Personal Notes of *Part I: Scalar Fields* of *Quantum Field Theory* by Srednicki

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1 Attempts at relativistic quantum mechanics

(1.5)

$$
\begin{aligned}
H = +\sqrt{\vec{p}^2 + m^2c^4} &= \sqrt{m^2c^4\left(1 + \frac{\vec{p}^2}{mc^2}\right)} \\
&= mc^2\left(1 + \frac{\vec{p}^2}{2mc^2} + \cdots\right) \\
&= mc^2 + \frac{1}{2m}\vec{p}^2.
\end{aligned}
\tag{1.5}
$$

(1.24) In the non-relativistic limit, the Shrödinger equation reads

$$
\begin{aligned}
& i\hbar \frac{\partial}{\partial t} \psi(\vec{x}, t) = H\psi(\vec{x}, t) \\
\overset{\dagger}{\Longleftrightarrow} \; & -i\hbar \frac{\partial}{\partial t} \psi^*(\vec{x}, t) = H\psi^*(\vec{x}, t).
\end{aligned}
$$

Therefore,

$$
\begin{aligned}
\frac{\mathrm{d}}{\mathrm{d}t} \langle \psi, t \,|\, \psi, t \rangle &= \frac{\mathrm{d}}{\mathrm{d}t} \int \mathrm{d}^3\vec{x} \, \langle \psi, t \,|\, \vec{x} \rangle \langle \vec{x} \,|\, \psi, t \rangle \\
&= \int \mathrm{d}^3\vec{x} \left[\psi(\vec{x}, t) \frac{\partial}{\partial t} \psi^*(\vec{x}, t) + \psi^*(\vec{x}, t) \frac{\partial}{\partial t} \psi(\vec{x}, t) \right] \\
&= \frac{1}{i\hbar} \int \mathrm{d}^3\vec{x} \left[-\psi(\vec{x}, t) H \psi^*(\vec{x}, t) + \psi^*(\vec{x}, t) H \psi(\vec{x}, t) \right] \\
&= \frac{1}{i\hbar} \int \mathrm{d}^3\vec{x} \left[+\psi(\vec{x}, t) \frac{\hbar^2}{2m} \nabla^2 \psi^*(\vec{x}, t) - \psi^*(\vec{x}, t) \frac{\hbar^2}{2m} \nabla^2 \psi(\vec{x}, t) \right] \\
&= \frac{i\hbar}{2m} \int \mathrm{d}^3\vec{x} \left[\psi(\vec{x}, t) \nabla^2 \psi^*(\vec{x}, t) - \psi^*(\vec{x}, t) \nabla^2 \psi(\vec{x}, t) \right] \\
&= \frac{i\hbar}{2m} \int \mathrm{d}^3\vec{x} \, \nabla \cdot \left[\psi(\vec{x}, t) \nabla \psi^*(\vec{x}, t) - \psi^*(\vec{x}, t) \nabla \psi(\vec{x}, t) \right] \\
&= \frac{i\hbar}{2m} \oint_{s \to \infty} \mathrm{d}\vec{\sigma} \cdot \left[\psi(\vec{x}, t) \nabla \psi^*(\vec{x}, t) - \psi^*(\vec{x}, t) \nabla \psi(\vec{x}, t) \right] \\
&= 0. \qquad (1.24)
\end{aligned}
$$

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