Homework_1

January 3, 2020

1 Assignment 1 - STAT 7730

1.1 Problem 1

1. I showed in the first class that the following two statements are false in R.

$$(.3 - .1) == .2$$
$$(.3-.1)/.2 == (.3/.2-.1/.2)$$

For each of them, specify the reason for being false and the sources of errors.

[2]:
```
import math
print((.3 - .1) == .2)
print((.3-.1)/.2 == (.3/.2-.1/.2))
```

```
False
False
```

[2]:
```
p_three = .3
p_two = .2
p_one = .1
diffr = .3 - .1

print('0.3  0.', end = '')
while (p_three != 0):
    p_three = p_three*2
    print(math.floor(p_three), end = '')
    if(math.floor(p_three) > 0):
        p_three-=1
print('')

print('0.1  0.', end = '')
while (p_one != 0):
    p_one = p_one*2
    print(math.floor(p_one), end = '')
    if(math.floor(p_one) > 0):
        p_one-=1
print('')
```

```
print('diff 0.', end = '')
while (diffr != 0):
    diffr = diffr*2
    print(math.floor(diffr), end = '')
    if(math.floor(diffr) > 0):
        diffr-=1
print('')

print('0.2  0.', end = '')
while (p_two != 0):
    p_two = p_two*2
    print(math.floor(p_two), end = '')
    if(math.floor(p_two) > 0):
        p_two-=1
print('')
```

```
0.3  0.0100110011001100110011001100110011001100110011001100110011
0.1  0.0001100110011001100110011001100110011001100110011001100110011001101
diff 0.0011001100110011001100110011001100110011001100110011001100110011001
0.2  0.0011001100110011001100110011001100110011001100110011001100110011001101
```

Observe that there is a rounding error in the binary representation of 0.2. The rounded version of this number differs from $0.3 - 0.1$ in the last two bits.

[3]:
```
div3 = .3/.2
div1 = .1/.2
diffr1 = (.3-.1)/.2
diffr2 = .3/.2-.1/.2

print('.3/.2 1.', end = '')
div3-=1
while (div3 != 0):
    div3 = div3*2
    print(math.floor(div3), end = '')
    if(math.floor(div3) > 0):
        div3-=1
print('')

print('.1/.2 0.', end = '')
while (div1 != 0):
    div1 = div1*2
    print(math.floor(div1), end = '')
    if(math.floor(div1) > 0):
        div1-=1
print('')

print('diff2 0.', end = '')
```

```
while (diffr2 != 0):
    diffr2 = diffr2*2
    print(math.floor(diffr2), end = '')
    if(math.floor(diffr2) > 0):
        diffr2-=1
print('')

print('diff1 0.', end = '')
while (diffr1 != 0):
    diffr1 = diffr1*2
    print(math.floor(diffr1), end = '')
    if(math.floor(diffr1) > 0):
        diffr1-=1
print('')
```

```
.3/.2 1.0111111111111111111111111111111111111111111111111111111111
.1/.2 0.1
diff2 0.1111111111111111111111111111111111111111111111111111111111
diff1 0.1111111111111111111111111111111111111111111111111111111111
```

We observe again that there is a difference in their last bit due to rounding.

1.2 Problem 1

2. Let $X \in A \equiv \{x \in \mathbb{Z} : 60 \leq x \leq 100\}$ be a discrete random variable with a probability mass function

$$p_T(x) = \frac{1}{Z} e^{-\frac{1}{T}\sqrt{x}}$$

where parameter $T > 0$ represents the temperature in Kelvin. This problem studies the distribution when the temperature is close to absolute zero.

(1) Let $T = 0.1$. Compute $\max_{x \in A} p_T(x)$

(2) Let $T = 0.01$. Compute $\max_{x \in A} p_T(x)$

[9]:
```
T = 0.1

sum_den = 1
for x in range(61, 100):
    sum_den += math.exp(10*math.sqrt(60)-10*math.sqrt(x))

print('Answer: ' + str(1/sum_den))
```

```
Answer: 0.47112016199953166
```

[11]:
```
T = 0.01

sum_den = 1
```

```
for x in range(61, 100):
    sum_den += math.exp(100*math.sqrt(60)-100*math.sqrt(x))

print('Answer: ' + str(1/sum_den))
```

```
Answer: 0.9983846621774002
```