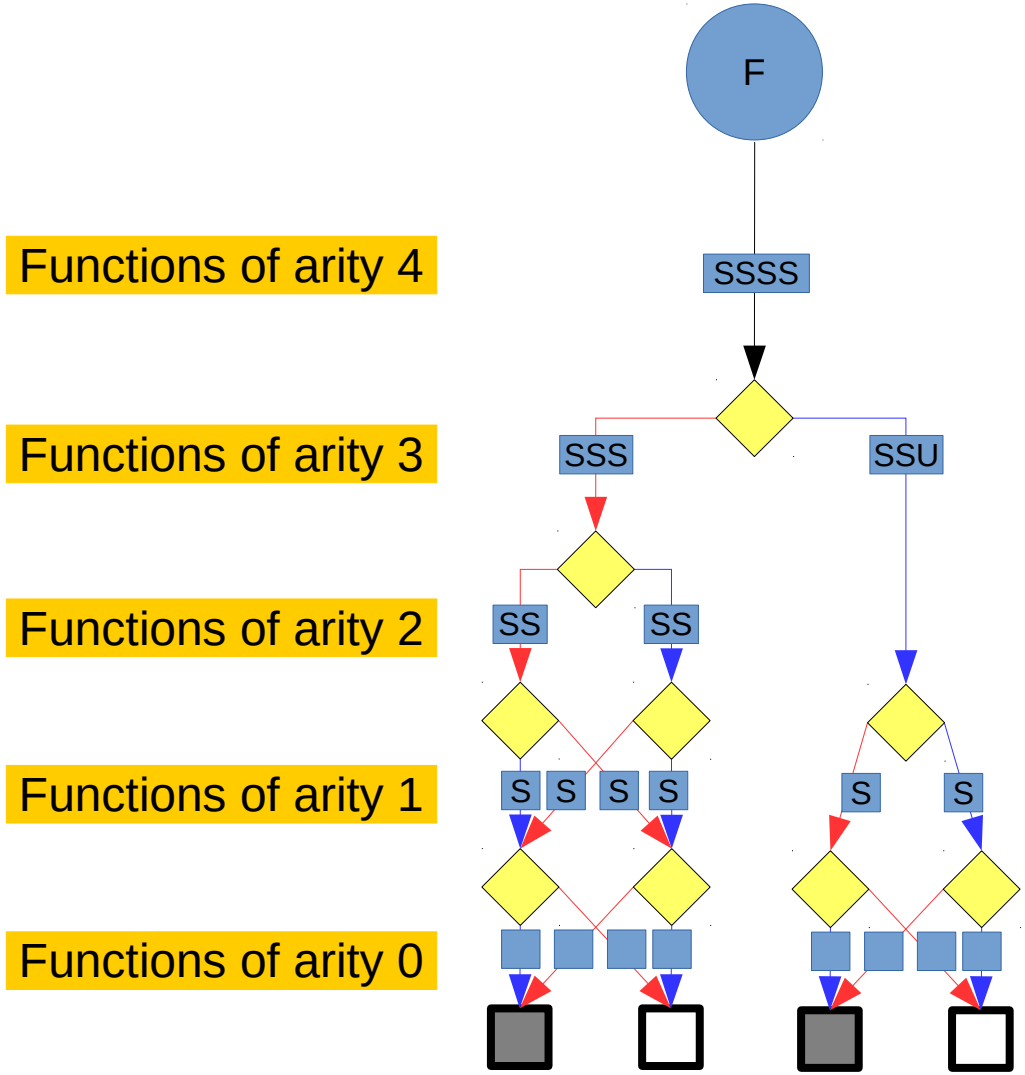
= False

= True

S = significant
U = useless

For aesthetic reasons, we do not merge all terminals

Section 1 : model U



= if False
= if True

= False

= True

S = significant
U = useless

Merge

For aesthetic reasons, we do not merge all terminals

Section 1 : model U

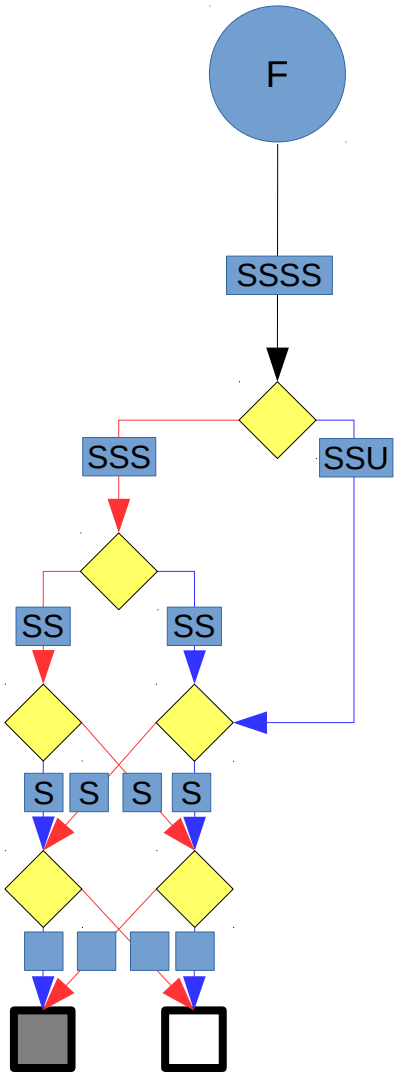
Functions of arity 4

Functions of arity 3

Functions of arity 2

Functions of arity 1

Functions of arity 0



= if False
= if True

= False

= True

S = significant
U = useless

For aesthetic reasons, we do not merge all terminals

Section 1 : model U

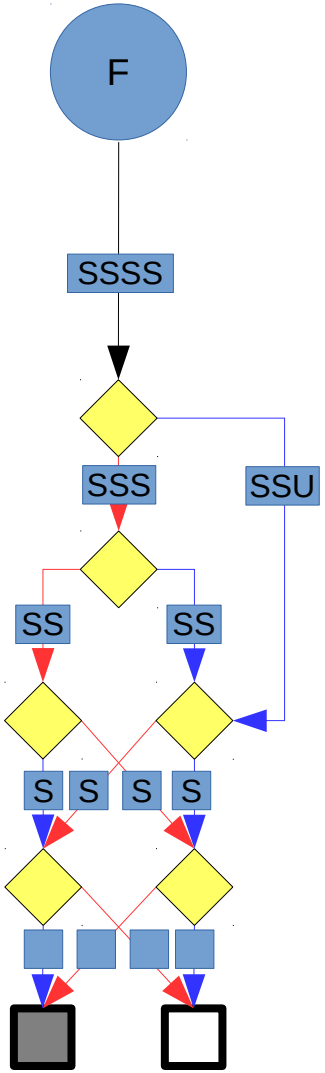
Functions of arity 4

Functions of arity 3

Functions of arity 2

Functions of arity 1

Functions of arity 0



= if False
= if True

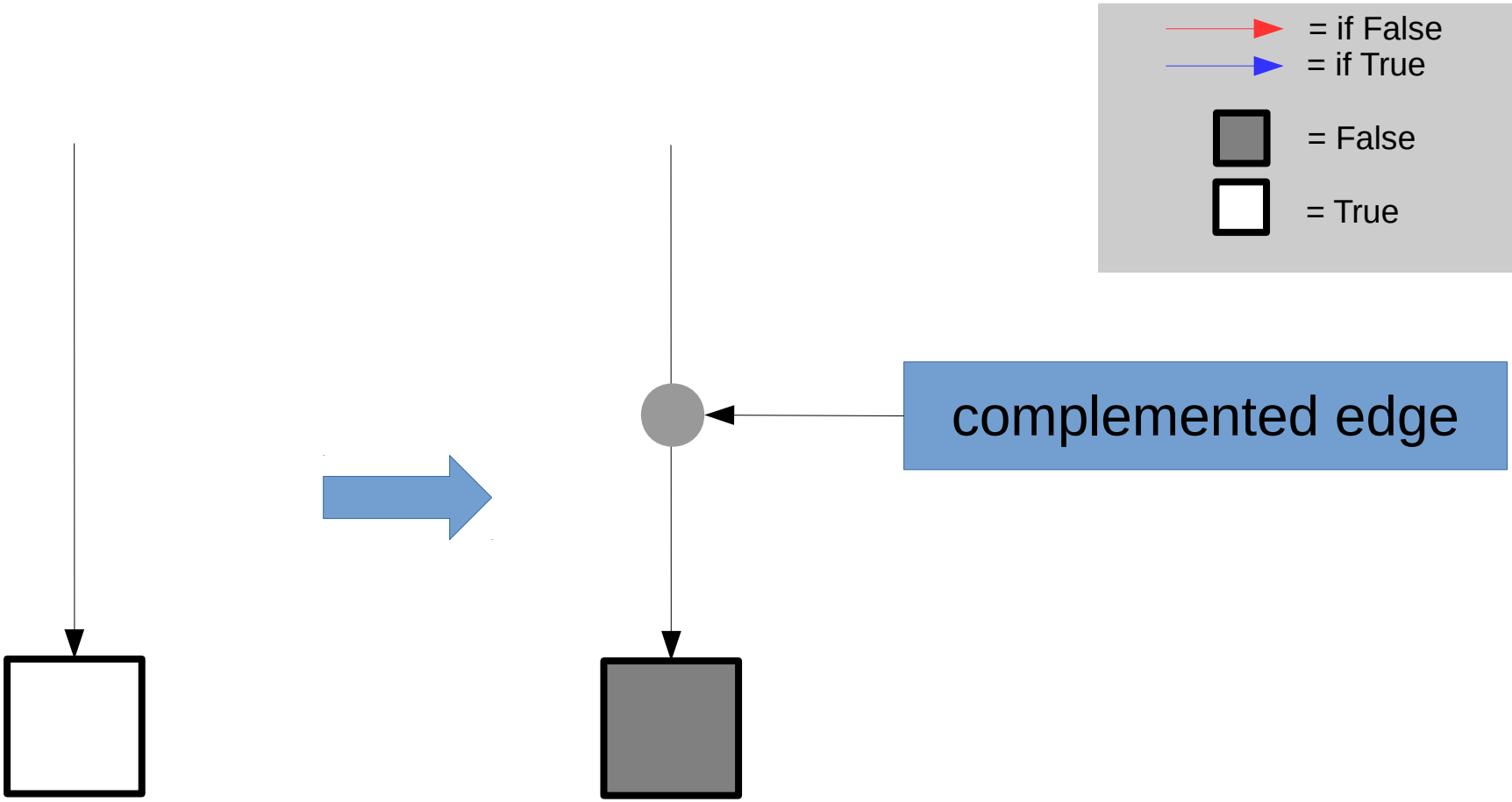
= False

= True

S = significant
U = useless

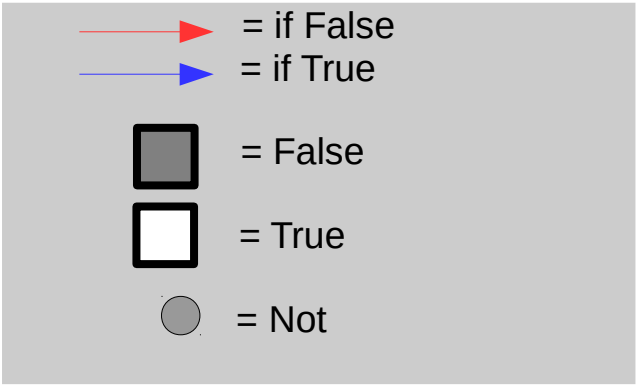
For aesthetic reasons, we do not merge all terminals

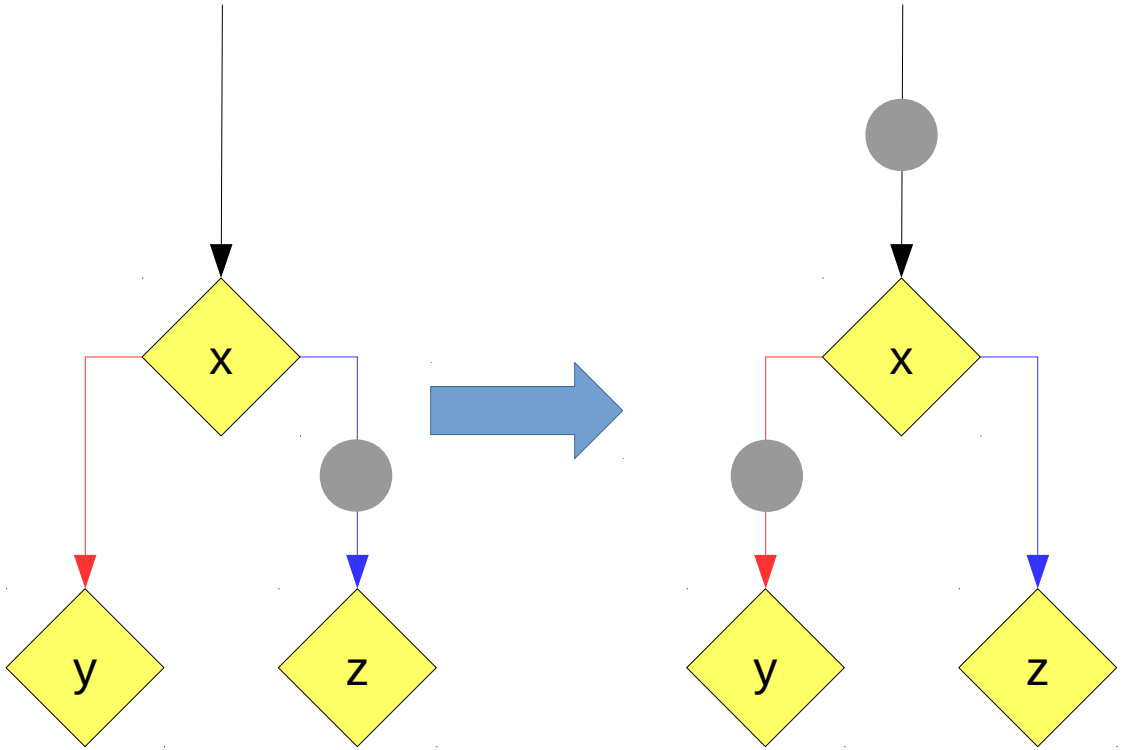
“output negation” : reduction rule (N1)



For aesthetic reasons, we do not merge all terminals

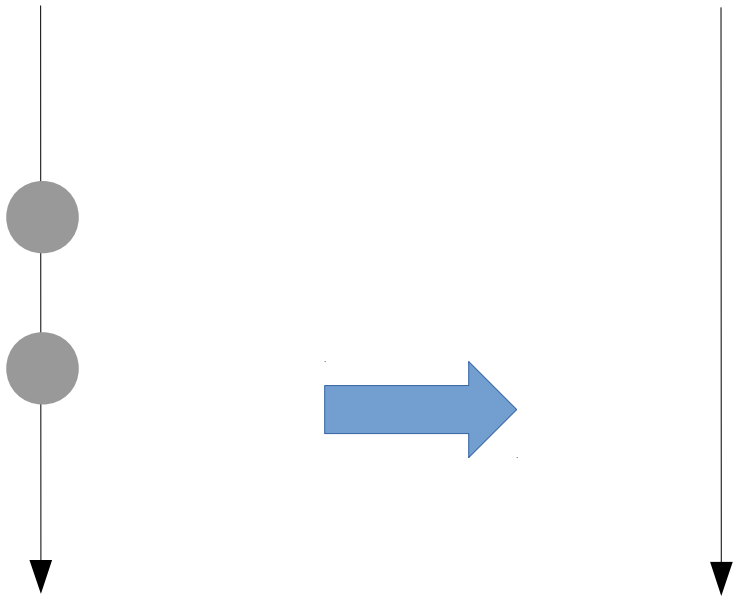
“output negation” : reduction rule (N2)

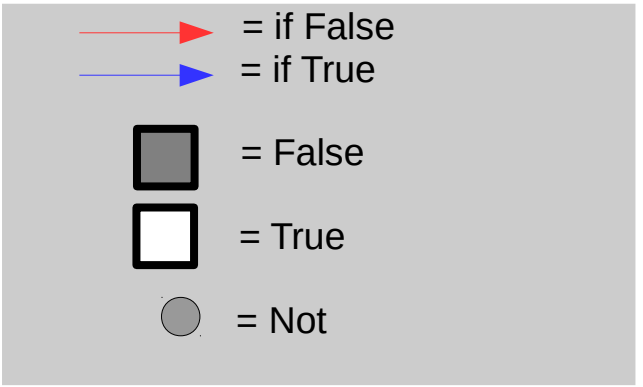




For aesthetic reasons, we do not merge all terminals

“output negation” : reduction rule (N3)





For aesthetic reasons, we do not merge all terminals

Section 2 : model NU

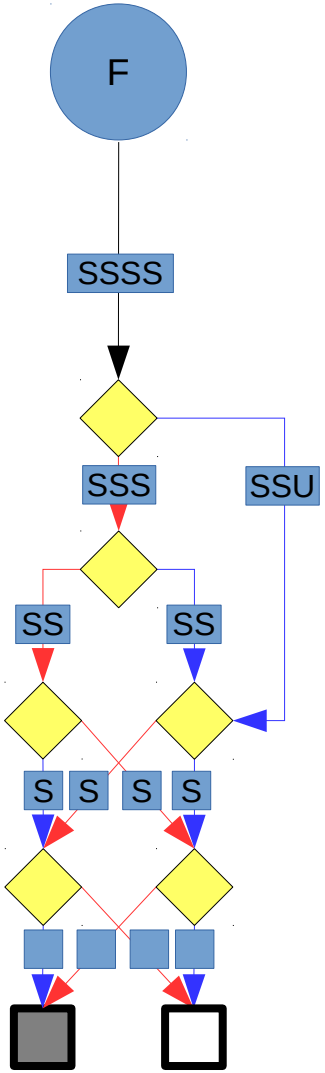
Functions of arity 4

Functions of arity 3

Functions of arity 2

Functions of arity 1

Functions of arity 0



= if False
= if True
= False
= True
= Not

S = significant
U = useless

Apply N1

For aesthetic reasons, we do not merge all terminals

Section 2 : model NU

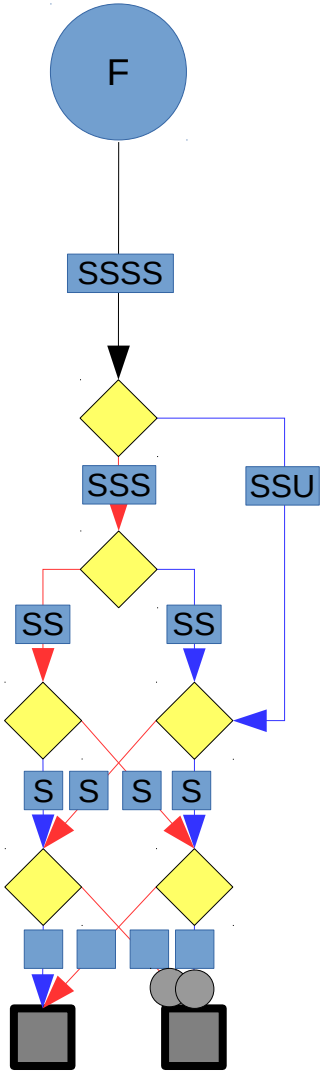
Functions of arity 4

Functions of arity 3

Functions of arity 2

Functions of arity 1

Functions of arity 0



= if False
= if True
= False
= True
= Not

S = significant
U = useless

Apply N2

For aesthetic reasons, we do not merge all terminals

Section 2 : model NU

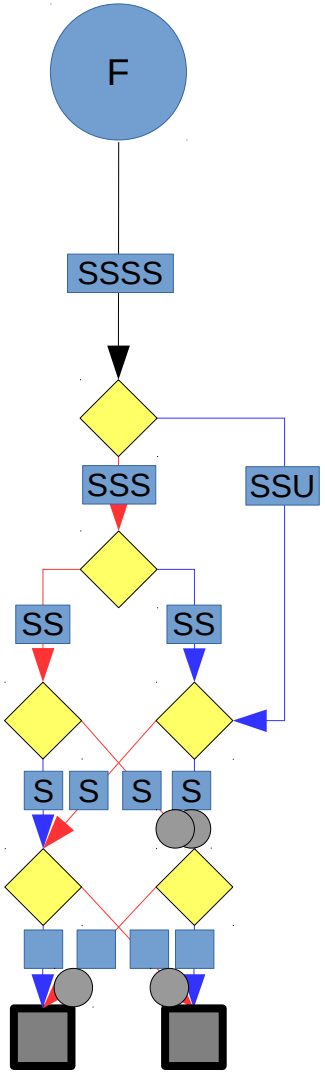
Functions of arity 4

Functions of arity 3

Functions of arity 2

Functions of arity 1

Functions of arity 0



= if False
= if True
= False
= True
= Not

S = significant
U = useless

Apply N2

For aesthetic reasons, we do not merge all terminals

Section 2 : model NU

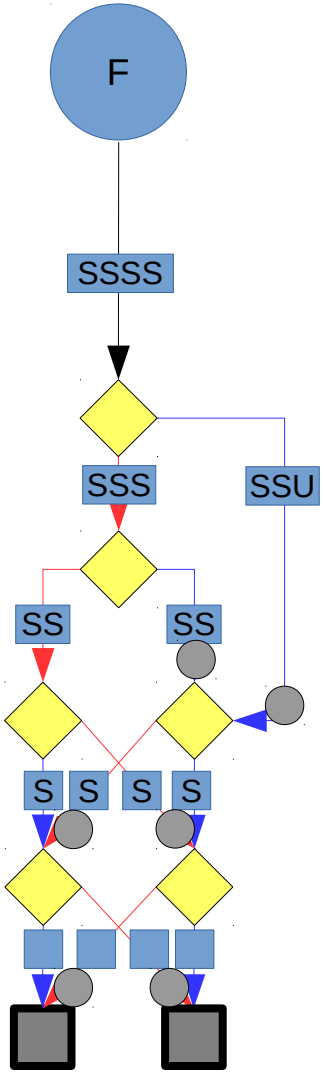
Functions of arity 4

Functions of arity 3

Functions of arity 2

Functions of arity 1

Functions of arity 0



= if False
= if True
= False
= True
= Not

S = significant
U = useless

Apply N2

For aesthetic reasons, we do not merge all terminals

Section 2 : model NU

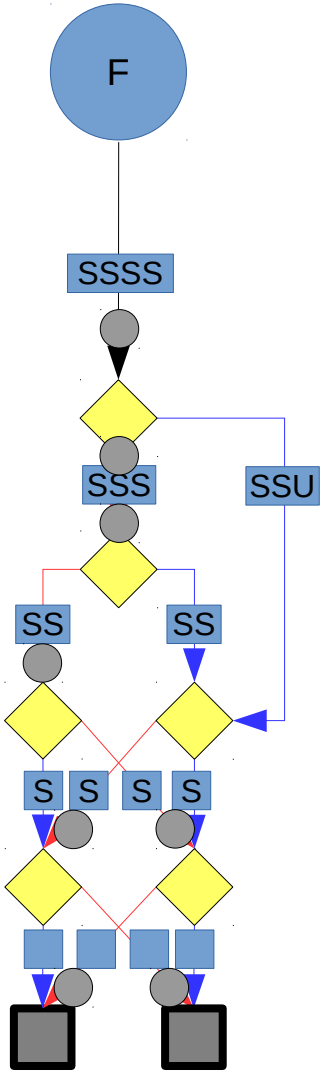
Functions of arity 4

Functions of arity 3

Functions of arity 2

Functions of arity 1

Functions of arity 0



= if False
= if True
= False
= True
= Not

S = significant
U = useless

Apply N3

For aesthetic reasons, we do not merge all terminals

Section 2 : model NU

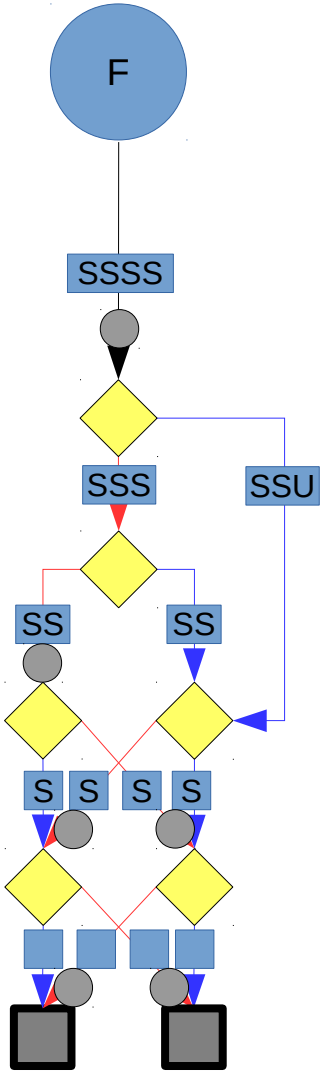
Functions of arity 4

Functions of arity 3

Functions of arity 2

Functions of arity 1

Functions of arity 0



= if False
= if True
= False
= True
= Not

S = significant
U = useless

Merge

For aesthetic reasons, we do not merge all terminals

Section 2 : model NU

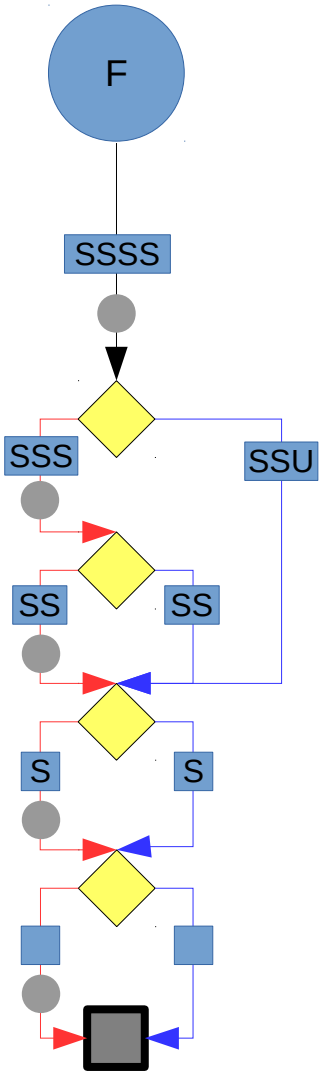
Functions of arity 4

Functions of arity 3

Functions of arity 2

Functions of arity 1

Functions of arity 0

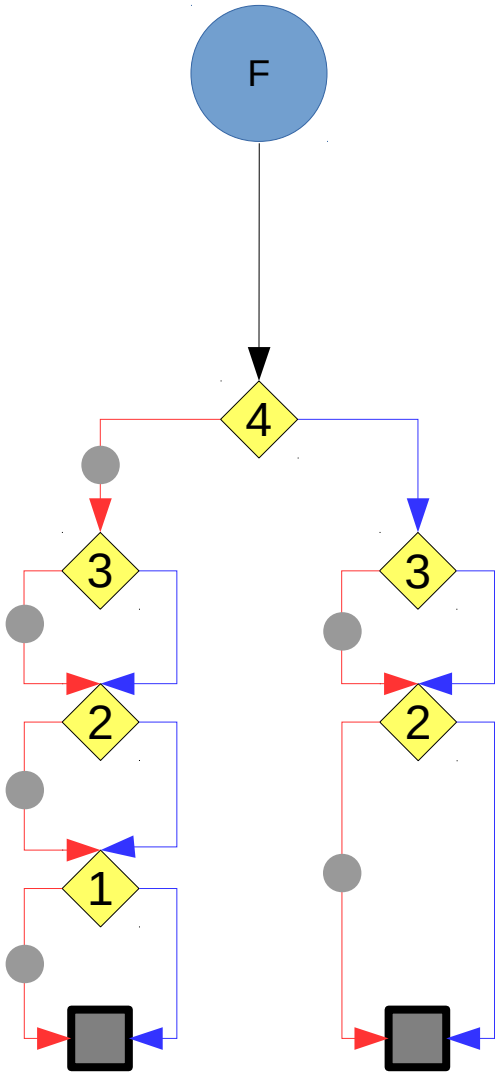


= if False
= if True
= False
= True
= Not

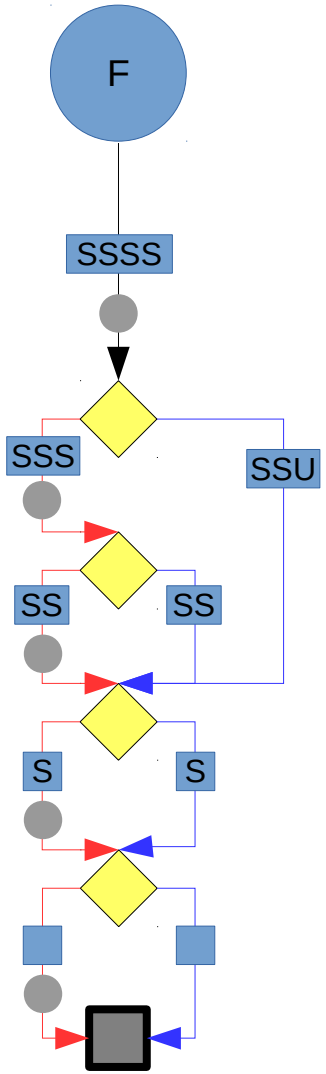
S = significant
U = useless

For aesthetic reasons, we do not merge all terminals

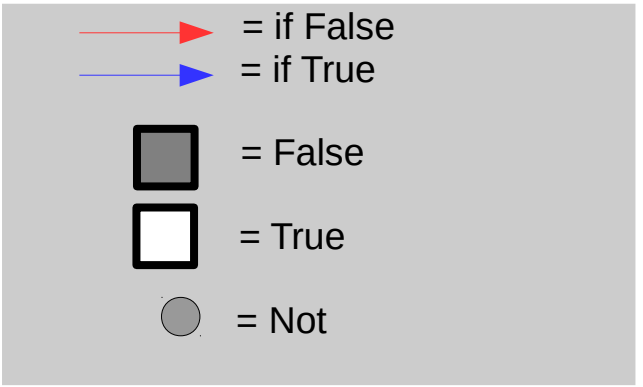
N vs NU



N =
ROBDD + “output negation”

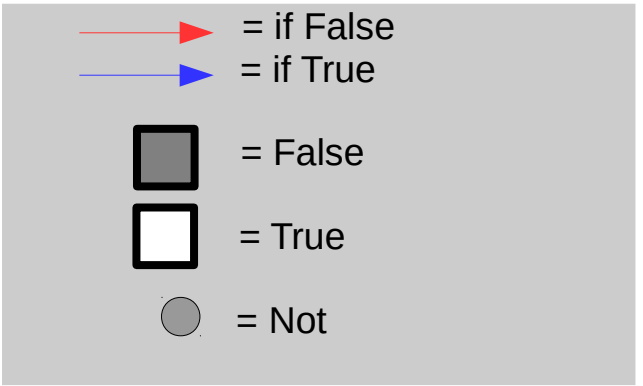


Model NU

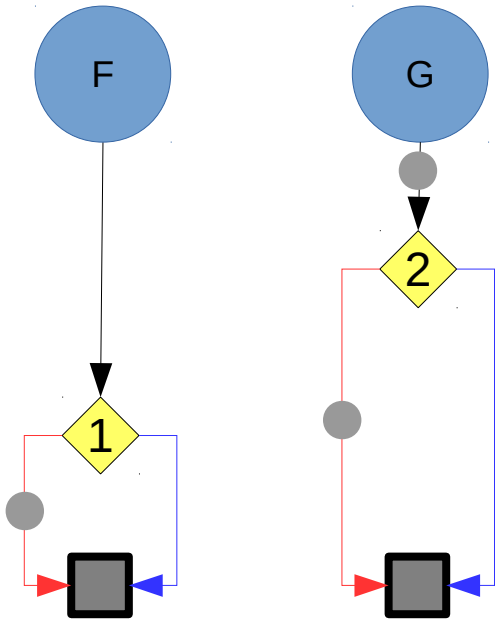


S = significant
U = useless

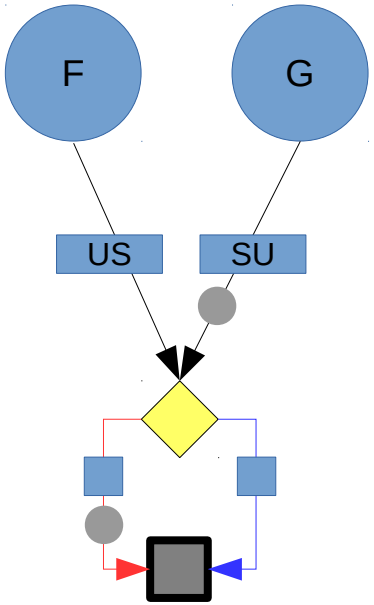
N vs NU: Example 2



S = significant
U = useless

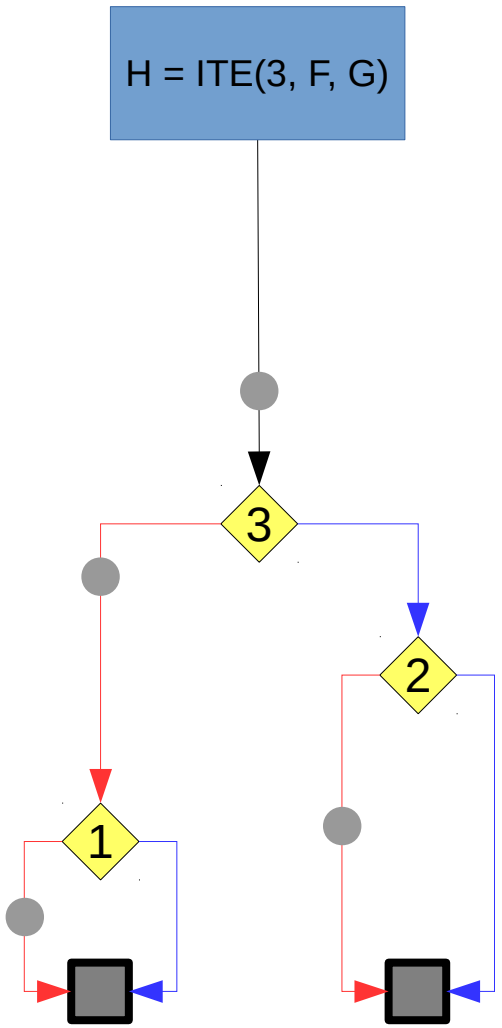


N =
ROBDD + “output negation”

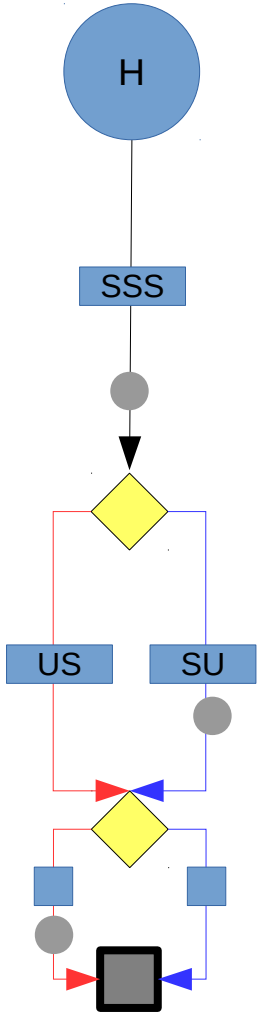


Model NU

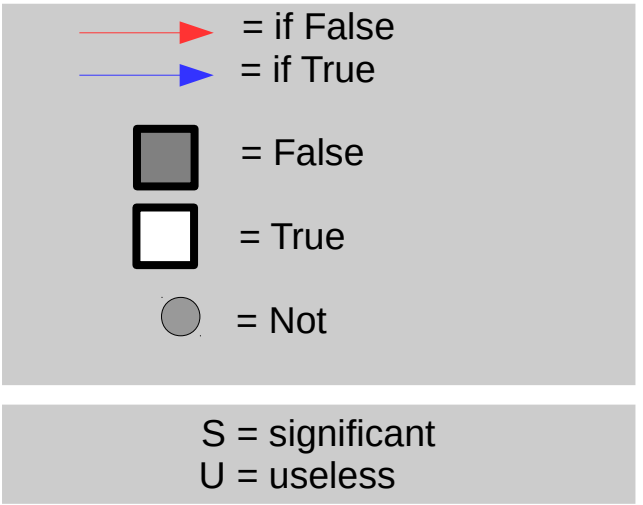
N vs NU: Example 3



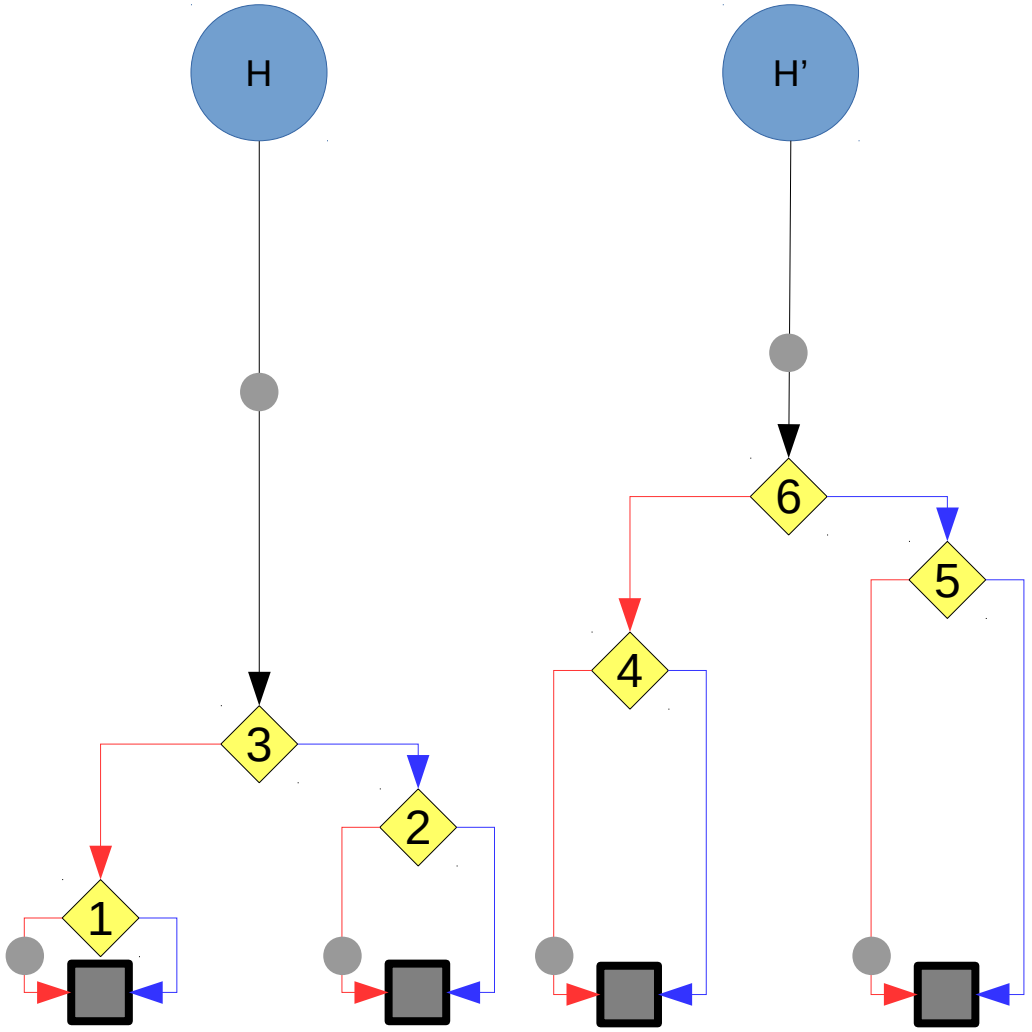
N =
ROBDD + “output negation”



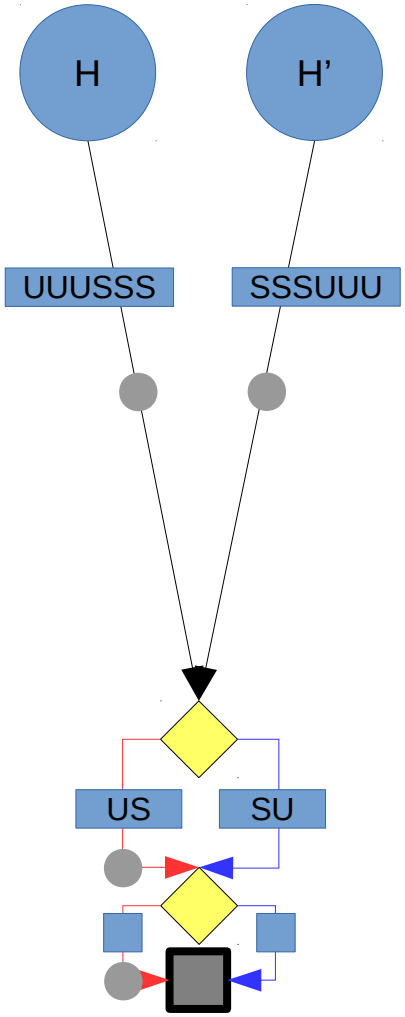
Model NU



N vs NU: Example 4

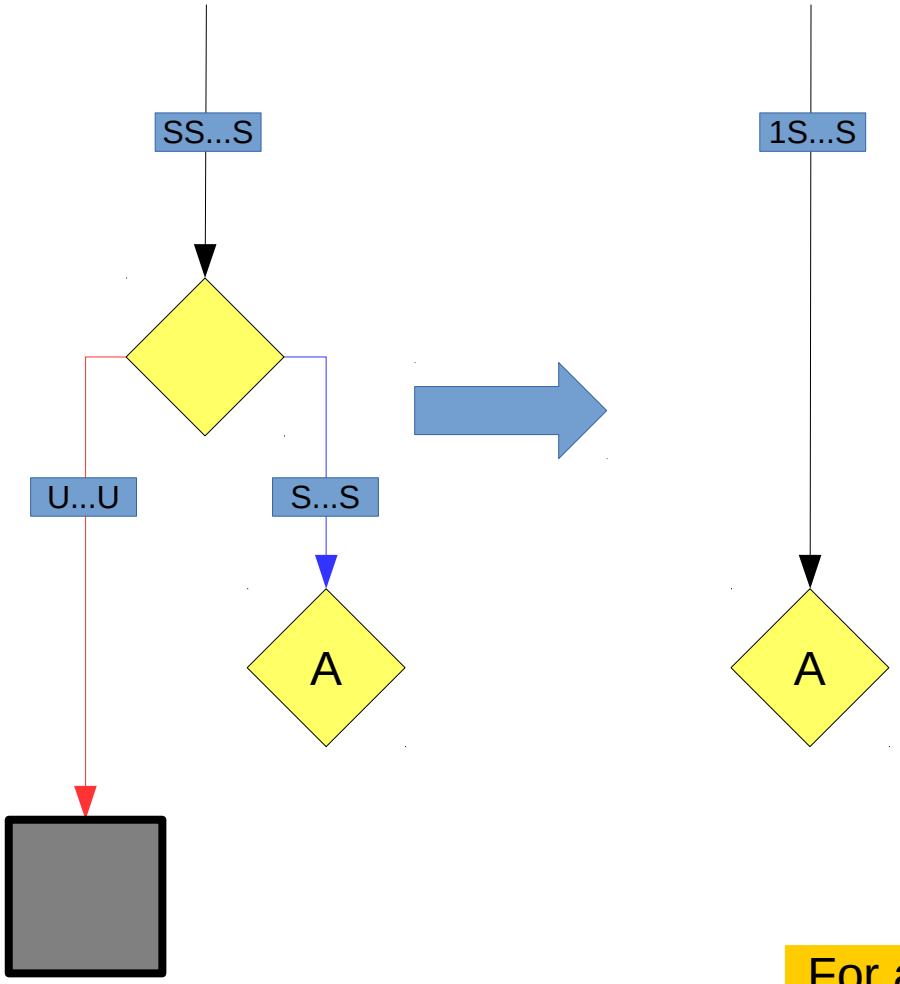


N =
ROBDD + “output negation”



Model NU

“1-prediction” : reduction rule (X1-’1’)

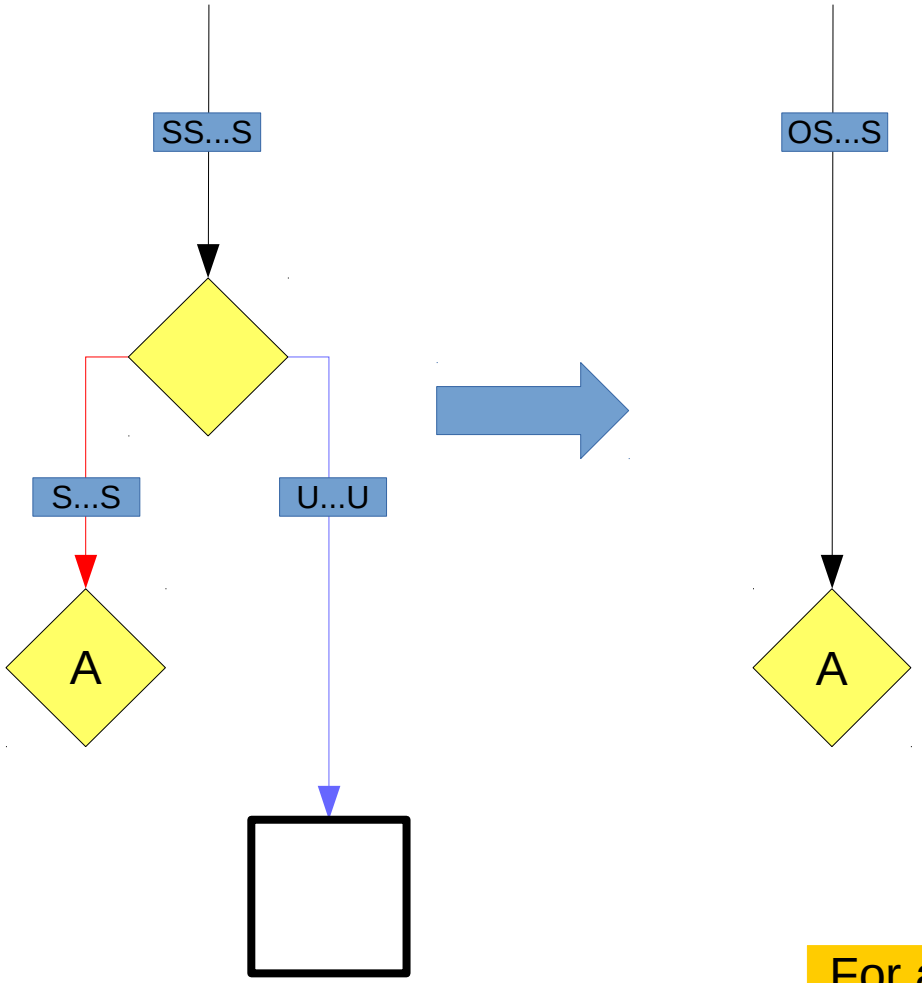


= if False
= if True
= False
= True
= Not

S = significant
U = useless
1 = If 0 then 0
0 = If 1 then 0
I = If 0 then 1
O = If 1 then 1

For aesthetic reasons, we do not merge all terminals

“1-prediction” : reduction rule (X1-’O’)

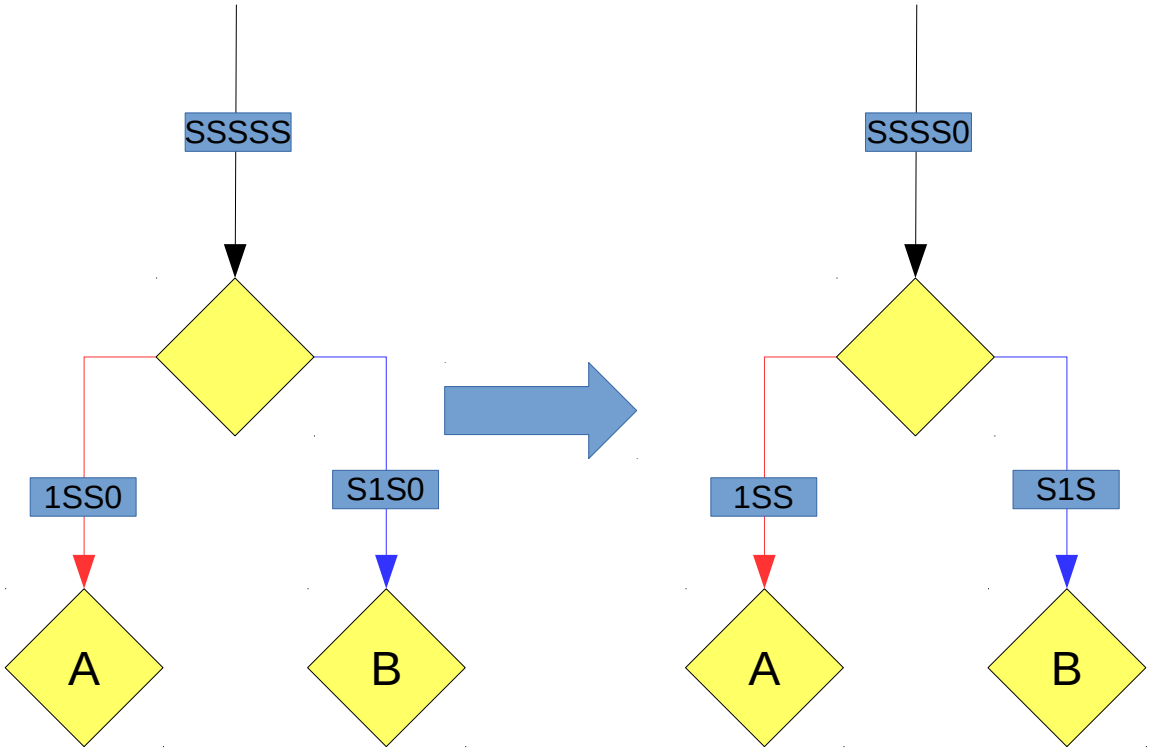


= if False
= if True
= False
= True
= Not

S = significant
U = useless
1 = If 0 then 0
0 = If 1 then 0
I = If 0 then 1
O = If 1 then 1

For aesthetic reasons, we do not merge all terminals

“1-prediction” : reduction rule (X2-’0’)



= if False
= if True
= False
= True
= Not

S = significant
U = useless
1 = If 0 then 0
0 = If 1 then 0
I = If 0 then 1
O = If 1 then 1

For aesthetic reasons, we do not merge all terminals

Section 2 : model NU

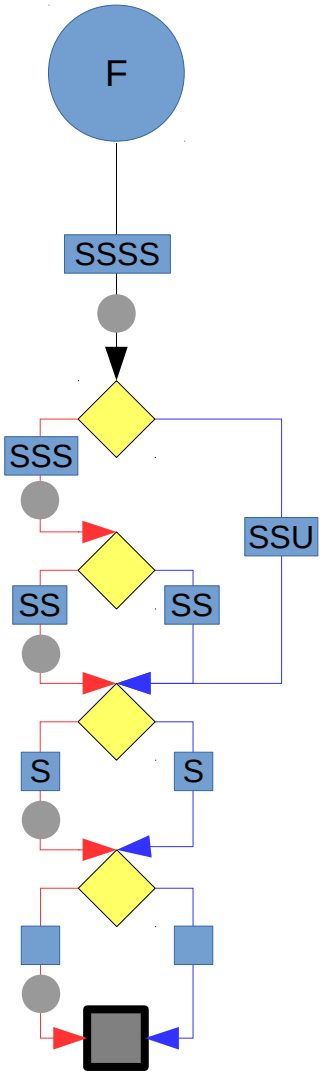
Functions of arity 4

Functions of arity 3

Functions of arity 2

Functions of arity 1

Functions of arity 0



= if False
= if True
= False
= True
= Not

S = significant
U = useless
1 = If 0 then 0
0 = If 1 then 0
I = If 0 then 1
O = If 1 then 1

Apply X1-'I'

For aesthetic reasons, we do not merge all terminals

Section 3 : model NU-X

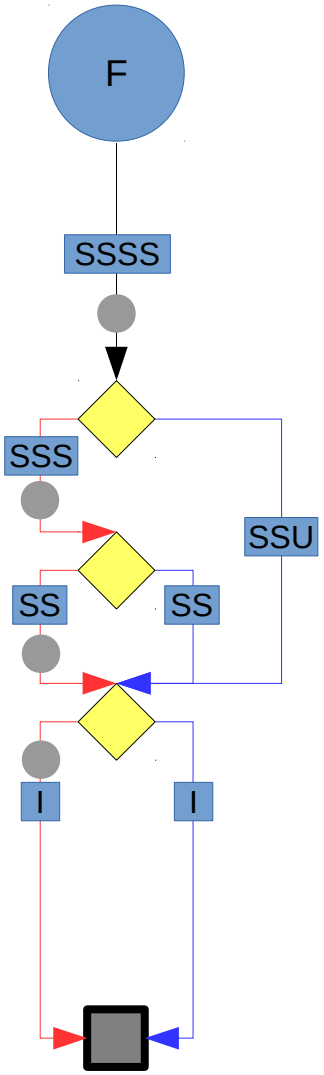
Functions of arity 4

Functions of arity 3

Functions of arity 2

Functions of arity 1

Functions of arity 0

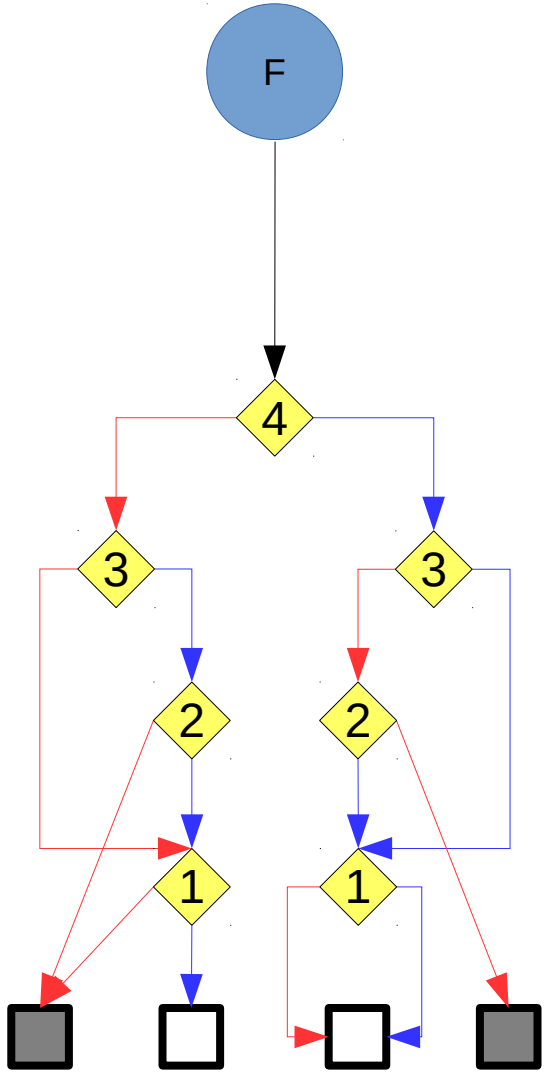


= if False
= if True
= False
= True
= Not

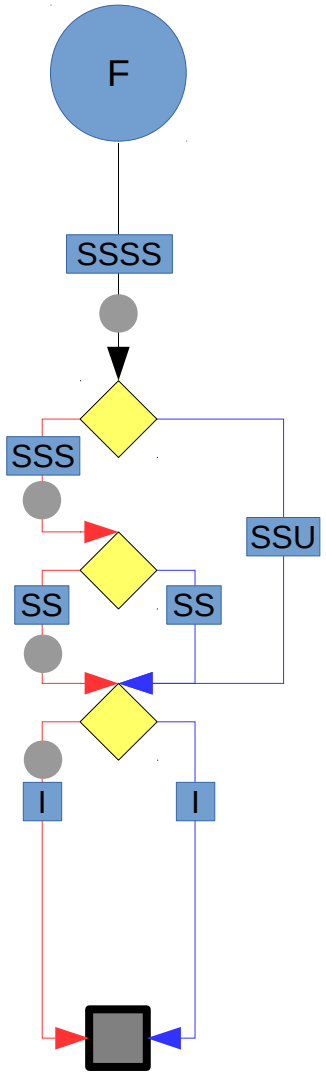
S = significant
U = useless
1 = If 0 then 0
0 = If 1 then 0
I = If 0 then 1
O = If 1 then 1

For aesthetic reasons, we do not merge all terminals

Section 3 : model NU-X



Z =
ZBDD

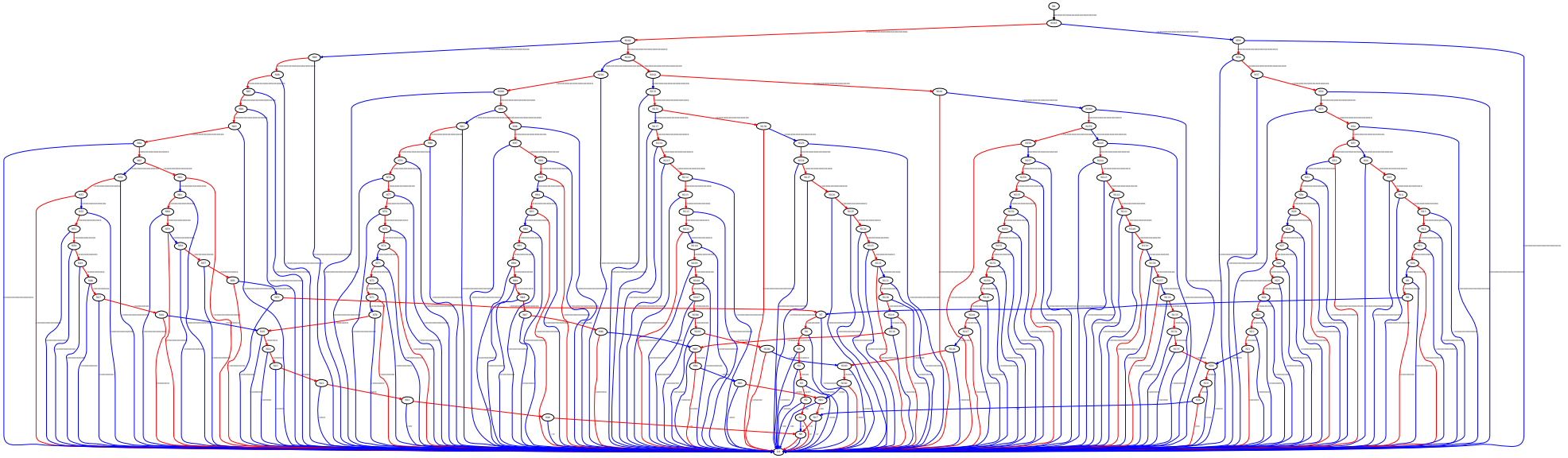


Model NU-X

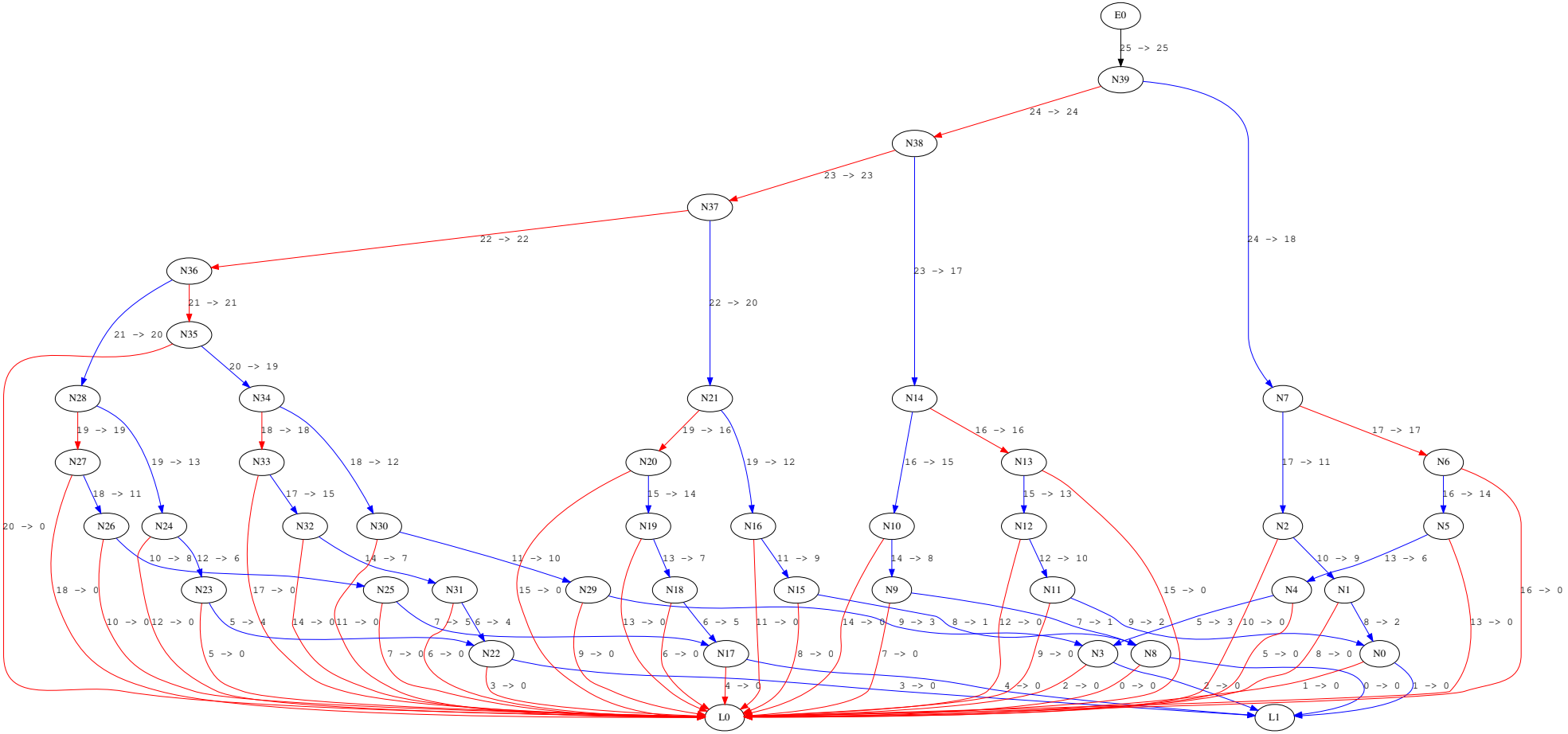
= if False
= if True
= False
= True
= Not

S = significant
U = useless
1 = If 0 then 0
0 = If 1 then 0
I = If 0 then 1
O = If 1 then 1

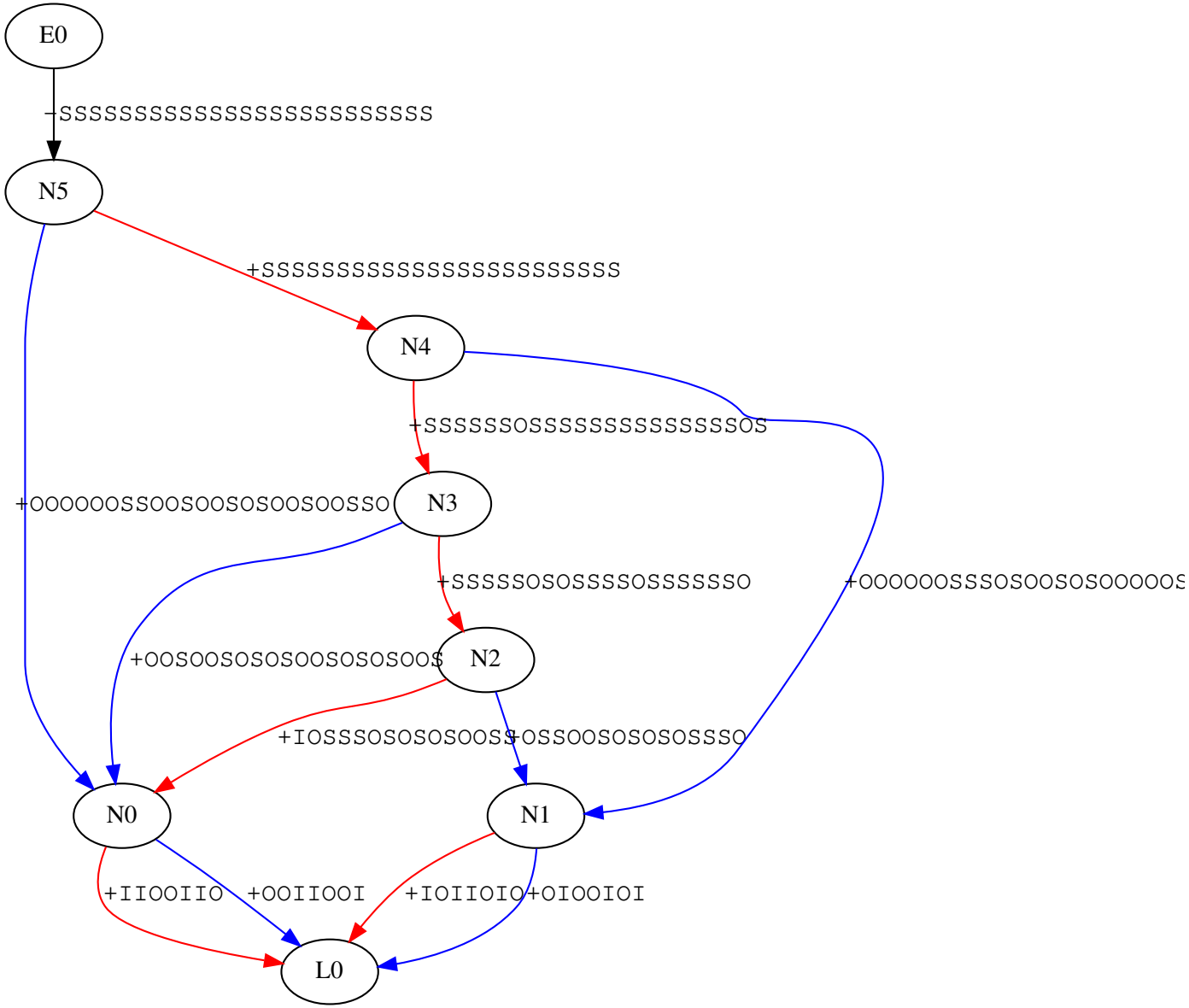
5-Queens: N or NU



5-Queens: Z


E0
25 -> 25
N39
24 -> 24
N38
23 -> 23
N37
22 -> 22
N36
21 -> 21
21 -> 20
N35
20 -> 19
N28
N34
19 -> 19
18 -> 18
N27
N33
19 -> 13
18 -> 12
18 -> 11
17 -> 15
N26
N24
N32
N30
20 -> 0
10 -> 8
12 -> 6
14 -> 7
11 -> 10
N23
N25
N31
N29
18 -> 0
17 -> 0
15 -> 0
10 -> 0
12 -> 0
5 -> 4
14 -> 0
11 -> 0
7 -> 5
6 -> 4
5 -> 0
7 -> 0
6 -> 0
N22
9 -> 0
3 -> 0
22 -> 20
N21
19 -> 16
N20
19 -> 12
15 -> 14
N19
N16
13 -> 7
11 -> 9
N18
N15
13 -> 0
6 -> 5
11 -> 0
6 -> 0
N17
8 -> 0
4 -> 0
23 -> 17
N14
16 -> 16
16 -> 15
N13
15 -> 13
N10
N12
14 -> 8
12 -> 10
N9
N11
14 -> 0
9 -> 3
8 -> 1
12 -> 0
7 -> 0
9 -> 0
N3
7 -> 1
9 -> 2
15 -> 0
24 -> 18
N7
17 -> 17
17 -> 11
N2
N6
16 -> 14
10 -> 9
13 -> 6
N4
N1
N5
16 -> 0
5 -> 3
10 -> 0
8 -> 2
13 -> 0
N8
N0
5 -> 0
8 -> 0
3 -> 0
4 -> 0
2 -> 0
0 -> 0
2 -> 0
1 -> 0
0 -> 0
1 -> 0
L0
L1

5-Queens: NU-X


E0
-SSSSSSSSSSSSSSSSSSSSSSSS
N5
+SSSSSSSSSSSSSSSSSSSSSSS
N4
+SSSSSSOSSSSSSSSSSSSSOS
N3
+OOOOOOSSOOSOOSOSOOSOOSSO
+SSSSSOSOSSSSOSSSSSSO
+OOOOOOSSSOSOOSOSOOOOOS
N2
+OOSOOSOSOSOOSOSOSOOS
+IOSSSOSOSOSOOSS+OSSOOSOSOSOSSSO
N0
N1
+IIOOIIO
+OOIIOOI
+IOIIOIO
+OIOOIOI
L0

Results

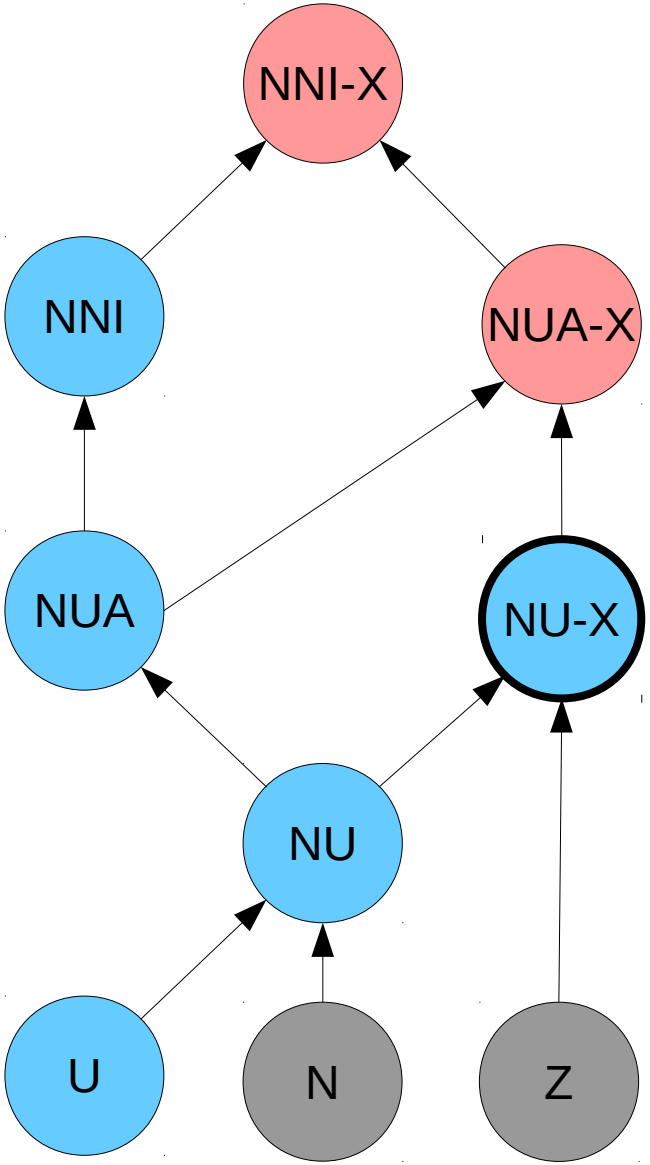
Average reduction of the {number of nodes / **estimated** memory cost} on four benchmarks

	Circuits (dense functions)				CNF formulas (sparse functions)			
	lgsynth91		iscas99		uf20-91		uf50-218	
variants	#node	mem	#node	mem	#node	mem	#node	mem
Z	+233%	+233%	+162%	+162%	-41%	-41%	-42%	-42%
NU	-26%	-21%	-25%	-20%	-3%	+7%	-3%	+22%
NU-X	-64%	-58%	-55%	-46%	-96%	-95%	-97%	-96%

Can we go further ?

“input/output negation”

“output negation” + “useless variables extraction” + “anti-variables extraction”



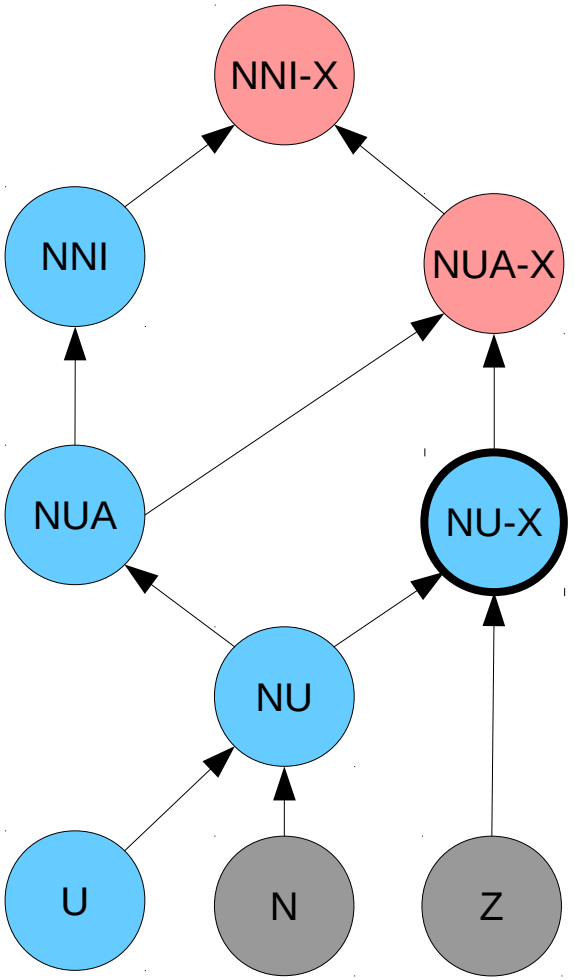
“input/output negation” + “1-prediction extraction”

“output negation” + “useless variables extraction” + “anti-variables extraction” + “1-prediction extraction”

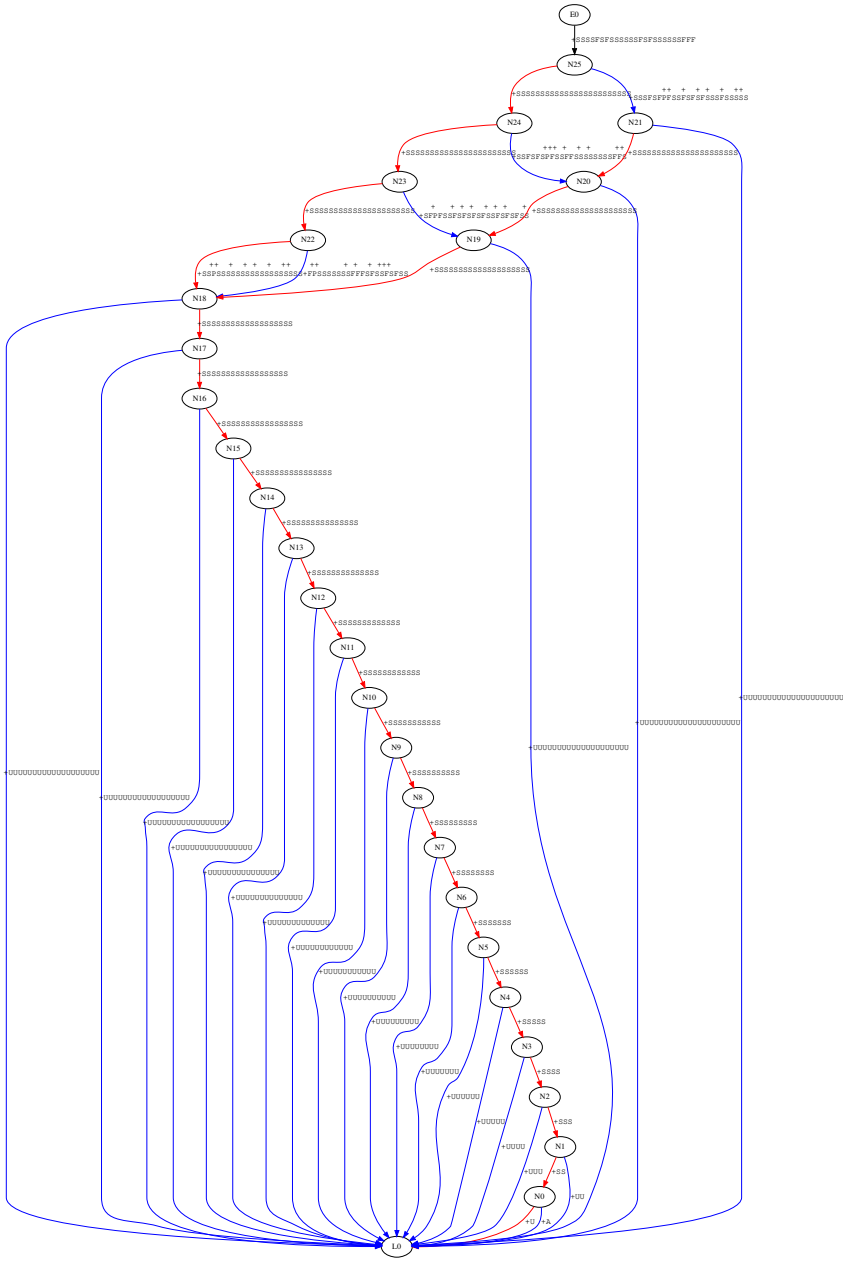
“output negation” + “useless variables extraction” + “1-prediction extraction”

Conclusion

- Software implemented in OCaml:
  - https://github.com/JoanThibault/DAGaml/tree/grobdd-dev
  - ~ 12 000 lines of OCaml
- Fewer nodes & Less memory
- Future Work
  - Quantify the dependency between variables' order and #node
  - Solve & Implement NUA-X and NNI-X versions
- TO DO
  - Quantification Operators
  - Variable Reordering
  - Parallelism & hardware acceleration
- Other Applications
  - Apply similar strategies to compress other DAG
    - DAG / Graph isomorphism



5-Queens: NNI


E0
N25
N24
N21
N23
N20
N22
N19
N18
N17
N16
N15
N14
N13
N12
N11
N10
N9
N8
N7
N6
N5
N4
N3
N2
N1
N0
L0

Results

Average reduction of the {number of nodes / **estimated** memory cost} on four benchmarks

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NU	-26%	-21%	-25%	-20%	-3%	+7%	-3%	+22%
NNI	-60%	-53%	-56%	-49%	-30%	-10%	-39%	+5%
NU-X	-64%	-58%	-55%	-46%	-96%	-95%	-97%	-96%