Θέμα 1: Bayes

$P(x|w_1) = N(\mu_1, \Sigma_1)$, $\quad P(x|w_2) = N(\mu_2, \Sigma_2)$

$\mu_1 = \begin{pmatrix} 3 \\ 3 \end{pmatrix} \quad \Sigma_1 = \begin{pmatrix} 1,2 & -0,4 \\ -0,4 & 1,2 \end{pmatrix}$

$\mu_2 = \begin{pmatrix} 6 \\ 6 \end{pmatrix} \quad \Sigma_2 = \begin{pmatrix} 1,2 & 0,4 \\ 0,4 & 1,2 \end{pmatrix}$

Εύρεση ορίου απόφασης:

$P(w_1|x) = P(w_2|x) \Leftrightarrow P(w_1)P(x|w_1) = P(w_2)P(x|w_2) \Leftrightarrow$

$\dfrac{P(x|w_1)}{P(x|w_2)} = \dfrac{P(w_2)}{P(w_1)} \Leftrightarrow$

$$\ln\left(\frac{\frac{1}{2\pi\sqrt{|\Sigma_1|}} \exp\left(-\frac{1}{2}(x-\mu_1)^T \Sigma_1^{-1}(x-\mu_1)\right)}{\frac{1}{2\pi\sqrt{|\Sigma_2|}} \exp\left(-\frac{1}{2}(x-\mu_2)^T \Sigma_2^{-1}(x-\mu_2)\right)}\right) = \ln\frac{P(w_2)}{P(w_1)} \Leftrightarrow$$

$$\ln\left(\frac{1}{2\pi\sqrt{|\Sigma_1|}} \exp\left(-\frac{1}{2}(x-\mu_1)^T \Sigma_1^{-1}(x-\mu_1)\right)\right) - \ln\left(\frac{1}{2\pi\sqrt{|\Sigma_2|}} \exp\left(-\frac{1}{2}(x-\mu_2)^T \Sigma_2^{-1}(x-\mu_2)\right)\right) =$$
$$= \ln\left(\frac{P(w_2)}{P(w_1)}\right)$$

$$\ln\left(\frac{1}{2\pi\sqrt{|\Sigma_1|}}\right) - \frac{1}{2}(x-\mu_1)^T \cdot \Sigma_1^{-1}(x-\mu_1) - \ln\left(\frac{1}{2\pi\sqrt{|\Sigma_2|}}\right) + \frac{1}{2}(x-\mu_2)^T \cdot \Sigma_2^{-1}(x-\mu_2) =$$
$$= \ln\left(\frac{P(w_2)}{P(w_1)}\right)$$

όπου $|\Sigma_1| = \begin{vmatrix} 1,2 & -0,4 \\ -0,4 & 1,2 \end{vmatrix} = (1,2)^2 - (0,4)^2 = 1,28$

και $|\Sigma_2| = \begin{vmatrix} 1,2 & 0,4 \\ 0,4 & 1,2 \end{vmatrix} = (1,2)^2 - (0,4)^2 = 1,28$

$\Rightarrow |\Sigma_1| = |\Sigma_2|$

άρα

$$\boxed{(x-\mu_2)^T \Sigma_2^{-1}(x-\mu_2) - (x-\mu_1)^T \Sigma_1^{-1}(x-\mu_1) = 2\ln\left(\frac{P(w_2)}{P(w_1)}\right)}$$

το όριο απόφασης