

Θεωρία: Minimum risk

$$P(x|w_i) = \begin{cases} \frac{x}{\sigma^2} e^{-\frac{x}{2\sigma^2}}, & x \geq 0 \\ 0, & x < 0 \end{cases}$$

$$L = \begin{pmatrix} 0 & 0.5 \\ 1 & 0 \end{pmatrix}$$

$$\sigma_1 = 1, \sigma_2 = 2$$

$$\begin{aligned} l_1 &= \lambda_{11} P(x|w_1) P(w_1) + \lambda_{21} P(x|w_2) P(w_2) \\ l_2 &= \lambda_{12} P(x|w_1) P(w_1) + \lambda_{22} P(x|w_2) P(w_2) \end{aligned}$$

Για $x > 0$:

όριο ανόγασης x_0 :

$$l_1 = l_2 \Leftrightarrow \lambda_{21} P(x|w_2) P(w_2) = \lambda_{12} P(x|w_1) P(w_1) \Leftrightarrow$$

$$P(x|w_2) = \frac{1}{2} P(x|w_1) \Leftrightarrow 2 P(x|w_2) = P(x|w_1) \Leftrightarrow$$

$$2 \cdot \frac{x}{\sigma_2^2} \cdot \exp\left(-\frac{x^2}{2\sigma_2^2}\right) = \frac{x}{\sigma_1^2} \cdot \exp\left(-\frac{x^2}{2\sigma_1^2}\right) \Leftrightarrow$$

$$2 \cdot \frac{x}{4} \cdot \exp\left(-\frac{x^2}{8}\right) = x \cdot \exp\left(-\frac{x^2}{2}\right) \Leftrightarrow$$

$$\frac{x}{2} \cdot e^{-\frac{x^2}{8}} = x \cdot e^{-\frac{x^2}{2}} \Leftrightarrow x e^{-\frac{x^2}{8}} - 2x e^{-\frac{x^2}{2}} = 0 \Leftrightarrow$$

$$x_0 (e^{-\frac{x_0^2}{8}} - 2x_0 e^{-\frac{x_0^2}{2}}) = 0 \Leftrightarrow e^{-\frac{x_0^2}{8}} - 2e^{-\frac{x_0^2}{2}} = 0 \Leftrightarrow$$

$$\ln(e^{-\frac{x^2}{8}}) = \ln(2e^{-\frac{x^2}{2}}) (\Leftrightarrow)$$

$$-\frac{x^2}{8} = \ln 2 - \frac{x^2}{2} (\Leftrightarrow)$$

$$\frac{x^2}{2} - \frac{x^2}{8} = \ln 2 (\Leftrightarrow)$$

$$\frac{3}{8} x^2 = \ln 2 (\Leftrightarrow)$$

$$x^2 = \frac{8 \ln 2}{3}$$

$$x = \pm 2 \sqrt{\frac{2 \ln 2}{3}} \quad \text{donc } x > 0$$

à p a

$$x_0 = 2 \sqrt{\frac{2 \ln 2}{3}}$$