Personal Notes of *Part I: Scalar Fields* of *Quantum Field Theory* by Srednicki

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Contents

1 Attempts at relativistic quantum mechanics

$$i\hbar\frac{\partial}{\partial t}\left|\psi, t\right\rangle = H\left|\psi, t\right\rangle. \tag{1.1}$$

$$H = \frac{1}{2m}\mathbf{P}^2. \tag{1.2}$$

$$i\hbar\frac{\partial}{\partial t}\psi(\mathbf{x},t) = -\frac{\hbar^2}{2m}\nabla^2\psi(\mathbf{x},t). \tag{1.3}$$

$$H = +\sqrt{\mathbf{P}^2c^2 + m^2c^4}. \tag{1.4}$$

$$H = mc^2 + \frac{1}{2m}\mathbf{P}^2 + \cdots . \tag{1.5}$$

$$i\hbar\frac{\partial}{\partial t}\psi(\mathbf{x},t) = +\sqrt{-\hbar^2c^2\nabla^2 + m^2c^4}\;\psi(\mathbf{x},t). \tag{1.6}$$

$$-\hbar^2\frac{\partial^2}{\partial t^2}\psi(\mathbf{x},t) = \left(-\hbar^2c^2\nabla^2 + m^2c^4\right)\,\psi(\mathbf{x},t). \tag{1.7}$$

$$g_{\mu\nu} = \begin{pmatrix} -1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix}. \tag{1.8}$$

$$g^{\mu\nu} = \begin{pmatrix} -1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix}. \tag{1.9}$$

$$\bar{x}^\mu = \Lambda^\mu{}_\nu x^\nu + a^\mu. \tag{1.10}$$

$$g_{\mu\nu}\Lambda^\mu{}_\rho\Lambda^\nu{}_\sigma = g_{\rho\sigma}. \tag{1.11}$$

$$
\begin{aligned}
(x-x')^2 &\equiv g_{\mu\nu}(x-x')^\mu(x-x')^\nu \\
&= (\mathbf{x}-\mathbf{x}')^2 - c^2(t-t')^2.
\end{aligned} \tag{1.12}
$$

$$
\begin{aligned}
(\bar{x}-\bar{x}')^2 &= g_{\mu\nu}(\bar{x}-\bar{x}')^\mu(\bar{x}-\bar{x}')^\nu \\
&= g_{\mu\nu}\Lambda^\mu{}_\rho\Lambda^\nu{}_\sigma(x-x')^\rho(x-x')^\sigma \\
&= g_{\rho\sigma}(x-x')^\rho(x-x')^\sigma \\
&= (x-x')^2.
\end{aligned} \tag{1.13}
$$

$$
\partial_\mu \equiv \frac{\partial}{\partial x^\mu} = \left(+\frac{1}{c}\frac{\partial}{\partial t}, \nabla\right), \tag{1.14}
$$

$$
\partial^\mu \equiv \frac{\partial}{\partial x_\mu} = \left(-\frac{1}{c}\frac{\partial}{\partial t}, \nabla\right). \tag{1.15}
$$

$$
\partial^\mu x^\nu = g^{\mu\nu}. \tag{1.16}
$$

$$
\bar{\partial}^\mu = \Lambda^\mu{}_\nu\partial^\nu. \tag{1.17}
$$

$$
\bar{\partial}^\rho\bar{x}^\sigma = (\Lambda^\rho{}_\mu\partial^\mu)(\Lambda^\sigma{}_\nu x^\nu + a^\sigma) = \Lambda^\rho{}_\mu\Lambda^\sigma{}_\nu(\partial^\mu x^\nu) = \Lambda^\rho{}_\mu\Lambda_{\sigma\nu}g^{\mu\nu} = g^{\rho\sigma}. \tag{1.18}
$$

$$
-\hbar^2c^2\partial_0^2\psi(x) = (-\hbar^2c^2\nabla^2 + m^2c^4)\psi(x). \tag{1.19}
$$

$$
(-\partial^2 + m^2c^2/\hbar^2)\psi(x) = 0. \tag{1.20}
$$

$$(-\bar{\partial}^2 + m^2c^2/\hbar^2)\bar{\psi}(\bar{x}) = 0. \tag{1.21}$$

$$\bar{\partial}^2 = g_{\mu\nu}\bar{\partial}^\mu\bar{\partial}^\nu = g_{\mu\nu}\Lambda^\mu{}_\rho\Lambda^\nu{}_\sigma\partial^\rho\partial^\sigma = \partial^2. \tag{1.22}$$

$$\langle\psi, t|\psi, t\rangle = \int \mathrm{d}^3x\, \langle\psi, t|\mathbf{x}\rangle\, \langle\mathbf{x}|\psi, t\rangle = \int \mathrm{d}^3x\psi^*(x)\psi(x). \tag{1.23}$$

$$i\hbar\frac{\partial}{\partial t}\psi_a(x) = \left(-i\hbar c\, \left(\alpha^j\right)_{ab}\partial_j + mc^2(\beta)_{ab}\right)\psi_b(x). \tag{1.24}$$

$$H_{ab} = cP_j\left(\alpha^j\right)_{ab} + mc^2(\beta)_{ab}. \tag{1.25}$$