Personal Notes of Part I: Scalar Fields of Quantum Field Theory by Srednicki

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$$i\hbar \frac{\partial}{\partial t} |\psi, t\rangle = H |\psi, t\rangle.$$
 (1.1)

$$H = \frac{1}{2m} \mathbf{P}^2. \tag{1.2}$$

$$i\hbar \frac{\partial}{\partial t} \psi(\mathbf{x}, t) = -\frac{\hbar^2}{2m} \nabla^2 \psi(\mathbf{x}, t).$$
 (1.3)

$$H = +\sqrt{\mathbf{P}^2 c^2 + m^2 c^4}. ag{1.4}$$

$$H = mc^2 + \frac{1}{2m}\mathbf{P}^2 + \cdots$$
 (1.5)

$$i\hbar \frac{\partial}{\partial t}\psi(\mathbf{x},t) = +\sqrt{-\hbar^2 c^2 \nabla^2 + m^2 c^4} \,\psi(\mathbf{x},t).$$
 (1.6)

$$-\hbar^2 \frac{\partial^2}{\partial t^2} \psi(\mathbf{x}, t) = \left(-\hbar^2 c^2 \nabla^2 + m^2 c^4\right) \psi(\mathbf{x}, t). \tag{1.7}$$

$$g_{\mu\nu} = \begin{pmatrix} -1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix}. \tag{1.8}$$

$$g^{\mu\nu} = \begin{pmatrix} -1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix}. \tag{1.9}$$

$$\bar{x}^{\mu} = \Lambda^{\mu}{}_{\nu} x^{\nu} + a^{\mu}. \tag{1.10}$$

$$g_{\mu\nu}\Lambda^{\mu}{}_{\rho}\Lambda^{\nu}{}_{\sigma} = g_{\rho\sigma}. \tag{1.11}$$

$$(x - x')^{2} \equiv g_{\mu\nu}(x - x')^{\mu}(x - x')^{\nu}$$
$$= (\mathbf{x} - \mathbf{x}')^{2} - c^{2}(t - t')^{2}.$$
 (1.12)

$$(\bar{x} - \bar{x}')^{2} = g_{\mu\nu}(\bar{x} - \bar{x}')^{\mu}(\bar{x} - \bar{x}')^{\nu}$$

$$= g_{\mu\nu}\Lambda^{\mu}{}_{\rho}\Lambda^{\nu}{}_{\sigma}(x - x')^{\rho}(x - x')^{\sigma}$$

$$= g_{\rho\sigma}(x - x')^{\rho}(x - x')^{\sigma}$$

$$= (x - x')^{2}.$$
(1.13)

$$\partial_{\mu} \equiv \frac{\partial}{\partial x^{\mu}} = \left(+\frac{1}{c} \frac{\partial}{\partial t}, \nabla \right),$$
 (1.14)

$$\partial^{\mu} \equiv \frac{\partial}{\partial x_{\mu}} = \left(-\frac{1}{c} \frac{\partial}{\partial t}, \nabla \right). \tag{1.15}$$

$$\partial^{\mu}x^{\nu} = g^{\mu\nu}.\tag{1.16}$$

$$\bar{\partial}^{\mu} = \Lambda^{\mu}{}_{\nu}\partial^{\nu}. \tag{1.17}$$

$$\bar{\partial}^{\rho}\bar{x}^{\sigma} = (\Lambda^{\rho}{}_{\mu}\partial^{\mu})(\Lambda^{\sigma}{}_{\nu}x^{\nu} + a^{\sigma}) = \Lambda^{\rho}{}_{\mu}\Lambda^{\sigma}{}_{\nu}(\partial^{\mu}x^{\nu}) = \Lambda^{\rho}{}_{\mu}\Lambda_{\sigma\nu}g^{\mu\nu} = g^{\rho\sigma}. \tag{1.18}$$

$$-\hbar^2 c^2 \partial_0^2 \psi(x) = (-\hbar^2 c^2 \nabla^2 + m^2 c^4) \psi(x). \tag{1.19}$$

$$(-\partial^2 + m^2 c^2 / \hbar^2) \psi(x) = 0. \tag{1.20}$$

$$(-\bar{\partial}^2 + m^2 c^2/\hbar^2)\bar{\psi}(\bar{x}) = 0. \tag{1.21}$$

$$\bar{\partial}^2 = g_{\mu\nu}\bar{\partial}^{\mu}\bar{\partial}^{\nu} = g_{\mu\nu}\Lambda^{\mu}{}_{\rho}\Lambda^{\nu}{}_{\sigma}\partial^{\rho}\partial^{\sigma} = \partial^2. \tag{1.22}$$

$$\langle \psi, t | \psi, t \rangle = \int d^3x \, \langle \psi, t | \mathbf{x} \rangle \, \langle \mathbf{x} | \psi, t \rangle = \int d^3x \psi^*(x) \psi(x).$$
 (1.23)

$$i\hbar \frac{\partial}{\partial t} \psi_a(x) = \left(-i\hbar c \left(\alpha^j \right)_{ab} \partial_j + mc^2(\beta)_{ab} \right) \psi_b(x). \tag{1.24}$$

$$H_{ab} = cP_j \left(\alpha^j\right)_{ab} + mc^2(\beta)_{ab}. \tag{1.25}$$