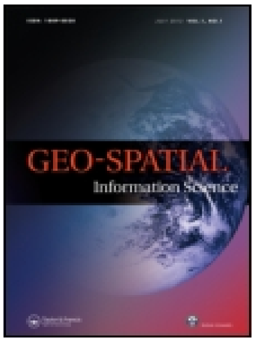

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Image Zernike Moments Shape Feature Evaluation Based on Image Reconstruction

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Abstract The evaluation approach to the accuracy of the image feature descriptors plays an important role in image feature extraction. We point out that the image shape feature can be described by the Zernike moments set while briefly introducing the basic concept of the Zernike moment. After talking about the image reconstruction technique based on the inverse transformation of Zernike moment, the evaluation approach to the accuracy of the Zernike moments shape feature via the dissimilarity degree and the reconstruction ratio between the original image and the reconstructed image is proposed. The experiment results demonstrate the feasibility of this evaluation approach to image Zernike moments shape feature.

Keywords feature evaluation; Zernike moment; image reconstruction; reconstruction ratio

CLC number TP753

Introduction

With the development and maturity of the image acquisition and storage technology, humans are deluged by image data. They need the techniques and tools to efficiently retrieve, analyze and understand the image data. Then the image feature which represents the image becomes more and more important. In fact, the image feature extraction and description is the most critical phase in the image retrieval, image analysis and image understanding.

Besides the image text features, the basic image features include color, texture, shape, edge, shadows, temporal detail and so on[1]. The shape feature is much more important than the others, so an effective and efficient shape descriptor is the key component of the description of the image shape feature.

There are two types of shape feature descriptors: contour-based and region-based. The region-based shape feature descriptors, for example the moment, are more reliable for shapes that have complex boundaries, because they rely on not only the contour pixels but also all pixels constituting the shapes[2].

The moments, especially geometric moment, centric moment, orthogonal invariance moments, have already been used in image shape description and content-based image retrieval[2-5]. The Zernike moment, one kind of the orthogonal invariance moments, is the most commonly used technique in image shape feature extraction and description. Many researchers have paid much more attention to its invariant characteristics, including translation invariance, rotation invariance and scale invariance[3,6-10].

In fact, the first problem has to be solved is whether the Zernike moments can accurately describe the image shape feature. The accuracy of the image

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feature description directly affects the further image processing. By this observation, we try to solve this problem based on image reconstruction technique.

In this paper, we firstly talk about the definition of the Zernike moment and discuss the calculation and the representation of the image Zernike moments shape feature set. Then, we point out that the image can be reconstructed from the image Zernike moments shape feature set. Finally, we put forward the approach using the dissimilarity degree and the reconstruction ratio between the original image and the reconstructed image to evaluate the accuracy of the Zernike moment descriptor.

The remainder of this paper is organized as follows. Section 1 gives a brief review of the Zernike moment and the image Zernike moments shape feature set. Section 2 gives the basic principle of the image reconstruction from the Zernike moments shape feature set and discusses the evaluation approach to the Zernike shape descriptor based on the image reconstruction. Section 3 presents the experiment and demonstrates the feasibility of the evaluation approach. Section 4 concludes the paper.

1 Basic concepts

1.1 Definition of Zernike moment

Zernike moment is a kind of orthogonal complex moments and its kernel is a set of Zernike complete orthogonal polynomials defined over the interior of the unit disc in the polar coordinates space. Let $f(r,\theta)$ be the image intensity function, and the two-dimensional Zernike moment of order m with repetition n is defined as:

$$Z_{mn} = \frac{m+1}{\pi}\int_0^{2\pi}\int_0^1 f(r,\theta)V_{mn}^*(r,\theta)r\mathrm{d}r\mathrm{d}\theta,\ r \leqslant 1 \quad (1)$$

where $V_{mn}^*(r,\theta)$ is the complex conjugate of Zernike polynomial $V_{mn}(r,\theta)$; and m and n both are integer and the relation between m and n can be described as:

$$(m-|n|) \text{ is even and } |n| \leqslant m \quad (2)$$

The Zernike polynomial $V_{mn}(r,\theta)$ is defined as:

$$V_{mn}(r,\theta) = R_{mn}(r)\exp(jn\theta) \quad (3)$$

where $j=\sqrt{-1}$; and the orthogonal radial polynomial $R_{mn}(r)$ is given by:

$$R_{mn}(r) = \sum_{s=0}^{\frac{m-|n|}{2}}(-1)^s \frac{(m-s)!}{s!(\frac{m+|n|}{2}-s)!(\frac{m-|n|}{2}-s)!} r^{m-2s} \quad (4)$$

From the formula mentioned above and the Euler's complex number formula, the equations listed as follows are available.

$$R_{m(-n)}(r) = R_{mn}(r) \quad (5)$$

$$V_{mn}^*(r,\theta) = V_{m(-n)}(r,\theta) \quad (6)$$

For the computer digital image, let $P(r,\theta)$ be the intensity of the image pixel, and Eq.(1) can be represented as:

$$Z_{mn} = \frac{m+1}{\pi}\sum_r\sum_\theta P(r,\theta)V_{mn}^*(r,\theta) \qquad r \leqslant 1 \quad (7)$$

1.2 Zernike moments shape feature set

The shape feature extracted and described by the Zernike moments is not sensitive to the noises and the values of the Zernike moments are hardly redundant because the kernel of the Zernike moment is the set of the orthogonal radial polynomials. The low-order Zernike moments can represent the whole shape of the image and the high-order Zernike moments can describe the detail. So the shape feature of the image can be represented by a set of the values of the Zernike moments.

The different order of the Zernike moments can be computed via Eq.(7) according to the variance of the order. In the same way, the different values of the Zernike moments can be calculated according to the variance of the repetition in the case of the order invariance. The results can be looked on as the Zernike moments set of the specified image shape feature.

The Zernike moments set Z of the image shape feature can be given easily after sorted the values of the Zernike moments each order firstly ascending and then sorted those of each repetition ascending according to the order.

$$Z = \{z_i \mid i = 0,1,2,...\} \quad (8)$$

2 Image reconstruction and feature evaluation

2.1 Image reconstruction

The Zernike moments set can represent and describe the image shape feature. In fact, the image can also be reconstructed from the Zernike moments shape feature set via the inverse transformation of the Zernike moments in the unit disc because it is only the mapping transformation from the image intensive space to the Zernike moment space.

The image reconstruction can be defined as:

$$\hat{P}(r,\theta)=\sum_{m=0}^{m_{\max}}\sum_{n} Z_{mn}V_{mn}(r,\theta) \tag{9}$$

where $\hat{P}(r,\theta)$ is the reconstructed image and $m_{\max}$ is the maximum of the order of the Zernike moment. It is proved that the reconstructed image is the same as the original image while $m_{\max}$ is infinite in theory.

If expand Eq.(9) and use Eqs.(5)-(6), the following formula is given.

$$\begin{aligned}
&\hat{P}(r,\theta)=\sum_{m=0}^{m_{\max}}\sum_{n<0} Z_{mn}V_{mn}(r,\theta)+\sum_{m=0}^{m_{\max}}\sum_{n\geqslant 0} Z_{mn}V_{mn}(r,\theta)= \\
&\sum_{m=0}^{m_{\max}}\sum_{n>0} Z_{m(-n)}V_{m(-n)}(r,\theta)+\sum_{m=0}^{m_{\max}}\sum_{n\geqslant 0} Z_{mn}V_{mn}(r,\theta)= \\
&\sum_{m=0}^{m_{\max}}\sum_{n>0} Z^{*}_{mn}V^{*}_{mn}(r,\theta)+\sum_{m=0}^{m_{\max}}\sum_{n\geqslant 0} Z_{mn}V_{mn}(r,\theta)= \\
&\sum_{m=0}^{m_{\max}}\left[\sum_{n>0}\left[Z^{*}_{mn}V^{*}_{mn}(r,\theta)+Z_{mn}V_{mn}(r,\theta)\right]+Z_{m0}V_{m0}(r,\theta)\right]= \\
&\sum_{m=0}^{m_{\max}}\left[\sum_{n>0}\{[\mathrm{Re}[Z_{mn}]-j\,\mathrm{Im}[Z_{mn}]]R_{mn}(r)[\cos(n\theta)-\right. \\
&j\sin(n\theta)]+[\mathrm{Re}[Z_{mn}]+j\,\mathrm{Im}[Z_{mn}]]R_{mn}(r)[\cos(n\theta)+ \\
&\left. j\sin(n\theta)]\}+[\mathrm{Re}[Z_{m0}]+j\,\mathrm{Im}[Z_{m0}]]R_{m0}(r)\right]
\end{aligned} \tag{10}$$

The final result can be represented as：

$$\hat{P}(r,\theta)=\sum_{m=0}^{m_{\max}}\left\{\sum_{n>0}[C_{mn}\cos(n\theta)+S_{mn}\sin(n\theta)]R_{mn}(r)+\frac{C_{m0}}{2}R_{m0}(r)\right\} \tag{11}$$

$$C_{mn}=2\,\mathrm{Re}(Z_{mn})=\frac{2m+2}{\pi}\sum_{r}\sum_{\theta}P(r,\theta)R_{mn}(r)\cos(n\theta) \quad (r\leqslant 1) \tag{12}$$

$$S_{mn}=-2\,\mathrm{Im}(Z_{mn})=\frac{-2m-2}{\pi}\sum_{r}\sum_{\theta}P(r,\theta)R_{mn}(r)\sin(-n\theta) \quad (r\leqslant 1) \tag{13}$$

2.2 Feature evaluation

According to the definition of the Zernike moment, the values of the Zernike moments of each order least than $m_{\max}$ can be calculated of the image once the maximum of the order $m_{\max}$ is specified. Then the Zernike moments shape feature set Z of the original image can be given. Correspondingly, if the Zernike moments shape feature set Z of the original image is known, the image can be reconstructed based on the principle of the image reconstruction mentioned above.

Of course, there is some difference between the reconstructed image and the original image. The difference can be measured via the dissimilarity degree between the reconstructed image and the original image.

$$H(P,\hat{P})=\sum_{r}\sum_{\theta}|P(r,\theta)-\hat{P}(r,\theta)| \tag{14}$$

where $P(r,\theta)$ and $\hat{P}(r,\theta)$ are the original image and the reconstructed image respectively and $H(P,\hat{P})$ is the dissimilarity degree. The meaning of the dissimilarity degree is the number of the pixels not reconstructed completely if the original image is a binary one.

In fact, after the image reconstruction mentioned above, the reconstructed image should be taken the further processes, such as image quantifying, intensive equalization and thresholding.

According to difference between the reconstructed image and the original image, the image reconstruction ratio R can be defined as:

$$R=(1-\frac{H(P,\hat{P})}{\sum_{r}\sum_{\theta}P(r,\theta)})\times 100\,\% \tag{15}$$

The image reconstruction ratio can be used to evaluate the description ability of the image Zernike moments shape feature.

3 Experiment results

In this paper, experiments in the MATLAB environment have been made on the trademark image database ITEM S8, the MPEG-7 standard testing image database which has around 3 000 trademark logo images. The experiment results of mark1279 trademark image in the trademark logo image database are listed to demonstrate the feasibility of the evaluation approach and the experiment results of the other trademark images used in the experiments can drive the same conclusion.

The image of the Fig.1 is the testing image, i.e., the original mark1279 trademark image. The Fig.2(a) to Fig.2(l) are the reconstructed intensive images based on the Zernike moments shape feature set of the original image. We use the magnitude of complex Zernike moments value as the Zernike moments value for convenient calculation in the following experiments, which does not affect the experiment results [4].

Fig.1 Testing image

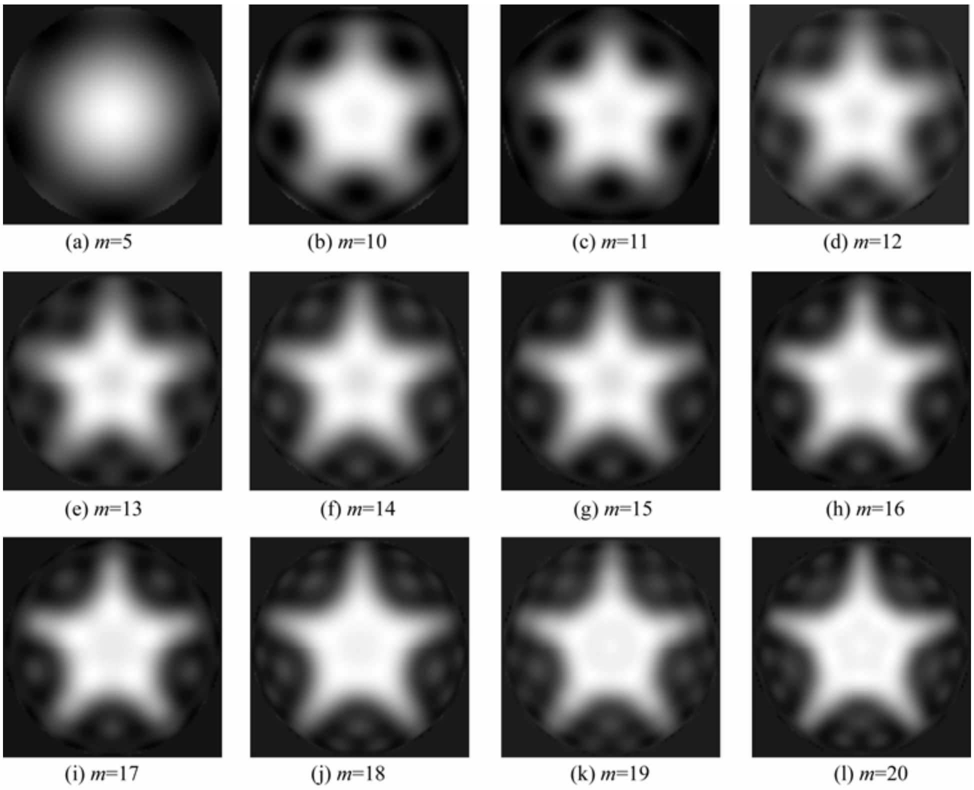

Fig.2 Reconstructed images based on Zernike moments shape feature set

From the results mentioned above, the conclusion is easily driven that the reconstructed image is very close to the original image when the order of the Zernike moment is 19. The reconstructed image will contain more detail of the original image and be closer to the original image as the order is higher.

The detail of the reconstruction ratios between the original image and the reconstructed images are showed in the Fig.3. There are 110 values in the image Zernike moments shape feature set when the order of the Zernike moment is 19. The fluctuation of the reconstruction ratios between moment order 19 and moment order 20 is mainly attributes to the further processes, especially the threshold value selection.

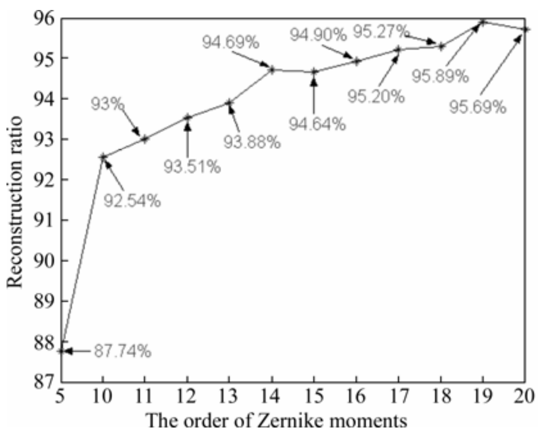

Fig.3 Reconstruction ratio of Zernike moments shape feature set (m=5, 10-20)

The comparative experiment is made between Zernike moment descriptor and Fourier descriptor since Fourier descriptor is usually used to represent and describe image shape feature. The Fourier descriptor matrix has the same size as the original image. The trademark image mark1279 and its Fourier descriptor matrix have the same size 111×111, and the size is obviously much more than 110. If only adopting the top 110 elements of the Fourier descriptor matrix, the image reconstruction based on the Fourier descriptor does not work at all. When selecting the top 20×111 elements of the Fourier descriptor matrix, the reconstructed image based on the Fourier descriptor is shown in Fig.4(a) and apparently only part of the whole original testing image and much worse than Fig.2(k). Only when selecting the top 30×111 elements of the Fourier descriptor matrix, the shape of the original testing image can be approximately reconstructed, and the reconstructed result is listed in Fig.4(b) and also much worse than Fig.2(k).

(a) (b)

Fig.4 Reconstructed image based on Fourier descriptor

The same reconstruction and comparative experiments results can be made on the other images in the trademark image database ITEM S8.

The values of the Zernike moments of the testing image are illustrated in Fig.5 when the value of the order is from 0 to 19. There are totally 110 values in the Fig.5.

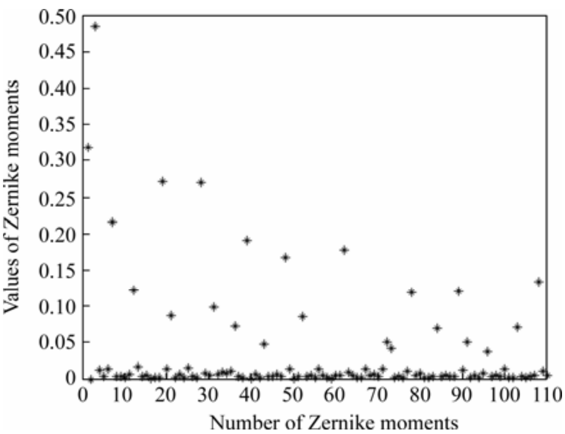

Fig.5 Values of Zernike moments of testing image (m=0-19)

4 Conclusions

The evaluation approach to the Zernike moments shape feature proposed in this paper can also be used to evaluate the other orthogonal invariance moments, such as Legendre moment, Pseudo-Zernike moment, Fourier-Mellon moment, and so on. The values of the Zernike moments can also be improved by neural network or evolutionary computation to increase the reconstruction ratio and the shape feature description accuracy of the Zernike moments.

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