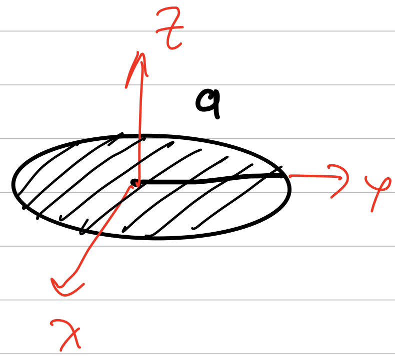
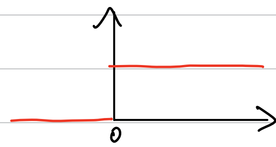
Ejercicios
métodos



$$\sigma(r) = a_0\left(\frac{r}{a}\right)$$



a) Cilíndricas $\qquad \Theta(a-r)$

$$\vec{\rho}(r) = \frac{k}{r}\,\delta(z)\{\Theta(r) - \Theta(r-a)\}\,\Theta(\theta)$$

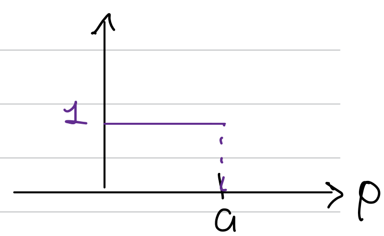
$$\int \rho(r)\,dV = dq$$

$$dq = \int_{-\infty}^{\infty}\int_0^{2\pi}\int_0^{\infty} \frac{k}{\cancel{r}}\,\delta(z)\{\Theta(r)-\Theta(r-a)\}\;\Theta(\theta)\,\cancel{r}\,dr\,d\theta\,dz$$

$$dq = k\underbrace{\int_{-\infty}^{\infty}\delta(z)\,dz}_{1}\int_0^{\infty}\Theta(r)-\Theta(r-a)\,dr\int_0^{2\pi}\Theta(\theta)\,d\theta$$

$$dq = k\left(\int_0^a[\Theta(r)-\Theta(r-a)]\,dr + \underbrace{\int_a^{\infty}[\Theta(r)-\Theta(r-a)]\,dr}_{0}\right)\int_0^{2\pi}\Theta(\theta)\,d\theta$$

$$Q = k a 2\pi$$



Por otro lado

$$dq = \lambda a\,d\theta \qquad \text{con} \qquad \lambda = \sigma_0\left(\frac{r}{a}\right)$$

Luego $\quad dq = a_0\left(\frac{r}{a}\right) a\,d\theta$

$$\Rightarrow k = a_0\left(\frac{r}{a}\right)$$

λ

$$-\int_0^a \Theta(\rho-a)\,d\rho - \int_a^\infty \cancel{\Theta(\rho}-a)\,d\rho$$

$$\downarrow$$

$$\int_0^a 1\,d\rho = a-0 = a$$

① $\int_0^1 \sqrt{x-x^2}\,dx = \int_0^1 \sqrt{x(1-x)}\,dx$

$= \int_0^1 x^{1/2}(1-x)^{1/2}\,dx$

Luego por definición de Beta

$$\boxed{B(x,y) = \int_0^1 t^{x-1}(1-t)^{y-1}\,dt}$$

$\Rightarrow \quad x-1 = \frac{1}{2} \Rightarrow x = \frac{3}{2} \qquad y-1 = 1/2 \Rightarrow y = \frac{3}{2}$

entonces

$$\int_0^1 \sqrt{x-x^2}\,dx = B\left(\frac{3}{2}, \frac{3}{2}\right)$$

Y por la propiedad

$$\boxed{B(x,y) = \frac{\Gamma(x)\Gamma(y)}{\Gamma(x+y)}}$$

$$= B\left(\frac{3}{2}, \frac{3}{2}\right) = \frac{\Gamma\left(\frac{3}{2}\right)\Gamma\left(\frac{3}{2}\right)}{\Gamma(3)} = \frac{\Gamma\left(\frac{1}{2}+1\right)\Gamma\left(\frac{1}{2}+1\right)}{\Gamma(2+1)}$$

$$= \frac{\frac{1}{2}\Gamma\left(\frac{1}{2}\right)\frac{1}{2}\Gamma\left(\frac{1}{2}\right)}{2!} \qquad \boxed{\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}}$$

$$= \frac{\pi}{4 \cdot 2!} = \frac{\pi}{8}$$

(2)

$$\Gamma(z) = \int_0^1 \left(\ln \frac{1}{x}\right)^{z-1} dx \qquad \frac{d}{du}(\ln u) = \frac{1}{u}$$

$$\frac{1}{1/x} = x$$

$$\boxed{t = \ln\left(\frac{1}{x}\right)} \quad dt = \frac{d}{dt}\left[\ln\left(\frac{1}{x}\right)\right] = -\frac{1}{x^2}\, x\, dx$$

$x=0 \to \ln\left(\frac{1}{0}\right) = \infty$ (*)

$x=1 \to \ln(1) = 0$

$$dt = -\frac{1}{x}dx$$

$\Rightarrow dx = -x\,dt$ luego $t = \ln\left(\frac{1}{x}\right) \Rightarrow e^t = \frac{1}{x}$

$\Rightarrow x = e^{-t}$

$t \to 0 \quad e^{-t} \to 1$

Entonces $t \to \infty$

$\boxed{dx = -e^{-t}\,dt}$ sustituyendo consid los lím (*)

$$-\int_\infty^0 t^{z-1} e^{-t} dt = \int_0^\infty t^{z-1} e^{-t} dt$$

Como $\Gamma(z) = \int_0^\infty t^{z-1} e^{-t} dt$

$$\therefore \Gamma(z) = \int_0^1 \left(\ln \frac{1}{x}\right)^{z-1} dx$$

$$\int_0^{\pi/2} \sqrt{\tan\theta}\, d\theta = \int_0^{\pi/2} \operatorname{sen}^{1/2}\theta \cos^{-1/2}\theta\, d\theta$$

$$= \frac{1}{2}\left(2\int_0^{\pi/2} \operatorname{sen}^{1/2}\theta \cos^{-1/2}\theta\, d\theta\right)$$

Y como

$$\boxed{B(x,y) = 2\int_0^{\pi/2} (\cos\theta)^{2x-1}(\sin\theta)^{2y-1}\, d\theta}$$

$$2x-1 = \frac{1}{2} \Rightarrow x = \frac{3}{4} \qquad 2y-1 = -\frac{1}{2} \Rightarrow y = \frac{1}{4}$$

$$\int_0^{\pi/2} \sqrt{\tan\theta}\, d\theta = \frac{1}{2} B\left(\frac{3}{4}, \frac{1}{4}\right) = \frac{1}{2}\,\frac{\Gamma\left(\frac{3}{4}\right)\Gamma\left(\frac{1}{4}\right)}{\Gamma(1)}$$

$$= \frac{1}{2}\Gamma\left(\frac{3}{4}\right)\Gamma\left(\frac{1}{4}\right)$$

Y como $\boxed{\Gamma(z)\Gamma(1-z) = \frac{\pi}{\operatorname{sen}\pi z}}$

$$z = \frac{3}{4} \Rightarrow \int_0^{\pi/2} \sqrt{\tan\theta}\, d\theta = \frac{1}{2}\,\frac{\pi}{\operatorname{sen}\left(\pi\frac{3}{4}\right)} = \frac{\pi}{2}\,\frac{1}{\sqrt{2}/2}$$

$$\int_0^{\pi/2} \sqrt{\tan\theta}\, d\theta = \frac{\pi}{\sqrt{2}}$$