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$$i\hbar \frac{\partial}{\partial t} |\psi, t\rangle = H |\psi, t\rangle.$$
 (1.1)

$$H = \frac{1}{2m} \mathbf{P}^2. \tag{1.2}$$

$$i\hbar \frac{\partial}{\partial t} \psi(\mathbf{x}, t) = -\frac{\hbar^2}{2m} \nabla^2 \psi(\mathbf{x}, t). \tag{1.3}$$

$$H = +\sqrt{\mathbf{P}^2 c^2 + m^2 c^4}. ag{1.4}$$

$$H = mc^2 + \frac{1}{2m}\mathbf{P}^2 + \cdots {1.5}$$

$$i\hbar \frac{\partial}{\partial t} \psi(\mathbf{x}, t) = +\sqrt{-\hbar^2 c^2 \nabla^2 + m^2 c^4} \, \psi(\mathbf{x}, t). \tag{1.6}$$

$$-\hbar^2 \frac{\partial^2}{\partial t^2} \psi(\mathbf{x}, t) = \left(-\hbar^2 c^2 \nabla^2 + m^2 c^4\right) \psi(\mathbf{x}, t). \tag{1.7}$$

$$g_{\mu\nu} = \begin{pmatrix} -1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix}. \tag{1.8}$$

$$g^{\mu\nu} = \begin{pmatrix} -1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix}. \tag{1.9}$$

$$\bar{x}^{\mu} = \Lambda^{\mu}_{\ \nu} x^{\nu} + a^{\mu}. \tag{1.10}$$

$$g_{\mu\nu}\Lambda^{\mu}{}_{\rho}\Lambda^{\nu}{}_{\sigma} = g_{\rho\sigma}. \tag{1.11}$$

$$(x - x')^{2} \equiv g_{\mu\nu}(x - x')^{\mu}(x - x')^{\nu}$$

= $(\mathbf{x} - \mathbf{x}')^{2} - c^{2}(t - t')^{2}$. (1.12)

$$(\bar{x} - \bar{x}')^{2} = g_{\mu\nu}(\bar{x} - \bar{x}')^{\mu}(\bar{x} - \bar{x}')^{\nu}$$

$$= g_{\mu\nu}\Lambda^{\mu}{}_{\rho}\Lambda^{\nu}{}_{\sigma}(x - x')^{\rho}(x - x')^{\sigma}$$

$$= g_{\rho\sigma}(x - x')^{\rho}(x - x')^{\sigma}$$

$$= (x - x')^{2}.$$
(1.13)

$$\partial_{\mu} \equiv \frac{\partial}{\partial x^{\mu}} = \left(+\frac{1}{c} \frac{\partial}{\partial t}, \nabla \right),$$
 (1.14)

$$\partial^{\mu} \equiv \frac{\partial}{\partial x_{\mu}} = \left(-\frac{1}{c} \frac{\partial}{\partial t}, \nabla \right). \tag{1.15}$$

$$\partial^{\mu} x^{\nu} = g^{\mu\nu}. \tag{1.16}$$

$$\bar{\partial}^{\mu} = \Lambda^{\mu}{}_{\nu}\partial^{\nu}. \tag{1.17}$$

$$\bar{\partial}^{\rho}\bar{x}^{\sigma}=(\Lambda^{\rho}{}_{\mu}\partial^{\mu})(\Lambda^{\sigma}{}_{\nu}x^{\nu}+a^{\sigma})=\Lambda^{\rho}{}_{\mu}\Lambda^{\sigma}{}_{\nu}(\partial^{\mu}x^{\nu})=\Lambda^{\rho}{}_{\mu}\Lambda_{\sigma\nu}g^{\mu\nu}=g^{\rho\sigma}. \eqno(1.18)$$